2.10 (5 points)

Let f = m(1, 2, 3, 4, 5, 6, 7).

The canonical sum-of-products for f is given by

$$f = x_1' x_2' x_3 + x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3$$

It can be manipulated as follows:

$$\begin{aligned}
f &= m(4, 5, 6, 7) + m(2, 3, 6, 7) + m(1, 3, 5, 7) \\
&= (x_1 x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3) \\
&\quad + (x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2 x_3' + x_1 x_2 x_3) \\
&\quad + (x_1' x_2' x_3 + x_1' x_2 x_3 + x_1 x_2' x_3 + x_1 x_2 x_3) \\
&= x_1 (x_2' x_3' + x_2' x_3 + x_2 x_3' + x_2 x_3) \\
&\quad + x_2 (x_1' x_3' + x_1' x_3 + x_1 x_3' + x_1 x_3) \\
&\quad + x_3 (x_1' x_2' + x_1' x_2 + x_1 x_2' + x_1 x_2) \\
&= x_1 (x_2' (x_3' + x_3) + x_2 (x_3' + x_3)) \\
&\quad + x_2 (x_1' (x_3' + x_3) + x_1 (x_3' + x_3)) \\
&\quad + x_3 (x_1' (x_2' + x_2) + x_1 (x_2' + x_2)) \\
&= x_1 + x_2 + x_3
\end{aligned}$$

2.11 (5 points)

Let f = M(0, 1, 2, 3, 4, 5, 6).

The canonical sum-of-products for f is given by

$$\begin{aligned}
f &= (x_1 + x_2 + x_3) (x_1 + x_2 + x_3') (x_1 + x_2' + x_3) (x_1 + x_2' + x_3') (x_1' + x_2 + x_3) \bullet \\
&\quad (x_1' + x_2 + x_3') (x_1' + x_2' + x_3)
\end{aligned}$$

It can be manipulated as follows:

$$\begin{aligned}
f &= M(0, 1, 2, 3) \bullet M(0, 1, 4, 5) \bullet M(0, 2, 4, 6) \\
&= ((x_1 + x_2 + x_3) (x_1 + x_2 + x_3') (x_1 + x_2' + x_3) (x_1 + x_2' + x_3')) \bullet \\
&\quad ((x_1 + x_2 + x_3) (x_1 + x_2 + x_3') (x_1' + x_2 + x_3) (x_1' + x_2 + x_3')) \bullet \\
&\quad ((x_1 + x_2 + x_3) (x_1 + x_2' + x_3) (x_1' + x_2 + x_3)(x_1' + x_2' + x_3)) \\
&= ((x_1 + x_2 + x_3) (x_1 + x_2 + x_3') (x_1 + x_2' + x_3) (x_1 + x_2' + x_3'))'' \bullet \\
&\quad ((x_1 + x_2 + x_3) (x_1 + x_2 + x_3') (x_1' + x_2 + x_3) (x_1' + x_2 + x_3'))'' \bullet \\
&\quad ((x_1 + x_2 + x_3) (x_1 + x_2' + x_3) (x_1' + x_2 + x_3)(x_1' + x_2' + x_3))'' \\
&= (x_1' x_2' x_3' + x_1' x_2' x_3 + x_1' x_2 x_3' + x_1' x_2 x_3)' \bullet \\
&\quad (x_1' x_2' x_3' + x_1' x_2' x_3 + x_1 x_2' x_3' + x_1 x_2' x_3)' \bullet \\
&\quad (x_1' x_2' x_3' + x_1' x_2 x_3' + x_1 x_2' x_3' + x_1 x_2 x_3')' \qquad \textbf{obtained by using DeMorgan's Theorem} \\
&= (x_1' (x_2' x_3' + x_2' x_3 + x_2 x_3' + x_2 x_3))' \bullet \\
&\quad (x_2' (x_1' x_3' + x_1' x_3 + x_1 x_3' + x_1 x_3))' \bullet \\
&\quad (x_3' (x_1' x_2' + x_1' x_2 + x_1 x_2' + x_1 x_2))' \\
&= x_1'' \bullet x_2'' \bullet x_3'' \\
&= x_1 x_2 x_3
\end{aligned}$$

2.12 (5 points)

Derivation of the minimum sum-of-products expression:

$$\begin{aligned} f &= x_1x_3 + x_1\overline{x}_2 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1(\overline{x}_2 + x_2)x_3 + x_1\overline{x}_2(\overline{x}_3 + x_3) + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1\overline{x}_2x_3 + x_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1x_3 + (x_1 + \overline{x}_1)x_2x_3 + (x_1 + \overline{x}_1)\overline{x}_2\overline{x}_3 \\ &= x_1x_3 + x_2x_3 + \overline{x}_2\overline{x}_3 \end{aligned}$$

2.13 (5 points)

Derivation of the minimum sum-of-products expression:

$$\begin{aligned} f &= x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3(\overline{x}_4 + x_4) + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3x_4 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2(\overline{x}_3 + x_3)\overline{x}_4 + x_1x_2x_4 \\ &= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2\overline{x}_4 + x_1x_2x_4 \end{aligned}$$

2.14 (5 points)

The simplest POS expression is derived as

$$\begin{aligned} f &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\ &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\ &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)((x_1 + \overline{x}_2 + x_4)(x_3 + \overline{x}_3)) \\ &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4) \cdot 1 \\ &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4) \end{aligned}$$

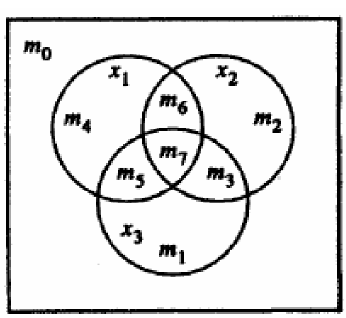
2.15 (5 points)

The simplest POS expression is derived as

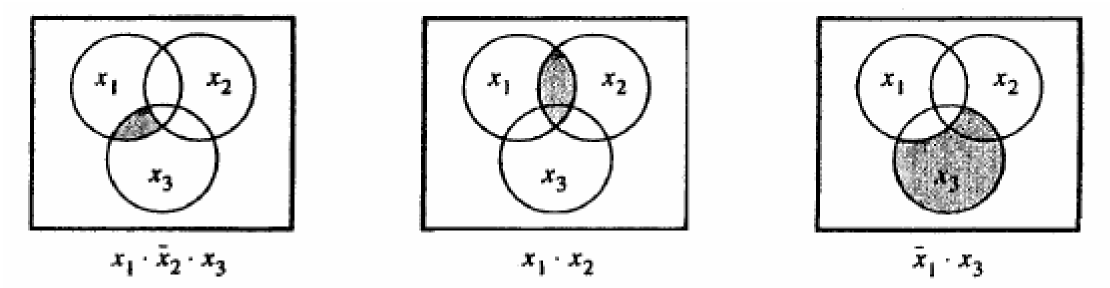
$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)(x_1 + (\overline{x}_2 + x_3))(\overline{x}_1 + (\overline{x}_2 + x_3)) \\ &= (x_1 + x_2)(\overline{x}_2 + x_3) \end{aligned}$$

2.16

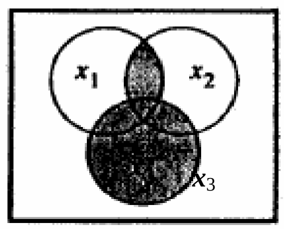
(a) (5 points) Location of all minterms in a 3-varialbe Venn diagram:



(b) (5 points) We have



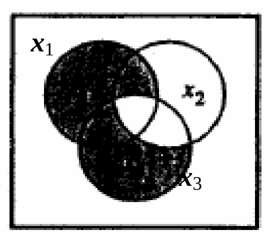
Therefore, we get the Venn Diagram for f as follows:



,

which implies that the minimal sum-of-products for f is given by f = x_1 x_2 + x_3.

2.17 (5 points)



which implies that the minimal sum-of-products is given by f = x_1 x_3' + x_2' x_3.

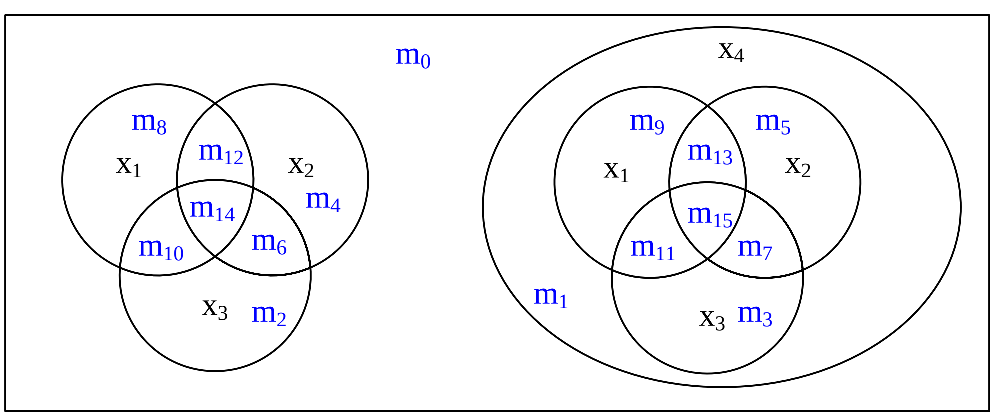
2.18 (5 points)

In Figure P2.1a, it is possible to represent only 14 minterms. It is impossible to represent the minterms x_1' x_2' x_3 x_4 and x_1 x_2 x_3' x_4'.

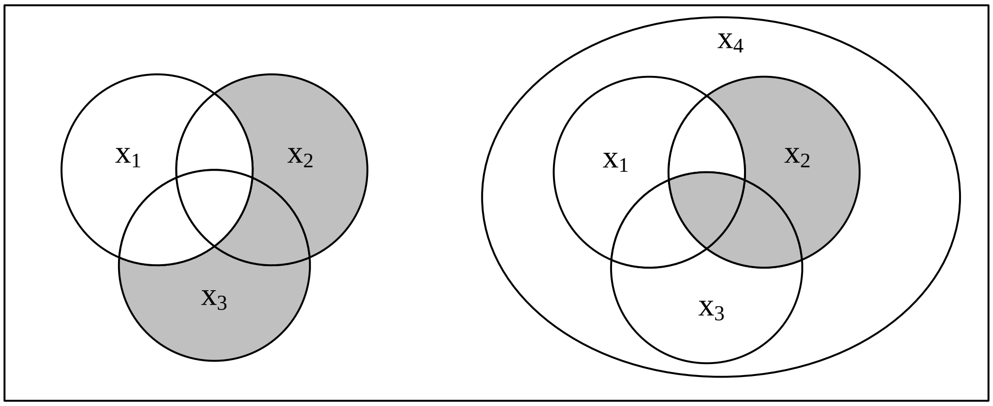
In Figure P2.1b it is impossible to represent the minterms x_1 x_2 x_3' x_4' and x_1 x_2 x_3 x_4'.

2.19

(5 points) Locations of minterms:



(5 points) Representation of f:



2.20 (5 points)

The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= (\overline{x}_1 + x_1) x_2 x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 \\ &= x_2 x_3 + x_1 \overline{x}_3 \end{aligned}$$

2.21 (5 points)

The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 \overline{x}_2 x_3 + \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= \overline{x}_1 (\overline{x}_2 + x_2) x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 + (\overline{x}_1 + x_1) x_2 x_3 \\ &= \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_2 x_3 \end{aligned}$$

Another possibility is

$$f = \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_1 x_2$$

2.22 (5 points)

The simplest SOP implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \end{aligned}$$

2.23 (5 points)

The simplest SOP implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)((\overline{x}_1 + x_3) + x_2)((\overline{x}_1 + x_3) + \overline{x}_2) \\ &= (x_1 + x_2)(\overline{x}_1 + \overline{x}_3) \end{aligned}$$

2.24 (5 points)

The simplest sum-of-products expression for f is derived as follows:

$f = x_1 x_3' x_4' + x_2 x_3' x_4 + x_1 x_2' x_3'$

$= x_3' (x_1 x_4' + x_2 x_4 + x_1 x_2')$

$= x_3' (x_2 x_4 + (x_1 x_4' + x_1 x_2'))$

$= x_3' (x_2 x_4 + x_1 (x_4' + x_2')' ')$

$= x_3' (x_2 x_4 + x_1 (x_2 x_4)')$ **obtained by using DeMorgan's Theorem**

Letting $g = x_2 x_4$, we get

$f = x_3' (g + x_1 g')$

$= x_3' (g + x_1)$ **obtained by using 16a**

$= x_3' (x_2 x_4 + x_1)$

$= x_2 x_3' x_4 + x_1 x_3'$

2.25 (5 points)

$f = x_1' x_3' x_5' + x_1' x_3' x_4' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' (x_3' x_5' + x_3' x_4' + x_4 x_5) + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

Letting $x = x_5'$, $y = x_3'$, and $z = x_4$, we get

$f = x_1' ((x y + x' z) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x y + y z + x' z) + x_3' x_4') + x_1 x_2' x_3' x_5$ **obtained by using the consensus property**

$= x_1' ((x_5' x_3' + x_3' x_4 + x_5 x_4) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_3' x_4 + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' (x_3' x_5' + x_3' x_4 + x_4 x_5 + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' (x_3' (x_5' + x_4 + x_4') + x_4 x_5) + x_1 x_2' x_3' x_5$

$= x_1' x_3' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' x_4 x_5 + x_3' (x_1' + x_1 x_2' x_5)$

$= x_1' x_4 x_5 + x_3' (x_1' + x_2' x_5)$

$= x_1' x_3' + x_1' x_4 x_5 + x_2' x_3' x_5$

An alternative manipulation by using DeMorgan's Theorem

$f = x_1' x_3' (x_4' + x_5')' ' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' x_3' (x_4 x_5)' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

Letting $g = x_4 x_5$, we get

$f = x_1' x_3' g' + x_1' g + x_1 x_2' x_3' x_5$

$= x_1' (x_3' g' + g) + x_1 x_2' x_3' x_5$

$= x_1' (x_3' + g) + x_1 x_2' x_3' x_5$

$= x_1' x_3' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' x_4 x_5 + x_3' (x_1' + x_1 x_2' x_5)$

$= x_1' x_4 x_5 + x_3' (x_1' + x_2' x_5)$

$= x_1' x_3' + x_1' x_4 x_5 + x_2' x_3' x_5$

2.26 (5 points)

The simplest product-of-sums expression for f is derived as follows:

$$
\begin{aligned}
f &= (x_1' + x_3' + x_4')(x_2' + x_3' + x_4)(x_1 + x_2' + x_3') \\
&= (x_1 + x_2' + x_3')(x_1' + x_3' + x_4')(x_2' + x_3' + x_4)
\end{aligned}
$$

Letting $x = x_1$, $y = x_2' + x_3'$, $z = x_3' + x_4'$, we get

$$
\begin{aligned}
f &= (x + y)(x' + z)(x_2' + x_3' + x_4) \\
&= (x + y)(x' + z)(y + z)(x_2' + x_3' + x_4) \qquad \textbf{obtained by using the consensus property 17b} \\
&= (x_1 + x_2' + x_3')(x_1' + x_3' + x_4')(x_2' + x_3' + x_4')(x_2' + x_3' + x_4) \\
&= (x_1 + x_2' + x_3')(x_1' + x_3' + x_4')(((x_2' + x_3') + x_4')(x_2' + x_3') + x_4)) \\
&= (x_1 + x_2' + x_3')(x_1' + x_3' + x_4')(x_2' + x_3') \\
&= ((x_1 + (x_2' + x_3'))(x_2' + x_3')(x_1' + x_3' + x_4') \\
&= (x_2' + x_3')(x_1' + x_3' + x_4')
\end{aligned}
$$

2.27 (5 points)

The simplest product-of-sums expression for f is derived as follows:

$$
\begin{aligned}
f &= (x_2' + x_3 + x_5)(x_1 + x_3' + x_5)(x_1 + x_2 + x_5)(x_1 + x_4' + x_5') \\
&= ((x_2' + x_3 + x_5)(x_1 + x_3' + x_5))(x_1 + x_2 + x_5)(x_1 + x_4' + x_5')
\end{aligned}
$$

Letting $x = x_3$, $y = x_2' + x_5$, and $z = x_1 + x_5$, we get

$$
\begin{aligned}
f &= ((x + y)(x' + z))(x_5 + x_1 + x_2)(x_5' + x_1 + x_4') \\
&= ((x + y)(y + z)(x' + z))(x_5 + x_1 + x_2)(x_5' + x_1 + x_4') \\
&= (x_2' + x_3 + x_5)(x_1 + x_2' + x_5)(x_1 + x_3' + x_5)(x_5 + x_1 + x_2)(x_5' + x_1 + x_4') \\
&= ((x_1 + x_2' + x_5)(x_1 + x_2 + x_5)(x_1 + x_3' + x_5))(x_2' + x_3 + x_5)(x_5' + x_1 + x_4') \\
&= (x_1 + x_5)(x_2' + x_3 + x_5)(x_5' + x_1 + x_4') \\
&= (x_2' + x_3 + x_5)((x_1 + x_5)(x_5' + x_1 + x_4'))
\end{aligned}
$$

Letting $u = x_5$, $v = x_1$, $w = x_1 + x_4'$, we get

$$
\begin{aligned}
f &= (x_2' + x_3 + x_5)((u + v)(u' + w)) \qquad \textbf{using the consensus property 17b} \\
&= (x_2' + x_3 + x_5)((u + v)(v + w)(u' + w)) \\
&= (x_2' + x_3 + x_5)(x_1 + x_5)(x_1 + x_4')(x_5' + x_1 + x_4') \\
&= (x_2' + x_3 + x_5)(x_1 + x_5)((x_1 + x_4')(x_5' + x_1 + x_4')) \\
&= (x_2' + x_3 + x_5)(x_1 + x_5)(x_1 + x_4')
\end{aligned}
$$