ROBUST CONTROL
The Parametric Approach

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THIS BOOK IS DEDICATED TO

Krishna Lee, Mohadev and Shona Lee

S.P. Bhattacharyya

My beloved family: Pierre, Jacqueline, Laetitia, Patrick, Véronique et Pascale

H. Chapellat

Sook, Allen and Jessica

L.H. Keel

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FOREWORD

Control systems need to be designed so that certain essential properties remain unchanged under perturbations. The design problem of maintaining invariance of system properties under small perturbations of the system parameters were considered in the initial stages of the development of Control Theory. These properties included the stability and performance of systems. It was understood later that even more important was the requirement that a control system function satisfactorily under large perturbations. Thus the problems of stability and performance validation for a family of controlled systems with parameters or frequency responses lying within admissible sets, have attracted considerable attention. This field has come to be known as Robust Control.

The theory of Robust Control has been elegantly developed within the H_∞ framework. This framework can effectively deal with robust stability and performance problems under unstructured perturbations. It is however quite deficient in addressing the same issues when parameter uncertainty is considered.

The treatment of robust stability problems under parameter uncertainty has been pioneered by the Italian mathematician Faedo (1953) and the Russian scientist Kharitonov (1978). A rich array of useful results have been developed over the last ten years and at present there exists an extensive literature in this specific field.

The authors of the present book are well known as experts in the area of Robust Control under parameter uncertainty. They have actively participated in the development of the theory, and their personal contribution cannot be overestimated. It suffices to mention their developments of the Generalized Kharitonov Theorem, the theory of disk polynomials, the extremal properties of interval systems, the calculation of the real parametric stability margin and their contributions to design problems under simultaneous parametric and nonparametric uncertainty, among others.

The book contains a complete account of most of the fundamental results in this field. The proofs of the results are elegant, simple and insightful. At the same time careful attention is paid to basic aspects of control problems such as design, performance, and synthesis and their relationship to the theory developed. The organization and sequence of the material has many significant merits and the entire book is written with great expertise, style, rigour and clarity.

The reader will find here a unified approach to robust stability theory. It includes both linear and nonlinear systems. A systematic use of frequency domain methods allowed the authors to combine different types of uncertainty and to link parametric robust stability with nonlinear perturbations and the results of H_∞ theory. Numerous examples of various origins, as well as exercises are included. It will be helpful to a broad audience of control theorists and students, and indispensable to the specialist in Robust Control.

The book should stand as an outstanding and fundamental contribution to the science of automatic control and I commend the authors for their effort. I felt a true pleasure when reading this book, and I believe the reader will share my feelings.

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PREFACE

The subject of robust control began to receive worldwide attention in the late 1970's when it was found that Linear Quadratic Optimal Control (H_2 optimal control), state feedback through observers, and other prevailing methods of control system synthesis such as Adaptive Control, lacked any guarantees of stability or performance under uncertainty. Thus, the issue of robustness, prominent in Classical Control, took rebirth in a modern setting.

H_∞ optimal control was proposed as a first approach to the solution of the robustness problem. This elegant approach, and its offshoots, such as μ theory, have been intensely developed over the past 12 years or so, and constitutes one of the triumphs of control theory. The theory provides a precise formulation and solution of the problem of synthesizing an output feedback compensator that minimizes the H_∞ norm of a prescribed system transfer function. Many robust stabilization and performance problems can be cast in this formulation and there now exists effective, and fairly complete theory for control system synthesis subjected to perturbations, in the H_∞ framework.

The H_2 or H_∞ theory delivers an "optimal" feedback compensator for the system. Before such a compensator can be deployed in a physical (real-world) system it is natural to test its capabilities with regard to additional design criteria, not covered by the optimality criterion used. In particular the performance of any controller under real parameter uncertainty, as well as mixed parametric-unstructured uncertainty, is an issue which is vital to most control systems. However, optimal H_2 or H_∞ theory is incapable of providing a direct and nonconservative answer to this important question.

The problem of robustness under parametric uncertainty received a shot in the arm in the form of Kharitonov's Theorem for interval polynomials, which appeared in the mid-1980's in the Western literature. It was originally published in 1978 in a Russian journal. With this surprising theorem the entire field of robust control under real parametric uncertainty came alive and it can be said that Kharitonov's Theorem is the most important occurrence in this area after the development of the Routh-Hurwitz criterion. A significant development following Kharitonov's Theorem was the calculation, in 1985, by Soh, Berger and Dabke of the radius of the stability ball in the space of coefficients of a polynomial.

From the mid-1980's rapid and spectacular developments have taken place in this field. As a result we now have a rigorous, coherent, and comprehensive theory to deal directly and effectively with real parameter uncertainty in control systems. This theory nicely complements the optimal H_2 and H_∞ theories as well as Classical Control and considerably extends the range of possibilities available to the control specialist.

The main accomplishment of this theory is that it allows us to determine if a linear time invariant control system, containing several uncertain real parameters remains stable as the parameters vary over a set. This question can be answered in a precise manner, that is, nonconservatively, when the parameters appear linearly or multilinearly in the characteristic polynomial. In developing the solution to the above problem, several important control system design problems are answered. These are 1) the calculation of the real parametric stability margin, 2) the determination of stability and stability margins under mixed parametric and unstructured (norm-bounded or nonlinear) uncertainty 3) the evaluation of the worst case or robust performance measured in the H_∞ norm, over a prescribed parametric uncertainty set and 4) the extension of classical design techniques involving Nyquist, Nichols and Bode plots and root-loci to systems containing several uncertain real parameters.

These results are made possible because the theory developed provides *built-in* solutions to several extremal problems. It identifies apriori the critical subset of the uncertain parameter set over which stability or performance will be lost and thereby reduces to a very small set, usually points or lines, the parameters over which robustness must be verified. This built-in optimality of the parametric theory is its main strong point particularly from the point of view of applications. It allows us, for the first time, to devise methods to effectively carry out robust stability and performance analysis of control systems under parametric and mixed uncertainty. To balance this rather strong claim we point out that a significant deficiency of control theory at the present time is the lack of nonconservative synthesis methods to achieve robustness under parameter uncertainty. Nevertheless, even here the sharp analysis results obtained in the parametric framework can be exploited in conjunction with synthesis techniques developed in the H_∞ framework to develop design techniques to partially cover this drawback.

The objective of this book is to describe the parametric theory in a self-contained manner. The book is suitable for use as a graduate textbook and also for self-study. The entire subject matter of the book is developed from the single fundamental fact that the roots of a polynomial depend continuously on its coefficients. This fact is the basis of the Boundary Crossing Theorem developed in Chapter 1 and is repeatedly used throughout the book. Surprisingly enough this simple idea, used systematically is sufficient to derive even the most mathematically sophisticated results. This economy and transparency of concepts is another strength of the parametric theory. It makes the results accessible and appealing to a wide audience and allows for a unified and systematic development of the subject. The contents of the book can therefore be covered in one semester despite the size of the book. In accordance with our focus we do not develop any results in H_∞ or H_2 theory although some results from H_∞ theory are used in the chapter on synthesis. In Chapter 0, which serves as an extension of this preface, we rapidly overview some basic aspects of control systems, uncertainty models and robustness issues. We also give a brief historical sketch of Control Theory, and then describe the contents of the rest of the chapters in some detail.

The theory developed in the book is presented in mathematical language. The results described in these theorems and lemmas however are completely oriented towards control systems applications and in fact lead to effective algorithms and graphical displays for design and analysis. We have throughout included examples to illustrate the theory and indeed the reader who wants to avoid reading the proofs can understand the significance and utility of the results by reading through the examples. A MATLAB based software package, the Robust Parametric Control ToolBox, has been developed by the authors in collaboration with Samir Ahmad, our graduate student. It implements most of the theory presented in the book. In fact, all the examples and figures in this book have been generated by this ToolBox. We gratefully acknowledge Samir's dedication and help in the preparation of the numerical examples given in the book. A demonstration diskette illustrating this package is included with this book.

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We are indeed honored that Academician Ya. Z. Tsypkin, one of the leading control theorists of the world, has written a Foreword to our book. Professor Tsypkin's pioneering contributions range from the stability analysis of time-delay systems in the 1940's, learning control systems in the 1960's to robust control under parameter uncertainty in the 1980's and 1990's. His observations on the contents of the book and this subject based on this wide perspective are of great value.

The first draft of this book was written in 1989. We have added new results of our own and others as we became aware of them. However, because of the rapid pace of developments of the subject and the sheer volume of literature that has been published in the last few years, it is possible that we have inadvertently omitted some results and references worthy of inclusion. We apologize in advance to any authors or readers who feel that we have not given credit where it is due.

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