ECEN 605

LINEAR SYSTEMS

Lecture 4

Dynamic response of discrete time systems

Discrete time signals and systems

A discrete time signal is a sequence $x(k\,T)$, $k = 0, 1, 2, \ldots$ denoted as $x[k]$ when the sampling period T is known and fixed. The input and output of a discrete time system consists of sequences $u[k]$ (inputs) and $y[k]$ (outputs), respectively. The system is typically described by **difference equations** such as:

$$y(k+1) = \alpha\, y(k) + \beta\, u(k) \quad \text{(1st order system)} \tag{1}$$

$$y(k+2) = \alpha_1\, y(k+1) + \alpha_0\, y(k) + \beta_0\, u(k) + \beta_1\, u(k+1) \quad \text{(2nd order system)} \tag{2}$$

represented below, using delays, multipliers and summers.

Discrete time signals and systems (cont.)

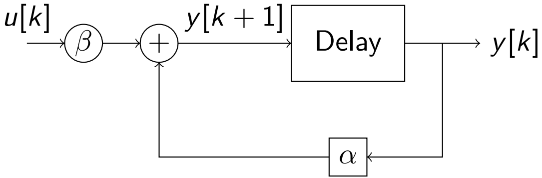


Figure: First order discrete time system

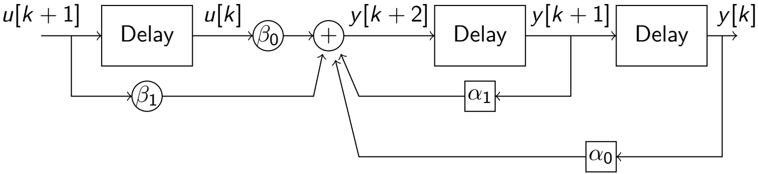


Figure: Second order discrete time system

Discrete time signals and systems (cont.)

To solve (1) and obtain the output sequence $y[k]$ it is necessary to know the sequence $u[k]$ and $y(0)$, whereas to solve (2) one needs to know the sequence $u[k]$ as well as $y(0)$ and $y(1)$.

z-Transform

The solution of difference equations with constant coefficients is facilitated by the z-**transform**, just as Laplace transforms help in solving differential equations. The z-transform of the sequence

$$x[k] = \{x(0), x(1), x(2), \cdots\} \tag{3}$$

denoted $X(z)$, is defined by

$$X(z) := \mathcal{Z}\{x[k]\} := x(0) + x(1)z^{-1} + \cdots + x(k)z^{-k} + \cdots . \tag{4}$$

Note that $x[k]$ may have real or complex elements.

z-Transform (cont.)

Example

Find the z-transform of following sequence.

$$x[k] = \{1, 0, -1, 2, 4, 0, 0, 0, 0, \cdots\}. \tag{5}$$

Solution:

$$X(z) = 1 + 0\,z^{-1} + (-1)\,z^{-2} + 2\,z^{-3} + 4\,z^{-4}. \tag{6}$$

z-Transform (cont.)

Example (**Unit Pulse**)

$$x[k] = \{1, 0, 0, 0, 0, \cdots\} =: \delta[k]. \tag{7}$$

Solution:

$$X(z) = 1 + 0\,z^{-1} + 0\,z^{-2} + \cdots + k\,z^{-k} + \cdots . \tag{8}$$

z-Transform (cont.)

Example (**Shifted Pulse**)

$$x[k] = \{0, 0, 0, 1, 0, 0, \cdots\}. \tag{9}$$

Solution:

$$X(z) = z^{-3}. \tag{10}$$

z-Transform (cont.)

Example (**Geometric Series**)

$$x[k] = \left\{a^k\right\} = \{1, a, a^2, a^3, \cdots\}\,. \tag{11}$$

Solution:

$$\begin{aligned} X(z) &= 1 + a\,z^{-1} + a^2\,z^{-2} + a^3\,z^{-3} + \cdots \\ &= \frac{1}{1 - a\,z^{-1}} = \frac{z}{z-a}, \quad \left|a\,z^{-1}\right| < 1. \end{aligned} \tag{12}$$

Stable Region: Note that $x[k]$ converges if and only if $|a| < 1$ and thus the asymptotic stability corresponds to all poles lying strictly inside the unit circle. Poles on the unit circle or exterior to it represent an unstable system.

z-Transform (cont.)

Example (**Repeated Poles**)

$$x[k] = \left\{1, 2\,a, 3\,a^2, 4\,a^3, \cdots, (k+1)a^k, \cdots\right\}. \tag{13}$$

Solution:

$$\begin{aligned} X(z) &= 1 + 2\,a\,z^{-1} + 3\,a^2\,z^{-2} + \cdots + (k+1)a^k z^{-k} + \cdots \\ &= \frac{1}{(1 - a\,z^{-1})^2}. \end{aligned} \tag{14}$$

z-Transform (cont.)

Example (**Unit Step**)

$$x[k] = U[k] = \{1, 1, 1, 1, \cdots\}. \tag{15}$$

Solution:

$$\begin{aligned} \mathcal{Z}\{U[k]\} = U(z) &= 1 + z^{-1} + z^{-2} + \cdots \\ &= \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}. \end{aligned} \tag{16}$$

z-Transform (cont.)

Example (**Unit Ramp**)

$$x[k] = R[k] = \{0, 1, 2, 3, 4, \cdots\}\,. \tag{17}$$

Solution:

$$\begin{aligned} \mathcal{Z}\{R[k]\} = R(z) &= 0 + 1\,z^{-1} + 2\,z^{-2} + 3\,z^{-3} + \cdots \\ &= \frac{z^{-1}}{(1 - z^{-1})^2} = \frac{z}{(z-1)^2}\,. \end{aligned} \tag{18}$$

z-Transform (cont.)

Example (**Sampled Exponential**)

$$\begin{aligned}\left[e^{\alpha\, t}\right]_{t=k\,T} &= \left\{e^{\alpha\, k\, T}\right\} \\ &= \left\{1, e^{\alpha\, T}, e^{\alpha\, 2\, T}, e^{\alpha\, 3\, T}, \cdots\right\}.\end{aligned} \tag{19}$$

$$\mathcal{Z}\left\{e^{\alpha\, k\, T}\right\} = \frac{z}{\left(z - e^{\alpha\, T}\right)}. \tag{20}$$

z-Transform (cont.)

From (20) we can derive the z-transforms of $[\cos \omega\, k\, T]$ and $[\sin \omega\, k\, T]$, the sampled cosine and sine waves. Also if α in (20) is $\sigma + j\,\omega$ we obtain the sampled complex exponential $e^{\sigma\, k\, T + j\,\omega\, k\, T}$ with z-transform

$$\mathcal{Z}\left\{ e^{(\sigma + j\,\omega) k\, T} \right\} = \frac{z}{z - e^{(\sigma + j\,\omega) T}}. \tag{21}$$

z-Transform (cont.)

Example

Find the z-transforms of following signals.
a) $\sin(2\,k)$ for $k = 0,\, 1,\, 2,\, \ldots,$
Solution.

$$\sin(2\,k) = \frac{1}{2\,j}\left[e^{2\,k\,j} - e^{-2\,k\,j}\right]. \tag{22}$$

$$\begin{aligned} \mathcal{Z}\left[\sin(2\,k)\right] &= \frac{1}{2\,j}\left[\frac{z}{z - e^{2j}} - \frac{z}{z - e^{-2j}}\right] \\ &= \frac{1}{2\,j}\,\frac{z\,e^{2j} - z\,e^{-2j}}{z^2 - z\left(e^{2j} + e^{-2j}\right) + 1} \\ &= \frac{z\,\sin(2)}{z^2 - 2\,z\,\cos(2) + 1}. \end{aligned} \tag{23}$$

z-Transform (cont.)

Example (cont.)

b) $\{0, 2, 8, 24, 64, 160, \ldots\} = \left[k(2)^k\right]$

Solution.

$$\mathcal{Z}\left[k\,s^k\right] = \frac{2\,z}{(z-2)^2}. \tag{24}$$

z-Transform (cont.)

Example (cont.)

c) $x[k]$ as below,

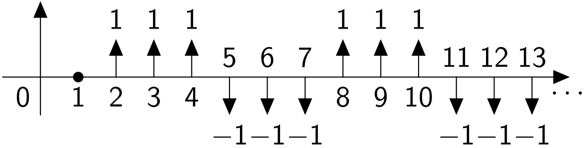


Figure:

z-Transform (cont.)

Example (cont.)

Solution. The function is periodic with period 6, and shifted 2 units to the right:

$$x[k] = \delta[k-2]+\delta[k-3]+\delta[k-4]-\delta[k-5]-\delta[k-6]-\delta[k-7]+\dots. \tag{25}$$

$$\begin{aligned}\mathcal{Z}\left[x[k]\right] &= \left\{1+z^{-1}+z^{-2}-z^{-3}-z^{-4}-z^{-5}\right\}\left[\frac{1}{1-z^{-6}}\right]z^{-2} \\ &= \left\{z^{-2}+z^{-3}+z^{-4}-z^{-5}-z^{-6}-z^{-7}\right\}\left[\frac{1}{1-z^{-6}}\right].\end{aligned} \tag{26}$$

z-Transform (cont.)

Example

Find the inverse z-transforms of following:

a) z^{-5}

Solution.

$$\mathcal{Z}^{-1}\left[z^{-5}\right] = \delta\left[k-5\right]. \tag{27}$$

z-Transform (cont.)

Example (cont.)

b) $\dfrac{1}{z(z+1)(z+2)}$

Solution. Multiplying by $\dfrac{1}{z}$ we have

$$\frac{1}{z}\frac{1}{z(z+1)(z+2)} = \frac{A_0}{z} + \frac{A_1}{z^2} + \frac{B_1}{z+1} + \frac{B_2}{z+2} \qquad (28)$$

where

$$A_0 = -\frac{3}{4}, \quad A_1 = \frac{1}{2}, \quad B_1 = 1, \quad B_2 = -\frac{1}{4}. \qquad (29)$$

z-Transform (cont.)

Example (cont.)

Therefore,

$$\frac{1}{z(z+1)(z+2)} = -\frac{3}{4} + \frac{\frac{1}{2}}{z} + \frac{z}{z+1} + \frac{-\frac{1}{4}z}{z+2} \tag{30}$$

and

$$\mathcal{Z}^{-1}\left[\frac{1}{z(z+1)(z+2)}\right] = -\frac{3}{4}\delta[k] + \frac{1}{2}\delta[k-1] + [-1]^k - \frac{1}{4}[-2]^k . \tag{31}$$

z-Transform (cont.)

Example (cont.)

c) $\dfrac{z^2}{z^2+1}$

Solution.

$$\frac{z^2}{z^2+1} = \frac{z^2+1-1}{z^2+1} = -\frac{1}{z^2+1} + 1. \tag{32}$$

$$\begin{aligned} \frac{1}{z}\frac{1}{z^2+1} &= \frac{A_0}{z} + \frac{A_1}{z-j} + \frac{A_2}{z+j} \\ &= \frac{A_0(z^2+1) + A_1(z^2+j\,z) + A_2(z^2-j\,z)}{z(z^2+1)}. \end{aligned} \tag{33}$$

$$\begin{aligned} A_0 + A_1 + A_2 &= 0 \\ A_1 - A_2 &= 0 \\ A_0 &= 1. \end{aligned} \tag{34}$$

Example (cont.)

So,

$$A_0 = 1, \quad A_1 = -\frac{1}{2}, \quad A_2 = -\frac{1}{2}. \tag{35}$$

Thus,

$$\frac{1}{z^2+1} = 1 + \frac{-\frac{1}{2}z}{z-j} + \frac{-\frac{1}{2}z}{z+j}. \tag{36}$$

$$\begin{aligned} \mathcal{Z}^{-1}[1] - \mathcal{Z}^{-1}\left[\frac{1}{z^2+1}\right] &= \delta[k] - \left(\delta[k] - \frac{1}{2}[j]^k - \frac{1}{2}[-j]^k\right) \\ &= \frac{1}{2}[j]^k + \frac{1}{2}[-j]^k. \end{aligned} \tag{37}$$

Shifted sequences

Consider the sequences

$$x[k] = \{x(0), x(1), x(2), \cdots\}, \tag{38}$$
$$x[k+1] = \{x(1), x(2), x(3), \cdots\}, \tag{39}$$
$$x[k+2] = \{x(2), x(3), x(4), \cdots\}, \tag{40}$$

and their z-transforms

$$\mathcal{Z}\{x[k]\} = x(0) + x(1)\,z^{-1} + x(2)\,z^{-2} + \cdots, \tag{41}$$
$$\mathcal{Z}\{x[k+1]\} = x(1) + x(2)\,z^{-1} + x(3)\,z^{-2} + \cdots, \tag{42}$$
$$\mathcal{Z}\{x[k+2]\} = x(2) + x(3)\,z^{-1} + x(4)\,z^{-2} + \cdots. \tag{43}$$

We see that

$$X(z) = z^{-1}\,\mathcal{Z}\{x[k+1]\} + x(0) \tag{44}$$

so that

$$\mathcal{Z}\{x[k+1]\} = z\,X(z) - z\,x(0). \tag{45}$$

Shifted sequences (cont.)

Similarly

$$\mathcal{Z}\{x[k+2]\} = z^2\, X(z) - z^2\, x(0) - z\, x(1). \qquad (46)$$

Relations (45), (46) are useful for solving difference equations as we show next.

Solution of difference equations

In this section we describe how the z transform can be used to solve linear difference equations with constant coefficients.
Difference equations describe a discrete linear time invariant system (discrete LTI system) by relating the output $y[k]$ to the input $u[k]$, for $k \geq 0$.
Consider a discrete LTI system

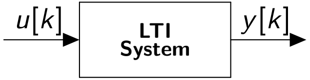


Figure: a discrete LTI system

Solution of difference equations (cont.)

where $y[k]$ and $u[k]$ are related by the difference equation

$$\begin{aligned} a_n y[k+n] + a_{n-1} y[k+n-1] + \cdots + a_1 y[k+1] + a_0 y[k] \\ = b_m u[k+m] + b_{m-1} u[k+m-1] + \cdots + b_1 u[k+1] + b_0 u[k]. \end{aligned} \tag{47}$$

Given the known input

$$u[k], \quad k \geq 0, \tag{48}$$

the output

$$y[k], \quad k \geq 0 \tag{49}$$

can be determined if the initial conditions, denoted by $\mathbf{u}(0)$, $\mathbf{y}(0)$

$$\mathbf{u}(0) := [u(0), u(1), \ldots, u(m-1)], \tag{50}$$

$$\mathbf{y}(0) := [y(0), y(1), \ldots, y(n-1)] \tag{51}$$

Solution of difference equations (cont.)

are known.
Taking the z transform of (47) and using the notation $y[k] \leftrightarrow Y(z)$, $u[k] \leftrightarrow U(z)$, we have

$$
\begin{aligned}
& a_n \left[z^n Y(z) - z^n y(0) - z^{n-1} y(1) - \cdots - z\, y(n-1)\right] \\
& \quad + a_{n-1} \left[z^{n-1} Y(z) - z^{n-1} y(0) - \cdots - z\, y(n-2)\right] \\
& \quad + \cdots + a_1 [z\, Y(z) - z\, y(0)] + a_0 Y(z) \\
& = b_m \left[z^m U(z) - z^m u(0) - z^{m-1} u(1) - \cdots - z\, u(m-1)\right] \\
& \quad + b_{m-1} \left[z^{m-1} U(z) - z^{m-1} u(0) - \cdots - z\, u(m-2)\right] \\
& \quad + \cdots + b_1 [z\, U(z) - z\, u(0)] + b_0 U(z).
\end{aligned}
\tag{52}
$$

Solution of difference equations (cont.)

Introduce the polynomials

$$A(z) := a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 \tag{53}$$

$$B(z) := b_m z^m + b_{m-1} z^{m-1} + \cdots + b_1 z + b_0 \tag{54}$$

$$\begin{aligned} P(z, \mathbf{y}(0)) := {} & a_n \left[z^n y(0) + z^{n-1} y(1) + \cdots + z\, y(n-1) \right] \\ & + a_{n-1} \left[z^{n-1} y(0) + z^{n-2} y(1) + \cdots + z\, y(n-2) \right] \\ & + \cdots + a_1 y(0) \end{aligned} \tag{55}$$

$$\begin{aligned} Q(z, \mathbf{y}(0)) := {} & b_m \left[z^m u(0) + z^{m-1} u(1) + \cdots + z\, u(m-1) \right] \\ & + b_{m-1} \left[z^{m-1} u(0) + z^{m-2} u(1) + \cdots + z\, u(m-2) \right] \\ & + \cdots + b_1 u(0). \end{aligned} \tag{56}$$

Now solving for $Y(z)$ from (52) we obtain

$$Y(z) = \underbrace{\frac{P\left(z, \mathbf{y}(0)\right) - Q\left(z, \mathbf{u}(0)\right)}{A(z)}}_{Y_0(z)} + \underbrace{\frac{B(z)}{A(z)} U(z)}_{Y_u(z)} \tag{57}$$

Solution of difference equations (cont.)

or

$$Y(z) = Y_0(z) + Y_u(z). \tag{58}$$

Using the notation $Y_0(z) \leftrightarrow y_0[k]$, $Y_u(z) \leftrightarrow y_u[k]$ we have, from (58), taking inverse z transforms and using linearity of the inverse transform,

$$y[k] = y_0[k] + y_u[k]. \tag{59}$$

In (59) we see that the **total** response $y[k]$ is the sum of $y_0[k]$ which depends only on the initial conditions $\mathbf{y}(0)$, $\mathbf{u}(0)$, (see (57)) and $y_u[k]$ which depends only on the input $u[k]$, $k > 0$. Therefore $y_0[k]$ is called the **initial condition response** and $y_u[k]$ is called the **forced response**. Alternatively $y_0[k]$ is also called the **zero input response** and $y_u[k]$ is the **zero state response**, that is, the response to $u[k]$ under zero initial conditions.

Solution of difference equations (cont.)

Finally, $\dfrac{B(z)}{A(z)} =: G(z)$ is called the **system transfer function**, and the roots of $A(z)$ and $B(z)$ are called the **poles** and **zeros** of the system.

Solution of difference equations (cont.)

Example

Solve

$$y(k+1) = \alpha\, y(k) + \beta\, u(k), \quad k = 0,\, 1,\, 2,\, \cdots . \tag{60}$$

Solution of difference equations (cont.)

Example (cont.)

Solution. Convert (60) to an equation in sequences $y[k]$, $u[k]$

$$y[k+1] = \alpha\, y[k] + \beta\, u[k]. \tag{61}$$

Taking z-transforms of (61) we get

$$z\,Y(z) - z\,y(0) = \alpha\,Y(z) + \beta\,U(z) \tag{62}$$

and

$$\begin{aligned} Y(z) &= \frac{z}{z-\alpha} y(0) + \frac{\beta}{z-\alpha} U(z) \\ &= Y_0(z) + G(z)\,u(z) \end{aligned} \tag{63}$$

where $G(z) = \dfrac{\beta}{z-\alpha}$ is the transfer function.

Solution of difference equations (cont.)

Example (cont.)

We see that

$$y_0[k] = \mathcal{Z}^{-1}\{Y_0(z)\} = \alpha^k\, y(0) \tag{64}$$

is the zero input response and

$$y_u[k] = \mathcal{Z}^{-1}\{G(z)\,U(z)\} \tag{65}$$

is the zero state response of the system. Now suppose that the input is a unit step. Then

$$U(z) = \frac{z}{z-1} \tag{66}$$

and

$$Y_u(z) = \beta \frac{z}{(z-\alpha)(z-1)}. \tag{67}$$

Solution of difference equations (cont.)

Example (cont.)

Write

$$\frac{Y_u(z)}{z} = \frac{\beta}{(z-\alpha)(z-1)} = \frac{\beta/(\alpha-1)}{z-\alpha} + \frac{\beta/(1-\alpha)}{z-1} \qquad (68)$$

so that

$$Y_u(z) = \frac{\beta}{\alpha-1}\frac{z}{z-\alpha} + \frac{\beta}{1-\alpha}\frac{z}{z-1} \qquad (69)$$

and the inverse transform of $Y_0(z)$ is

$$y_u[k] = \frac{\beta}{\alpha-1}(\alpha)^k + \frac{\beta}{1-\alpha}U[k]. \qquad (70)$$

Discretization of continuous systems

In this section we discuss how to convert a continuous time system to its discrete time equivalents. Consider first the discretization of an integrator.

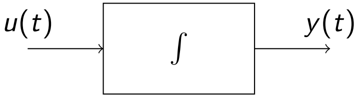


Figure: An integrator.

$$y(t) = \int_0^t u(\tau)\, d\tau + y(0) \tag{71}$$

$$\begin{aligned} y\left((k+1)T\right) &= \int_0^{(k+1)T} u(\tau)\, d\tau + y(0) \\ &= \underbrace{\int_0^{k\,T} u(\tau)\, d\tau + y(0)}_{y(kT)} + \int_{k\,T}^{(k+1)T} u(\tau)\, d\tau \end{aligned} \tag{72}$$

Discretization of continuous systems (cont.)

$$y\left((k+1)T\right) \cong y(kT) + u(kT)T. \tag{73}$$

Taking z-transforms of (73), with zero initial conditions

$$z\,Y(z) = Y(z) + T\,U(z) \tag{74}$$

$$\frac{Y(z)}{U(z)} = \frac{T}{z-1}. \tag{75}$$

Since continuous time integration has the transfer function

$$\frac{Y(s)}{U(s)} = \frac{1}{s} \tag{76}$$

we have the mapping

$$s \to \frac{z-1}{T} \tag{77}$$

to determine a digital integrator.

Discretization of continuous systems (cont.)

Example

Discretize the following transfer function with $T = 1$.

$$H(s) = \frac{s+1}{s\,(s+2)} \tag{78}$$

Substituting (77) into (78) we get

$$H(z) = \frac{z}{(z-1)(z+1)}. \tag{79}$$

The mapping (77) is shown in the figure next.

Discretization of continuous systems (cont.)

Example (cont.)

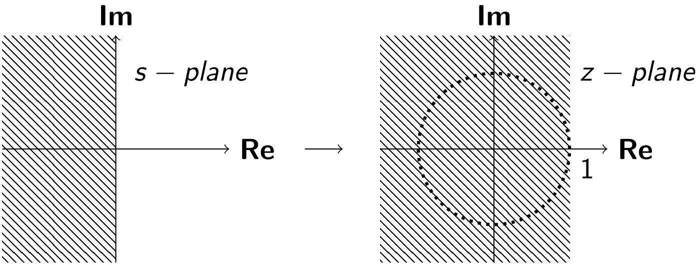


Figure:

Note that LHP poles can be mapped to unstable regions in the z plane.

Tustin approximation

Consider the integrator

$$y(t) = \int_0^t u(\tau)\, d\tau + y(0) \tag{80}$$

so that, as before,

$$y\left((k+1)T\right) = y(kT) + \int_{k\,T}^{(k+1)\,T} u(\tau)\, d\tau. \tag{81}$$

The integral in (81) can be approximated by a **trapezoidal** area

$$\int_{k\,T}^{(k+1)\,T} u(\tau)\, d\tau \cong \frac{u((k+1)\,T) + u(kT)}{2}\, T, \tag{82}$$

so that

$$y\left((k+1)T\right) = y(kT) + \frac{u((k+1)\,T) + u(kT)}{2}\, T. \tag{83}$$

Tustin approximation (cont.)

Taking z-transforms of (83)

$$z\,Y(z) - Y(z) = (z\,U(z) + U(z))\,\frac{T}{2} \tag{84}$$

and so

$$\frac{Y(z)}{U(z)} = \frac{T}{2}\,\frac{z+1}{z-1}. \tag{85}$$

Since an integrator has the transfer function $\frac{1}{s}$ we have the mapping

$$s \to \frac{2}{T}\,\frac{z-1}{z+1} \tag{86}$$

to determine the digital integrator. The inverse of (86) is

$$z \to \frac{1 + s\,T/2}{1 - s\,T/2}, \tag{87}$$

and is shown in the figure below.

Tustin approximation (cont.)

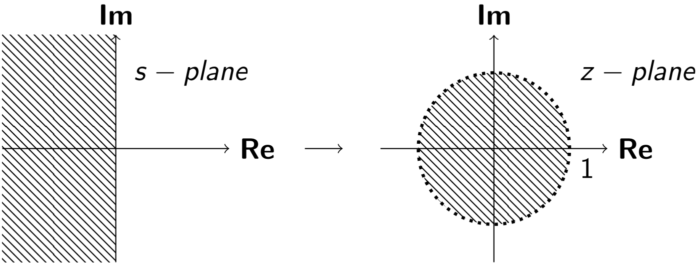


Figure:

Tustin approximation (cont.)

Example

Find the Tustin equivalent of

$$G(s) = \frac{s-1}{s+1} \tag{88}$$

with $T = 0.1\, sec$.

Solution. Since

$$\frac{1}{s} \rightleftarrows \frac{T}{2}\frac{z+1}{z-1} \tag{89}$$

we get

$$s \rightleftarrows \frac{2}{T}\frac{z-1}{z+1} = 20\frac{z-1}{z+1}. \tag{90}$$

Tustin approximation (cont.)

Example (cont.)

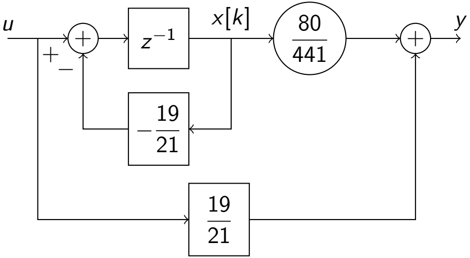
So, the Tustin equivalent of $G(s)$ is:

$$\begin{aligned}
\hat{G}(z) &= \frac{20\frac{z-1}{z+1} - 1}{20\frac{z-1}{z+1} + 1} \\
&= \frac{20\,z - 20 - z - 1}{20\,z - 20 + z + 1} \\
&= \frac{19\,z - 21}{21\,z - 19} \\
&= \frac{\frac{19}{21}z - 1}{z - \frac{19}{21}} \\
&= \frac{\frac{19}{21}\left(z - \frac{19}{21}\right) + \left(\frac{19}{21}\right)^2 - 1}{z - \frac{19}{21}} \\
&= \frac{\frac{80}{441}}{z - \frac{19}{21}} + \frac{19}{21}.
\end{aligned} \tag{91}$$

Tustin approximation (cont.)

Example (cont.)

This can be represented by the following realization and state variable equations:



$$\begin{aligned} x[k+1] &= \frac{19}{21}x[k] + u[k] \\ y[k] &= \frac{80}{441}x[k] + \frac{19}{21}u[k]. \end{aligned} \tag{92}$$

Step response equivalent

This method produces a digital equivalent with discrete time step input similar to the step input of the continuous time system.

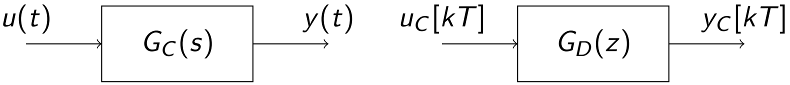


Figure:

Suppose $u(t) = U(t)$. Then

$$Y_C(s) = G_C(s)\,\frac{1}{s} \tag{93}$$

and

$$y_C(t) = \mathcal{L}^{-1}\left\{\frac{G_C(s)}{s}\right\}. \tag{94}$$

Step response equivalent (cont.)

Sampling $y_C(t)$ we get, equating to $y_D[kT]$

$$y_C[kT] = y_D[kT]. \tag{95}$$

Therefore

$$G_D(z) = \frac{\mathcal{Z}\left\{y_C[kT]\right\}}{\frac{z}{z-1}}. \tag{96}$$

Step response equivalent (cont.)

Example

$$G_C(s) = \frac{s+1}{s+2}. \tag{97}$$

$$y_C(s) = \frac{s+1}{s\,(s+2)} = \frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}}{s+2}. \tag{98}$$

$$y_C(t) = \frac{1}{2} U(t) - \frac{1}{2}\, e^{-2\,t}. \tag{99}$$

Take $T = 0.1\, secs$.

$$y_C[kT] = \frac{1}{2} U[k] - \frac{1}{2}\, \mathcal{Z}^{-1} \left\{ \frac{z}{z - e^{-0.2}} \right\}. \tag{100}$$

$$Y_D(z) = \frac{1}{2} \frac{z}{z-1} - \frac{1}{2} \frac{z}{z - e^{-0.2}}. \tag{101}$$

$$G_D(z) = \frac{1}{2} - \frac{1}{2} \left(\frac{z-1}{z - e^{-0.2}} \right). \tag{102}$$