ECEN689: Special Topics in Optical Interconnects Circuits and Systems
Spring 2022

Lecture 6: Limiting Amplifiers (LAs)

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Announcements

- Exam 1 Mar 10
  - In class
  - One double-sided 8.5x11 notes page allowed
  - Bring your calculator
  - Covers through Lecture 6
- Reading
  - Sackinger Chapter 6
  - Razavi Chapter 5

Announcements & Agenda

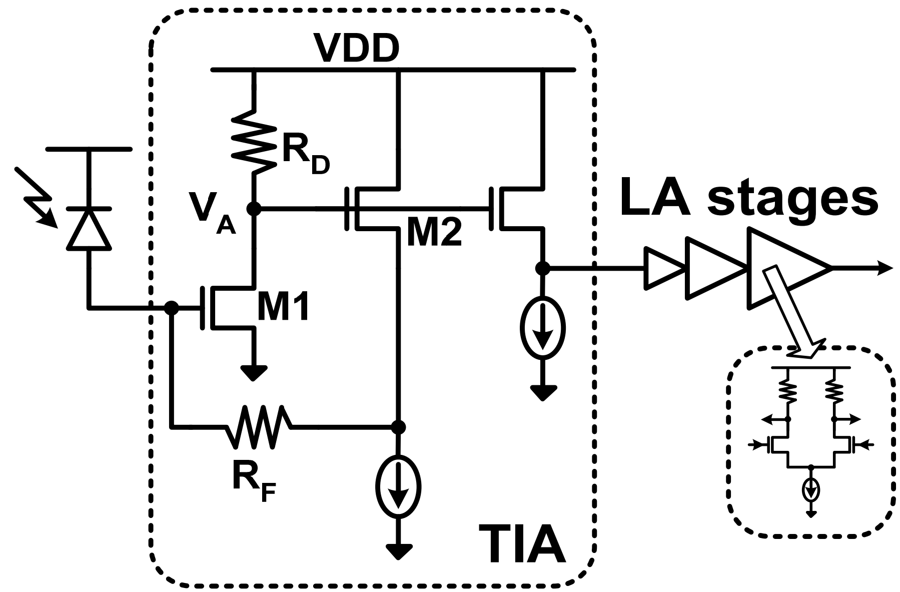
- Multi-stage limiting amplifiers
- Bandwidth extension techniques
- Offset compensation

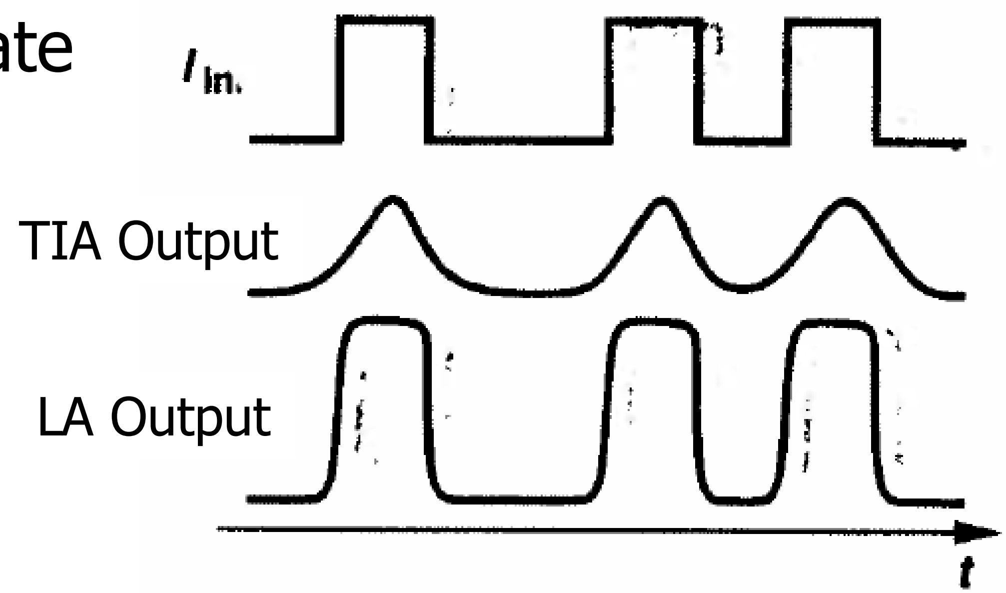
Limiting Amplifiers

- Limiting amplifier amplifies the TIA output to a reliable level to achieve a given BER with a certain decision element (comparator)

- Typically designed with a bandwidth of 1-1.2X data rate

- Want group delay variation <±10% over bandwidth of interest to limit DDJ





How to Achieve an Ampilfier GBW > f_T?

Assume for a 10Gb/s system that we need to build an amplifer with $A_v = 30dB$ and $f_{3dB} = 10GHz$.

$$GBW_{tot} = (31.6)(10GHz) = 316GHz$$

However, the peak f_T of our technology is only 200GHz, and generally we can

only achieve a single-stage amplifier GBW of

$$\text{Max Single-Stage GBW}_s \approx \frac{f_T}{3} = \frac{200GHz}{3} = 66.7GHz$$

with $A_v = 30dB \Rightarrow f_{3dB} = \dfrac{66.7GHz}{31.6} = 2.11GHz$, well below our 10GHz spec.

Instead of using a single-stage, let's break the amplifier into multiple stages with

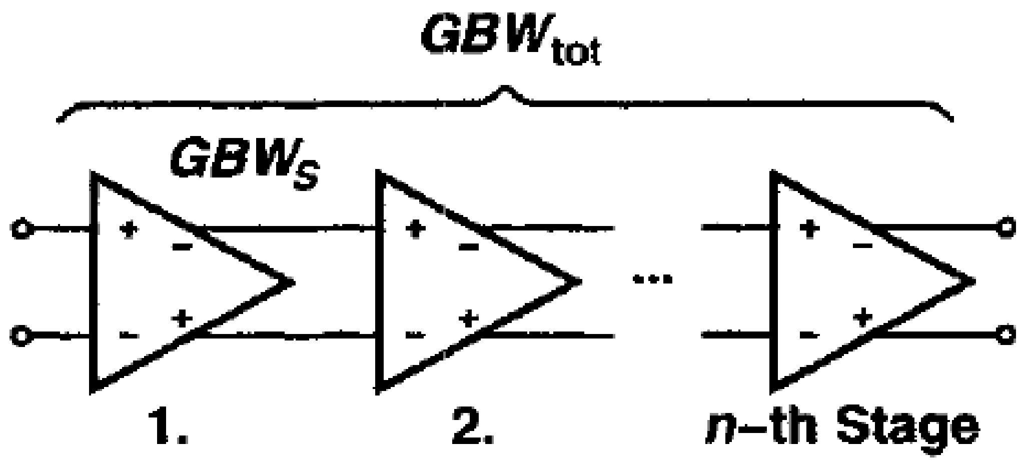
lower gain, but higher bandwidth. An optimal choice, from a maximum GBW perspective, is

$n = 7$ stages, with $A_{vs} = \sqrt[7]{31.6} = 1.64$ and $f_{3dB} = 31GHz$, or $GBW_s = 50.8GHz$

After multi-stage bandwidth compression, this will yield a total GBW $\approx$ 316GHz

with our target gain of 31.6 and with a single-stage $\text{GBW}_s = 50.8\text{GHz}$ that our technology can support.

Multi-Stage Amplifier GBW



If every stage is a single - pole amplifier

$$\frac{A_{vs}}{1+\frac{s}{\omega_{3dBs}}}$$

The total multi - stage amplifier transfer function will be

$$\frac{v_{out}}{v_{in}} = \left(\frac{A_{vs}}{1+\frac{s}{\omega_{3dBs}}}\right)^n = A_{vs}^n \frac{1}{\left(1+\frac{s}{\omega_{3dBs}}\right)^n}$$

The gain has increased significantly, but the bandwidth does compress relative to a single stage.

Multi-Stage Amplifier Bandwidth Compression

The total amplifier 3 - dB bandwidth, ω_{3dBtot}, is where

$$\left|\frac{v_{out}}{v_{in}}\right| = \left|\frac{A}{1+\frac{j\omega_{3dBtot}}{\omega_{3dBs}}}\right|^n = \frac{A^n}{\sqrt{2}}$$

$$\left(\frac{A_{vs}}{\sqrt{1+\left(\frac{\omega_{3dBtot}}{\omega_{3dBs}}\right)^2}}\right)^n = \frac{A_{vs}^n}{\sqrt{2}}$$

$$\left(1+\left(\frac{\omega_{3dBtot}}{\omega_{3dBs}}\right)^2\right)^n = 2$$

$$\omega_{3dBtot} = \omega_{3dBs}\sqrt{2^{1/n}-1}$$

The total multi - stage bandwidth does compress, although at a much slower rate than the increase in gain.

Thus, a significant increase in GBW can be achieved with a multi - stage amplifier approach.

Optimum Number of Gain Stages

Assuming that there is a maximum per-stage GBW_s that the technology can support

$$GBW_s = A_{vs}\omega_{3dBs} \Rightarrow \omega_{3dBs} = \frac{GBW_s}{A_{vs}} \qquad \text{(Note, here GBW is in rad/s)}$$

If we need to achieve a high bandwidth, we have to reduce the per-stage gain and increase the number of stages. However, the bandwidth will compress with cascaded stages. Thus, there must be an optimum number of stages for a maximum potentail gain bandwidth.

Recall that the total bandwidth is

$$\omega_{3dBtot} = \omega_{3dBs}\sqrt{2^{1/n} - 1} = \frac{GBW_s}{A_{vs}}\sqrt{2^{1/n} - 1}$$

and we will achieve a total gain G_{tot} with n stages

$$A_{vs} = G_{tot}^{1/n} \Rightarrow \omega_{3dBtot} = \frac{GBW_s}{G_{tot}^{1/n}}\sqrt{2^{1/n} - 1}$$

Optimum Number of Gain Stages

For a given total gain, we would like to maximize the bandwidth. In order to do this, let's make the following approximation

$$\omega_{3dBtot} = \frac{GBW_s}{G_{tot}^{1/n}}\sqrt{2^{1/n}-1} \approx \frac{GBW_s}{G_{tot}^{1/n}}\sqrt{\frac{1}{n}\ln 2}$$

Also, instead of maximizing this expression, let's minimize its reciprocal w.r.t the number of stages

$$\frac{1}{\omega_{3dBtot}} = \left(\frac{\sqrt{n}}{GBW_s\sqrt{\ln 2}}\right)G_{tot}^{1/n}$$

$$\frac{d}{dn}\left(\frac{1}{\omega_{3dBtot}}\right) = \frac{d}{dn}\left(\left(\frac{\sqrt{n}}{GBW_s\sqrt{\ln 2}}\right)G_{tot}^{1/n}\right) = 0$$

Moreover, to make this easier, let's minimize the natural log of the denominator, as this should yield the same optimum.

$$\frac{d}{dn}\left(\ln\left(\frac{1}{\omega_{3dBtot}}\right)\right) = \frac{d}{dn}\left(\ln\left(\left(\frac{\sqrt{n}}{GBW_s\sqrt{\ln 2}}\right)G_{tot}^{1/n}\right)\right) = \frac{d}{dn}\left(\frac{1}{2}\ln(n) + \frac{1}{n}\ln(G_{tot}) - \ln\left(GBW_s\sqrt{\ln 2}\right)\right) = 0$$

Optimum Number of Gain Stages

$$\frac{d}{dn}\left(\ln\left(\frac{1}{\omega_{3dBtot}}\right)\right)=\frac{d}{dn}\left(\ln\left(\left(\frac{\sqrt{n}}{GBW_s\sqrt{\ln 2}}\right)G_{tot}^{1/n}\right)\right)=\frac{d}{dn}\left(\frac{1}{2}\ln(n)+\frac{1}{n}\ln(G_{tot})-\ln\left(GBW_s\sqrt{\ln 2}\right)\right)=0$$

$$\frac{1}{2n}-\frac{1}{n^2}\ln(G_{tot})=0$$

$$\frac{1}{n}\ln(G_{tot})=\frac{1}{2}$$

Thus, the optimum number of stages is

$$n_{opt}=2\ln(G_{tot})$$

and the optimum stage gain is

$$A_{vs,opt}{}^{2\ln(G_{tot})}=G_{tot}$$

$$2\ln(G_{tot})\ln\left(A_{vs,opt}\right)=\ln(G_{tot})$$

$$A_{vs,opt}=\sqrt{e}=1.65$$

Optimum Number of Gain Stages

For example, a multi - stage amplifier with $G_{tot} = 100$ should have

$$n_{opt} = 2\ln(G_{tot}) = 2\ln(100) = 9.21$$

Assuming 9 stages results in

$$A_{vs} = \sqrt[9]{100} = 1.67$$

which is close to $\sqrt{e} = 1.65$

Relative to the per - stage bandwidth, the total amplifier bandwidth will compress to

$$\omega_{\text{3dBtot}} = \omega_{3dBs}\sqrt{2^{1/9} - 1} = 0.283\omega_{3dBs}$$

- Note, while this is the optimum number of stages from a maximum GBW perspective, the bandwidth doesn't falloff too dramatically with lower n
- Thus, from a power and noise perspective, it may make sense to use a lower number of LA stages
- Typically high-gain LAs use between 3-7 stages

Bandwidth Extension Techniques

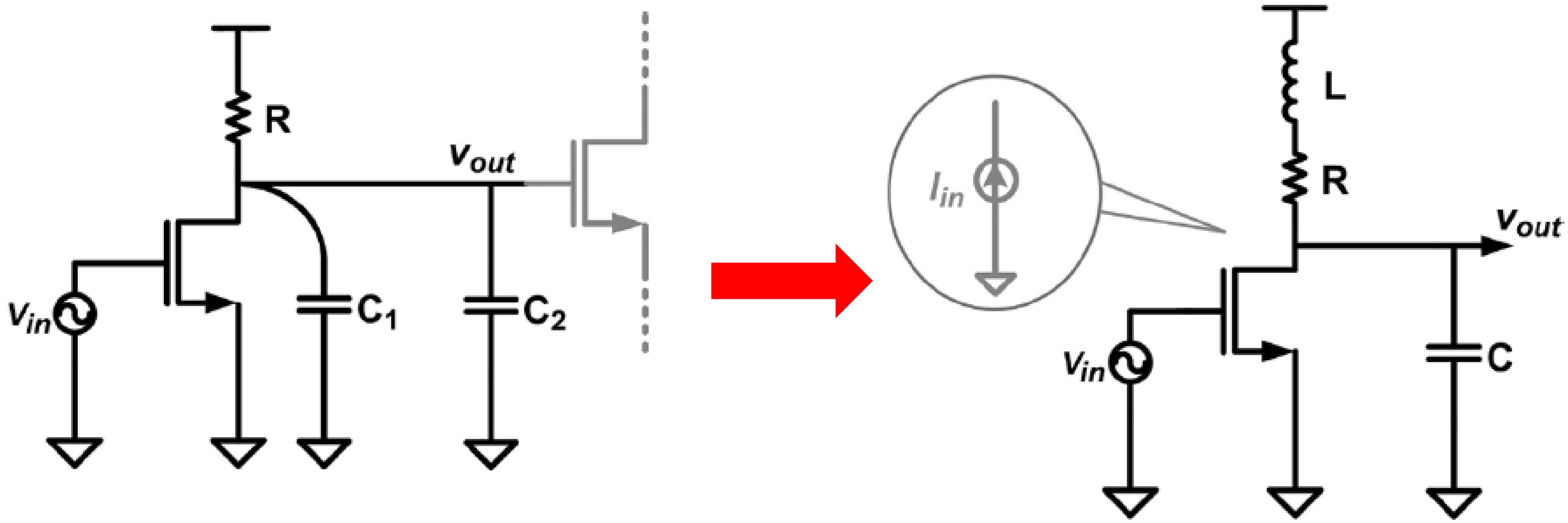
- In order to increase the bandwidth of our multi-stage amplifiers, we need to increase the bandwidth of the individual stages
- Passive bandwidth extension techniques
  - Shunt Peaking
  - Series Peaking
  - T-coil Peaking
- An excellent reference

2424 IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 41, NO. 11, NOVEMBER 2006

Bandwidth Extension Techniques for CMOS Amplifiers

Sudip Shekhar, *Student Member, IEEE*, Jeffrey S. Walling, *Student Member, IEEE*, and David J. Allstot, *Fellow, IEEE*

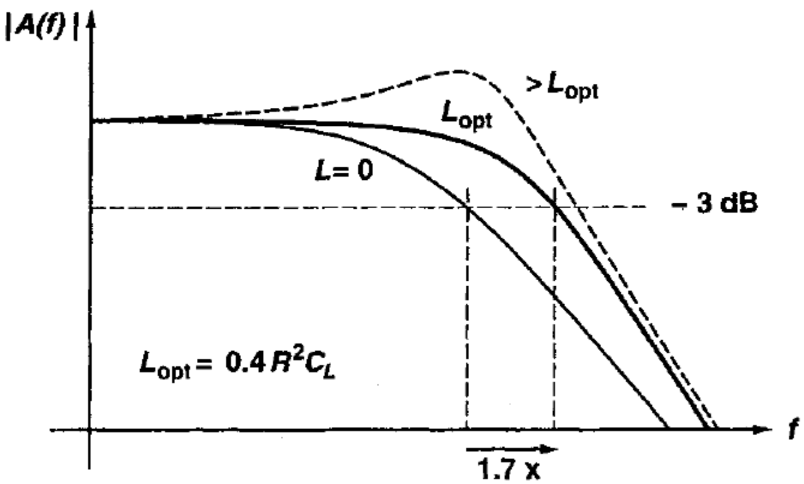
Shunt Peaking



$$Z(s) = \frac{V_{\text{out}}}{I_{\text{in}}} = \left(\frac{1}{sC}\right) \| (R + sL) = \frac{R + sL}{1 + sRC + s^2 LC}$$

- Adding an inductor in series with the load resistor introduces a zero in the impedance transfer function
- This zero increases the impedance with frequency, compensating the decrease caused by the capacitor, and extending the bandwidth

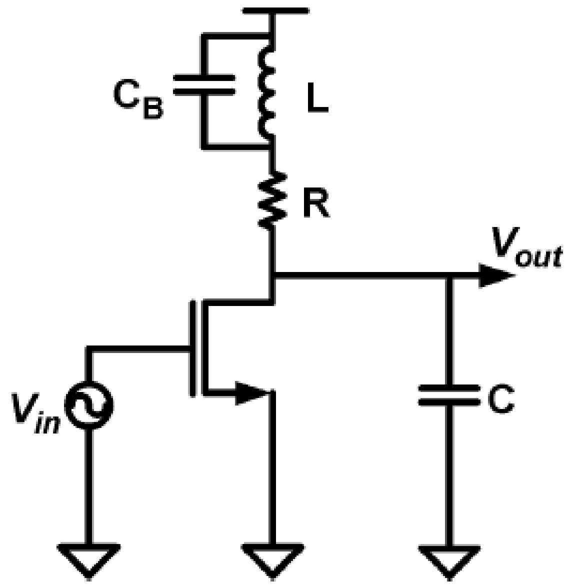
Shunt Peaking

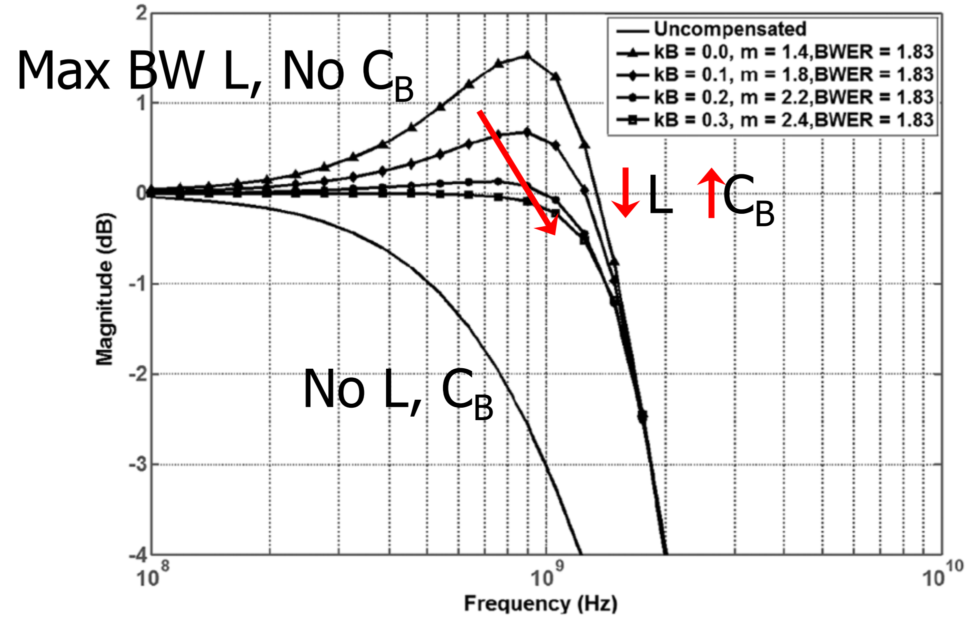


- While the inductor can increase the bandwidth significantly, frequency peaking can occur if the inductor is too big
- For a flat frequency response, ~70% bandwidth increase can be achieved
- A maximum 85% bandwidth increase is possible with 1.5dB of peaking

Condition	Ratio of $\left(\frac{RC}{L/R}\right)$ time constants $m = R^2C/L$	Normalized bandwidth	Normalized peak frequency response
Maximum bandwidth	~1.41	~1.85	1.19
$\lvert Z\rvert = R$ @ $\omega = 1/RC$	2	~1.8	1.03
Maximally flat frequency response	~2.41	~1.72	1
Best group delay	~3.1	~1.6	1
No shunt peaking	∞	1	1

Bridged-Shunt Peaking



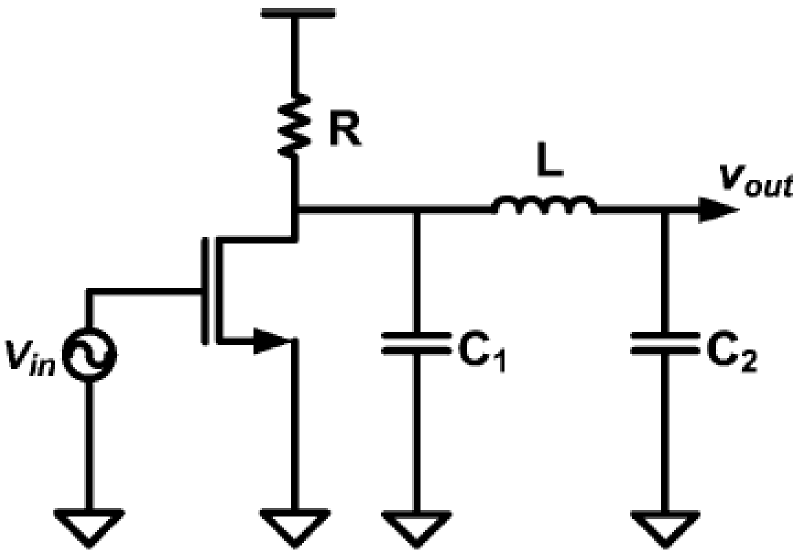


$$Z_N(s) = \frac{1 + \left(\frac{1}{m}\right)\frac{s}{\omega_0} + \left(\frac{k_B}{m}\right)\frac{s^2}{\omega_0^2}}{1 + \frac{s}{\omega_0} + \left(\frac{k_B + 1}{m}\right)\frac{s^2}{\omega_0^2} + \left(\frac{k_B}{m}\right)\frac{s^3}{\omega_0^3}}$$

$$k_B = C_B/C,\ \omega_0 = 1/RC,\ \text{and } m = R^2C/L$$

- Adding a bridge capacitor in parallel with the inductor allows for compensation of the frequency peaking with the possible maximum shunt peaking bandwidth increase
- A real inductor will always have some parasitic C_B, and thus k_B will be >0 in practice even without an extra cap

Series Peaking

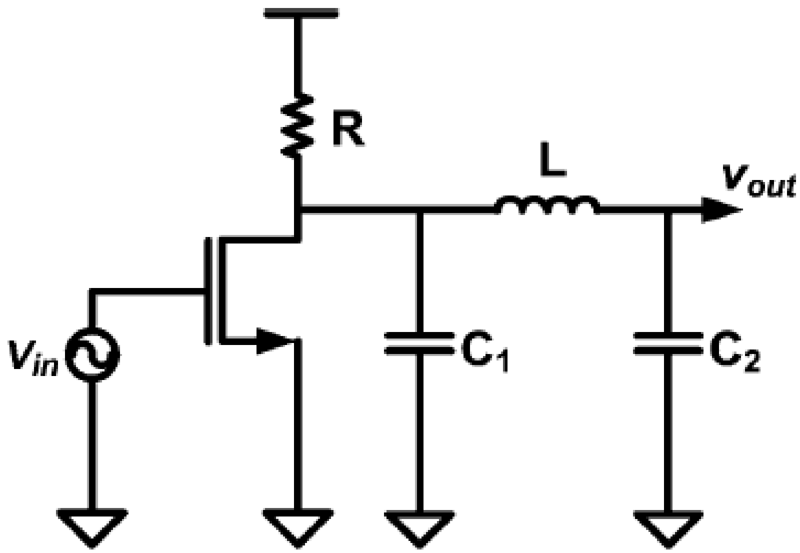


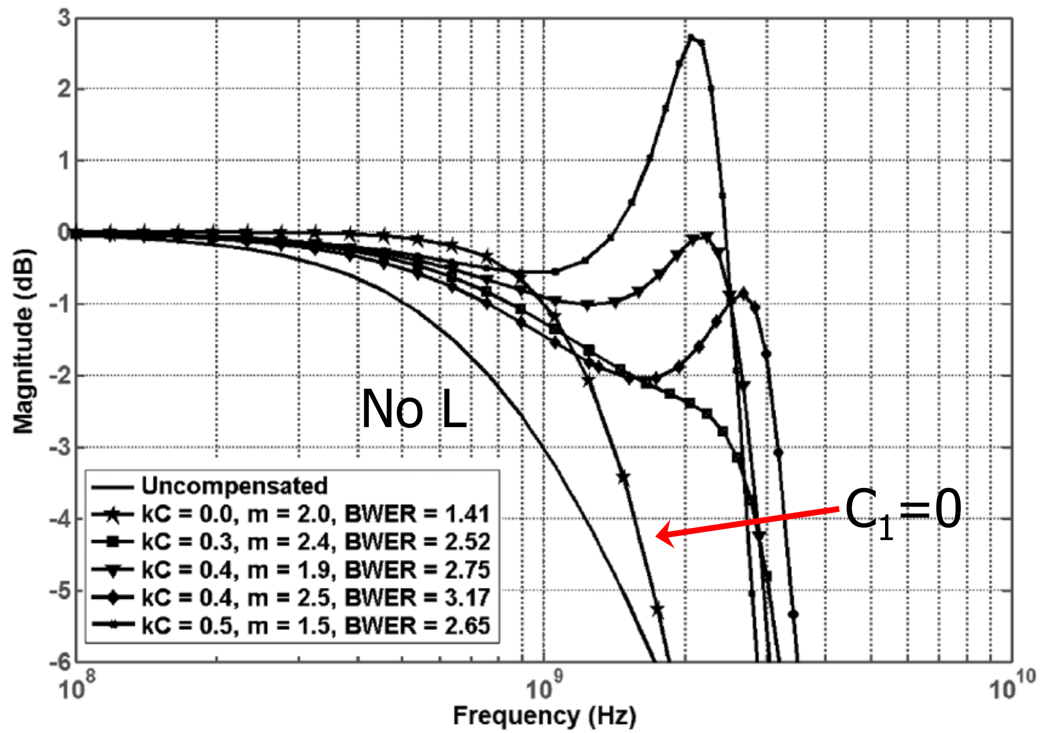
$$Z_N(s) = \frac{1}{1 + \frac{s}{\omega_0} + \left(\frac{1-k_C}{m}\right)\frac{s^2}{\omega_0^2} + \left(\frac{k_C(1-k_C)}{m}\right)\frac{s^3}{\omega_0^3}}$$

$$k_C = C_1/C \qquad m = R^2C/L$$

- Introducing a series peaking inductor is useful to “split” the load capacitance between the amplifier drain capacitance and the next stage gate capacitance
- Without L, the transistor has to charge the total capacitance at the same time
- With L, initially only C_1 is charged, reducing the risetime at the drain and increasing bandwidth

Series Peaking



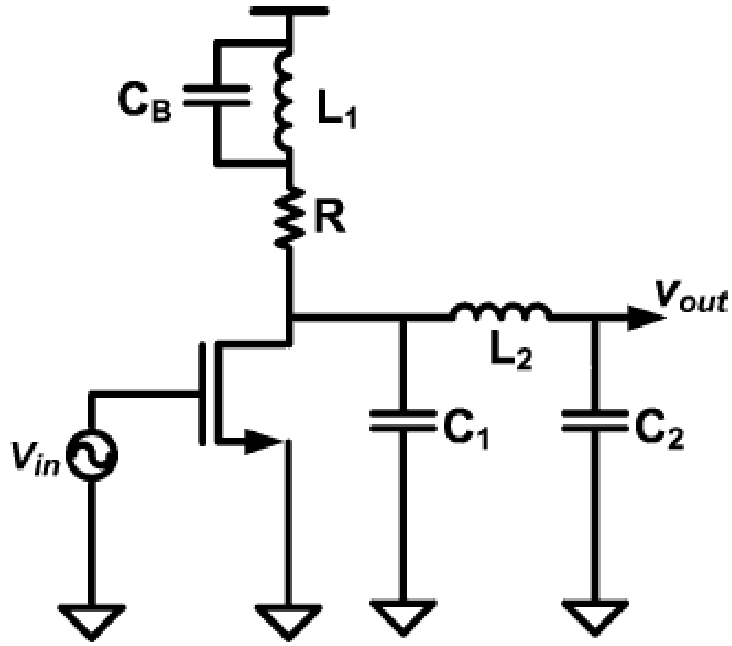


- As the capacitance is more distributed with a higher k_C value, a higher BWER is achieved
- Up to 2.5x bandwidth increase is achieved with no peaking
- Higher BWER is possible with some frequency peaking

$k_C=C_1/C$	Ripple (dB)	$m=R^2C/L$	BWER
0	0	2	1.41
0.1	0	1.8	1.58
0.2	0	1.8	1.87
0.3	0	2.4	2.52
0.4	1	1.9	2.75
	2	2.5	3.17
0.5	3.3	1.5	2.65

Bridged-Shunt-Series Peaking

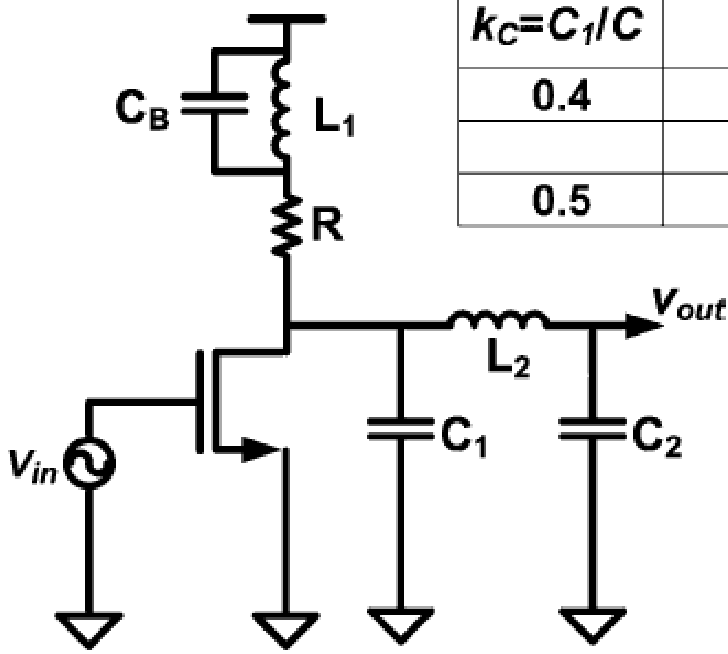
- Combining both shunt and series peaking can yield even higher bandwidth extension



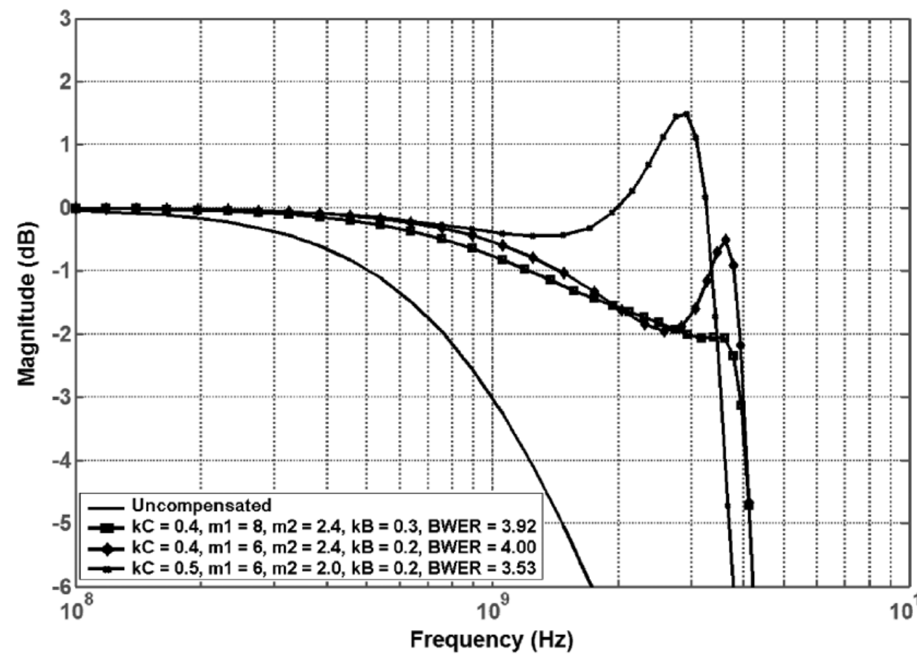
$$Z_N(s) = \frac{1+\left(\frac{1}{m_1}\right)\frac{s}{\omega_0}+\left(\frac{k_B}{m_1}\right)\frac{s^2}{\omega_0^2}}{1+\frac{s}{\omega_0}+\left(\frac{1+k_B}{m_1}+\frac{1-k_C}{m_2}\right)\frac{s^2}{\omega_0^2}+\left(\frac{k_B}{m_1}+\frac{k_C(1-k_C)}{m_2}\right)\frac{s^3}{\omega_0^3}+\left(\frac{(k_C+k_B)(1-k_C)}{m_1 m_2}\right)\frac{s^4}{\omega_0^4}+\left(\frac{k_B k_C(1-k_C)}{m_1 m_2}\right)\frac{s^5}{\omega_0^5}}$$

$$m_1 = R^2 C/L_1 \qquad m_2 = R^2 C/L_2$$

Bridged-Shunt-Series Peaking



$k_C=C_1/C$	Ripple (dB)	$m_1=R^2C/L_1$	$m_2=R^2C/L_2$	$k_B=C_B/C$	BWER
0.4	0	8	2.4	0.3	3.92
	2	6	2.4	0.2	4
0.5	2	6	2	0.2	3.53

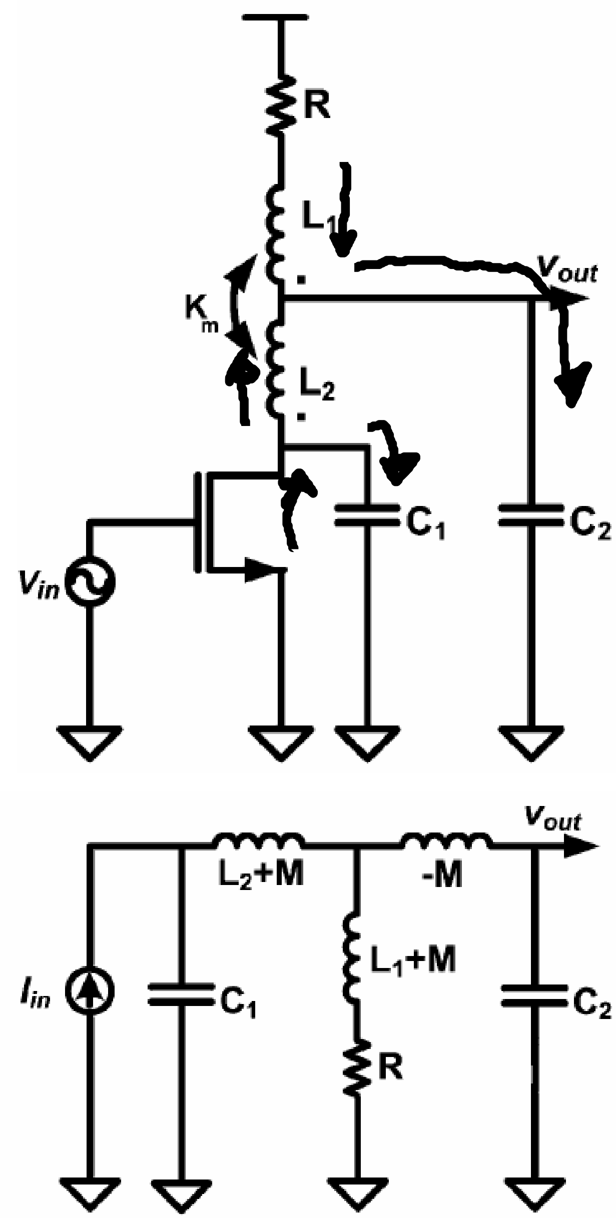


- Proper choice of component values can yield close to 4x increase in bandwidth with no peaking
- However, this requires tight control of these components, which can be difficult with PVT variations

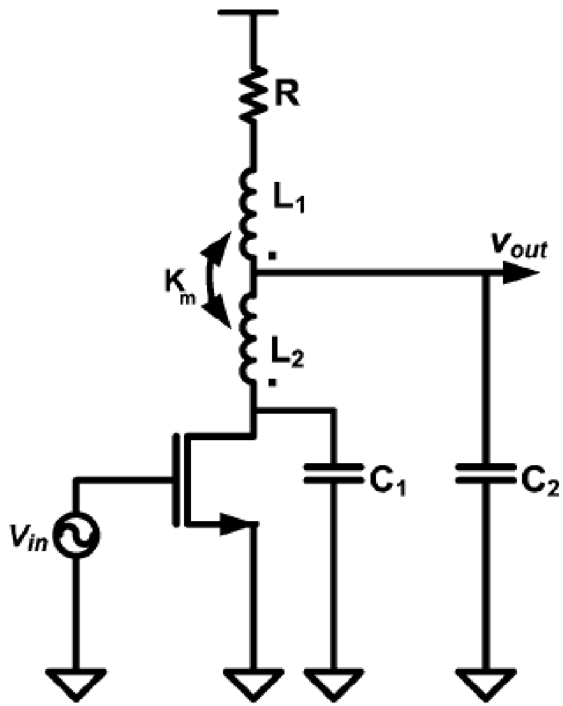
T-Coil Peaking

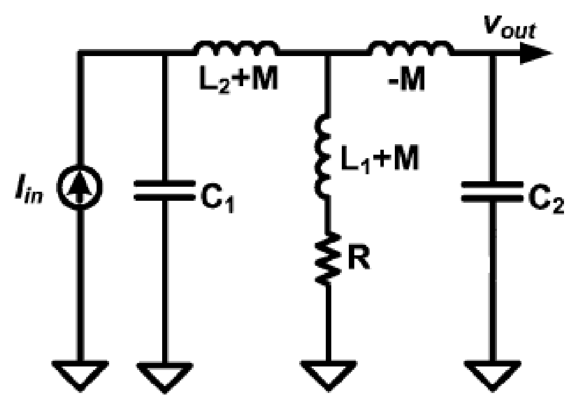
- If the input transistor drain capacitance (C_1) is relatively small, then the bandwidth extension through shunt-series peaking is limited

- T-coil peaking, which utilizes the magnetic coupling of a transformer, provides better bandwidth extension in this case
    - L_2 performs capacitive splitting, such that the initial current charges only C_1
    - As current begins to flow through L_2, magnetically coupled current also flows through L_1, providing increased current to charge C_2 which improves bandwidth and transition times



T-Coil Peaking

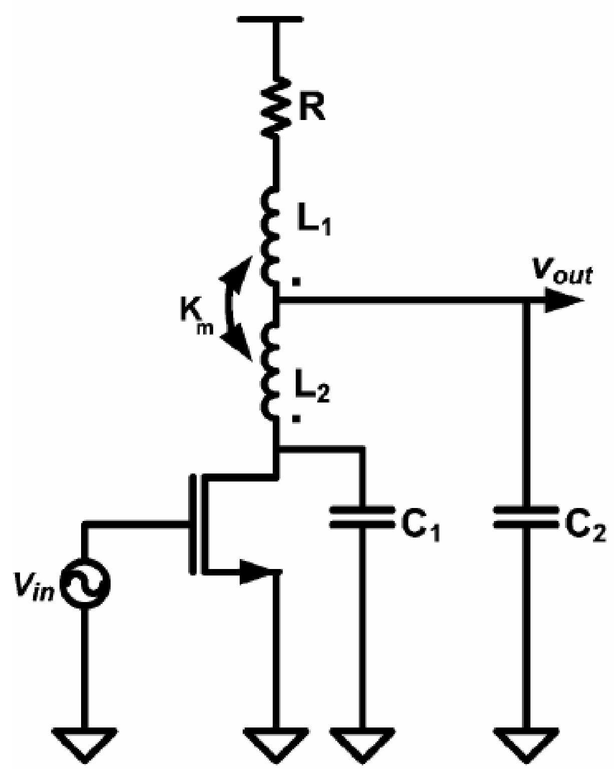


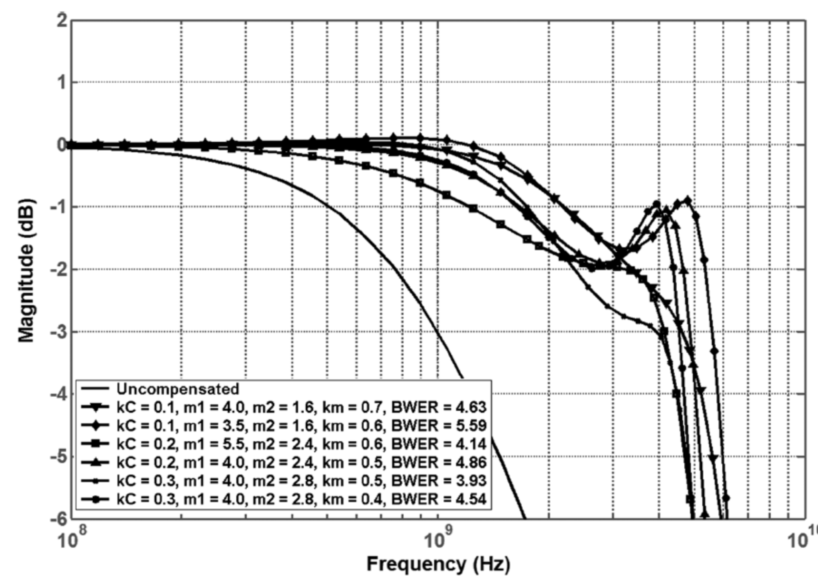


$$Z_N(s) = \frac{1 + \left(\frac{1}{m_1} + \frac{k_m}{\sqrt{m_1 m_2}}\right)\frac{s}{\omega_0}}{1 + \frac{s}{\omega_0} + \left(\frac{1}{m_1} + \frac{k_C}{m_2} + \frac{2k_C k_m}{\sqrt{m_1 m_2}}\right)\frac{s^2}{\omega_0^2} + \left(\frac{k_C(1-k_C)}{m_2}\right)\frac{s^3}{\omega_0^3} + \left(\frac{k_C(1-k_C)\left(1-k_m^2\right)}{m_1 m_2}\right)\frac{s^4}{\omega_0^4}}$$

$$k_m = M/\sqrt{L_1 L_2}$$

T-Coil Peaking





$k_C=C_1/C$	Ripple (dB)	$m_1=R^2C/L_1$	$m_2=R^2C/L_2$	$k_m = M/\sqrt{L_1L_2}$	BWER
0.1	0	4	1.6	-0.7	4.63
	1	3.5	1.2	-0.6	4.92
	2	3.5	1.6	-0.6	5.59
0.2	0	5.5	2.4	-0.6	4.14
	1	3	2	-0.6	4.51
	2	4	2.4	-0.5	4.86
0.3	0	4	2.8	-0.5	3.93
	1	3.5	2	-0.4	3.98
	2	4	2.8	-0.4	4.54

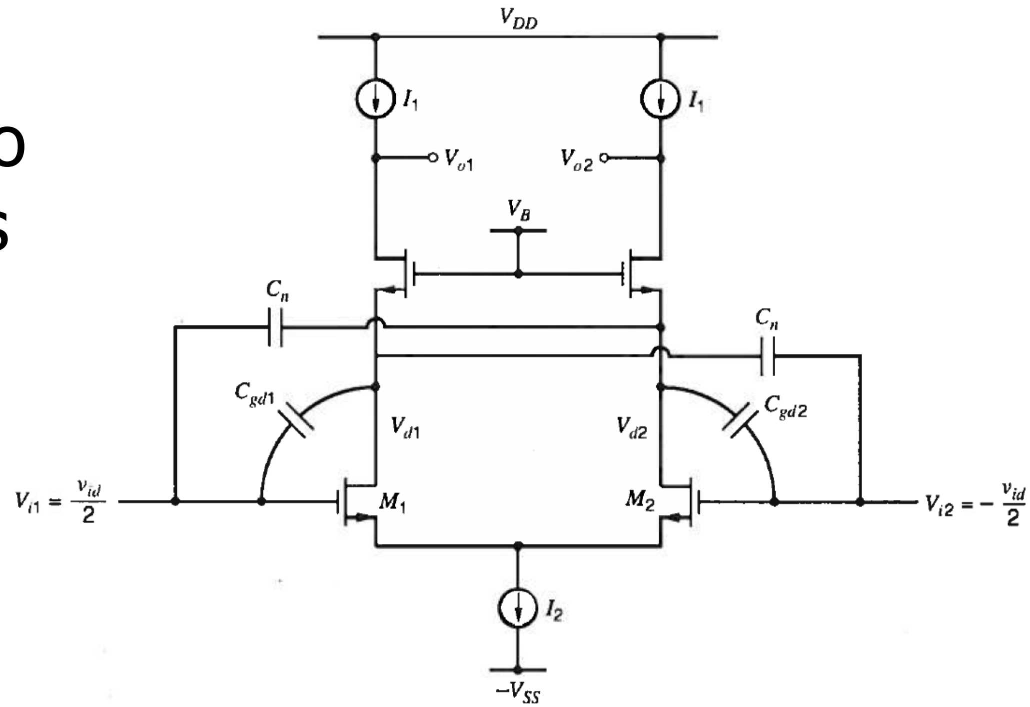
- A bandwidth extension of 4x is possible without any frequency peaking
- If peaking is acceptable, then a BWER near 5 can be achieved, depending on the size of C_1

Active Bandwidth Extension Techniques

- While passive techniques offer excellent bandwidth extension at near zero power cost, there are some disadvantages
  - Generally large area
  - Process support/characterization of inductors/transformers
- Active circuit techniques can also be employed to extend amplifier bandwidth
- Some active bandwidth extension techniques
  - Negative Miller Capacitance
  - TIA Load
  - Active Negative Feedback

Negative Miller Capacitance

- In modern technologies, Cgd is a significant (50% to near 100%) fraction of Cgs
- Amplifier effective input capacitance can increase significantly due to the Miller multiplication of Cgd
- Without additional Cn:

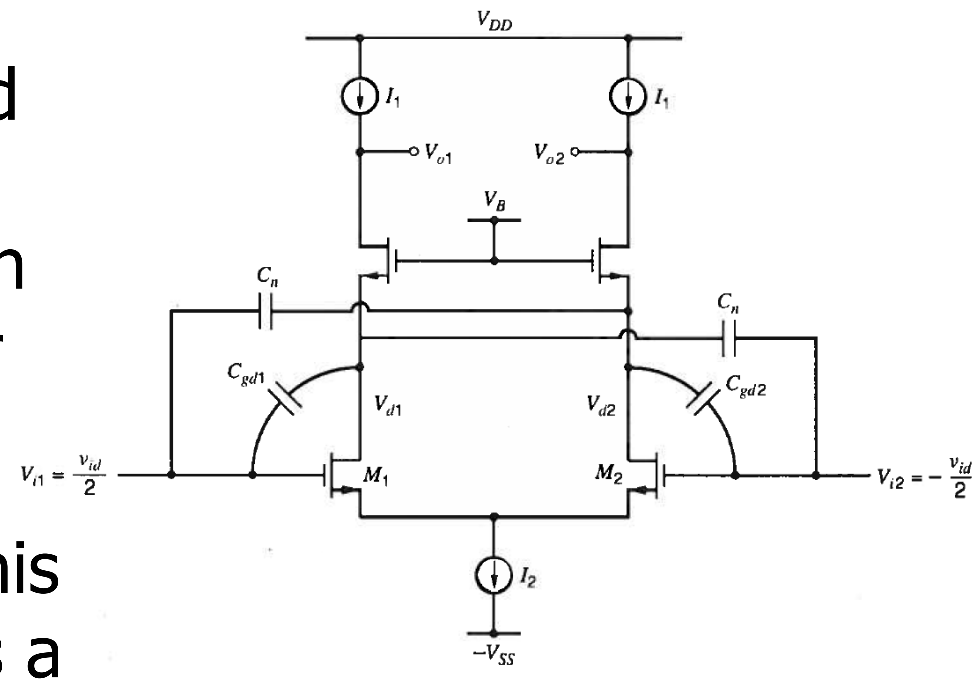


$$C_{in} = C_{gs1} + C_{gd1}\left(1 - A_{gd}\right)$$

As A_{gd} is negative, and often is the differential gain of the amplifier, this can result in significant increase in the effective input capacitance

Negative Miller Capacitance

- In order to mitigate this Cgd multiplication, additional cross-coupled capacitors can be added from the amplifier inputs to the outputs
- Effectively, the charge on this additional capacitor charges a (large) portion of the Cgd capacitor



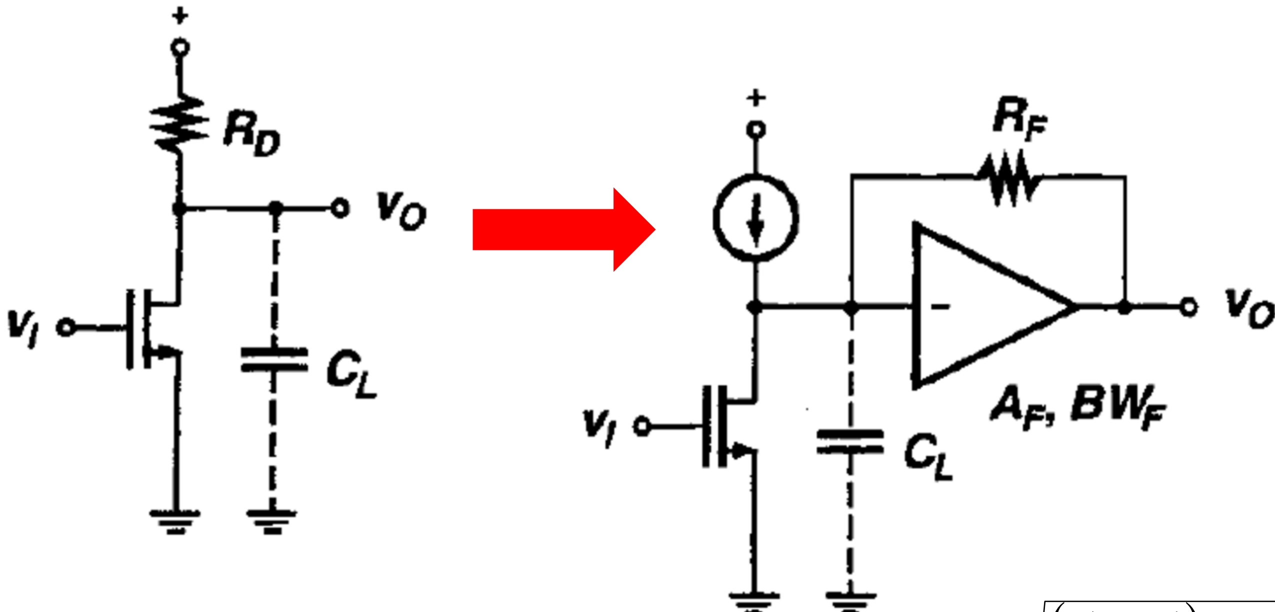
$$C_{in} = C_{gs1} + C_{gd1}\left(1 - A_{gd}\right) + C_n\left(1 - \left(-A_{gd}\right)\right)$$

If C_n is set equal to C_{gd1}

$$C_{in} = C_{gs1} + 2C_{gd1}$$

Thus, as long as the amplifier gain is > 1, a reduction in the effective input capacitance is achieved

TIA Load



$$A_v = -g_m R_D$$
$$C_{in} = C_{gs} + C_{gd}\left(1 + g_m R_D\right)$$
$$\omega_p = \frac{1}{R_D C_L}$$

$$A_v = -g_m Z_T(s)$$
$$Z_T(s) = \frac{\left(\dfrac{R_F A_F}{1 + A_F}\right)}{1 + \dfrac{s}{\omega_o Q} + \dfrac{s^2}{\omega_o^2}}$$

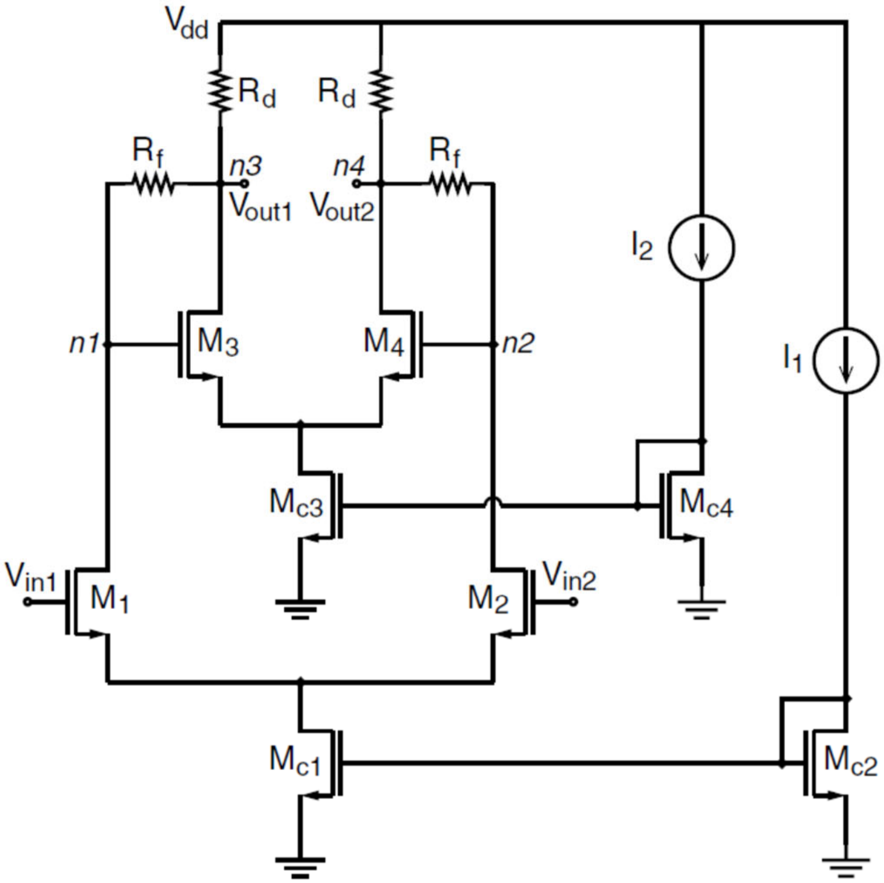
$$\omega_o = \sqrt{\frac{(A_F + 1) BW_F}{R_F C_L}} \qquad Q = \frac{\sqrt{(A_F + 1) R_F C_L \left(\dfrac{1}{BW_F}\right)}}{R_F C_L + \dfrac{1}{BW_F}}$$

with a Butterworth response, can show that

$$\omega_{3\mathrm{dB}} \approx \frac{\sqrt{A}}{R_D C_L} \quad \text{for the same gain.}$$

$$C_{in} = C_{gs} + C_{gd}\left(1 + \frac{g_m R_D}{A_F}\right)$$

Cherry Hooper Amplifier



$$A_{CH} = A_{CH,0} \frac{1 - s\frac{C_{gd,M3}}{g_{m,M3}}}{s^2 \frac{R_f}{g_{m,M3}} C^2 + s(RC)_{CH} + 1},$$

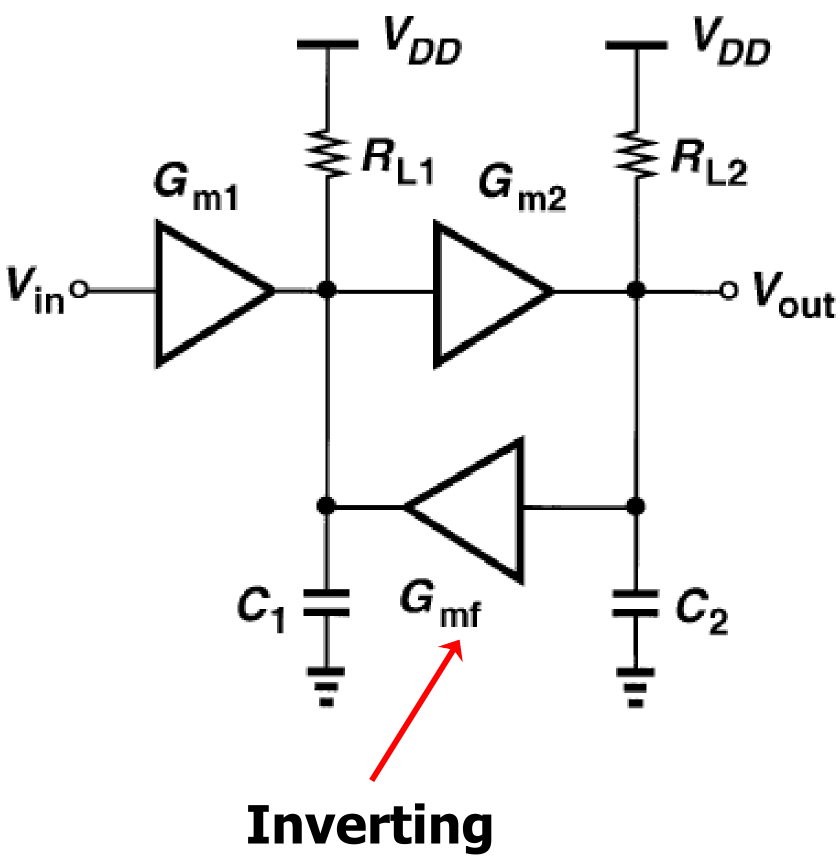
$$A_{CH,0} = g_{m,M1} R_f,$$

$$C^2 = C_1 C_{gd,M3} + C_1 C_L + C_{gd,M3} C_L,$$

$$(RC)_{CH} = R_f C_{gd,M3} + \frac{R_f + R_d}{R_d g_{m,M3}} C_1 + \frac{C_L}{g_{m,M3}}.$$

Active Negative Feedback

- Instead of using simple first-order amplifier cells, a second-order cell with active negative feedback can provide bandwidth enhancement



$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A_{vo}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

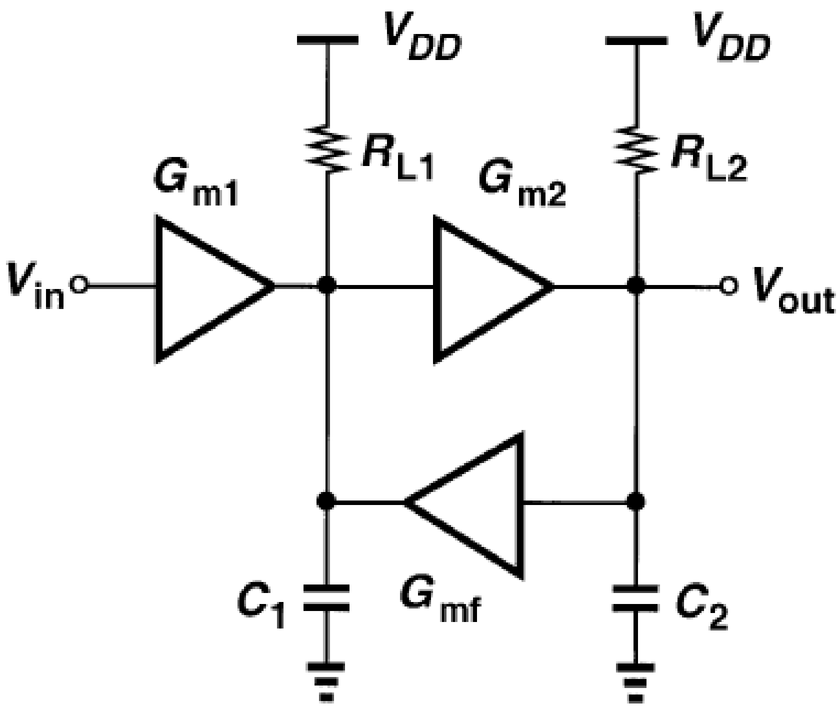
$$A_{vo} = \frac{G_{m1}G_{m2}R_{L1}R_{L2}}{1 + G_{m2}G_{mf}R_{L1}R_{L2}}$$

$$\zeta = \frac{1}{2}\frac{R_{L1}C_1 + R_{L2}C_2}{\sqrt{R_{L1}R_{L2}C_1C_2(1 + G_{mf}G_{m2}R_{L1}R_{L2})}}$$

$$\omega_n^2 = \frac{1 + G_{mf}G_{m2}R_{L1}R_{L2}}{R_{L1}R_{L2}C_1C_2}.$$

Active Negative Feedback

- This second-order amplifier cell can be optimized for different objectives, but G_{mf} can be set to yield a Butterworth response with a maximally-flat frequency response



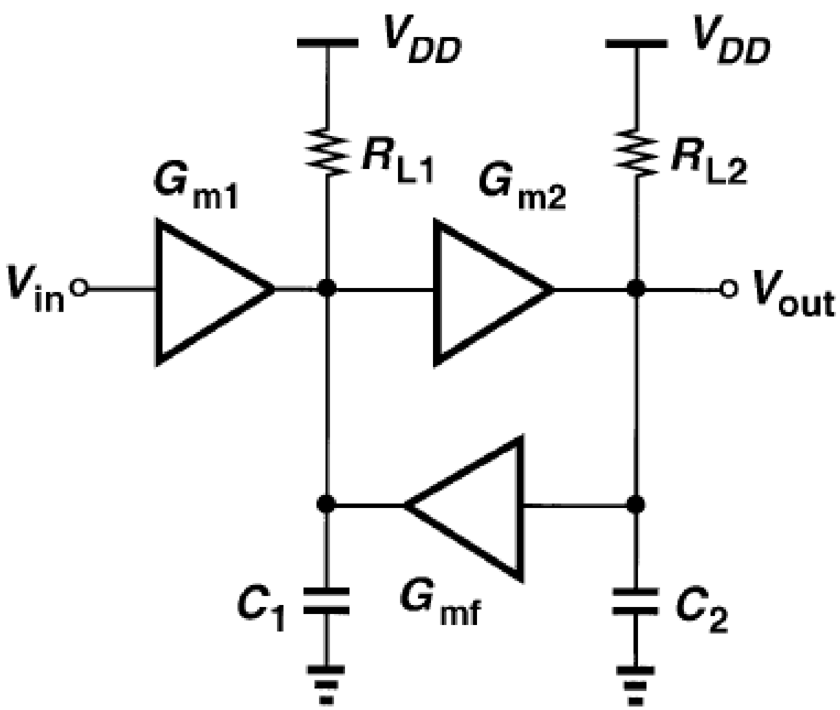
$$\zeta = \sqrt{2}/2$$

$$\omega_{-3dB} = \omega_n$$

$$A_{vo}\omega_{-3\text{dB}}{}^2 = \frac{G_{m1}G_{m2}}{C_1C_2}$$

$$A_{vo}\omega_{-3\text{dB}} = \frac{G_{m1}G_{m2}}{C_1C_2}\frac{1}{\omega_{-3\text{dB}}}.$$

Active Negative Feedback



$$A_{vo}\omega_{-3\text{dB}} = \frac{G_{m1}G_{m2}}{C_1C_2}\frac{1}{\omega_{-3\text{dB}}}.$$

The ratio $\frac{G_m}{C}$ is proportional to the technology ω_T

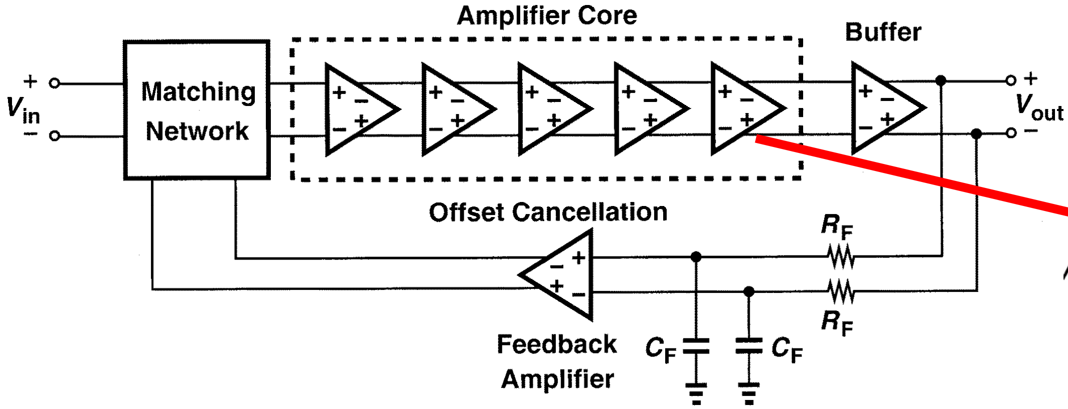
$$\frac{G_m}{C} = \alpha\omega_T$$

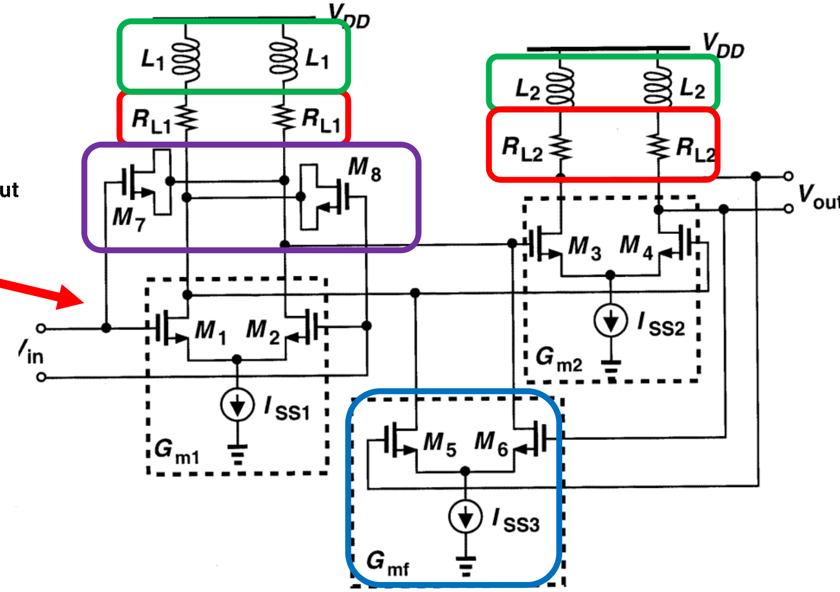
Assuming $\frac{G_{m1}}{C_1} \approx \frac{G_{m2}}{C_2} \approx \alpha\omega_T$

$$A_{vo}\omega_{-3dB} = \frac{(\alpha\omega_T)^2}{\omega_{-3dB}} = \omega_T\left(\frac{\alpha^2\omega_T}{\omega_{-3dB}}\right)$$

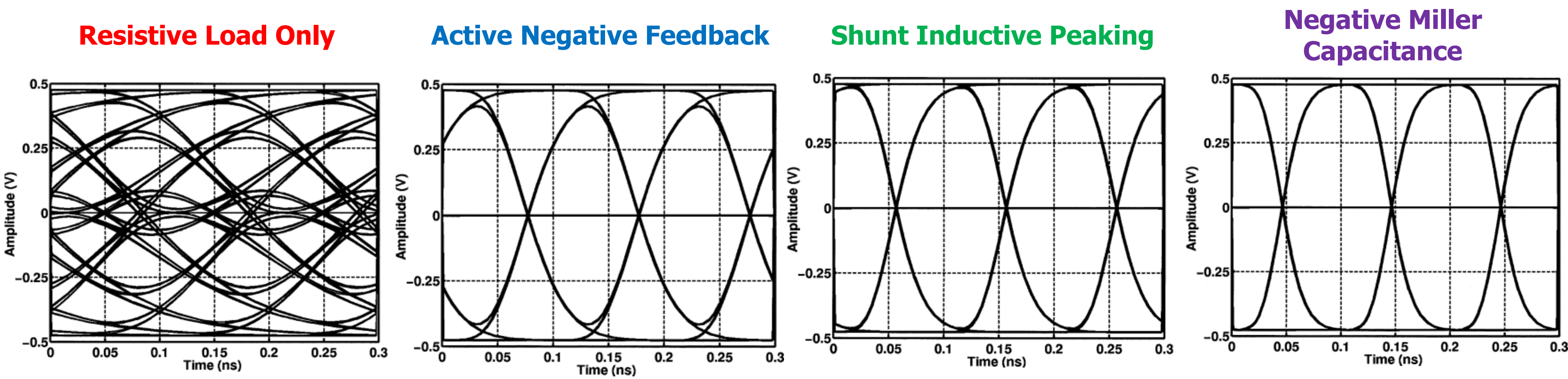
- The second-order cell gain-bandwidth can potentially achieve a value greater than the technology f_T

Limiting Amplifier Example 1



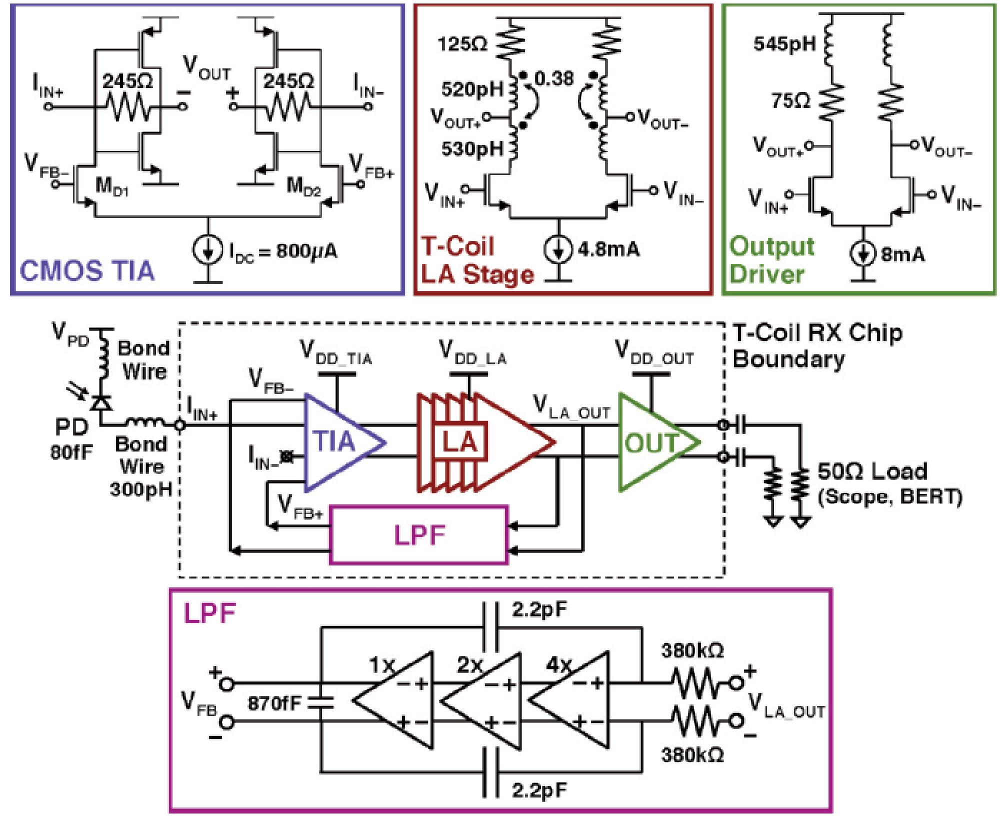


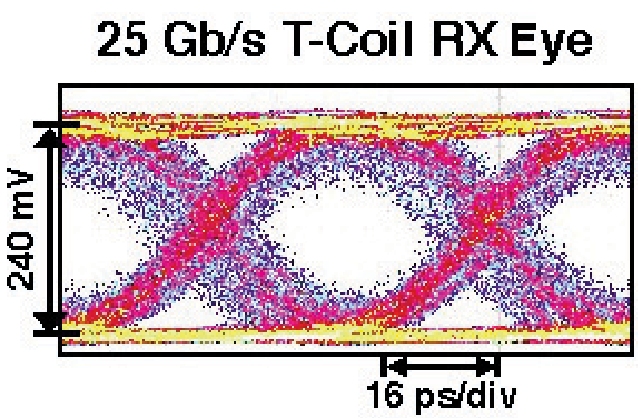
[Galal JSSC 2003]

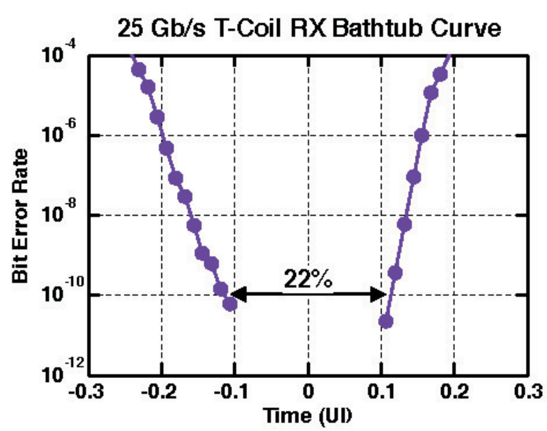


Limiting Amplifier Example 2

- T-coils in LA stages allow for a combination of series and shunt peaking and close to 3x bandwidth extension







[Proesel ISSCC 2012]

Offset Compensation

- The receiver sensitivity is degraded if the limiting amplifier has an input-referred offset
- This is often quantified in terms of a Power Penalty, PP

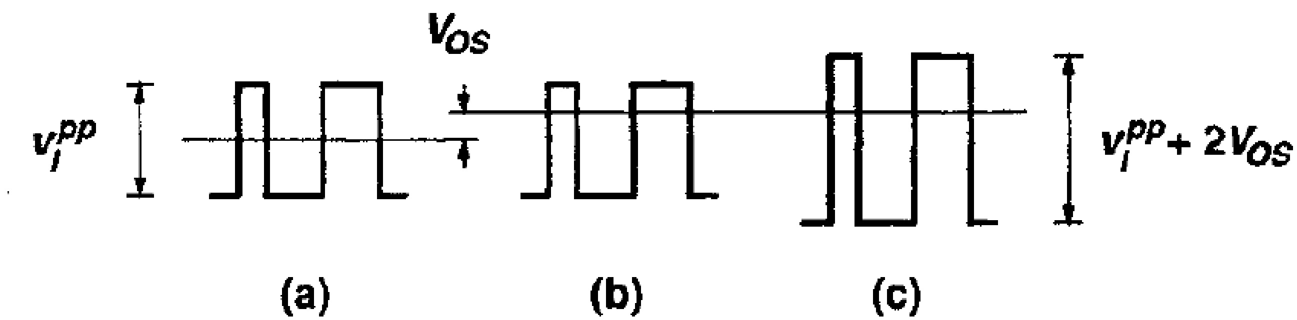
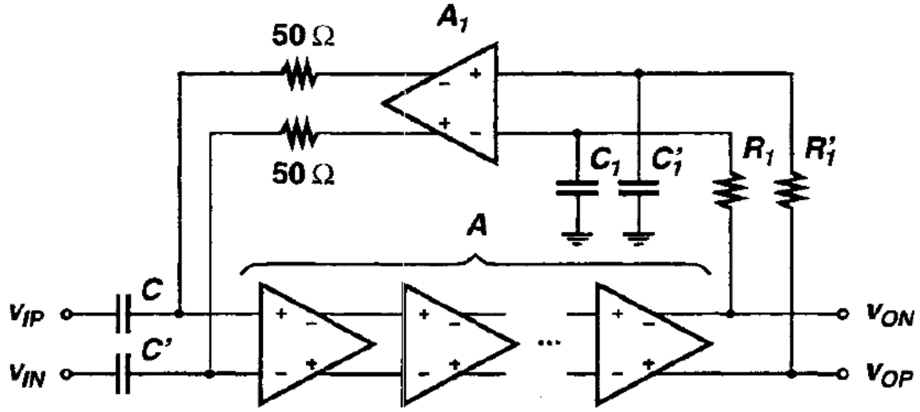


Fig. 6.5 Effect of an input offset voltage in the LA: (a) without offset, (b) with offset, and (c) with offset and increased signal swing to restore the original bit-error rate.

$$PP = \frac{v_I^{pp} + 2V_{OS}}{v_I^{pp}} = 1 + \frac{2V_{OS}}{v_I^{pp}}$$

- It is important to minimize the offset of these multi-stage limiting amplifiers!

Offset Compensation



- The DC offset, V_{os}, of the limiting amplifier is compensated by a low-frequency negative feedback loop

Ideally, this reduces the offset to

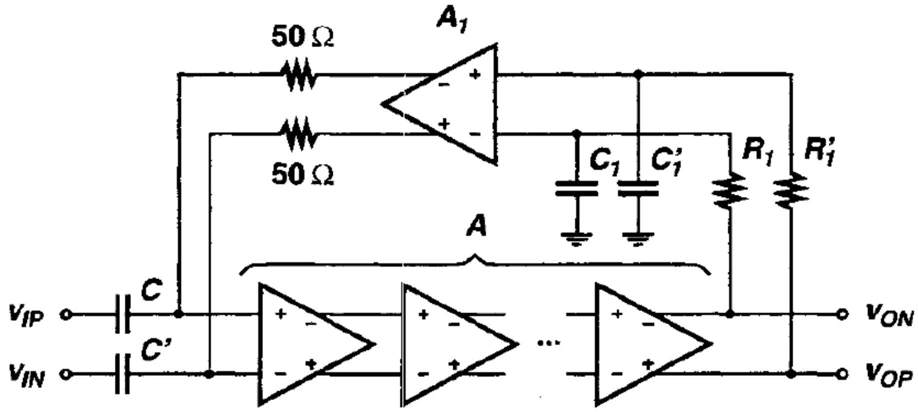
$$\frac{V_{os}}{1+AA_1} \approx \frac{V_{os}}{AA_1}$$

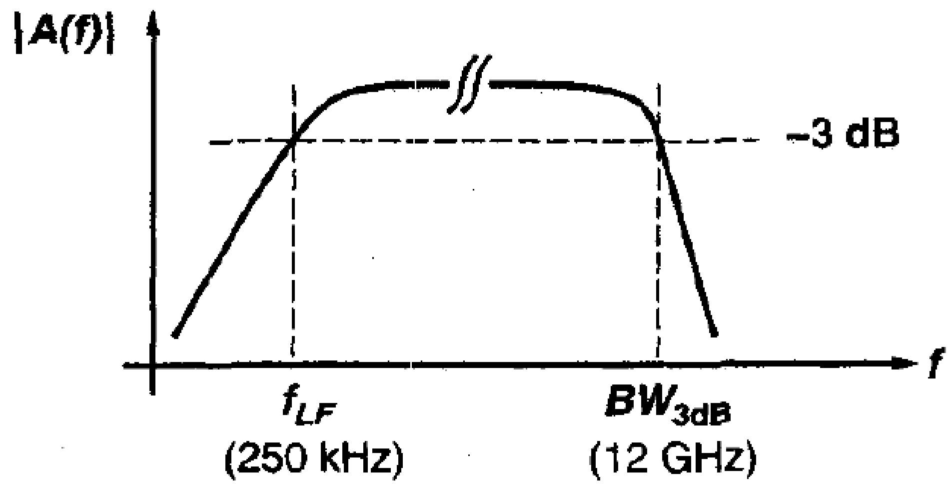
However, if the error amplifier has an offset, V_{os1}, the offset becomes

$$\sqrt{\left(\frac{V_{os}}{AA_1}\right)^2 + \left(\frac{V_{os1}}{A}\right)^2}$$

with uncorrelated offset voltages

Offset Compensation





- The low-pass filtering in the feedback loop causes a low-frequency cutoff

$$f_{LF} = \frac{1}{2\pi}\frac{AA_1/2+1}{R_1C_1}$$

Note, the $AA_1/2$ factor assume a 50Ω driver source

- Thus, the feedback loop bandwidth should be made much lower than the lowest frequency content of the input data
- This may lead to large-area passive in the offset correction feedback
- Some designs leverage Miller capacitive multiplication with the error amplifier to reduce this filter area

Next Time

- High-Speed Transmitters