Modeling Throughput Gain of Network Coding in Multi-Channel Multi-Radio Wireless Ad Hoc Networks

Hang Su, *Student Member, IEEE,* and Xi Zhang, *Senior Member, IEEE*

***Abstract*— In this paper, we model the network throughput gains of two types of wireless network coding (NC) schemes, including the conventional NC and the analog NC schemes, over the traditional non-NC transmission scheduling schemes in multi-hop, multi-channel, and multi-radio wireless ad hoc networks. In particular, we first show that the network throughput gains of the conventional NC and analog NC are $(2n)/(2n-1)$ and $n/(n-1)$, respectively, for the n-way relay networks where $n \geq 2$. Second, we propose an analytical framework for deriving the network throughput gain of the wireless NC schemes over general wireless network topologies. By solving the problem of maximizing the network throughput subject to the fairness requirements under our proposed framework, we quantitatively analyze the network throughput gains of these two types of wireless NC schemes for a variety of wireless ad hoc network topologies with different routing strategies. Finally, we develop a heuristic joint link scheduling, channel assignment, and routing algorithm that aims at approaching the optimal solution to the optimization problem under our proposed framework.**

***Index Terms*— Wireless network coding, optimization, link scheduling, channel assignment, routing, wireless ad hoc networks.**

I. INTRODUCTION

NETWORK coding (NC) is receiving more and more research attention since it is a promising technique to increase the network throughput for both wired and wireless networks. By exploiting the broadcast nature of the wireless channel, the conventional wireless NC proposed in [1], [2] can significantly increase the network throughput as compared with the traditional non-NC transmission scheduling based scheme in multi-hop wireless ad hoc networks.

The essential idea of conventional wireless NC can be explained as follows using a simple example. As shown in Fig. 1(a), node A wants to send a single packet to node C, while node C wants to send a single packet to node A. Due to transmission range limitations, both of these two paths go through the relay node B. This is the simplest two-way relay topology. Suppose that the time axis is divided into time slots and the transmission of each packet spends one time slot. Then, if we adopt the non-NC scheduling based scheme, four time slots will be needed to complete these two-way relay transmissions. The following is a possible sequence of these transmissions: 1) node A sends a packet to node B while node C remains silent; 2) node B relays A's packet to node C; 3) node C sends its packet to node B while A remains silent; 4) node B relays C's packet to node A. In contrast to the non-NC scheme, the conventional wireless NC scheme only needs three time slots to complete the two-way relay transmissions. The following is a possible sequence of these transmissions: 1) node A sends a packet to node B; 2) node C sends a packet to node B; 3) node B transmits a new packet obtained by performing an XOR of A's and C's packets. Node A can XOR the received new packet from node B with its own packet to obtain C's packet. In the same way, node C can get A's packet.

Besides the broadcast nature of the wireless channel, is there anything else that can be exploited by the NC to further increase the network throughput? The answer is positive. The authors of [3], [4] developed the analog NC, which can even take advantage of the native physical-layer coding ability by analogously mixing simultaneously arrived radio waves at the relay nodes, to further increase the network throughput. Specifically, under the analog NC, the two-way relay transmissions can be completed in just *two time slots*. In the first time slot, A and C transmit their packets to node B simultaneously, resulting in interfere of their transmissions at the relay node B. Due to interference, the relay node receives the sum of A's and C's analog signals. This is a collision and the relay node B cannot decode the bits. In the second time slot, the relay node B simply amplifies and forwards the received interfered signal at the physical layer without decoding it. Since node A knows the packet it sent, it also knows the packet's corresponding analog signal. It can thus subtract its original signal from the received interfered signal to get the signal of C's packet, from which it can decode C's packet. Likewise, C can decode A's packet.

The promising analog NC technique motivates us to investigate how to practically apply the analog NC and how well the analog NC can perform in the multi-hop, multi-channel and multi-radio wireless ad hoc networks with multiple unicast sessions. The main goal of this paper is to analytically model the network throughput improvements of the above mentioned two types of wireless NC over wireless ad hoc networks. To our best knowledge, there was no existing work reported yet in the literature, which compares the network throughputs achieved by the non-NC scheme, conventional NC scheme,

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The authors are with the Networking and Information Systems Laboratory, Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX 77843 USA (e-mails: hangsu@ece.tamu.edu; xizhang@ece.tamu.edu).

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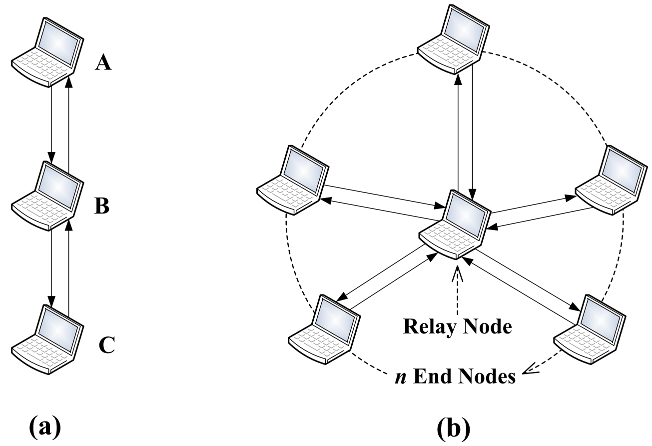


Fig. 1. Diagrams of the relay network topologies. (a) An example of the 2-way relay network. (b) An example of the n-way relay network.

and analog NC scheme, respectively, from a theoretical perspective. Our contributions can be summarized as follows:

- We show that the network throughput gains of the conventional NC and the analog NC are $(2n)/(2n-1)$ and $n/(n-1)$, respectively, for the n-way relay networks where $n \geq 2$.
- Applying the linear programming/optimization technique, we formulate a general framework, which is applicable to *any* transmission schemes with or without NC, to maximize the network throughput for *any* wireless network topologies. Our framework is featured with multi-path routing that efficiently seizes the wireless NC opportunities. Under the developed framework, we propose a joint link scheduling, channel assignment and routing algorithm to approach the obtained optimal network throughput.
- We utilize the developed framework to quantitatively characterize the network throughput gains of the conventional NC and the analog NC in various network topologies where the type of routing schemes, the number of channels, and the number of radios may vary. We also conduct extensive simulations to evaluate the performance of our proposed joint link scheduling, channel assignment, and routing algorithm.

The rest of this paper is organized as follows. Section II discusses the related works. Section III investigates the throughput gain of wireless NC in the n-way relay networks with $n \geq 2$. Section IV describes the system models for the general wireless ad hoc network topology. Section V develops the general linear programming framework. Section VI designs the joint link scheduling, channel assignment, and routing algorithm for the wireless NC schemes. Section VII evaluates the network throughput gains of the wireless NC schemes over the non-NC scheme under a variety of general wireless ad hoc network topologies. The paper concludes with Section VIII.

II. Related Works

The notion of NC was first coined in [5] in the context of wired multicast communication. Since then, extensive research efforts [6]–[8] have been devoted to the area of design and analysis of NC for wired networks. However, the wireless networks cannot directly benefit from the NC schemes designed for the wired counterpart. The conventional wireless NC [1], [2] and analog NC [3], [4] were proposed for the wireless networks to enhance the network throughput. In [9], the joint superposition coding and convectional NC was proposed to improve the network throughput.

The linear programming technique has been widely utilized in previous work [10]–[14] to analyze the network throughput for wireless ad hoc networks. The authors of [10] studied the capacity region in multi-radio wireless networks using linear programming technique. Our paper extends the similar multi-channel multi-radio network modeling technique like that used in [10] to model and analyze the different application targets in multi-channel multi-radio networks. The authors of [11] formulated the linear program to analyze the throughput of NC for the wired networks with multiple unicast sessions. In [12], the authors analyzed the throughput of conventional wireless NC based on linear optimization. The authors of [13] developed the XOR-Sym NC scheme, which achieved a lower implementation complexity than the conventional wireless NC, under the linear optimization based framework. In [14], the authors formulated the optimization problem to seek the optimal network throughput for the conventional NC with multiple packet reception. None of the aforementioned work provides a general framework to analyze the network throughput gain for both the conventional and analog NC schemes in multi-hop, multi-channel, and multi-radio wireless ad hoc networks.

On the other hand, there has been recent work on the link scheduling, channel assignment, and routing in multi-hop wireless ad hoc networks. The link scheduling and channel assignment protocols in the link layer level for multi-channel wireless networks have been studied in [15]–[17]. The optimal routing and transmission scheduling approaches for multi-channel multi-radio networks were studied in [18]. Recent results in [19]–[22] provide various routing metrics to find the load-balanced high-throughput path between a source and a destination in the multi-channel multi-radio wireless ad hoc networks. The joint link scheduling, channel assignment, and routing without considering wireless NC have been investigated in [10], [23].

III. Throughput Gain in n-Way Relay Networks

We start with the n-way relay networks, where there are n, with $n \geq 2$, end nodes at outside and one relay node at inside, as shown in Fig. 1(b). There are no direct wireless links between any two end nodes. This implies that there is no opportunistic listening.[1] Notice that the opportunistic listening is not considered in this paper because the opportunistic listening is practically difficult to be implemented under analog NC schemes due to synchronization and power control problems.

Definition 1: One round of n-way relay is defined as the process that each of n end nodes successfully distributes one of its own packets to the all other $(n-1)$ end nodes.

We first study the performance of the conventional NC in the n-way relay network. As discussed in Section I, the

[1] With the opportunistic listening, the nodes in the promiscuous mode can use the packets overheard from the neighbors to help decode the network-coded packets [2].

conventional NC needs to spend 3 time slots to finish a round of the 2-way relay. The following Lemma extends the above result to the n-way relay network.

Lemma 1: For the n-way single-channel relay network, it takes at least $(2n-1)$ time slots for the conventional NC scheme without opportunistic listening to perform a round of the n-way relay.

Proof: In the conventional NC scheme, the n-way relay can be divided into two phases. In the first phase, each end node sends its own packet to the relay node. It will take n time slots to perform the first phase. In the second phase, the relay node linearly combines the end nodes' packets, and then broadcasts the linearly combined packets to all the end nodes. The process for any given end node to correctly decode the combined packets from the relay node is analogical to the process of solving a set of linear equations with $(n-1)$ pronumerals. It requires at least $(n-1)$ independent linear equations to obtain the values for the $(n-1)$ pronumerals, each of which represents the packet of each end node. Thus, it takes at least $(n-1)$ time slots to finish the second phase. Thus, it takes at least $n+(n-1)=2n-1$ time slots to accomplish a round of n-way relay, which completes the proof of Lemma 1. ■

Then, we focus on the analog NC scheme. As observed from Section I, the analog NC takes at least 2 time slots to finish a round of the 2-way relay. We then obtain the following lemma to extend the result from the 2-way relay network to the n-way relay network.

Lemma 2: For the n-way single-channel relay network, it takes at least $2(n-1)$ time slots for the analog NC scheme without opportunistic listening to accomplish one round of the n-way relay.

Proof: In the analog NC scheme, n end node can transmit its packet to the relay node simultaneously. Similar to the proof of Lemma 1, it also requires $(n-1)$ independent linearly combined packets for any given end node to correctly decode the other nodes' packets. That is, $(n-1)$ times of transmission from different combination of end nodes are needed. On the other hand, the relay node needs $(n-1)$ time slots to broadcast the $(n-1)$ linearly combined packets to the n end nodes. Therefore, the total number of time slots that is needed to complete one round of relay is $2(n-1)$. ■

Theorem 1: For the n-way single-channel relay network, the throughput gain of the conventional NC is $(2n)/(2n-1)$, while the throughput gain of the analog NC is $n/(n-1)$.

Proof: If non-NC scheme is employed, it takes $2n$ time slots to perform one round of n-way relay in the n-way relay network. More specifically, on one hand, it takes n time slots for each of n nodes to upload their own packets to the relay node. On the other hand, it takes another n time slots for the relay to broadcast the packets to every end node. Hence, according to Lemmas 1 and 2, we can obtain Theorem 1. ■

According to Theorem 1, in the n-way relay networks, the throughput gains of both the conventional NC and the analog NC are maximized when $n=2$. We continue to study the upper-bound throughput that can be achieved by the analog NC in the n-way single-channel relay network.

Theorem 2: For the n-way single-channel relay network, $\forall\ n \geq 2$, the analog NC without opportunistic listening can achieve the upper-bound capacity, which is 0.5 packets per time slot per node.

Proof: In one round of n-way relay, each end node can receive $(n-1)$ packets from the other $(n-1)$ end nodes. Based on Lemma 2, it takes at least $2(n-1)$ time slots for the analog NC scheme without opportunistic listening to finish one round of n-way relay. Thus, each end node of the network receives $(n-1)$ packets from the other $(n-1)$ end nodes in $2(n-1)$ time slots. In other words, at most $(n-1)/[2(n-1)]=0.5$ packets per time slot per node can be sent in the analog NC scheme. ■

IV. System Models for General Network Topologies

We then consider the wireless ad hoc networks with general network topologies, where there exist multiple unicast sessions. The wireless spectrum is divided into a set of orthogonal channels. Each node is equipped with either a single radio or multiple radios. If a node has multiple radios, it can communicate with more than one neighbor at the same time over different orthogonal channels. To analyze the throughput gain of wireless NC over the non-NC scheme for general network topologies, we start with the system models including network model and the wireless channel/interference models.

A. The Network Model

We model the wireless network topology, characterized by the nodes and the links corresponding to pairs of nodes within direct communication range and interference range, as a directed graph $G=(V,E,I)$, where V represents the set of nodes, E is the set of data links, and I is the set of interference links. Note that E is the set of links that can carry data, while I is the set of links that can sense signals but not decode the data. Let $E^{-}(v)$ and $E^{+}(v)$ be the sets of incoming and outgoing links of node v with $v \in V$, respectively. Denote by $e=(u,v)$ the directed link in the network from node u to node v with $u,v \in V$. Let $t(e)$ and $r(e)$ be the transmitting and receiving nodes, respectively, of link e. Also, let $\overline{e}=(v,u)$ be the reverse link of $e=(u,v)$.

There are multiple orthogonal channels over each link in the network. Let M and $\|M\|$ be the set and the number of these channels, respectively, over each link in the network. The network is exploited by a number of sessions to transport data packets. Denote the set of sessions by A. A session a, with $a \in A$, is characterized by a triplet $\{s(a), d(a), \theta(a)\}$, where $s(a)$, $d(a)$, and $\theta(a)$ are the source node, destination node, and throughput, respectively, of session a. Packets of session a with $a \in A$ are routed from $s(a)$ to $d(a)$ in multiple hops if there is no directed link between the source and destination nodes. Every node in the wireless network can be a source or destination, i.e., $s(a), d(a) \in V, \forall a \in A$. There may be multiple routes for session a from $s(a)$ to $d(a)$. Let $\mathcal{P}_a$ be the set of available routes/paths for session a. For a path $P \in \mathcal{P}_a$ of session a, it can be considered as an ordered subset of links, $P=\{e_0, e_1, \cdots, e_{N_a}\}$, such that $t(e_0)=s(a)$ and $r(e_{N_a})=d(a)$. For any given path P, link e, and node v, we use $e \in P$ to represent that link e is on path P and $v \in P$ to represent that node v is on path P. Furthermore, we use

$e_1e_2 \in P$ to denote that the path P includes links e_1 and e_2, and the link e_2 is immediately behind e_1, i.e., $r(e_1) = t(e_2)$.

B. Wireless Channel/Interference Model

We denote by D_T and D_I the transmission range and interference range, respectively. Because D_I is always larger than or equal to D_T in practice, let $D_I = \alpha D_T$ with $\alpha \geq 1$. Let $h(u, v)$ be the Euclidean distance between nodes u and v. This paper adopts the protocol model of interference [24]. There exists an edge $e = (u, v) \in E$, if and only if $h(u, v) \leq D_T$, which implies that nodes u and v can communicate directly in one hop. Let $c_m(e)$ be the date rate of link e over channel m. This is the maximum data rate at which node $t(e)$ can communicate with node $r(e)$. There exists an edge $i = (u, v) \in I$, if and only if $D_T \leq h(u, v) \leq D_I$, which implies that nodes u and v cannot communicate directly in one hop, but can interfere with each other. The definition of interference link set I captures the behavior of the carrier sense multiple access with collision avoidance (CSMA/CA) featured by IEEE 802.11 medium access control (MAC) [25]. In light of carrier sensing, a communication between nodes u and v can block all transmissions within distance D_I away from either u or v.

V. The Framework for General Network Topologies

In this section, we formulate a general linear programming framework to find the optimized flows for each link, which maximize the network throughput while ensuring fairness in the resource allocation among wireless nodes. Our framework adopts the linear programming technique similar to that used in [12]. However, significantly different from the work reported in [12], where the framework is constructed based on the conventional wireless NC scheme in single-channel single-radio networks, our proposed framework is applicable to the transmission scheduling schemes with or without NC including the non-NC scheme, the conventional NC scheme, and the analog NC scheme in multi-channel multi-radio networks.

A. NC Links Combinations

It is not difficult to perceive that n-way relay is more complicated to implement (e.g., due to the synchronization problems) than the 2-way relay. On the other hand, Theorem 2 in Section III shows that the throughput gain is maximized for the n-way relay networks without opportunistic listening when $n = 2$. Hence, we focus on the 2-way relay case only in the rest of this paper.

For the convenience of presentation, in Table I we summarize the important notations which will be used in the rest of the paper. We then formulate various sets of NC links combinations. In fact, an NC links combination is a set of data links which together can perform the NC operation. The existence of an NC links combination depends on the routes/paths of different sessions. First, we introduce two basic NC links combinations: incoming and outgoing NC links combinations. Figs. 2(a) and (b) show the examples of an incoming NC links combination and an outgoing NC links combination, respectively. Under the analog NC scheme, the pair of links in the incoming/outgoing NC links combination can and must be active at the same time for the two-way relay traffic. In particular, based on the analog NC, nodes B and D transmit packets to node C simultaneously, as shown in Fig. 2(a), which implies that the incoming NC links combination is active. In the next time slot, node C broadcasts the mixed analog signal of nodes B and D, and thus, nodes B and D can decode each other's packet, as shown in Fig. 2(b). This NC operation is characterized by an outgoing NC links combination. Definitions 2 and 3 provide the ways to construct the incoming NC links combinations and outgoing NC links combinations, respectively.

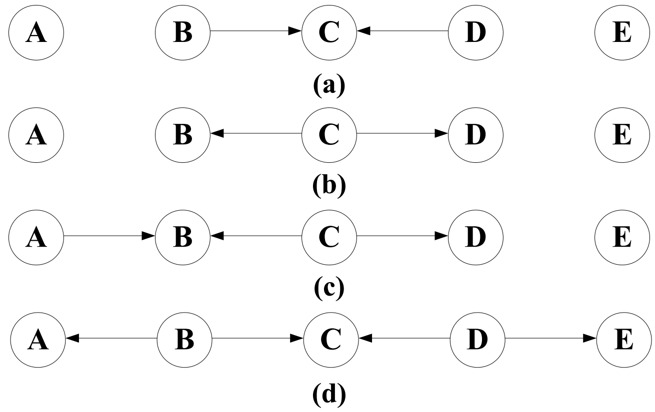


Fig. 2. Illustrations of various NC links combinations. (a) an incoming NC links combination. (b) an outgoing NC links combination. (c) a 4-node NC links combination. (d) a 5-node NC links combination.

Definition 2: $L^- = \{e_1, e_2\}$, the set of two directed links, e_1 and e_2, is an incoming NC links combination, if $e_1\overline{e}_2 \in P_1$, $e_2\overline{e}_1 \in P_2$, $\forall P_1 \in \mathcal{P}_{a_1}, P_2 \in \mathcal{P}_{a_2}, \forall a_1 \neq a_2 \in A$.

Definition 3: $L^+ = \{e_1, e_2\}$, the set of two directed links, e_1 and e_2, is an outgoing NC links combination, if $\overline{e}_1e_2 \in P_1$, $\overline{e}_2e_1 \in P_2$, $\forall P_1 \in \mathcal{P}_{a_1}, P_2 \in \mathcal{P}_{a_2}, \forall a_1 \neq a_2 \in A$.

Briefly speaking, the incoming NC links combination characterizes the native physical-layer coding ability of the analog NC, while the outgoing NC links combination represents the broadcast nature of wireless channel. Let $\mathcal{L}^-$ and $\mathcal{L}^+$ be the sets of the incoming and outgoing NC links combinations, respectively. In addition, let $\mathcal{L}_3 \triangleq \mathcal{L}^- \cup \mathcal{L}^+$, which is the set of the three-node NC links combinations.

Lemma 3: If $L = \{e_1, e_2\} \in \mathcal{L}_3$, there exists $P_a \in \mathcal{P}_a$ and $P_b \in \mathcal{P}_b$, $a \neq b \in A$, such that $e_1\overline{e}_2 \in P_a$ and $e_2\overline{e}_1 \in P_b$.

Proof: According to the rules on how to construct the incoming and outgoing NC links combinations (i.e., Definitions 2 and 3), the proof for Lemma 3 is straightforward, and is thus omitted. ■

Remark 1: Lemma 3 indicates that links in the NC links combination are traversed by at least two different sessions which have reverse directions of each other.

Definition 4: Given $L \in \mathcal{L}_3$, let (a, b) with $a \neq b \in A$ be the session pair that enables the L. Let $S(L)$ be the set of those session pairs of L.

It is worth noting that Lemma 3 guarantees that $S(L)$ is non-empty. The previously constructed $\mathcal{L}_3$, where there are

TABLE I
THE VARIABLES AND NOTATIONS USED IN THE PAPER

G	network graph	V	set of vertices
E	set of directed data links	I	set of interference links
$E(v)$	set of directed data links incident on node v	$I(v)$	set of interference links incident on node v
$E^-(v)$	set of incoming data links incident on node v	$E^+(v)$	set of outgoing data links incident on node v
M	set of channels over each link in the network	N	number of nodes in an N-node NC links combination
A	set of sessions	$\mathcal{P}_a$	set of available paths for session a, $a \in A$
$c_m(e)$	data rate of link e over channel m	$f(e)$	total flow rate over link e
$f_m^{NC}(L)$	flow rate of NC links combinations L over channel m	$f_m^U(e)$	flow rate of unicast traffic on link e over channel m
$F_a(p)$	flow rate of session a over path p	$\overline{e}$	reverse link of directed link e
$\mathcal{L}$	set of all NC links combinations	$\mathcal{L}_N$	set of N-node NC links combinations
$\mathcal{L}^-$	set of incoming NC links combinations	$\mathcal{L}^+$	set of outgoing NC links combinations
$S(L)$	set of session pairs associated with $L, L \in \mathcal{L}$	$\mathcal{L}(e)$	set of NC link associated with link e
$E^C(e)$	set of conflict links for link e	$\mathcal{L}^C(e)$	set of conflict NC links combinations for link e
$t(e)$	transmitting node of link e	$r(e)$	receiving node of link e
$s(a)$	source node of session a	$d(a)$	destination node of session a
$W(v)$	the number of radios in node v	$k(a)$	predetermined flow weight for session a

three nodes involved, represents the basic NC scenario. We notice that the outgoing and incoming NC pair may be further combined into a more powerful NC links combination, as shown in Figs. 2(c) and (d). Thus, we can further explore the NC opportunity involving more than three nodes. Note that the generalized packet-forwarding mechanism to decode and code the analog NC packets for any NC links combinations can be found in [3].

Theorem 3: $L = \bigcup_{i=1}^{N-2} L_i$ is an N-node NC links combination (consisting of N nodes), if and only if there exists $L_i \neq L_j \in \mathcal{L}_3, \forall i \neq j = 1, \cdots, N-2$, which satisfies the following conditions: **Condition 1:** $\bigcap_{i=1}^{N-2} S(L_i) \neq \emptyset$; **Condition 2:** $L_i \cap L_{i+1} \neq \emptyset$, $\forall i = 1, \cdots, N-3$; and **Condition 3:** $h(r(e_1), t(e_2)) > D_I$, where $e_1 \in L_i, e_2 \in L_j$, and $t(e_1) \neq t(e_2)$, $(r(e_1), t(e_2)) \notin L_i \cap L_j$.

Proof: We prove the sufficient condition of Theorem 3 first. If $L = \bigcup_{i=1}^{N-2} L_i$ where $L_i \neq L_j \in \mathcal{L}_3, \forall i \neq j = 1, \cdots, N-2$ is an N-node NC links combination, then all links in L can be active simultaneously. Since the links together perform NC operations, the links carry data for at least one common session, which leads to **Condition 1**. Note that the nodes in an NC links combination should be concatenated one by one. In other words, L_i and L_{i+1} should have one common links. We thus have **Condition 2**. In an NC links combination, a given sender should not interfere with the receiver that does not perform the analog NC operations with the given sender. Thus, the distance between the sender and the receiver that do not perform NC operations together should be larger than the interference range (D_I), which is equivalent to **Condition 3**. Hence, the sufficient condition follows.

Then, we prove the necessary condition of Theorem 3. Suppose that there exists $(N-2)$ distinct three-node NC links combinations, i.e., $L_i \neq L_j \in \mathcal{L}_3, \forall i \neq j = 1, \cdots, N-2$. If **Condition 1**, $\bigcap_{i=1}^{N-2} S(L_i) \neq \emptyset$ holds, then there is at least one common session that is shared by all $L_i, i = 1, \cdots, N-2$. **Condition 2**, $L_i \cap L_{i+1} \neq \emptyset$, $\forall i = 1, \cdots, N-3$, implies that a link in L_i overlaps with a link in L_{i+1}. In other words, L_i and L_{i+1} share two common nodes and the transmission direction of the common nodes in L_i and L_{i+1} is the same. Under **Condition 1**, if there are $(N-2)$ distinct three-node NC links combinations, there are N distinct nodes that are concatenated into a continuous path. According to **Condition 3**, $h(r(e_1), t(e_2)) > D_I$, where $e_1 \in L_i, e_2 \in L_j$, and $t(e_1) \neq t(e_2)$, $(r(e_1), t(e_2)) \notin L_i \cap L_j$, ensures that the sender $(t(e_2))$ does not interfere with the receiver $(r(e_1))$ which does not perform the analog NC operations with the node $t(e_2)$. Therefore, all links included in $L = \bigcup_{i=1}^{N-2} L_i$ can be active simultaneously to perform the analog NC operations, and thus constitute an NC links combination, which completes the proof of necessary condition for Theorem 3. ■

According to Theorem 3, we can use the NC links combinations in $\mathcal{L}_3$ as elements to obtain the set of N-node NC links combinations, denoted by $\mathcal{L}_N$, with $N > 3$. Let $\mathcal{L} = \bigcup_{N \geq 3} \mathcal{L}_N$ be the super set of all types of NC links combinations. Let $\mathcal{L}(e)$ be the subset of $\mathcal{L}$ such that for any $L \in \mathcal{L}(e)$, L contains link e.

Based on the idea of analog NC, every link in the NC links combination can be active at the same time. Thus, theoretically, more links being included in the NC links combination implies the higher network efficiency that the network can achieve. However, the larger the number of involved NC links, the more complicated the system is. This is because the nodes associated with the NC links need to be accurately synchronized together to transmit/receive packets to take advantage of the analog NC. It is not realistic to achieve the accurate synchronization for a large number of nodes. Hence, in this paper, we only consider the NC links combination that contains no more than one element in $\mathcal{L}^-$. In other words, in an NC links combination there is at most a pair of incoming links that are incident in a single node. Therefore, the most complicated and realistic case we consider is the 5-node NC links combination. Fig. 2 shows all the possibly realistic NC scenarios that we consider.

B. Sets of Conflict Links and NC Links Combinations

We consider the popular IEEE 802.11 MAC with Request-To-Send/Clear-To-Send (RTS/CTS) mode as the MAC protocol. Under this protocol, for the unicast communications, all neighbors of a pair of transmitter and receiver, which is associated with link e, have to prohibit from transmitting/receiving to/from the transmitter and receiver. We call the links that are incident on the transmitter $t(e)$ and receiver $r(e)$ as the conflict links of link e. We denote by $E^C(e)$ the set of conflict links of link e. Every link except itself in $E^C(e)$ interferences with link e, which implies that any link in $E^C(e)$ cannot be active at the time when the link e is active. The set of conflict links of e with $e \in E \cup I$ can be given by

$$E^C(e) \supseteq \{E(t(e)) \cup E(r(e)) \cup I(t(e)) \cup I(r(e))\}. \quad (1)$$

Besides the conflict links for the unicast traffic, we also need to take into account the conflict NC links combinations for the NC traffic. For any given link $e \in E$, any L which is associated with one of conflict links of link e can not be active as link e is active. Thus, the set of conflict NC links combinations, denoted by $\mathcal{L}^C(e)$, of link e can be obtained by

$$\mathcal{L}^C(e) = \bigcup_{e' \in E^C(e)} \mathcal{L}(e'). \quad (2)$$

C. Problem Formulation

Let $k(a)$ be the predetermined flow weight for session a, with $a \in A$. The larger the $k(a)$ for session a is, the more flow rate the session a requests. Let λ be the scaling factor by which the flows of each session can be scaled up. We seek to maximize λ where at least $\lambda k(a)$ amount of throughput is guaranteed for any session a, with $a \in A$. We have the following six constraints:

1. Fairness constraint

Let $F_a(P)$ be the amount of flow on path P for session a, where $P \in \mathcal{P}_a, a \in A$. Thus, the throughput $(\theta(a))$ of session a is equal to $\sum_{P\in\mathcal{P}_a} F_a(P)$. To ensure the fairness of resource allocation among different sessions, we have the following constraint:

$$\theta(a) = \sum_{P\in\mathcal{P}_a} F_a(P) = k(a)\lambda, \;\; \forall a \in A, \quad (3)$$

where $k(a), a \in A$ is the predetermined flow weight parameter for session a.

2. Link flow constraint

There are two types of traffic on link e, with $e \in E$: i) the unicast traffic and ii) the NC traffic. For each link e, with $e \in E$, let $f_m^U(e)$ and $f_m^{NC}(L)$ be the flow of unicast traffic on link e and the flow of the NC traffic on link e associated with the NC links combination L with $L \in \mathcal{L}(e)$, respectively, over channel m. Thus, the total amount of flow, denoted by $f(e)$, of link e, with $e \in E$, is the sum of all unicast traffic and NC traffic, i.e.,

$$f(e) = \sum_{m\in M} f_m^U(e) + \sum_{m\in M}\sum_{L\in\mathcal{L}(e)} f_m^{NC}(L). \quad (4)$$

At the same time, the amount of flow on link e should also be equal to the total flow of all sessions which have routes going through link e. Thus, we have

$$f(e) = \sum_{a\in A}\sum_{P\in\mathcal{P}_a : P\ni e} F_a(P). \quad (5)$$

3. Flow conservation constraint

We have the following constraint to maintain flow balance at every wireless node for each session.

$$\sum_{e\in E^+(v)} f(e) - \sum_{e\in E^-(v)} f(e) = \begin{cases} 0, & \forall v \neq s(a), d(a), \forall a \in A \\ \sum_{a\in A, s(a)=v}\sum_{P\in\mathcal{P}_a} F_a(P) \\ \quad - \sum_{a\in A, d(a)=v}\sum_{P\in\mathcal{P}_a} F_a(P), & \forall v = s(a), d(a), \forall a \in A \end{cases} \quad (6)$$

4. Transmission interference constraint

For any given NC links combination $L \in \mathcal{L}$ over channel m, the fraction of time, denoted by $x_m^{NC}(L)$, during which L over channel m is active, can be given by

$$x_m^{NC}(L) = \begin{cases} \frac{f_m^{NC}(L)}{\min_{e\in L}\{c_m(e)\}}, & \min_{e\in L}\{c_m(e)\} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where the term of $\min_{e\in L}\{c_m(e)\}$ is to guarantee that all nodes associated with L can correctly decode the received signals over channel m. The fraction of time, during which link e over channel m for unicast traffic is active, is written as:

$$x_m^U(e) = \begin{cases} \frac{f_m^U(e)}{c_m(e)}, & c_m(e) > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Thus, considering the unicast and the NC traffic together, we derive the transmission interference constraint as follows:

$$\sum_{e'\in E^C(e)} x_m^U(e') + \sum_{L\in\mathcal{L}_m^C(e)} x_m^{NC}(L) \leq 1, \quad \forall m \in M \text{ and } \forall e \in E \cup I. \quad (9)$$

It is worth noting that the constraint given in Eq. (9) is applied not only to the data links (E), but also to the interference links (I). This is because given an interference link e, with $e \in I$, between two nodes u and v, if one of the data links incident on either node u or v is active, then the other node has to be silent for that time slot.

5. Link capacity constraint

Since the link rate is the upper bound of the link flow, we get

$$0 \leq f_m^U(e) \leq c_m(e), \forall m \in M \text{ and } \forall e \in E. \quad (10)$$

Similarly, the NC flow should be less than the minimum link rate among all links associated with this NC links combination. Then, we obtain

$$0 \leq f_m^{NC}(L) \leq \min_{e\in L}\{c_m(e)\}, \forall m \in M \text{ and } \forall L \in \mathcal{L}. \quad (11)$$

6. Node radio constraint

Denote by $W(v)$ the number of radios that a node v has. Since a node v with $W(v)$ radios can work on at most $W(v)$ channels simultaneously, in any interference free schedule, the fraction of time, during which the unicast and NC traffics are active in node v, is constrained by the following:

$$\sum_{m\in M}\left(\sum_{e\in E(v)} x_m^U(e) + \sum_{L\in\bigcup_{e\in E(v)}\mathcal{L}(e)} x_m^{NC}(L)\right) \leq W(v). \quad (12)$$

The objective of the node radio constraint is to guarantee that the radios and channels assignments are feasible for any number of radios for each node and any number of channels. In addition, notice that the node radio constraint is applied to each individual node. Our framework can be readily applied to the case where different nodes are equipped with different numbers of radios.

After obtaining the constraints given in Eqs. (3)-(12), we thus can finalize a *linear program*, denoted by LP1, to find the flows which maximize λ subject to the above constraints. Table II shows the complete LP1. It is worth noting that

TABLE II
LINEAR PROGRAM 1 (LP1) USED TO FIND THE FLOWS THAT MAXIMIZE λ.

Maximize: λ
Subject to:

$$\sum_{P\in\mathcal{P}_a} F_a(P) = k(a)\lambda, \forall a \in A$$

$$f(e) = \sum_{m\in M} f_m^U(e) + \sum_{m\in M}\sum_{L\in\mathcal{L}(e)} f_m^{NC}(L), \forall e \in E$$

$$f(e) = \sum_{a\in A}\sum_{P\in\mathcal{P}_a : P\ni e} F_a(P), \forall e \in E$$

$$x_m^U(e) = \begin{cases} \frac{f_m^U(e)}{c_m(e)}, & c_m(e) > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x_m^{NC}(L) = \begin{cases} \frac{f_m^{NC}(L)}{\min_{e\in L}\{c_m(e)\}}, & \min_{e\in L}\{c_m(e)\} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{e'\in E^C(e)} x_m^U(e') + \sum_{L\in\mathcal{L}_m^C(e)} x_m^{NC}(L) \le 1, \forall m \in M \text{ and } \forall e \in E\cup I$$

$$\sum_{m\in M}\left(\sum_{e\in E(v)} x_m^U(e) + \sum_{L\in\bigcup_{e\in E(v)}\mathcal{L}(e)} x_m^{NC}(L)\right) \le W(v), \forall v \in V$$

$$0 \le f_m^{NC}(L) \le \min_{e\in L}\{c_m(e)\}, \forall m \in M \text{ and } \forall L \in \mathcal{L}$$

$$0 \le f_m^U(e) \le c_m(e), \forall m \in M \text{ and } \forall e \in E$$

$$\sum_{e\in E^+(v)} f(e) - \sum_{e\in E^-(v)} f(e) = \begin{cases} 0, & \forall v \ne s(a), d(a), \forall a \in A \\ \sum_{a\in A, s(a)=v}\sum_{P\in\mathcal{P}_a} F_a(P) \\ \quad -\sum_{a\in A, d(a)=v}\sum_{P\in\mathcal{P}_a} F_a(P), & \forall v = s(a), d(a), \forall a \in A \end{cases}$$

the LP1 represents the general form for the throughput optimization problems of the non-NC scheme, conventional NC scheme, and analog NC scheme. In particular, if we set $\mathcal{L}(e) = \emptyset, \forall e \in E$, then the LP1 becomes the throughput optimization problem for the non-NC scheme. On the other hand, when we redefine $\mathcal{L} = \mathcal{L}^+$, the set of NC links combinations includes the conventional NC links only. That is, only the NC scenario as shown in Fig. 2(b) is considered. After we recalculate $\mathcal{L}(e)$ and $\mathcal{L}^C(e), \forall e \in E$, the new LP1 characterizes the throughput optimization problem of the conventional NC scheme. After solving the LP1s for the non-NC scheme, conventional NC scheme, and analog NC scheme, respectively, we can obtain the throughput gains of the conventional NC scheme and the analog NC scheme over the non-NC scheme for wireless ad hoc networks with any network topologies.

VI. Joint Link Scheduling, Channel Assignment, and Routing Algorithm

A. Routing Strategies

Our developed framework supports both the multi-path routing and single-path routing strategies. In other words, a given session can have either a single path or multiple paths. Intuitively, multi-path coding strategy can provide the most coding opportunities for the wireless NC schemes. However,

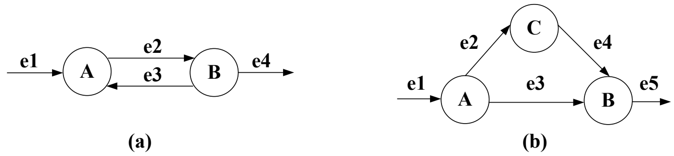


Fig. 3. The examples of direct cycles and redundant paths. (a) The example of direct cycle. (b) The example of redundant paths.

for some routing schemes, multi-path strategy is not applicable due to the high routing maintenance overhead. Thus, we also consider two types of single path routing strategies: i) shortest single-path routing and optimized single-path routing. In particular, the single-path routing can be obtained by Dijkstra's algorithm. On the other hand, the key idea of the optimized single-path routing is to select the path that provides the maximum flow in the multi-path routing for each session. The optimized single-path routing can be obtained by the following two steps. First, we solve the LP1 with multi-path routing. For each session, we select the path that achieves the highest flow, i.e., $p = \arg\max_{p\in\mathcal{P}_a} F_a(p)$. Second, we input these obtained optimized single-path routes for each session to the LP1 and re-solve the LP1.

B. Traffic Routing Optimization

The LP1 described in Table II gives the optimized flows on each link, which maximize λ for each session subject to the fairness constraint given in Eq. (3) in the wireless ad hoc networks. Let the optimal solution to this LP1 be λ^*. There may be multiple combinations of the flows for each link that can obtain λ^*. Although the solution from LP1 obtains the best possible λ, the obtained flows for unicast and NC traffic over each link are not necessarily the optimal in terms of interference, because we have the following two observations. i) The flow may have directed cycles. This may happen since the LP1 does not try to minimize the amount of interference directly. Fig. 3(a) shows an example of the directed cycle problems. Nodes A and B are not source or sink nodes. Assume that all links have the same capacity. The LP1 may give the following results: $f(e1) = f(e4) = 0.1$, $f(e2) = 0.4$, and $f(e3) = 0.3$. In fact, the actual flow between nodes A and B should be only 0.1 unit. By removing the flow on the directed cycle flow, we get the following results: $f(e1) = f(e2) = f(e4) = 0.1$ and $f(e3) = 0$. The interference is reduced while the flow conservation still holds. ii) The flow may go through redundant paths. Note that redundant paths imply more link transmissions, and thus more interference between links. Fig. 3(b) gives an example of this case. Suppose that all links have the same capacity. Nodes A, B, and C are not the source or sink. The solution to LP1 may be as follows: $f(e1) = f(e5) = 0.3$, $f(e2) = f(e4) = 0.1$, and $f(e3) = 0.2$. Clearly, two-hop path $\{e2, e4\}$ are redundant. The one hop path $\{e3\}$ can take all the flows that go through the two-hop path. Thus, after removing the redundant path, we get $f(e1) = f(e3) = f(e5) = 0.3$ and $f(e2) = f(e4) = 0$, which not only satisfies the conservation constraint, but also reduces the total interference.

Therefore, the above observations imply that it will be practical to find the flows for each link among all solutions such that the optimal λ^* is attained and the total interference is minimized. Thus, we introduce the second linear program, denoted by LP2. The objective of the LP2 is to minimize the total interference between any two links. Intuitively, we set the objective of LP2 as follows:

$$\text{minimize} \sum_{m \in M} \left[\sum_{e \in E} x_m^U(e) + \sum_{L \in \mathcal{L}} x_m^{NC}(L) \right] \tag{13}$$

where $x_m^U(e)$ and $x_m^{NC}(L)$ are given by Eqs. (8) and (7), respectively. LP2 has the similar constraints with LP1. The only difference of constraint conditions between LP1 and LP2 is the fairness constraint, which is modified as follows in LP2:

$$\sum_{P \in \mathcal{P}_a} F_a(P) = k(a)\lambda^*, \forall a \in A. \tag{14}$$

C. Link Scheduling and Channel Assignment

Solving LP2, we obtain the optimal flows for each link and NC links combination. We then develop the link scheduling and channel assignment algorithm based on the solution to the LP2, which aims at approximately attaining the optimal λ^*. Due to the node hardware constraints, the radio transceiver has to work over an assigned channel for a period of time before it can switch to another different channel. Let T_s ($T_s \geq 1$) be this period of time in the unit of time slots. Thus, we can update the channel assignment every T_s time slots.

Let $\pi(e)$ be the schedule for link e. The schedule of link e, $\pi(e)$, contains the information about that during which time slot (τ) and on which channel (m), what type of traffic (b) is active. Schedule $\pi(e)$ can be constructed by the following definition:

Definition 5: $\pi(e)$ is a set of triplet (τ, m, b), where τ, $\tau = 1, 2, \cdots$, represents time slots, m, $m \in M$ denotes channels, and b, $b = 0, 1$ indicates the traffic types. In particular, $(\tau, m, 1) \in \pi(e)$ if the link e is active at time slot τ over channel m for NC traffic. On the other hand, $(\tau, m, 0) \in \pi(e)$ if there is the unicast traffic carried on link e at time slot τ over channel m.

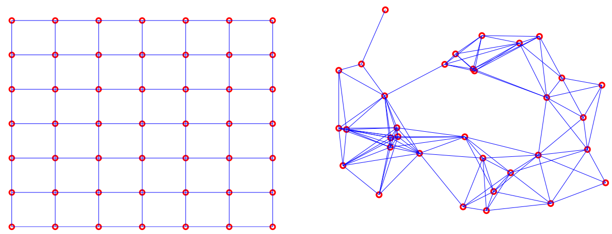
Note that the problem of optimal channel assignments is NP-hard. We therefore design a heuristic greedy algorithm to obtain the schedule $\pi(e), \forall e \in E$, which approximates the optimal solution derived by the LP2. The objective of our proposed algorithm is to provide a feasible solution to approximate the optimal network throughput. The solution to LP2 is the input of the link scheduling and channel assignment algorithm. Once we get the solution from the LP2, we scale all the link flows to make them integers. We scale the flows by an appropriately large value β so that the eliminated fractional portion is negligible. The key idea of our link scheduling and channel assignment algorithm is to assign the flows to channels in as few time slots as possible. We implement this in a greedy way every T_s time slots. Since we do not need to assign the unicast link flows or NC flows to channels that are given by the solution of LP2, we sum up all the unicast flows over different channels on the link e into a single unicast flow, denoted by $g^U(e) = \sum_{m \in M} f_m^U(e)$. Similarly, we aggregate all the NC flows associated with the same NC links L combinations over different channels to a single NC flow, denoted by $g^{NC}(L) = \sum_{m \in M} f_m^{NC}(L)$. Fig. 4 shows the pseudo code of our link scheduling and channel assignment algorithm.

Algorithm 1 The Link Scheduling and Channel Assignment Algorithm

Input: $f_m^U(e), \forall e \in E, m \in M$, $f_m^{NC}, \forall L \in \mathcal{L}, m \in M$
Output: $\pi(e), \forall e \in E$, T_p
$g^U(e) := \sum_{m \in M} f_m^U(e), \forall e \in E$
$g^{NC}(L) := \sum_{m \in M} f_m^{NC}(L), \forall L \in \mathcal{L}$; $\pi(e) := \emptyset, \forall e \in E$
$T_p := 0$; $Z_E := E$; $Z_{\mathcal{L}} := \mathcal{L}$; time_idx = 1
while $(Z_E \neq \emptyset \mid\mid Z_{\mathcal{L}} \neq \emptyset)$
  $B_E := Z_E$; $B_{\mathcal{L}} := Z_{\mathcal{L}}$
  $U(v) := W(v), \forall v \in V$ //reset the number of radios
  $\overline{c}_m(e) := c_m(e), \forall e \in E, m \in M$ //reset the capacity for each link
  while $B_{\mathcal{L}} \neq \emptyset$
    $L := \arg\max_{L' \in B_{\mathcal{L}}} g^{NC}(L')$
    $m := \arg\max_{m' \in M} [\min_{e \in L} \overline{c}_{m'}(e)]$
    if $\min_{e \in L} \overline{c}_m(e) > 0$ && $\min_{e \in L} U(t(e)) > 0$
        && $\min_{e \in L} U(r(e)) > 0$ **then**
      //assign channel m to L
      $\pi(e) := \pi(e) \cup \{\tau, m, 1\}$,
        $\forall e \in L, \tau \in [time\_idx, time\_idx + T_s - 1]$
      $g^{NC}(L) := g^{NC}(L) - T_s \min_{e \in L} \overline{c}_m(e)$
      //set all links associated with L over channel m unavailable
      $\overline{c}_m(e) := 0, \forall e \in E : \mathcal{L}^C(e) \ni L$
      $U(t(e)) := U(t(e)) - 1$, $U(r(e)) := U(r(e)) - 1$,
        $\forall v \in \bigcup_{e \in L} \{t(e) \cup r(e)\}$ //available radios reduce by 1
      if $(g^{NC}(L) < 0)$ **then** $Z_{\mathcal{L}} := Z_{\mathcal{L}} \backslash L$ **end if**
    end if
    $B_{\mathcal{L}} := B_{\mathcal{L}} \backslash L$
  end while
  while $B_E \neq \emptyset$
    $e := \arg\max_{e' \in B_E} g^U(e')$
    $m := \arg\max_{m' \in M} \overline{c}_{m'}(e)$
    if $\overline{c}_m(e) > 0$ && $U(t(e)) > 0$ && $U(r(e)) > 0$ **then**
      //assign channel m to link e
      $\pi(e) := \pi(e) \cup \{\tau, m, 0\}$,
        $\forall \tau \in [time\_idx, time\_idx + T_s - 1]$
      $g^U(e) := g^U(e) - T_s \overline{c}_m(e)$
      $\overline{c}_m(e') := 0, \forall e' \in E^C(e)$
      $U(t(e)) := U(t(e)) - 1$; $U(r(e)) := U(r(e)) - 1$
      if $(g^U(e) < 0)$ **then** $Z_E := Z_E \backslash e$ **end if**
    end if
    $B_E := B_E \backslash e$
  end while
  $T_p := T_p + T_s$
end while

Fig. 4. The pseudo code of the link scheduling and channel assignment algorithm for wireless NC schemes.

In every T_s time slots, we first schedule for the NC flows, and then for the unicast flows. When we schedule for the NC flows, we sort the NC links combinations in a decreasing order of the unassigned flows (g^{NC}). We assign the first NC links combination to the channel that can provide maximum flow rate. Since the flow rate of an NC links combination L relies on all the links associated with L, the channel m that satisfies $\max_{m \in M}\{min_{e \in L} c_m(e)\}$ is assigned to the first NC links combinations. If channel m exists and every node associated with L has at least one radio available, then we assign the channel m to each link contained in the NC link combination L, i.e., $\pi(e) = \pi(e) \cup (\tau, m, 1)$. After assigning the channel to NC link combinations L, we need to set the capacity for all the links that are associated with L to zero in order to prevent the links from being assigned to other links in the same time slot. Furthermore, the radio of each node incident on the involved links should be decreased by 1. If there is no channel that can be assigned to the current NC links combination, we move on to the next NC links combination with second highest flow. We repeat the above procedure until we reach the last NC links combination. Then, we start the process of channel



(a) (b)

Fig. 5. The network topologies. (a) The grid network with 49 nodes. (b) The random graph network with 34 nodes.

assignments and link schedule for the unicast traffic, which is very similar to that for the NC traffic. After we go through all the NC links combinations for NC traffic and all the links for the unicast traffic. We check if there are still unassigned flows, $\sum_{L\in\mathcal{L}} g^{NC}(L) + \sum_{e\in E} g^{U}(e) > 0$. If so, we move on to the next T_s time slots and repeat the scheduling process for both NC traffic and unicast traffic until all unassigned flows become zero.

Since the solution to LP2 contains the routing information for each session, the final outputs of the link scheduling and channel assignment algorithm, as shown in Fig. 4, are the joint link scheduling, channel assignment, and routing results, which approximately obtain the optimal throughput for each session over the wireless ad hoc networks. Given the link resources assignments derived from LP1 or LP2, the link scheduling and channel assignment algorithm for wireless NC schemes can eventually find the feasible solution for the link resources assignments since the flow of each link is finite. Therefore, our proposed link scheduling and channel assignment algorithm ensures convergence and stability.

VII. Performance Evaluation

A. Evaluation Setups

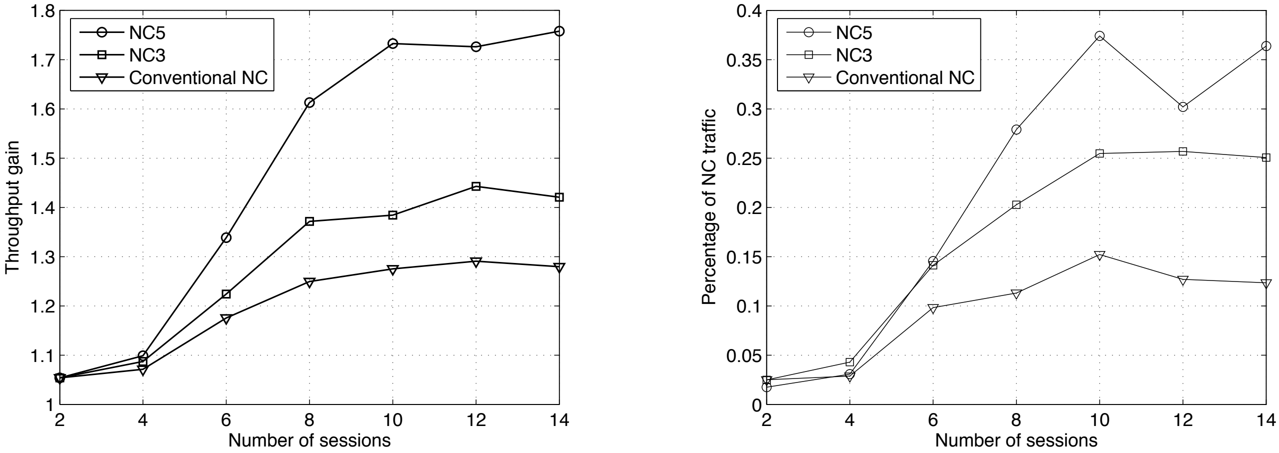
In this section, we evaluate the performance of various wireless NC schemes over the non-NC scheme in the multi-channel multi-radio wireless ad hoc networks, where there are multiple unicast sessions. We focus on three different network topologies including the linear network with 30 nodes, the 7×7 grid topology network with 49 nodes, and the random graph network with 34 nodes. More specifically, in the linear network, the 30 nodes are deployed along a line and the distance between two consecutive nodes is 1 unit. Fig. 5(a) shows the network topology of the 7×7 grid network in which there are 49 nodes. The distance between two adjacent grid points is 1 unit. Every wireless node is placed at a grid point. Fig. 5(b) shows the network topology of the random graph network where 34 nodes are arbitrarily deployed in the square region where each side is 3.3 units long.

For each network topology, we study the non-NC scheme, the conventional NC scheme, the analog NC scheme with only three-node NC links combinations, and the fully functional analog NC scheme. Specifically, the analog NC scheme with only three-node NC links combinations, referred to as NC3, is the NC scheme that takes advantage of the NC links combinations in which at most three nodes are involved. The possible NC links combinations of the NC3 scheme are those shown in Figs. 2(a) and (b). On the other hand, the fully functional analog NC scheme, referred to as NC5, is the NC scheme that utilizes all possible and realistic NC links combinations. The NC links combinations that are enabled by the NC5 scheme include those shown in Figs. 2(a)-(d).

We set the communications range of each node and the interference range to be 1 unit and 1.4 units, respectively. We assume a free-space wireless channel model in which the link data rate depends only on the distance between two nodes of the link. We vary the number of sessions, the number of radios, and the number of channels, respectively. For each session, two distinct nodes are randomly chosen as source and destination. For the sake of simplicity, let each channel have the same capacity and each session have the same flow weight, i.e., $k(a) = k(b) = 1, \forall a \neq b \in A$. Suppose that each node has the same number of radios, i.e., $W(u) = W(v), \forall u \neq v \in V$. For each network topology, we solve the linear programs, LP1 and LP2 using AMPL [26] with the LP_SOLVE solver [27] to obtain the theoretically optimized network throughputs and the corresponding flows for the non-NC scheme, the conventional NC scheme, the NC3 scheme, and NC5 scheme, respectively. Then, we calculate the network throughput gains of the three NC schemes over the non-NC scheme. Regarding the routing strategies, we will consider the shortest single-path routing, multiple path routing, and optimized single-path routing, as discussed in Section VI-A. We also compare the theoretical network throughput, i.e., the solution of LP1, with the experimental network throughput which is obtained by our designed joint link scheduling, channel assignment, and routing algorithm. The value of T_s in the algorithm is set to 1 in the following evaluations.

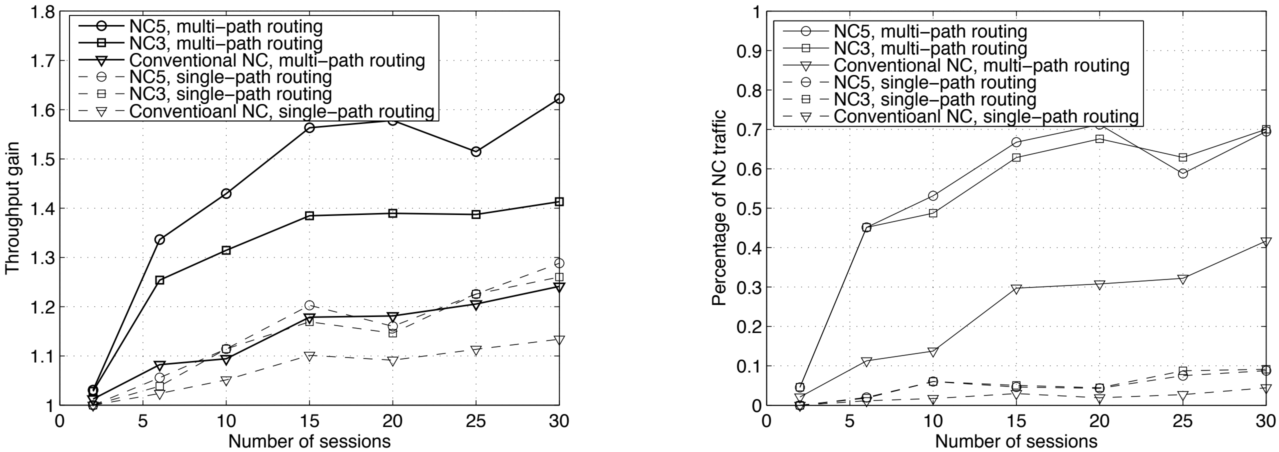
B. The Throughput Gain of Wireless NC Schemes

We first study the network throughput gain of wireless NC schemes over the non-NC scheme in different network topologies. We focus on the case where each node has one radio and there is only one channel in the networks. In the linear network, we consider only the single-path routing because there is only one route for any given session. Figs. 6(a) and (b) show the network throughput gains of wireless NC schemes over the non-NC scheme and the percentages of NC traffic of different wireless NC schemes, respectively, in the linear network. As shown in Fig. 6(a), the network throughput gain of the NC 5 scheme significantly outperforms the other two NC schemes. The NC3 scheme achieves better network throughput gain than the conventional NC scheme in the linear network topology. As the number of sessions increases, the network throughput gain gets larger for all of the wireless NC schemes. This can be explained by Fig. 6(b) which shows the percentage of the NC traffic, which is the ratio of the total amount of NC flows to the total amount of all flows, for the three wireless NC schemes. As shown in Fig. 6(b), the percentage of NC traffic gets larger when the number of sessions increases. Note that the NC traffic can utilize the network bandwidth more efficiently than the unicast traffic. The more the NC opportunities exist, the higher the throughput the network can support.



(a) (b)

Fig. 6. The performance of wireless NC in the linear network with 30 nodes. (a) The network throughput gain. (b) The percentage of NC traffic.

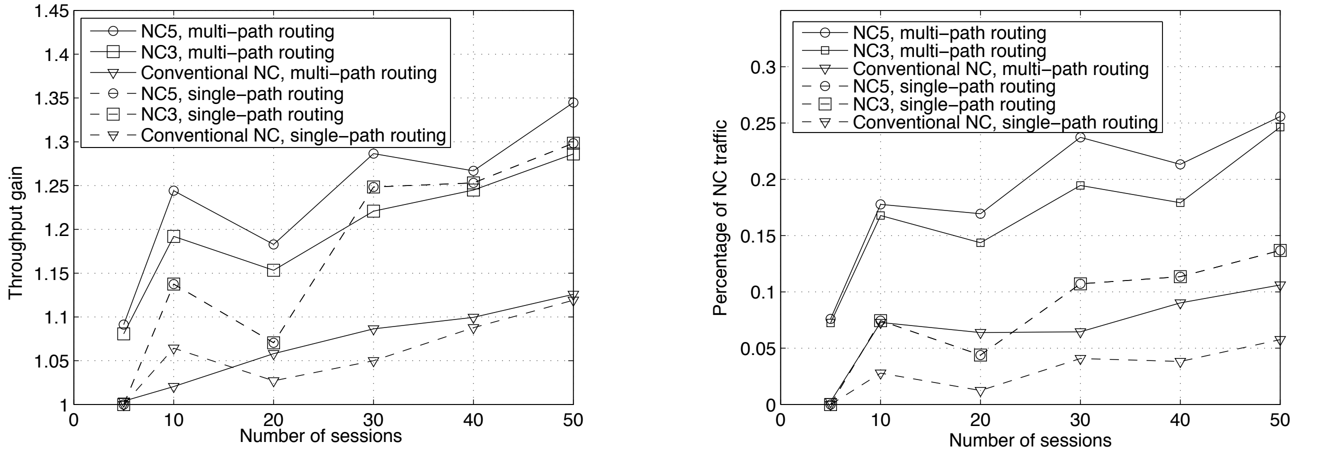


(a) (b)

Fig. 7. The performance of wireless NC in the grid network with 49 nodes. (a) The network throughput gain. (b) The percentage of NC traffic.

Figure 7 shows the performance of the wireless NC in the grid network. From Fig. 7(a), for any given wireless NC scheme, the multi-path routing can significantly increase the network throughput gain as compared to the single-path routing. It is expected because more routes per session mean that there are more NC opportunities along the routes. It is interesting to note that when the multi-path routing is adopted, the performance gap between the NC5 and the NC3 is much larger than that when the single-path routing is adopted. The reason behind this is that when the multi-path routing is adopted, the NC5 can find much more NC links combinations than the NC3, especially in the grid network. When the single-path routing is employed, the throughput gain of the NC5 is close to that of the NC3, because there are few NC links combinations that belong to $\mathcal{L}_4$ and $\mathcal{L}_5$. Correspondingly, the percentage of NC traffic under the single-path routing strategy is much lower than that under the multi-path routing strategy, as shown in Fig. 7(b). The higher the percentage of NC traffic, the larger the throughput gain. Overall, when the multi-path routing is used, the NC5 and NC3 can increase the network throughput by about 60% and 39%, respectively, as compared to the conventional NC scheme in the grid network.

In contrast, the improvement brought by the multi-path routing is less in the random graph network as compared to the grid network, as shown in Fig. 8(a). Fig. 8(a) also shows that the throughput gain of NC5 is exactly the same as the NC3, which implies that no NC links combinations in the NC5 belong to neither $\mathcal{L}_4$ nor $\mathcal{L}_5$, and thus the NC5 is degraded to the NC3. In Fig. 8(a), the network throughput gain of the wireless scheme fluctuates as the number of sessions varies. This is because the number of NC links combinations is not persistent when the number of sessions changes. As shown in Fig. 8(b), in the random graph network, the percentage of NC traffic is lower than that in the grid network given the same number of sessions and wireless NC scheme. In the random network, the NC5 and NC3 can increase the network throughput by 24% and 18%, respectively, on average as compared to the conventional NC scheme.



(a) (b)

Fig. 8. The performance of wireless NC in the random graph network with 34 nodes. (a) The network throughput gain. (b) The percentage of NC traffic.

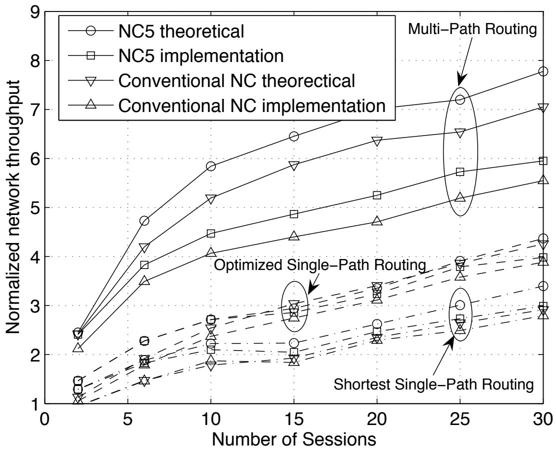


Fig. 9. The impact of routing strategies on the network throughput in the single-channel single-radio grid network. The solid lines, dotted lines and dotted-dashed lines with different symbols represent the different schemes under the multi-path routing, optimized single-path routing, and shortest single-path routing, respectively.

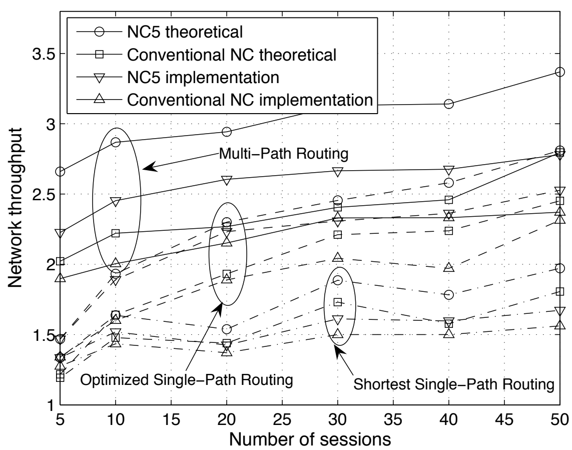


Fig. 10. The impact of routing strategies on the network throughput in the single-channel single-radio random graph network. The solid lines, dotted lines and dotted-dashed lines with different symbols represent the different schemes under the multi-path routing, optimized single-path routing, and shortest single-path routing, respectively.

It is worth noting that even though we focus on the single-channel single-radio case, our analytical models can be also applied to the multi-channel multi-radio case. The numerical results, which are omitted in this paper due to lack of space, show that the trends of the throughput gain for wireless NC schemes in multi-channel multi-radio case are similar to those in single-channel and single-radio case. Furthermore, we will analyze the performance impact of multi-radio and multi-channel in the following section.

C. Performance Impact of Routing Strategies

Figures 9 and 10 show that the different performance impacts when the different routing strategies are used in the grid network and the random graph network, respectively. In the grid network, the multi-path routing can greatly benefit the network throughput for the wireless NC schemes. The optimized single-path routing strategy achieves better network throughput than the shortest single-path routing strategy. The improvement of the multi-path routing over the optimized single-path routing in the grid network is more noticeable than that in the random graph network.

As shown in Figs. 9 and 10, we also compare the theoretical network throughput obtained by solving LP1 with the experimental network throughput achieved by implementing our joint link scheduling, channel assignment, and routing algorithm. There is a gap between the theoretical network throughput and the experimental one. It is more difficult for our heuristic algorithm to obtain the optimal channel assignments when the percentage of NC traffic is higher. The efficiency of our algorithm is about 0.77 on average when NC5 and multi-path routing strategy are adopted. When the single-path routing is used, the efficiency of our algorithm can reach up to 0.9. On the other hand, given the same routing strategies, the efficiency of our algorithm is higher for the conventional NC scheme than the NC5 scheme.

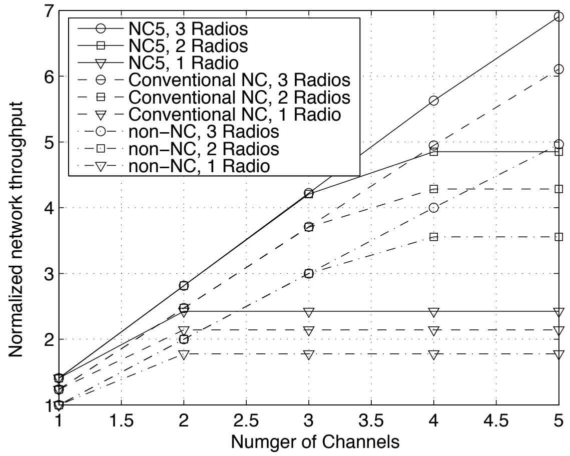


Fig. 11. The performance impact of multi-channel and multi-radio in the grid network where the number of unicast sessions is 30.

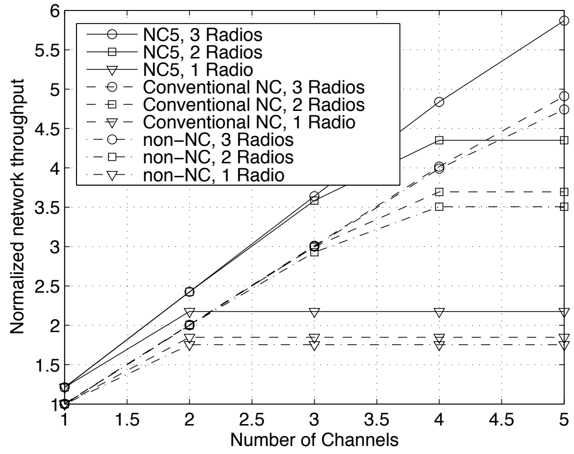


Fig. 12. The performance impact of multi-channel and multi-radio in the random graph network where there are 20 unicast sessions.

D. Performance Impact of Multi-Radio and Multi-Channel

Then, we study how the multi-radio and multi-channel affect the performance of the wireless NC schemes when multi-path routing is adopted. The performance evaluations of multi-radio and multi-channel under single-path routing strategies is omitted here since they are similar to those under the multi-path routing. Fig. 11 shows the performance impact of multi-radio and multi-channel in the grid network. The impacts of multi-radio and multi-channel on the wireless NC schemes and non-NC scheme are similar. The network throughput increases linearly with the number of channels when the number of radios is larger than the number of channels. This implies that if there are Y, with $2 \leq Y \leq \|M\|$, channels in the network, at least $(Y-1)$ radios per node are needed in order to make the best use of these Y channels. Given the same number of radios and number of channels, the NC5 achieves the highest network throughput while the non-NC scheme achieves the lowest. It is worth noting that, given the same number of radios of each node, the slopes of the network throughput plots for different schemes are different. When the number of channels gets larger, the network throughput of NC5 increases faster than that of conventional NC scheme, while the network throughput of conventional NC scheme increases faster than that of non-NC scheme. It suggests that the wireless NC schemes are more sensitive to the number of channels, as compared to the non-NC scheme. The more the channels we have, the more the benefits the wireless NC schemes can get. From Fig. 12, we can observe the similar facts in the random graph network as those in the grid network.

VIII. Conclusions

We modeled the throughput gains of both the conventional NC and analog NC schemes over the traditional non-NC scheme in multi-channel multi-radio wireless ad hoc networks. We first showed that the throughput gains of the conventional NC and analog NC are $(2n)/(2n-1)$ and $n/(n-1)$, respectively, for the n-way relay networks where $n \geq 2$. Then, we formulated a general linear programming framework for solving the throughput optimization problems for the traditional non-NC scheme and the two types of wireless NC schemes. Under this framework, we quantitatively analyzed the network throughput gains of two types of wireless NC in different network topologies for the wireless ad hoc networks with multiple unicast sessions. We also developed a joint link scheduling, channel assignment, and routing algorithm for the wireless NC schemes to closely approximate the optimal solutions. The extensive simulations showed that the network throughput achieved by the analog NC scheme increases 24% and 18% on average as compared to the conventional NC scheme and non-NC scheme, respectively, in the random graph network, while 60% and 39% in the grid network.

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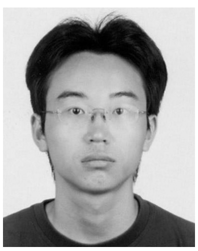

Hang Su (S’04) received B.S. and M.S. degrees in electrical engineering from Zhejiang University, Hangzhou, China, in 2002 and 2005, respectively. He is currently working toward a Ph.D. degree at Department of Electrical and Computer Engineering, Texas A&M University, College Station. He worked as a software engineer with Nokia Research Center, Hangzhou, China, in 2005.

His research interests lie in cognitive radio networks, wireless mesh networks, vehicular ad hoc networks, and wireless sensor networks with emphasis on design and analysis of MAC and routing protocols.

Xi Zhang (S’89-SM’98) received the B.S. and M.S. degrees from Xidian University, Xi’an, China, the M.S. degree from Lehigh University, Bethlehem, PA, all in electrical engineering and computer science, and the Ph.D. degree in electrical engineering and computer science (Electrical Engineering-Systems) from The University of Michigan, Ann Arbor.

He is currently an Associate Professor and the Founding Director of the Networking and Information Systems Laboratory, Department of Electrical and Computer Engineering, Texas A&M University, College Station. He was an Assistant Professor and the Founding Director of the Division of Computer Systems Engineering, Department of Electrical Engineering and Computer Science, Beijing Information Technology Engineering Institute, China, from 1984 to 1989. He was a Research Fellow with the School of Electrical Engineering, University of Technology, Sydney, Australia, and the Department of Electrical and Computer Engineering, James Cook University, Australia, under a Fellowship from the Chinese National Commission of Education. He worked as a Summer Intern with the Networks and Distributed Systems Research Department, AT&T Bell Laboratories, Murray Hills, NJ, and with AT&T Laboratories Research, Florham Park, NJ, in 1997. He has published more than 140 research papers in the areas of wireless networks and communications systems, mobile computing, network protocol design and modeling, statistical communications, random signal processing, information theory, and control theory and systems.

Prof. Zhang received the U.S. National Science Foundation CAREER Award in 2004 for his research in the areas of mobile wireless and multicast networking and systems. He received the Best Paper Award in the IEEE Globecom 2007. He also received the TEES Select Young Faculty Award for Excellence in Research Performance from the Dwight Look College of Engineering at Texas A&M University, College Station, in 2006. He is currently serving as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS for the special issue on “wireless video transmissions”, an Associate Editor for the IEEE COMMUNICATIONS LETTERS, an Editor for the JOHN WILEY’S JOURNAL ON WIRELESS COMMUNICATIONS AND MOBILE COMPUTING, an Editor for the JOURNAL OF COMPUTER SYSTEMS, NETWORKING, AND COMMUNICATIONS, and an Associate Editor for the JOHN WILEY’S JOURNAL ON SECURITY AND COMMUNICATIONS NETWORKS, and is also serving as the Guest Editor for the IEEE WIRELESS COMMUNICATIONS MAGAZINE for the special issue on “next generation of CDMA versus OFDMA for 4G wireless applications” and a Guest Editor for JOHN WILEY’S JOURNAL ON WIRELESS COMMUNICATIONS AND MOBILE COMPUTING for the special issue on “next generation wireless communications and mobile computing”. He has frequently served as the Panelist on the U.S. National Science Foundation Research-Proposal Review Panels. He is serving or has served as the TPC Vice-Chair for IEEE INFOCOM 2010, TPC Chair for IEEE Globecom 2011, Co-Chair for IEEE INFOCOM 2009 - Mini-Conference, Co-Chair for IEEE Globecom 2008 - Wireless Communications Symposium, Co-Chair for the IEEE ICC 2008 - Information and Network Security Symposium, Symposium Chair for IEEE/ACM International Cross-Layer Optimized Wireless Networks Symposium 2006, 2007, and 2008, respectively, the TPC Chair for IEEE/ACM IWCMC 2006, 2007, and 2008, respectively, the Poster Chair for IEEE INFOCOM 2008, the Student Travel Grants Co-Chair for IEEE INFOCOM 2007, the Panel Co-Chair for IEEE ICCCN 2007, the Poster Chair for IEEE/ACM MSWiM 2007 and IEEE QShine 2006, Executive Committee Co-Chair for QShine, the Publicity Chair for IEEE/ACM QShine 2007 and IEEE WirelessCom 2005, and the Panelist on the Cross-Layer Optimized Wireless Networks and Multimedia Communications at IEEE ICCCN 2007 and WiFi-Hotspots/WLAN and QoS Panel at IEEE QShine 2004. He has served as the TPC members for more than 70 IEEE/ACM conferences, including IEEE INFOCOM, IEEE Globecom, IEEE ICC, IEEE WCNC, IEEE VTC, IEEE/ACM QShine, IEEE WoWMoM, IEEE ICCCN, etc.

Prof. Zhang is a Senior Member of the IEEE and a Member of the Association for Computing Machinery (ACM).