ECEN689: Special Topics in Optical Interconnects Circuits and Systems

Spring 2022

Lecture 4: Receiver Analysis

1876

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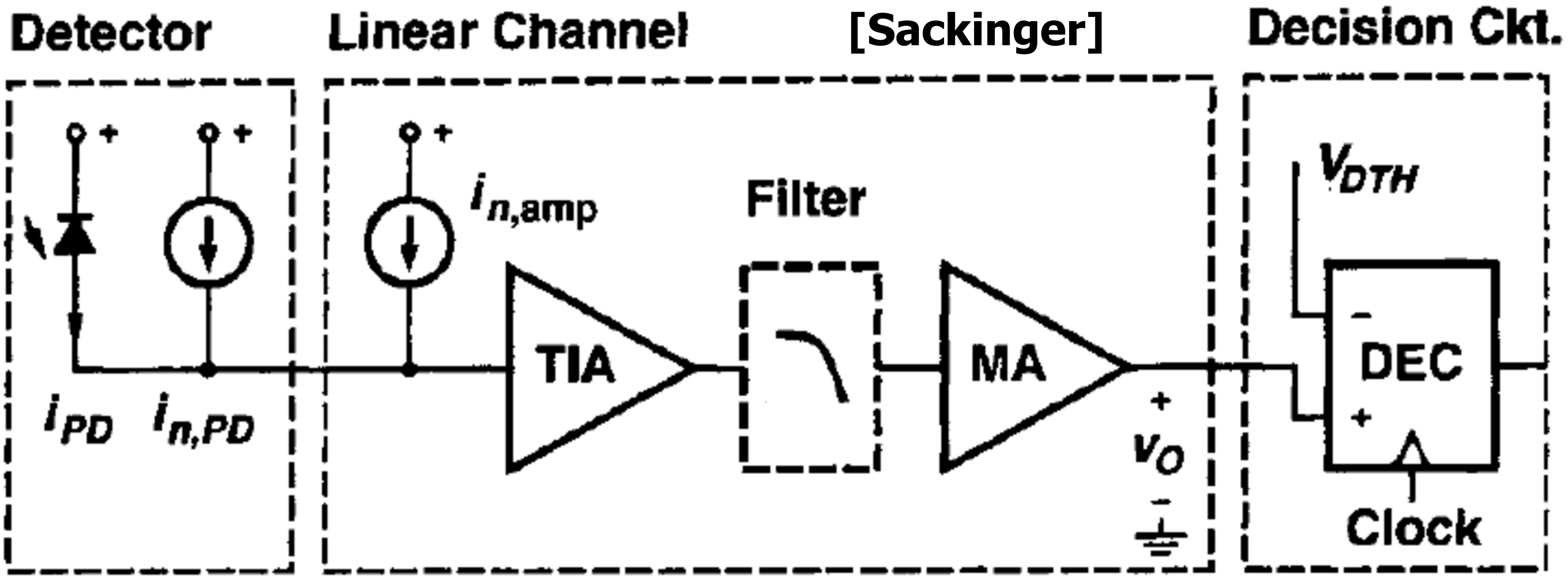
Announcements

- Homework 2 is posted on website and due Mar 1
- Majority of material follows Sackinger Chapter 4

Agenda

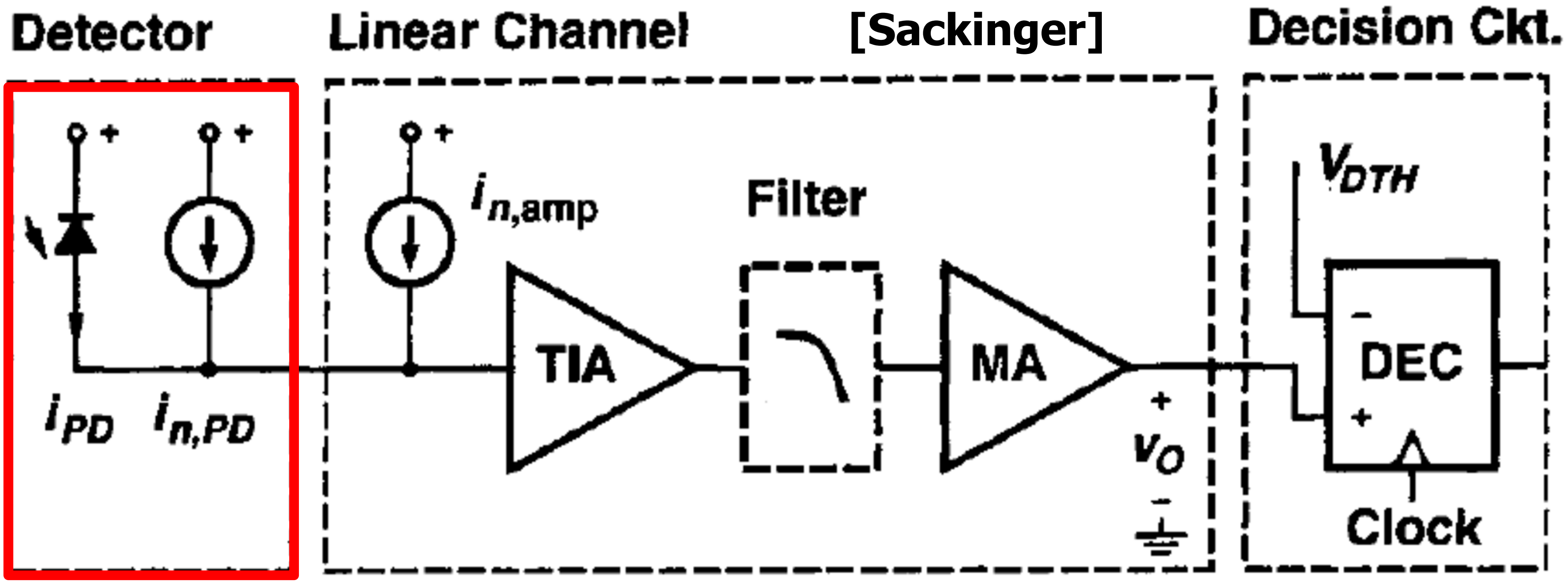
- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

Receiver Model



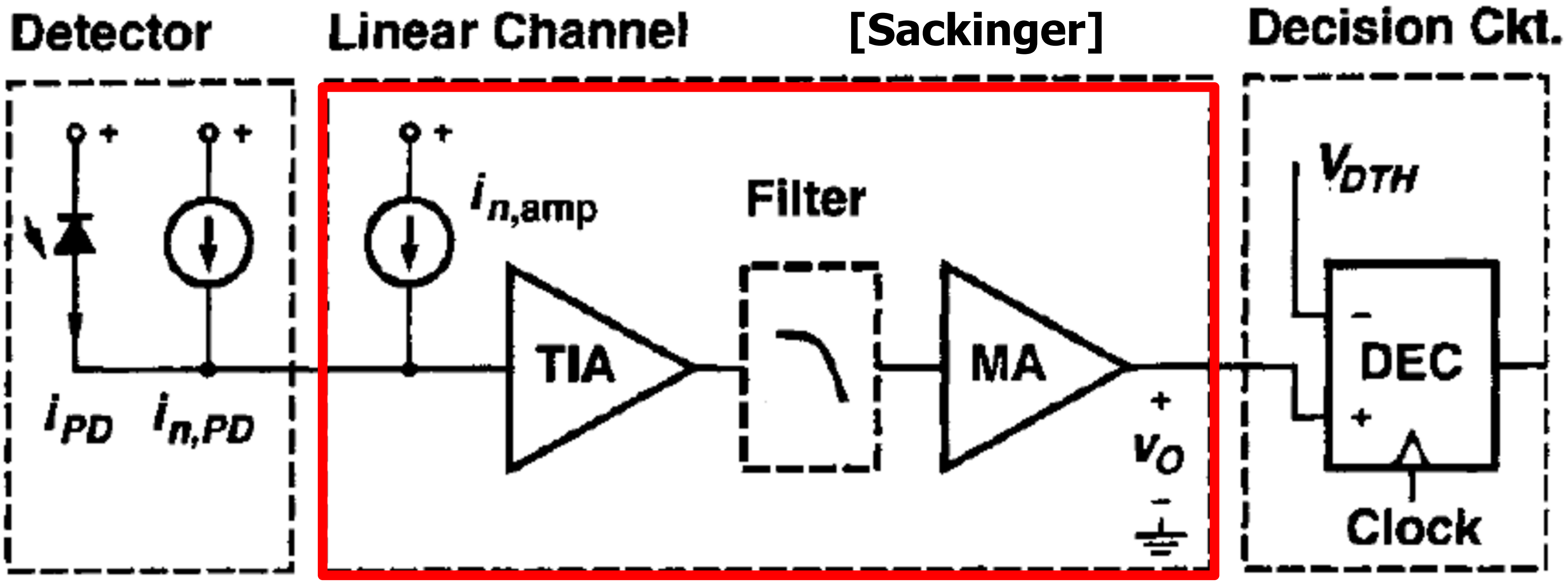
- Photodetector model
- Linear channel representing the transimpedance amplifier (TIA) and main amplifier (MA) gain and an optional low-pass filter
- Detector with a decision threshold, V_{DTH}

Receiver Detector Model



- Signal current source i_{PD} which is linearly related to the optical power
- Noise current source $i_{n,AMP}$ whose spectrum is approximated as uniform and signal dependent

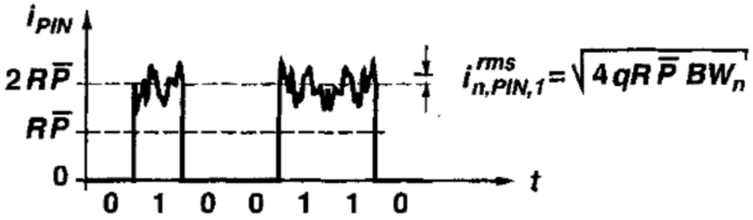
Receiver Linear Channel (Front-End)



- Modeled with a linear transfer function *H(f)* relating the output voltage v_O amplitude & phase with input current i_{PD}
  - From a sensitivity perspective, the signals are small & linearity generally holds
- Single input-referred noise current source with a spectrum that produces the correct output-referred noise spectrum after passing through *H(f)*
- Generally, the TIA’s input-referred noise dominates

Detector and Amplifier Noise

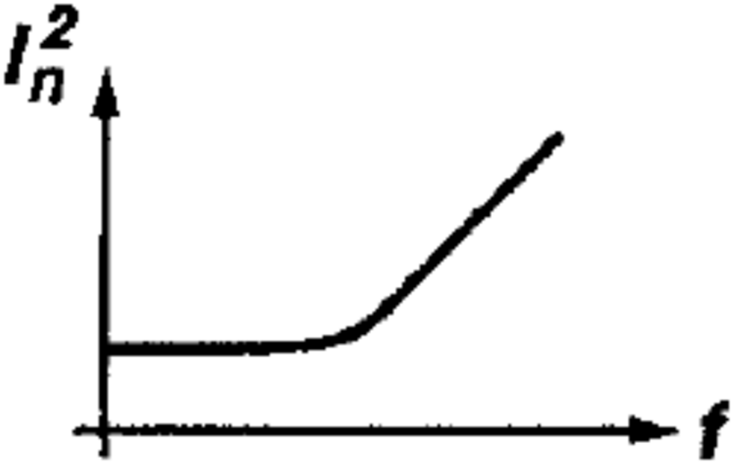
- Detector noise
  - Nonstationary – rms value changes with the bit value
  - Uniform (white) frequency spectrum
  - Noise power spectral density must formally be written as a time-varying function



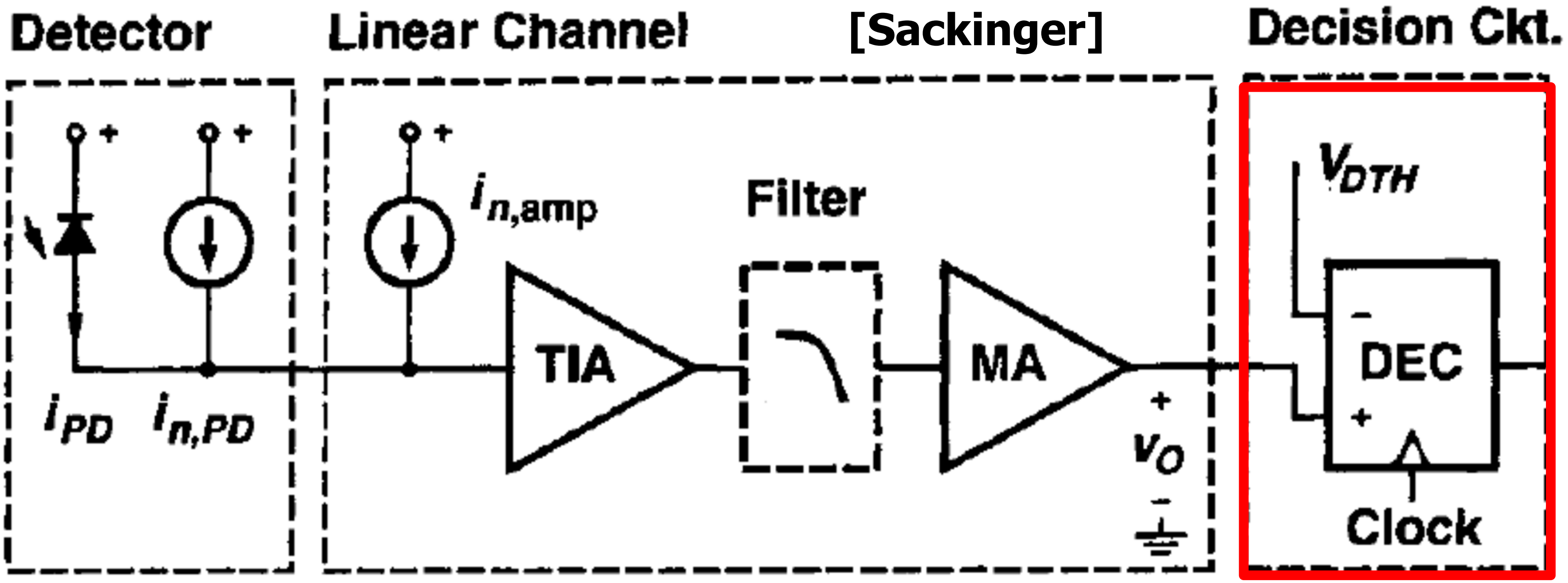
$$I^2_{n,PD}(f,t) \sim \text{bit\_value}(t)$$

- Amplifier noise
  - Stationary – rms independent of time
  - Non-white frequency spectrum which is well modeled as having a white component and a component that increases $\propto$ to f^2

$$I^2_{n,amp}(f) = \alpha_0 + \alpha_2 f^2 + \ldots$$



Receiver Decision Circuit



- Compares the linear channel output v_O with a decision threshold V_{DTH}
- For binary (OOK) modulation
    - Above $V_{DTH} \rightarrow$ "One" bit
    - Below $V_{DTH} \rightarrow$ "Zero" bit

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Bit Errors

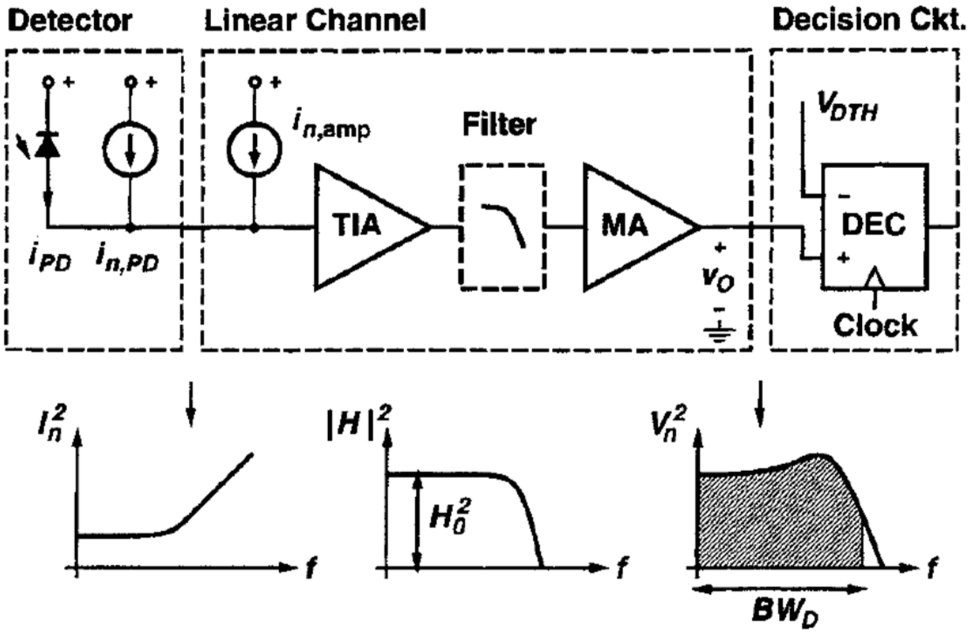
The receiver front - end output before the decision element can be modeled as the superposition of the desired signal and the noise

$$v_O(t) = v_S(t) + v_n(t)$$

Occasionally, the instantaneous noise voltage $v_n(t)$ can suffiently corrupt the output and exceed the decision threshold V_{DTH} to cause a bit error.

Ideally, this happens at a low - probability or bit - error rate (BER)

Output Noise – Amplifier Component



[Sackinger]

$$\text{Output Noise Power Spectrum}: V_{n,amp}^2(f) = |H(f)|^2 \cdot I_{n,amp}^2(f)$$

Integrating this noise spectrum over the decision circuit bandwidth BW_D gives the total noise power experienced by the decision circuit

$$\overline{v_{n,amp}^2} = \int_0^{BW_D} |H(f)|^2 \cdot I_{n,amp}^2(f)\,df$$

- Note that since *H(f)* generally rolls-off quickly, the exact upper bound is not too critical and could be set to a very high value (infinity)

Output Noise – Detector Component

Formally, because the detector noise is nonstationary, we should write it as

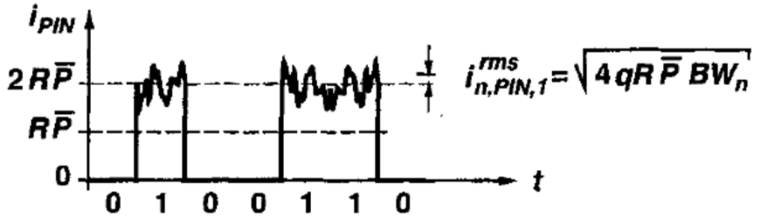
$$V_{n,PD}^{2}\left(f,t\right)=H\left(f\right)\cdot\int_{-\infty}^{\infty}I_{n,PD}^{2}\left(f,t-t'\right)\cdot h\left(t'\right)\cdot e^{j2\pi ft'}dt'$$

where $h\left(t\right)$ is the front - end impulse response.

- This effective convolution implies that the noise can impact not only it’s bits, but can also spread to impact other bits
- However, we generally assume that the noise varies slowly relate to *h(t)* and we can simplify the detector noise analysis

$$V_{n,PD}^{2}\left(f,t\right)=\left|H\left(f\right)\right|^{2}\cdot I_{n,PD}^{2}\left(f,t\right)$$

$$\overline{v_{n}^{2}}_{,PD(t)}=\int_{0}^{BW_{D}}\left|H\left(f\right)\right|^{2}\cdot I_{n,PD}^{2}\left(f,t\right)df$$



- For simple OOK modulation, we use 2 values of the time-dependent output noise power

Total Output Noise

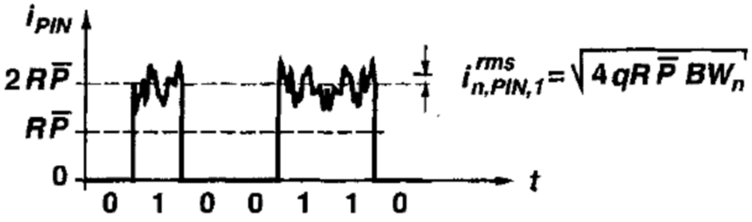
- The total output rms noise value is the root-sum-of-squares of the uncorrelated detector and amplifier noise components

$$v_n^{rms}(t) = \sqrt{\overline{v_{n,PD}^2}(t) + \overline{v_{n,amp}^2}(t)}$$

$$= \sqrt{\int_0^{BW_D} \left|H(f)\right|^2 \cdot \left[I_{n,PD}^2(f,t) + I_{n,amp}^2(f)\right] df}$$

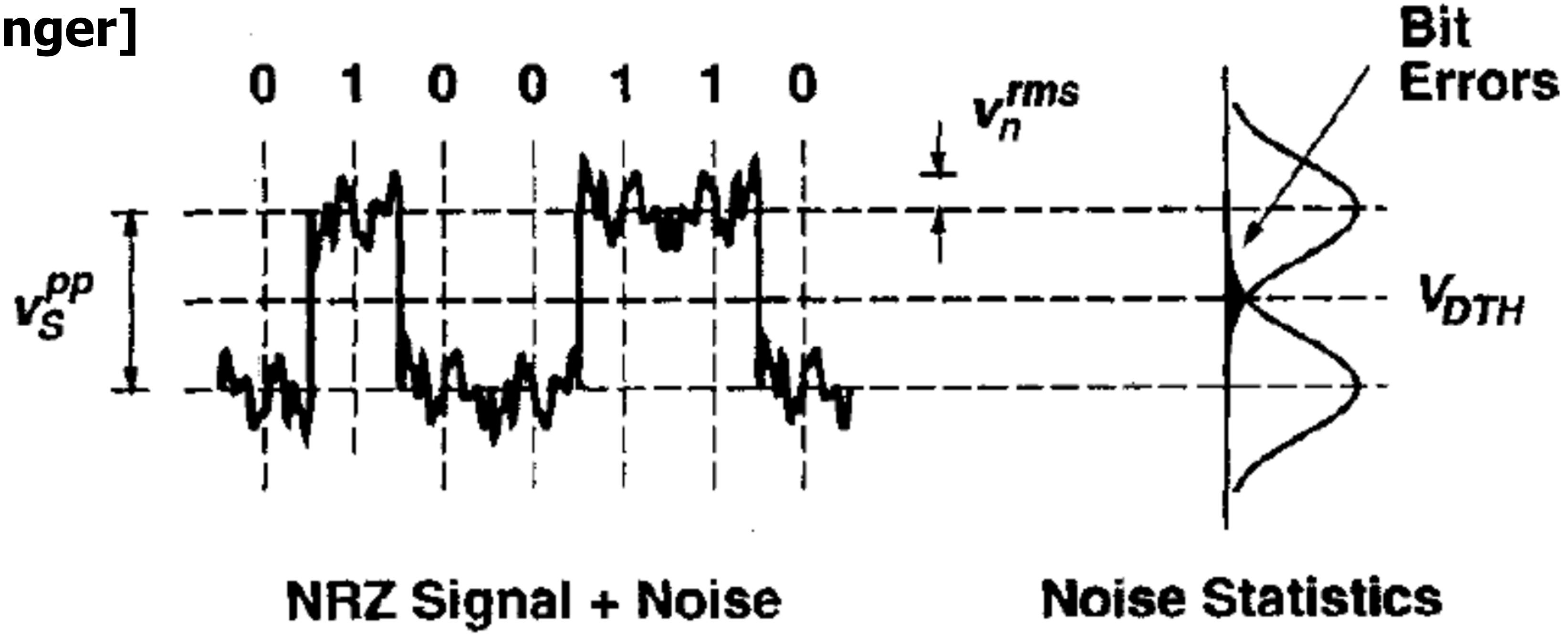
- For simple OOK modulation, we will have two rms values

$$v_{n,0}^{rms} \quad \text{and} \quad v_{n,1}^{rms}$$



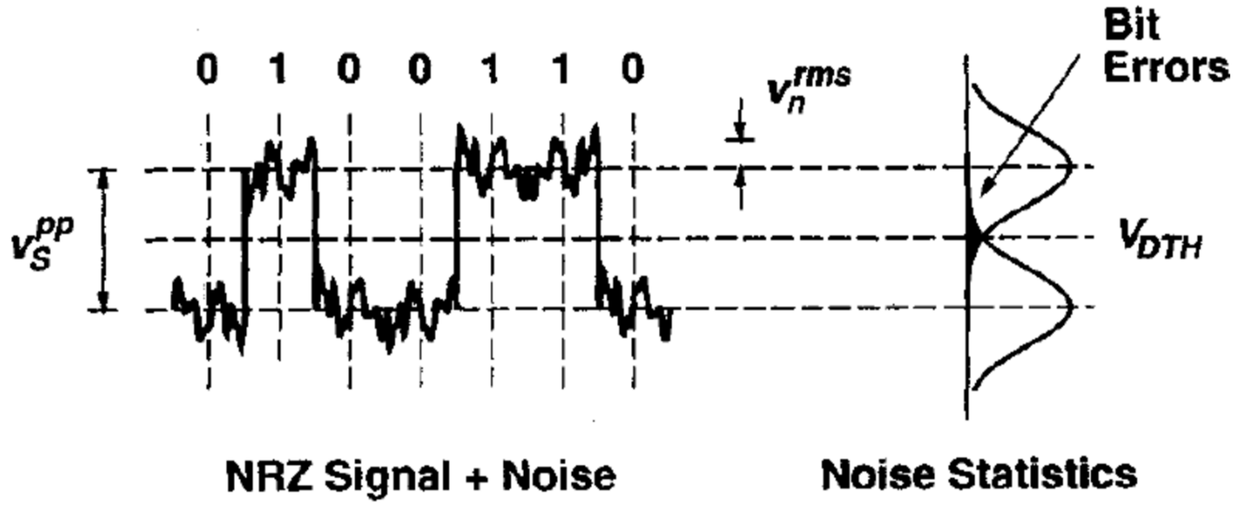
Signal, Noise, and Bit-Error Rate (BER)

[Sackinger]



- The noise is Gaussian with a standard deviation equal to the noise voltage rms value
- With an equal distribution of 1s and 0s, setting V_{DTH} at the crossover of the two distributions yields the fewest bit errors
- The bit-error rate (BER) is defined as the probability that a 0 is misinterpreted as a 1 or vice-versa

BER Calculation



- For BER, we should calculate the area under the Gaussian “tails”
- Assuming equal 0 and 1 noise statistics for now, the 2 tails should be equal and we just need to calculate 1 of them

$$BER = \int_Q^\infty Gauss(x)dx \quad \text{with} \quad Q = \frac{V_{DTH}}{v_n^{rms}} = \frac{v_S^{pp}}{2v_n^{rms}}$$

- Here Gauss(x) is a normalized Gaussian distribution (μ=0,σ=1)
- The lower bound *Q* is the difference between the levels and the decision threshold, normalized by the Gaussian distribution standard deviation, σ

Personick *Q* and BER

- The *Q* parameter is called the Personick *Q* and is a measure of the ratio between the signal and noise

$$\int_Q^\infty Gauss(x)dx = \frac{1}{\sqrt{2\pi}}\int_Q^\infty e^{-\frac{x^2}{2}}dx = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right)$$

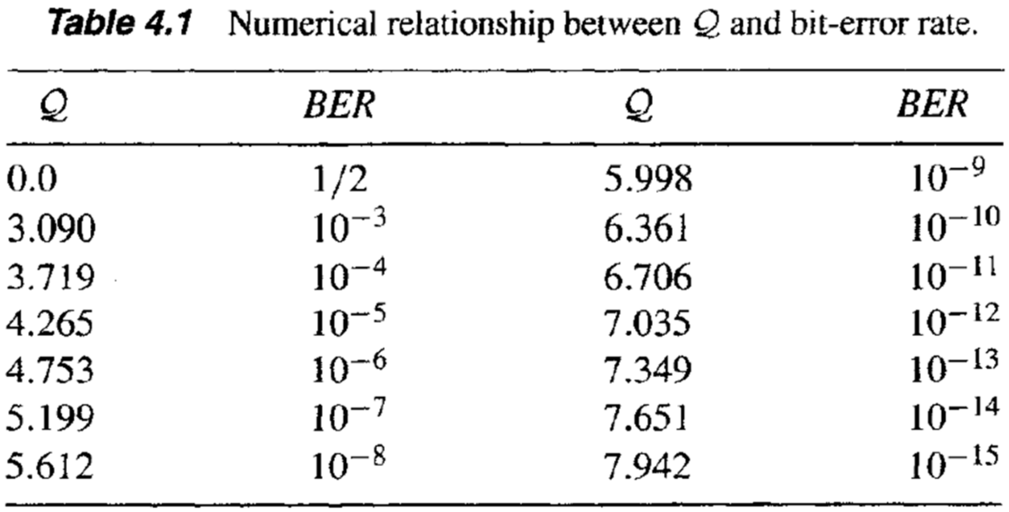


Table 4.1 Numerical relationship between Q and bit-error rate.

Q	BER	Q	BER
0.0	1/2	5.998	10^{-9}
3.090	10^{-3}	6.361	10^{-10}
3.719	10^{-4}	6.706	10^{-11}
4.265	10^{-5}	7.035	10^{-12}
4.753	10^{-6}	7.349	10^{-13}
5.199	10^{-7}	7.651	10^{-14}
5.612	10^{-8}	7.942	10^{-15}

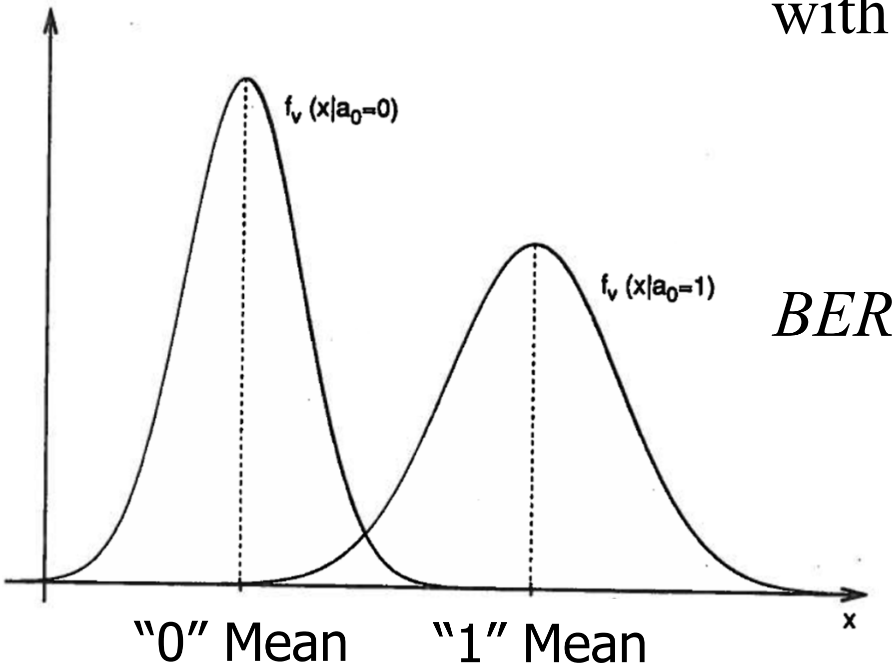
If we want BER $=10^{-12}$, then we need $Q = 7.035$ or

$v_S^{pp} = 14.07 v_n^{rms}$, assuming equal 1 and 0 noise statistics

What if I Have Unequal Noise Distributions?

Neglecting any noise memory effect, the rms noise simply alternates between $v_{n,0}^{rms}$ and $v_{n,1}^{rms}$

We have a relatively thinner, lower - noise distribution for the 0s, with $\sigma_{n,0} = v_{n,0}^{rms}$, and a thicker, higher - noise distribution for the 1s, with $\sigma_{n,1} = v_{n,1}^{rms}$.



$$BER = \int_Q^\infty Gauss(x)dx \quad \text{with} \quad Q = \frac{v_s^{pp}}{v_{n,0}^{rms} + v_{n,1}^{rms}}$$

Signal-to-Noise Ratio

- In optical receiver analysis, the signal-to-noise ratio (SNR) is often defined as the mean-free average signal power divided by the average noise power

$$\text{Mean - Free Average Signal Power: } \overline{v_s^2(t)} - \overline{v_S(t)}^2$$

$$\text{For a DC - balanced NRZ signal, this is } \left(\frac{v_s^{pp}}{2}\right)^2$$

$$\text{Noise Power: } \frac{\left(\overline{v_{n,0}^2} + \overline{v_{n,1}^2}\right)}{2}$$

$$\boxed{SNR = \frac{\left(v_s^{pp}\right)^2}{2\left(\overline{v_{n,0}^2} + \overline{v_{n,1}^2}\right)}}$$

Signal-to-Noise Ratio Extremes

1. Noise is dominated by the amplifier, with equal noise on 0s and 1s

$$SNR = \frac{\left(v_s^{pp}\right)^2}{2\left(\overline{v_{n,0}^2} + \overline{v_{n,1}^2}\right)} = \frac{\left(v_s^{pp}\right)^2}{2\left(2\left(v_n^{rms}\right)^2\right)} = Q^2, \quad \text{with} \quad v_{n,1}^{rms} = v_{n,0}^{rms}$$

$$\text{For a BER} = 10^{-12}\ (Q = 7.0) \Rightarrow\ SNR = (7.0)^2 = 49.0 = 16.9dB$$

2. Noise is dominated by the detector/optical amplifier, with un-equal noise on 0s and 1s

$$SNR = \frac{\left(v_s^{pp}\right)^2}{2\left(\overline{v_{n,0}^2} + \overline{v_{n,1}^2}\right)} \approx \frac{\left(v_s^{pp}\right)^2}{2\left(\overline{v_{n,1}^2}\right)} = \frac{Q^2}{2}, \quad \text{with} \quad v_{n,1}^{rms} >> v_{n,0}^{rms}$$

$$\text{For a BER} = 10^{-12}\ (Q = 7.0) \Rightarrow\ SNR = \frac{(7.0)^2}{2} = 24.5 = 13.9dB$$

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Electrical Receiver Sensitivity

- Sensitivity is the minimum input-referred signal necessary to achieve the desired bit-error rate

Electrical Receiver Sensitivity, i_{sens}^{pp}, is the minimum peak - to - peak signal current at the receiver input to achieve the desired BER.

An input current swing produces an output voltage swing

$$v_S^{pp} = H_0 i_S^{pp} = 2Qv_n^{rms}$$

where H_0 is the midband value of $H(f)$.

$$i_{sens}^{pp} = \frac{2Qv_n^{rms}}{H_0} \quad \text{for the } Q \text{ necessary for the BER}$$

$$\text{Input - Referred RMS Noise: } i_n^{rms} = \frac{v_n^{rms}}{H_0}$$

$$\boxed{i_{sens}^{pp} = 2Qi_n^{rms}}$$

Electrical Receiver Sensitivity

If $i_n^{rms} = 380\text{nA}$, what is the electrical receiver sensitivity for a BER $= 10^{-12}$?

$$i_{sens}^{pp} = 2Qi_n^{rms} = 2(7.035)(380\text{nA}) = 5.35\mu A$$

- What if I have unequal noise distributions on 0s and 1s?

$$v_S^{pp} = Q\left(v_{n,0}^{rms} + v_{n,1}^{rms}\right) \Rightarrow i_{sens}^{pp} = Q\left(i_{n,0}^{rms} + i_{n,1}^{rms}\right)$$

$$\text{where} \quad i_{n,0}^{rms} = \frac{v_{n,0}^{rms}}{H_0} \quad \text{and} \quad i_{n,1}^{rms} = \frac{v_{n,1}^{rms}}{H_0}$$

- Note that so far we have assumed an ideal slicer for the decision circuit. A real slicer’s minimum signal input and offset will degrade this sensitivity. More about this later.

Optical Receiver Sensitivity

Optical Receiver Sensitivity, $\overline{P}_{sens}$, is the minimum optical power, averaged over time, required to achieve the desired BER.

Assuming a DC - balanced signal with a high extinction ratio (more about this later), the average signal current is

$$\overline{i_S} = \frac{i_S^{pp}}{2} \Rightarrow \overline{P_S} = \frac{i_S^{pp}}{2R}$$

$$\boxed{\overline{P_{sens}} = \frac{i_{sens}^{pp}}{2R} = \frac{2Qi_n^{rms}}{2R} = \frac{Qi_n^{rms}}{R}}$$

or if we have different noise distributions

$$\boxed{\overline{P_{sens}} = \frac{Q\left(i_{n,0}^{rms} + i_{n,1}^{rms}\right)}{2R}}$$

Optical Receiver Sensitivity

If $i_n^{rms} = 380\text{nA}$ and $R = 0.8A/W$, what is the optical receiver sensitivity for a BER $= 10^{-12}$?

$$\overline{P_{sens}} = \frac{Q i_n^{rms}}{R} = \frac{(7.035)(380\text{nA})}{0.8A/W} = 3.34\mu W = -24.8dBm$$

- Note that the optical receiver sensitivity is based on the average signal value, whereas the electrical sensitivity is based on the peak-to-peak signal value

Optical RX Sensitivity w/ Ideal Photodetector

- In order to compare the relative performance of different electrical receivers, it is useful to normalize out the photodetector performance

- The sensitivity excluding the PD's quantum efficiency η is

$$\boxed{\eta\overline{P_{sens}} = \frac{hc}{\lambda q} \cdot Q \cdot i_n^{rms}}$$

or if we have different noise distributions

$$\boxed{\eta\overline{P_{sens}} = \frac{hc}{\lambda q} \cdot \frac{Q\left(i_{n,0}^{rms} + i_{n,1}^{rms}\right)}{2}}$$

Optical RX Sensitivity w/ Ideal Photodetector

- Previous example using PD with R=0.8A/W

If $i_n^{rms} = 380\text{nA}$ and $R = 0.8A/W$, what is the optical receiver sensitivity for a BER $= 10^{-12}$?

$$\overline{P_{sens}} = \frac{Qi_n^{rms}}{R} = \frac{(7.035)(380\text{nA})}{0.8A/W} = 3.34\mu W = -24.8dBm$$

- Now, normalizing (multiplying) by the quantum efficiency or dividing by an ideal responsivity at a given wavelength

If $i_n^{rms} = 380\text{nA}$ and we are operating at a wavelenth of 1550nm,

what is the optical receiver sensitivity for a BER $= 10^{-12}$ with an ideal photodetector?

$$\eta\overline{P_{sens}} = \frac{hc}{\lambda q} \cdot Qi_n^{rms} = \frac{(7.035)(380\text{nA})}{\left(8\times 10^5 (A/W \cdot m)\right)(1550nm)} = \frac{2.67\mu W}{1.24} = 2.16\mu W = -26.7dBm$$

Low and High Power Limits

- The sensitivity limit is the weakest signal for which we can achieve the desired BER
- However, if the signal is too large, we can also have bad effects that degrade BER
  - Pulse-width distortion
  - Data-dependent jitter
- The overload limit is the maximum signal for which we can achieve the desired BER

Dynamic Range

- Input overload current i_{ovl}^{pp}
  - This is the maximum peak-to-peak signal current for which a desired BER can be achieved
- Optical overload power $\overline{P}_{ovl}$
  - This is the maximum time-averaged optical power for which a desired BER can be achieved
- The dynamic range is the ratio of the overload limit and the sensitivity limit

$$\text{Dynamic Range} = \frac{i_{ovl}^{pp}}{i_{sens}^{pp}} = \frac{\overline{P}_{ovl}}{\overline{P}_{sens}}$$

Reference Bit-Error Rates Examples

- Sensitivity must be specified at a desired BER!

Assuming $i_n^{rms} = 380\text{nA}$ and $R = 0.8A/W$ for the following

- SONET OC-48 (2.5Gb/s) requires BER≤10^{-10} (*Q*=6.361)

$$\overline{P_{sens}} = \frac{Qi_n^{rms}}{R} = \frac{(6.361)(380\text{nA})}{0.8A/W} = 3.02\mu W = -25.2dBm$$

- SONET OC-192 (10Gb/s) requires BER≤10^{-12} (*Q*=7.035)

$$\overline{P_{sens}} = \frac{Qi_n^{rms}}{R} = \frac{(7.035)(380\text{nA})}{0.8A/W} = 3.34\mu W = -24.8dBm$$

- What about BER≤10^{-15} ? (*Q*=7.942)

$$\overline{P_{sens}} = \frac{Qi_n^{rms}}{R} = \frac{(7.942)(380\text{nA})}{0.8A/W} = 3.77\mu W = -24.2dBm$$

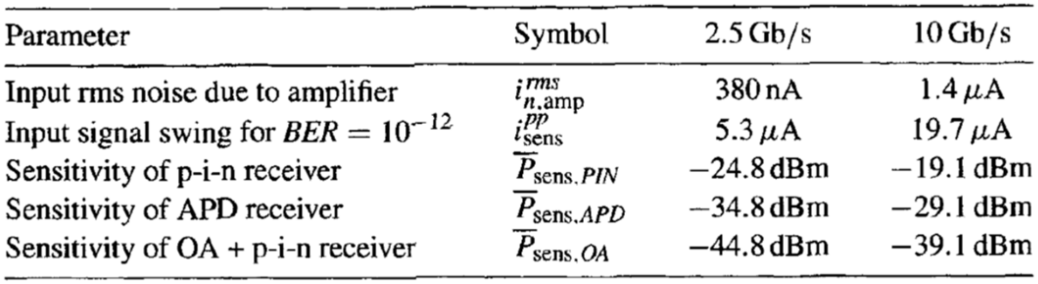
Sensitivity Analysis w/ Amplifier Noise Only

- Here we are assuming that amplifier noise dominates

$$i_n^{rms} = i_{n,amp}^{rms}$$

- With a p-i-n photodetector $\boxed{\overline{P}_{sens,PIN} = \frac{Q i_{n,amp}^{rms}}{R}}$
- With an APD $\boxed{\overline{P}_{sens,APD} = \frac{1}{M} \cdot \frac{Q i_{n,amp}^{rms}}{R}}$
- With an optically preamplified p-i-n detector $\boxed{\overline{P}_{sens,APD} = \frac{1}{G} \cdot \frac{Q i_{n,amp}^{rms}}{R}}$

Assuming R=0.8A/W, M=10, G=100, and BER=10^{-12}



Parameter	Symbol	2.5 Gb/s	10 Gb/s
Input rms noise due to amplifier	$i_{n,\mathrm{amp}}^{rms}$	380 nA	1.4 μA
Input signal swing for $BER = 10^{-12}$	i_{sens}^{pp}	5.3 μA	19.7 μA
Sensitivity of p-i-n receiver	$\overline{P}_{\mathrm{sens},PIN}$	−24.8 dBm	−19.1 dBm
Sensitivity of APD receiver	$\overline{P}_{\mathrm{sens},APD}$	−34.8 dBm	−29.1 dBm
Sensitivity of OA + p-i-n receiver	$\overline{P}_{\mathrm{sens},OA}$	−44.8 dBm	−39.1 dBm

If we neglect detector noise, the optically preamplified p-i-n detector only requires an average optical power of -39.1dBm or 123nW!

Now Let's Include the Detector Noise

- Starting with a p-i-n detector RX, because of the signal-dependent detector noise we need to consider 2 different noise values

$$\overline{i_{n,0}^2} = \overline{i_{n,PIN,0}^2} + \overline{i_{n,amp}^2} \quad \text{and} \quad \overline{i_{n,1}^2} = \overline{i_{n,PIN,1}^2} + \overline{i_{n,amp}^2}$$

- Here we assume that the detector noise is very small for a 0 bit and that we have a high extinction ratio, i.e. $P_1 = 2\overline{P}_{sens}$

$$i_{n,0}^{rms} = i_{n,amp}^{rms} \quad \text{and} \quad i_{n,1}^{rms} = \sqrt{4qR\overline{P}_{sens}BW_n + \left(i_{n,amp}^{rms}\right)^2}$$

Utilizing $\overline{P}_{sens} = \frac{Q\left(i_{n,0}^{rms} + i_{n,1}^{rms}\right)}{2R}$ we can derive that

$$\boxed{\overline{P}_{sens,PIN} = \frac{Qi_{n,amp}^{rms}}{R} + \frac{Q^2qBW_n}{R}}$$

Amplifier Noise

Shot Noise

Now Let's Include the Detector Noise

- With an APD receiver, we assume the following 2 different noise values

$$i_{n,0}^{rms} = i_{n,amp}^{rms} \quad \text{and} \quad i_{n,1}^{rms} = \sqrt{F \cdot M^2 4qR\overline{P}_{sens} BW_n + \left(i_{n,amp}^{rms}\right)^2}$$

$$\boxed{\overline{P}_{sens,APD} = \frac{1}{M} \cdot \frac{Q i_{n,amp}^{rms}}{R} + F \cdot \frac{Q^2 q BW_n}{R}}$$

- With an optically preamplified p-i-n detector receiver, we assume the following 2 different noise values

$$i_{n,0}^{rms} = i_{n,amp}^{rms} \quad \text{and} \quad i_{n,1}^{rms} = \sqrt{\eta F \cdot G^2 4qR\overline{P}_{sens} BW_n + \left(i_{n,amp}^{rms}\right)^2}$$

$$\boxed{\overline{P}_{sens,OA} = \frac{1}{G} \cdot \frac{Q i_{n,amp}^{rms}}{R} + \eta F \cdot \frac{Q^2 q BW_n}{R}}$$

- The amplifier noise is suppressed with increasing detector gain, while the shot noise increases with the excess noise factor

Sensitivity w/ Amplifier & Detector Noise

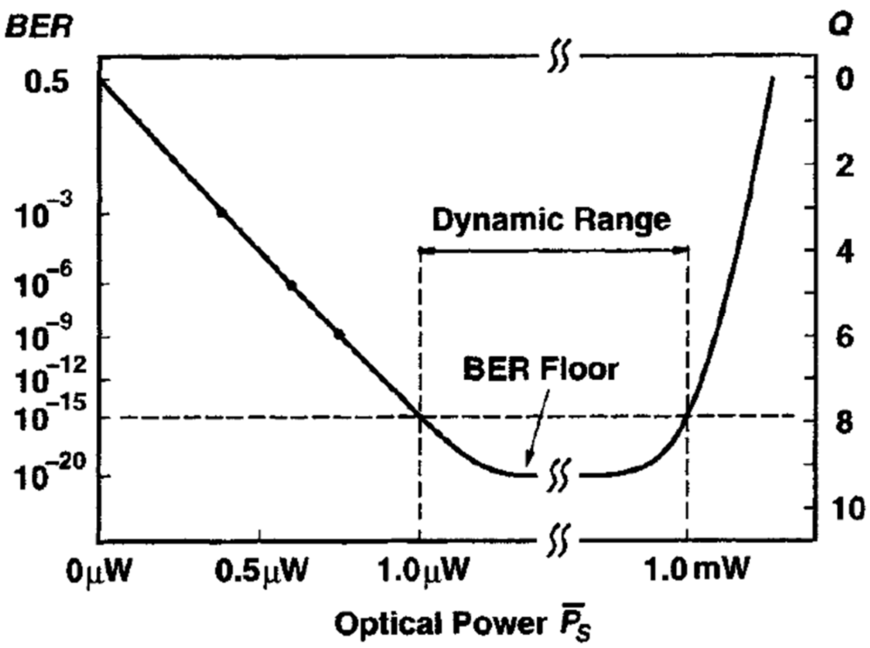
Assuming R=0.8A/W, M=10, G=100, and BER=10^{-12}
For the APD: F=6 (7.8dB)
For the OA+p-i-n: η=0.64, F=3.16 (5dB)

Parameter	Symbol	2.5 Gb/s	10 Gb/s	w/amp noise only (previous table) 10Gb/s Δ
Input rms noise due to amplifier	$i_{n,\text{amp}}^{rms}$	380 nA	1.4 μA	
Sensitivity of p-i-n receiver	$\overline{P}_{\text{sens},PIN}$	−24.7 dBm	−19.1 dBm	0dB
Sensitivity of APD receiver	$\overline{P}_{\text{sens},APD}$	−33.5 dBm	−27.8 dBm	1.3dB
Sensitivity of OA + p-i-n receiver	$\overline{P}_{\text{sens},OA}$	−41.5 dBm	−35.6 dBm	3.5dB

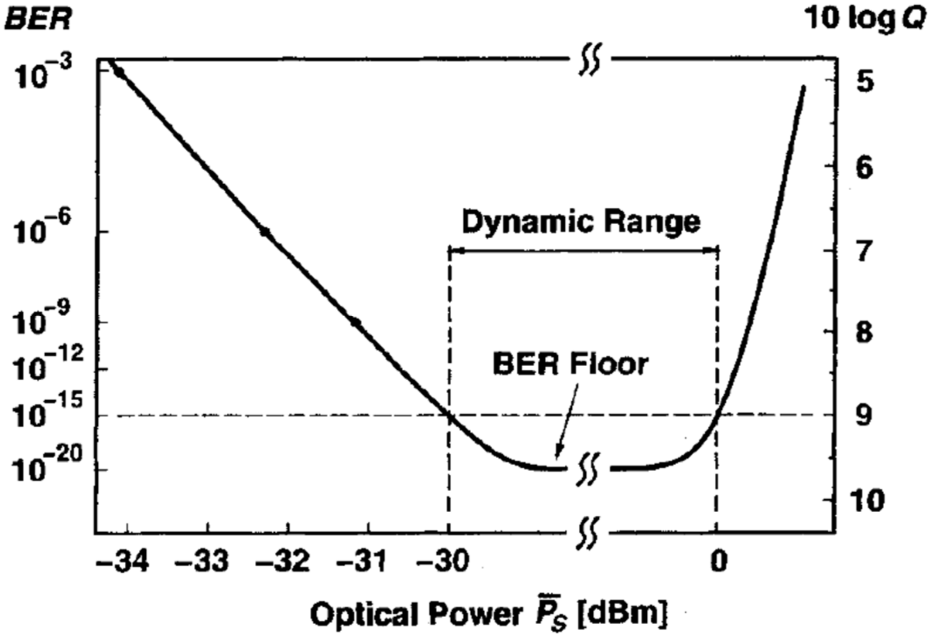
- For the 10Gb/s receivers, relative to amplifier noise only
  - p-i-n RX sensitivity is virtually unchanged ⇒ OK to ignore shot noise
  - APD RX sensitivity is degraded by ~1dB ⇒ ignoring shot noise gives you a reasonable estimate. Depending on the link budget margin, may or may not be able to neglect shot noise.
  - OA + p-i-n RX sensitivity degrades by >3dB ⇒ definitely need to include the shot noise

BER Plots

- To analyze RX performance, we often plot BER or Q versus the average optical power
- At low power levels, this should track $\overline{P}_{sens,PIN} = \frac{Q i_{n,amp}^{rms}}{R} + \frac{Q^2 q B W_n}{R}$

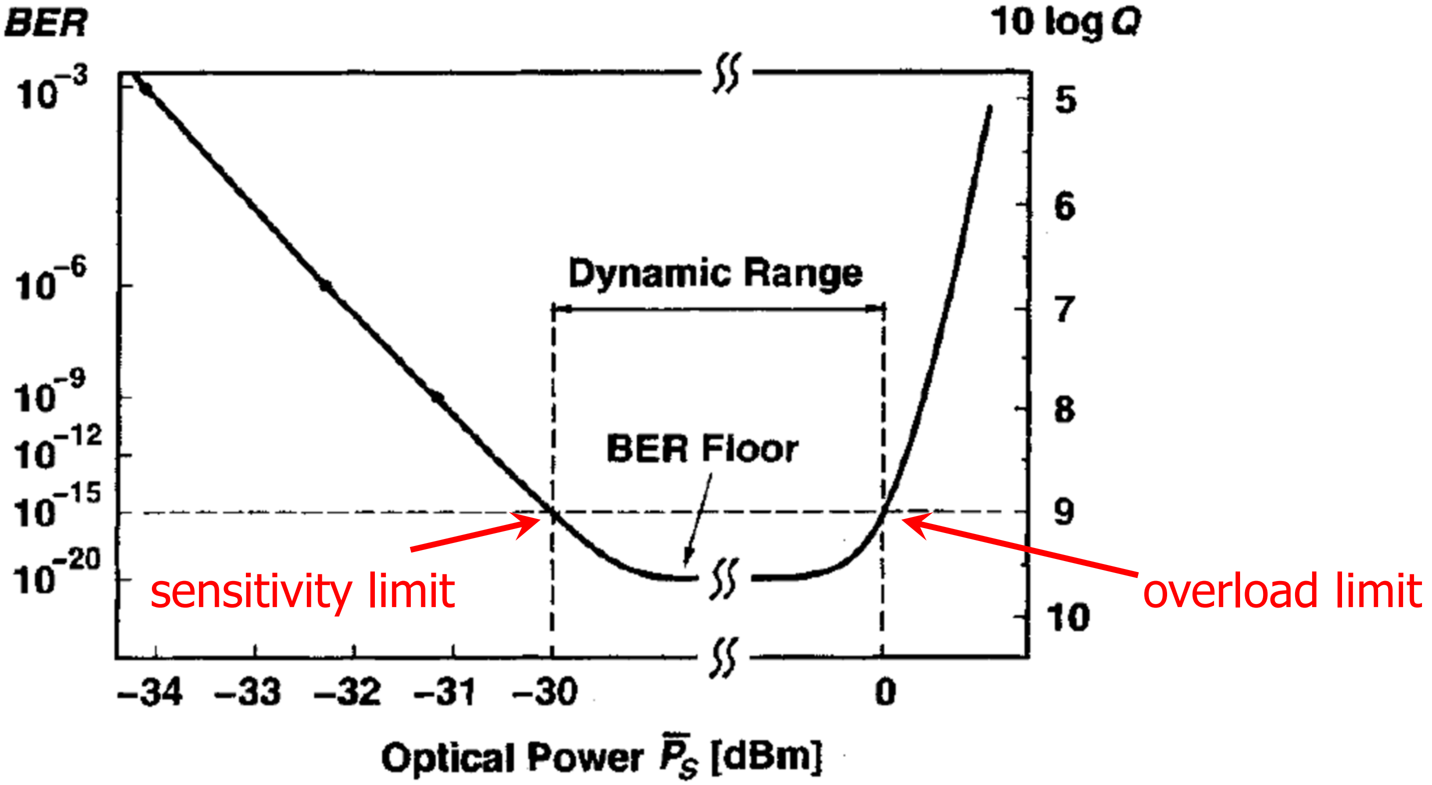


If we plot versus linear power, the Q function increases linearly and the BER improves ∝ erfc(Q)



If we plot versus power in dB, then 10log(Q) function increases linearly

BER Plots



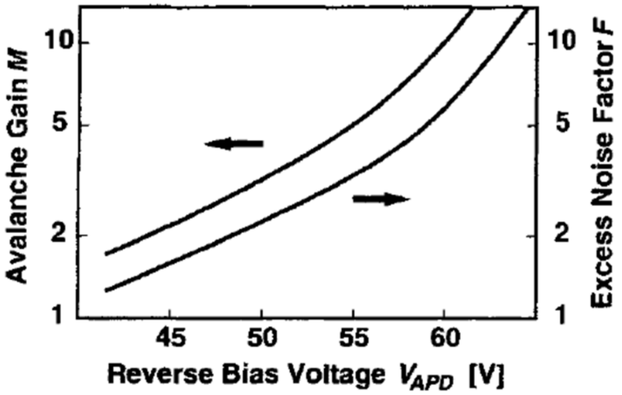
- The sensitivity limit occurs at the minimum power level for the desired BER
- The BER will improve if we increase the power further, until the shot noise term begins to dominate and we reach a BER floor
- As power is increased further, signal distortions occur and we reach the overload limit, beyond which the BER tends to degrade rapidly

Optimum APD Gain

- Recall for an APD, that as the avalanche gain M increases, so does the excess noise factor F and they are related by the ionization-coefficient ratio k_A

$$F = k_A M + (1 - k_A)\left(2 - \frac{1}{M}\right)$$

- Considering that the sensitivity is inversely proportional to M and proportional to F, there exists an optimum APD gain



$$\overline{P}_{sens,APD} = \frac{1}{M} \cdot \frac{Q i_{n,amp}^{rms}}{R} + F \cdot \frac{Q^2 q B W_n}{R}$$

$$\boxed{M_{opt} = \sqrt{\frac{i_{m,amp}^{rms}}{Q k_A q B W_n} - \frac{1 - k_A}{k_A}}}$$

- The optimum APD gain increases with more amplifier noise, as the APD gain suppresses this noise

- Note for an optically preamplified p-i-n RX, the noise figure goes down with increased gain G, and thus higher G always improves sensitivity

What If We Had a Perfect Noiseless Amplifier?

- If we can somehow reduce our amplifier noise to be very low, we will ultimately be limited by the detector noise

$$\overline{P}_{sens,PIN} = \frac{Q^2 qBW_n}{R} \qquad \overline{P}_{sens,APD} = F \cdot \frac{Q^2 qBW_n}{R} \qquad \overline{P}_{sens,OA} = \eta F \cdot \frac{Q^2 qBW_n}{R}$$

Assuming R=0.8A/W, M=10, G=100, and BER=10^{-12}
For the APD: F=6 (7.8dB)
For the OA+p-i-n: η=0.64, F=3.16 (5dB)

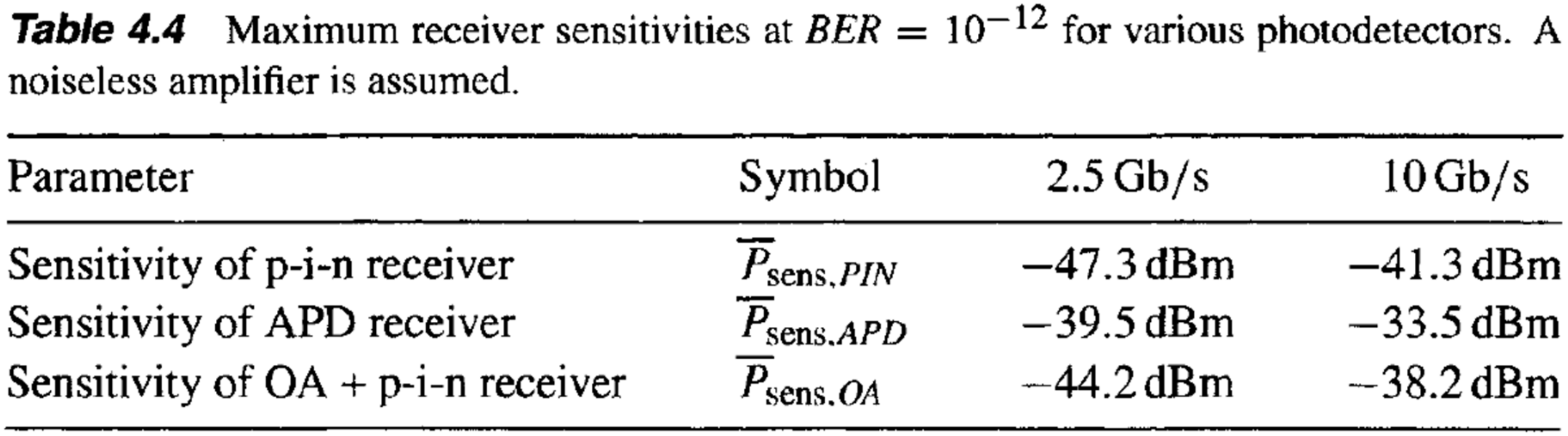


Table 4.4 Maximum receiver sensitivities at $BER = 10^{-12}$ for various photodetectors. A noiseless amplifier is assumed.

Parameter	Symbol	2.5 Gb/s	10 Gb/s
Sensitivity of p-i-n receiver	$\overline{P}_{sens,PIN}$	−47.3 dBm	−41.3 dBm
Sensitivity of APD receiver	$\overline{P}_{sens,APD}$	−39.5 dBm	−33.5 dBm
Sensitivity of OA + p-i-n receiver	$\overline{P}_{sens,OA}$	−44.2 dBm	−38.2 dBm

- As evident by the equations above, the p-i-n RX performs best
- The APD RX sensitivity is degraded by F (7.8dB)
- The OA+p-i-n RX sensitivity is degraded by hF (-1.9dB+5dB=3.1dB)

What If Everything Is Perfect?

- If we have zero amplifier and detector noise, we can receive data with an infinitesimally amount of optical power, right?
- Uh no, as we still need to at least detect one photon to determine that we have a "1" bit, which is the quantum limit
- Photon count per "1" bit, *n*, follows a Poisson distribution

$$\text{Poisson}(n) = e^{-M} \cdot \frac{M^n}{n!}$$

where M is the mean of the distribution

- Assuming no power is sent for a "0", these bits will always be correct

Quantum Limit Sensitivity

- The error probability for a “1” is Poisson(0)

$$BER = \frac{1}{2}\text{Poisson}(0) = \frac{1}{2}e^{-M}$$

Thus, we need an average number of M photons per "1" bit

$$M = -\ln(2BER)$$

Per bit, we need $M/2$ photons, which results in an average power of

$$\boxed{\overline{P}_{sens,quant} = \frac{-\ln(2BER)}{2}\cdot\frac{hc}{\lambda}\cdot B}$$

where B is the bit rate

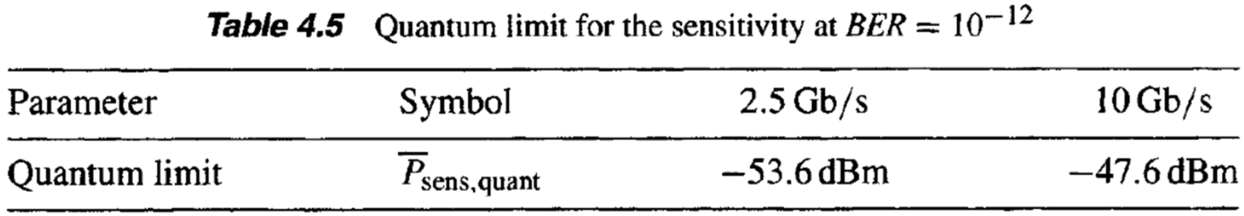


Table 4.5 Quantum limit for the sensitivity at $BER = 10^{-12}$

Parameter	Symbol	2.5 Gb/s	10 Gb/s
Quantum limit	$\overline{P}_{\text{sens,quant}}$	−53.6 dBm	−47.6 dBm

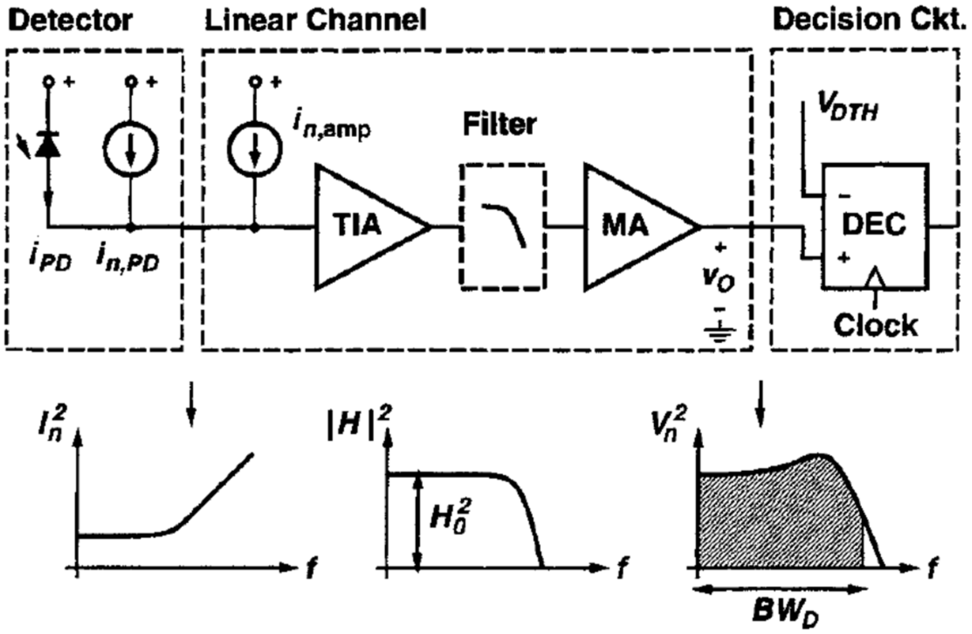
- How do the previous example RX sensitivities with amplifier and detector noise compare relative to the 10Gb/s quantum limit sensitivity?
  - p-i-n RX = +28.5dB
  - APD RX = +19.8dB
  - OA + p-i-n = +12dB

Agenda

- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

Total Input-Referred Noise

- In order to calculate the RX sensitivity, we need the input-referred rms current noise
- The easiest way to obtain this (in simulations) is to integrate the output noise spectrum over the decision element bandwidth and divide by the midband gain H_0



$$\overline{v_n^2} = \int_0^{BW_D} |H(f)|^2 \cdot I_n^2(f)\,df$$

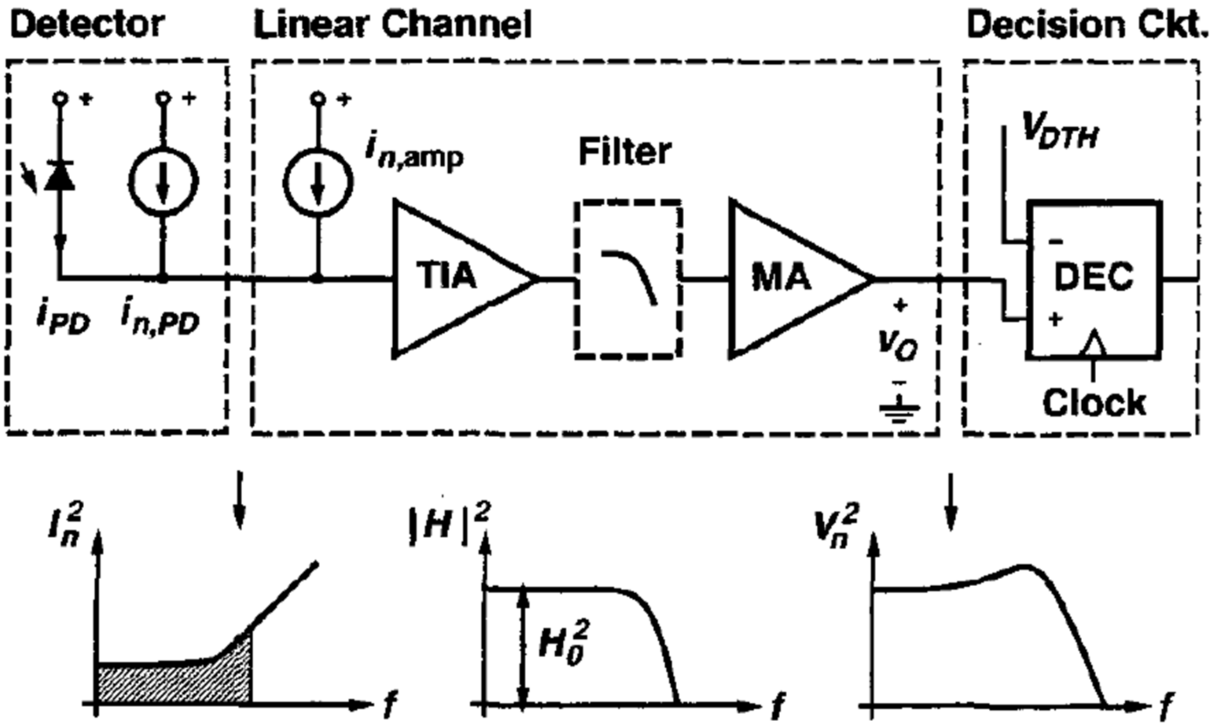
$$\overline{i_n^2} = \frac{\overline{v_n^2}}{H_0^2} = \frac{1}{H_0^2}\int_0^{BW_D} |H(f)|^2 \cdot I_n^2(f)\,df$$

where $I_n^2(f) = I_{n,PD}^2(f) + I_{n,amp}^2(f)$ are the input-referred noise power spectrum of the detector and amplifier noise.

$$i_n^{rms} = \sqrt{\overline{i_n^2}} = \sqrt{\frac{\overline{v_n^2}}{H_0^2}} = \frac{v_n^{rms}}{H_0}$$

- Note that since *H(f)* generally rolls-off quickly, the exact upper bound is not too critical and could be set to a very high value (infinity)

How to Get the Input RMS Noise from the Input Noise Power Spectrum?

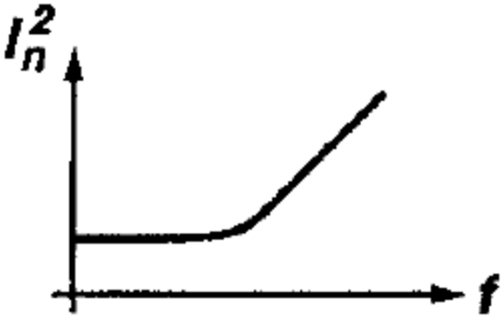


- If we cannot simulate the output noise spectrum, we can get the input-referred rms noise from the input noise spectrum through integration
- However, we must be very careful regarding the bounds of the integral due to the rapidly rising f^2 component

$$\overline{i_n^2} = \int_0^{?} I_n^2(f)\,df$$

Noise Bandwidths

The input - noise spectrum can be expressed as



$$I_n^2(f) = \alpha_0 + \alpha_2 f^2$$

$$\overline{i_n^2} = \frac{1}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 \left(\alpha_0 + \alpha_2 f^2\right) df$$

$$= \frac{\alpha_0}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 df + \frac{\alpha_2}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 f^2 df$$

$$= \alpha_0 BW_n + \frac{\alpha_2}{3} BW_{n2}^3$$

where

$$BW_n = \frac{1}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 df \quad \text{and} \quad BW_{n2}^3 = \frac{1}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 f^2 df$$

BW_n is identical to the noise bandwidth of the receiver's frequency response.

BW_{n2} is the second - order noise bandwidth for the f^2 noise component.

Noise Bandwidths

$$\overline{i_n^2} = \alpha_0 BW_n + \frac{\alpha_2}{3} BW_{n2}^3$$

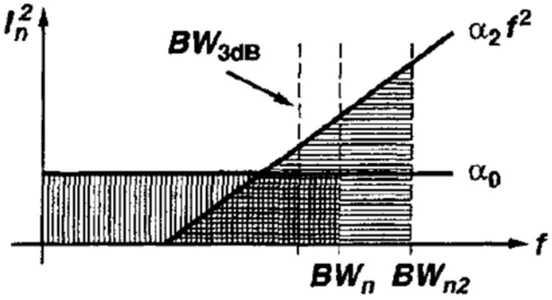
where

$$BW_n = \frac{1}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 df \quad \text{and} \quad BW_{n2}^3 = \frac{1}{H_0^2}\int_0^{BW_D} \left|H(f)\right|^2 f^2 df$$

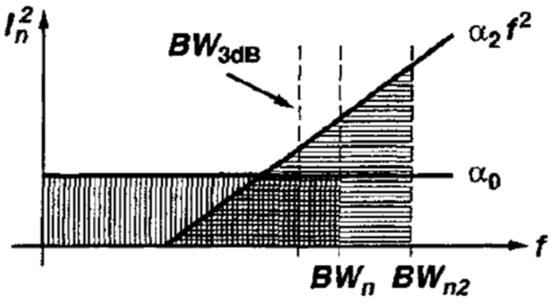
- The bandwidths BW_n and BW_{n2} depend only on the receiver's frequency response and the decision circuit's bandwidth BW_D
- Note that BW_D is not too critical if it is larger than the receiver bandwidth
- Assuming $BW_D=\infty$, BW_n and BW_{n2} are calculated for typical receiver frequency responses

Table 4.6 Numerical values for BW_n and BW_{n2}.

$H(f)$	BW_n	BW_{n2}
1st-order low pass	$1.57 \cdot BW_{3dB}$	∞
2nd-order low pass, crit. damped ($Q = 0.500$)	$1.22 \cdot BW_{3dB}$	$2.07 \cdot BW_{3dB}$
2nd-order low pass, Bessel ($Q = 0.577$)	$1.15 \cdot BW_{3dB}$	$1.78 \cdot BW_{3dB}$
2nd-order low pass, Butterworth ($Q = 0.707$)	$1.11 \cdot BW_{3dB}$	$1.49 \cdot BW_{3dB}$
Brick wall low pass	$1.00 \cdot BW_{3dB}$	$1.00 \cdot BW_{3dB}$
Rectangular (impulse response) filter	$0.500 \cdot B$	∞
NRZ to full raised-cosine filter	$0.564 \cdot B$	$0.639 \cdot B$



What if I Just Integrate Up To the 3dB Bandwidth?



- What we should do is use the table data and calculate

$$\overline{i_n^2} = \alpha_0 BW_n + \frac{\alpha_2}{3} BW_{n2}^3$$

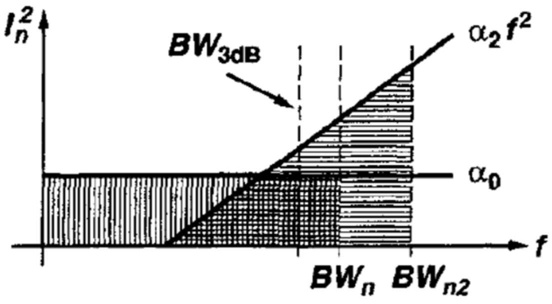
- But, what if we simply integrate up to the 3dB bandwidth, which is equivalent to using

$$\overline{i_n^2} = \alpha_0 BW_{3dB} + \frac{\alpha_2}{3} BW_{3dB}^3$$

- Referring to the table, this is only correct for a Brick Wall Low Pass response and can lead to significant error
- For example, with a 2nd-order Butterworth response, this underestimates the white noise component by 1.11x and the f^2 component by 3.33x

Personick Integrals

- Optical receiver literature often uses constants from Personick Integrals



$$\overline{i_n^2} = \alpha_0 BW_n + \frac{\alpha_2}{3} BW_{n2}^3 = \alpha_0 \cdot I_2 B + \alpha_2 \cdot I_3 B^3$$

where B is the bit rate

$$I_2 = \frac{BW_n}{B} \quad \text{and} \quad I_3 = \frac{BW_{n2}^3}{3B^3}$$

- The Personick Integrals I_2 and I_3 are normalized noise bandwidths

Agenda

- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

Power Penalty

- So far we have primarily been considering random noise sources and assumed that we have had an ideal transmitter, receiver decision circuit, etc…
- The actual receiver sensitivity will be degraded by impairments throughout the optical link and is quantified by power penalties
- The power penalty *PP* is the increase in average transmit power necessary to maintain the desired BER, relative to an ideal case where we don't have the impairment
- This is quantified in dBs, 10log(*PP*)

Typical Impairments

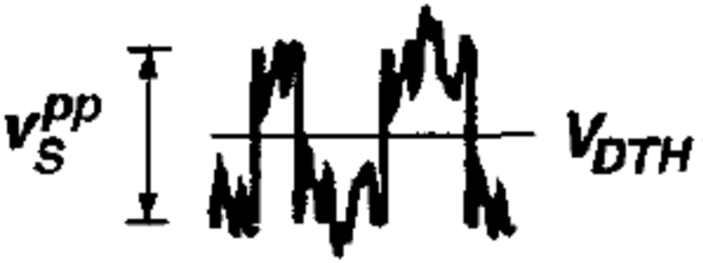
- Transmitter
  - Extinction ratio
  - Relative intensity noise (RIN)
  - Output power variations
- Fiber
  - Dispersion
  - Nonlinear effects
- Detector
  - Dark current
- TIA
  - Distortions (ISI)
  - Offset
- MA
  - Distortions (ISI)
  - Offset
  - Noise figure
  - Low-frequency cutoff
- CDR
  - Decision-threshold offset
  - Decision-threshold ambiguity
  - Sampling-time offset
  - Sampling-time jitter

Decision-Threshold Offset PP

- So far we have assumed that the decision threshold is in the ideal place

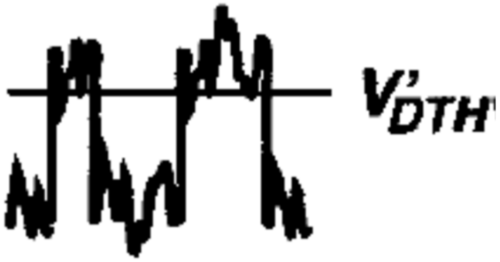
Assuming equal noise distributions and DC - balanced data

$$V_{DTH} = \frac{v_S^{pp}}{2}$$

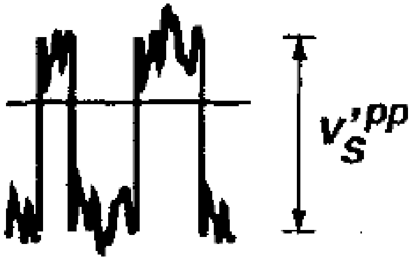


- What if there is an offset?

$$V'_{DTH} = V_{DTH} + \delta v_S^{pp}$$



Depending on the polarity on the offset, we must increase the distance of one of the levels (high level) from the offset threshold. This implies a new peak - to - peak signal level $v_S'^{pp}$ with



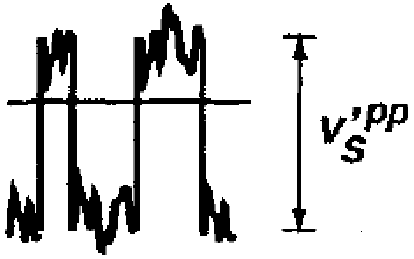
$$\frac{v_S'^{pp}}{2} = \frac{v_S^{pp}}{2} + \delta v_S^{pp}$$

$$v_S'^{pp} = v_S^{pp} + 2\delta v_S^{pp} = v_S^{pp}\left(1 + 2\delta\right)$$

Decision-Threshold Offset PP

Thus the signal swing must be increased by

$$\frac{v'^{pp}_S}{v^{pp}_S} = 1 + 2\delta$$



and the power penalty is

$$\boxed{PP = 1 + 2\delta}$$

- Note that we are neglecting the improved BER on one of the levels (low level), but formally considering this has only a small impact on the resulting PP

Example 1: $v_n^{rms} = 1mV$ and the decision - threshold offset is 1mV

$$\text{For a BER} = 10^{-12} \Rightarrow v_S^{PP} = 14.07mV$$

$$\delta = \frac{1mV}{14.07mV} = 0.071$$

$$PP = 1 + 2\delta = 1.142 = 0.577dB$$

Decision-Threshold Offset PP

Example 2: What should the offset be for only a 0.1dB power penalty?

$$\boxed{\delta = \frac{PP\text{-}1}{2}}$$

$$\delta = \frac{10^{\frac{0.1}{10}} - 1}{2} = 0.012$$

Thus the offset should be

$$\delta v_S^{pp} = 0.012\left(14.07mV\right) = 164\mu V$$

- Good receiver offset control is necessary to minimize this power penalty!

Dark Current PP

- Dark current by itself isn’t a major issue, as we generally assume that the receiver can somehow subtract it out

- However, a potential problem is the shot noise that it induces, which can be quantified as a power penalty

$$\overline{i_{n,DK}^2} = 2qI_{DK}BW_n$$

Dark Current PP

- To keep things simple, let’s assume that the receiver noise is dominated by the amplifier noise. Note, this will slightly overestimate the dark current PP.
- The dark current noise increases the total noise by

$$\frac{\overline{i^2_{n,amp}} + \overline{i^2_{n,DK}}}{\overline{i^2_{n,amp}}} = 1 + \frac{2qI_{DK}BW_n}{\overline{i^2_{n,amp}}}$$

As the sensitivity is proportional to i_n^{rms}

$$\boxed{PP = \sqrt{1 + \frac{2qI_{DK}BW_n}{\overline{i^2_{n,amp}}}}}$$

Dark Current PP

Example 1: Assume a 2.5Gb/s receiver with $i_{n,amp}^{rms} = 380nA$, $BW_n = 1.9GHz$, and $I_{DK} = 5nA$.

$$PP = \sqrt{1 + \frac{2\left(1.6\times10^{-19}C\right)(5nA)(1.9GHz)}{(380nA)^2}} = 1.0000105 = 4.6\times10^{-5}dB$$

Example 2: What if I have an APD RX with $F = 6$ and $M = 10$?

$$PP = \sqrt{1 + \frac{F\cdot M^2 2qI_{DK}BW_n}{i_{n,amp}^2}} = \sqrt{1 + \frac{6(10)^2(2)\left(1.6\times10^{-19}C\right)(5nA)(1.9GHz)}{(380nA)^2}}$$

$$= 1.0063 = 0.027dB$$

Dark Current PP

Example 3 : What must the dark current be for a 0.05dB power penalty?

$$I_{DK} < \left(PP^2 - 1\right)\frac{\overline{i^2_{n,amp}}}{2qBW_n}$$

Using the 2.5Gb/s receiver numbers

$$I_{DK} < \left(\left(10^{\frac{0.05}{10}}\right)^2 - 1\right)\frac{(380nA)^2}{2\left(1.6\times10^{-19}C\right)\left(1.9GHz\right)} = 5.53\mu A$$

- As long as the effective dark current is in the low μA or less, the power penalty is generally negligible

Agenda

- Receiver Model
- Bit-Error Rate
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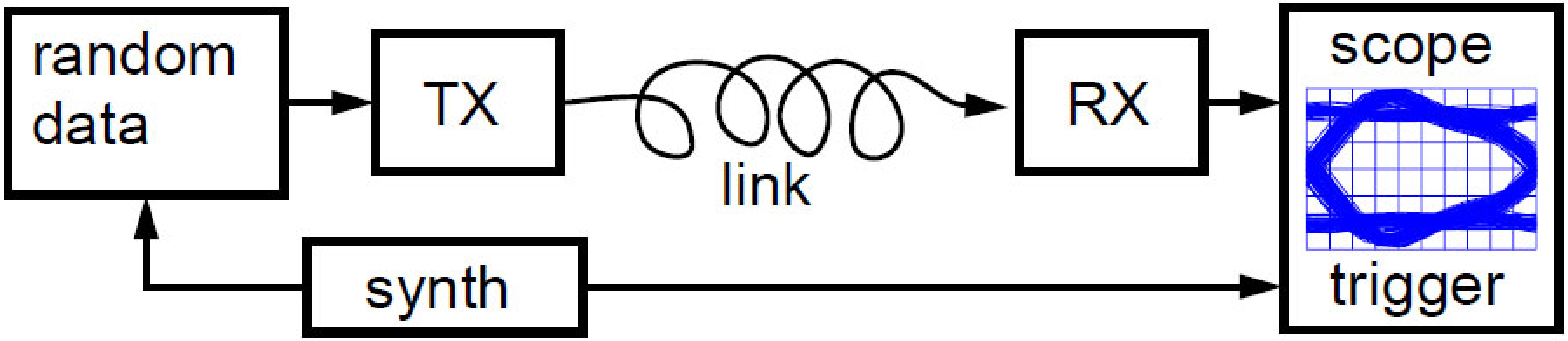
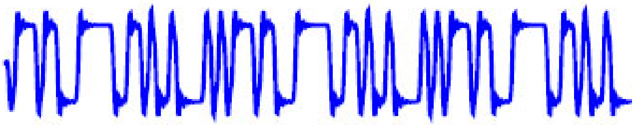
Noise vs ISI Bandwidth Trade-Offs

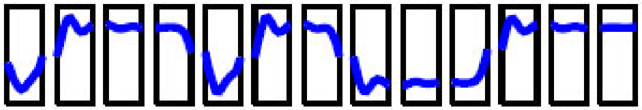
- If we design our receiver to have a very wide bandwidth, then we will receive the signal with minimal distortion
- However, noise will grow as bandwidth increases
- From a basic sensitivity perspective, decreasing bandwidth results in ever-improving sensitivity
- However, this neglects the filtering of the high-frequency pulses (bits) which causes intersymbol interference (ISI)
- Thus, there is an optimum bandwidth from a sensitivity perspective to balance noise and ISI
- This optimum bandwidth is generally about (2/3)B

Eye Diagrams

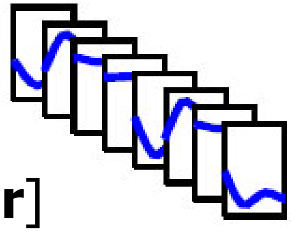




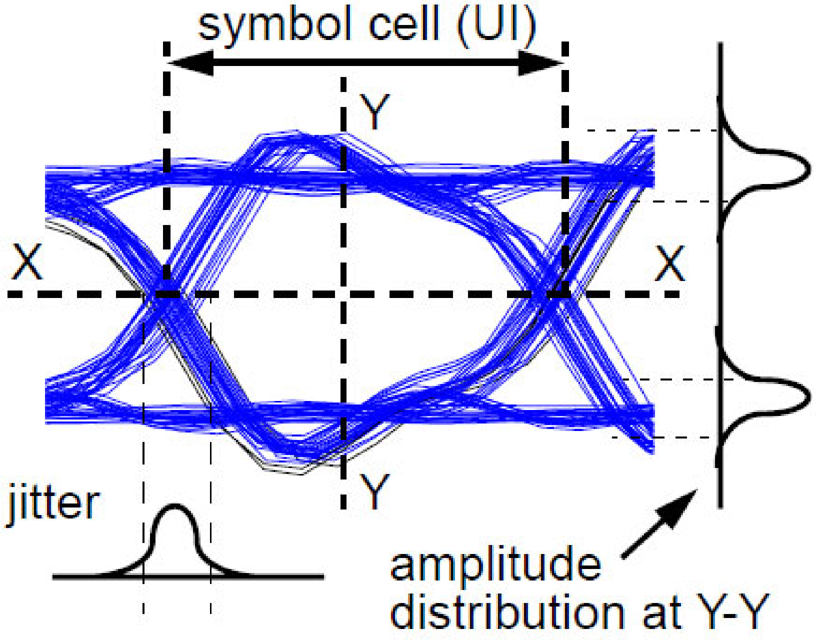
Use a precise clock to chop the data into equal periods



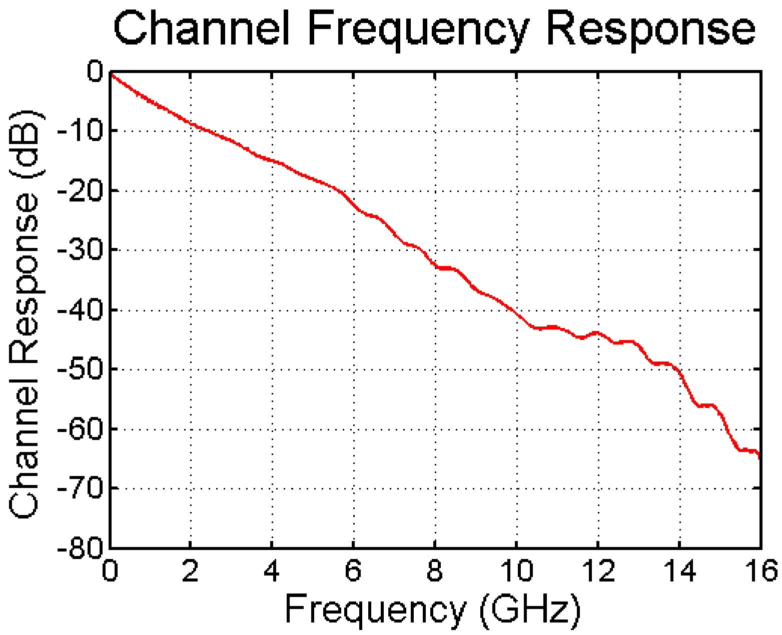
overlay each period onto one plot

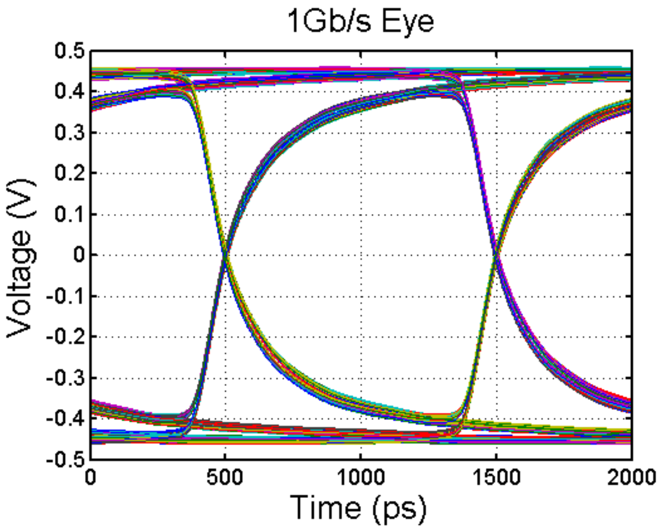


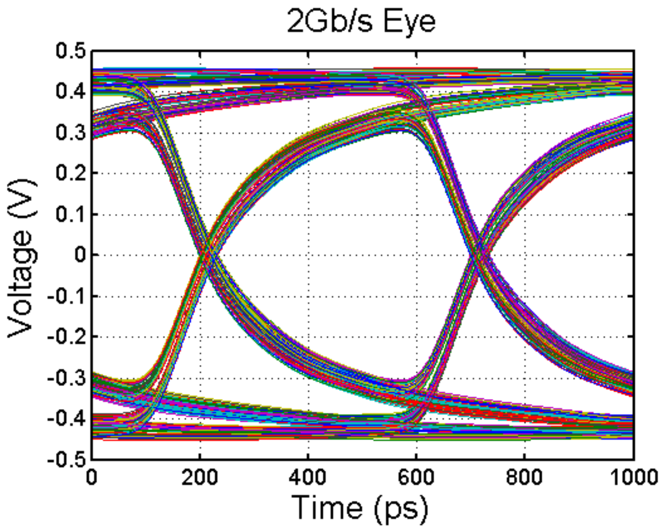
[**Walker**]

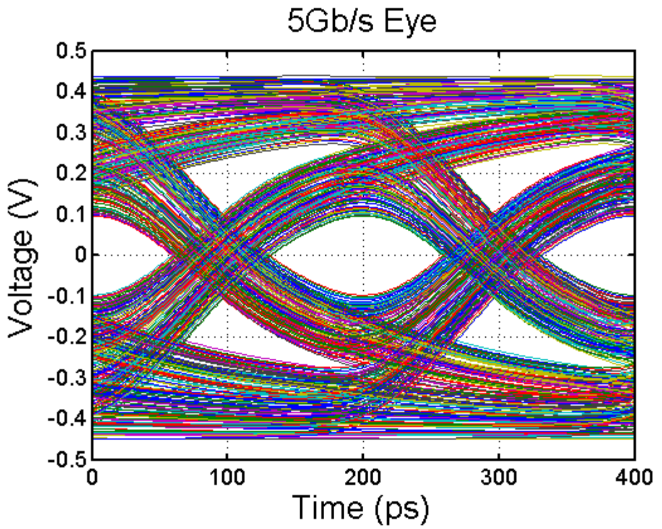


Eye Diagrams vs Data Rate

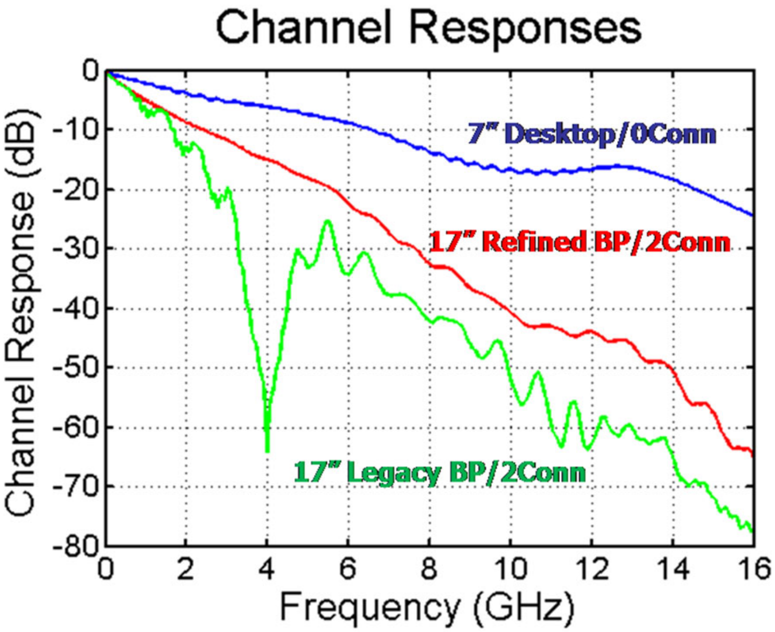

Channel Frequency Response

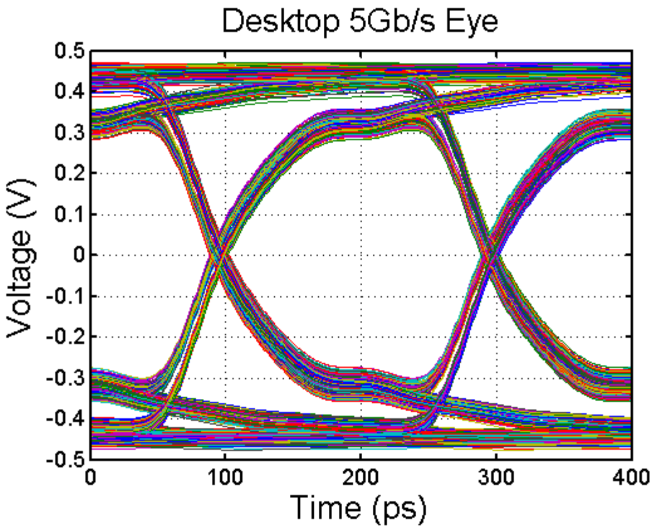

1Gb/s Eye

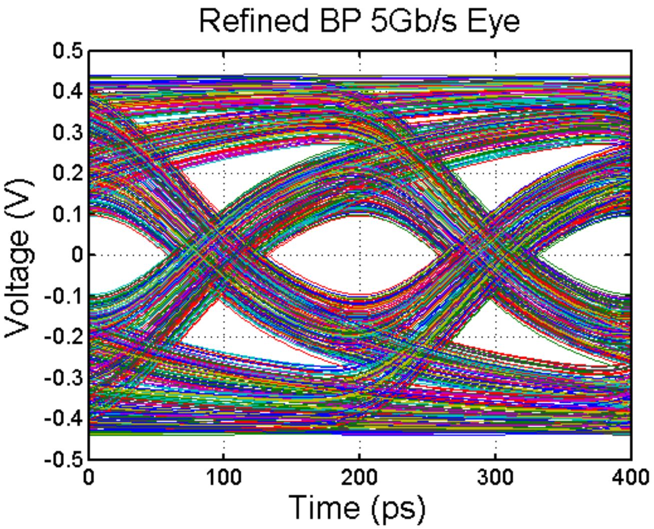

2Gb/s Eye

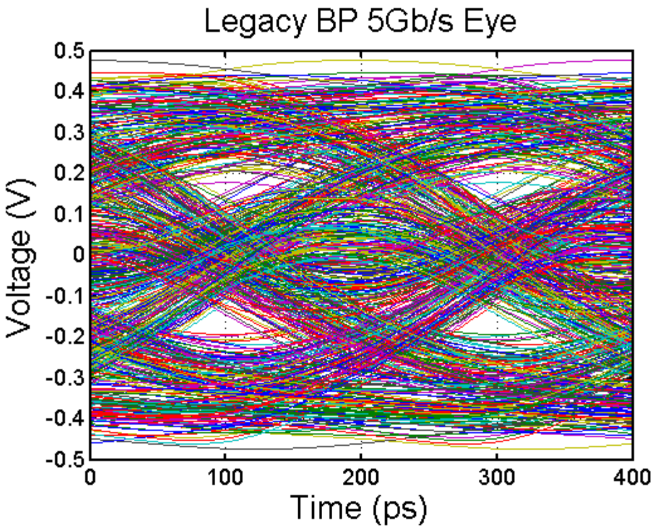

5Gb/s Eye

Eye Diagrams vs Channel



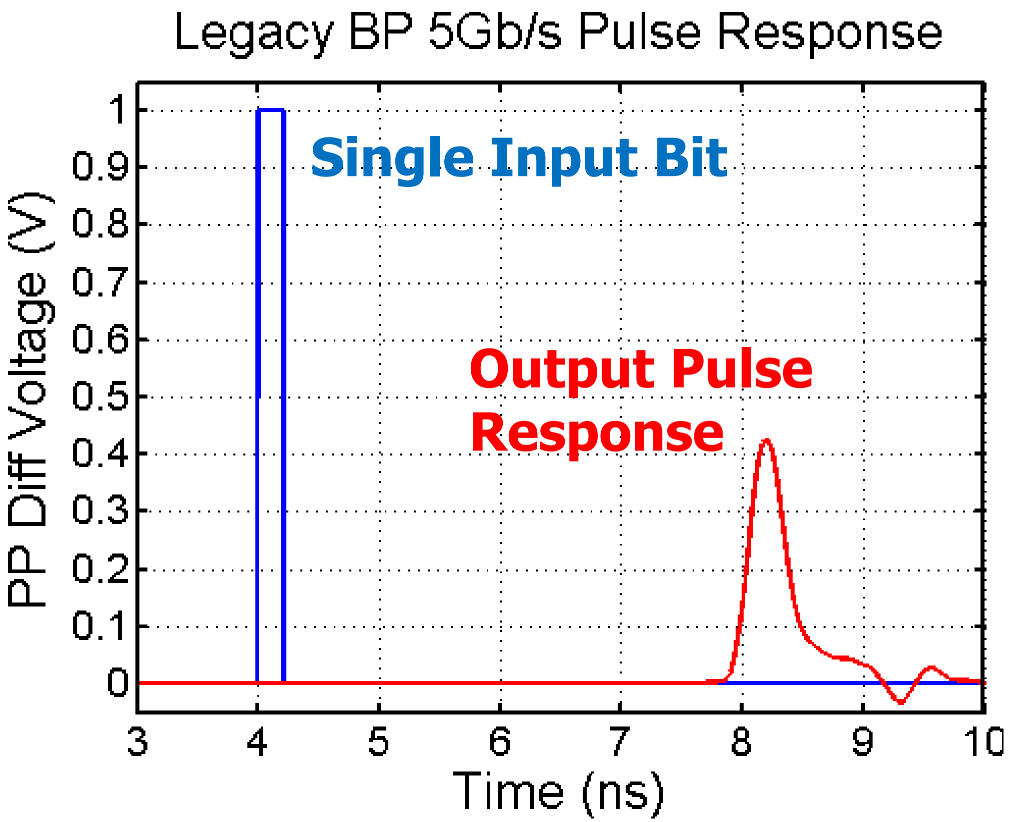


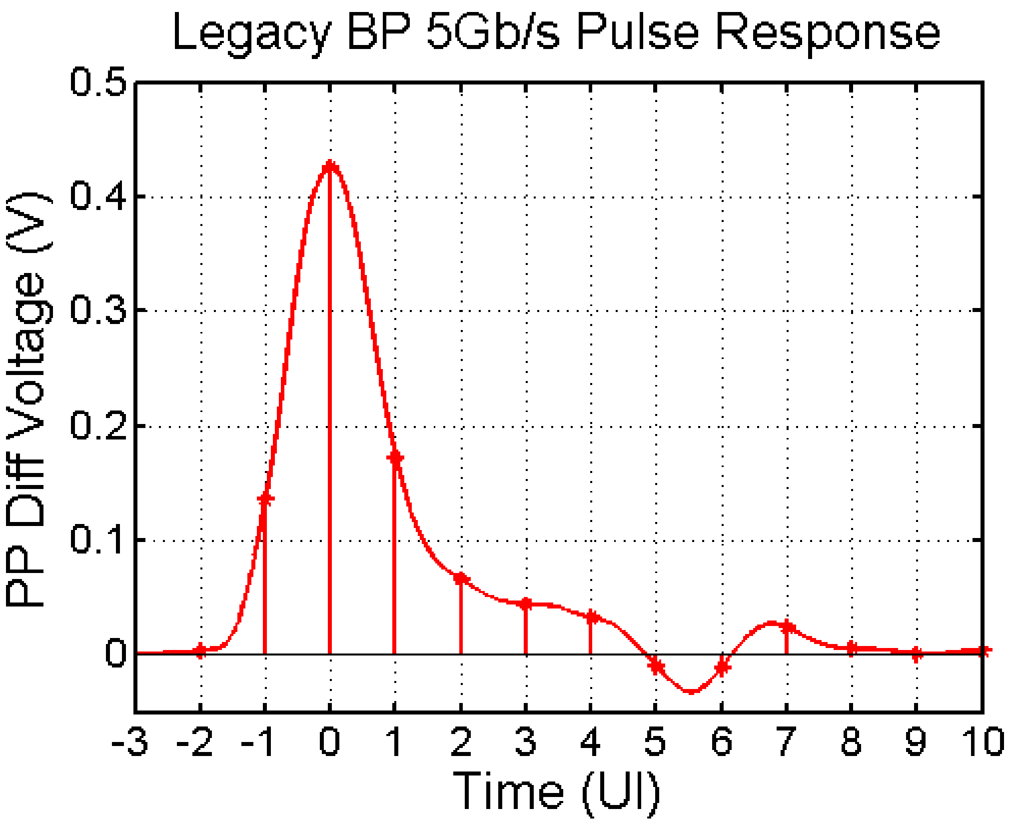




Inter-Symbol Interference (ISI)

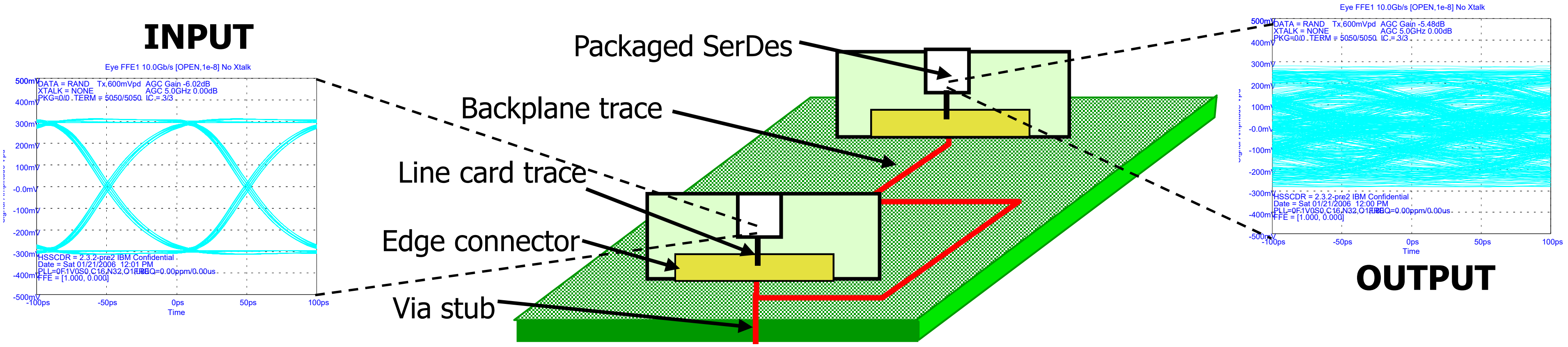
- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
    - Reflections, Channel resonances, Channel loss (dispersion)





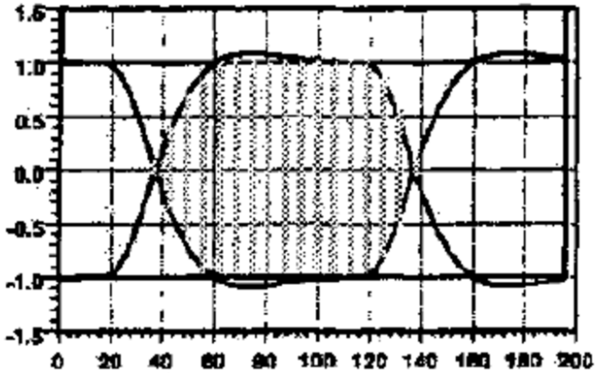
ISI Impact

- At channel input (TX output), eye diagram is wide open
- As data pulses propagate through channel, they experience dispersion and have significant ISI
  - Result is a closed eye at channel output (RX input)



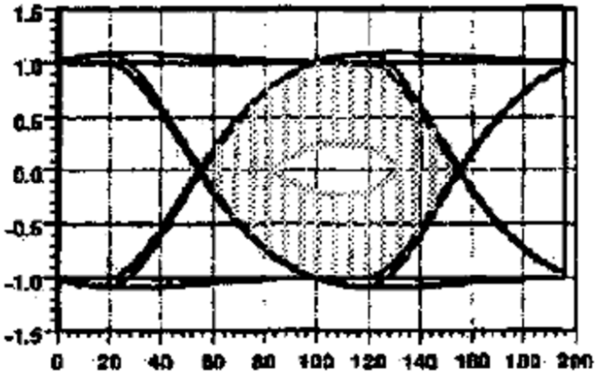
[Meghelli (IBM) ISSCC 2006]

Eye Diagrams w/ a 2nd-Order Butterworth RX



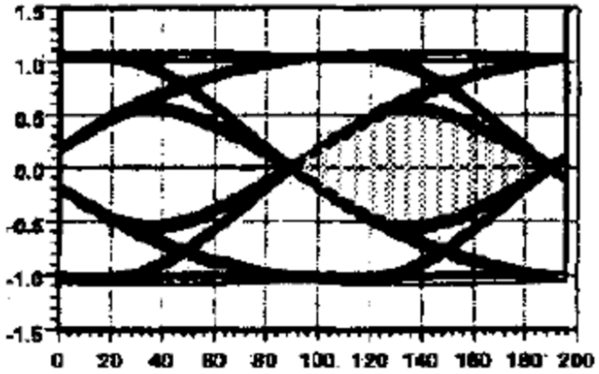
$$BW_{3dB} = \frac{4}{3}B$$

- No ISI present
- Assume that the noise (BER=10^{-12}) is exactly equal to the eye height, and we have no margin



$$BW_{3dB} = \frac{2}{3}B$$

- Still minimal (no) ISI present
- Assuming white noise dominates, we have a sqrt(2) reduction in rms noise
- We could reduce our optical power by the same sqrt(2) factor and obtain the same BER!

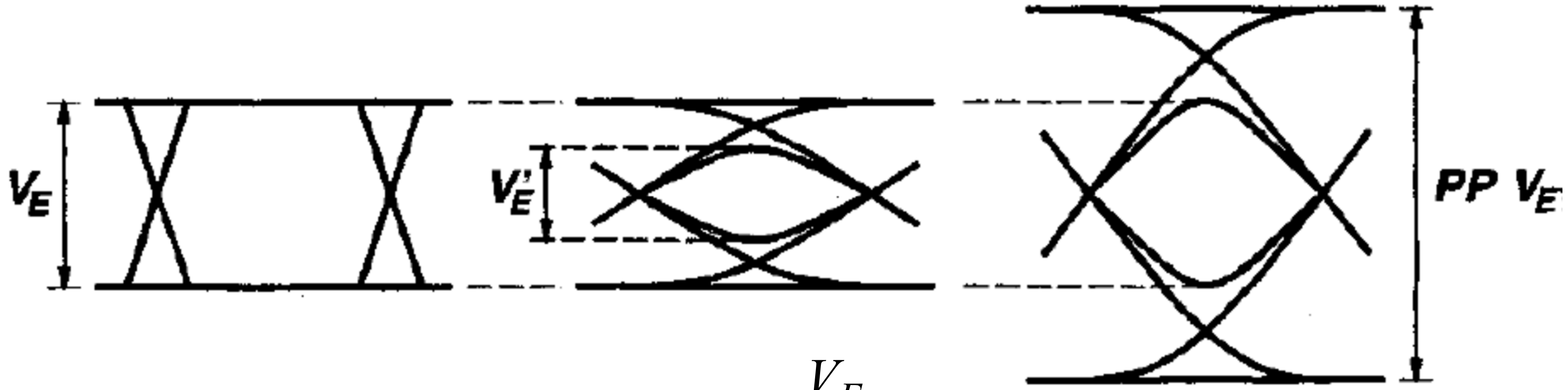


$$BW_{3dB} = \frac{1}{3}B$$

- Severe ISI (~1/2 eye height)
- While the rms noise is reduced by 2x, the overall vertical margin is the same as the 4/3*B* RX
- Note that if we are off in time (horizontally), we won’t achieve our desired BER!

ISI Power Penalty

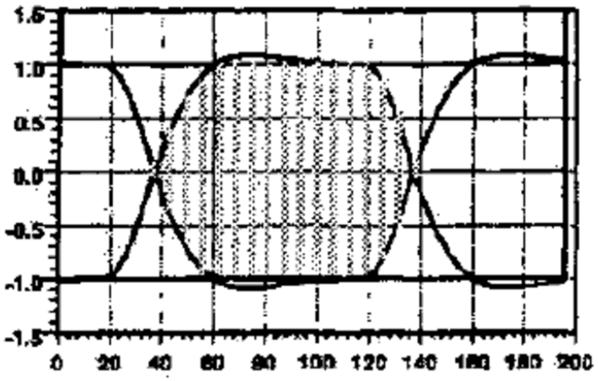
- In order to get the same effective (vertical) eye opening, we have to increase our optical signal power to overcome the ISI



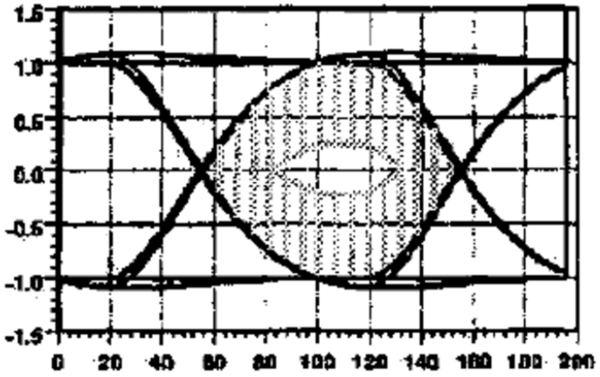
$$PP = \frac{V_E}{V_E'}$$

- Note, this power penalty is a bit conservative, as the worst-case data pattern, which produces the eye closure can occur at a low probability. This is a peak-distortion analysis power penalty.

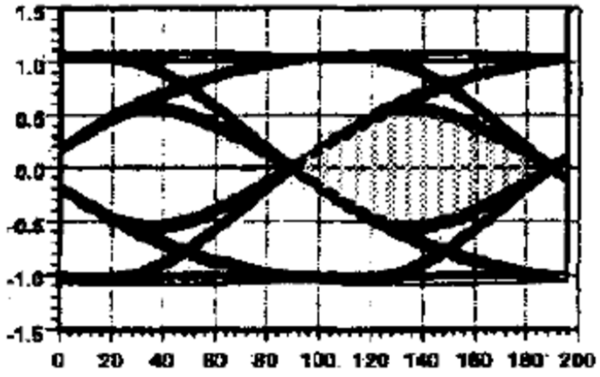
Optimum Receiver Bandwidth



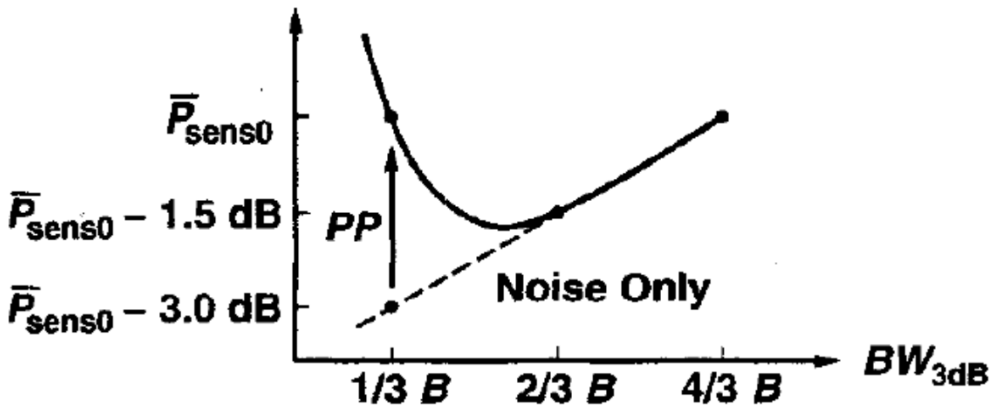
$$BW_{3dB} = \frac{4}{3}B$$



$$BW_{3dB} = \frac{2}{3}B$$



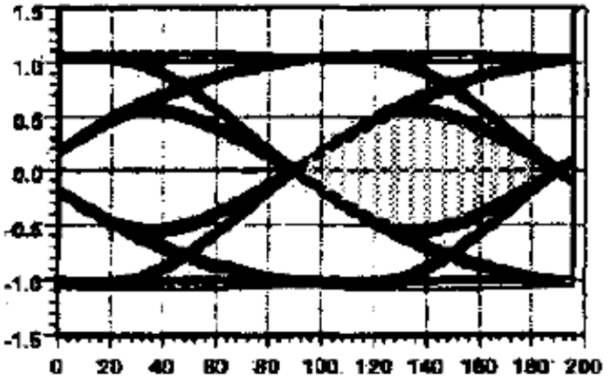
$$BW_{3dB} = \frac{1}{3}B$$



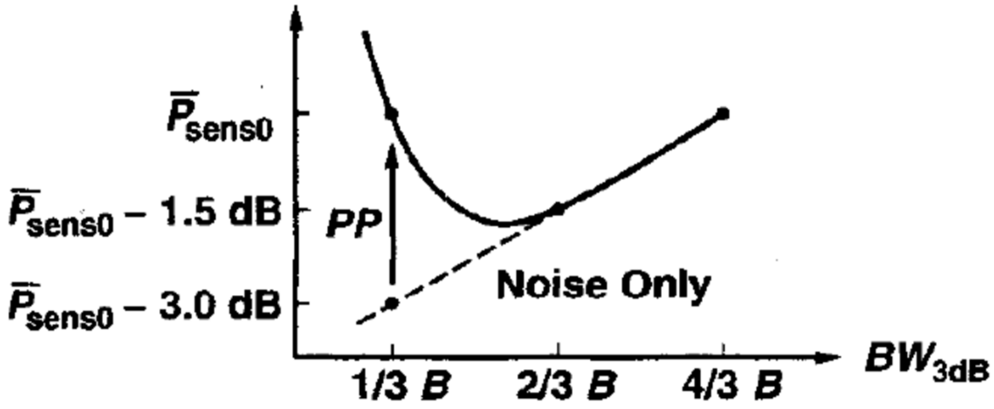
- Assuming white noise dominates, the sensitivity improves by a sqrt factor as bandwidth decreases
- However, around (2/3)*B* the ISI power penalty increases rapidly
- Overall, the optimum bandwidth is near 60%-70% of the bit rate

Will a *B*/3 Bandwidth RX Work?

- If I am willing to live with a 1.5dB degradation in sensitivity, can I design my receiver with *B*/3 bandwidth?
  - 13.3GHz for a 40Gb/s RX!

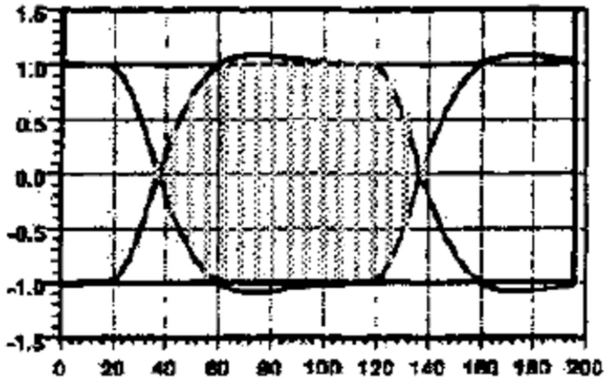


$$BW_{3dB} = \frac{1}{3}B$$



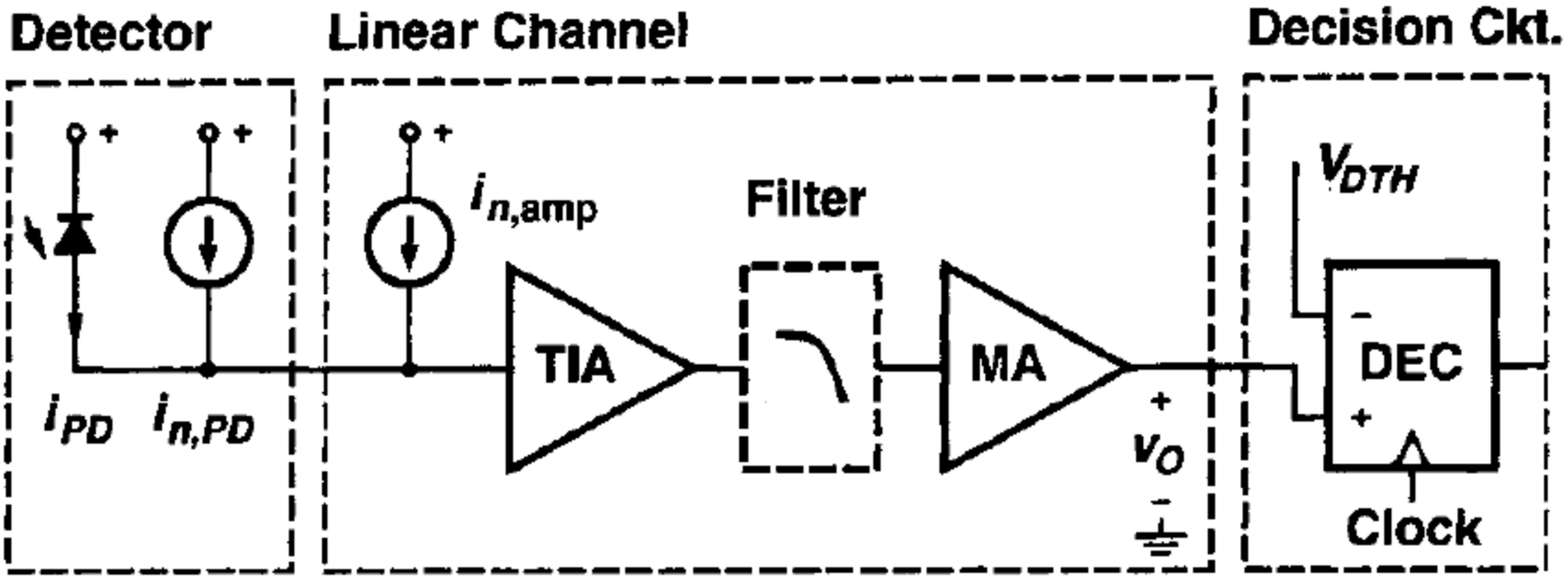
- Maybe, there is much more sensitivity to timing noise (jitter)

- Note that while the (4/3)B receiver has theoretically the same sensitivity, it maintains the same effective eye height over a much wider time window



$$BW_{3dB} = \frac{4}{3}B$$

Bandwidth Allocation



- Note that the equivalent bandwidth of the entire receiver front-end must be close to (2/3)*B*
- Thus, each individual block must have a larger bandwidth

$$\frac{1}{BW^2} \approx \frac{1}{BW_1^2} + \frac{1}{BW_2^2} + \ldots$$

Bandwidth Allocation Strategies

- Wide bandwidth circuits and a precise low-pass filter
  - Often a Bessel-Thompson filter is used to limit the noise
  - Applicable for low-speed receivers (<2.5Gb/s)

- TIA sets the receiver bandwidth
  - Allows for a higher TIA gain and better noise performance
  - This means that the subsequent MA stages need to have a much wider bandwidth
  - Higher bandwidth than a fixed filter, but also less controlled

- All blocks have similar bandwidths
  - If we are designing at the highest speeds, then we can’t afford to overdesign any of the blocks
  - Applicable for higher-speed receivers (>10Gb/s)

Optimum Receiver Response

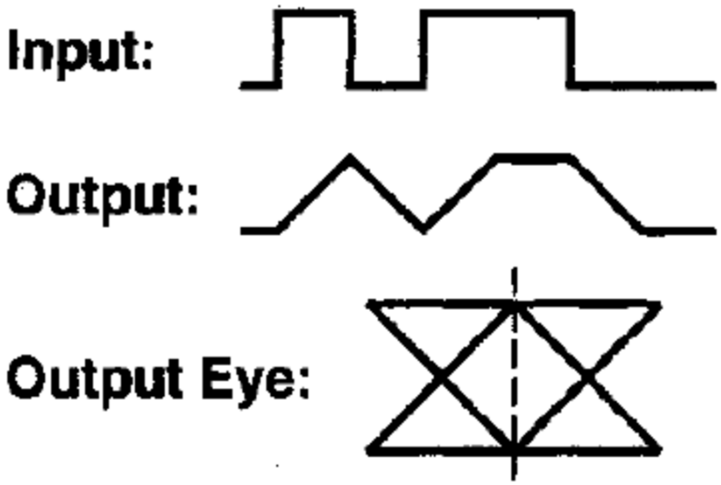
- While we have shown that a bandwidth of ~(2/3)B is optimum from a receiver-induced ISI and noise perspective, is this truly the optimal response when we consider other factors?

- Important factors
  - Received signal ISI
  - Input-referred noise spectrum
  - RX clock jitter
  - Bit estimation technique

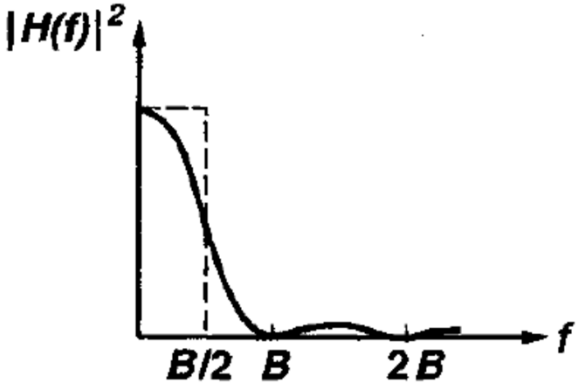
Low ISI Input Optimal RX Response

- A matched filter receiver maximizes the sampled signal-to-noise ratio if the input ISI is minimal
- This has an impulse response h(t) which is proportional to a time-reversed copy of the received pulses x(t)
- For NRZ signals, this is a simple rectangular filter with an impulse response being a rectangular pulse with length of one bit period

Rectangular Filter



$$H\left(f\right)=\frac{\sin\left(\pi f/B\right)}{\pi f/B}e^{-j\pi f/B}$$

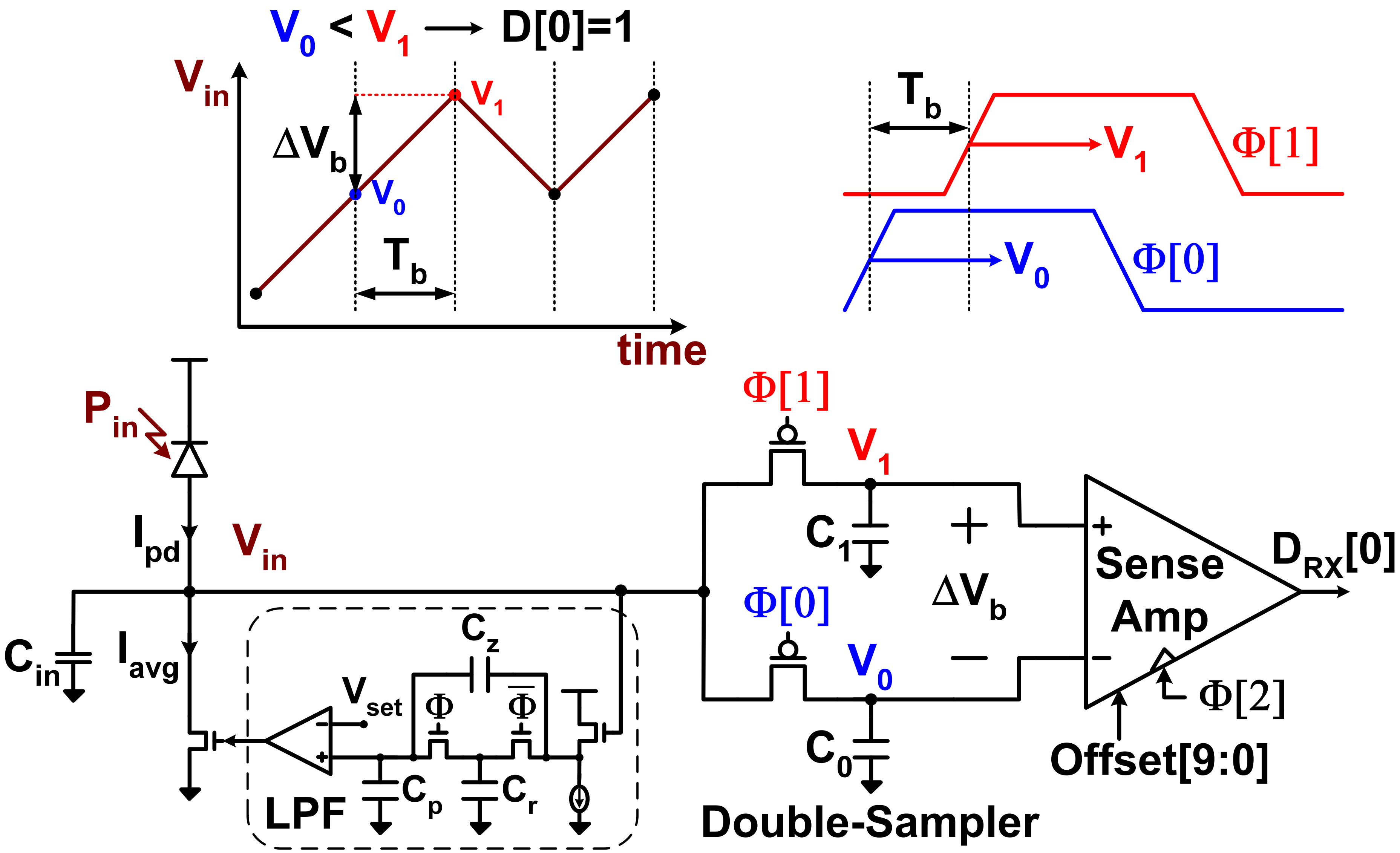


- If we convolve the NRZ input with the rectangular filter impulse response, we get a triangular output waveform
  - Not sampling exactly in the center of the eye will result in a power penalty
- In the frequency domain, the rectangular filter has

$$\text{Noise Bandwidth: } BW_n = B/2$$

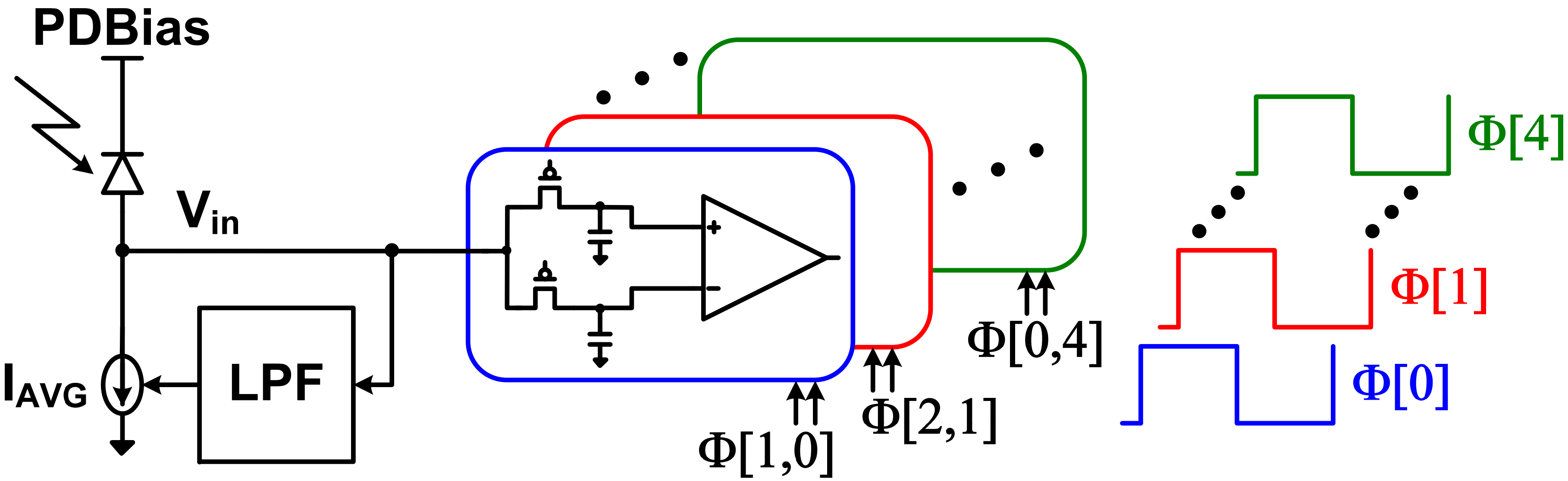
$$\text{3-dB Bandwidth: } BW_{3dB} = 0.443B$$

Integrating Receiver Block Diagram



[Emami VLSI 2002]

Demultiplexing Receiver



- Demultiplexing with multiple clock phases allows higher data rate
  - Data Rate = #Clock Phases x Clock Frequency
  - Gives sense-amp time to resolve data
  - Allows continuous data resolution

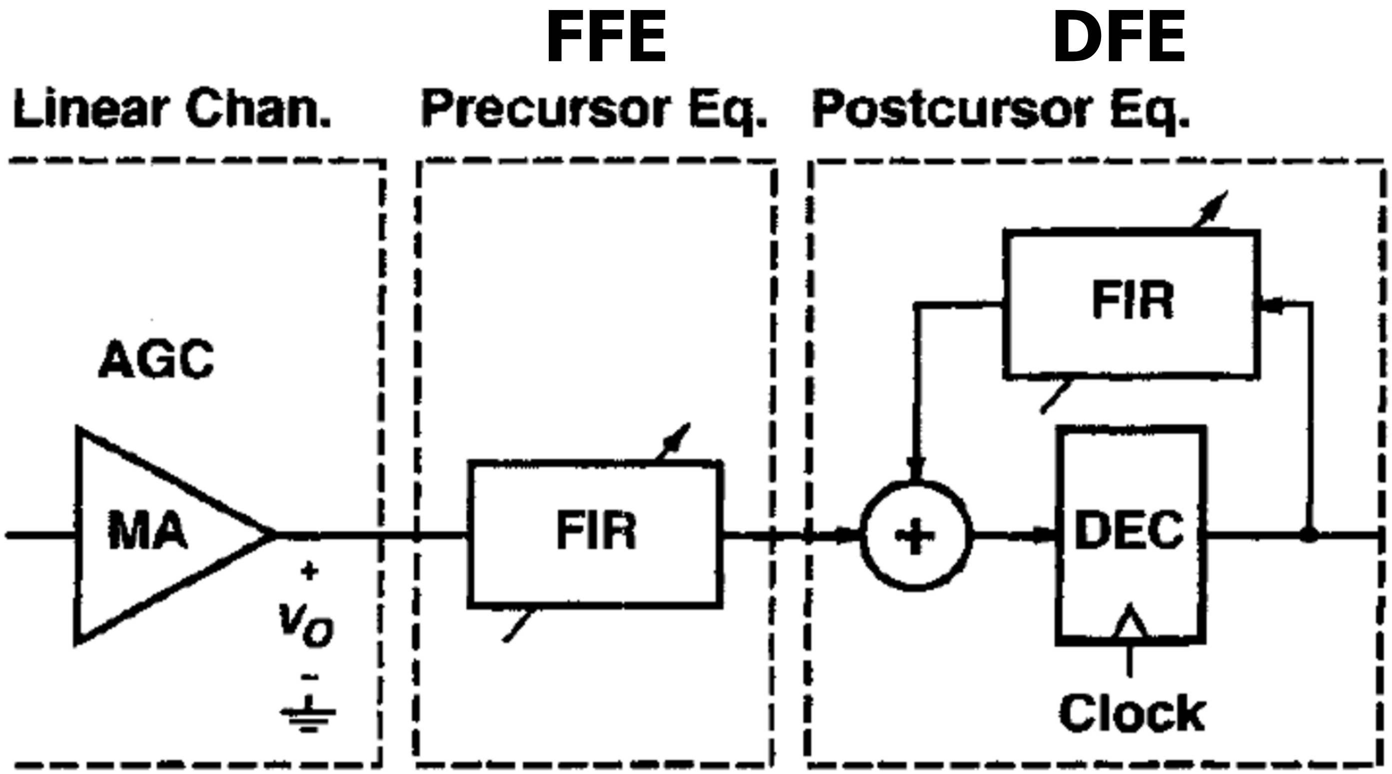
Agenda

- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

What If We Have Significant ISI?

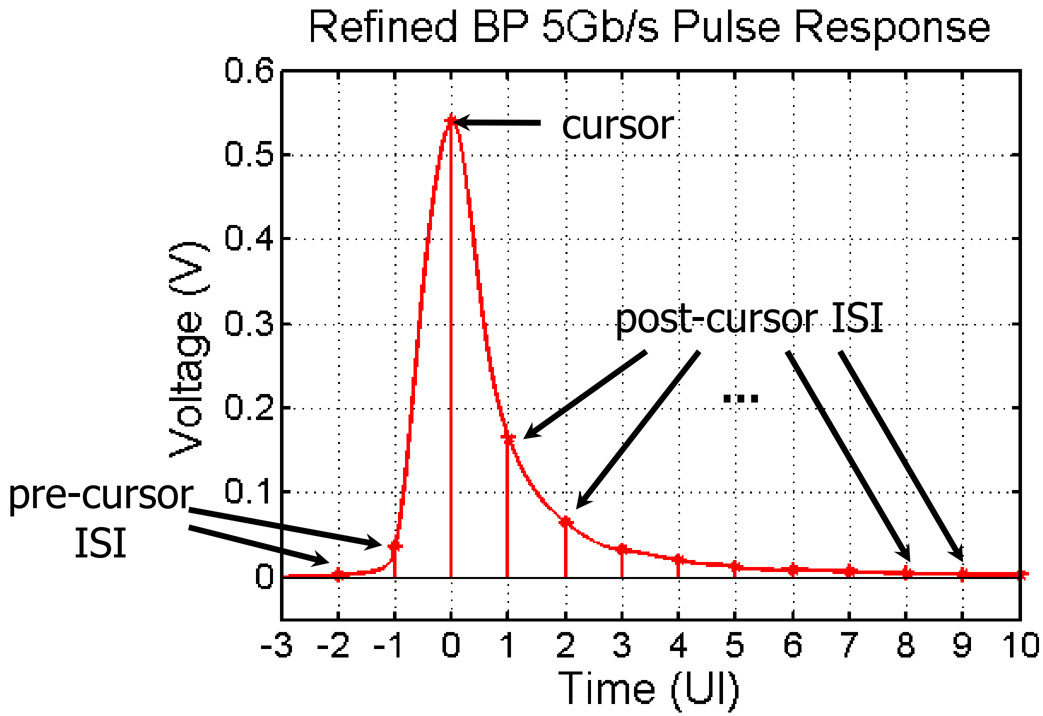
- If we have significant ISI in our system, then an integrating receiver is not optimal
- It is preferred to have a receiver with bandwidth ~(2/3)B to filter the noise, and then have circuitry which cancels the ISI
- A Viterbi decoder, which performs a maximum-likelihood sequence detection, is an optimum realization of an ISI canceller. However, this is generally too complex (power/area).
- Instead, an equalizer is often used to cancel ISI

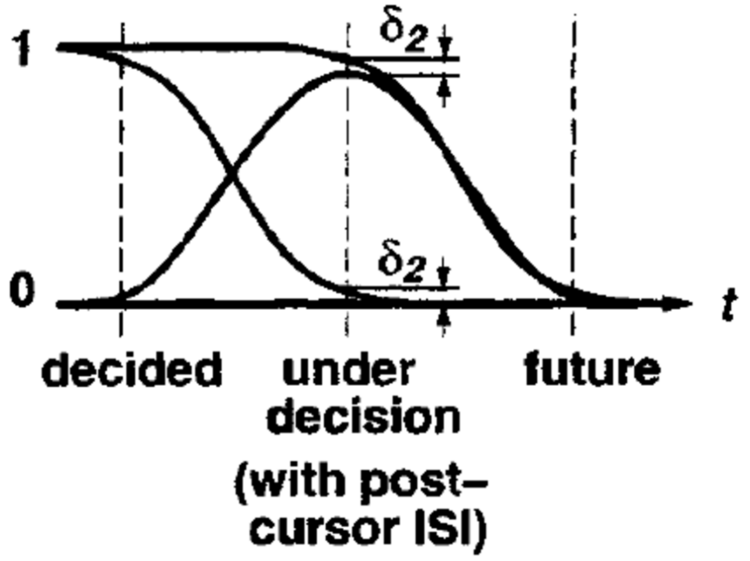
Receiver with Equalization

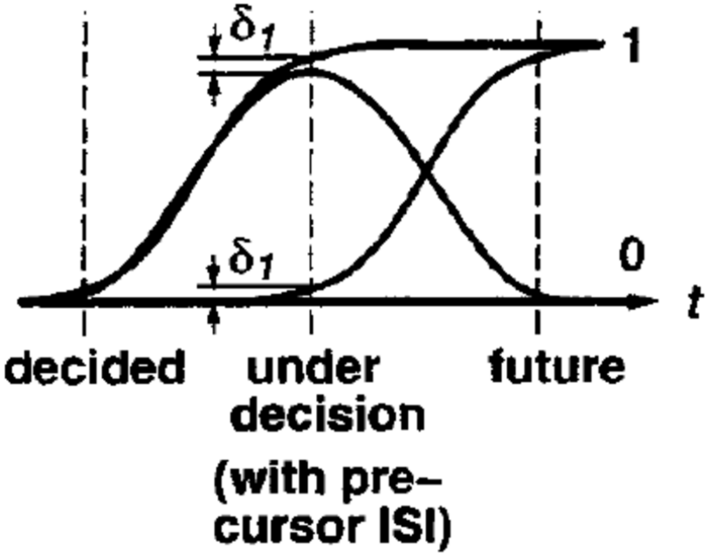


- An FIR filter, also called a feed-forward equalizer (FFE), is used to (primarily) cancel pre-cursor ISI
- A decision feedback equalizer (DFE) cancels post-cursor ISI

Pre- and Post-Cursor ISI



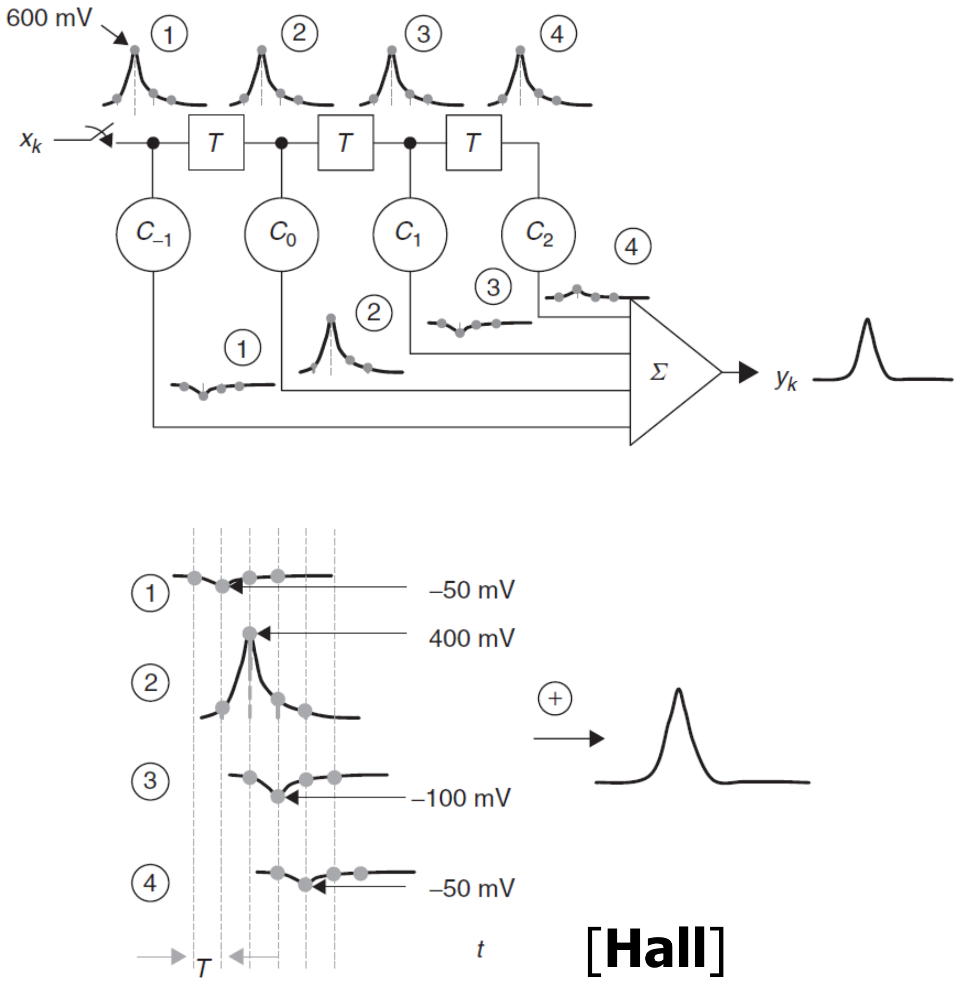




- With post-cursor ISI, the bits before our current bit induces some error in the detected level
- With pre-cursor ISI, the bits after our current bit induce the error
- ISI can span over multiple bit periods

RX FIR Equalization

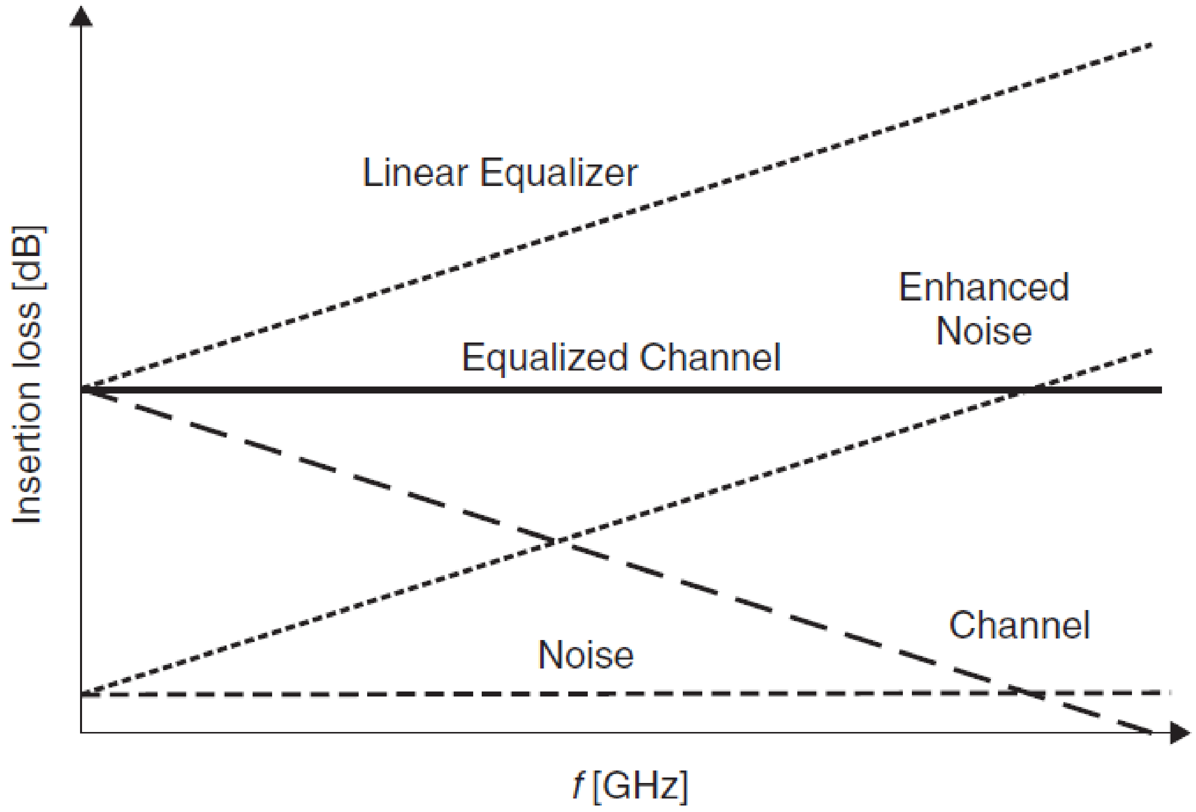
- Delay analog input signal and multiply by equalization coefficients
- Pros
  - With sufficient dynamic range, can amplify high frequency content (rather than attenuate low frequencies)
  - Can cancel ISI in pre-cursor and beyond filter span
  - Filter tap coefficients can be adaptively tuned without any back-channel
- Cons
  - Amplifies noise/crosstalk
  - Implementation of analog delays
  - Tap precision



[Hall]

RX Equalization Noise Enhancement

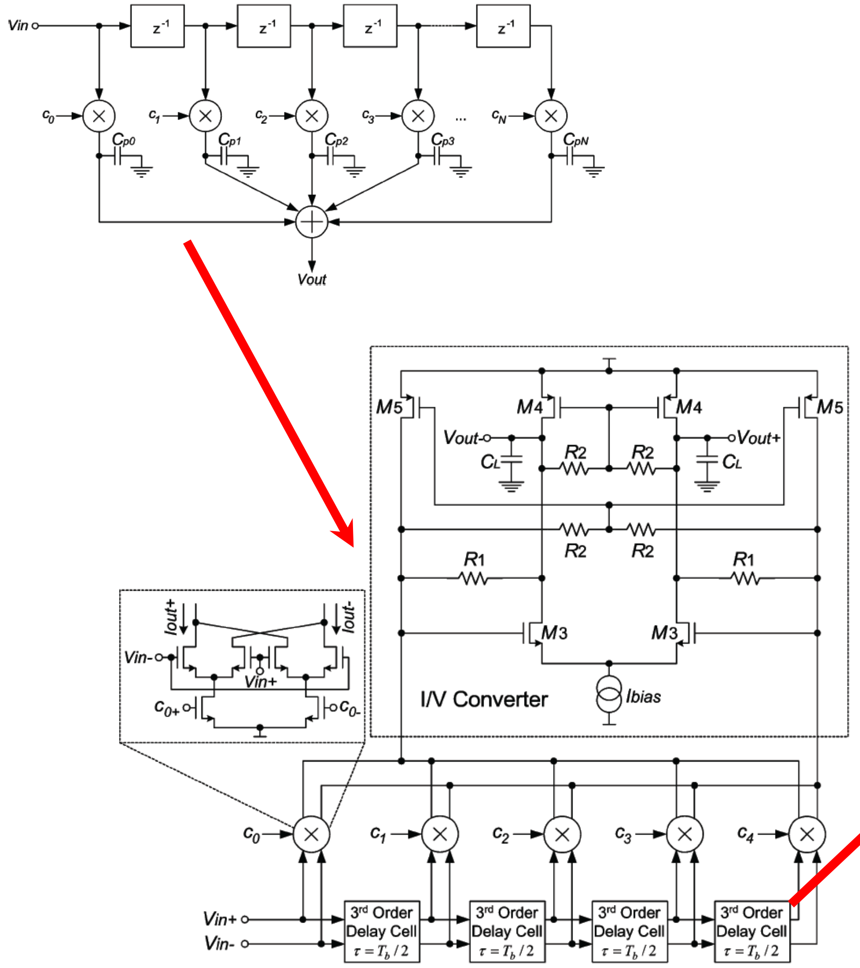
- Linear RX equalizers don’t discriminate between signal, noise, and cross-talk
  - While signal-to-distortion (ISI) ratio is improved, SNR remains unchanged



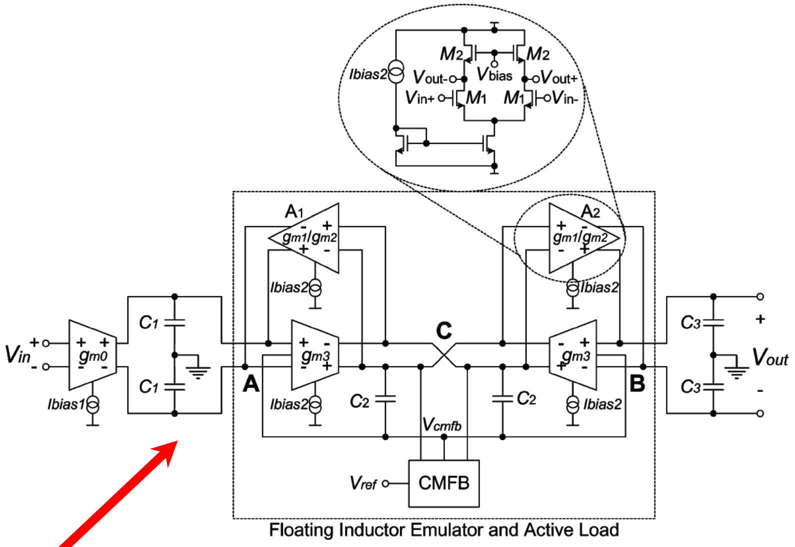
[Hall]

Analog RX FIR Equalization Example

- 5-tap equalizer with tap spacing of $T_b/2$

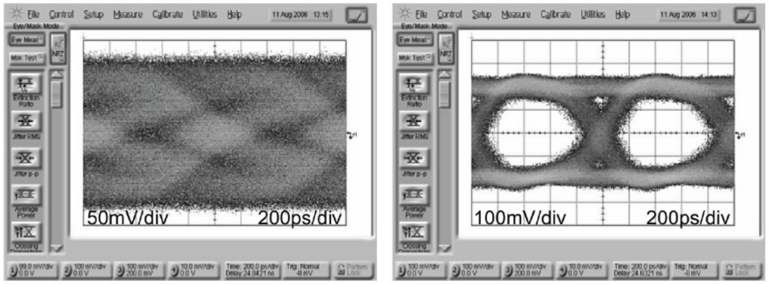


3rd-order delay cell



Floating Inductor Emulator and Active Load

1Gb/s experimental results



Before Equalizer: 23meters

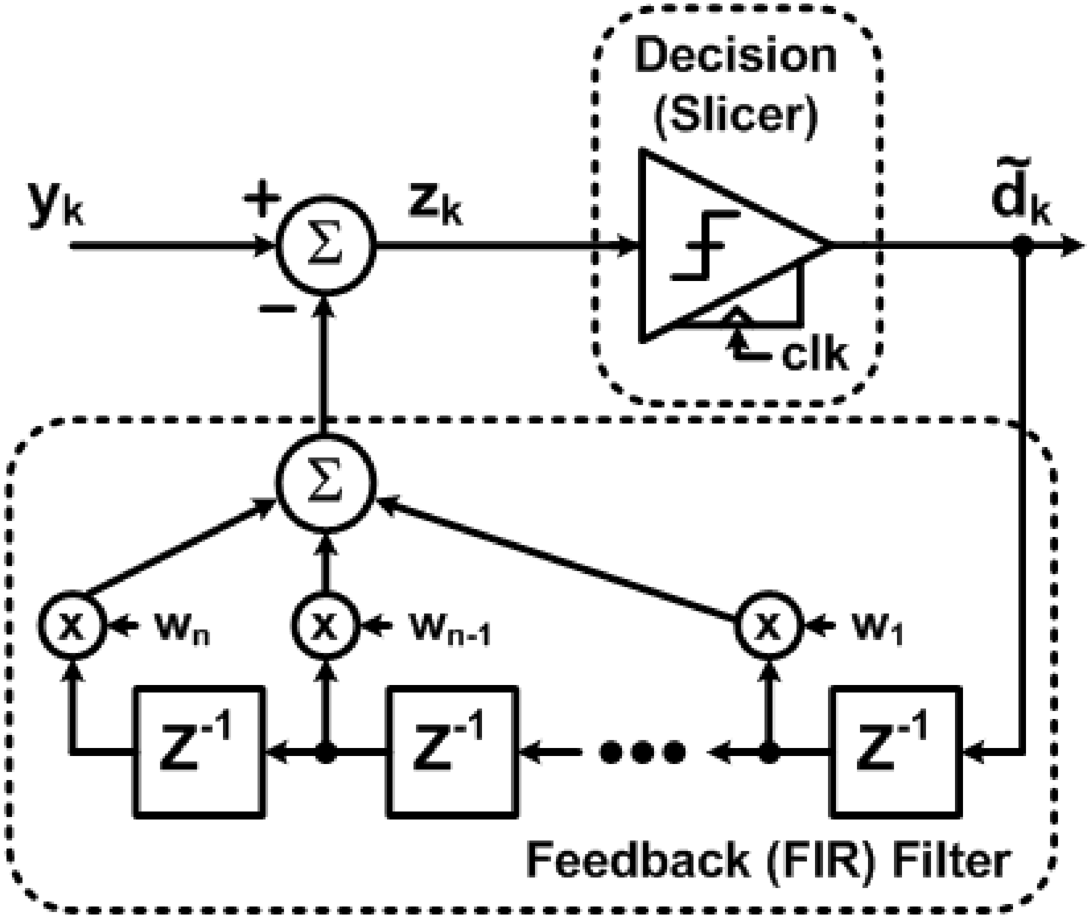
After Equalizer: 23meters

D. Hernandez-Garduno and J. Silva-Martinez, “A CMOS 1Gb/s 5-Tap Transversal Equalizer based on 3rd-Order Delay Cells," ISSCC, 2007.

RX Decision Feedback Equalization (DFE)

- DFE is a **non-linear** equalizer
- Slicer makes a **symbol decision**, i.e. quantizes input
- ISI is then directly subtracted from the incoming signal via a feedback FIR filter

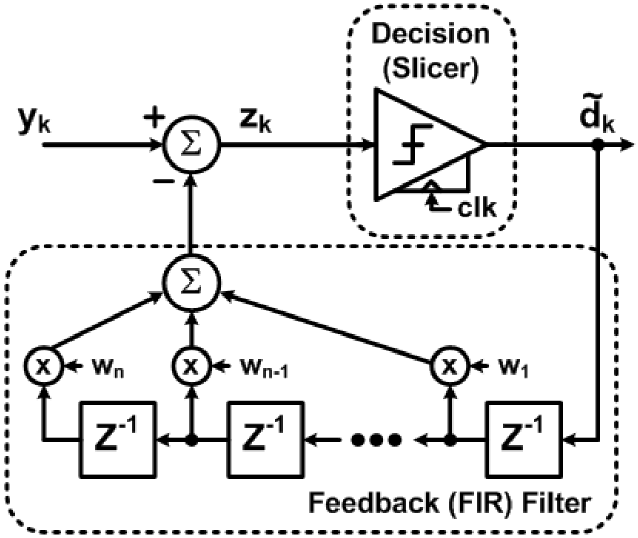
$$z_k = y_k - w_1 \tilde{d}_{k-1} \cdots - w_{n-1} \tilde{d}_{k-(n-1)} - w_n \tilde{d}_{k-n}$$

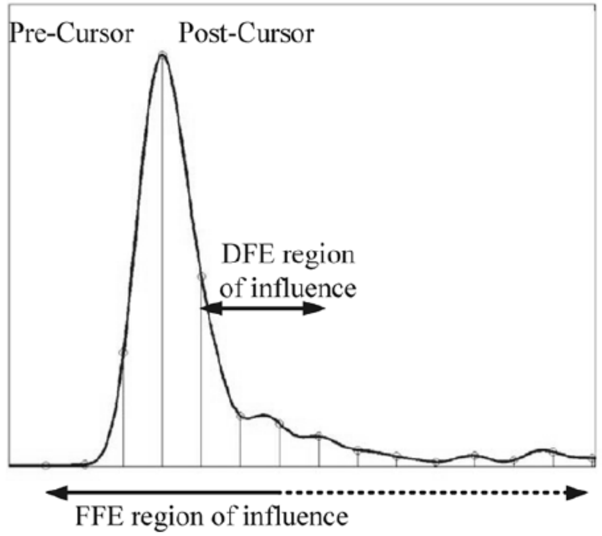


RX Decision Feedback Equalization (DFE)

- Pros
  - Can boost high frequency content without noise and crosstalk amplification
  - Filter tap coefficients can be adaptively tuned without any back-channel
- Cons
  - Cannot cancel pre-cursor ISI
  - Chance for error propagation
    - Low in practical links (BER=10^{-12})
  - Critical feedback timing path
  - Timing of ISI subtraction complicates CDR phase detection

$$z_k = y_k - w_1 \tilde{d}_{k-1} \cdots - w_{n-1} \tilde{d}_{k-(n-1)} - w_n \tilde{d}_{k-n}$$

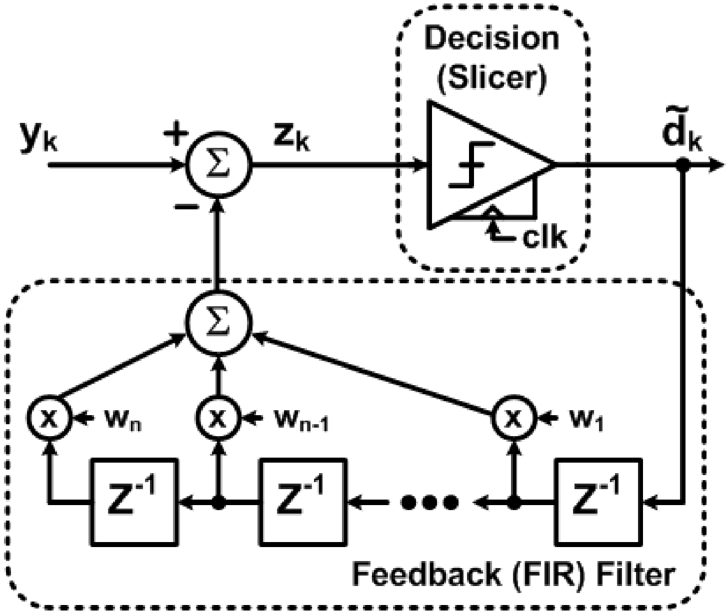




[Payne]

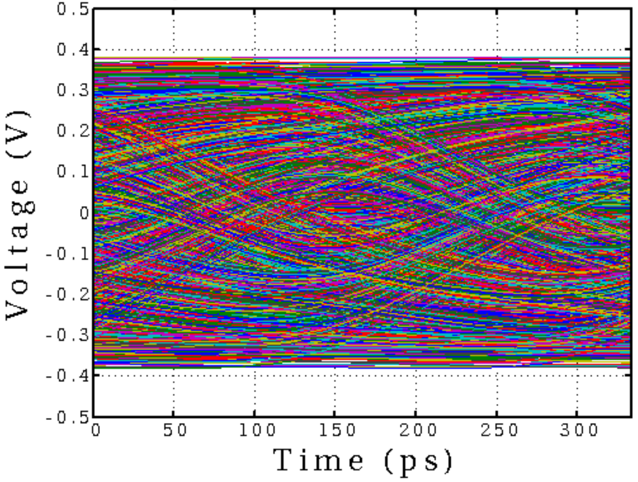
DFE Example

- If only DFE equalization, DFE tap coefficients should equal the unequalized channel pulse response values $[a_1\ a_2\ \ldots\ a_n]$
- With other equalization, DFE tap coefficients should equal the pre-DFE pulse response values
  - DFE provides flexibility in the optimization of other equalizer circuits
  - i.e., you can optimize a TX equalizer without caring about the ISI terms that the DFE will take care of

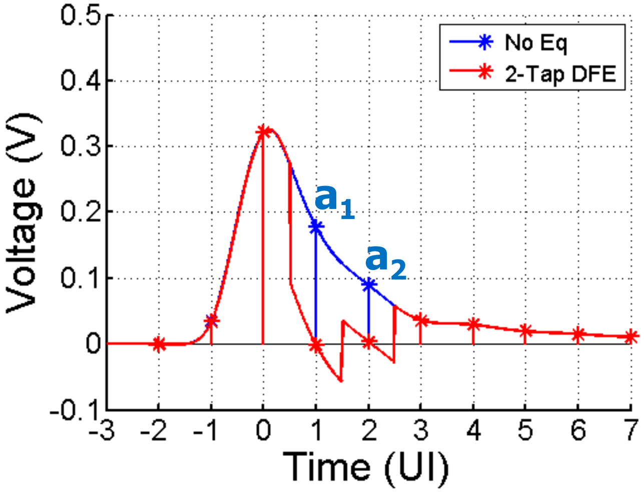


$$[w_1\ w_2]=[a_1\ a_2]$$

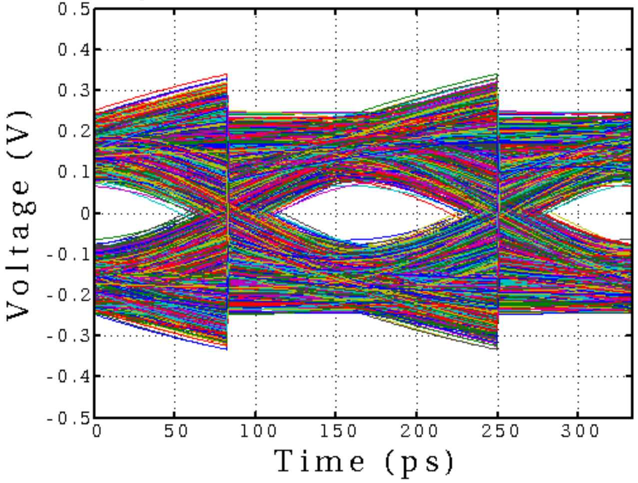
6Gb/s Eye - Refined BP Channel w/ No Eq



Refined BP Channel 6Gb/s Pulse Responses

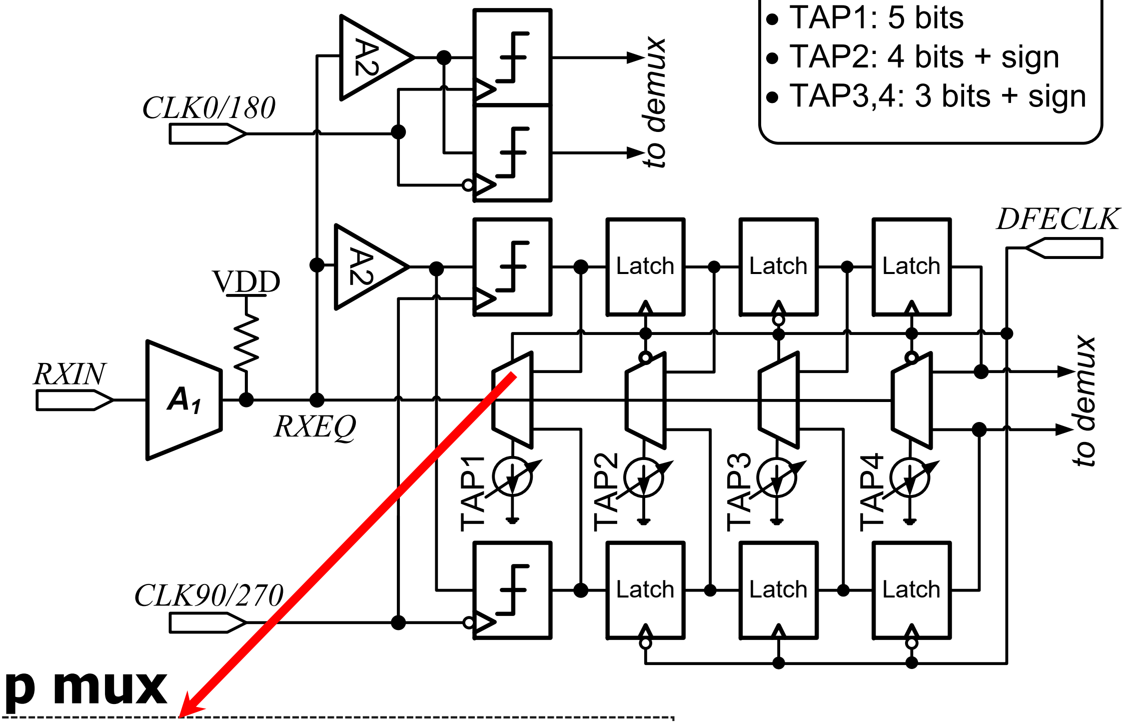


6Gb/s Eye - Refined BP Channel w/ RX DFE Eq



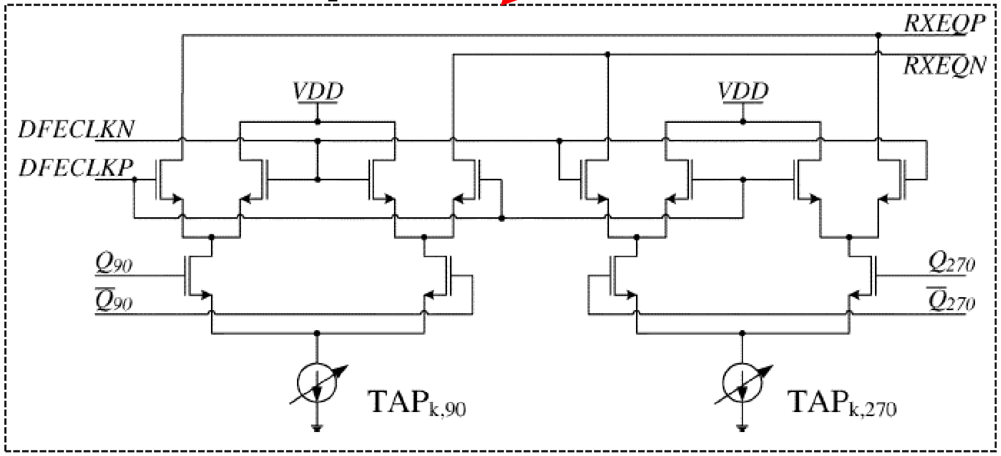
Direct Feedback DFE Example (TI)

- 6.25Gb/s 4-tap DFE
  - ½ rate architecture
  - Adaptive tap algorithm
  - Closes timing on 1st tap in **½ UI** for convergence of both adaptive equalization tap values and CDR



- TAP1: 5 bits
- TAP2: 4 bits + sign
- TAP3,4: 3 bits + sign

Feedback tap mux



R. Payne *et al*, "A 6.25-Gb/s Binary Transceiver in 0.13-um CMOS for Serial Data Transmission Across High Loss Legacy Backplane Channels," *JSSC*, vol. 40, no. 12, Dec. 2005, pp. 2646-2657

Setting Equalizer Values

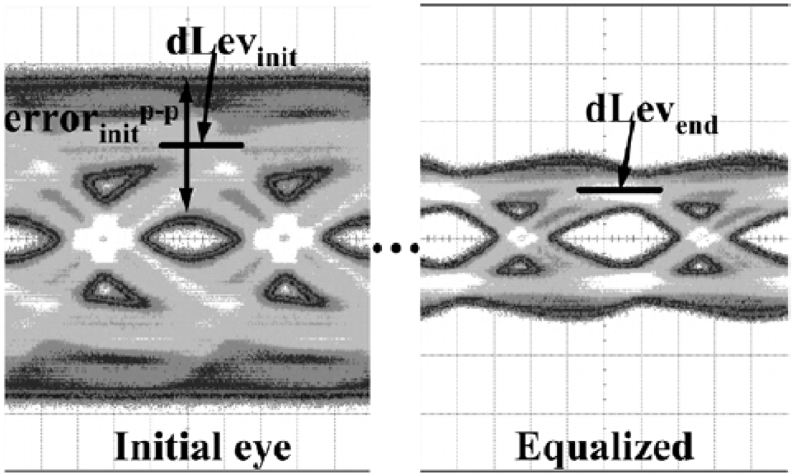
- Simplest approach to setting equalizer values (tap weights, poles, zeros) is to fix them for a specific system
  - Choose optimal values based on lab measurements
  - Sensitive to manufacturing and environment variations

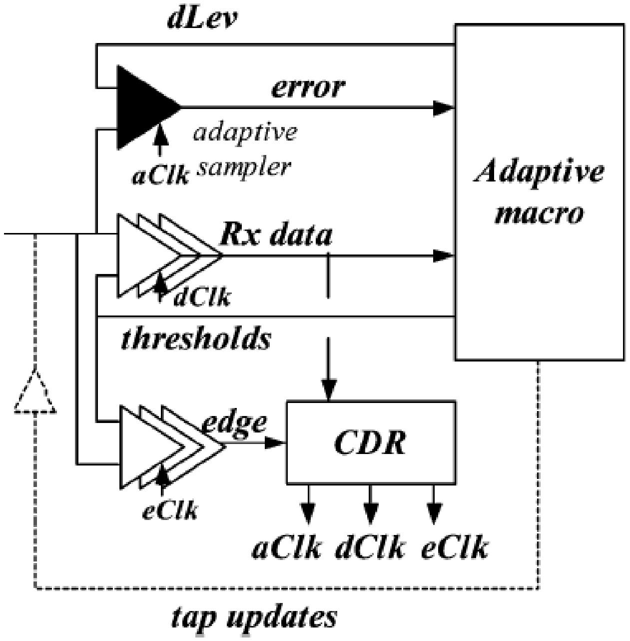
- An adaptive tuning approach allows the optimization of the equalizers for varying channels, environmental conditions, and data rates

- Important issues with adaptive equalization
  - Extracting equalization correction (error) signals
  - Adaptation algorithm and hardware overhead
  - Communicating the correction information to the equalizer circuit

FIR Adaptation Error Extraction

- In order to adapting the FIR filter, we need to measure the response at the receiver input
- Equalizer adaptation (error) information is often obtained by comparing the receiver input versus the desired symbol levels, dLev
- This necessitates additional samplers at the receiver with programmable threshold levels





[**Stojanovic JSSC 2005**]

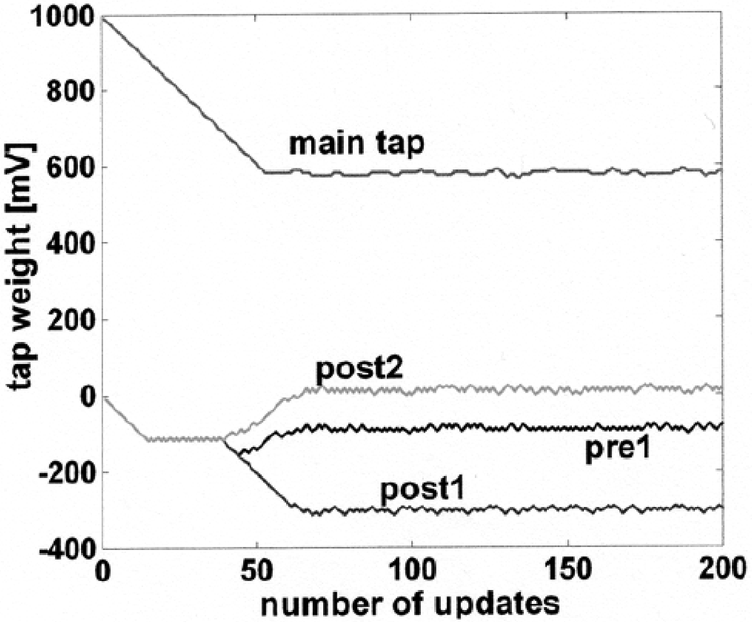
FIR Adaptation Algorithm

- The sign-sign LMS algorithm is often used to adapt equalization taps due to implementation simplicity

$$w_{n+1}^{k} = w_{n}^{k} + \Delta_{w}\text{sign}(d_{n-k})\text{sign}(e_{n})$$

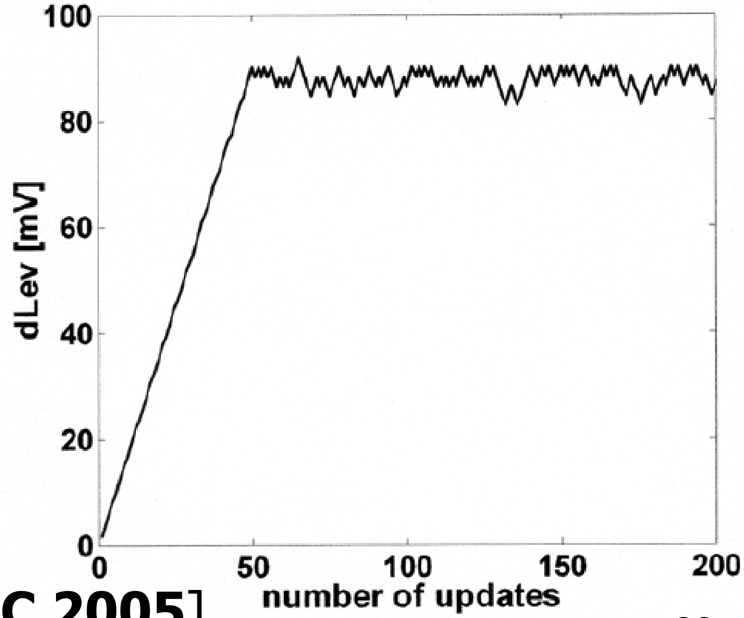
$w =$ tap coefficients, $n =$ time instant, $k =$ tap index, $d_n =$ received data,

$e_n =$ error with respect to desired data level, $dLev$



- As the desired data level is a function of the transmitter swing and channel loss, the desired data level is not necessarily known and should also be adapted

$$dLev_{n+1} = dLev_{n} - \Delta_{dLev}\text{sign}(e_{n})$$



[**Stojanovic JSSC 2005**]

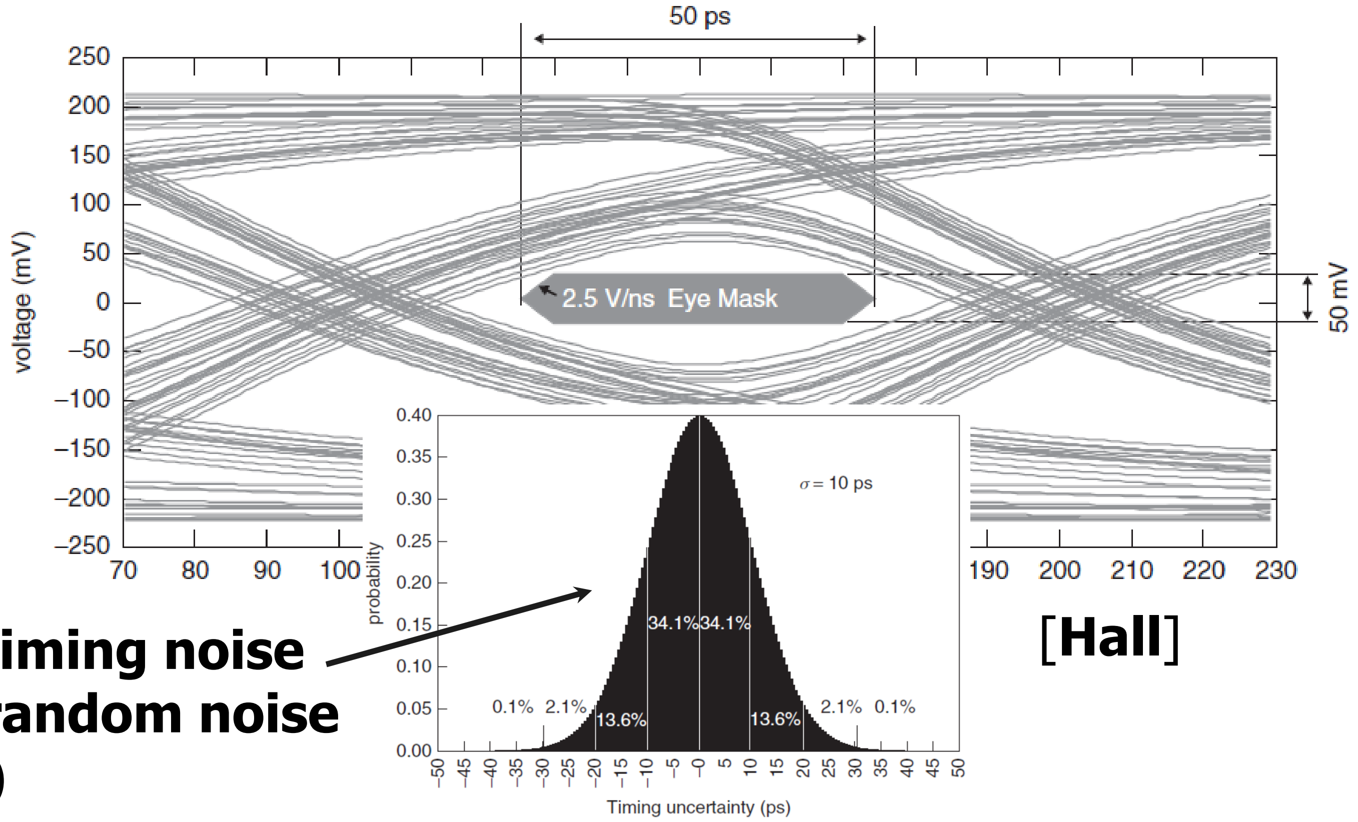
Agenda

- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

Eye Diagram and Spec Mask

- Links must have margin in both the voltage AND timing domain for proper operation
- For independent design (interoperability) of TX and RX, a spec eye mask is used

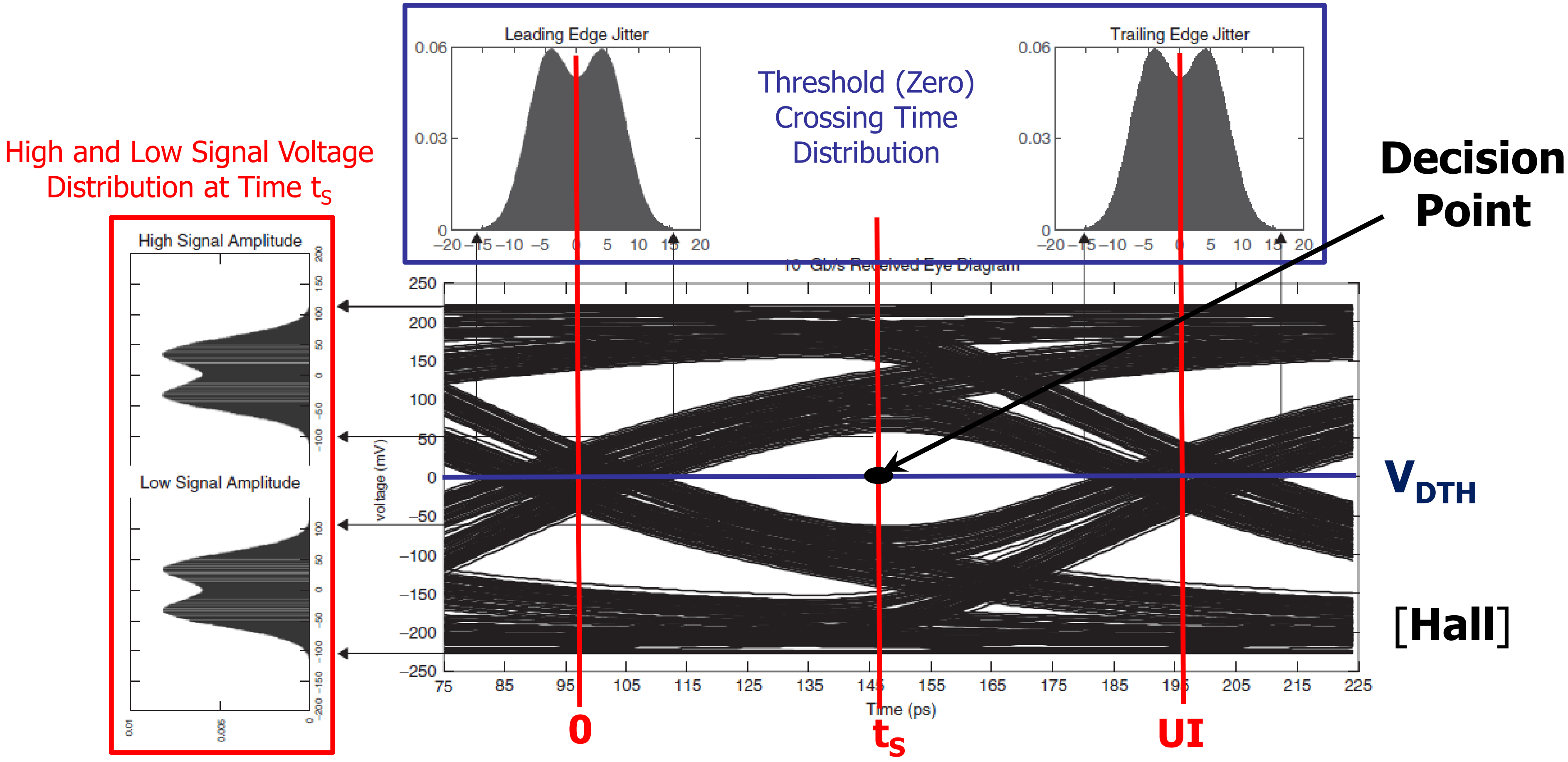
Eye at RX sampler



RX clock timing noise or jitter (random noise only here)

[Hall]

Jitter Histogram


High and Low Signal Voltage Distribution at Time t_S

Threshold (Zero) Crossing Time Distribution

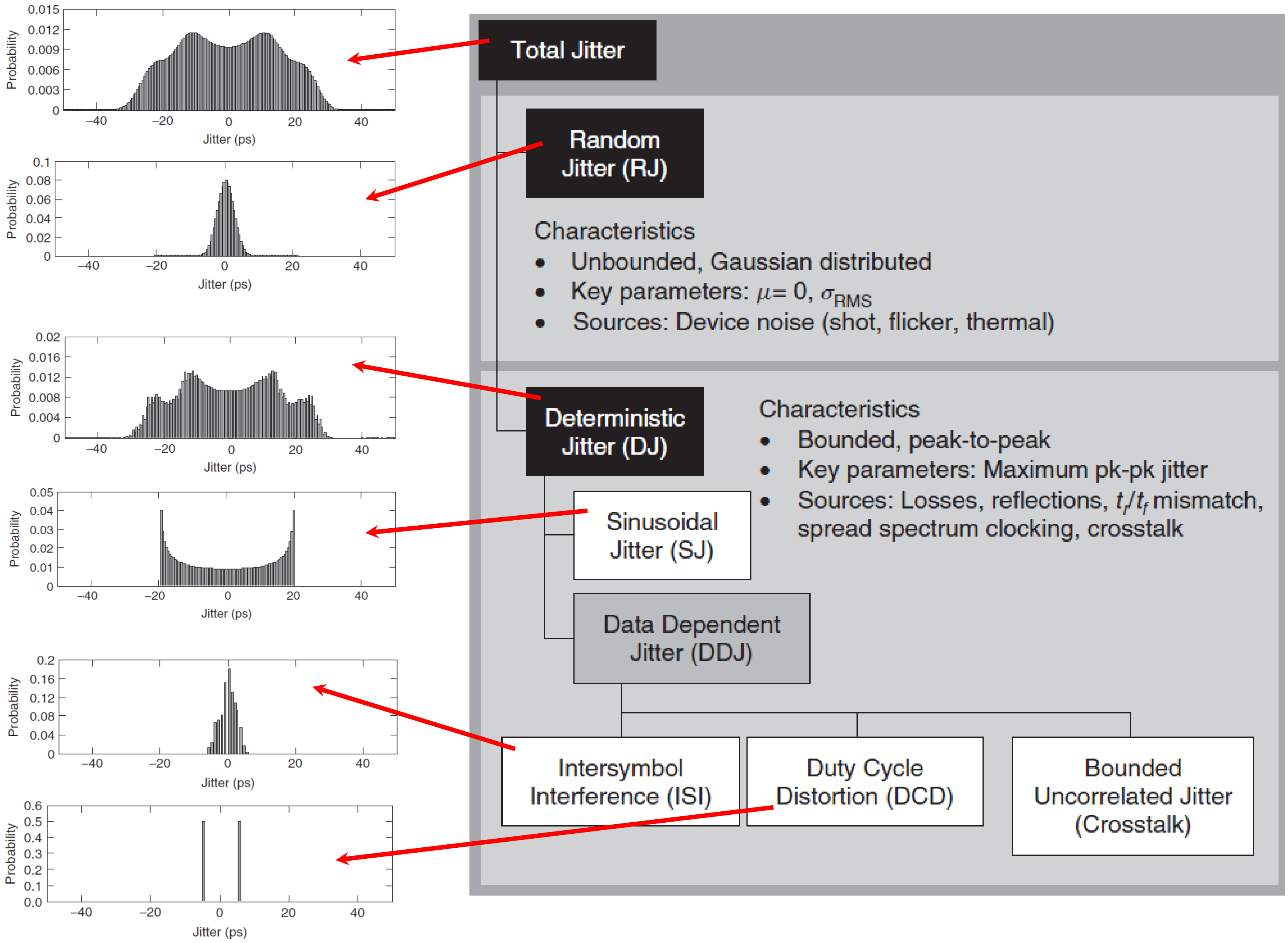
Decision Point

V_{DTH}

[Hall]

- Used to extract the jitter PDF
- Consists of both deterministic and random components
  - Need to decompose these components to accurately estimate jitter at a given BER

Jitter Categories



Total Jitter

Random Jitter (RJ)

Characteristics

- Unbounded, Gaussian distributed
- Key parameters: μ= 0, σ_{RMS}
- Sources: Device noise (shot, flicker, thermal)

Deterministic Jitter (DJ)

Characteristics

- Bounded, peak-to-peak
- Key parameters: Maximum pk-pk jitter
- Sources: Losses, reflections, t_r/t_f mismatch, spread spectrum clocking, crosstalk

Sinusoidal Jitter (SJ)

Data Dependent Jitter (DDJ)

Intersymbol Interference (ISI)

Duty Cycle Distortion (DCD)

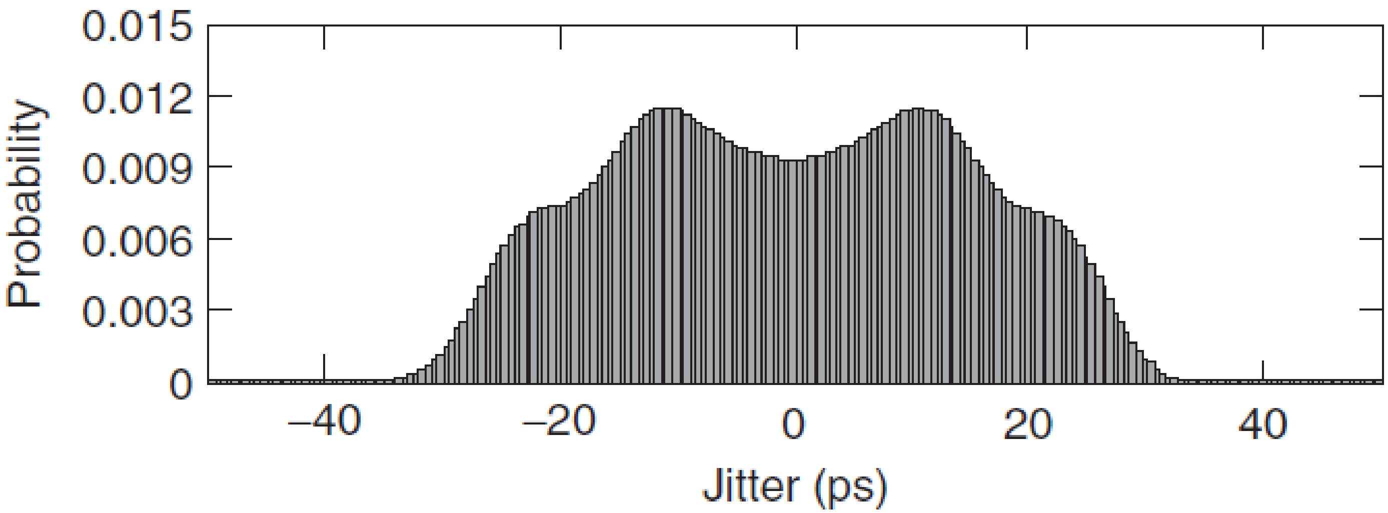
Bounded Uncorrelated Jitter (Crosstalk)

Total Jitter (TJ)

- The total jitter PDF is produced by convolving the random and deterministic jitter PDFs

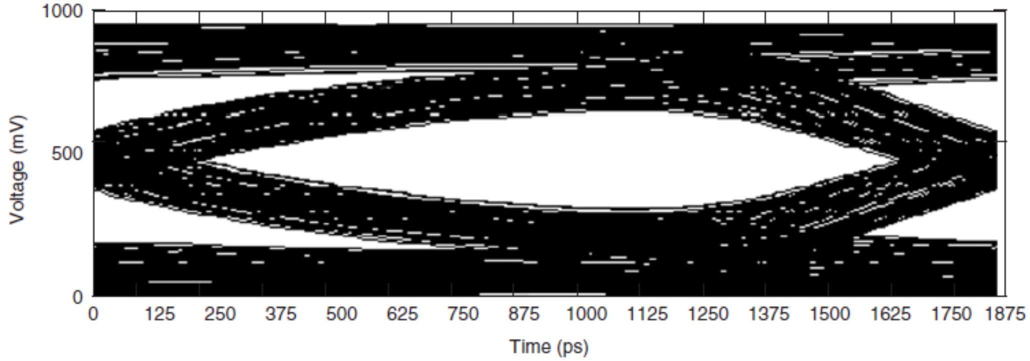
$$PDF_{JT}(t) = PDF_{RJ}(t) * PDF_{DJ}(t)$$

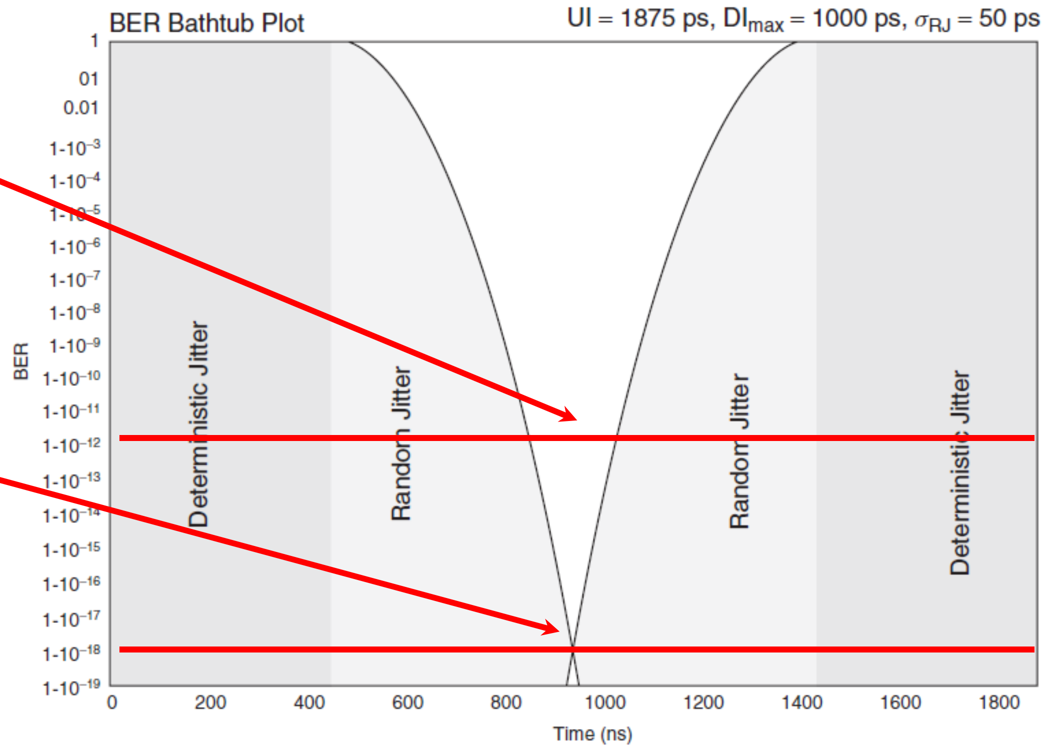
where $PDF_{DJ}(t) = PDF_{SJ}(t) * PDF_{DCD}(t) * PDF_{ISI}(t) * PDF_{BUJ}(t)$



Jitter and Bit Error Rate

- Jitter consists of both deterministic and **random** components
- Total jitter must be quoted at a given BER
  - At BER=10^{-12}, jitter ~1675ps and eye width margin ~200ps
  - System can potentially achieve BER=10^{-18} before being jitter limited





System Jitter Budget

- For a system to achieve a minimum BER performance

$$UI \geq DJ_{\delta\delta}(sys) + 2Q\sigma_{RMS}(sys)$$

- The convolution of the individual deterministic jitter components is approximated by linear addition of the terms

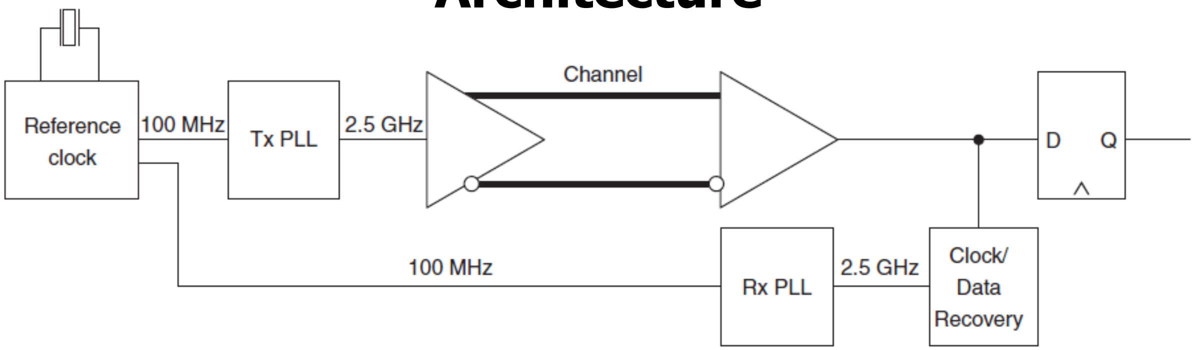
$$DJ_{\delta\delta}(sys) = \sum_i DJ_{\delta\delta}(i)$$

- The convolution of the individual random jitter components results in a root-sum-of-squares system rms value

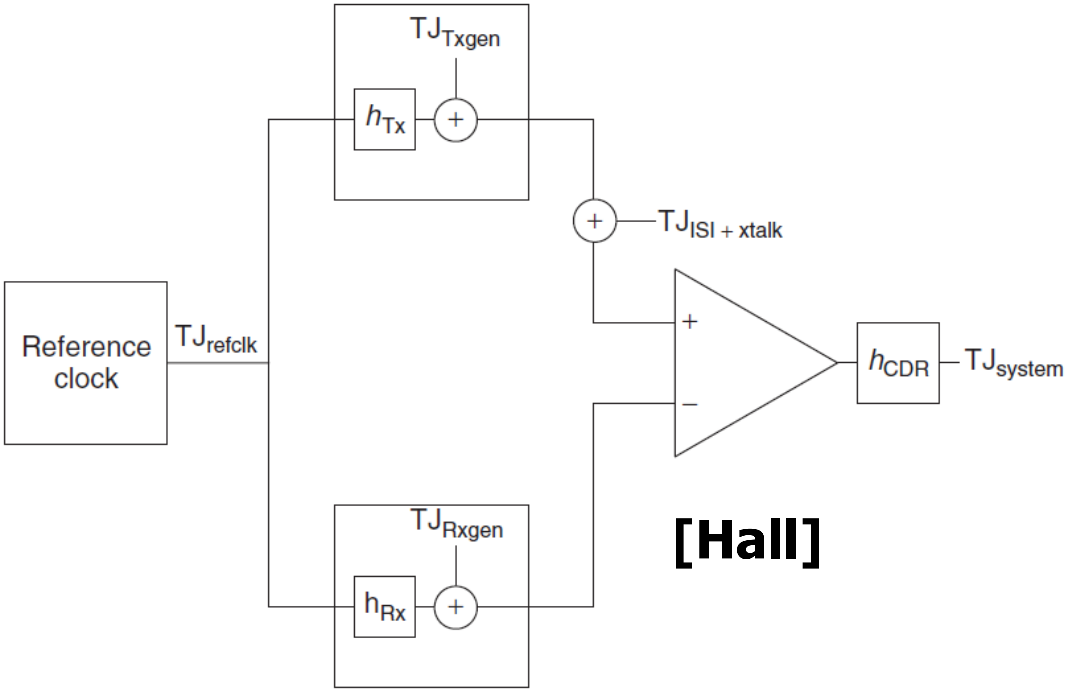
$$\sigma_{RMS}(sys) = \sqrt{\sum_i \sigma_{RMS}^2(i)}$$

Jitter Budget Example – PCI Express System

Architecture

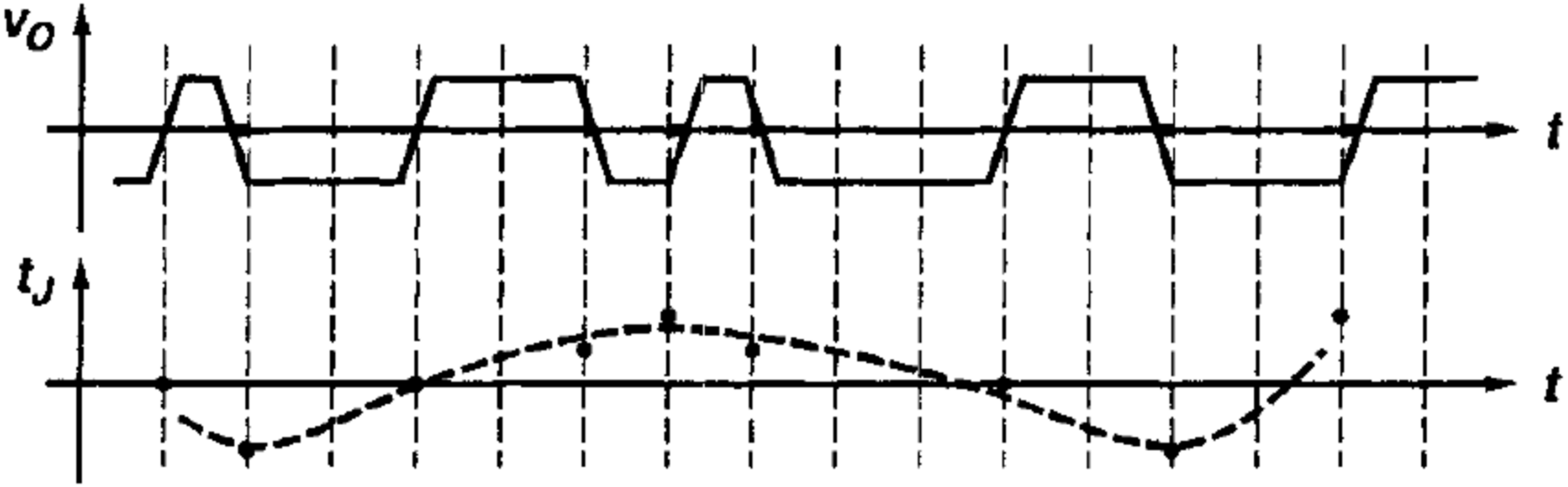


Jitter Model



[Hall]

Jitter Frequency Content



$$\text{Time Interval Error} \quad TIE(i) = \frac{\varphi_n(iT_c)}{2\pi f_c}$$

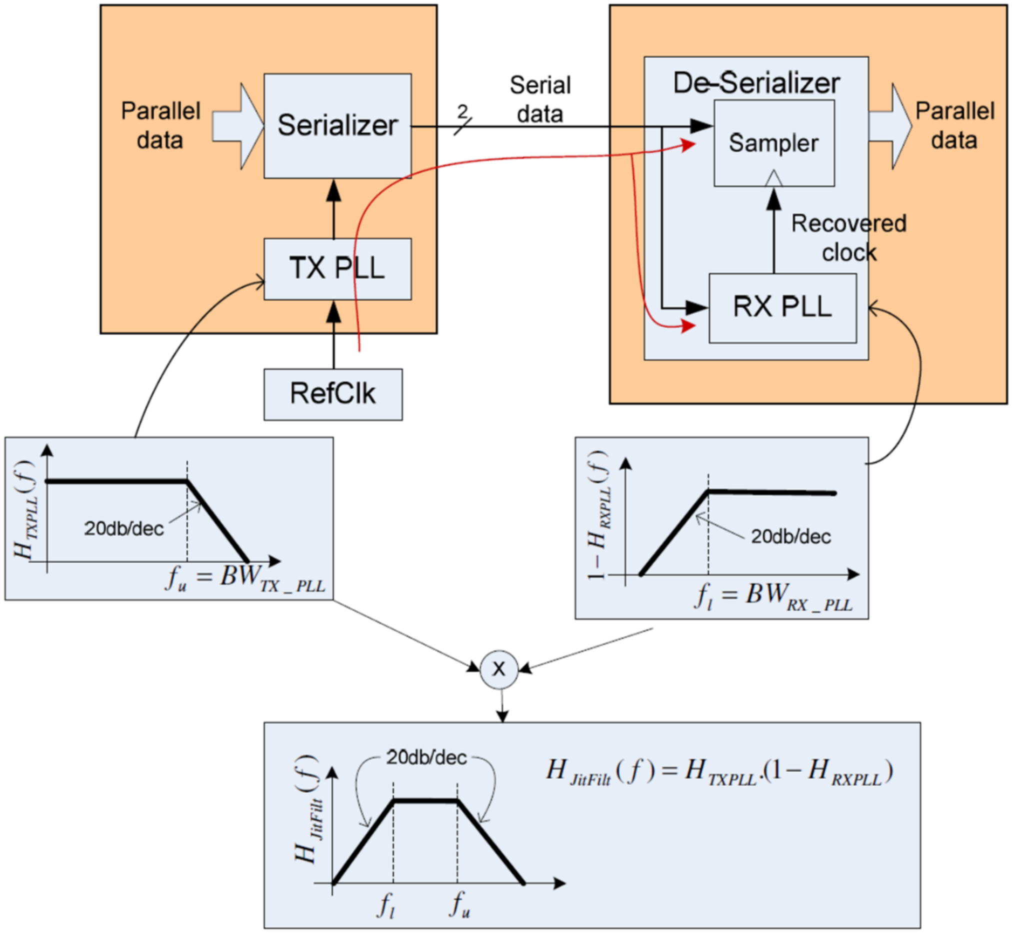
$$\text{where } T_c = \frac{1}{f_c} \text{ is the ideal bit/clock period.}$$

$$PN(f) = 20\log_{10}\left(2\pi f_c \cdot F\{TIE\}\right)$$

System Jitter Filtering

- Jitter sources get shaped/filtered differently depending where they are in the clocking system

CDR (Embedded Clocking) System



- Reference clock jitter gets low-pass filtered by the TX PLL and high-pass filtered by the RX PLL/CDR when we consider the phase error between the sample clock and incoming data

$$\text{Filtered RMS Jitter} = \sqrt{2\int_{f_1}^{f_2} \left|F\{TIE\}\right|^2 \cdot \left|H(f)\right|^2 df}$$

Jitter Budget Example – PCI Express System

$$DJ_{\delta\delta}(sys) = DJ_{\delta\delta}(TX) + DJ_{\delta\delta}(channel) + DJ_{\delta\delta}(RX) + DJ_{\delta\delta}(clock)$$

$$\sigma_{RMS}(sys) = \sqrt{\sigma_{RMS}^2(TX) + \sigma_{RMS}^2(channel) + \sigma_{RMS}^2(RX) + \sigma_{RMS}^2(clock)}$$

TABLE 13-2. PCI Express 2.5-Gb/s Jitter Budget at 10^{-12} BER

Component	Term	σ_{RJ} (ps)	$DJ_{\delta\delta}$ (ps)	TJ (ps)
Reference clock	TJ_{clock}	4.7	41.9	108
Transmitter	TJ_{TX}	2.8	60.6	100
Channel	$TJ_{channel}$	0	90	90
Receiver	TJ_{Rx}	2.8	120.6	~~147~~ → 160
Linear TJ				458
RSS TJ	6.15 * 14.07 =	86.5	313.1	399.6

[Hall]

Table 4.1 Numerical relationship between Q and bit-error rate.

Q	BER	Q	BER
0.0	1/2	5.998	10^{-9}
3.090	10^{-3}	6.361	10^{-10}
3.719	10^{-4}	6.706	10^{-11}
4.265	10^{-5}	7.035	10^{-12}
4.753	10^{-6}	7.349	10^{-13}
5.199	10^{-7}	7.651	10^{-14}
5.612	10^{-8}	7.942	10^{-15}

Agenda

- Receiver Model
- Bit-Error Rate
- Sensitivity
- Personick Integrals
- Power Penalties
- Bandwidth
- Equalization
- Jitter
- Forward Error Correction

Forward Error Correction

- From previous analysis, we found that we need a certain SNR for a given BER
  - w/ NRZ it is Q^2 or ~17dB for BER=10^{-12} (equal noise statistics)
- Can we do better?
- Yes, if we add some redundancy in the bits that we transmit and use this to correct errors at the receiver
- This is called forward error correction (FEC)
- Common codes are Reed-Solomon (RS) and Bose-Chaudhuri-Hocquenghem (BCH)

Shannon's Channel Capacity Theorem

- If sufficient coding is employed, error-free transmission over a channel with additive white Gaussian noise is possible for

$$B \leq BW \log_2(1 + SNR)$$

Here B is the information bit rate, which is lower than the channel bit rate with coding. If we assume ideal Nyquist signaling, we need a minimum channel bandwidth

$$BW = \frac{B}{2r}$$

where r is the code rate and $\frac{B}{r}$ is the channel bit rate. Thus, with coding

$$B \leq \frac{B}{2r} \log_2(1 + SNR)$$

$$SNR = 2^{2r} - 1$$

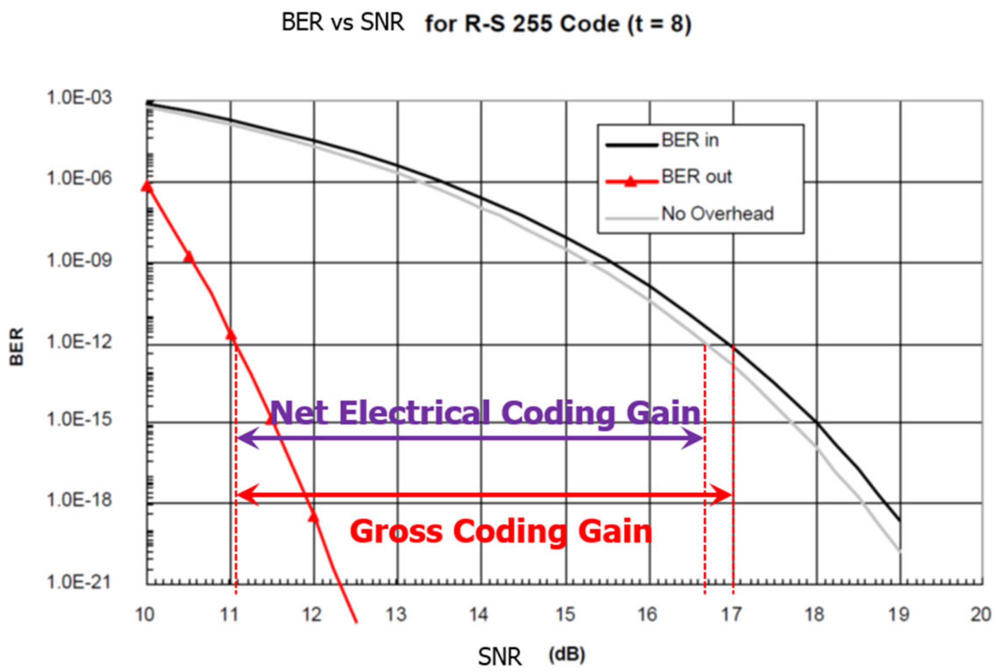
For example, if we have r = 0.8 (which is a 25% data rate overhead) then

$$SNR = 2^{2(0.8)} - 1 = 2.03 = 3.08dB \Rightarrow \text{ Much smaller than 17dB!!}$$

Reed-Solomon Code Example

- Reed-Solomon codes are often used in the Synchronous Optical Networking (SONET) standard
- An important parameter in any error-correcting code is it’s overhead or redundancy, with a RS(255,239) code having n=255 symbols/codeword, but only k=239 information symbols (although 1 is used for framing and isn’t considered in the data payload)
  - The overhead is 255/(239-1)=1.071 or 7.1%
  - This is equivalent to r=238/255=0.933
- A RS(n,k) code can correct for (n-k)/2 symbol errors in a codeword
  - RS(255,239) can correct for 8 symbols/codeword

Coding Gain



At BER = 10^{-12}

Gross Coding Gain ~5.9dB

Net Electrical Coding Gain ~5.6dB

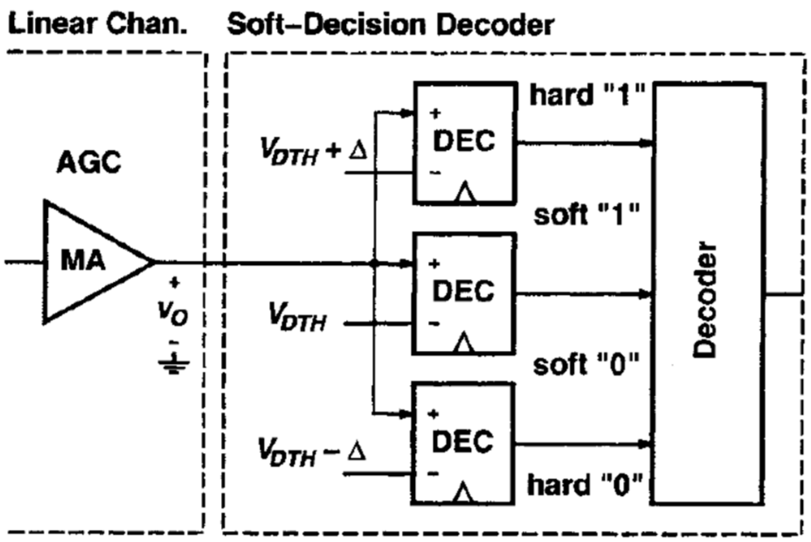
$$\text{Gross Coding Gain at the desired BER} = \frac{SNR_{out}}{SNR_{in}} = \frac{Q_{out}^2}{Q_{in}^2}$$

However, for a fairer comparison we should consider the $\frac{1}{r}$ increase in bandwidth necessary for the code, which will yield $\frac{1}{\sqrt{r}}$ more noise.

$$\text{Net Electrical Coding Gain (NECG)} = r \cdot \frac{Q_{out}^2}{Q_{in}^2}$$

Soft-Decision Decoding

- So far we have talked about codes which use binary "hard-decisions"
- Superior performance (~2dB) occurs if we use more "analog" information in the form of "soft-decisions"
- Soft decisions are utilized in turbo codes and low-density parity check codes (LDPC)
- In an NRZ system, soft decisions can be realized with 2 additional comparators with some Δ offset or with an ADC front-end
- An AGC loop may be necessary to maintain required linearity



Next Time

- Transimpedance Amplifier (TIA) Circuits