ECEN 605

LINEAR SYSTEMS

Lecture 16

State Feedback and Observers IV
– A Structure of Robust Observer

A Structure of Robust Observer

Let us recall the error equation

$$\dot{e} = Me + (M - A + LC)x + (G - B)u.$$

For $e(t) \longrightarrow 0$, it is required that

$$\begin{aligned} M &= A - LC \\ G &= B. \end{aligned}$$

However, it is unlikely that L could be computed such that $M = A + LC$ exactly. It means that small mismatch error between M and $A - LC$ will cause the term $(M - A + LC)$ nonzero. Furthermore, if the state $x(t)$ is unstable, $e(t)$ will certainly blow up for a large value of t. This can be prevented if we design an observer for the following from.

$$\begin{aligned} \dot{z} &= Az + Bu + L(y - Cz) \\ \hat{x} &= z. \end{aligned}$$

A Structure of Robust Observer (cont.)

Let us write the error equation again.

$$\begin{aligned}\dot{e} &= \dot{x} - \dot{z} \\ &= Ax + Bu - Az - Bu - LC(x - z) \\ &= (A - LC)(x - z) \\ &= (A - LC)e.\end{aligned}$$

As seen, the error signal $e(t)$ will converge even if L is inaccurate as long as it maintains the stability of $A - LC$. However, this mechanism requires exact duplication of the system model (A, B, C) into the observer. This is called *a robust observer*.

A Structure of Robust Observer (cont.)

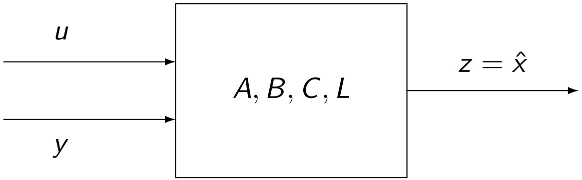

Figure 1: An Observer

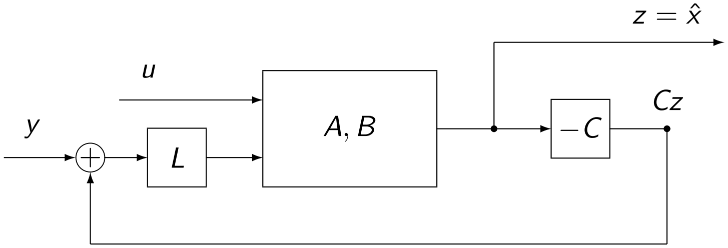

Figure 2: A Robust Observer

A Structure of Robust Observer (cont.)

Remark

The case we treated in the previous example would not be a problem if we use an observer for stabilization. Once the loop is closed by using the estimated states, $x(t)$ *will no longer blow up since the closed loop system is stable. In other words,* $e(t)$ *will converge.*

Of course, for the same argument, the state estimation will be okay without loop closure if the system is stable. Therefore, if you want to use an observer as a state estimator alone (without loop closure) for the unstable system, you will have the problem. In this case, you may want to consider to use the different structure of observer, for example a robust observer we discuss here.

A Structure of Robust Observer (cont.)

Case 1: Deasinging a Robust Obasever From

$$\begin{aligned} \dot{z} &= Az + Bu + L(y - Cz) \\ \hat{x} &= z, \end{aligned}$$

we have an observer

$$\begin{aligned} \dot{z} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} z + \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} u + \begin{bmatrix} 1.1998 & -0.0008 \\ -0.0148 & 0.0301 \\ -0.0644 & 0.4002 \end{bmatrix} \left(y - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} z \right) \\ \hat{x} &= z. \end{aligned}$$

A Structure of Robust Observer (cont.)

Case 2: Closing the Loop Consider first the observer in the last example of the previous lecture and let the desired poles of the closed loop system be

$$\Lambda_p = \{-1, -2, -3\},$$

then we have a state feedback

$$F = \begin{bmatrix} -4.5636 & 1.5970 & 0.9457 \\ -3.3733 & 5.4395 & 2.0574 \end{bmatrix}$$

which places the poles of $A + BF$ at Λ_p.

A Structure of Robust Observer (cont.)

Now we close the loop.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ c &= Cx \end{aligned}$$

$$\dot{z} = Mx + Ly + Bu$$

$$u = F\hat{x} = Fz.$$

We have

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} A & BF \\ LC & M + BF \end{bmatrix}}_{A_{cl}} \begin{bmatrix} x \\ z \end{bmatrix} \tag{1}$$

A Structure of Robust Observer (cont.)

By $Q^{-1}A_{cl}Q$ where

$$Q = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix},$$

we have another expression

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} A+BF & BF \\ 0 & M \end{bmatrix}}_{A_{cl}} \begin{bmatrix} x \\ z \end{bmatrix}. \qquad (2)$$

We can plot the states of the closed loop system by using either (1) or (2) which are the same. Using (1) with initial conditions

$$x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad z(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

we have the following figures which show the convergence (see Figures 3 and 4).

A Structure of Robust Observer (cont.)

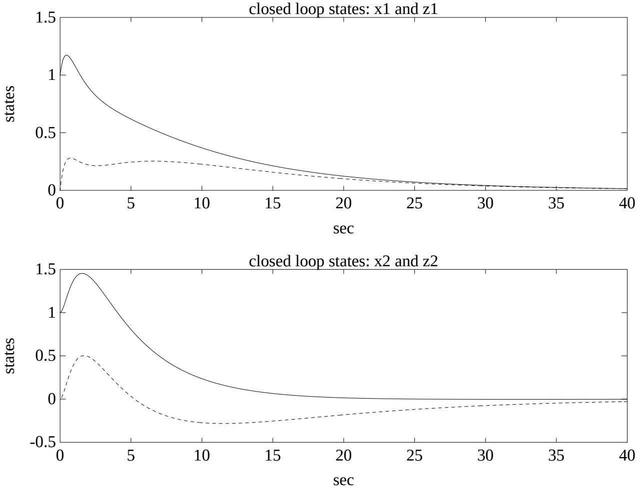

Figure 3: Closed Loop System: $(x_1(t), \hat{x}_1(t))$ and $(x_2(t), \hat{x}_2(t))$ (Problem 3: Robust Observer)

A Structure of Robust Observer (cont.)

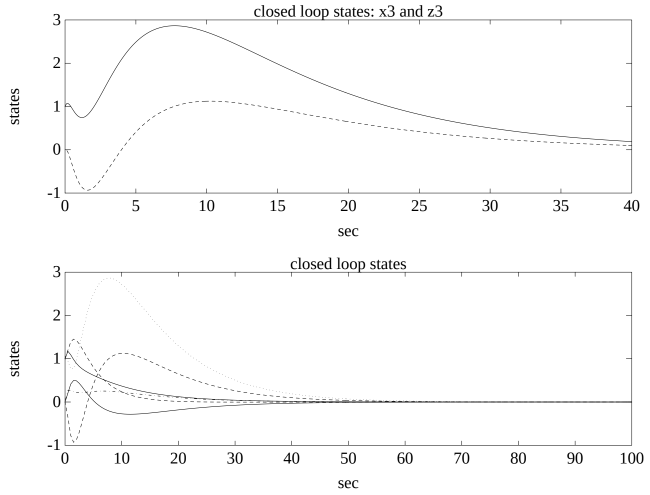

Figure 4: Closed Loop System: $(x_3(t), \hat{x}_3(t))$ and $(x(t)$ and $\hat{x}(t))$ (Robust Observer.)