Error Probability Analysis of TAS/MRC-Based Scheme for Wireless Networks

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***Abstract*— We develop the framework to analyze the symbol-error probability (SEP) for the scheme integrating transmit antenna selection (TAS) with maximal-ratio combining (MRC) used in wireless networks. Applying this scheme, the transmitter always selects an optimal antenna out of all possible antennas based on channel state information (CSI) feedbacks. Over flat-fading Rayleigh channel, we develop the closed-form SEP expressions for a set of commonly used constellations when assuming the perfect and delayed CSI feedbacks, respectively. We also derive the Chernoff-bounds of the SEP's for both perfect and delayed feedbacks. Our analyses show that while the antenna diversity improves the system performance, the feedback delay can significantly impact the SEP of TAS/MRC schemes.**

***Index Terms*—Symbol-error probability (SEP), order statistics, transmit antenna selection (TAS), wireless networks.**

I. INTRODUCTION

THE EXPLOSIVE demand for high speed wireless networking motivates an unprecedented revolution in wireless communications [1]. This presents great challenges in designing the next-generation wireless systems since the multipath fading channel has a significant impact on reliable transmissions over wireless networks. To overcome this problem, multiple-input-multiple-output (MIMO) architecture emerges as one of the most important technical breakthroughs in modern wireless communications [1] [2].

Based on whether or not the channel state information (CSI) is available at the transmitter, the MIMO systems can be classified into open-loop systems (i.e., without CSI feedback) and closed-loop systems (i.e., with CSI feedback). For the open-loop systems, space-time block coding (STBC) is a simple and powerful approach to achieve the diversity gain [3] [4]. For the closed-loop systems, the smart-antenna (SA) technique using transmit beamforming is shown to be optimal [5] [6]. The STBC-based scheme cannot achieve the performance as good as SA-based scheme, while the processing complexity of SA, e.g., *full* CSI feedback, eigenvalue or singular-value decomposition, etc, is much higher than that of STBC. Moreover, both STBC and SA require multiple RF chains, which are typically very expensive [7]. In contrast, the scheme using transmit antenna selection (TAS) provides a good tradeoff among cost, complexity, and performance.

As compared to STBC- and SA-based schemes, TAS receives relatively much less research attention. TAS was first proposed in [8] and thereafter studied in, e.g., [6] [9] [10]. Unlike SA-based scheme using the full CSI feedback, the TAS-based scheme only employs the *partial* CSI feedback, which costs much less feedback channel bandwidth. Using this partial CSI feedback in TAS scheme, a single antenna out of all possible transmit antennas, which maximizes the signal-to-noise ratio (SNR) at the receiver, is dynamically selected to transmit data. When integrating TAS with maximal-ratio combining (MRC) at the receiver, the scheme is called TAS/MRC [10]. Also, TAS scheme needs only one RF chain, and does not impose complicated signal processing at the transmitter, which can significantly reduce the transmitter's hardware and software complexity.

As a closed-loop MIMO scheme, the accuracy of CSI feedback will significantly impact the performance of TAS/MRC-based system. In [9] and [10], the authors derived closed-form bit-error rate (BER) expressions for BPSK modulation with the perfect feedbacks. In [9], the authors also *numerically* analyzed the impact of feedback delays on the BER of BPSK modulation without obtaining any closed-form BER expressions. In this paper, we analyze the symbol-error probability (SEP) of a set of commonly used signal constellations for TAS/MRC scheme with both perfect and imperfect feedbacks. For perfect feedbacks, we obtain the closed-form SEP expressions, which are more general and explicit than the previous results. Dealing with the imperfect feedbacks, we focus on feedback delays. Under a number of special cases, we obtain the closed-form SEP expressions to take the feedback delays into account. In all above situations, we develop the closed-form Chernoff-bounds of the SEP's. Our analyses show that while the antenna diversity improves the system performance, the feedback delays can significantly affect the SEP of the TAS/MRC-based scheme.

The rest of the paper is organized as follows. Section II describes the system model. Section III derives the SEP when the feedback is perfect. Section IV derives the SEP with the delayed feedbacks. Section V presents the numerical results of the SEP for TAS/MRC scheme. The paper concludes with Section VI.

II. SYSTEM DESCRIPTION

We consider a point-to-point wireless link over flat-fading Rayleigh channel in a MIMO wireless networks with N_t trans-

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mit antennas at the transmitter and N_r receive antennas at the receiver. The complex channel gain between the ith transmit antenna and the jth receive antenna is denoted by $h_{ij}[n]$, where $i \in [1, N_t]$, $j \in [1, N_r]$, and n is the discrete time-index. The channel gains are modeled as stationary and ergodic random processes, the marginal distributions of which follow independent identically distributed (i.i.d.) Gaussian with zero-mean and variance of $\Omega/2$ per dimension. The receiver is assumed to have the perfect knowledge of the CSI. Thus, the index of the optimal transmit antenna, which maximizes the total received power, can be obtained and then fed back to the transmitter by the receiver. Based on this partial CSI feedback, the transmitter selects the optimal antenna out of N_t candidates to send data. Since our discussion focuses on the level of the symbol duration, we omit the time-index n in the rest of the paper for simplicity.

If the transmitter selects an arbitrary antenna i for data transmission, the received signal vector $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \mathbf{h}_i s + \mathbf{w} \tag{1}$$

where $\mathbf{r} = (r_1 \ ... \ r_{N_r})^T$ is the received signal vector, $(\cdot)^T$ denotes the transpose of $(\cdot)$, $\mathbf{h}_i = (h_{i1} \ ... \ h_{iN_r})^T$ is the channel gain vector, s denotes the transmitted symbol, and $\mathbf{w} = (w_1 \ ... \ w_{N_r})^T$ represents the zero-mean additive white Gaussian noise (AWGN) vector, which are modeled as i.i.d. and having the single-sided power-spectral density of N_0. At the receiver side, the maximal-ratio combining (MRC) is employed. Then, the post-processing SNR at the output of the MRC combiner when using an arbitrary transmit antenna i, denoted by $\gamma_{(i)}$, can be expressed as

$$\gamma_{(i)} = \frac{E_s}{N_0} \sum_{j=1}^{N_r} \left| h_{ij} \right|^2 \tag{2}$$

where E_s denotes the average energy per symbol. We sort $\{\gamma_{(i)}\}_{i=1}^{N_t}$ from the highest SNR to the lowest SNR, as follows:

$$\gamma_{[N_t]} \geq \gamma_{[N_t-1]} \geq ... \geq \gamma_{[1]}, \tag{3}$$

such that $\{\gamma_{[i]}\}_{i=N_t}^{1}$ forms the permutation of the original $\{\gamma_{(i)}\}_{i=1}^{N_t}$ sorted in a descending order. When employing TAS, the transmitter always selects the antenna which can maximize the SNR, i.e., the highest SNR as

$$\gamma_{[N_t]} = \max_{i \in \{1, \cdots, N_t\}} \left\{ \gamma_{(i)} \right\}. \tag{4}$$

III. SEP WITH PERFECT CSI FEEDBACK

A. Post-Processing SNR

In this section, we assume that the CSI feedback is perfect, i.e., there is no feedback delay considered. Over the flat-fading Rayleigh channel, the post-processing SNR $\gamma_{(i)}$ follows the central χ^2 distribution with degree of freedom equal to $2N_r$. The probability density function (PDF) and cumulative density function (CDF) of $\gamma_{(i)}$, denoted by $g(\gamma)$ and $G(\gamma)$, respectively, can be expressed as [11]

$$g(\gamma) = \frac{\gamma^{N_r-1}}{\overline{\gamma}^{N_r}(N_r-1)!} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right), \tag{5}$$

$$G(\gamma) = 1 - \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \sum_{m=0}^{N_r-1} \left(\frac{1}{m!}\right) \left(\frac{\gamma}{\overline{\gamma}}\right)^m \tag{6}$$

where $\overline{\gamma} = E_s\Omega/N_0$ denotes the average SNR per symbol. Using the order statistics [12], the PDF of the kth ordered statistics $\gamma_{[k]}$, denoted by $f_{[k]}(\gamma)$, can be expressed as

$$f_{[k]}(\gamma) = \frac{N_t! \, g(\gamma)}{(k-1)!(N_t-k)!} \big[G(\gamma)\big]^{k-1} \big[1 - G(\gamma)\big]^{N_t-k} \tag{7}$$

where $g(\gamma)$ and $G(\gamma)$ are given by Eqs. (5) and (6), respectively. Employing TAS, the transmitter selects the antenna with the highest SNR $\gamma_{[N_t]}$. Thus, Eq. (7) generates the PDF of $\gamma_{[N_t]}$, which is specified by

$$f_{[N_t]}(\gamma) = N_t \, g(\gamma) \big[G(\gamma)\big]^{N_t-1}. \tag{8}$$

B. SEP Derivations

Based on the PDF of $\gamma_{[N_t]}$ given in Eq. (8), the average symbol error probability (SEP), denoted by P_M, for a set of commonly used signal constellations, including BPSK, M-PSK, and M-PAM, can be expressed, or approximated, as

$$P_M = \int_0^{+\infty} \alpha \, Q\left(\sqrt{\beta\gamma}\right) f_{[N_t]}(\gamma) d\gamma \tag{9}$$

where α and β are determined by the specific constellations [11]. For example, for BPSK modulation, $\alpha = 1$ and $\beta = 2$; for M-PSK, $\alpha \approx 2$ and $\beta \approx 2\sin^2(\pi/M)$; for M-PAM, $\alpha = 2(M-1)/M$ and $\beta = 6/(M^2-1)$.

For square M-QAM, the SEP $P'_M = 1 - (1 - P_{\sqrt{M}})^2$, where $P_{\sqrt{M}}$ is the SEP of $\sqrt{M}$-PAM with $\alpha = 2(1 - 1/\sqrt{M})$ and $\beta = 3/(M-1)$. Since P'_M can be easily obtained from $P_{\sqrt{M}}$, we only focus on the derivations of P_M in Eq. (9) in the rest of the paper. To further derive the SEP, we introduce the following integral function [13]:

$$\begin{aligned} \psi(L) &\triangleq \int_0^{+\infty} Q(\sqrt{ax}) x^L \exp\left(-\frac{x}{b}\right) \\ &= \frac{1}{2} b^{L+1} L! \left[1 - \sum_{k=0}^{L} \mu \left(\frac{1-\mu^2}{4}\right)^k \binom{2k}{k}\right] \end{aligned} \tag{10}$$

$$P_M = \frac{\alpha N_t!}{(N_r-1)!} \sum_{k=0}^{N_t-1} \left\{ \sum_{i_0+\ldots+i_{N_r-1}=k} \frac{(-1)^k p! \left[1 - \mu_k \sum_{j=0}^{p} \left(\frac{1-\mu_k^2}{4}\right)^j \binom{2j}{j}\right]}{2(k+1)^{p+1}(N_t-k-1)! \left(\prod_{m=0}^{N_r-1} i_m! \, (m!)^{i_m}\right)} \right\} \tag{11}$$

where $\mu = \sqrt{ab/(2+ab)}$. Substituting Eqs. (5) and (6) into Eq. (8) and using Eq. (10), P_M can be derived as Eq. (11), which is shown at the bottom of this page, where $p = \sum_{m=0}^{N_r-1} m i_m + N_r - 1$, $\mu_k = \sqrt{\beta\overline{\gamma}/\left(2k+\beta\overline{\gamma}+2\right)}$, and the terms $\{i_m\}_{m=0}^{N_r-1}$ are *all possible* combinations satisfying:

$$\begin{cases} 0 \le i_m \le k \\ i_0 + \ ... \ + i_{N_r-1} = k, \end{cases} \tag{12}$$

with each combination satisfying Eq. (12) corresponding to one term within the second summation operation in Eq. (11).

Note that both [9] and [10] addressed the derivations of P_M for BPSK modulation. In contrast with these existing results, our expression is more general and more explicit for computer calculations. Furthermore, Eq. (11) can be significantly simplified under some special cases as follows:

CASE I: $N_t = 1$.

When $N_t = 1$, in terms of Eq. (8), we have $f_{[N_t]}(\gamma) = g(\gamma)$. The SEP given by Eq. (11) reduces to:

$$P_M = \frac{\alpha}{2}\left[1 - \mu_0 \sum_{j=0}^{N_r-1}\left(\frac{1-\mu_0^2}{4}\right)^j \binom{2j}{j}\right] \tag{13}$$

which corresponds to the case where no TAS is employed.

CASE II: $N_r = 1$.

Substituting $N_r = 1$ into Eq. (11), SEP can be simplified to

$$P_M = \alpha \sum_{k=0}^{N_t-1} (-1)^k \binom{N_t}{k+1}\left(\frac{1-\mu_k}{2}\right) \tag{14}$$

which corresponds to the case where no MRC is employed.

CASE III: $N_r = 2$.

When $N_r = 2$, the SEP given by Eq. (11) can be derived as

$$\begin{aligned} P_M \ = \ & \sum_{k=0}^{N_t-1}\sum_{m=0}^{k} \frac{\alpha N_t!(-1)^k(m+1)}{2(k+1)^{m+2}(N_t-k-1)!(k-m)!} \\ & \cdot \left[1 - \mu_k \sum_{j=0}^{m+1}\left(\frac{1-\mu_k^2}{4}\right)^j \binom{2j}{j}\right]. \end{aligned} \tag{15}$$

For the cases with more than 2 receive antennas, the expression of P_M is complicated. Moreover, due to the hardware constraints at the mobile users or handsets, the number of receive antennas at the mobile terminals is typically limited. Therefore, the cases with $N_r \le 2$ are particularly attractive for the practical wireless-network implementations.

C. The Chernoff-Bound of SEP

Although Eq. (11) can be simplified under some special cases, it still suffers from the complexity. To remedy this problem, we derive the Chernoff-bound of SEP P_M for *any* numbers N_t and N_r of transmit and receive antennas, respectively, as follows:

$$\begin{aligned} P_M \ \le \ & \alpha \int_0^{+\infty} \exp\left(-\frac{\beta\gamma}{2}\right) f_{[N_t]}(\gamma) d\gamma \\ = \ & \frac{\alpha N_t!}{(N_r-1)!} \sum_{k=0}^{N_t-1} \left\{ \sum_{i_0+\ldots+i_{N_r-1}=k} \frac{(-1)^k p\,!}{(N_t-k-1)!} \right. \\ & \left. \cdot \frac{\left(\frac{2}{2(k+1)+\beta\overline{\gamma}}\right)^{p+1}}{\left(\prod_{m=0}^{N_r-1} i_m!\,(m!)^{i_m}\right)} \right\}. \end{aligned} \tag{16}$$

Using Eq. (16), we obtain the simplified expressions under some special cases as follows:

CASE I: $N_t = 1$.

$$P_M \le \alpha \left(\frac{2}{2+\beta\overline{\gamma}}\right)^{N_r}. \tag{17}$$

CASE II: $N_r = 1$.

$$P_M \le \sum_{k=0}^{N_t-1} \frac{2(-1)^k \alpha N_t!}{(N_t-k-1)!k!\left[2(k+1)+\beta\overline{\gamma}\right]}. \tag{18}$$

CASE III: $N_r = 2$.

$$P_M \le \sum_{k=0}^{N_t-1}\sum_{m=0}^{k} \frac{(-1)^k \alpha N_t!(m+1)\left(\frac{2}{2(k+1)+\beta\overline{\gamma}}\right)^{m+2}}{(N_t-k-1)!(k-m)!}. \tag{19}$$

It is worth noting that the Chernoff-bound of P'_M for M-QAM is valid only when the Chernoff-bound of $P_{\sqrt{M}}$ for $\sqrt{M}$-PAM is less than or equal to 1, because $P'_M = 1 - \left(1 - P_{\sqrt{M}}\right)^2$.

IV. SEP WITH FEEDBACK DELAYS

A. Induced Order Statistics Analysis

In this section, we assume that the transmitter receives the feedback with a time-delay, denoted by τ. Due to the time-varying nature of the wireless channel, the current optimal SNR $\gamma_{[N_t]}$ may have changed already at the moment when the transmitter receives the feedback after the delay τ, which can significantly degrade the performance of TAS/MRC scheme.

Let $\widetilde{\gamma}_{[N_t]}$ denote the time-delayed SNR of the original $\gamma_{[N_t]}$, and $\widetilde{\gamma}_{(i)}$ denote the time-delayed SNR of the original $\gamma_{(i)}$. According to the order statistics [12], $\widetilde{\gamma}_{[N_t]}$ is called the *induced order statistics* (or the *concomitant*) of the original ordered $\gamma_{[N_t]}$. The PDF of $\widetilde{\gamma}_{[N_t]}$, denoted by $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$, is determined by

$$\begin{aligned} \widetilde{f}_{[N_t]}(\widetilde{\gamma}) \ = \ & \int_0^{+\infty} f\left(\widetilde{\gamma}|\gamma\right) f_{[N_t]}(\gamma) d\gamma \\ = \ & \int_0^{+\infty} \left[\frac{f\left(\gamma,\widetilde{\gamma}\right)}{g(\gamma)}\right] f_{[N_t]}(\gamma) d\gamma \end{aligned} \tag{20}$$

where $f\left(\widetilde{\gamma}|\gamma\right)$ denotes the PDF of $\widetilde{\gamma}_{(i)}$ conditioned on $\gamma_{(i)}$, and $f\left(\gamma,\widetilde{\gamma}\right)$ is the joint PDF of $\gamma_{(i)}$ and $\widetilde{\gamma}_{(i)}$. Then, the SEP considering feedback delay, denoted by $P_M^{(d)}$, can be expressed as

$$P_M^{(d)} = \int_0^{+\infty} \alpha Q\left(\sqrt{\beta \widetilde{\gamma}}\right) \widetilde{f}_{[N_t]}(\widetilde{\gamma}) d\widetilde{\gamma}. \tag{21}$$

where $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ is given by Eq. (20), which can be derived by using the following proposition:

Proposition 1: For any two random variables γ_1 and γ_2 following the central χ^2 distribution with PDF's specified by Eq. (5), the joint PDF $f(\gamma_1, \gamma_2)$ can be expressed as follows:

$$f(\gamma_1, \gamma_2) = \frac{g(\gamma_1)\, g(\gamma_2)\, (N_r - 1)!}{(1-\rho)} \; I_{N_r - 1}\left(\frac{2\sqrt{\rho \gamma_1 \gamma_2}}{(1-\rho)\sqrt{\overline{\gamma}_1 \overline{\gamma}_2}}\right) \cdot \left(\frac{\rho \gamma_1 \gamma_2}{\overline{\gamma}_1 \overline{\gamma}_2}\right)^{-\frac{N_r - 1}{2}} \exp\left(-\frac{\rho}{1-\rho}\left(\frac{\gamma_1}{\overline{\gamma}_1} + \frac{\gamma_2}{\overline{\gamma}_2}\right)\right) \tag{22}$$

where $I_\nu(\cdot)$ denotes the modified Bessel function of the first kind with order ν, and ρ is the correlation coefficient of γ_1 and γ_2.

Proof: Using the Kibble's bivariate gamma distribution [14], Eq. (22) can be obtained. ■

Using Proposition 1, the conditional PDF $f(\gamma_2|\gamma_1)$ can be derived as

$$f(\gamma_2|\gamma_1) = \frac{\left(\frac{\overline{\gamma}_1 \gamma_2}{\rho \gamma_1 \overline{\gamma}_2}\right)^{\frac{N_r - 1}{2}}}{(1-\rho)\overline{\gamma}_2} \; I_{N_r - 1}\left(\frac{2\sqrt{\rho \gamma_1 \gamma_2}}{(1-\rho)\sqrt{\overline{\gamma}_1 \overline{\gamma}_2}}\right) \cdot \exp\left(-\frac{1}{1-\rho}\left(\frac{\rho \gamma_1}{\overline{\gamma}_1} + \frac{\gamma_2}{\overline{\gamma}_2}\right)\right). \tag{23}$$

Substituting $\gamma_1 = \gamma$, $\gamma_2 = \widetilde{\gamma}$, and $\overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$ into Eq. (23), we obtain the expression of $f(\widetilde{\gamma}|\gamma)$ in Eq. (20), which can be expressed as

$$f(\widetilde{\gamma}|\gamma) = \frac{\left(\frac{\widetilde{\gamma}}{\rho \gamma}\right)^{\frac{N_r - 1}{2}} \exp\left(-\frac{\rho\gamma + \widetilde{\gamma}}{(1-\rho)\overline{\gamma}}\right)}{(1-\rho)\overline{\gamma}} \; I_{N_r - 1}\left(\frac{2\sqrt{\rho \gamma \widetilde{\gamma}}}{(1-\rho)\overline{\gamma}}\right) \tag{24}$$

where the correlation coefficient ρ is determined by [15]

$$\rho = J_0^2(2\pi f_D \tau) \tag{25}$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind and f_D is the Doppler frequency.

Note that Eq. (24) follows the non-central χ^2 distribution when $\rho \neq 0$. On the other hand, when $\rho = 0$, by expanding the Bessel function, $f(\widetilde{\gamma}|\gamma)$ can be expressed as

$$f(\widetilde{\gamma}|\gamma) = g(\widetilde{\gamma}). \tag{26}$$

Then, for $\rho = 0$, the PDF $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ of the concomitant $\widetilde{\gamma}_{[N_t]}$ in Eq. (20) can also be simplified as

$$\widetilde{f}_{[N_t]}(\widetilde{\gamma}) = \int_0^{+\infty} g(\widetilde{\gamma}) f_{[N_t]}(\gamma) d\gamma = g(\widetilde{\gamma}) \tag{27}$$

which is expected since $\rho = 0$ means that the delayed SNR $\widetilde{\gamma}_{[N_t]}$ is independent of the original $\gamma_{[N_t]}$, making TAS have no effect (effectively $N_t = 1$). Thus, when $\rho = 0$ the SEP and the Chernoff-bounds of SEP are determined by Eqs. (13) and (17), respectively.

B. SEP Derivations

When $\rho \neq 0$, substituting Eq. (24) into Eq. (20) and solving the integral, we can derive the PDF $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ as Eq. (28), which is shown at the bottom of this page, where ${}_1F_1(\cdot)$ denotes the confluent hypergeometric (Kummer) function [16].

Note that the authors in [9] also focused on deriving the expression of $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ (see [9], Eq. (13)). However, we obtain the similar expression by using the different approaches. The authors in [9] started from the conditional Gaussian distribution, while we derive Eq. (28) from Proposition 1. As compared to [9], our approach is more general. Also, our expression, which explicitly contains ρ, can offer more insights. Furthermore, the authors in [9] did not obtain any closed-form expression for Eq. (21) (with $\alpha = 1$ and $\beta = 2$) and thus they only solved it numerically. In contrast, we obtain the closed-form expressions of Eq. (21) for some special cases as follows.

CASE I: $N_t = 1$.

Substituting $N_t = 1$ into Eq. (28) and using the properties of Kummer function [16], $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ can be simplified as

$$\widetilde{f}_{[N_t]}(\widetilde{\gamma}) = \frac{\widetilde{\gamma}^{N_r - 1}}{\overline{\gamma}^{N_r}(N_r - 1)!} \exp\left(-\frac{\widetilde{\gamma}}{\overline{\gamma}}\right). \tag{29}$$

Note that Eq. (29) is independent of ρ and has the same expression as $g(\gamma)$ in Eq. (5), which is expected since no TAS is employed ($N_t = 1$), the feedback delay will not affect the performance of the system. Thus, the SEP $P_M^{(d)}$ also has the same expression as Eq. (13).

CASE II: $N_r = 1$.

The PDF $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ when $N_r = 1$ can be simplified as

$$\widetilde{f}_{[N_t]}(\widetilde{\gamma}) = \sum_{k=0}^{N_t - 1} \frac{(-1)^k N_t! \, \exp\left(-\frac{\widetilde{\gamma}}{(1-\rho)\overline{\gamma}}\right)}{[k(1-\rho)+1]\,\overline{\gamma}\,(N_t - k - 1)! k!} \cdot {}_1F_1\left(1; 1; \frac{\rho \widetilde{\gamma}}{[k(1-\rho)+1][(1-\rho)\overline{\gamma}]}\right). \tag{30}$$

Substituting Eq. (30) into Eq. (21) and using the properties of Kummer function, $P_M^{(d)}$ can be derived as

$$P_M^{(d)} = \alpha \sum_{k=0}^{N_t - 1} (-1)^k \binom{N_t}{k+1} \left(\frac{1 - \widetilde{\mu}_k}{2}\right) \tag{31}$$

$$\widetilde{f}_{[N_t]}(\widetilde{\gamma}) = \frac{N_t! \, \widetilde{\gamma}^{N_r - 1} \exp\left(-\frac{\widetilde{\gamma}}{(1-\rho)\overline{\gamma}}\right)}{[(1-\rho)\overline{\gamma}]^{N_r} [(N_r - 1)!]^2} \sum_{k=0}^{N_t - 1} \left\{ \sum_{i_0 + \ldots + i_{N_r - 1} = k} \frac{(-1)^k \left(\frac{1-\rho}{k(1-\rho)+1}\right)^{p+1} p!\, {}_1F_1\left(p+1; N_r; \frac{\rho \widetilde{\gamma}}{[k(1-\rho)+1][(1-\rho)\overline{\gamma}]}\right)}{(N_t - k - 1)! \left(\prod_{m=0}^{N_r - 1} i_m! \, (m!)^{i_m}\right)} \right\} \tag{28}$$

where

$$\widetilde{\mu}_k = \sqrt{\frac{[k(1-\rho)+1]\beta\overline{\gamma}}{2(k+1)+[k(1-\rho)+1]\beta\overline{\gamma}}}. \tag{32}$$

Note that Eq. (31) has the similar structure to Eq. (14) except that $\widetilde{\mu}_k$ is different from μ_k. However, when $\rho = 1$, we have $\widetilde{\mu}_k = \mu_k$, making the two SEP expressions exactly same with each other, which is expected since $\rho = 1$ corresponds to the time delay $\tau = 0$, the perfect feedback case. This also verifies the correctness of Eq. (31).

CASE III: $N_t = N_r = 2$.

The system structure with $N_t = N_r = 2$ is attractive since it can be used for the Ad Hoc wireless networks. Similar to the above two cases, we can derive the SEP $P_M^{(d)}$ by expanding $\widetilde{f}_{[N_t]}(\widetilde{\gamma})$ and solving the integral, as follows:

$$\begin{aligned} P_M^{(d)} \;=\; & \frac{\alpha}{2}(1-\widetilde{\mu}_0)^2(2+\widetilde{\mu}_0) - \frac{\alpha(4-3\rho)}{8(2-\rho)}(1-\widetilde{\mu}_1)^2(2+\widetilde{\mu}_1) \\ & - \frac{\alpha\rho}{32(2-\rho)}(1-\widetilde{\mu}_1)^3(3\widetilde{\mu}_1^2+9\widetilde{\mu}_1+8) \end{aligned} \tag{33}$$

where $\widetilde{\mu}_0$ and $\widetilde{\mu}_1$ are the special forms of $\widetilde{\mu}_k$ in Eq. (32).

C. The Chernoff-Bound of SEP

In Section IV-B, we derive the closed-form expressions of SEP $P_M^{(d)}$ under a number of special cases. For the more complex cases, deriving the closed-form expressions are also possible. However, it is tedious and cannot be expressed by general expressions. Alternatively, in this section we derive the Chernoff-bound of $P_M^{(d)}$ for *any* numbers N_t and N_r of transmit and receive antennas, respectively. The Chernoff-bound of $P_M^{(d)}$ is determined by

$$P_M^{(d)} \le \alpha \int_0^{+\infty} \exp\left(-\frac{\beta\widetilde{\gamma}}{2}\right) \widetilde{f}_{[N_t]}(\widetilde{\gamma}) d\widetilde{\gamma} \tag{34}$$

Substituting Eq. (28) into Eq. (34) and solving the integral, the Chernoff-bound of SEP $P_M^{(d)}$ can be derived as Eq. (35), which is shown at the bottom of this page. Note that when $\rho = 1$, Eq. (35) becomes Eq. (16), verifying the correctness of our derivations. Also, we provide the results under some special cases as follows:

CASE I: $N_t = 1$.

$$P_M^{(d)} \le \alpha \left(\frac{2}{2+\beta\overline{\gamma}}\right)^{N_r}. \tag{36}$$

CASE II: $N_r = 1$.

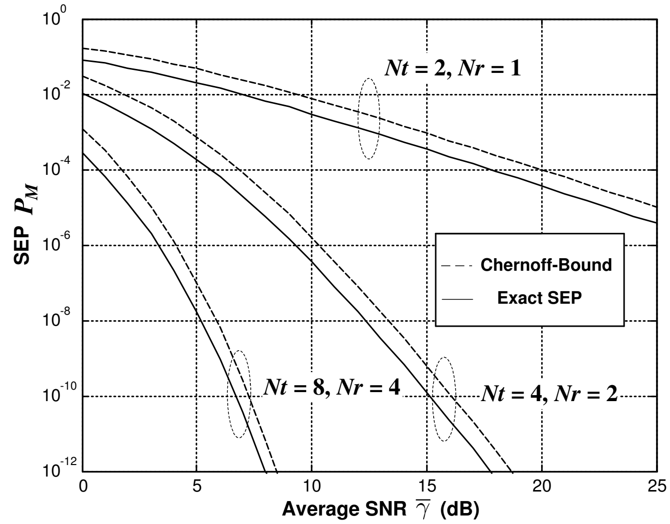

Fig. 1. The SEP P_M's and the corresponding Chernoff-bounds with perfect CSI feedback using different numbers of transmit and receive antennas.

$$P_M^{(d)} \le \sum_{k=0}^{N_t-1} \frac{\alpha N_t!(-1)^k \left(\frac{2}{\beta\overline{\gamma}[k(1-\rho)+1]+2(k+1)}\right)}{(N_t-k-1)!k!} \tag{37}$$

CASE III: $N_r = 2$.

$$\begin{aligned} P_M^{(d)} \;\le\; & \sum_{k=0}^{N_t-1}\sum_{m=0}^{k} \frac{(-1)^k 4\alpha N_t!(m+1)}{(N_t-k-1)!(k-m)!\,[(1-\rho)\beta\overline{\gamma}+2]^2} \\ & \cdot \left(\frac{(1-\rho)\beta\overline{\gamma}+2}{\beta\overline{\gamma}[k(1-\rho)+1]+2(k+1)}\right)^{m+2}. \end{aligned} \tag{38}$$

V. Performance Evaluations

Without loss of generality, we evaluate the SEP performance of TAS/MRC scheme using BPSK modulation ($\alpha = 1, \beta = 2$). In all figures, we present the exact SEP's and the corresponding Chernoff-bounds, which are plotted by the solid and dotted lines, respectively. All results used in the figures are calculated by using the closed-form expressions for both exact SEP's and Chernoff-bounds derived in the previous sections.

Using the general and the special forms of Eqs. (11) and Eq. (16), Fig. 1 shows the SEP P_M's and the corresponding Chernoff-bounds with perfect CSI feedbacks when using different numbers N_t and N_r of transmit and receive antennas. As shown in Fig. 1, both the SEP and the Chernoff-bound decreases as the product of N_t and N_r increases, which verifies the fact that TAS/MRC can achieve the diversity order of N_tN_r [7] [10], as if all the transmit antennas were used.

$$P_M^{(d)} \le \frac{\alpha N_t!}{(N_r-1)!}\left(\frac{2}{(1-\rho)\beta\overline{\gamma}+2}\right)^{N_r} \sum_{k=0}^{N_t-1} \left\{ \sum_{i_0+\ldots+i_{N_r-1}=k} \frac{(-1)^k p!\left(\frac{(1-\rho)\beta\overline{\gamma}+2}{\beta\overline{\gamma}[k(1-\rho)+1]+2(k+1)}\right)^{p+1}}{(N_t-k-1)!\left(\prod_{m=0}^{N_r-1} i_m!\,(m!)^{i_m}\right)} \right\} \tag{35}$$

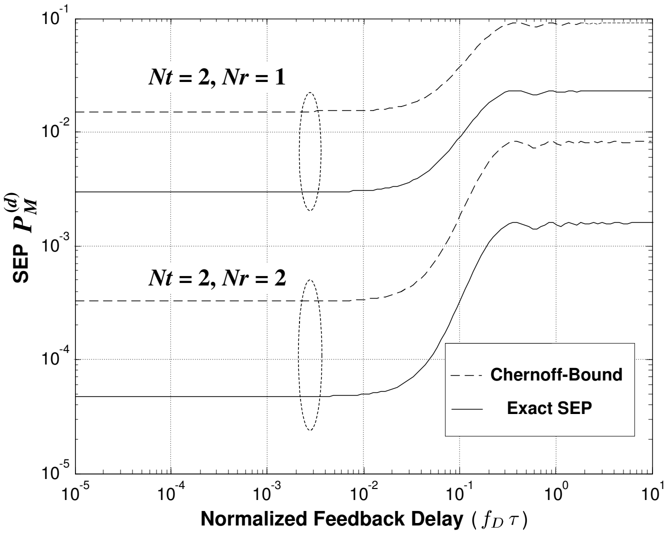

Fig. 2. The SEP $P_M^{(d)}$ and the corresponding Chernoff-bounds when considering feedback delays using different numbers of transmit and receive antennas ($\overline{\gamma} = 10$ dB).

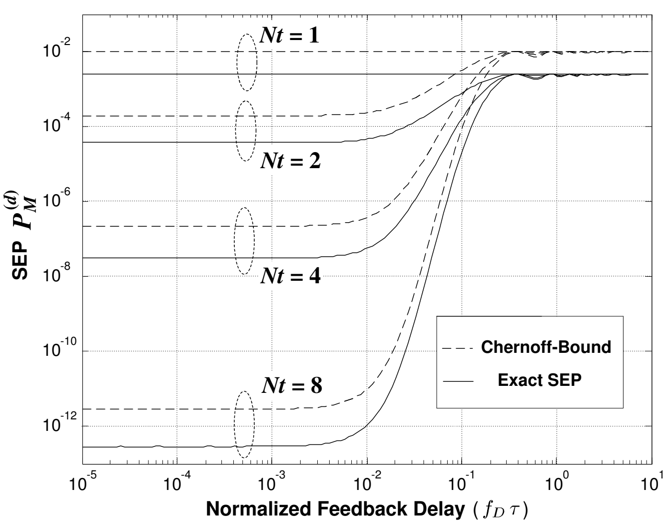

Fig. 3. The SEP $P_M^{(d)}$ and the corresponding Chernoff-bounds when considering feedback delays using different numbers of transmit antennas ($N_r = 1, \overline{\gamma} = 20$ dB).

Also we can see from Fig. 1 that the Chernoff-bounds are tight, especially when the average SNR $\overline{\gamma}$ or the diversity order $N_t N_r$ is high. For example, when $N_t = 8$ and $N_r = 4$, the difference between the exact SEP and its Chernoff-bound is only about 1 dB at $P_M = 10^{-6}$.

Finally, Figs. 2 and 3 plot the SEP $P_M^{(d)}$'s and their Chernoff-bounds when considering feedback delays, where the average SNR is set to $\overline{\gamma} = 10$ dB in Fig. 2 and $\overline{\gamma} = 20$ dB in Fig. 3, respectively. As shown by both of the figures, the SEP $P_M^{(d)}$'s increase with the increase of the normalized delay $f_D\tau$. The system can tolerate about $f_D\tau \leq 10^{-2}$ when $\overline{\gamma} = 10$ dB (see Fig. 2) and about $f_D\tau \leq 10^{-2.5}$ when $\overline{\gamma} = 20$ dB (see Fig. 3) to keep the SEP virtually invariant. When the normalized delay $f_D\tau$ further increases, the SEP first grows up, then fluctuates, and finally approaches to a constant. The fluctuation is due to the tail of the Bessel function [see Eq. (25)]. In Fig. 3, all numbers of receive antennas are set to $N_r = 1$. With the increase of delay, all SEP's approach to the constant equal to the SEP with $N_t = 1$, which is also expected since when $f_D\tau$ is too large, the correlation coefficient $\rho \to 0$, making the feedback loop of the TAS-based scheme virtually broken. This observation also verifies the correctness of our analytical results for the delayed feedbacks.

VI. Conclusions

We developed the framework to analyze the SEP for TAS/MRC scheme with perfect and delayed CSI feedbacks used in wireless networks. Specifically, we derived the general closed-form SEP expressions when considering perfect feedbacks. For the system with delayed feedbacks, we obtained a set of closed-form SEP expressions under a number of special cases. In all above situations, the Chernoff-bounds of SEP's are derived. Our analyses showed that while the antenna diversity improves the system performance, the feedback delay can significantly impact the SEP of TAS/MRC scheme.

References

[1] T. Rappaport, A. Annamalai, R. Buehrer, and W. Tranter, "Wireless Communications: Past Events and a Future Perspective," *IEEE Comm. Magazine*. May, 2002, pp. 148-161.
[2] D. Gesbert, M. Shafi, D. Shiu, P. Smith, and A. Naguib, "From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems," *IEEE J. Select Areas Comm.*, vol. 21, no. 3, April 2003, pp. 281-302.
[3] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Select Areas Comm.*, vol. 16, no. 8, Oct. 1998, pp. 1451-1458.
[4] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans. Inform. Theory*, Vol. 45, No. 5, Jul. 1999, pp. 1456-1467.
[5] G. Giannakis, Y. Hua, P. Stoica, and L. Tong, *Signal processing advances in communications*, Prentise Hall, New Jersey, 2001.
[6] Z. Chen, B. Vucetic, and J. Yuan, "Comparison of three closed-loop transmit diversity schemes," *IEEE VTC* 2003, Vol. 1, pp. 22-25.
[7] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Processing*, vol. 50, no. 10, Oct. 2002, pp. 2580-2588.
[8] T. Skinner and J. Cavers, "Selective Diversity for Rayleigh Fading Channels with a Feedback Link," *IEEE Trans. Commun.*, vol. 21, no. 2, Feb. 1973, pp. 117-126.
[9] S. Thoen, L. V. Perre, B. Gyselinckx, and M. Engels, "Performance analysis of combined transmit-SC/receive-MRC," *IEEE Trans. Commun..*, vol. 49, no. 1, Jan. 2001, pp. 5-8.
[10] Z. Chen; B. Vucetic, J. Yuan, and K. L. Lo, "Analysis of transmit antenna selection/maximal-ratio combining in Rayleigh fading channels," *IEEE ICCT*, Vol. 2, 2003, pp. 1532 - 1536.
[11] J. Proakis, *Digital Communications*, 3rd Edition. New York: McGraw-Hill, 1995.
[12] H. A. David, *Order Statistics*, 2nd Edition, New York: Wiley, 1981
[13] H. Zhang and T. A. Gulliver, "Capacity and error probability analysis of space-time block codes for DS-CDMA over fading channels," *IEEE Milcom*. 2003, pp. 1095 - 1100.
[14] W. F. Kibble, "A two-variate gamma type distribution," *Sankhyā*, 5, 1941, pp. 137-150.
[15] W. C. Jake, Jr., *Microwave Communications*. New York: Wiley, 1974.
[16] M. Abramowitz, *Handbook of mathematical functions*, New York: Dover, 1965.