Solution to Homework #2

Note: In the following solutions, the notation x' is equivalent to $\overline{x}$.

2.1 (10 points)

Proof:

$$\begin{aligned}(x + y) \bullet (x + z) &= xx + xz + xy + yz \\ &= x + xz + xy + yz \\ &= x(1 + z + y) + yz \\ &= x \bullet 1 + yz \\ &= x + yz\end{aligned}$$

2.2 (10 points)

Proof:

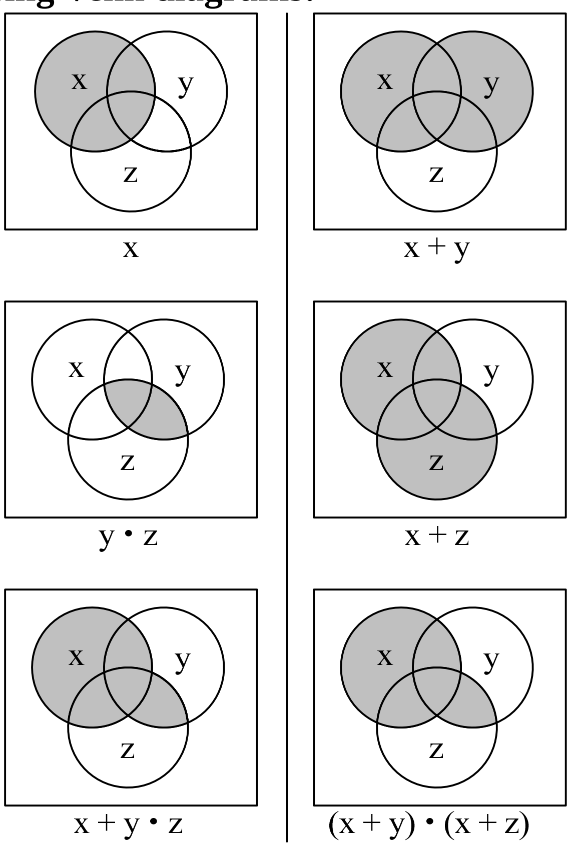
$$\begin{aligned}(x + y) \bullet (x + y') &= xx + xy + xy' + yy' \\ &= x + xy + xy' + 0 \\ &= x(1 + y + y') \\ &= x\end{aligned}$$

2.3 (10 points)

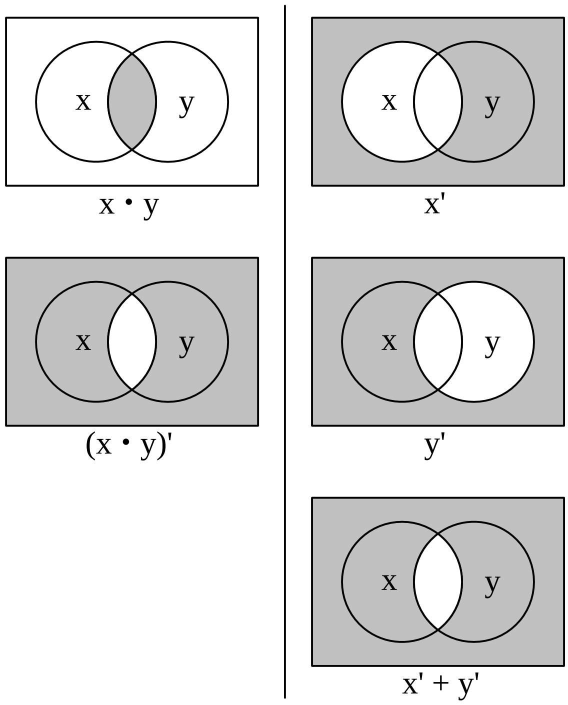
Proof:

$$\begin{aligned}xy + yz + x'z &= xy + (x + x')yz + x'z \\ &= xy + xyz + x'yz + x'z \\ &= xy(1+z) + x'z(y+1) \\ &= xy + x'z\end{aligned}$$

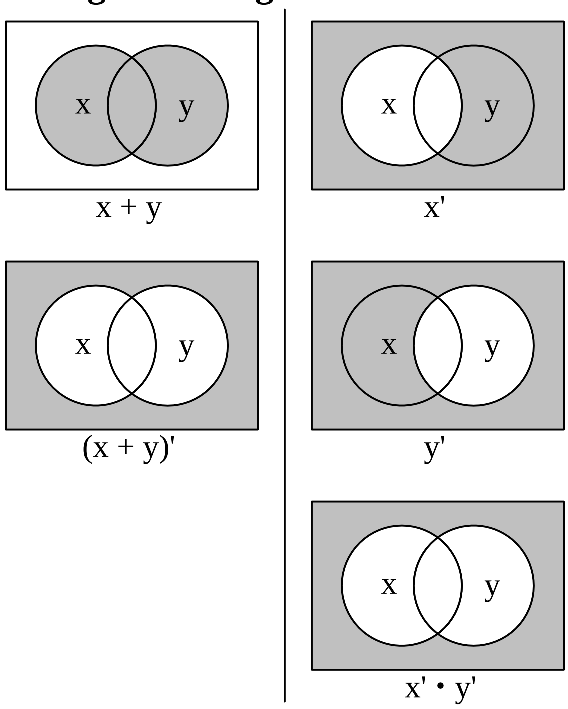
2.4 (10 points) Proof using Venn diagrams:



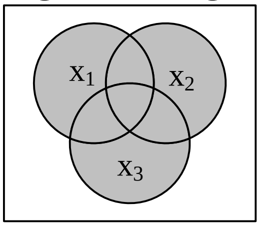
2.5 (10 points) Proof of 15a using Venn diagrams:



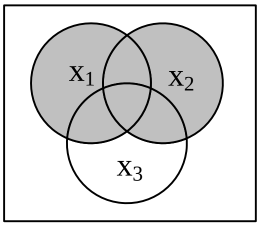
(10 points) Proof of 15b using Venn diagrams:



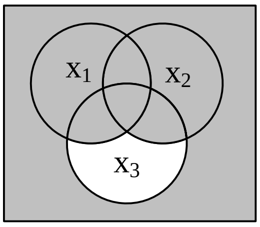
2.6 (10 points) Proof using Venn diagrams:



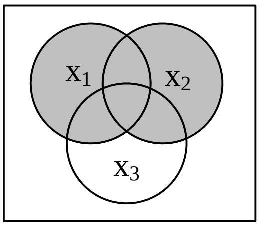
$x_1 + x_2 + x_3$



$x_1 + x_2$



$x_1 + x_2 + x_3'$



$(x_1+x_2+x_3)\cdot(x_1+x_2+x_3')$

2.7 Note: Colors are used to help you understand the algebraic manipulation. Also, the manipulation process is not unique.

(a) (10 points. To get full points, you need to include the detailed algebraic manipulation. Proof by using truth table gets at most half points. If neither detailed manipulation nor truth table is included, no points will be given.)

$$x_1' x_3 + x_1 x_2 x_3' + x_1' x_2 + x_1 x_2'$$

$$= x_1' (x_2' + x_2) x_3 + x_1 x_2 x_3' + x_1' x_2 (x_3' + x_3) + x_1 x_2' (x_3' + x_3)$$

write twice

$$= x_1' x_2' x_3 + x_1' x_2 x_3 + x_1 x_2 x_3' + x_1 x_2 x_3' + x_1' x_2 x_3' + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1 x_2' x_3$$

$$= (x_1' x_2' x_3 + x_1 x_2' x_3) + (x_1 x_2 x_3' + x_1 x_2' x_3') + (x_1 x_2 x_3' + x_1' x_2 x_3') + x_1' x_2 x_3$$

$$= (x_1' + x_1) x_2' x_3 + x_1 (x_2 + x_2') x_3' + (x_1 + x_1') x_2 x_3' + x_1' x_2 x_3$$

$$= x_2' x_3 + x_1 x_3' + x_2 x_3' + x_1' x_2 x_3$$

Therefore, expression (a) is valid.

(b) (10 points. To get full points, you need to include the detailed algebraic manipulation. Proof by using truth table gets at most half points. If neither detailed manipulation nor truth table is included, no points will be given.)

$$\text{RHS} = (x_1 + x_2' + x_3)(x_1 + x_2 + x_3')(x_1' + x_2 + x_3')$$

$$= ((x_2 + x_3') + x_1)((x_2+x_3') + x_1')(x_1 + x_2' + x_3)$$

$$= (x_2 + x_3')(x_1 + x_2' +x_3)$$

$$= x_1 x_2 + x_1 x_3' + x_2 x_2' + x_2' x_3' + x_2 x_3 + x_3 x_3'$$

$$= x_1 x_2 + x_1 x_3' + x_2' x_3' + x_2 x_3$$

$$= \quad x_1 x_2 (x_3 + x_3') + x_1 x_3' + x_2' x_3' + x_2 x_3$$
$$= \quad (x_1 x_3' x_2 + x_1 x_2 x_3) + x_1 x_3' + x_2' x_3' + x_2 x_3$$
$$= \quad (x_1 x_3' x_2 + x_1 x_3') + (x_1 x_2 x_3 + x_2 x_3) + x_2' x_3'$$
$$= x_1 x_3' + x_2 x_3 + x_2' x_3' = \text{LHS}$$

Therefore, expression (b) is valid.

(c) (10 points. You can get full points by just using truth table)

x_1	x_2	x_3	$(x_1 + x_3)(x_1' + x_2' + x_3')(x_1' + x_2)$	$(x_1 + x_2)(x_2 + x_3)(x_1' + x_3')$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

Therefore, expression (c) is not valid.