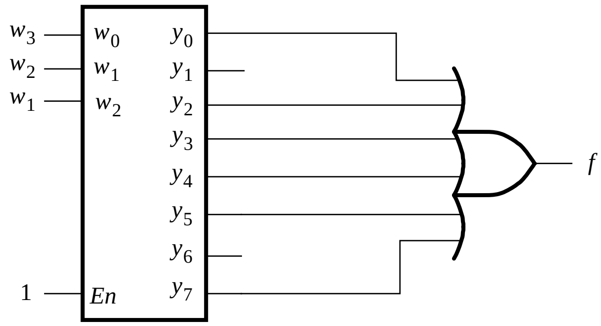
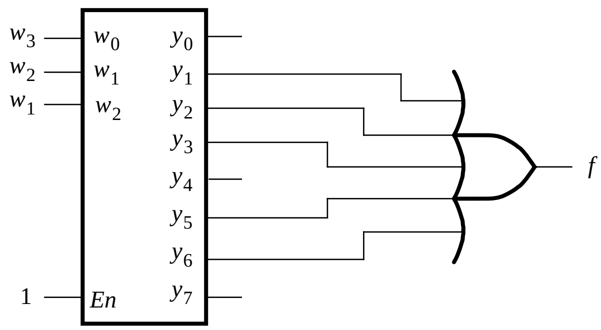
Chapter 6

6.1.



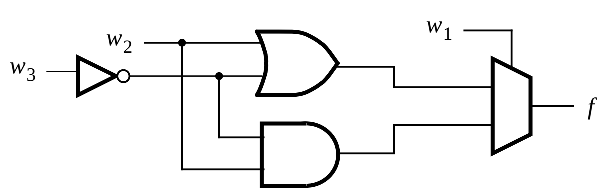
6.2.



6.3.

w_1	w_2	w_3	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

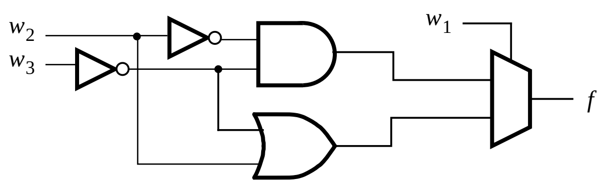
w_1	f
0	$w_2 + \overline{w}_3$
1	$w_2\overline{w}_3$



6.4.

w_1	w_2	w_3	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

w_1	f
0	$\overline{w}_2\overline{w}_3$
1	$w_2 + \overline{w}_3$



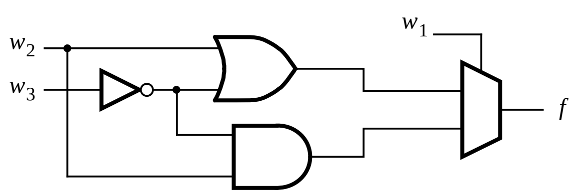
6.5. The function f can be expressed as

$$f = \overline{w}_1\overline{w}_2\overline{w}_3 + \overline{w}_1 w_2\overline{w}_3 + \overline{w}_1 w_2 w_3 + w_1 w_2\overline{w}_3$$

Expansion in terms of w_1 produces

$$f = \overline{w}_1(w_2 + \overline{w}_3) + w_1(w_2\overline{w}_3)$$

The corresponding circuit is



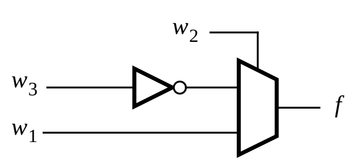
6.6. The function f can be expressed as

$$f = \overline{w}_1\overline{w}_2\overline{w}_3 + w_1\overline{w}_2\overline{w}_3 + w_1 w_2\overline{w}_3 + w_1 w_2 w_3$$

Expansion in terms of w_2 produces

$$f = \overline{w}_2(\overline{w}_3) + w_2(w_1)$$

The corresponding circuit is



6.7. Expansion in terms of w_2 gives

$$\begin{aligned} f &= \overline{w}_2(1 + \overline{w}_1\overline{w}_3 + w_1w_3) + w_2(\overline{w}_1\overline{w}_3 + w_1w_3) \\ &= \overline{w}_1\overline{w}_2\overline{w}_3 + w_1\overline{w}_2w_3 + \overline{w}_2 + \overline{w}_1w_2\overline{w}_3 + w_1w_2w_3 \end{aligned}$$

Further expansion in terms of w_1 gives

$$\begin{aligned} f &= \overline{w}_1(w_2\overline{w}_3 + \overline{w}_2\overline{w}_3 + \overline{w}_2) + w_1(w_2w_3 + \overline{w}_2w_3 + \overline{w}_2) \\ &= \overline{w}_1w_2\overline{w}_3 + \overline{w}_1\overline{w}_2\overline{w}_3 + \overline{w}_1\overline{w}_2 + w_1w_2w_3 + w_1\overline{w}_2w_3 + w_1\overline{w}_2 \end{aligned}$$

Further expansion in terms of w_3 gives

$$\begin{aligned} f &= \overline{w}_3(\overline{w}_1w_2 + \overline{w}_1\overline{w}_2 + \overline{w}_1\overline{w}_2 + w_1\overline{w}_2) + w_3(w_1w_2 + w_1\overline{w}_2 + w_1\overline{w}_2 + \overline{w}_1\overline{w}_2) \\ &= \overline{w}_1w_2\overline{w}_3 + \overline{w}_1\overline{w}_2\overline{w}_3 + w_1\overline{w}_2\overline{w}_3 + w_1w_2w_3 + w_1\overline{w}_2w_3 + \overline{w}_1\overline{w}_2w_3 \end{aligned}$$

6.8. Expansion in terms of w_1 gives

$$f = \overline{w}_1w_2 + \overline{w}_1\overline{w}_3 + w_1w_2$$

Further expansion in terms of w_2 gives

$$\begin{aligned} f &= \overline{w}_2(\overline{w}_1\overline{w}_3) + w_2(w_1 + \overline{w}_1 + \overline{w}_1\overline{w}_3) \\ &= \overline{w}_1w_2 + \overline{w}_1w_2\overline{w}_3 + \overline{w}_1\overline{w}_2\overline{w}_3 + w_1w_2 \end{aligned}$$

Further expansion in terms of w_3 gives

$$\begin{aligned} f &= \overline{w}_3(\overline{w}_1\overline{w}_2 + w_1w_2 + \overline{w}_1w_2 + \overline{w}_1w_2) + w_3(w_1w_2 + \overline{w}_1w_2) \\ &= \overline{w}_1\overline{w}_2\overline{w}_3 + w_1w_2\overline{w}_3 + \overline{w}_1w_2\overline{w}_3 + \overline{w}_1w_2w_3 + w_1w_2w_3 \end{aligned}$$

6.9. Proof of Shannon's expansion theorem

$$f(x_1, x_2, ..., x_n) = \overline{x}_1 \cdot f(0, x_2, ..., x_n) + x_1 \cdot f(1, x_2, ..., x_n)$$

This theorem can be proved using *perfect induction*, by showing that the expression is true for every possible value of x_1. Since x_1 is a boolean variable, we need to look at only two cases: $x_1 = 0$ and $x_1 = 1$.

Setting $x_1 = 0$ in the above expression, we have:

$$\begin{aligned} f(0, x_2, ..., x_n) &= 1 \cdot f(0, x_2, ..., x_n) + 0 \cdot f(1, x_2, ..., x_n) \\ &= f(0, x_2, ..., x_n) \end{aligned}$$

Setting $x_1 = 1$, we have:

$$\begin{aligned} f(1, x_2, ..., x_n) &= 0 \cdot f(0, x_2, ..., x_n) + 1 \cdot f(1, x_2, ..., x_n) \\ &= f(1, x_2, ..., x_n) \end{aligned}$$

This proof can be performed for any arbitrary x_i in the same manner.

6.10. Derivation using $\overline{f}$:

$$\begin{aligned} \overline{f} &= \overline{w}\overline{f}_{\overline{w}} + w\overline{f}_w \\ f &= \left(\overline{\overline{w}\overline{f}_{\overline{w}} + w\overline{f}_w}\right) \\ &= \left(\overline{\overline{w}\overline{f}_{\overline{w}}}\right) \cdot \left(\overline{w\overline{f}_w}\right) \\ &= (w + f_{\overline{w}})(\overline{w} + f_w) \end{aligned}$$