Subchannel-Allocation Algorithms and Performance Analysis for Space-Time OFDM-CDMA Based Systems in Wireless Networks

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***Abstract*—Both Space-Time (ST) processing and Orthogonal Frequency Division Multiplexing (OFDM) are becoming increasingly important techniques used in physical layer to support the Quality of Service (QoS) for different applications in wireless networks. However, the employment of such techniques affects the scheme designs at higher layers such as channel allocation algorithms at Media Access Control (MAC) layer. Therefore, a crossing-layer development is required. In this paper, we propose the subchannel allocation algorithms based on space-time OFDM-CDMA systems, and develop the throughput-performance analysis frameworks to evaluate the proposed algorithms. Our analyses show that the proposed subchannel allocation scheme can significantly increase the system throughput over wireless networks, while achieving a much lower implementation and computation complexity as compared to the smart-antenna-based systems. Also presented are the simulation results that verify the analytical findings and observations.**

Keywords—Subchannel Allocation, Space-Time Processing, OFDM-CDMA, Wireless Networks.

I. INTRODUCTION

The demand for wireless network services such as wireless Internet access, mobile computing, and broadband wireless communications continues to grow rapidly. The provision and control of QoS (Quality of Service) plays a critical role in wireless networking as the QoS requirements vary for different users of wireless services. This presents a great challenge in designing the wireless networks in which the multipath fading over wireless channels has a significant impact on QoS. The QoS-requirement variations of different applications over fading channels require the developments of higher-layer protocols to take the physical-layer designs into account when optimizing the wireless network performance [1]. In physical layer, the QoS for reliable data communications over the lossy wireless channels due to multipath fading can be quantitatively characterized by Signal-to-Interference-and-Noise Ratio (SINR) or Bit Error Rate (BER).

The Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising network-accessing techniques in dealing with the channel fading and reducing BER for wireless networks [1][2][5][6]. Consequently, OFDM has been included in IEEE 802.11a, 802.11h, ARIB HiSWAN and ETSI HIPERLAN2 standards for Wireless Local Area Networks (WLAN). OFDM uses a number of lower-rate orthogonal subcarriers to transmit a single high-speed data stream, efficiently alleviating Inter-Symbol Interference (ISI) and thus significantly improving the transmission throughputs [2][3][7]. The combination of the Code Division Multiple Access (CDMA) with OFDM, called OFDM-CDMA, provides us with not only the frequency diversity, but also the multiple access control, such that multiple users can be assigned into a single subchannel with different signature sequences. As a result, OFDM-CDMA receives a great deal of research efforts for the next generation wireless networks as it can significantly increase the system throughput.

To further improve the spectrum efficiency of current wireless systems, the recently proposed Space-Time (ST) processing and Multiple-Input-Multiple-Output (MIMO) architecture are attracting much research attention [1][3][8]. Among many space-time processing schemes developed, Space-Time Block Coding (STBC), especially Alamouti's 2-transmitter scheme [4], becomes an attractive way to improve the current wireless systems because of its high efficiency and low complexity. Clearly, further combining space-time processing with OFDM-CDMA can achieve the integrated diversities from spatial, temporal, frequency, and code domains, which will result in significant improvements in the system throughputs and QoS for different users of wireless networks.

While there have already been a large body of literature on both OFDM-CDMA and space-time processing at physical layer, the impact of such architectures on higher-layer developments, such as resource allocations at MAC layer, its throughput analysis, and crossing-layer optimizations, has received relatively much less attention. Koutsopoulos and Tassiulas addressed this issue by proposing a novel OFDM smart antenna system which can improve the throughput of wireless networks [5]. Unfortunately, a major concern with their scheme is the high implementation and computation complexities which are inherently associated with the smart antenna-based systems. To overcome these problems, we propose the subchannel allocation algorithms based on an alternative multi-antenna OFDM system, which is the space-time OFDM-CDMA system. Furthermore, we develop the analytical models to analyze the throughput-performance improvement of our proposed scheme, and also conduct simulation experiments which validate our analytical analyses. Both analytical and simulation results show that our proposed scheme can significantly increase system throughputs while achieving a much lower implementation and computation complexity as compared to the smart-antenna-based systems.

The rest of this paper is organized as follows. Section II formalizes the system model for the above-described space-time OFDM-CDMA scheme. Section III presents the proposed subchannel-allocation algorithms. Section IV derives the analytical analyses evaluating the average throughput performance, and also investigates the complexity, of the proposed scheme. Section V describes the analytical analyses and simulation results confirming our analytical findings. The paper concludes with Section VI.

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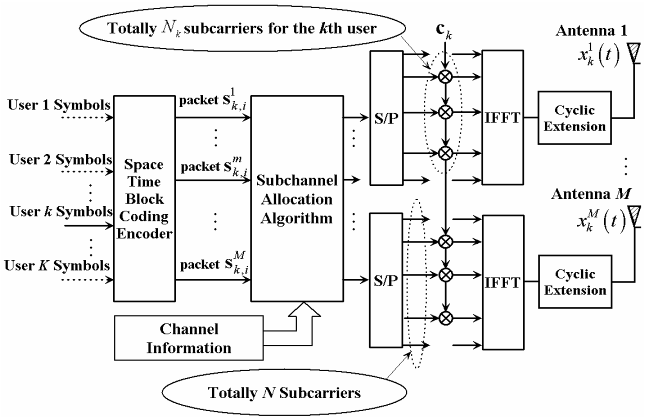

Fig. 1 System model of the proposed space-time OFDM-CDMA system.

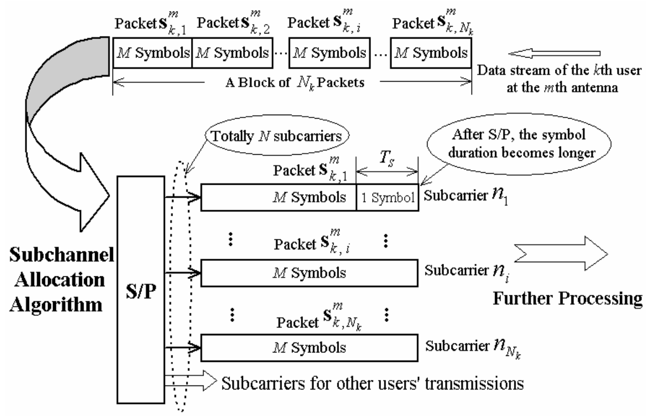

Fig. 2 Data stream of the *k*th user transmitted through the *m*th antenna.

II. SYSTEM MODEL

A. Transmitting Scheme

The structure of our proposed space-time OFDM-CDMA system is shown in Fig. 1. We consider the downlink of the system with *M* antennas at the basestation. Let *K* denote the number of active users and *N* the total number of subcarriers or subchannels – we use the terms "subcarrier" and "subchannel" interchangeably in the following discussions.

A-I. Space-Time Block Coding

We focus on the normalized symbol stream belonging to the *k*th user, where $k \in \Omega \triangleq \{1,2,...,K\}$ and Ω is the index set of all active users. First, the *k*th user's symbol stream passes through the space-time coding encoder as shown in Fig. 1. Each time, there are *M* symbols being coded and assigned to each antenna. We define the coded *M* symbols as a *packet*. Denote this *M*-symbol packet transmitted through the *m*th antenna by:

$$\mathbf{s}_{k,i}^m = \begin{bmatrix} s_{k,i}^{1,m} & s_{k,i}^{2,m} & \cdots & s_{k,i}^{M,m} \end{bmatrix}^T \tag{1}$$

where $[\bullet]^T$ is the transpose of matrix $[\bullet]$, $m \in \{1,2,...,M\}$, i denotes the packet index in a transmission block with $i \in \{1,2,...,N_k\}$, and N_k is the total number of subcarriers assigned to the *k*th user. See Fig. 2 and Section A-II for more details. Then, the entire transmission matrix of the *k*th user over all antennas is given by:

$$\mathbf{S}_{k,i} = \begin{bmatrix} \mathbf{s}_{k,i}^1 & \mathbf{s}_{k,i}^2 & \ldots & \mathbf{s}_{k,i}^M \end{bmatrix} = \begin{bmatrix} s_{k,i}^{1,1} & s_{k,i}^{1,2} & \cdots & s_{k,i}^{1,M} \\ s_{k,i}^{2,1} & s_{k,i}^{2,2} & \cdots & s_{k,i}^{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ s_{k,i}^{M,1} & s_{k,i}^{M,2} & \cdots & s_{k,i}^{M,M} \end{bmatrix} \tag{2}$$

which corresponds to the space-time coding matrix. For example, when $M = 2$, the coding matrix given by Eq. (2) becomes:

$$\mathbf{S}_{k,i} = \begin{bmatrix} \mathbf{s}_{k,i}^1 & \mathbf{s}_{k,i}^2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \triangleq \begin{bmatrix} s_{k,i}^{1,1} & s_{k,i}^{1,2} \\ s_{k,i}^{2,1} & s_{k,i}^{2,2} \end{bmatrix} \tag{3}$$

A-II. OFDM-CDMA

After the space-time coding, the data stream of the *k*th user transmitted through the *m*th antenna is shown in Fig. 2. In terms of the subchannel allocation algorithm, the *k*th user is assigned N_k ($N_k \le N$) subcarriers for transmission. All symbols in the same packet will be transmitted by the same subcarrier, but different packets will be transmitted by the different subcarriers. A block of N_k packets is converted into N_k parallel data streams and transmitted simultaneously on the subcarriers which are assigned to the *k*th user. Then, the spectral spreading is applied on these parallel data streams by multiplying them with the spreading code $\mathbf{c}_k$ (see Fig. 1), which is given by:

$$\mathbf{c}_k = \begin{bmatrix} c_k^1 & c_k^2 & \cdots & c_k^G \end{bmatrix}^T \tag{4}$$

where *G* denotes the spreading gain of the spreading codes.

As shown in Fig. 1, the spreaded parallel data streams are transmitted simultaneously on N_k subchannels by modulating different subcarriers and then added together during the operation of IFFT. Following the IFFT, a cyclic extension is added to eliminate the effect of Inter-Symbol Interference (ISI). Finally, the signal is transmitted by the *m*th antenna.

Assume that the symbol duration after serial-to-parallel (S/P) conversion is T_S (see Fig. 2). Since the spreading gain is *G*, the chip duration of the signal is $T_C = T_S / G$, and thus $\Delta = 1/T_C$ represents the frequency spacing between two adjacent subcarriers. If each symbol of the *k*th user comprises b_k bits of information, then the total transmission rate (bandwidth) for the *k*th user is $b_k N_k / T_S$.

To transmit a block of N_k packets, we need *M* slots of symbol duration T_S. Within each time slot, there are N_k symbols being transmitted simultaneously. Denoting the index set of subcarriers for the *k*th user by $\{n_i \mid i = 1,2,...,N_k\}$, the transmitted signal $x_k^m(t)$ through the *m*th antenna within this block can be expressed by:

$$x_k^m(t) = \frac{1}{\sqrt{G}} \sum_{i=1}^{N_k} \sum_{j=1}^{M} s_{k,i}^{j,m} \sum_{u=1}^{G} c_k^u g(t - uT_C - jT_S) e^{j\frac{2\pi n_i (t - T_G)}{T_C}} \tag{5}$$

where $g(t)$ is a normalized pulse-shaping function with a finite duration $[0, T_C)$ and T_G is the guard interval for cyclic extension.

Thus, the transmitted signal of the *k*th user over all antennas given by Eq. (5) can be expressed in its vector form as follows:

$$\mathbf{x}_k(t) = \begin{bmatrix} x_k^1(t) & x_k^2(t) & \ldots & x_k^M(t) \end{bmatrix}^T \tag{6}$$

which is also shown in Fig. 1.

B. Downlink Decoding

Using OFDM, the original frequency selective channel can be considered as the combination of *N* subchannels which have flat fading characteristics. Denote the flat fading factor at the *n*th subcarrier, $n \in \{1,2,...,N\}$ between the kth user and the basestation's mth antenna by $h_{k,m}^n$. We assume that $\{h_{k,m}^n \mid \forall k,m,n\}$ are independent complex Gaussian random variables with the mean and variance equal to 0 and 0.5, respectively, at each dimension.

For simplicity, we will mainly focus on Alamouti's space-time block coding scheme with $M=2$ in the rest of the paper. And we make the following assumptions: (1) the downlink channel information is known by the receiver; (2) the basestation can obtain the downlink channel information and statistical characteristics of the noise by the receiver's feedback; (3) the channel is invariant within 2 symbol durations.

Since the decoding delay is $2T_S$, we need to receive two successive symbols and then exert decoding. After removing cyclic extension and applying FFT, the received signal changes back into N parallel data streams. Within duration $2T_S$, the original packet consisting of 2 symbols, transmitted at the nth subcarrier from the basestation to the kth user, is expressed by $\begin{bmatrix} s_k(1) & s_k(2) \end{bmatrix}$, where we remove the superscripts n for simplicity. Thus, the transmission matrix corresponding to Eqs. (2) and (3) becomes:

$$\mathbf{S}_{k,i} = \begin{bmatrix} \mathbf{s}_{k,i}^1 & \mathbf{s}_{k,i}^2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} = \begin{bmatrix} s_k(1) & s_k(2) \\ -s_k^*(2) & s_k^*(1) \end{bmatrix} \quad (7)$$

Let Φ_n denote the index set of users which are already assigned at the nth subcarrier. The sampled signal vectors $\mathbf{r}_k(1)$ and $\mathbf{r}_k(2)$ received by the kth user at the nth subcarrier within 2 symbol durations are determined by:

$$\begin{aligned} \mathbf{r}_k(1) = & \left[h_{k,1} s_k(1)\mathbf{c}_k + h_{k,2} s_k(2)\mathbf{c}_k \right] \\ & + \sum_{\substack{j\in\Phi_n \\ j\neq k}} \left[h_{k,1} s_j(1)\mathbf{c}_j + h_{k,2} s_j(2)\mathbf{c}_j \right] + \mathbf{v}_k(1) \end{aligned} \quad (8)$$

$$\begin{aligned} \mathbf{r}_k(2) = & \left[h_{k,2} s_k^*(1)\mathbf{c}_k - h_{k,1} s_k^*(2)\mathbf{c}_k \right] \\ & + \sum_{\substack{j\in\Phi_n \\ j\neq k}} \left[h_{k,2} s_j^*(1)\mathbf{c}_j - h_{k,1} s_j^*(2)\mathbf{c}_j \right] + \mathbf{v}_k(2) \end{aligned} \quad (9)$$

where vectors $\mathbf{v}_k(1)$ and $\mathbf{v}_k(2)$ denote the sampled Additive Gaussian White Noise (AGWN) with variance of σ_v^2, and $h_{k,1}$ and $h_{k,2}$ denote the nth subchannel fading factors. Notice that we remove the superscript n in $h_{k,1}^n$ and $h_{k,2}^n$ in Eqs. (8)-(9) and in the rest of the paper to simplify the presentation. In Eqs. (8)-(9), the first term represents the useful signal of the kth user, the second term (the summation term) represents the Multi-User Interference (MUI) in the nth subcarrier, and the last term denotes the noise. Combining Eqs. (8)-(9) into a matrix form, we obtain:

$$\mathbf{y}_k = \mathbf{W}_{k,k}\mathbf{s}_k + \sum_{\substack{j\in\Phi_n \\ j\neq k}} \mathbf{W}_{j,k}\mathbf{s}_j + \mathbf{n}_k \quad (10)$$

where $\mathbf{y}_k = \begin{bmatrix} \mathbf{r}_k(1) & \mathbf{r}_k^*(2) \end{bmatrix}^T$, $\mathbf{W}_{j,k} = \begin{bmatrix} \mathbf{c}_j & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_j \end{bmatrix} \begin{bmatrix} h_{k,1} & h_{k,2} \\ h_{k,2}^* & -h_{k,1}^* \end{bmatrix}$, $\mathbf{s}_k = \begin{bmatrix} s_k(1) & s_k(2) \end{bmatrix}^T$, and $\mathbf{n}_k = \begin{bmatrix} \mathbf{v}_k(1) & \mathbf{v}_k^*(2) \end{bmatrix}^T$.

We integrate the operations of de-spreading and space-time decoding, which combines the received signals into the following expression:

$$\begin{aligned} \mathbf{u}_k &= \begin{bmatrix} u_k(1) & u_k(2) \end{bmatrix}^T = \mathbf{W}_{k,k}^H \mathbf{y}_k \\ &= \mathbf{W}_{k,k}^H \mathbf{W}_{k,k}\mathbf{s}_k + \sum_{\substack{j\in\Phi_n \\ j\neq k}} \mathbf{W}_{k,k}^H \mathbf{W}_{j,k}\mathbf{s}_j + \mathbf{W}_{k,k}^H \mathbf{n}_k \end{aligned} \quad (11)$$

where $[\bullet]^H$ is the conjugate-transpose of matrix $[\bullet]$. Then, we can simplify the maximum-likelihood detections of the received signal estimations $\hat{s}_k(1)$ and $\hat{s}_k(2)$ to the linear combinations as follows:

$$\begin{cases} \hat{s}_k(1) = \arg\min_{s\in S} \left\{ \delta^2\left(u_k(1), s\right) \right\} \\ \hat{s}_k(2) = \arg\min_{s\in S} \left\{ \delta^2\left(u_k(2), s\right) \right\} \end{cases} \quad (12)$$

where $\delta^2(a,b)$ denotes the square Euclidean distance between symbol a and b, and S denotes the entire symbol set.

III. THE SUBCHANNEL ALLOCATION ALGORITHMS

The QoS criteria vary with different wireless-networking applications. For reliable data transmissions, while imposing no strict delay bounds, each user's SINR must be higher than its minimum threshold. On the other hand, although the audio/video streams can tolerate a certain degree of losses or errors, they need to meet the stringent delay bounds, requiring that each user must be assigned the minimum number of subcarriers for minimum bandwidth-guarantee of QoS. This paper focuses mainly on the reliable wireless data transmission, and thus our subchannel-allocation algorithms need to ensure that every user's SINR at each subcarrier must be higher than its minimum threshold. The scheme for real-time wireless transmissions is treated in our another paper.

As described earlier in Section II-B, we assume that the basestation knows the information and the statistical characteristics about the downlink channel and the noise. Consequently, the basestation can pre-compute the SINR of decoded signals for every user at each subchannel. When a new user attempts to join in the communication system, the basestation must evaluate the feasibility of the request and determine the subcarrier allocation algorithm based on the SINR information and current bandwidth allocations.

Let us reconsider Eq. (11) in more details. Define the correlation factor $\rho_{k,j}$ of different spreading codes by:

$$\rho_{k,j} \triangleq \frac{1}{G}\mathbf{c}_k^T\mathbf{c}_j = \frac{1}{G}\sum_{i=1}^{G} c_k^i c_j^i, \text{ for } k,j \in \{1,2,...,K\} \quad (13)$$

Thus, the random variable SINR_k corresponding to the kth user at the nth subcarrier is determined by:

$$\begin{aligned} \text{SINR}_k &= \frac{\left(\mathbf{W}_{k,k}^H\mathbf{W}_{k,k}\mathbf{s}_k\right)^H \left(\mathbf{W}_{k,k}^H\mathbf{W}_{k,k}\mathbf{s}_k\right)}{\sum_{\substack{j\in\Phi_n \\ j\neq k}} \left(\left(\mathbf{W}_{k,k}^H\mathbf{W}_{j,k}\mathbf{s}_j\right)^H \left(\mathbf{W}_{k,k}^H\mathbf{W}_{j,k}\mathbf{s}_j\right) \right) + \mathbf{W}_{k,k}^H\mathbf{W}_{k,k}\sigma_v^2} \\ &= \frac{\left|h_{k,1}\right|^2 + \left|h_{k,2}\right|^2}{\sum_{\substack{j\in\Phi_n \\ j\neq k}} \left(\left|h_{k,1}\right|^2 + \left|h_{k,2}\right|^2 \right)\rho_{k,j}^2 + \sigma_v^2}. \end{aligned} \quad (14)$$

The goal of our subchannel allocation scheme is to assign as many users as possible into a single subcarrier based on the criterion that each user within the subcarrier meets its SINR-threshold requirement.

Let the kth user, $k \in \Omega \setminus \Phi_n$, be the candidate which attempts to join the nth subcarrier at the next step (this case is different from the ones discussed in Section-II, where $k \in \Phi_n$). Denote the SINR-

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00.  Procedure: subchannel allocation for the kth user at the nth subcarrier
01.  for ( i := 1 to K ) {                          // Test the SINR of all the users in Φ_n .
02.    if ( i = k ) { compute SINR_i ;              // Using Eq. (15).
03.      if (SINR_i < γ_i) {return 0;}}             // Subchannel allocation fails.
04.    elseif ( i ∈ Φ_n ) { compute SINR_i ; // Using Eq. (16).
05.      if (SINR_i < γ_i) {return 0;}}}            // Subchannel allocation fails.
06.  Φ_n := Φ_n + {k} ;                             // k becomes a new element in Φ_n .
07.  return 1;                                      // Subchannel allocation succeeds.
```

Fig. 3. Pseudocode of channel allocation for the *k*th user at the *n*th subcarrier.

threshold of the *k*th user by γ_k. We describe the algorithm of subchannel allocation for the *k*th user at the *n*th subcarrier by the pseudocode given in Fig. 3.

As shown Fig. 3, the algorithms compute the value of SINR_k and all other SINR_j's, $j \in \Phi_n$, to see if they are all larger than their thresholds γ_k and γ_j 's, which can be expressed by:

$$\text{SINR}_k \triangleq Z_k = \frac{|h_{k,1}|^2 + |h_{k,2}|^2}{\sum_{j\in\Phi_n}\left(|h_{k,1}|^2 + |h_{k,2}|^2\right)\rho_{k,j}^2 + \sigma_\nu^2} \geq \gamma_k \tag{15}$$

where $k \in \Omega \setminus \Phi_n$; and

$$\text{SINR}_j \triangleq Z_j = \frac{|h_{j,1}|^2 + |h_{j,2}|^2}{\left(|h_{j,1}|^2 + |h_{j,2}|^2\right)\left(\sum_{\substack{i\in\Phi_n \\ i\neq j}}\rho_{j,i}^2 + \rho_{j,k}^2\right) + \sigma_\nu^2} \geq \gamma_j \tag{16}$$

where $\forall j \in \Phi_n$, and to simplify the presentation, the random variables SINR_k and SINR_j are denoted by Z_k and Z_j in Eqs. (15)-(16), respectively, and in the rest of the paper.

If both Eqs. (15) and (16) are satisfied, the *k*th user is qualified to be assigned at the *n*th subcarrier. Then, it becomes a new element in Φ_n (as shown in Step-06, see Fig. 3). Otherwise, if any of the SINR is lower than its threshold, the *k*th user is rejected to join the *n*th subcarrier (Step-03 and Step-05, see Fig. 3). The procedure continues and repeats itself until all *N* subcarriers have been examined. Thus, the *k*th user achieves the maximum transmission bandwidth under the condition that its SINR constraint is satisfied.

IV. Performance and Complexity Analyses

A. Average Throughput Derivations

We first derive the average throughput of the proposed scheme analytically. We assume that the symbol of different users comprises the same number of bits of information. Then the system average throughput can be simplified into the average number of users per subcarrier.

If the *n*th subcarrier can be assigned to the *k*th user, the SINRs of users in the *n*th subcarrier must satisfy the Eqs. (15) and (16). As mentioned in Section II-B, we assume that the subchannel fading factors are independent complex Gaussian variables with the mean and variance equal to 0 and 0.5, respectively, at each dimension. Therefore, the random variable X_k :

$$X_k \triangleq \left(|h_{k,1}|^2 + |h_{k,2}|^2\right) \tag{17}$$

follows χ^2 distribution with the degree of freedom $\upsilon = 4$. The *pdf* $f(x_k)$ of X_k with $\upsilon = 4$ is given by:

$$f(x_k) = \begin{cases} xe^{-x} & x_k \geq 0 \\ 0 & x_k < 0 \end{cases} \tag{18}$$

In the following, we consider 2 different coding schemes to analyze the system average throughput, respectively.

A-I. The Special Spreading-Code Scheme

For simplicity, in this case we assume that the different spreading codes selected have the same square correlation factors, that is:

$$\rho_{k,j}^2 = \begin{cases} 1, & \text{if } k = j \\ 1/G, & \text{if } k \neq j \end{cases} \tag{19}$$

where $\forall k, j \in \Omega$. Let the number of users in Φ_n be d . Then, Eq. (15) becomes:

$$Z_k = \frac{X_k}{X_k d / G + \sigma_\nu^2} \geq \gamma_k \tag{20}$$

Given that d users have already been assigned in the *n*th subcarrier, the probability that the *k*th user satisfies its SINR-threshold is determined by:

$$\Pr\{Z_k \geq \gamma_k\} = \Pr\left\{\frac{X_k}{X_k d / G + \sigma_\nu^2} \geq \gamma_k\right\} = \begin{cases} \Pr\left\{X_k \geq \dfrac{\sigma_\nu^2 \gamma_k}{1 - \gamma_k d / G}\right\}, & \text{if } 1 - \gamma_k d / G \geq 0 \\ 0, & \text{if } 1 - \gamma_k d / G < 0 \end{cases} \tag{21}$$

where the condition $(1 - \gamma_k d / G \geq 0)$ implies that the number d of co-subchannel users must satisfy $d \leq G/\gamma_k$ to have a non-zero $\Pr\{Z_k \geq \gamma_k\} > 0$. Since the *k*th user can be the one extra user accepted to join the *n*th subcarrier, the upper-bound of the total number d' of co-subchannel users can be expressed as:

$$d' = d + 1 \leq D_{\max} = \left\lfloor \frac{G}{\gamma_k} + 1 \right\rfloor \tag{22}$$

where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to *x*, because $D_{\max}$ must be an integer.

As long as $d < D_{\max}$, Eq. (21) can be simplified as follows:

$$\Pr\left\{X_k \geq \frac{\sigma_\nu^2 \gamma_k}{1 - \gamma_k d / G}\right\} = \int_{x = (\sigma_\nu^2 \gamma_k)/(1 - \gamma_k d / G)}^{\infty} \left(xe^{-x}\right) dx \tag{23}$$

We define this probability given in Eq. (23) by $\Pr\{Z_k(d) \geq \gamma_k\}$, where d also represents the number of cochannel interferences.

At the same time, other users in the *n*th subcarrier must remain satisfying their SINR-threshold constraints specified by Eq. (16). Thus, the probability that the *j*th user $(j \in \Phi_n)$ satisfies its SINR-threshold constraint becomes a conditional probability. The condition is that the *j*th user has already satisfied its SINR-threshold constraint before the *k*th user joins in (there are $(d-1)$ interferences for the *j*th user before the *k*th user joins in). This conditional probability can be expressed as follows:

$$\begin{aligned} &\Pr\{Z_j(d) \geq \gamma_j \mid Z_j(d-1) \geq \gamma_j\} \\ &= \Pr\{Z_j(d) \geq \gamma_j, Z_j(d-1) \geq \gamma_j\} \big/ \Pr\{Z_j(d-1) \geq \gamma_j\} \\ &= \Pr\{Z_j(d) \geq \gamma_j\} \big/ \Pr\{Z_j(d-1) \geq \gamma_j\} \end{aligned} \tag{24}$$

where the last equation of (24) follows because the inequality $Z_j(d) < Z_j(d-1)$ always holds. If the kth user is qualified for joining the nth subcarrier, both Eqs. (23) and (24) must hold. Since the statistics of all users' subchannels are independent, the probability $\phi_k(d)$ that the kth user successfully joins the nth subchannel is determined by:

$$\begin{aligned}\phi_k(d) &\triangleq \Pr\left\{Z_k(d) \ge \gamma_k, \bigcap_{j\in\Phi_n}\left\{Z_j(d)\ge\gamma_j\right\}\right\} \\ &= \Pr\left\{Z_k(d)\ge\gamma_k\right\}\prod_{j\in\Phi_n}\Pr\left\{Z_j(d)\ge\gamma_j \mid Z_j(d-1)\ge\gamma_j\right\} \\ &= \Pr\left\{Z_k(d)\ge\gamma_k\right\}\prod_{j\in\Phi_n}\left[\Pr\left\{Z_j(d)\ge\gamma_j\right\}/\Pr\left\{Z_j(d-1)\ge\gamma_j\right\}\right] \end{aligned} \quad (25)$$

where the last equation of (25) is due to Eq. (24). Clearly, $(1-\phi_k(d))$ represents the probability that the kth user is rejected to join the nth subchannel.

If all the SINR-thresholds are equal to the same value γ, we can remove the subscript k and j. Then, Eq. (25) becomes:

$$\phi(d) = \phi_k(d) = \left[\Pr\left\{Z(d)\ge\gamma\right\}\right]^{(d+1)} \Big/ \left[\Pr\left\{Z(d-1)\ge\gamma\right\}\right]^{d} \quad (26)$$

The proposed subchannel allocation algorithm allows all users to attempt to join the nth subcarrier, but the attempt can be accepted or rejected with a certain probability. The probability $\varphi(m,K)$ that m users out of total K users can be successfully accepted to join the nth subcarrier is derived by the following equation:

$$\varphi(m,K) = \underset{\substack{0\le k_0,k_1,\ldots,k_m\le K-m \\ k_0+k_1+\ldots+k_m=K-m}}{\sum_{k_0}\sum_{k_1}\cdots\sum_{k_m}} \left\{\prod_{i=0}^{m-1}\phi(i)\left[1-\phi(i)\right]^{k_i}\left[1-\phi(m)\right]^{k_m}\right\} \quad (27)$$

Eq. (27) gives the probability that **any** m of K users are accepted to join the nth subcarrier while the rest $(K-m)$ users are rejected for joining in the nth subcarrier, where $m = 1,2,\ldots,K$. Thus, the average system throughput $\overline{R}$ can be described as follows:

$$\overline{R} = \begin{cases} \sum_{k=1}^{K} k\,\varphi(k,K), & \text{if } K \le D_{\max} \\ \sum_{k=1}^{D_{\max}} k\,\varphi(k,K), & \text{if } K > D_{\max} \end{cases} \quad (28)$$

Similarly, the derivations of Eqs. (24)-(28) can be applied to different antenna combinations infrastructures such as 1-transmit and 1-receive antennas, 1-transmit and 2-receive antennas, and 2-transmit and 2-receive antennas, etc. The forms of Eqs. (24)-(28) remain the same except that the freedom degree of χ^2 distribution of the random variable X_k specified by Eq. (17) varies with different antenna combinations infrastructures. In general, with the antennas infrastructure using M_T transmit antennas and M_R receive antennas, the random variable X_k follows the χ^2 distribution with the degree of freedom $\upsilon = 2M_T M_R$.

A-II. The Random Spreading-Code Scheme

In most applications, the square correlation factors of different spreading codes are not fixed as we described in Eq. (19). In this section, we analyze the situation where the spreading codes are randomly selected such that $\rho_{k,j}^2$ given by Eq. (19) becomes a random variable. We assume that the elements of the spreading codes are *i.i.d.* variables each with the probability satisfying:

$$\Pr\left\{c_k^i = 1\right\} = \Pr\left\{c_k^i = -1\right\} = 0.5\,, \text{ for } k=1,2,\ldots,K;\ i=1,2,\ldots,G \quad (29)$$

According to the *Central Limit Theorem*, the correlation factor $\rho_{k,j}$ defined in Eq. (13) satisfies the Gaussian distribution with mean and variance equal to 0 and $1/G$, respectively, if $k \ne j$ and the spreading gain G is large enough. Thus, $\rho_{k,j}^2$ satisfies χ^2 distribution with the degree of freedom 1. The accurate derivations for the closed-form expression of Z_k's *pdf* given by Eqs. (15) and (16) turn out to be complicated and lengthy, which are omitted for lack of space, but available in the full version of this paper. Instead, in this paper we use the numerical solutions to obtain the *pdf* of Z_k. Then, we apply the similar procedures as described in Eqs. (24)-(28) to derive the system average throughputs as shown in Figs. 4-7. Notice that for this scheme there is no upper bound as given by Eq. (22) described in Section IV-A-I. Then, Eq. (28) becomes:

$$\overline{R} = \sum_{k=1}^{K} k\,\varphi(k,K) \quad (30)$$

Similarly, we can also derive the system average throughputs for the other cases of different combinations of antenna infrastructures.

B. Implementation Complexity Analysis

In our subchannel allocation algorithm, we only use matrix multiplications without involving in any higher-order computations such as eigen-vector decompositions and matrix inversions such that our scheme can achieve a much lower computational complexity when compared with the smart-antenna-based schemes. For N subcarriers, K users, and M antennas, the computational complexity of our algorithm is in order of $O(NKM)$ based on its calculation intensity. Compared with the scheme based on the OFDM smart antenna systems [5] whose computational complexity is in order of $O(NKM^4)$. Thus, the computational complexity of our algorithm is much lower than the smart-antenna-based systems, making our scheme much easier to implement.

Furthermore, the smart-antenna-based schemes demand a great deal of costly efforts in dealing with the multi-antenna inconsistence and intercoupling problems, which are inherently associated with the smart-antenna-based systems. In contrast, our schemes based on the transmit diversity techniques do not have these high hardware-complexity problems at all.

V. Numerical and Simulation Analysis

We use both numerical analyses and simulation experiments to evaluate our proposed algorithms. In all the experiments, the number N of subcarriers of the system is set to 16 and spreading gain G set to 32. The spreading codes that we use are random signature sequences. In Figs. 4-5 and Fig. 7, the letters "A" and "S" in the legends represent the analytical and simulation results, respectively.

Fig. 4 plots the average throughputs versus the number of users with different numbers of transmit and receive antennas. All users have the same threshold $\gamma = 10$dB, and also the same

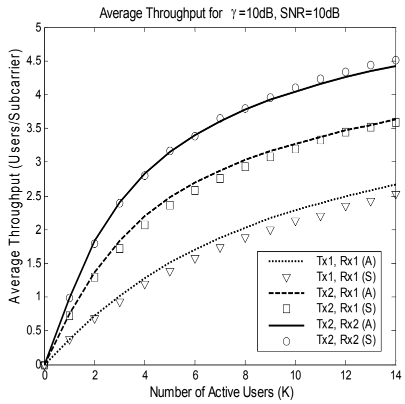

Fig. 4 Average throughput versus number of users.

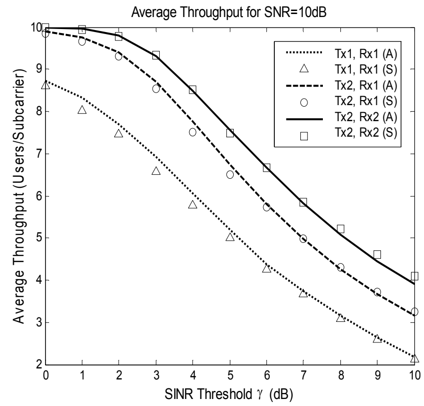

Fig. 5 Average throughput versus SINR-threshold.

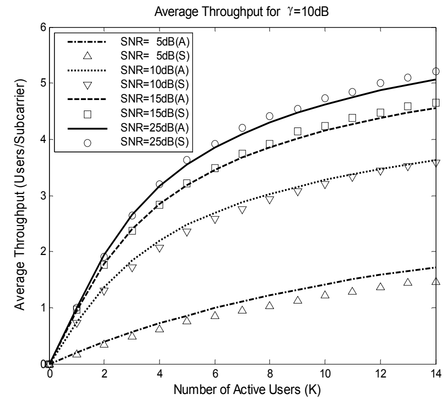

Fig. 7 Average throughput versus SNR (2Tx, 1Rx).

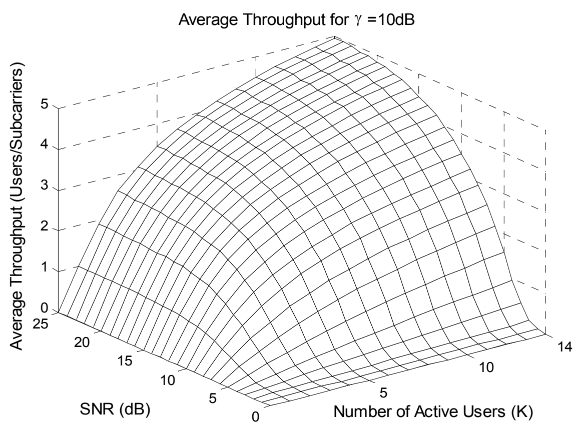

Fig. 6 Average throughput versus number of users and SNR (2Tx, 1Rx).

SNR = 10dB. If only one transmit antenna is used, we do not employ space-time block coding schemes. From Fig. 4, we observe that the average throughputs increase as the number of active users increases. Fig. 4 also shows that the space-time OFDM-CDMA infrastructure at physical layer can significantly increase the throughput of the system as the number of transmit antenna increases. When the receive diversity is also applied, the throughput can be further improved as the number of receive antenna increases.

Fig. 5 plots the average throughputs versus SINR-threshold γ with different infrastructures. The SNR value is set to 10dB. When the SINR-threshold γ increases, the average throughputs decrease significantly. This implies that if we need to guarantee the reliability requirement of QoS, we cannot assign too many users into a single subchannel. Similar to the results observed from Fig. 4, the space-time OFDM-CDMA infrastructures and transmit- and receive-diversity significantly increase the average throughputs for any given γ .

To comprehensively study how multiple parameters affect the throughput simultaneously, Fig. 6 plots the average throughput against two independent variables: SNR and K (number of active users) for a given SINR-threshold γ = 10dB. Based on the numerical solutions of analytical results, Fig. 6 shows that the average throughput is a monotonic-increasing function of both SNR and K. This is expected since increasing either SNR or user number K improves the system throughput by the more reliable wireless channel or potentially more users transmitting data.

Fig. 7 plots the average throughputs versus the number of users with SNR's varying. From Fig. 7 we observe that the rate of throughput improvement slows down as the SNR increases. This trend indicates that there is an upper-bound of system throughput, which is reasonable because the channel capacity is always finite.

Fig. 4, Fig. 5, and Fig. 7 also compare the average throughputs obtained from analytical solutions with simulation experiments. We observe that the analytical and simulation results agree well with each other, verifying the correctness of our analytical derivations.

VI. Conclusion

We proposed the subchannel-allocation algorithms for the space-time OFDM-CDMA systems and developed the performance analysis frameworks to evaluate the proposed algorithms. Also, we conducted simulation experiments to verify the analytical results. Both analytical analyses and simulation experiments show that the proposed infra-structures and algorithms can significantly increase the throughput over the wireless networks, while achieving a much lower implementation and computation complexities as compared to the smart-antenna-based systems. All analytical results are confirmed by simulations.

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