ECEN 605

LINEAR SYSTEMS

Lecture 19

Characteristics of Feedback Control Systems I

Introduction

In engineering systems one often needs to be able to set variables such as temperature, velocity, position and flow rate, to prescribed or desired values despite the presence of unknown disturbances and large uncertainties. It is the task of a control system to provide this capability.

In biological systems there exist built in control systems that regulate for instance, body temperature, blood pressure and blood glucose levels despite varying external weather, exercise level and sugar intake. In economic systems the Federal Reserve tries to maintain buying power despite inflationary tendencies.

A common structural characteristic of all the above systems is the presence of feedback, which allows corrective action to be taken on the error when deviations from desired values occur.

Black's Amplifier (Reliable Gains Using Reliable and Unreliable Components)

Consider the problem of designing an amplifier which is to provide an accurate gain of say $A = 100$, using unreliable components, which may vary by as much as, say 50%.

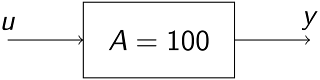

Figure 1: An open loop system

With an input $u = 1$, $y = A\,u$ may vary between 150 and 50, in Figure 1, which is an **open loop** system.

Black's Amplifier (Reliable Gains Using Reliable and Unreliable Components) (cont.)

Now consider the feedback system below,

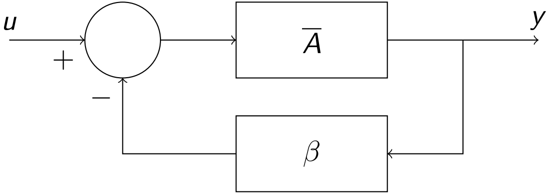

Figure 2: A feedback system

in Figure 2 where $\overline{A}$ is made with the same unreliable components as before but with the nominal value $\overline{A}_0 = 10,000$, and $\beta = 0.01$.

Black's Amplifier (Reliable Gains Using Reliable and Unreliable Components) (cont.)

It is easy to see that

$$y = \frac{\overline{A}_0}{1 + \overline{A}_0\,\beta} u \tag{1a}$$

$$= \frac{10,000}{1 + 10,000 \times 0.01} = \frac{10,000}{101} = 99.001 \cong 100. \tag{1b}$$

Moreover under $\pm 50\%$ variation in $\overline{A}_0$, we have

$$y = \frac{15,000}{1 + 15,000 \times 0.01} = \frac{15,000}{151} = 99, \tag{2}$$

and

$$y = \frac{5,000}{1 + 5,000 \times 0.01} = \frac{5,000}{51} = 98. \tag{3}$$

Black's Amplifier (Reliable Gains Using Reliable and Unreliable Components) (cont.)

We see from (1b), (2) and (3) that despite 50% variation in $\overline{A}$, the variation in y is only about 1% ! Compared to the 50% variation in y in the open loop case (Figure 1) this is a very significant improvement.

Black's Amplifier (Reliable Gains Using Reliable and Unreliable Components) (cont.)

To complete the above analysis we note that the gain of the feedback system, denoted A_f, is given by,

$$A_f = \frac{\overline{A}}{1 + \beta \overline{A}} = \frac{1}{\frac{1}{\overline{A}} + \beta} \tag{4}$$

so that as $\overline{A} \nearrow \infty$, $A_f \to \frac{1}{\beta}$. By setting $\beta = 0.01 = \frac{1}{100}$, we achieve $A_f \approx 100$.

We note that the above robustness result with respect to $\overline{A}$ was achieved by using **high gain** and also that there is **no robustness** of A_f with respect to β.

Unity Closed Loop Gain

As another application consider the feedback system

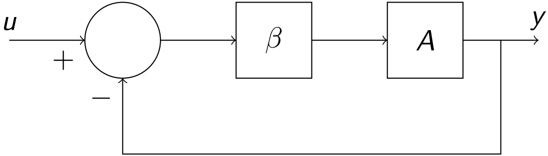

Figure 3: A unity feedback system

where a unity gain between u and y is to be achieved.

Unity Closed Loop Gain (cont.)

Since

$$y = \underbrace{\frac{A\,\beta}{1 + A\,\beta}}_{A_f} u \tag{5}$$

one achieves **unity gain** by letting $\beta \nearrow \infty$ so that

$$\lim_{\beta \nearrow \infty} = A_f = \lim_{\beta \nearrow \infty} \frac{1}{1 + \dfrac{1}{A\,\beta}} = 1, \tag{6}$$

independent of A, or robustly with respect to A.

Unity Closed Loop Gain (cont.)

The reader can convince oneself that the system in Figure 3 is far more robust than the open loop solution, where

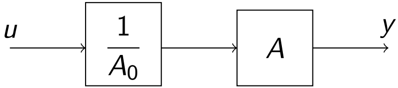

Figure 4: A unity gain open loop system

and A_0 is the nominal value of A which is subject to say 50% variation.

Robust Linearization

In many instances a physical element may have a nonlinear characteristic f, $y = f(u)$. Of may

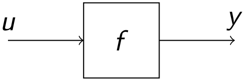

Figure 5: A nonlinear relationship

be desirable, for calibration or measurement purposes to have a linear input output relationship.

Robust Linearization (cont.)

This could be achieved by the high-gain feedback system

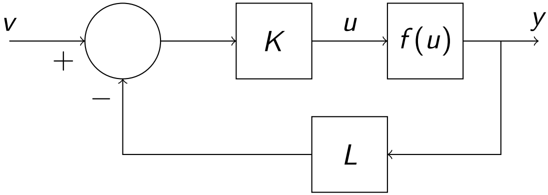

Figure 6: A high gain feedback system

since

$$v = L\,y + \frac{1}{K} f^{-1}(y) \tag{7}$$

so that as $K \nearrow \infty$, $v \nearrow L\,y$.

Robust Linearization (cont.)

Example

$$f(u) = 10\,u^3 = y. \tag{8}$$

Choose $L = 10$, $K = 100$ so that in Figure 6

$$v = 10\,y + \frac{1}{100}\sqrt[3]{\frac{y}{10}}. \tag{9}$$

If y varies between 0 and 10 we have v varying between 0 and 100 linearly with y with an error less than 1%.

Feedback Control Systems

A typical feedback control system is represented by the block diagram below; representing a unity feedback system:

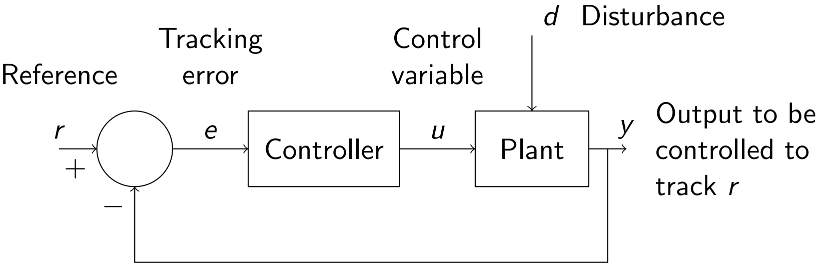

Figure 7: Unity Feedback Control System

Feedback Control Systems (cont.)

The main features of Figure 7 are:

Plant

This is the system (such as a DC Motor) whose output y (say, speed of output shaft) is to be controlled by manipulating the control input u (say, armature voltage), so that y equals the reference r (desired speed).

Controller

The controller input is the error $e = r - y$, which is the signal processed by it to determine the control signal u in such a way that y is driven into correspondence with r, despite the presence of the disturbance d (load torque).

Feedback Control Systems (cont.)

Assuming all systems are linear and time-invariant and using transfer function matrix representations we have

$$Y(s) = P(s)\,U(s) + Q(s)\,D(s) \tag{10a}$$

$$U(s) = C(s)\,E(s) \tag{10b}$$

$$E(s) = R(s) - Y(s), \tag{10c}$$

where $P(s)$ ($Q(s)$) is the transfer function between y and u (y and d). So,

$$E(s) = \underbrace{[I + P(s)\,C(s)]^{-1}}_{A(s)} R(s) + \underbrace{[I + P(s)\,C(s)]^{-1}\,Q(s)}_{B(s)} D(s). \tag{11}$$

Feedback Control Systems (cont.)

Equation (11) may be represented as

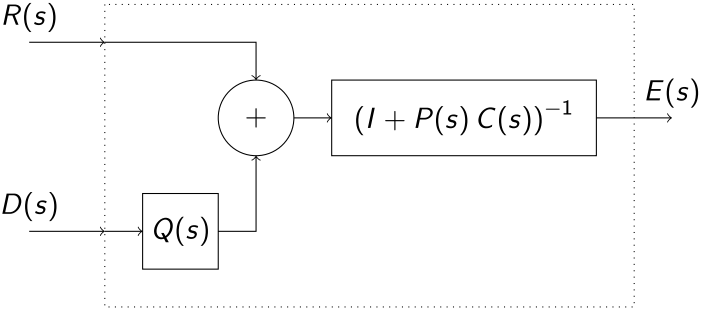

Figure 8: Diagram of Equation (11)

Feedback Control Systems (cont.)

In the representation of Figure 8 we see that the task of the control system is to maintain $e(t)$ "close" to zero even though $r(t)$ and $d(t)$ may work against it. The figure also shows that $e(t)$ may be maintained "close" to zero as long as $(I + P(s)\, C(s))^{-1}$ is "small". Since $P(s)$ is given, the latter condition means that $C(s)$ must be "large" or equivalently that $P(s)\, C(s)$, called the **loop gain**, must be "large".

We summarize this conclusion by noting that, qualitatively speaking, in a unity feedback system, tracking and disturbance rejection, that is, "small" errors with respect to reference inputs and disturbance inputs may be simultaneously achieved by using "large" loop gain, $P(s)\, C(s) =: L(s)$.

Feedback Control Systems (cont.)

The statement above is a “rough” statement since $L(s)$ is a transfer function matrix and not a number, or even a numerical matrix. In the following section we elaborate on “large” loop gain and its precise meaning. For now it suffices to note that “large” loop gain is needed over the frequency bands in which r and d have significant energy. For instance if they are slowly varying signals, “large” loop gain must be achieved over a suitable low frequency band.