Quality-of-Service Driven Power and Rate Adaptation for Multichannel Communications over Wireless Links

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Abstract— We propose a quality-of-service (QoS) driven power and rate adaptation scheme for multichannel communications systems over wireless links. In particular, we use *multichannel communications* to model the conceptual architectures for either *diversity* or *multiplexing* systems, which play a fundamental role in physical-layer evolutions of mobile wireless networks. By integrating information theory with the concept of *effective capacity*, our proposed scheme aims at maximizing the multichannel-systems throughput subject to a given delay-QoS constraint. Under the framework of convex optimization, we develop the optimal adaptation algorithms. Our analyses show that when the QoS constraint becomes loose, the optimal power-control policy converges to the well-known water-filling scheme, where the Shannon (or ergodic) capacity can be achieved. On the other hand, when the QoS constraint gets stringent, the optimal policy converges to the scheme operating at a constant-rate (i.e., the zero-outage capacity), which, by using only a limited number of subchannels, approaches the Shannon capacity. This observation implies that the optimal effective capacity function decreases from the ergodic capacity to the zero-outage capacity as the QoS constraint becomes more stringent. Furthermore, unlike the single-channel communications, which have to trade off the throughput for QoS provisioning, the multichannel communications can achieve both high throughput and stringent QoS at the same time.

Index Terms— Mobile wireless networks, quality-of-service (QoS), effective capacity, information theory, convex optimization, diversity, multiplexing, multicarrier, multiple input multiple output (MIMO), cross-layer design and optimization.

I. INTRODUCTION

THE INCREASING demand for wireless network services such as wireless Internet accessing, mobile computing, and cellular telephoning motivates an unprecedented revolution in wireless broadband communications [1]. This also imposes great challenges in designing the wireless networks since the time-varying fading channel has a significant impact on supporting diverse *quality-of-service* (QoS) requirements for heterogeneous mobile users. In response to these challenges, a great deal of research has been devoted to the techniques that can enhance the spectral-efficiency of the wireless communications systems [2]. The framework used

to evaluate these techniques is mainly based on information theory, using the concept of Shannon capacity [3]. While this framework is suitable for an analysis of maximizing the system throughput, it may overlook the mobile users' QoS requirements, since Shannon theory does not place any restrictions on the complexity and delay [4]. Consequently, to provide QoS guarantees for diverse mobile users, it is necessary to take the QoS metrics into account when applying the prevalent information theory to mobile wireless network designs.

In a companion paper [5], we proposed a QoS-driven power and rate adaptation scheme for *single-input-single-output* (SISO) systems over flat-fading channels. The proposed scheme aims at maximizing the system throughput subject to a given delay-QoS constraint. Specifically, by integrating information theory with the concept of *effective capacity* [6]–[9], we convert the original problem to the one with the target at maximizing the effective capacity, by which the delay-QoS constraint is characterized by the QoS exponent θ. Using the effective capacity, a smaller θ corresponds to a looser QoS guarantee, while a larger θ implies a more stringent QoS requirement. In the limiting case, when $\theta \to 0$, the system can tolerate an arbitrarily long delay, which is the scenario studied in information theory. On the other hand, when $\theta \to \infty$, the system cannot tolerate any delay, which corresponds to an extremely stringent delay-bound. In [5], we derived the optimal power-control policy which is adaptive to the QoS exponent θ. The results obtained in [5] show that when the QoS constraint becomes loose ($\theta \to 0$), the optimal power-control policy converges to the well-known *water-filling* scheme [3], [4], where the Shannon (or ergodic) capacity can be achieved. In contrast, when the QoS constraint gets stringent ($\theta \to \infty$), the optimal policy converges to the *total channel inversion* scheme [4], [10] under which the system operates at a constant rate. Our analyses also demonstrate that there exists a *fundamental tradeoff* between the throughput and the QoS provisioning. For instance, over a flat-fading Rayleigh channel, the SISO system cannot support stringent delay QoS ($\theta \to \infty$), no matter how much power and spectral bandwidth resources are assigned for the transmission [5].

As the sequel of [5], in this paper we focus on QoS provisioning for *multichannel* communications over wireless networks. The motivation of this paper is mainly based on recent advances in physical layer developments, where a large number of promising schemes can be considered as
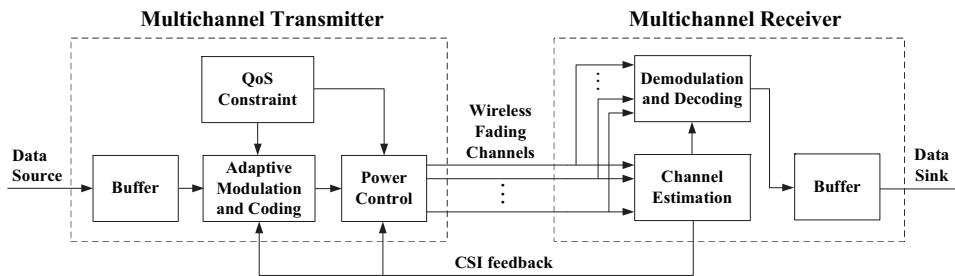
Fig. 1. The system model.

utilizing multichannels to enhance the system performance. The multichannel communications architecture discussed in this paper is in a broad sense, which models either multiple diversity branches for *diversity combining* or a number of parallel subchannels for *multiplexing* [11]. Examples of diversity-based systems include code-division-multiple-access (CDMA) RAKE receivers which take the advantage of frequency diversity [12] and multiple input multiple output (MIMO) diversity systems which utilize spatial diversity [13]. On the other hand, examples of multiplexing-based systems include multicarrier systems employing orthogonal-frequency-division-multiplexing (OFDM) mechanism [14] and MIMO multiplexing systems [15].

In this paper, we show that multichannel transmission can significantly improve the delay-QoS provisioning for wireless communications. In particular, when the QoS constraint is loose ($\theta \to 0$), the optimal power-control policy also converges to the water-filling scheme that achieves the Shannon (ergodic) capacity. By contrast, when the QoS constraint is stringent ($\theta \to \infty$), the optimal policy converges to a scheme which operates at a constant rate (the zero-outage capacity), where an important observation is that, by using only a limited number of subchannels, the above resulting constant rate (the zero-outage capacity) is close to the Shannon capacity. This implies that the optimal effective capacity function connects the ergodic capacity and the zero-outage capacity as the QoS constraint varies. Furthermore, unlike the single channel transmission scheme which has to tradeoff the throughput for QoS provisioning [5], the multichannel transmission scheme can achieve both high throughput and stringent QoS at the same time. For instance, our simulation results show that over the Rayleigh fading channel, a multicarrier system with 64 independent subchannels can achieve more than 99% of the Shannon capacity, while still guaranteeing a constant rate transmission, as if the transmission was over a wireline network. The above observation demonstrates from another perspective that the zero-outage capacity approaches the ergodic capacity as the number of parallel subchannels increases [16].

The rest of the paper is organized as follows. Section II describes our general multichannel wireless system model. Sections III derives the optimal power adaptation for diversity-based systems. Section IV formulates the optimization problem for the multiplexing-based systems, and Section V develops the corresponding optimal solutions. Section VI further investigates the special cases of our proposed optimal power-control scheme. Section VII conducts simulations to evaluate

the performance of our proposed scheme. The paper concludes with Section VIII.

II. THE SYSTEM MODEL

The general multichannel system model over a wireless link is shown in Fig. 1. We concentrate on a discrete-time system with a point-to-point link between the transmitter and the receiver in mobile wireless networks. Let us denote the system total spectral-bandwidth by B and the mean transmit power by $\overline{P}$, respectively. The power spectral density (PSD) of the complex additive white Gaussian noise (AWGN) is denoted by $N_0/2$ per dimension. We assume that AWGN is independent identically distributed (i.i.d.) on each subchannel. Unless otherwise stated, throughout this paper we use "subchannels" to represent either diversity or multiplexing branches in our multichannel system model.

First, the upper-layer packets are divided into *frames* at the datalink layer, which forms the "data source" as shown in Fig. 1. The frame duration is denoted by T_f, which is assumed to be less than the fading coherence time, but sufficiently long so that the information-theoretic assumption of infinite code-block length is meaningful [17]. The frames are stored at the transmit buffer and split into bit streams at the physical layer. Then, based on the QoS constraint and channel-state information (CSI) fed back from the receiver, adaptive modulation and coding (AMC), as well as power control are applied, respectively, at the transmitter. Depending on the specific transmission mechanism, the bit streams are transmitted through N subchannels to the receiver. The reverse operations are executed at the receiver side. Finally, the frames are recovered at the "data sink" for further processing. We also make the following two assumptions:

A1: The discrete-time channel is assumed to be block fading. The path gains are invariant within a frame's time duration T_f, but vary independently from one frame to another. Making such an assumption is mainly based on the following reasons. First, the effective capacity expression in a block fading channel [5, eq. (4)] only depends on marginal statistics of a service process, which is much simpler than the general expression given by [5, eq. (3)], where higher order statistics of a service process are required. Second, more importantly, through the study of [5] we observe that there exists a simple and efficient approach to convert the power adaptation policy obtained in block-fading channels to that over correlated-fading channels, making the investigation of power adaptation in block-fading channels more applicable.

A2: We further assume that given the transmit power, the specific multichannel transmission scheme, and an instantaneous channel gain, the AMC scheme can achieve the Shannon capacity. Based on the above two assumptions, for each given power-control policy, the resulting effective capacity reaches its maximum for all modulation/coding schemes and all channel realizations.

The wireless channel may be modeled as being frequency-selective (e.g., in the context of multicarrier OFDM system or CDMA RAKE receiver-based system), but each subchannel experiences the flat-fading. Denote the nth subchannel envelope process by $\{\alpha_n[i], n \in \mathcal{N}_0, i = 1, 2, ...\}$, where $\mathcal{N}_0 = \{1, 2, ..., N\}$ represents the index-set of the subchannels and $i \in \{1, 2, ...\}$ is time-index of the frame. Let $\lambda_n[i] \triangleq \alpha_n^2[i]$ denote the path-gain process. Then, the joint probability density function (pdf) of the path-gains $\boldsymbol{\lambda}[i] \triangleq (\lambda_1[i], \lambda_2[i], ..., \lambda_N[i])$ can be expressed as $p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) = p_{\Lambda_1, \Lambda_2, ..., \Lambda_N}(\lambda_1, \lambda_2, ..., \lambda_N)$. Although the scheme discussed in this paper can be applied to any channel distribution models, we just assume the Rayleigh channel model for convenience.

Throughout this paper, we also assume that the CSI is perfectly estimated at the receiver and reliably fed back to the transmitter without delay. Moreover, the datalink-layer buffer size is assumed to be infinite. In the following discussions, since the block-fading channel process is stationary and ergodic, its instantaneous-time marginal statistics is independent of the time-index i, and thus we may omit the time-index i for simplicity.

III. DIVERSITY-BASED SYSTEMS

We first focus on the QoS-driven power adaptation for *diversity-based systems*. The key idea of diversity-based systems is to transmit multiple copies of the same data through different subchannels. At the receiver side, the multiple copies are combined together such that the transmission reliability can be enhanced.

For diversity combining systems, the system performance is determined by the combined signal-to-noise ratio (SNR) at the receiver. If we assume that no power control is used, then the SNR at the receiver combiner can be denoted by $\gamma[i]$, which depends not only on the instantaneous channel condition, but also on the specific diversity scheme used. For instance, in a maximal-ratio combining (MRC) system with N diversity branches [18], the SNR at the output of the combiner can be expressed as $\gamma[i] = \sum_{n=1}^{N} \overline{P} \lambda_n[i] / (N_0 B)$, with its mean $\overline{\gamma} = \overline{P} \sum_{n=1}^{N} \mathbb{E}\{\lambda_n[i]\} / (N_0 B)$, where $\mathbb{E}\{\cdot\}$ denotes the expectation. On the other hand, in a selection combining (SC) system [18], the SNR at the output of the combiner is given by $\gamma[i] = \overline{P} \lambda_{\max}[i] / (N_0 B)$, with the mean $\overline{\gamma} = \overline{P} \mathbb{E}\{\lambda_{\max}[i]\} / (N_0 B)$, where $\lambda_{\max}[i] = \max\{\lambda_n[i], n \in \mathcal{N}_0\}$. Let the pdf of $\gamma[i]$ be denoted by $p_\Gamma(\gamma)$. It is well known that there have been a great deal of research efforts in deriving the analytical expressions of the pdf $p_\Gamma(\gamma)$ under different channel conditions and different diversity combining techniques.

Using diversity combining, the original vector channel (i.e., multichannel) transmission problem is converted into a scalar channel (i.e., single channel) transmission problem. Therefore, the scheme discussed in [5] can be directly applied to obtain

the optimal power-adaptation policy, where the only difference is at the pdf $p_\Gamma(\gamma)$ of the SNR at the combiner output. Specifically, the optimal policy, denoted by $\mu_{\text{opt}}(\theta, \gamma)$, can be expressed as [5, eq. (8)]

$$\mu_{\text{opt}}(\theta, \gamma) = \begin{cases} \dfrac{1}{\gamma_0^{\frac{1}{\beta+1}} \gamma^{\frac{\beta}{\beta+1}}} - \dfrac{1}{\gamma}, & \gamma \geq \gamma_0 \\ 0, & \gamma < \gamma_0 \end{cases} \quad (1)$$

where we define

$$\beta \triangleq \frac{\theta T_f B}{\log 2} \quad (2)$$

as the normalized QoS exponent and γ_0 as the cutoff SNR threshold, which can be numerically obtained by meeting the following mean power constraint:

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0^{\frac{1}{\beta+1}} \gamma^{\frac{\beta}{\beta+1}}} - \frac{1}{\gamma} \right) p_\Gamma(\gamma) d\gamma = 1. \quad (3)$$

Note that the threshold $\gamma_0 = \gamma_0(\theta, p_\Gamma(\gamma))$ depends not only on the fading distribution $p_\Gamma(\gamma)$, but also on the QoS exponent θ. Similar to the conclusion obtained in [5], we can observe that when the QoS constraint is loose ($\theta \to 0$), the optimal power-control law converges to the water-filling scheme, where the Shannon capacity can be achieved. On the other hand, when the QoS constraint is stringent ($\theta \to \infty$), the optimal power-control law converges to the total channel inversion scheme[1] such that the system operates at a constant service rate.

Once obtaining γ_0, we can derive the optimal effective capacity, denoted by $\text{E}_C^{\text{opt}}(\theta)$, as [5]:

$$\begin{aligned} \text{E}_C^{\text{opt}}(\theta) &= -\frac{1}{\theta} \log \Bigg(\int_0^{\gamma_0} p_\Gamma(\gamma) d\gamma \\ &\quad + \int_{\gamma_0}^{\infty} \left(\frac{\gamma}{\gamma_0} \right)^{-\frac{\beta}{(\beta+1)}} p_\Gamma(\gamma) d\gamma \Bigg). \end{aligned} \quad (4)$$

Given the specific diversity scheme and channel statistics, the optimal effective capacity given in (4) can be calculated either by the closed-form expression or by numerical solution.

IV. MULTIPLEXING-BASED SYSTEMS: OPTIMIZATION PROBLEM FORMULATION

In the following, we consider QoS-driven power adaptation for *multiplexing-based systems*. The core idea of multiplexing systems is to transmit different data streams through different subchannels. At the receiver side, the parallel data steams are recovered separately. This transmission strategy can either combat the frequency selective fading channel (e.g., in multicarrier systems) or increase the throughput (e.g., in MIMO multiplexing systems), which are elaborated on, respectively, as follows.

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[1]In this scheme, the transmission power is proportional to the reciprocal of the channel power gain.

A. Multicarrier Systems

Consider a multicarrier system with N subchannels corresponding to N subcarriers. If we assume a constant equal-power distribution among all subcarriers, then the instantaneous transmit power for the nth subchannel at the ith frame, denoted by $P_n[i]$, is equal to $P_n[i] = \overline{P}/N$ for all n and i. The corresponding instantaneous received SNR, denoted by $\gamma_n[i]$, can be expressed as

$$\gamma_n[i] = \frac{\lambda_n[i](\overline{P}/N)}{N_0(B/N)} = \frac{\overline{P}\lambda_n[i]}{N_0 B}, \text{ for } n \in \mathcal{N}_0. \quad (5)$$

Denote the joint pdf of the SNR vector $\boldsymbol{\gamma}[i] = (\gamma_1[i], \gamma_2[i], ..., \gamma_N[i])$ for all subchannels by $p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) = p_{\Gamma_1, \Gamma_2, ..., \Gamma_N}(\gamma_1, \gamma_2, ..., \gamma_N)$, and the corresponding power-adaptation policy for the nth subchannel by $\mu_n(\theta, \boldsymbol{\gamma}[i])$, respectively. Then, the instantaneous transmit power for the nth subchannel becomes $P_n[i] = \mu_n(\theta, \boldsymbol{\gamma}[i])\overline{P}/N$. Note that we limit the mean transmit power by $\overline{P}$. Therefore, the power-control policy needs to satisfy the mean power constraint as follows:

$$\sum_{n=1}^{N} \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \mu_n(\theta, \boldsymbol{\gamma}) p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N = N \quad (6)$$

where

$$\mu_n(\theta, \boldsymbol{\gamma}) \geq 0, \text{ for all } n \in \mathcal{N}_0. \quad (7)$$

Recall that we assume that the AMC scheme can achieve the Shannon capacity. Thus, the instantaneous service rate of the frame i, denoted by $R[i]$, can be expressed as

$$R[i] = \sum_{n=1}^{N} \left(\frac{T_f B}{N}\right) \log_2\left(1 + \mu_n(\theta, \boldsymbol{\gamma}[i])\gamma_n[i]\right). \quad (8)$$

Thus, from [5, eq. (4)], the effective capacity, denoted by $\mathrm{E_C}(\theta)$, can be expressed as follows:

$$\begin{aligned}
\mathrm{E_C}(\theta) &= -\frac{1}{\theta} \log\left(\mathbb{E}\left\{e^{-\theta R[i]}\right\}\right) \\
&= -\frac{1}{\theta} \log\left(\underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{n=1}^{N} [1 + \mu_n(\theta, \boldsymbol{\gamma})\gamma_n]^{-\frac{\beta}{N}} \right. \\
&\qquad\qquad\qquad\qquad \left. \cdot p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N\right) \quad (9)
\end{aligned}$$

where β is also given by (2). To maximize the effective capacity, we can formulate an optimization problem as follows:

$$\begin{aligned}
&\mathrm{E_C^{opt}}(\theta) \\
&= \max_{\mu_n(\theta, \boldsymbol{\gamma}), n \in \mathcal{N}_0} \left\{ -\frac{1}{\theta} \log\left(\underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{n=1}^{N} [1 + \mu_n(\theta, \boldsymbol{\gamma})\gamma_n]^{-\frac{\beta}{N}} \right.\right. \\
&\qquad\qquad\qquad\qquad\qquad \left.\left. \cdot p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N\right)\right\} \quad (10)
\end{aligned}$$

subject to constraints given by (6) and (7).

B. MIMO Systems

Let N_t and N_r denote the number of transmit and receive antennas, respectively, and let $\mathbb{C}$ denote the space of complex numbers. Then, the MIMO multiplexing-based transmission can be expressed as $\mathbf{y}[i] = \mathbf{H}[i]\mathbf{x}[i] + \mathbf{n}[i]$, where $\mathbf{y}[i] \in \mathbb{C}^{N_r}$ denotes the received signal, $\mathbf{H}[i] \in \mathbb{C}^{N_r \times N_t}$ represents the complex channel matrix, $\mathbf{x}[i] \in \mathbb{C}^{N_t}$ stands for the input signal, and $\mathbf{n}[i] \in \mathbb{C}^{N_r}$ is the complex AWGN, where, without loss of generality, we assume $\mathbb{E}\{\mathbf{n}[i]\mathbf{n}[i]^\dagger\} = \mathbf{I}_{N_r}$, with $\dagger$ denoting the conjugate transpose. It is well known that for MIMO multiplexing systems, the data streams are equivalent to transmitting through N parallel singular-value channels [20], where $N = \min\{N_t, N_r\}$. Mathematically, the transmitted signals can be modeled as [20]

$$\widetilde{y}_\ell[i] = \sqrt{\lambda_\ell[i]}\,\widetilde{x}_\ell[i] + \widetilde{n}_\ell[i], \text{ for all } \ell \in \mathcal{N}_0 \quad (11)$$

where $\{\sqrt{\lambda_\ell[i]}\}_{\ell=1}^N$ are nonzero singular values of the channel matrix $\mathbf{H}[i]$. Corresponding to our system description in Section II, $\{\sqrt{\lambda_\ell[i]}\}_{\ell=1}^N$ and $\{\lambda_\ell[i]\}_{\ell=1}^N$ can be considered as the *virtual* envelope process and path-gain process for MIMO multiplexing system, respectively. There have been abundant literatures investigating the joint pdf $p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda})$ for $\boldsymbol{\lambda}[i] = (\lambda_1[i], \lambda_2[i], ..., \lambda_N[i])$. For instance, when the channels between all transmit and receive antenna pairs are i.i.d. Rayleigh distributed with unit energy, the pdf $p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda})$ follows the well-known Wishart distribution as [20]

$$\begin{aligned}
p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) &= \left[N!\left(\prod_{i=1}^{N}(N-i)!(M-i)!\right)\right]^{-1} \exp\left(-\sum_{i=1}^{N}\lambda_i\right) \\
&\quad \cdot \prod_{i=1}^{N}\lambda_i^{M-N} \prod_{1 \leq i < j \leq N}(\lambda_i - \lambda_j)^2 \quad (12)
\end{aligned}$$

where $M = \max\{N_t, N_r\}$. Using [5, eq. (4)], we also derive the effective capacity $\mathrm{E_C}(\theta)$ for MIMO multiplexing system as follows:

$$\begin{aligned}
\mathrm{E_C}(\theta) &= -\frac{1}{\theta} \log\left(\mathbb{E}\left\{e^{-\theta R[i]}\right\}\right) \\
&= -\frac{1}{\theta} \log\left(\underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{\ell=1}^{N} [1 + \mu_\ell(\theta, \boldsymbol{\lambda})\lambda_\ell]^{-\beta} \right. \\
&\qquad\qquad\qquad\qquad \left. \cdot p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) d\lambda_1 \cdots d\lambda_N\right) \quad (13)
\end{aligned}$$

where $\mu_\ell(\theta, \boldsymbol{\lambda})$ denotes the power-adaptation policy and β is also given by (2). To maximize the effective capacity, we formulate the optimization problem as follows:

$$\begin{aligned}
&\mathrm{E_C^{opt}}(\theta) \\
&= \max_{\mu_\ell(\theta, \boldsymbol{\lambda}), \ell \in \mathcal{N}_0} \left\{ -\frac{1}{\theta} \log\left(\underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{\ell=1}^{N} [1 + \mu_\ell(\theta, \boldsymbol{\lambda})\lambda_\ell]^{-\beta} \right.\right. \\
&\qquad\qquad\qquad\qquad\qquad \left.\left. \cdot p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) d\lambda_1 \cdots d\lambda_N\right)\right\} \quad (14)
\end{aligned}$$

subject to the mean power constraint:

$$\sum_{\ell=1}^{N} \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \mu_\ell(\theta, \boldsymbol{\lambda}) p_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) d\lambda_1 \cdots d\lambda_N = \overline{P} \qquad (15)$$

and also the constraint:

$$\mu_\ell(\theta, \boldsymbol{\lambda}) \geq 0, \ \text{for all } \ell \in \mathcal{N}_0. \qquad (16)$$

Comparing (10) with (14), we can observe that the two optimization problems have the same structure except for certain constant-scalar differences. Therefore, we can develop a unified approach to derive the optimal power-adaptation policy. To simplify the presentation, in the next section, we will mainly focus on multicarrier systems. The detailed derivations for MIMO multiplexing systems are similar to those of the multicarrier systems, but omitted in this paper for lack of space.

C. Independent Optimization

Before getting into details of maximizing the effective capacity expressed in (10) and (14), we first consider an alternative strategy, namely, the independent optimization approach for the following reasons. Since we already obtain the optimal power-adaptation policy for the single channel transmission [5], can we directly apply this strategy to multiplexing systems? For instance, in a multiplexing system with N i.i.d. subchannels, one possible solution is to maximize the effective capacity at each subchannel *independently* using the optimal single channel power-adaptation policy. Is this resulting scheme optimal?

Surprisingly, the answer to the above questions is *no*. In fact, this independent optimization approach turns out to be optimal in maximizing the Shannon capacity (e.g., water-filling power control for multichannel transmissions). Note that when $\theta \to 0$, the maximum effective capacity approaches the Shannon capacity. Therefore, this strategy can maximize the effective capacity as $\theta \to 0$. However, as will be shown in the following sections, the independent optimization approach is *not* the optimal policy to maximize the effective capacity for an arbitrary θ.

To characterize the performance of independent power adaptation over i.i.d. subchannels, we have the following proposition:

Proposition 1: Under the same power and spectral-bandwidth constraints, **if** we apply an arbitrary power-adaptation policy to a single channel transmission system, and apply the same power-adaptation policy to each of N i.i.d. subchannels of a multichannel transmission system *independently*, **then** the resulting effective capacities, denoted by $\mathrm{E_C}^{(1)}(\theta)$ for the single channel system and $\mathrm{E_C}^{(N)}(\theta)$ for the multichannel system, respectively, satisfy $\mathrm{E_C}^{(N)}(\theta) = \mathrm{E_C}^{(1)}(\theta/N)$.

Proof: Denote the service rate of the nth subchannel of multichannel transmission by $R_n[i]$ and the service rate of single channel transmission by $R[i]$, respectively. When the channel condition is the same, we have $R_n[i] = R[i]/N$, $\forall n$ and $\forall i$, which is because the nth subchannel only occupies

$1/N$ of the total spectral-bandwidth. Then, the following equations hold:

$$\begin{aligned} \mathrm{E_C}^{(N)}(\theta) &= -\frac{1}{\theta} \log \left(\mathbb{E}\left\{ e^{-\theta \sum_{n=1}^N R_n[i]} \right\} \right) \\ &= -\frac{1}{\theta} \log \left(\mathbb{E}\left\{ e^{-\theta R_n[i]} \right\} \right)^N \\ &= -\frac{1}{\left(\frac{\theta}{N}\right)} \log \left(\mathbb{E}\left\{ e^{-\left(\frac{\theta}{N}\right) R[i]} \right\} \right) \\ &= \mathrm{E_C}^{(1)}\left(\frac{\theta}{N} \right). \end{aligned} \qquad (17)$$

Thus, the proof follows. ∎

Remark 1: Proposition 1 says that as compared to the single channel transmission, the effective capacity gain of the multichannel transmission using the independent power control is $10 \log_{10} N$ dB. In other words, $\mathrm{E_C}^{(N)}(\theta)$ is a right-shifted version of $\mathrm{E_C}^{(1)}(\theta)$ along θ-axis using the logarithmic scale, where the difference between these two is $10 \log_{10} N$ dB. Over the single channel Rayleigh fading environment, we prove in [5] that the effective capacity always approaches zero as $\theta \to \infty$. Therefore, according to Proposition 1, by using the independent power-adaptation policies, as long as the number of subchannels N is finite, the effective capacity $\mathrm{E_C}^{(N)}(\theta)$ also approaches zero as $\theta \to \infty$. In the following sections, we propose a joint optimization approach, which performs much better than the independent optimization.

V. MULTIPLEXING-BASED SYSTEMS: OPTIMAL POWER-ADAPTATION STRATEGY

Since $\log(\cdot)$ is a monotonically increasing function, for each given $\theta > 0$, the original maximization problem of (10) is equivalent to the following minimization problem:

$$\min_{\mu_n(\theta,\boldsymbol{\gamma}), n \in \mathcal{N}_0} \left\{ \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{n=1}^N [1 + \mu_n(\theta, \boldsymbol{\gamma})\gamma_n]^{-\frac{\beta}{N}} \right.$$
$$\left. \cdot p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N \right\} \qquad (18)$$

which is subject to the same set of constraints given by (6) and (7). As derived in Appendix I, we prove that the objective function in (18) is strictly convex on the space spanned by $\left(\mu_1(\theta,\boldsymbol{\gamma}), ..., \mu_N(\theta,\boldsymbol{\gamma})\right)$. In addition, it is clear that the constraints given by (6) and (7) are linear with respect to $\left(\mu_1(\theta,\boldsymbol{\gamma}), ..., \mu_N(\theta,\boldsymbol{\gamma})\right)$. Therefore, the problem can be considered as a convex optimization problem which has the unique optimal solution. Then, using standard optimization technique, we can construct the Lagrangian function as follows:

$$\begin{aligned} \mathcal{J} &= \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{n=1}^N [1 + \mu_n(\theta, \boldsymbol{\gamma})\gamma_n]^{-\frac{\beta}{N}} p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N \\ &+ \kappa_0 \left\{ \sum_{n=1}^N \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \mu_n(\theta, \boldsymbol{\gamma}) p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma}) d\gamma_1 \cdots d\gamma_N - N \right\} \\ &- \sum_{n=1}^N \kappa_n \mu_n(\theta, \boldsymbol{\gamma}) \end{aligned} \qquad (19)$$

where all the Lagrangian multipliers $\{\kappa_n\}_{n=0}^{N}$ satisfy $\kappa_n \geq 0$. Differentiating the Lagrangian function and setting the derivative equal to zero [21, Sec. 4.2.4], we obtain a set of N equations:

$$\frac{\partial \mathcal{J}}{\partial \mu_n(\theta, \gamma)} = -\frac{\beta \gamma_n}{N} [1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}-1}$$
$$\cdot \prod_{i \in \mathcal{N}_0, i \neq n} [1 + \mu_i(\theta, \gamma)\gamma_i]^{-\frac{\beta}{N}} p_{\Gamma}(\gamma)$$
$$+ \kappa_0 p_{\Gamma}(\gamma) - \kappa_n = 0, \text{ for all } n \in \mathcal{N}_0. \quad (20)$$

According to the concept of *complementary slackness* [22, Sec. 5.5.2], if the *strict* inequality $\mu_j(\theta, \gamma) > 0$ holds for a certain $j \in \mathcal{N}_0$, then the Lagrangian multiplier κ_j corresponding to $\mu_j(\theta, \gamma)$ must be equal to zero. Based on this fact, we consider two different scenarios, respectively, as follows.

A. Scenario-1: The strict inequality $\mu_n(\theta, \gamma) > 0$ holds for all $n \in \mathcal{N}_0$.

Under the conditions of the above Scenario-1, all subchannels are assigned with power for data transmission. Then, according to the complementary slackness, except for κ_0, all the other Lagrangian multipliers $\{\kappa_n\}_{n=1}^{N}$ must be equal to zero. Thus, (20) reduces to:

$$[1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}-1} \prod_{i \in \mathcal{N}_0, i \neq n} [1 + \mu_i(\theta, \gamma)\gamma_i]^{-\frac{\beta}{N}}$$
$$= \frac{N\gamma_0}{\gamma_n}, \text{ for all } n \in \mathcal{N}_0 \quad (21)$$

where we define $\gamma_0 \triangleq \kappa_0/\beta$, which is a cutoff threshold to be optimized later. Solving (21), we can obtain the optimal power-adaptation policy as follows:

$$\mu_n(\theta, \gamma) = \frac{1}{\gamma_0^{\frac{1}{\beta+1}} \prod_{i \in \mathcal{N}_0} \gamma_i^{\frac{\beta}{N(\beta+1)}}} - \frac{1}{\gamma_n}, \; n \in \mathcal{N}_0. \quad (22)$$

Note that the policy given by (22) is optimal only if $\mu_n(\theta, \gamma) > 0$ holds for all $n \in \mathcal{N}_0$. Specifically, define $\mathcal{N}_1$ as the index-set of SNRs which satisfy this strict inequality as follows:

$$\mathcal{N}_1 \triangleq \left\{ n \in \mathcal{N}_0 \; \middle| \; \frac{1}{\gamma_0^{\frac{1}{\beta+1}} \prod_{i \in \mathcal{N}_0} \gamma_i^{\frac{\beta}{N(\beta+1)}}} - \frac{1}{\gamma_n} > 0 \right\}. \quad (23)$$

Then, (22) is the optimal solution only if $\mathcal{N}_1 = \mathcal{N}_0$. Otherwise, if $\mathcal{N}_1 \subset \mathcal{N}_0$, we need to consider the following scenario.

B. Scenario-2: There exist some $\mu_n(\theta, \gamma)$ such that $\mu_n(\theta, \gamma) = 0$.

If $\mathcal{N}_1 \subset \mathcal{N}_0$, there must exist certain $\mu_n(\theta, \gamma)$ such that $\mu_n(\theta, \gamma) = 0$. In other words, some subchannels are not assigned with any power. In order to identify the set of subchannels to which the system do not assign power, we introduce the following lemma:

Lemma 1: If $n \notin \mathcal{N}_1$, then $\mu_n(\theta, \gamma) = 0$.
Proof: The proof is provided in Appendix II. ∎

Algorithm: **QoS − driven power adaptation**.
(1) **Initialization.**
    a) Obtain $\mathcal{N}_1$, and N_1 by (23) and (25), respectively.
    b) $k = 1$.
(2) **While** ($\mathcal{N}_k \neq \mathcal{N}_{k-1}$) **do**

    a) $\mathcal{N}_{k+1} = \left\{ n \in \mathcal{N}_k \; \middle| \; \frac{1}{\gamma_0^{\frac{N}{N_k \beta + N}} \prod_{i \in \mathcal{N}_k} \gamma_i^{\frac{\beta}{N_k \beta + N}}} - \frac{1}{\gamma_n} > 0 \right\}$.

    b) $N_{k+1} = |\mathcal{N}_{k+1}|$.
    c) $k = k + 1$.
(3) **Obtain the optimal adaptation policy.**
    a) Denote $\mathcal{N}^* = \mathcal{N}_k$ and $N^* = N_k$, respectively.

    b) $\mu_n(\theta, \gamma) = \begin{cases} \frac{1}{\gamma_0^{\frac{N}{N^*\beta+N}} \prod_{i \in \mathcal{N}^*} \gamma_i^{\frac{\beta}{N^*\beta+N}}} - \frac{1}{\gamma_n}, & n \in \mathcal{N}^* \\ 0, & \text{otherwise.} \end{cases}$

Fig. 2. Algorithm of optimal power adaptation for the multicarrier system.

Lemma 1 states that *all the power is assigned to the subchannels which belong to $\mathcal{N}_1$*. Thus, the original minimization problem of (18) reduces to

$$\min_{\mu_n(\theta, \gamma), \, n \in \mathcal{N}_1} \left\{ \underbrace{\int_0^\infty \cdots \int_0^\infty}_{N-\text{fold}} \prod_{n \in \mathcal{N}_1} [1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}} \right.$$
$$\left. \cdot p_{\Gamma}(\gamma) d\gamma_1 \cdots d\gamma_N \right\}. \quad (24)$$

Comparing (24) with (18), we can observe that the two minimization problems have the same structure except that the optimization space shrinks from $\mathcal{N}_0$ to $\mathcal{N}_1$. The above observation suggests us to solve this minimization problem in a *recursive* manner.

Following the same procedure as that used in Section V-A, if the strict inequality $\mu_n(\theta, \gamma) > 0$ holds for all $n \in \mathcal{N}_1$, we can obtain the optimal power-control policy as follows:

$$\mu_n(\theta, \gamma) = \begin{cases} \frac{1}{\gamma_0^{\frac{N}{N_1\beta+N}} \prod_{i \in \mathcal{N}_1} \gamma_i^{\frac{\beta}{N_1\beta+N}}} - \frac{1}{\gamma_n}, & n \in \mathcal{N}_1 \\ 0, & \text{otherwise} \end{cases}$$

where N_1 denotes the number of subchannels belonging to $\mathcal{N}_1$, or, the cardinality of $\mathcal{N}_1$, i.e.,

$$N_1 \triangleq |\mathcal{N}_1|. \quad (25)$$

Otherwise, if not all of the subchannels $n \in \mathcal{N}_1$ satisfy the strict inequality $\mu_n(\theta, \gamma) > 0$, we need to further divide $\mathcal{N}_1$ and repeat this procedure itself again. In summary, the QoS-driven optimal power-adaptation algorithm is described as the algorithm shown in Fig. 2.

The principle of the optimal power-adaptation algorithm is to search for the maximum set of SNRs which can simultaneously satisfy the strict inequality $\mu_n(\theta, \gamma) > 0$, i.e., the maximum set of subchannels which can be assigned power simultaneously. Once we successfully identify such a set ($\mathcal{N}_k = \mathcal{N}_{k-1} = \mathcal{N}^*$), the optimal power-adaptation policy is obtained. Otherwise, we exclude those undesired SNRs from current optimization space and repeat this searching procedure
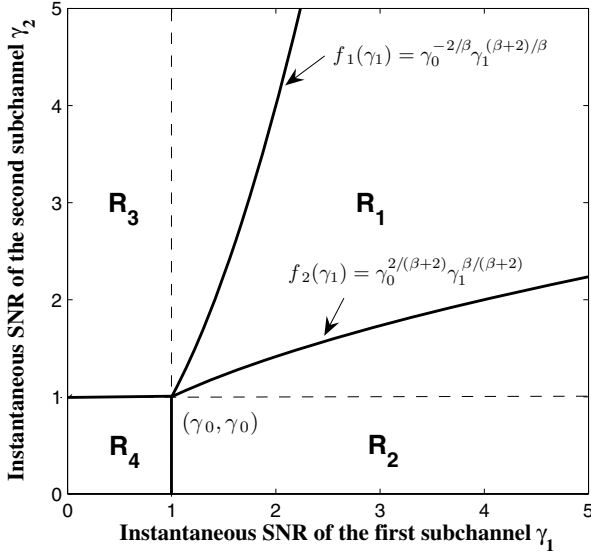
Fig. 3. The policy regions of different power-adaptation strategies when the number of subchannels $N = 2$. The solid lines depicts the result of joint optimization and the dashed lines depicts the result of independent optimization. In this example, we set $\gamma_0 = 1$ and $\beta = 2$. The four policy regions are: (R_1) both subchannels are allocated power; (R_2) only the first subchannel is allocated power; (R_3) only the second subchannel is allocated power; and (R_4) the system is in outage state.

itself again. If the "while-loop" ends up with ($\mathcal{N}_{k-1} = \mathcal{N}_k = \varnothing$), then no subchannel can satisfy the strict inequality condition $\mu_n(\theta, \gamma) > 0$. In this case, we set $\mu_n(\theta, \gamma) = 0$ for all $n \in \mathcal{N}_0$. Thus, the system falls into an outage state and cannot send any data. Finally, we obtain the optimal resource allocation policy for multicarrier systems as follows:

$$
\mu_n(\theta, \gamma) = \begin{cases} \dfrac{1}{\gamma_0^{\frac{N}{N^*\beta+N}} \prod_{i\in\mathcal{N}^*} \gamma_i^{\frac{\beta}{N^*\beta+N}}} - \dfrac{1}{\gamma_n}, & n \in \mathcal{N}^* \\ 0, & \text{otherwise.} \end{cases}
$$
(26)

Similarly, for MIMO multiplexing system, we can show that the optimal power-adaptation policy can be expressed as

$$
\mu_\ell(\theta, \lambda) = \begin{cases} \dfrac{1}{\lambda_0^{\frac{1}{N^*\beta+1}} \prod_{i\in\mathcal{N}^*} \lambda_i^{\frac{\beta}{N^*\beta+1}}} - \dfrac{1}{\lambda_\ell}, & \ell \in \mathcal{N}^* \\ 0, & \text{otherwise} \end{cases}
$$

where $\mathcal{N}^*$ and N^* can be obtained by a similar algorithm as shown in Fig. 2.

Given the optimal power-adaptation algorithm, the cutoff threshold γ_0 is determined by meeting the mean power constraint (6). Note that γ_0 is jointly determined by the QoS exponent θ and channel model distribution $p_\Gamma(\gamma)$. After obtaining the cutoff threshold, the optimal effective capacity can be calculated by (10).

VI. SPECIAL CASES OF THE OPTIMAL POWER CONTROL

A. Two-Subchannel Case ($N = 2$)

To demonstrate the execution procedure of our proposed algorithm, let us consider a particular case when the number of subcarriers $N = 2$. Using the algorithm described in

Fig. 2, we can see that the joint optimal power-adaptation policy partitions the SNR-plane (γ_1, γ_2) into four exclusive regions by the solid lines as shown in Fig. 3. If (γ_1, γ_2) falls into region R_1, both subchannels will be assigned with power for data transmission, where the boundaries of region R_1 is determined by $f_1(\gamma_1) = \gamma_0^{-2/\beta} \gamma_1^{(\beta+2)/\beta}$ and $f_2(\gamma_1) = \gamma_0^{2/(\beta+2)} \gamma_1^{\beta/(\beta+2)}$.[2] On the other hand, if (γ_1, γ_2) falls into either region R_2 or R_3, then only one of the subchannels will be assigned with power. Otherwise, if (γ_1, γ_2) belongs to region R_4, the system will be in an outage state. As shown by Fig. 3, the four regions are functions of γ_0 and β, which change as the values of γ_0 and β vary. Thus, based on (6), the cutoff threshold γ_0 is determined by satisfying the following power constraint:

$$
\int_{R_1} \left[\mu_1^{(1)}(\theta, \gamma) + \mu_2^{(1)}(\theta, \gamma) \right] p_\Gamma(\gamma) d\gamma_1 d\gamma_2
$$

$$
+ \int_{R_2} \mu_1^{(2)}(\theta, \gamma) p_\Gamma(\gamma) d\gamma_1 d\gamma_2
$$

$$
+ \int_{R_3} \mu_2^{(2)}(\theta, \gamma) p_\Gamma(\gamma) d\gamma_1 d\gamma_2 = 2 \quad (27)
$$

where

$$
\mu_n^{(1)}(\theta, \gamma) = \frac{1}{\gamma_0^{\frac{1}{\beta+1}} (\gamma_1 \gamma_2)^{\frac{\beta}{2(\beta+1)}}} - \frac{1}{\gamma_n} \quad (28)
$$

and

$$
\mu_n^{(2)}(\theta, \gamma) = \frac{1}{\gamma_0^{\frac{2}{\beta+2}} \gamma_n^{\frac{\beta}{\beta+2}}} - \frac{1}{\gamma_n} \quad (29)
$$

for $n = 1$ and $n = 2$, respectively. After obtaining γ_0 and using (10), the optimal effective capacity can be derived as follows:

$$
\begin{aligned}
E_C^{opt}(\theta) = -\frac{1}{\theta} \log \Bigg(& \int_{R_4} p_\Gamma(\gamma) d\gamma_1 d\gamma_2 \\
& + \int_{R_1} \prod_{n=1}^2 \left[1 + \mu_n^{(1)}(\theta, \gamma) \gamma_n \right]^{-\frac{\beta}{2}} p_\Gamma(\gamma) d\gamma_1 d\gamma_2 \\
& + \int_{R_2} \left[1 + \mu_1^{(2)}(\theta, \gamma) \gamma_n \right]^{-\frac{\beta}{2}} p_\Gamma(\gamma) d\gamma_1 d\gamma_2 \\
& + \int_{R_3} \left[1 + \mu_2^{(2)}(\theta, \gamma) \gamma_n \right]^{-\frac{\beta}{2}} p_\Gamma(\gamma) d\gamma_1 d\gamma_2 \Bigg). (30)
\end{aligned}
$$

From the above example, we can find that even for a simple case of $N = 2$, the cutoff threshold γ_0 and the optimal effective capacity $E_C^{opt}(\theta)$ generally do not have simple closed-form solutions. For the case with $N > 2$, the situation becomes even more complicated. However, by executing the proposed algorithm, γ_0 and $E_C^{opt}(\theta)$ can be easily found through simulations for any given joint channel distribution $p_\Gamma(\gamma)$. Thus, in this paper, except for the trivial case of $N = 1$, we use simulation to find γ_0 and $E_C^{opt}(\theta)$ for multiplexing-based systems. It is also worth noting that by using independent optimization approach, the power-adaptation policy partitions the SNR-plane (γ_1, γ_2) into four exclusive regions by the dashed lines as shown in Fig. 3.

[2]The functions $f_1(\gamma_1)$ and $f_2(\gamma_1)$ are obtained by solving the boundary condition $\mathcal{N}_1 = \mathcal{N}_0$, where $\mathcal{N}_1$ is given by (23).
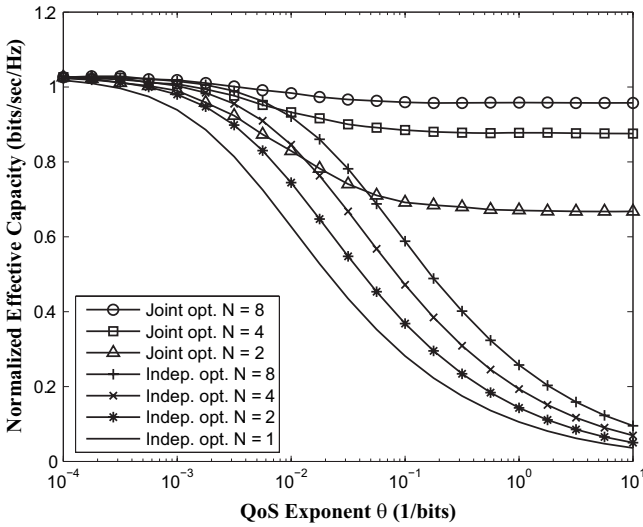
Fig. 4.   The effective capacity comparisons between the joint optimization-based and independent optimization-based power-adaptation policies for N i.i.d. subchannels in a multicarrier system.

Fig. 5.   The optimal effective capacity comparisons for MIMO systems using the different numbers of antennas.

B. Limiting Cases

One of the most significant differences between our proposed QoS-driven power adaptation and most other existing power-control approaches, such as the conventional water-filling algorithm, constant power scheme, and the independent optimization approach mentioned above, is that our proposed algorithm is executed in a *joint* fashion. Specifically, *the power assigned to one subchannel depends not only on its own channel quality, but also on the other subchannels' qualities*, by which the statistics of the aggregate service rate from all subchannels can be controlled to meet a certain delay-QoS requirement. In the following, we further study some limiting cases of our proposed optimal power-adaptation algorithms.

CASE I: When $N = 1$, or equivalently, all the subchannels are fully correlated, i.e., $\gamma_1 = \gamma_2 = \cdots = \gamma_N = \gamma$, the multi-channel transmission reduces to single channel transmission. In this case, the joint pdf $p_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma})$ reduces to $p_{\Gamma}(\gamma)$. Then, the optimal power-adaptation policy and power constraint turn out to be the ones reducing to our previous results [5, eqs. (8) and (9)], which is expected since single channel transmission is a special case of our multichannel communications.

CASE II: When the QoS exponent $\theta \to 0$, indicating that the system can tolerate an arbitrarily long delay, the optimal power-adaptation policy reduces to:

$$\lim_{\theta \to 0} \mu_n(\theta, \boldsymbol{\gamma}) = \begin{cases} \dfrac{1}{\gamma_0} - \dfrac{1}{\gamma_n}, & \gamma_n \geq \gamma_0, \\ 0, & \text{otherwise} \end{cases} \quad (31)$$

for all $n \in \mathcal{N}_0$, which is the water-filling formula for multichannel communications, where, as expected, the joint optimization reduces to the independent optimization. This observation verifies that the independent optimization approach is optimal to maximize the effective capacity as $\theta \to 0$. Thus, our QoS-driven power-adaptation scheme converges to water-filling algorithm when the system can tolerate an arbitrarily long delay. It also follows that the optimal effective capacity converges to the Shannon capacity as $\theta \to 0$.
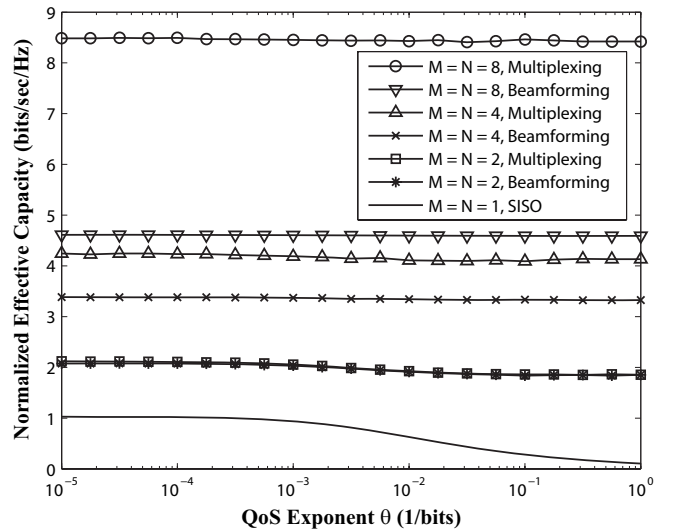
CASE III: When the QoS exponent $\theta \to \infty$, then the system cannot tolerate any delay. In this case, the cutoff threshold $\gamma_0 \to 0$ (note that $\gamma_0 = \kappa_0/\beta$), which implies that the system does not enter the outage state almost surely. Letting $\theta \to \infty$ in (26) [i.e., Step (3)-b) in Fig. 2], we obtain the corresponding optimal strategy as follows:

$$\lim_{\theta \to \infty} \mu_n(\theta, \boldsymbol{\gamma}) = \begin{cases} \dfrac{1}{\phi^{\frac{N}{N^*}} \prod_{i \in \mathcal{N}^*} \gamma_i^{\frac{1}{N^*}}} - \dfrac{1}{\gamma_n}, & n \in \mathcal{N}^* \\ 0, & \text{otherwise} \end{cases}$$
$$(32)$$

where $\phi \triangleq \lim_{\theta \to \infty} \gamma_0^{\frac{1}{\beta+1}}$ and $\mathcal{N}^* \neq \varnothing$ almost surely. The power-control law given by (32) is just the policy to achieve the zero-outage capacity of the system [16], [23]. Thus, when the QoS exponent $\theta \to \infty$, the optimal throughput approaches the zero-outage capacity of the system. In summary, *as the QoS exponent θ increases from zero to infinity, the optimal effective capacity decreases accordingly from the ergodic capacity to zero-outage capacity*.

Plugging the power-control strategy given by (32) into (8), we can derive the resulting instantaneous service rate $R = R[i]$ when $\theta \to \infty$ as follows:

$$\begin{aligned} R &= \sum_{n=1}^{N^*} \left(\frac{T_f B}{N} \right) \log_2 \left(1 + \mu_n(\theta, \boldsymbol{\gamma})\gamma_n \right) \\ &= \left(\frac{T_f B}{N} \right) \log_2 \left(\prod_{n \in \mathcal{N}^*} \left[\frac{\gamma_n}{\phi^{\frac{N}{N^*}} \prod_{i \in \mathcal{N}^*} \gamma_i^{\frac{1}{N^*}}} \right] \right) \\ &= \left(\frac{T_f B}{N} \right) \log_2 \left(\frac{\prod_{n \in \mathcal{N}^*} \gamma_n}{\phi^N \prod_{i \in \mathcal{N}^*} \gamma_i} \right) \\ &= T_f B \log_2 \left(\frac{1}{\phi} \right). \end{aligned} \quad (33)$$

That is, no matter what the channel realization is, the system maintains a constant service rate $T_f B \log_2(1/\phi)$. This result is also consistent with our previous work on single channel transmissions [5], where as the delay-QoS constraint becomes
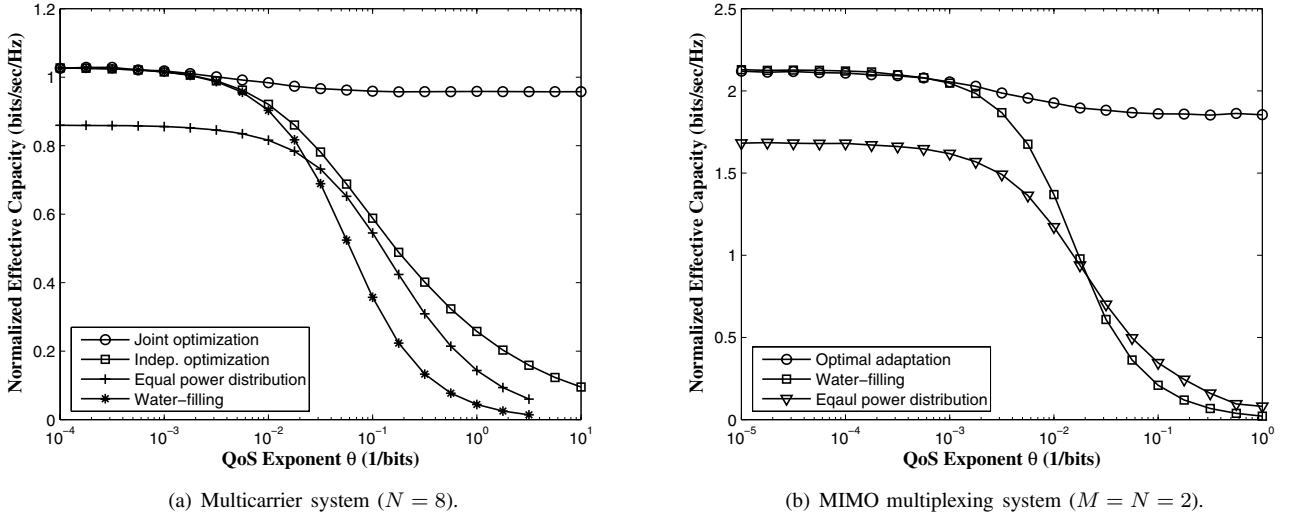
(a) Multicarrier system ($N = 8$).

(b) MIMO multiplexing system ($M = N = 2$).

Fig. 6. The effective capacity comparisons among different power-adaptation strategies for multiplexing-based systems.

(a) Multicarrier system.

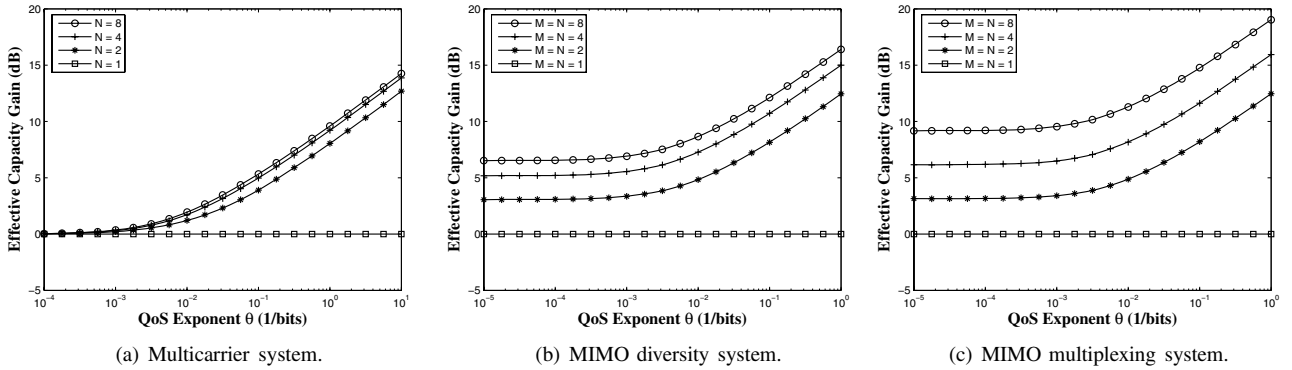(b) MIMO diversity system.

(c) MIMO multiplexing system.

Fig. 7. The effective capacity gains compared to single channel (SISO) transmissions.

stringent, the optimal power control operates at a constant service rate. Since the service rate is constant, the effective capacity is also equal to this constant, i.e.,

$$\lim_{\theta \to \infty} \mathrm{E}_{\mathrm{C}}^{\mathrm{opt}}(\theta) = T_f B \log_2 \left(\frac{1}{\phi} \right). \qquad (34)$$

From (34), we can observe that the smaller the value ϕ is, the larger the effective capacity $\mathrm{E}_{\mathrm{C}}^{\mathrm{opt}}(\theta)$ becomes. Our numerical results show that ϕ is a monotonic decreasing function of N. Consequently, when $\theta \to \infty$, the optimal effective capacity $\mathrm{E}_{\mathrm{C}}^{\mathrm{opt}}(\theta)$ increases as the number of sub-channels N increases. In contrast, as mentioned in Remark 1 for Proposition 1, by using the independent power-control policies, as long as the number N of subchannels is finite, the effective capacity $\mathrm{E}_{\mathrm{C}}(\theta)$ always approaches zero as $\theta \to \infty$. Thus, our proposed joint optimization-based power control shows significant advantages over all the other independent power-control strategies as the delay-QoS constraint becomes stringent. For the MIMO multiplexing system, by using a similar procedure, we can show that

$$\lim_{\theta \to \infty} \mathrm{E}_{\mathrm{C}}^{\mathrm{opt}}(\theta) = N T_f B \log_2 \left(\frac{1}{\varphi} \right) \qquad (35)$$

where $\varphi \triangleq \lim_{\theta \to \infty} \lambda_0^{\frac{1}{N\beta+1}}$. From (35), we can observe that

when the QoS exponent $\theta \to \infty$, the effective capacity of the MIMO multiplexing system is almost a linearly increasing function of the number of subchannels $N = \min\{N_t, N_r\}$, which implies the significant superiority of employing the MIMO infrastructure for the QoS provisioning in mobile wireless networks.

VII. SIMULATION EVALUATIONS

We evaluate the performance of proposed QoS-driven power-adaptation algorithms by simulations. In this section, we mainly focus on three different diversity-based and multiplexing-based multichannel systems. We first simulate the multicarrier system which utilizes frequency domain multiplexing. The fading statistics of different subcarriers are assumed to be i.i.d. Rayleigh distributed with average SNR $\overline{\gamma} = 0$ dB. We then simulate two MIMO systems which apply either diversity combining or multiplexing. For simplicity, we also assume that the fading statistics between all transmit and receive antenna pairs are i.i.d. Rayleigh distributed with average SNR $\overline{\gamma} = 0$ dB per receive antenna. The diversity combining MIMO scheme is Tx-beamforming/Rx-MRC (briefly termed as "beamforming" in the following for convenience) since this scheme provides the maximum spectral-efficiency

among all MIMO diversity schemes. Furthermore, the system total spectral-bandwidth B is fixed to $B = 100$ KHz and the frame duration T_f is set to $T_f = 2$ ms for all simulations.

Fig. 4 plots the optimal effective capacity of multicarrier system against the QoS exponent θ with different number of subcarriers, where for comparison purpose, we also plot the effective capacity using independent optimization approach. As mentioned in Section IV, independent optimization of N subcarriers can right-shift the effective capacity curves for $10 \log_{10} N$ dB, compared to single-carrier system. Consequently, all the effective capacity curves approach zero as the QoS exponent θ increases. In contrast, based on our proposed joint optimization, the effective capacities are significantly larger than those of independent optimizations. As the QoS exponent θ increases, the effective capacity approaches a nonzero constant, where the larger the number of subcarriers, the higher the effective capacity. For example, by using only $N = 8$ i.i.d. subcarriers, the proposed scheme can achieve more than 90% of the Shannon capacity while still guaranteeing a constant rate transmission (as $\theta \to \infty$).

Fig. 5 plots the optimal effective capacities of MIMO diversity and multiplexing systems with different numbers of transmit and receive antennas. We can observe from Fig. 5 that the effective capacity increases as the number of antennas increases. When $M = N = 2$, where as defined in the above, $M = \max\{N_t, N_r\}$ and $N = \min\{N_t, N_r\}$, the performance loss of beamforming system compared to multiplexing system is virtually indistinguishable. However, as the number of antennas increases, the diversity gain is limited, but the multiplexing gain almost linearly increases with N. On the other hand, we can observe that just using a small number of transmit and receive antennas, the effective capacity of MIMO transmission is close to the Shannon capacity as $\theta \to \infty$, since all effective capacities are virtually constants, which implies that the MIMO system can guarantee stringent QoS with the service rate near Shannon-capacity.

To compare the impact of different power adaptations on QoS provisioning, Fig. 6 plots the effective capacities of multicarrier system and MIMO multiplexing system under different power-control policies. The power-adaptation schemes shown in Fig. 6 include our proposed optimal optimization, independent optimization for i.i.d multicarrier system, water-filling scheme, and equal power distribution scheme. As expected, our proposed optimal power adaptation achieves the maximum effective capacity among all power-control policies. The optimal scheme converges to the water-filling for a small θ and converges to a constant for a large θ, where the effective capacity of all other schemes converges to zero for a large θ, which implies the significant advantage of our proposed scheme on supporting stringent QoS over other existing schemes.

Fig. 7 compares the effective-capacity gain of multichannel ($N > 1$) transmission with the single channel ($N = 1$) transmission. We can observe from Fig. 7 that by using the optimal power adaptation, our multichannel transmission-based scheme has the significant advantage over single channel transmission-based scheme, where the larger the QoS exponent θ, the higher the effective capacity gain. This means that multichannel transmission can support much more stringent
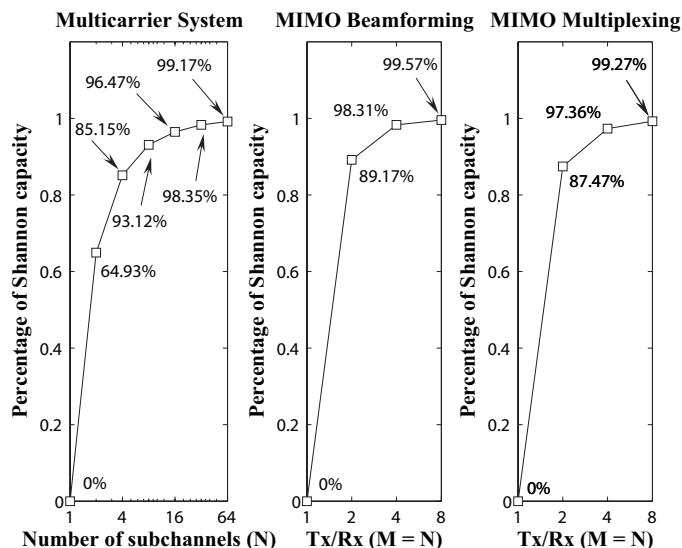

Fig. 8. The optimal effective capacity improvements as the function of the system diversity order compared to the Shannon capacity when the QoS exponent $\theta \to \infty$.

QoS than single channel transmission. In particular, since the effective-capacity gain at $\theta \to 0$ is actually the spectral-efficiency gain, we can observe that for MIMO diversity and multiplexing system, the superiority of employing MIMO infrastructure in terms of enhancing QoS-guarantees is even more significant than that in terms of improving the spectral-efficiency.

Finally, Fig. 8 shows how much percentage of the Shannon capacity that the constant service rate can achieve by using our proposed optimal power adaptation (as $\theta \to \infty$). As expected, when the number of subchannels increases, the service rate gets closer and closer to the Shannon capacity. The percentage of Shannon capacity achieved is approximately proportional to the diversity order of the system, where for multicarrier systems, the diversity order is N, but for MIMO systems, the diversity order is $M \times N$. We can observe from Fig. 8 that when the system diversity order is 64, all multichannel systems can achieve more than 99% of the Shannon capacity, while still guaranteeing a constant rate transmission. In this case, a simple and efficient approach is to just use the fixed power-adaptation policy of our proposed scheme with $\theta \to \infty$, no matter what the delay-QoS constraint is, since this fixed power-adaptation policy can support both loose and stringent QoS requirements with only a slight throughput loss compared to the optimal Shannon capacity.

VIII. CONCLUSION

We have proposed and analyzed the QoS-driven power and rate adaptation schemes for diversity and multiplexing systems by integrating information theory with the effective capacity. The proposed resource allocation policies are general and applicable to different fading channel distributions. Our results showed that as the QoS exponent increases from zero to infinity, the optimal effective capacity decreases accordingly from the ergodic capacity to zero-outage capacity. Moreover, the multichannel transmission provides a significant advantage

over single channel transmission for the stringent delay-QoS guarantees. Compared to the single channel transmission which has to deal with the tradeoff between throughputs and delay, the multichannel transmissions can achieve high throughput and stringent QoS at the same time.

APPENDIX I
PROOF OF THE STRICT CONVEXITY OF THE OBJECTIVE FUNCTION IN EQ. (18)

Proof: To show the strict convexity of the objective function in (18), we introduce the following proposition:

Proposition 2: If $\mathbf{x} = (x_1, x_2, ..., x_n)$ and $f(\mathbf{x}) = \prod_{i=1}^{n} x_i^{-\alpha}$, where $x_i > 0$ for all $i = 1, 2, ..., n$ and $\alpha > 0$, then $f(\mathbf{x})$ is strictly convex on the domain where $\mathbf{x} = (x_1, x_2, ..., x_n)$ is defined.

Proof: It is easy to show that

$$\frac{\partial^2 f(\mathbf{x})}{\partial x_k^2} = \frac{\alpha(\alpha + 1)}{x_k^2} \prod_{i=1}^{n} x_i^{-\alpha} \quad (36)$$

and

$$\frac{\partial^2 f(\mathbf{x})}{\partial x_k \partial x_l} = \frac{\alpha^2}{x_k x_l} \prod_{i=1}^{n} x_i^{-\alpha}, \text{ for } k \neq l. \quad (37)$$

Thus, the Hessian of $f(\mathbf{x})$ can be expressed as

$$\nabla^2 f(\mathbf{x}) = \alpha \prod_{i=1}^{n} x_i^{-\alpha} \left[\alpha \mathbf{y}^T \mathbf{y} + \text{diag}\left(\frac{1}{x_1^2}, ..., \frac{1}{x_n^2} \right) \right] \quad (38)$$

where $\mathbf{y} = (1/x_1, 1/x_2, ..., 1/x_n)$. For any nonzero $\mathbf{v} = (v_1, v_2, ..., v_n)$, we have

$$\mathbf{v} \left(\nabla^2 f(\mathbf{x}) \right) \mathbf{v}^T$$
$$= \alpha \prod_{i=1}^{n} x_i^{-\alpha} \left[\alpha \left(\mathbf{v} \mathbf{y}^T \right)^2 + \sum_{i=1}^{n} \left(\frac{v_i}{x_i} \right)^2 \right] > 0. \quad (39)$$

The Hessian of $f(\mathbf{x})$ is positive definite and therefore $f(\mathbf{x})$ is strictly convex on the domain where $\mathbf{x}$ is defined. ∎

By Proposition 2, since $[1 + \mu_n(\theta, \gamma)\gamma_n] > 0$ always holds, $\prod_{n=1}^{N} [1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}}$ is strictly convex on the space spanned by $([1 + \mu_1(\theta, \gamma)\gamma_1], ..., [1 + \mu_N(\theta, \gamma)\gamma_N])$. Also, since $\mu_n(\theta, \gamma)$ is just a *linear variety* of $[1 + \mu_n(\theta, \gamma)\gamma_n]$, $(\mu_1(\theta, \gamma), ..., \mu_N(\theta, \gamma))$ preserves the convexity of $\prod_{n=1}^{N} [1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}}$ [22, Sec. 3.2.2]. Thus, $\prod_{n=1}^{N} [1 + \mu_n(\theta, \gamma)\gamma_n]^{-\frac{\beta}{N}}$ is strictly convex on the space spanned by $(\mu_1(\theta, \gamma), ..., \mu_N(\theta, \gamma))$. Furthermore, the integral in (18) is a linear operation, which also preserves the strict convexity. Thus, the objective function in (18) is strictly convex on the space spanned by $(\mu_1(\theta, \gamma), ..., \mu_N(\theta, \gamma))$. ∎

APPENDIX II
PROOF OF LEMMA 1

Proof: Without loss of generality, we assume $\mathcal{N}_1 = \{\gamma_1, \gamma_2, ..., \gamma_{N_1}\}$, where $N_1 < N$. Let us denote the complementary set of $\mathcal{N}_1$ by $\overline{\mathcal{N}}_1$, i.e., $\overline{\mathcal{N}}_1 = \{\gamma_{N_1+1}, \gamma_{N_1+2}, ..., \gamma_N\}$. To prove Lemma 1, we need to show that for any nonempty subset $\mathcal{C} \subseteq \overline{\mathcal{N}}_1$, there is no policy $\mu_n(\theta, \gamma)$ such that $\mu_n(\theta, \gamma) > 0$ for all $n \in \mathcal{C} \cup \mathcal{N}_1$.

If $\mathcal{C} = \overline{\mathcal{N}}_1$, we already know there is no such a policy, due to the condition of Scenario-2.

Otherwise, if $\mathcal{C} \subset \overline{\mathcal{N}}_1$, without loss of generality, we assume $\mathcal{C} = \{\gamma_{N_1+1}, \gamma_{N_1+2}, ..., \gamma_{N_1+G}\}$, where $1 \leq G \leq N - N_1 - 1$. Suppose there exists such a policy, from Section V-B we know that the policy can be expressed as

$$\mu_n(\theta, \gamma) = \begin{cases} \dfrac{1}{\gamma_0^{\frac{N}{\omega}} \prod_{i=1}^{N_1+G} \gamma_i^{\frac{\beta}{\omega}}} - \dfrac{1}{\gamma_n}, & n \in \mathcal{C} \cup \mathcal{N}_1 \\ 0, & \text{otherwise} \end{cases} \quad (40)$$

where $\omega = (N_1 + G)\beta + N$. In particular, we have $\mu_{N_1+G}(\theta, \gamma) > 0$ in (40), which is equivalent to the following:

$$\gamma_{N_1+G} > \left(\gamma_0^N \prod_{i=1}^{N_1+G-1} \gamma_i^{\beta} \right)^{\frac{1}{(N_1+G-1)\beta+N}}. \quad (41)$$

On the other hand, from the definition of $\mathcal{N}_1$, we know

$$\frac{1}{\gamma_0^{\frac{1}{\beta+1}} \prod_{i=1}^{N} \gamma_i^{\frac{\beta}{N(\beta+1)}}} \leq \frac{1}{\gamma_n} \quad (42)$$

where $n \in \{N_1 + 1, N_1 + 2, ..., N\}$. Plugging $n = N_1 + G$ into (42), we get

$$\gamma_{N_1+G} \leq \left(\gamma_0^N \prod_{i=1}^{N_1+G-1} \gamma_i^{\beta} \prod_{j=N_1+G+1}^{N} \gamma_j^{\beta} \right)^{\frac{1}{(N-1)\beta+N}}. \quad (43)$$

Furthermore, letting $n = (N_1+G+1), (N_1+G+2), ..., N$ in (42), respectively, we obtain a set of $(N-N_1-G)$ inequalities. Multiplying the left-hand sides and right-hand sides of these $(N - N_1 - G)$ inequalities, respectively, we generate a new inequality as follows:

$$\prod_{j=N_1+G+1}^{N} \gamma_j^{\beta} \leq \left(\gamma_0^N \prod_{i=1}^{N_1+G} \gamma_i^{\beta} \right)^{\frac{\beta(N-N_1-G)}{\omega}}. \quad (44)$$

Finally, substituting (44) into the right-hand side of (43) and re-arranging the expression, we get

$$\gamma_{N_1+G} \leq \left(\gamma_0^N \prod_{i=1}^{N_1+G-1} \gamma_i^{\beta} \right)^{\frac{1}{(N_1+G-1)\beta+N}} \quad (45)$$

which contradicts (41). Therefore, such a policy does not exist. The proof follows. ∎

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