A Boolean Satisfiability based Solution to the Routing and Wavelength Assignment Problem in Optical Telecommunication Networks

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Abstract

Wavelength Division Multiplexing (WDM) effectively multiplies the bandwidth of an optic fiber by transmitting data over several different wavelengths on the same fiber. WDM is widely used to handle the ever-increasing demand for bandwidth in fiber optic telecommunication networks. Routing and Wavelength Assignment (RWA) is a critical problem to be addressed in WDM optical telecommunication networks. The goal of RWA is to maximize throughput by optimally and simultaneously assigning routes and wavelengths for a given pattern of routing or connection requests. In this paper, we present a novel technique to solve the static RWA problem using Boolean satisfiability (SAT). After formulating the RWA problem as a SAT instance, we utilize a very efficient SAT solver to find a solution. We report results for networks with and without wavelength translation capabilities in the nodes. In both cases we obtain an assignment in significantly less than one second (which is 3-4 orders of magnitude faster than existing approaches) for a set of benchmark problems. Our technique can handle arbitrary network topologies, and, due to its efficiency, can be extended to handle dynamic RWA instances, in which the network topology, link capacities and connection requests are time-varying.

Keywords: Optical Networking, Dense Wavelength Division Multiplexing (DWDM), Routing and Wavelength Assignment (RWA), Boolean Satisfiability (SAT).

1. Introduction

In fiber optic telecommunication networks, a lightpath consists of a sequence of active links from an origin to a destination along with the hardware needed to support traffic along this path. The capacity of a link in a WDM network corresponds to the number of wavelengths of light that the fiber and node-terminating equipment comprising the link can support. A pair of lightpaths that share a common link must use different wavelengths on that link, but translators can be installed at nodes to switch wavelengths along a lightpath. Hence, a lightpath having wavelength translators at every node could use different wavelengths on each link.

Routing and wavelength assignment (RWA) is the problem of simultaneously selecting lightpaths and assigning wavelengths for each link to meet a set of point-to-point demands in fiber optic telecommunication networks. The RWA problem can either be *static* (off-line) or *dynamic* (on-line). This paper addresses the static RWA problem where the connection requests are known with certainty offline (before the network is online), while the network is planned. The objective is to assign a wavelength to each light path, such that light paths on a given link have different wavelengths.

The RWA problem has two variants, one that *allows wavelength translation* and the other that *does not allow wavelength translation*. This is equivalent to the Virtual Wavelength Path (VWP) and Wavelength Path (WP) notation [1] which is commonly used in the optical networking community. The latter version satisfies the wavelength continuity constraint where the same wavelength assignment must be used for a connection request from origin to destination. Such a problem is relevant when wavelength translators are not available at the nodes in the network.

In this paper we address both versions. We formulate the RWA problem as a SAT instance, and then use a state of the art SAT solver to produce an assignment. In both cases, an assignment is obtained in significantly less than a second. This is between 3 and 4 orders of magnitude faster than recently reported results [2]. Our formulation can be easily extended to perform dynamic RWA, on account of its efficiency and speed. It can be easily modified to handle networks with time-varying topologies, links with time-varying capacities, nodes with different wavelength translation capability as well as dynamically varying connection requests.

2. Previous Work

The RWA problem has received significant attention in recent times. An excellent overview of RWA research is found in [3]. Many RWA solution methods, such as [4] are formulated based on Integer Linear Programming (ILP) techniques. However, the run-times of the resulting problems are often very large, and these techniques can typically only handle small networks. As a result, several heuristic approaches to RWA [5, 6, 7, 8, 9] have been developed. In [2], the authors present a heuristic RWA approach based on tabu search for networks with wavelength translation. Another approach [10] utilizes a genetic algorithm formulation. An approach (for ring networks) formulated using an integer programming approach, is [11]. It is hard to compare the efficiency of these methods with ours since they utilize randomly generated data for testing. In [12], the authors propose an alternate solution technique based on the maximum edge disjoint path (EDP) problem. However, testing is again done on random graphs, making comparison difficult. Other approaches like [13] are

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formulated for specific network topologies (a mesh) with no wavelength conversion allowed. The problem is formulated as a spatio-temporal combinatorial optimization problem, and solved using branch-and-bound as well as tabu search. No run-time numbers are reported.

Boolean satisfiability (SAT) is the problem of determining whether a Boolean formula in conjunctive normal form (CNF) has a satisfying assignment. SAT is an NP complete problem [14]. Several heuristic approaches exist for efficient solution of SAT. Among these are Zchaff[15] and GRASP [16]. In GRASP, efficiency results from the use of non-chronological backtrack. Zchaff improves these results further by an efficient mechanism of 'watching' literals in the clauses. In all cases, if a SAT solver determines that a formula is satisfiable, it also returns the corresponding satisfying assignment.

The problem of 3-SAT has been reduced to RWA, to prove the NP-completeness for the problem of dynamic routing of shared-path-protected connection in WDM network [17, 18]. However, based on our detailed survey of existing solution approaches for the RWA problem, there has been no Boolean Satisfiability (SAT) based *solution* technique. As we will see in the sequel, this yields an elegant formulation of the RWA problem. By using the state-of-the-art SAT solver Zchaff [15], we are able to solve the RWA problem very efficiently.

3. Preliminaries and Terminology

DEFINITION 1. *A* **conjunctive normal form (CNF)** *Boolean formula f on n Boolean variables $x_1, x_2, ..., x_n$ is a conjunction (logical AND) of m clauses $c_1, c_2, ..., c_m$. Each clause c_i is the disjunction (logical OR) of its constituent literals.*

For example

$$f = (x_1 + x_3) \cdot (\overline{x_1} + \overline{x_2} + \overline{x_3})$$

is a CNF formula with two clauses, $c_1 = (x_1 + x_3)$ and $c_2 = (\overline{x_1} + \overline{x_2} + \overline{x_3})$.

DEFINITION 2. **Boolean satisfiability (SAT)** *is the problem of determining whether a Boolean formula in conjunctive normal form (CNF) has a satisfying assignment.*

In the above example, *a* satisfying assignment[1] of variables for the formula f is $x_1 = 1, x_2 = 0$.

In theory, there can be two outcomes of the SAT process – 1) the problem is not satisfiable and 2) the problem is satisfiable. In the latter case, the SAT solver returns an assignment of variables which satisfies the CNF formula. In practice, however, there may be a third outcome. If either a satisfiable or unsatisfiable verdict cannot be reached in a user-specified time limit, a SAT solver will return an *unknown* result.

DEFINITION 3. *An* **optical network** *N is modeled as a graph $G(V, E)$, with vertices (nodes) v_i and edges e_{ij}. Nodes v_i of G correspond to nodes i in N. An edge e_{ij} is present in G iff there is a fiber connecting nodes i and j in N.*

Our formulation is flexible in that it handles arbitrary network topologies.

DEFINITION 4. *A* **connection or routing request** *is a pair of nodes (i, j) in N, with i as the origin (start node) and j as the destination (end) node. We denote the k^{th} connection request as $R_k \equiv (v_i, v_j)$*

[1]A satisfying assignment of f is an assignment of values to some or all variables of f which makes the function f evaluate to *TRUE*

Note that we currently assume bidirectional links. This can easily be generalized so we can model unidirectional links.

In general, a network may have multiple connection requests, all of which have to be routed (and assigned wavelengths) simultaneously. The capacity of a link in a WDM network corresponds to the number of wavelengths of light that the fiber and node-terminating equipment comprising the link can support.

The variables used in the formulation of SAT clauses are given below:

DEFINITION 5. *The Boolean variable $v_i^{R_k \lambda_p}$ represents the logical condition of whether a node i is part of the k^{th} connection request R_k using wavelength λ_p. If $v_i^{R_k \lambda_p} = 1$, we refer to the node i as an* **active node**

Note that this definition of active-ness of a node is utilized for SAT clause formulation when wavelength translation is *not* allowed. In this case, a node is active with respect to a route as well as a wavelength.

DEFINITION 6. *The Boolean variable $e_{ij}^{R_k \lambda_p}$ represents the logical condition of whether the edge connecting nodes i and j utilizes wavelength λ_p for the k^{th} connection request R_k. If $e_{ij}^{R_k \lambda_p} = 1$, we refer to the edge e_{ij} as an* **active edge**.

DEFINITION 7. *The Boolean variable $v_i^{R_k}$ represents the logical condition of whether the node i is part of the connection request R_k. If $v_i^{R_k} = 1$, we refer to the node i as an* **active node**

Note that this definition of active-ness of a node is utilized for SAT clause formulation when wavelength translation *is* allowed. In this case, a node is active with respect to a route alone.

4. Problem Formulation

4.1 Overview

The formulation of the SAT based RWA problem comprises the task of constructing SAT clauses to encode the constraints and requirements of the RWA problem. Our clauses are written for a fixed number q of wavelengths. Several types of SAT clauses are written, which are explained in the rest of this section. The final CNF expression (consisting of the conjunction (logical AND) of all such clauses) comprises the SAT instance to be solved.

This SAT instance is supplied as input to a state-of-the-art SAT solver (Zchaff [15] in our case). If the SAT solver returns a *satisfiable* result, it also returns the variable assignments which result in the satisfiable result. From this variable assignment, we can determine the RWA solution trivially. If the SAT solver returns an *unsatisfiable* or *unknown* result, we increment the number of allowable wavelengths q, regenerate the SAT instance, and attempt to solve it again.

Suppose there exists an exact solution to the RWA problem with L wavelengths. Whenever the SAT solver returns an *unknown* value for $L-1$ wavelengths, then the final result returned by our technique may be worse than the number of wavelengths in an exact solution. However, for all the experiments we ran, this condition did not occur. As a consequence, a comparison of our method with previous exact techniques is a fair one, for the experiments we conducted.

In the rest of this section we explain the formulation of clauses which define the RWA problem, for both variants of the RWA problem (with and without wavelength translation).

4.2 Clauses for Wavelength Translation

If the RWA problem allows wavelength translation, the SAT clauses are as described below. Note that the notation "$a \Rightarrow b$" indicates that a logically implies b.

- The start node must have at least one active edge per route

$$\left(\bigvee_{p=1}^{q} \bigvee_{x\in adj\_nodes(i)} e_{ix}^{R_k\lambda_p}\right) \quad (1)$$

Such clauses are written for all routes R_k where v_i is the start node of the route.

- The end node must have at least one active edge per route

$$\left(\bigvee_{p=1}^{q} \bigvee_{x\in adj\_nodes(i)} e_{ix}^{R_k\lambda_p}\right) \quad (2)$$

Such clauses are written for all routes R_k where v_i is the end node of the route.

- The start node must have at most one active edge per route

$$\left(e_{ix}^{R_k\lambda_r} \Rightarrow (\prod_{\substack{t=1\\t\neq r}}^{q} \overline{e_{ix}^{R_k\lambda_t}}) \prod_{p=1}^{q} \prod_{y\in adj\_nodes(i)} \overline{e_{iy}^{R_k\lambda_p}}\right) \quad ;y\neq x \quad (3)$$

Such clauses are written for all routes R_k and all wavelengths λ_p where v_i is the start node of the route.

- The end node must have at most one active edge per route

$$\left(e_{ix}^{R_k\lambda_r} \Rightarrow (\prod_{\substack{t=1\\t\neq r}}^{q} \overline{e_{ix}^{R_k\lambda_t}}) \prod_{p=1}^{q} \prod_{y\in adj\_nodes(i)} \overline{e_{iy}^{R_k\lambda_p}}\right) \quad ;y\neq x \quad (4)$$

Such clauses are written for all routes R_k and all wavelengths λ_p where v_i is the end node of the route.

- If a light edge adjoining a node is active, then at least one other light edge adjoining the same node must be active (excluding start and end node)

$$\left(e_{ix}^{R_k\lambda_p} \Rightarrow \bigvee_{p=1}^{q} \bigvee_{y\in adj\_nodes(i)} e_{iy}^{R_k\lambda_p}\right) \quad ;x\neq y \quad (5)$$

Such clauses are written for all routes R_k where v_i is neither start nor end node.

- At most two edges adjoining a node can be active (excluding start and end node)

$$e_{ix}^{R_k\lambda_p} \cdot e_{iy}^{R_k\lambda_r} \Rightarrow (\prod_{\forall s\neq p} \overline{e_{ix}^{R_k\lambda_s}})(\prod_{\forall t\neq r} \overline{e_{iy}^{R_k\lambda_t}})$$
$$(\prod_{z\in adj\_nodes(i)} \prod_{u=1}^{q} \overline{e_{iz}^{R_k\lambda_u}}) \quad ;z\neq x, z\neq y \quad (6)$$

Such clauses are written for all routes R_k where v_i is neither start nor end node.

- The start node must be active

$$\left(v_i^{R_k} = 1\right) \quad (7)$$

Such clauses are written for all routes R_k where v_i is the start node.

- The end node must be active

$$\left(v_i^{R_k} = 1\right) \quad (8)$$

Such clauses are written for all routes R_k where v_i is the end node.

- If a node is active and an edge connected to it is active, then the node at the other end of the edge must also be active

$$\prod_{x\in adj\_nodes(i)} \left(v_i^{R_k} \cdot \bigvee_{p=1}^{q} e_{ix}^{R_k\lambda_p} \Rightarrow v_x^{R_k}\right) \quad (9)$$

Such clauses are written for all routes R_k.

- If two nodes are active, then the light edge connecting them must be active (in some wavelength)

$$\prod_{x\in adj\_nodes(i)} \left(v_i^{R_k} \cdot v_x^{R_k} \Rightarrow \bigvee_{p=1}^{q} e_{ix}^{R_k\lambda_p}\right) \quad (10)$$

- If a node is not active then all its adjoining edges are not active

$$\left(\overline{v_i^{R_k}} \Rightarrow \prod_{x\in adj\_nodes(i)} \prod_{p=1}^{q} \overline{e_{ix}^{R_k\lambda_p}}\right) \quad (11)$$

Such clauses are written for all routes R_k.

- If a light edge is chosen in one connection request, then it cannot be chosen in any other connection request

$$\prod_{p=1}^{q} \left(e_{ij}^{R_k\lambda_p} \Rightarrow \prod_{x=1,x\neq k}^{n} \overline{e_{ij}^{R_x\lambda_p}}\right) \quad (12)$$

Such clauses are written for all routes R_k.

4.3 Clauses without Wavelength Translation

If the RWA problem does not allow wavelength translation, the SAT clauses are as described below. Some of the clauses are identical to the case with wavelength translation, however, all clauses are written below to avoid confusion.

- The start node must have at least one adjoining edge which is active, per route

$$\left(\bigvee_{p=1}^{q} \bigvee_{x\in adj\_nodes(i)} e_{ix}^{R_k\lambda_p}\right) \quad (13)$$

Such clauses are written for all routes R_k where v_i is the start node of the route.

- The end node must have at least one adjoining edge which is active, per route

$$\left(\bigvee_{p=1}^{q} \bigvee_{x\in adj\_nodes(i)} e_{ix}^{R_k\lambda_p}\right) \quad (14)$$

Such clauses are written for all routes R_k where v_i is the end node of the route.

- The start node must have at most one adjoining edge which is active, per route

$$\left(e_{ix}^{R_k\lambda_r} \Rightarrow (\prod_{\substack{t=1\\t\neq r}}^{q} \overline{e_{ix}^{R_k\lambda_t}}) \prod_{p=1}^{q} \prod_{y\in adj\_nodes(i)} \overline{e_{iy}^{R_k\lambda_p}}\right) \quad ;y\neq x \quad (15)$$

Such clauses are written for all routes R_k where v_i is the start node of the route.

- The end node must have at most one adjoining edge which is active, per route

$$\left(e_{ix}^{R_k\lambda_r} \Rightarrow (\prod_{\substack{t=1\\t\neq r}}^{q} \overline{e_{ix}^{R_k\lambda_t}}) \prod_{p=1}^{q} \prod_{y\in adj\_nodes(i)} \overline{e_{iy}^{R_k\lambda_p}}\right) \quad ;y\neq x \quad (16)$$

Such clauses are written for all routes R_k where v_i is the end node of the route.

- If a light edge adjoining a node is active, then at least one other light edge adjoining the same node must be active

(excluding start and end node)

$$\left(e_{ix}^{R_k\lambda_p} \Rightarrow \bigvee_{y \in adj\_nodes(i)} e_{iy}^{R_k\lambda_p} \right) \quad ; x \neq y \tag{17}$$

Such clauses are written for all routes R_k and wavelengths λ_p where v_i is neither start nor end node.

- At most two edges adjoining a node can be active (excluding start and end node)

$$\left(e_{ix}^{R_k\lambda_p} \cdot e_{iy}^{R_k\lambda_p} \Rightarrow \prod_{z \in adj\_nodes(i)} \prod_{p=1}^{q} \overline{e_{iz}^{R_k\lambda_p}} \right) \quad ; z \neq x, z \neq y \tag{18}$$

Such clauses are written for all routes R_k where v_i is neither start nor end node.

- The start node must be active

$$\bigvee_{p=1}^{q} \left(v_i^{R_k\lambda_p} = 1 \right) \tag{19}$$

Such clauses are written for all routes R_k where v_i is the start node.

- The end node must be active

$$\bigvee_{p=1}^{q} \left(v_i^{R_k\lambda_p} = 1 \right) \tag{20}$$

Such clauses are written for all routes R_k where v_i is the end node.

- If a node is active and an edge connected to it is active, then the node at the other end of the edge must also be active

$$\prod_{x \in adj\_nodes(i)} \prod_{p=1}^{q} \left(v_i^{R_k\lambda_p} \cdot e_{ix}^{R_k\lambda_p} \Rightarrow v_x^{R_k\lambda_p} \right) \tag{21}$$

Such clauses are written for all routes R_k.

- If two nodes are active, then the light edge connecting them must be active

$$\prod_{x \in adj\_nodes(i)} \prod_{p=1}^{q} \left(v_i^{R_k\lambda_p} \cdot v_x^{R_k\lambda_p} \Rightarrow e_{ix}^{R_k\lambda_p} \right) \tag{22}$$

- If a node is not active then all its adjoining edges are not active

$$\prod_{p=1}^{q} \left(\overline{v_i^{R_k\lambda_p}} \Rightarrow \prod_{x \in adj\_nodes(i)} \overline{e_{ix}^{R_k\lambda_p}} \right) \tag{23}$$

Such clauses are written for all routes R_k

- If a light edge is chosen in one connection request, then it cannot be chosen in any other connection request

$$\prod_{p=1}^{q} \left(e_{ij}^{R_k\lambda_p} \Rightarrow \prod_{\substack{x=1 \\ x \neq k}}^{n} \overline{e_{ij}^{R_x\lambda_p}} \right) \tag{24}$$

Such clauses are written for all routes R_k

- If a light edge adjoining the start node and end node is active in one wavelength then all other edges for that connection request must have the same wavelength

$$\prod_{j \in adj\_nodes(i)} \prod_{p=1}^{q} \left(e_{ij}^{R_k\lambda_p} \Rightarrow \prod_{x \neq i} \prod_{y \in adj\_nodes(x)} \prod_{g \neq p} \overline{e_{xy}^{R_k\lambda_g}} \right) \tag{25}$$

4.4 Overall Flow

Depending on the problem variant being considered (RWA with or without wavelength translation), we write the SAT clauses as described above. After all the clauses have been written, we construct the SAT instance (which is simply the conjunction of all the above clauses).

Note that the clauses are written for a fixed number of wavelengths q. Initially, we write clauses for $q = 1$. If the resulting SAT instance has no solution, we re-generate clauses for $q = 2$ and so on. The first value of q for which we find a solution is the smallest number of wavelengths that we need to successfully route the given WDM network.

4.5 Possible Extensions

Due to its efficiency and speed (which we will demonstrate in Section 5), our formulation can be easily extended to perform dynamic RWA as well. In particular:

- It can be modified to handle networks with time-varying topologies, which may be important in the case of networks in which link reliabilities are imperfect. Given that our method handles arbitrary graphs, we would simply regenerate the SAT instance for the modified network and find a new solution for this SAT instance.
- Our method can handle networks with time-varying capacities. This may be important when links are upgraded or new node-terminating equipment is installed. Our method handles this aspect by modeling each logical network edge as a multitude of physical edges in the SAT formulation.
- Our method can handle dynamically varying connection requests as well. This is achieved with minimal perturbation to an existing solution by creating a new SAT instance S':

  $$S' = S \cdot S^* \cdot S^{sol}$$

  Note that S' is the conjunction of the original SAT instance S, the new clauses S^* corresponding to the new connection requests and the single-literal SAT clauses corresponding to the satisfying variable assignment of S (denoted by S^{sol}). When S' is solved, we are guaranteed that the existing solution of S is unperturbed. In this way, an incremental or dynamic RWA problem can be handled in our framework.

5. Experimental Results

We implemented our technique in C++. We used Zchaff [15] as the SAT solver. For our experiments we used the same networks as used in [2]. Our experimental procedure consisted of reading in a network along with all the connection requests. Encoding was done in order to convert the problem into a SAT instance. Next, we invoked Zchaff to find the solution. We invoke Zchaff iteratively, attempting to find solutions with n wavelengths (n is initially 1, incremented by 1 in each iteration until a solution is found). The CPU times were measured on an IBM IntelliStation running Linux with a 1 GHz Pentium-4 CPU and 512 MB of RAM.

Table 1 shows our results with wavelength translation allowed at every node while Table 2 shows our results with no wavelength translation allowed (satisfying the wavelength continuity constraint). For both tables, Column 1 lists the networks used for our experiments. Columns 2 and 3 list the number of variables and clauses respectively in the SAT instance. These values represent the variables and clauses used to solve the SAT instance with minimum number of wavelengths. Column 4 lists the number of light edges in the final solution for satisfying all the connection requests, while column 5 lists the minimum number of wavelengths used to satisfy the requests. Finally, column 6 lists the total run-time required to find the final solution. The total run-time includes run-times for all the iterations of the SAT solver.

For both the variants of the RWA problem (with or without wavelength translation), our approach found solutions in sig-

Network	With Wavelength Translation				
	Variables	Clauses	Edges	Wavelength	Time in secs
A01	405	5317	19	3	0.001
A02	405	5332	19	3	0.001
ATT01	1734	41346	37	3	0.02
J01	1120	20273	30	4	0.01
J02	3000	73832	50	5	0.05
J03	1320	23957	39	4	0.02
EURO1	4740	145998	60	5	0.03

Table 1: Results with Wavelength Translation

Network	Without Wavelength Translation				
	Variables	Clauses	Edges	Wavelength	Time in secs
A01	297	7068	15	3	0.001
A02	297	7022	14	3	0.001
ATT01	1360	96410	29	3	0.01
J01	784	32861	29	4	0.03
J02	2040	147522	47	5	0.02
J03	915	42548	33	4	0.01
EURO1	3474	573138	47	5	0.11

Table 2: Results without Wavelength Translation

nificantly less than one second. This is between 3 and 4 orders of magnitude faster than other approaches [2]. The reason for the improvement can be attributed to a compact formulation of the problem using SAT. Also, there have been significant recent advances in the heuristics used by SAT solvers, which our method benefits from.

6. Conclusions

In this paper, we have described a Boolean Satisfiability (SAT) based formulation and solution methodology for the RWA problem in WDM optical telecommunication networks. Our formulation translates the optical network (with no restriction on its topology) along with the connection requests into a SAT instance. These SAT instances are parameterized in q, the number of allowed wavelengths. We solve the instances (with increasing q) using the state-of-the-art SAT solver Zchaff [15]. Experimental results demonstrate that we are able to solve a set of benchmark examples in significantly less than a second, a tremendous speed-up compared to existing approaches.

Our formulation can be easily modified to handle a dynamic RWA problem, where connection requests, network topologies and link capacities can be time-varying. In the future, we plan to extend our work to handle such application scenarios as well.

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