ECEN720: High-Speed Links Circuits and Systems
Spring 2023

Lecture 9: Noise Sources

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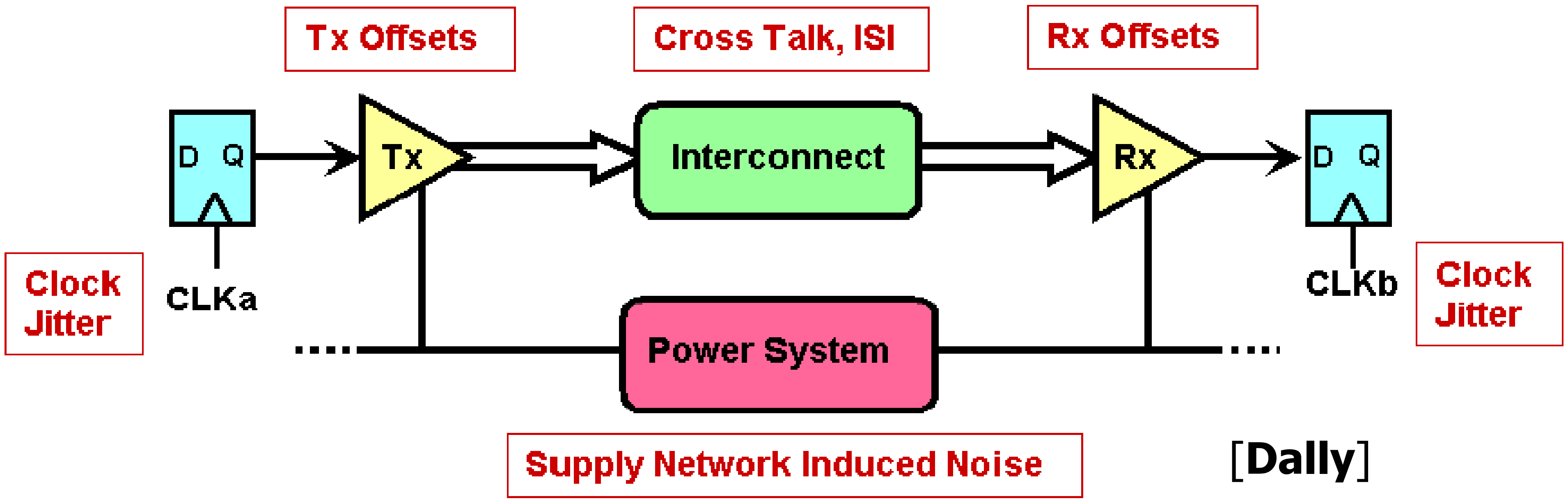
Announcements

- Lab 5 Report & Prelab 6 due Mar 27
- Stateye theory paper posted on website

Noise in High-Speed Link Systems



[**Dally**]

- Multiple noise sources can degrade link timing and amplitude margin

Noise Source Overview

- Common “noise” sources
  - Power supply noise
  - Receiver offset
  - Crosstalk
  - Inter-symbol interference
  - Random noise
- Power supply noise
  - Switching current through finite supply impedance causes supply voltage drops that vary with time and physical location
- Receiver offset
  - Caused by random device mismatches
- Crosstalk
  - One signal (aggressor) interfering with another signal (victim)
  - On-chip coupling (capacitive)
  - Off-chip coupling (t-line)
    - Near-end
    - Far-end
- Inter-symbol interference
  - Signal dispersion causes signal to interfere with itself
- Random noise
  - Thermal & shot noise
  - Clock jitter components

Bounded and Statistical Noise Sources

- Bounded or *deterministic* noise sources
  - Have theoretically predictable values with defined worst-case bounds
  - Allows for simple (but pessimistic) worst-case analysis
  - Examples
    - Crosstalk to small channel count
    - ISI
    - Receiver offset

- Statistical or *random* noise sources
  - Treat noise as a random process
    - Source may be psuedo-random
  - Often characterized w/ Gaussian stats
    - RMS value
    - Probability density function (PDF)
  - Examples
    - Thermal noise
    - Clock jitter components
    - Crosstalk to large channel count

- Understanding whether noise source is bounded or random is critical to accurate link performance estimation

Proportional and Independent Noise Sources

- Some noise is *proportional* to signal swing
  - Crosstalk
  - Simultaneous switching power supply noise
  - ISI
- Can't overpower this noise
  - Larger signal = more noise

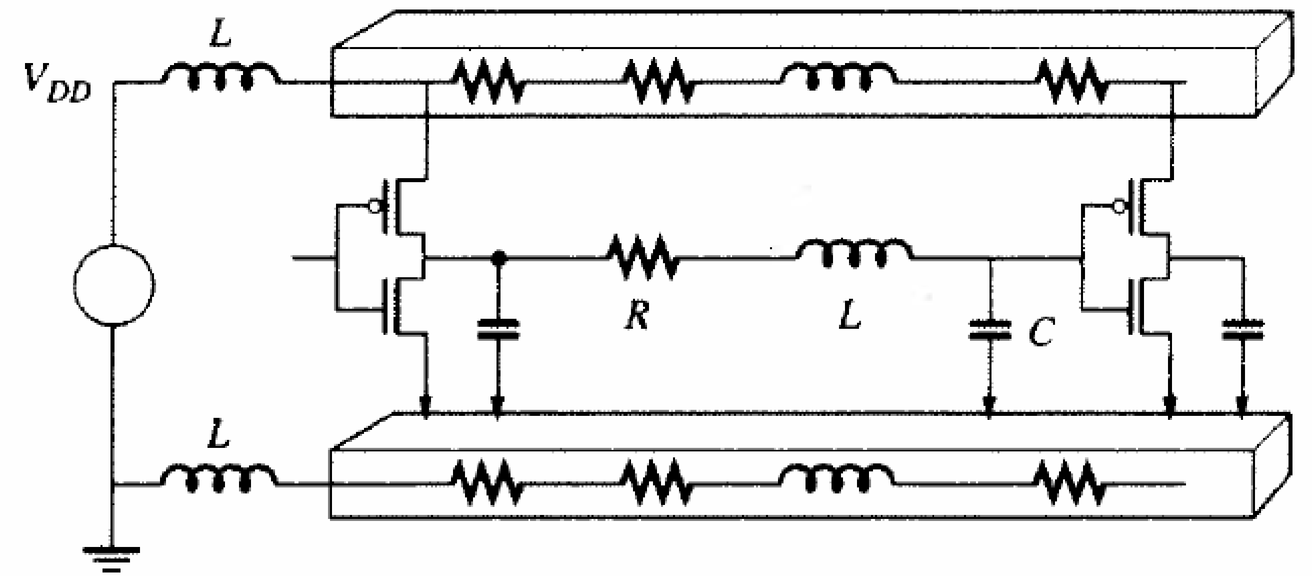
- Some noise is *independent* to signal swing
  - RX offset
  - Non-IO power supply noise
- Can overpower this noise

$$\underbrace{V_N}_{\text{Total noise}} = \underbrace{K_N}_{\text{Proportional noise constant}} \underbrace{V_S}_{\text{Signal swing}} + \underbrace{V_{NI}}_{\text{Independent noise}}$$

Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

Power Supply Noise

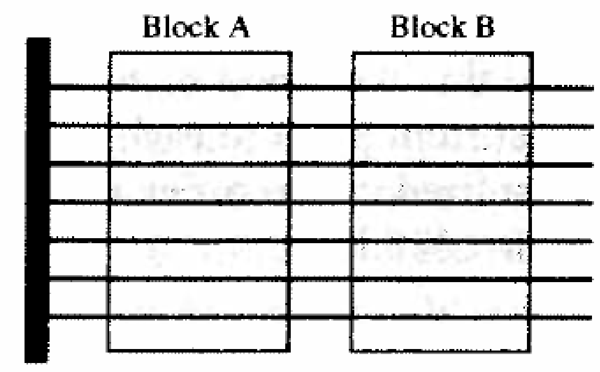


[**Hodges**]

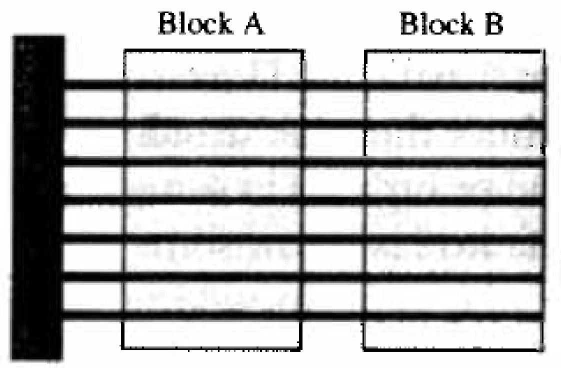
- Circuits draw current from the VDD supply nets and return current to the GND nets
- Supply networks have finite impedance
- Time-varying (switching) currents induce variations on the supply voltage
- Supply noise a circuit sees depends on its location in supply distribution network

Power Routing

Bad – Block B will experience excessive supply noise

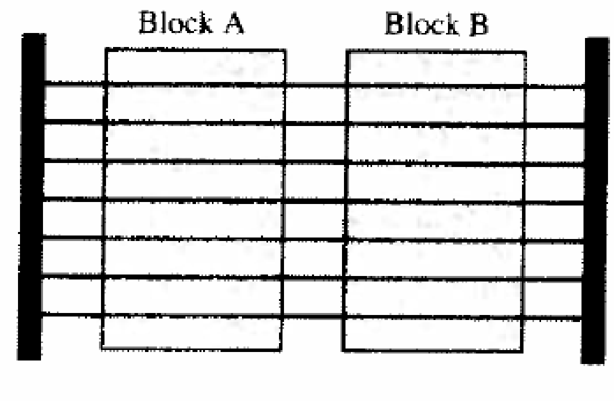


Better – Block B will experience 1/2 supply noise, but at the cost of double the power routing through blocks

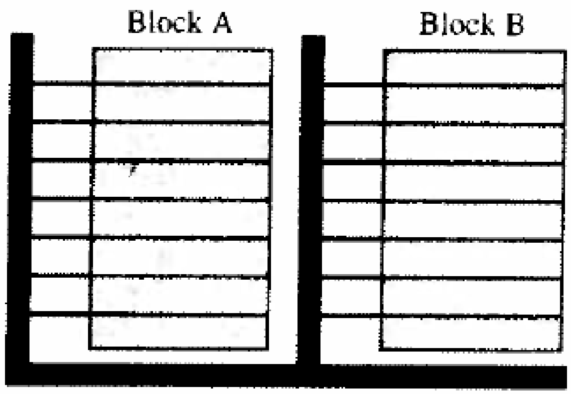


[**Hodges**]

Even Better – Block A & B will experience similar supply noise



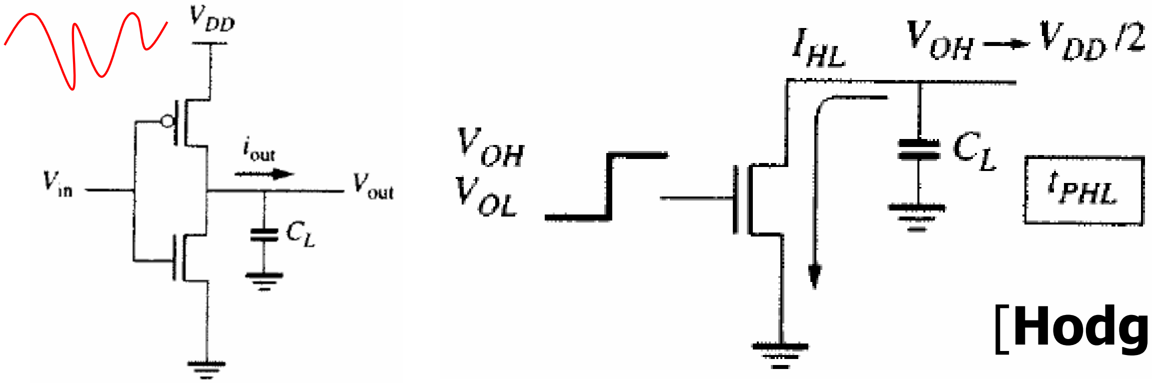
Best – Block A & B are more isolated



[**Hodges**]

Supply Induced Delay Variation

- Supply noise can induce variations in circuit delay
  - Results in deterministic jitter on clocks & data signals



[**Hodges**]

$$t_{PHL} = \frac{C_L(VDD/2)}{I_{DSATN}} = \frac{C_L(VDD/2)}{\left(\dfrac{W_N v_{sat} C_{ox}(VDD - V_{TN})^2}{VDD - V_{TN} + E_{CN} L_N}\right)} \approx \frac{C_L VDD}{2 W_N v_{sat} C_{ox}(VDD - V_{TN})}$$

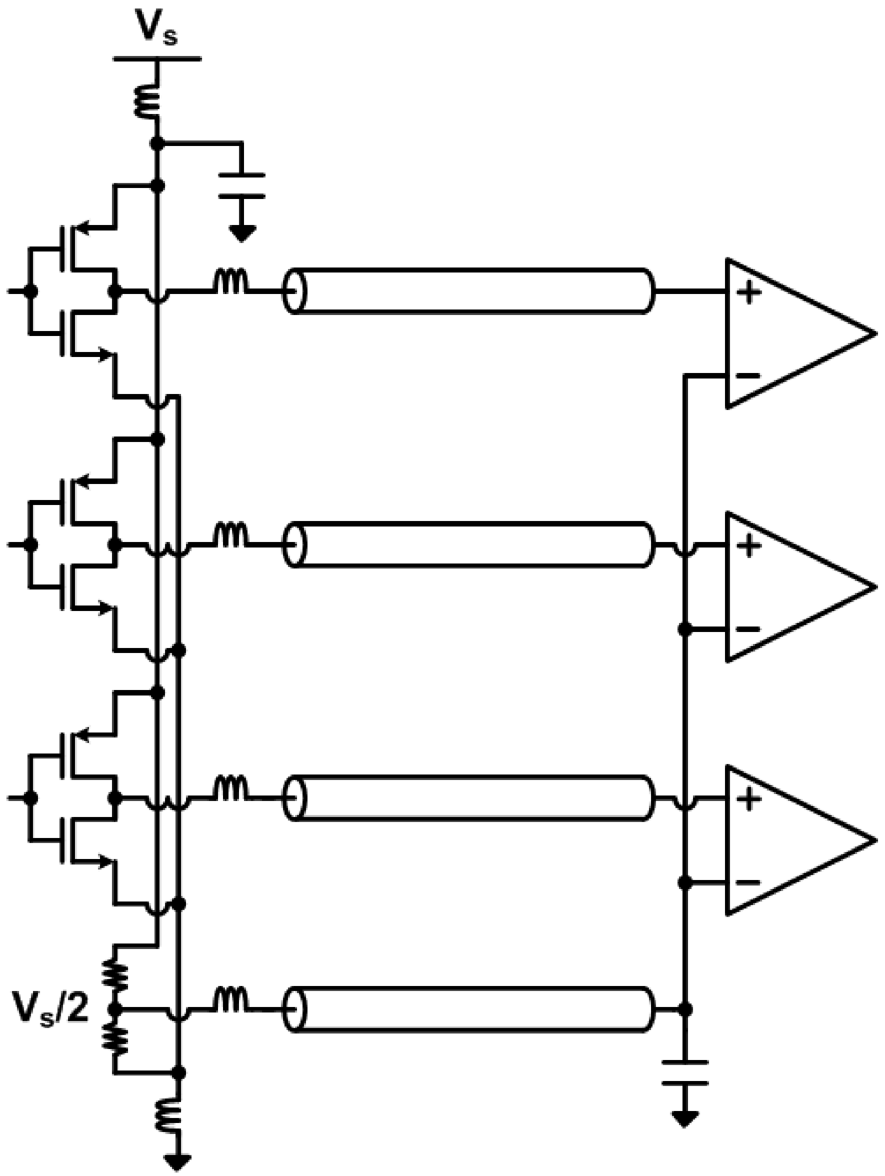
$$\textbf{Delay} \propto \frac{VDD}{VDD - V_{TN}} \approx\propto VDD$$

- CMOS delay is approximately directly proportional to VDD
  - More delay results in more deterministic jitter

Simultaneous Switching Noise

- Finite supply impedance causes significant Simultaneous Switching Output (SSO) noise (xtalk)
- SSO noise is proportional to number of outputs switching, n, and inversely proportional to signal transition time, t_r

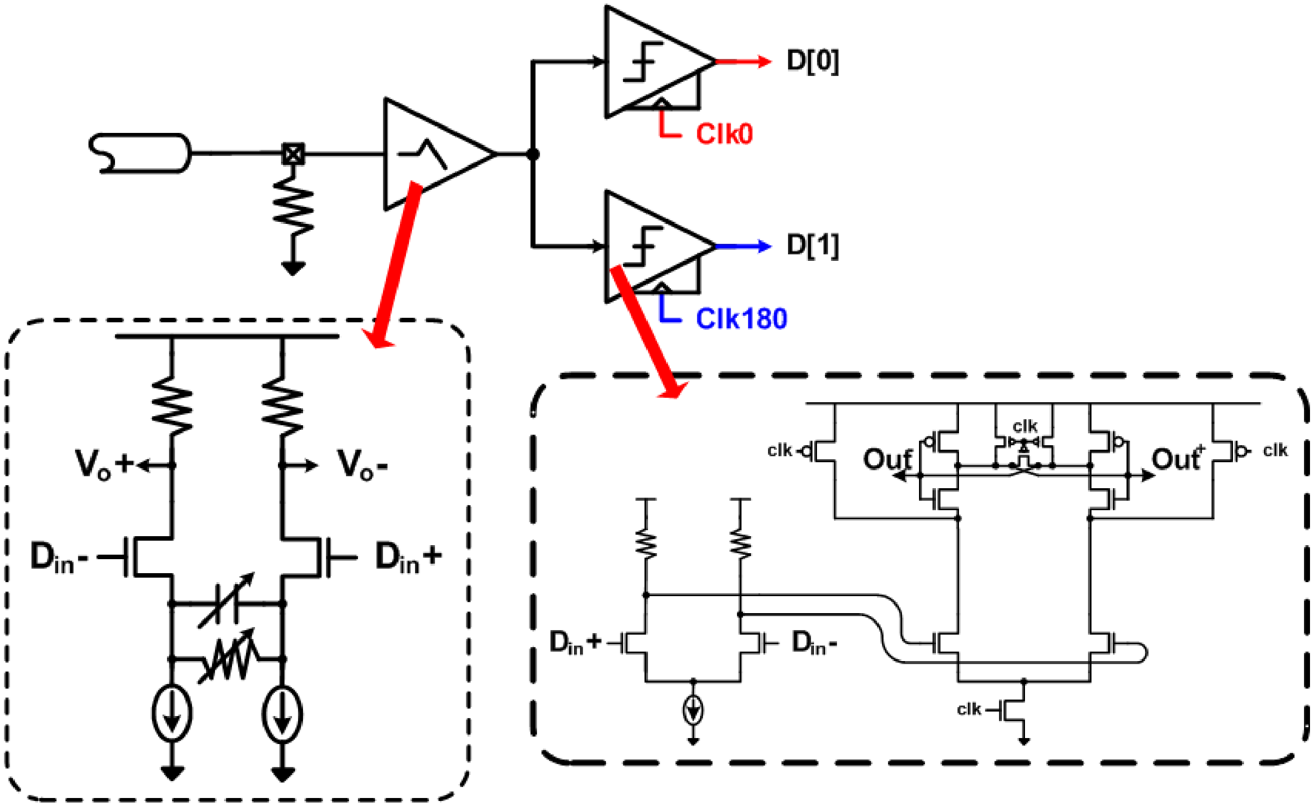
$$V_N = L\frac{i}{t_r} = n\frac{LV_s}{Z_0 t_r}$$



Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

Receiver Input Referred Offset



- The input referred offset is primarily a function of V_{th} mismatch and a weaker function of β (mobility) mismatch

$$\sigma_{V_t} = \frac{A_{V_t}}{\sqrt{WL}}, \quad \sigma_{\Delta\beta/\beta} = \frac{A_\beta}{\sqrt{WL}}$$

Receiver Input Referred Offset

$$\sigma_{V_t} = \frac{A_{V_t}}{\sqrt{WL}}, \quad \sigma_{\Delta\beta/\beta} = \frac{A_\beta}{\sqrt{WL}}$$

- To reduce input offset 2x, we need to increase area 4x
  - Not practical due to excessive area and power consumption
  - Offset correction necessary to efficiently achieve good sensitivity

- Ideally the offset "A" coefficients are given by the design kit and Monte Carlo is performed to extract offset sigma

- If not, here are some common values:
  - A_{Vt} = 1mVμm per nm of t_{ox}
    - For our default 90nm technology, t_{ox}=2.8nm → A_{Vt} ~2.8mVμm
  - A_β is generally near 2%μm

Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

Crosstalk

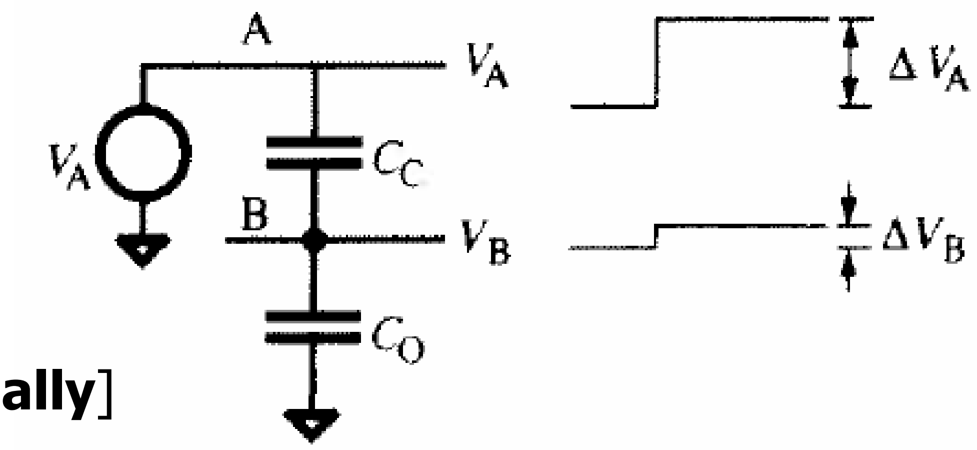
- Crosstalk is noise induced by one signal (aggressor) that interferes with another signal (victim)

- Main crosstalk sources
  - Coupling between on-chip (capacitive) wires
  - Coupling between off-chip (t-line/channel) wires
  - Signal return coupling

- Crosstalk is a proportional noise source
  - Cannot be reduced by scaling signal levels
  - Addressed by using proper signal conventions, improving channel and supply network, and using good circuit design and layout techniques

Crosstalk to Capacitive Lines

- **On-chip wires** have significant capacitance to adjacent wires both on same metal layer and adjacent vertical layers
- Floating victim
  - Examples: Sample-nodes, domino logic
  - When aggressor switches
    - Signal gets coupled to victim via a capacitive voltage divider
    - Signal is not restored



$$\Delta V_B = k_c \Delta V_A$$

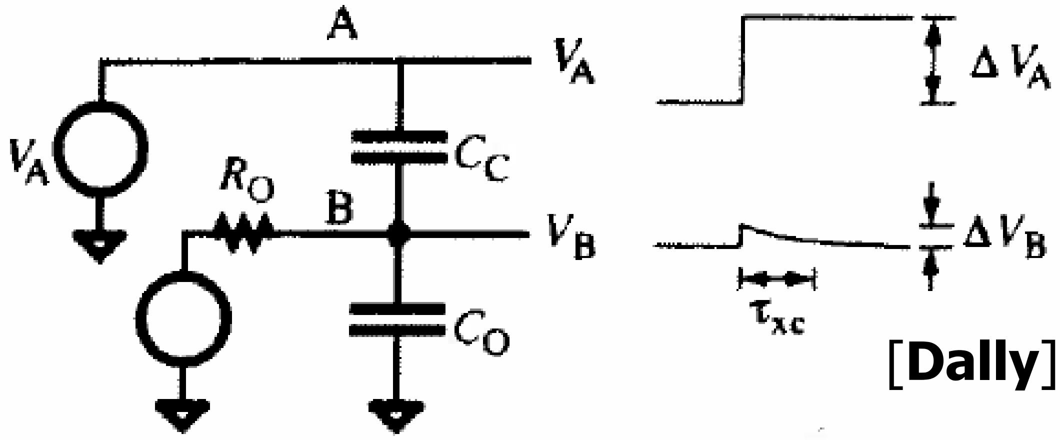
$$k_c = \frac{C_C}{C_C + C_O}$$

[Dally]

Crosstalk to Driven Capacitive Lines

- Crosstalk to a driven line will decay away with a time-constant

$$\tau_{xc} = R_O\left(C_C + C_O\right)$$



[Dally]

- Peak crosstalk is inversely proportional to aggressor transition times, t_r, and driver strength ($1/R_O$)

Ideal Unit Step :

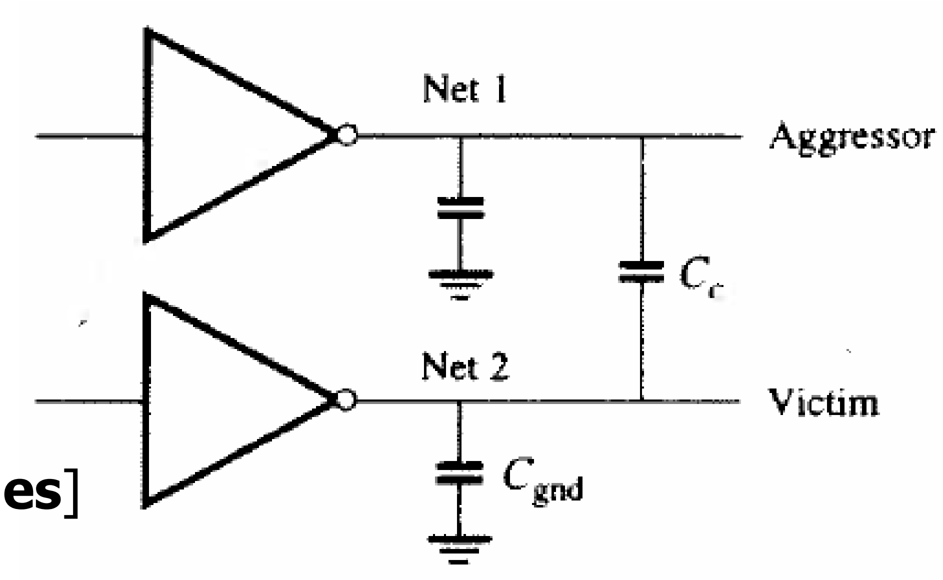
$$\Delta V_B(t) = k_c \exp\left(-\frac{t}{\tau_{xc}}\right)$$

Step with Finite Rise Time, t_r :

$$\Delta V_B(t) = \begin{cases} k_c\left(\frac{\tau_{xc}}{t_r}\right)\left[1 - \exp\left(-\frac{t}{\tau_{xc}}\right)\right] & \textbf{if } t < t_r \\ k_c\left(\frac{\tau_{xc}}{t_r}\right)\left[\exp\left(-\frac{t-t_r}{\tau_{xc}}\right) - \exp\left(-\frac{t}{\tau_{xc}}\right)\right] & \textbf{if } t \geq t_r \end{cases}$$

Capacitive Crosstalk Delay Impact

- Aggressor transitioning near victim transition can modulate the victim's effective load capacitance
- This modulates the victim signal's delay, resulting in deterministic jitter



[**Hodges**]

Aggressor Static : $C_L = C_{gnd} + C_C$

Aggressor Switching Same Way : $C_L = C_{gnd}$

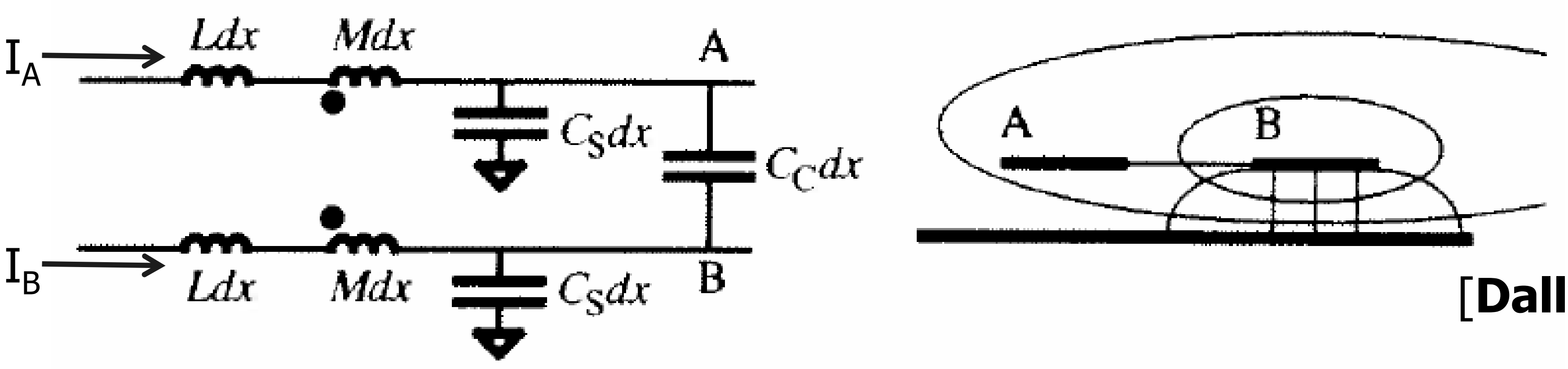
Aggressor Switching Opposite Way : $C_L = C_{gnd} + 2C_C$

Mitigating Capacitive (On-Chip) Crosstalk

- Adjacent vertical metal layers should be routed perpendicular (Manhattan routing)
- Limit maximum parallel routing distance
- Avoid floating signals and use keeper transistors with dynamic logic
- Maximize signal transition time
  - Trade-off with jitter sensitivity
- For differential signals, periodically “twist” routing to make cross-talk common-mode
- Separate sensitive signals
- Use shield wires
- Couple DC signals to appropriate supply

Transmission Line Crosstalk

- 2 coupled lines:



[Dally]

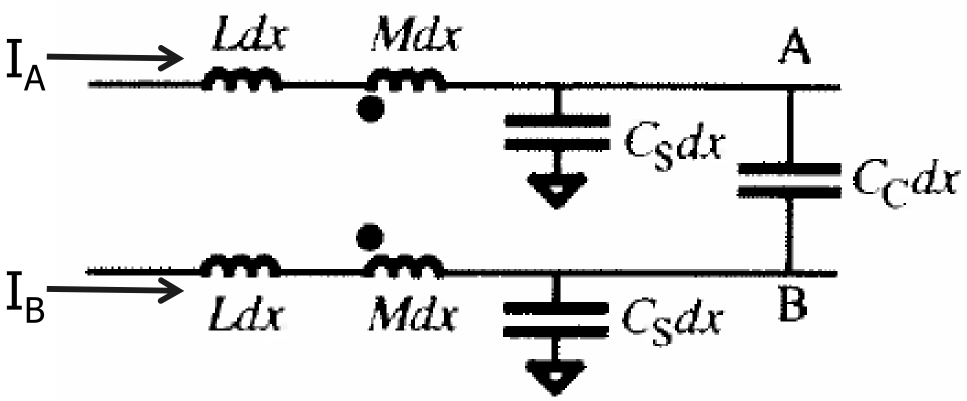
- Transient voltage signal on A is coupled to B capacitively

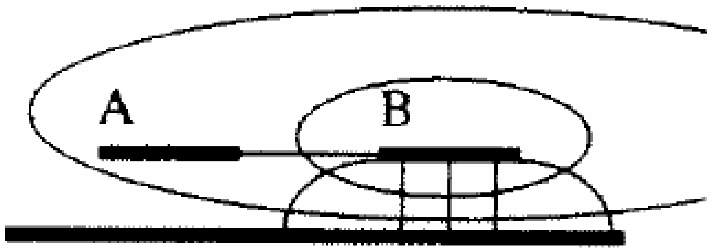
$$\frac{dV_B(x,t)}{dt} = k_{cx}\frac{dV_A(x,t)}{dt} \quad \text{where} \quad \boxed{k_{cx} = \frac{C_C}{C_S + C_C}}$$

- Capacitive coupling sends half the coupled energy in each direction with equal polarity

Transmission Line Crosstalk

- 2 coupled lines:





[Dally]

- Transient current signal on A is coupled to B through mutual inductance

$$\frac{\partial I_A(x,t)}{\partial t} = -\frac{\partial V_A(x,t)}{L\partial x}$$

$$\frac{dV_B(x,t)}{dx} = -M\frac{dI_A(x,t)}{dt} = \frac{M}{L}\left[\frac{dV_A(x,t)}{dx}\right] = k_{lx}\frac{dV_A(x,t)}{dx} \quad \text{where} \quad \boxed{k_{lx} = \frac{M}{L}}$$

- Inductive coupling sends half the coupled energy in each direction with a negative forward traveling wave and a positive reverse traveling wave

Near- and Far-End Crosstalk

[Hall]

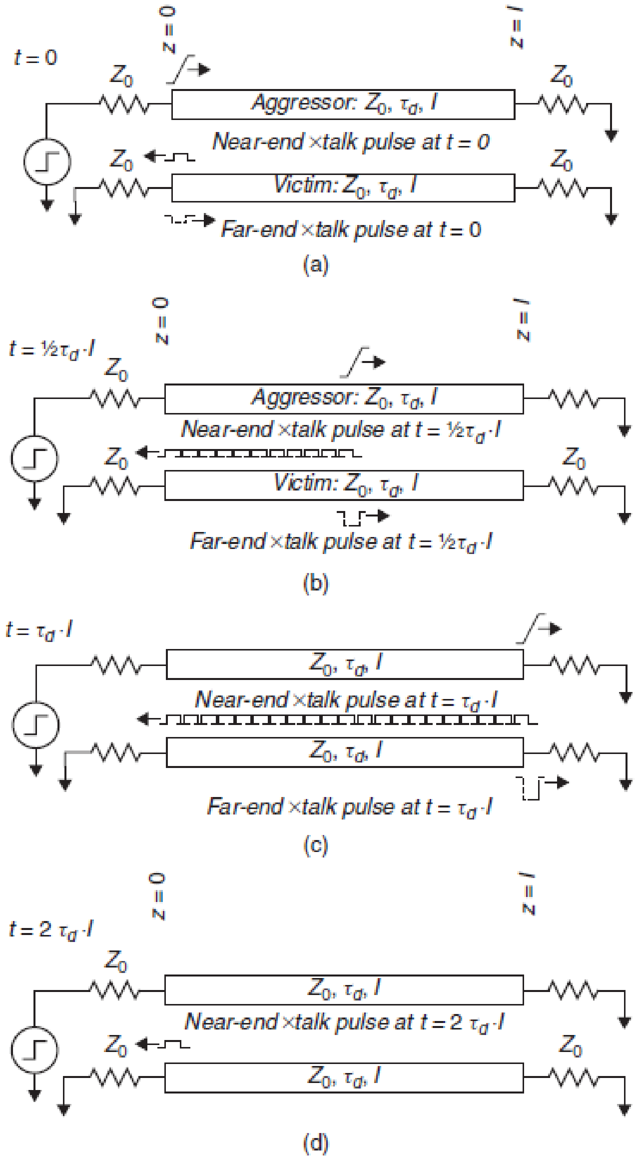
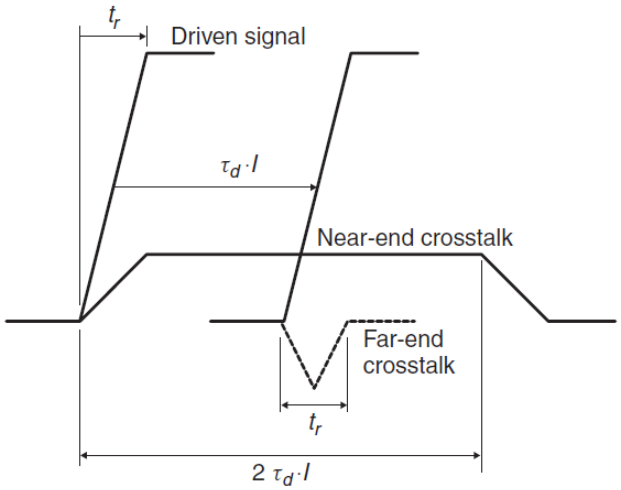
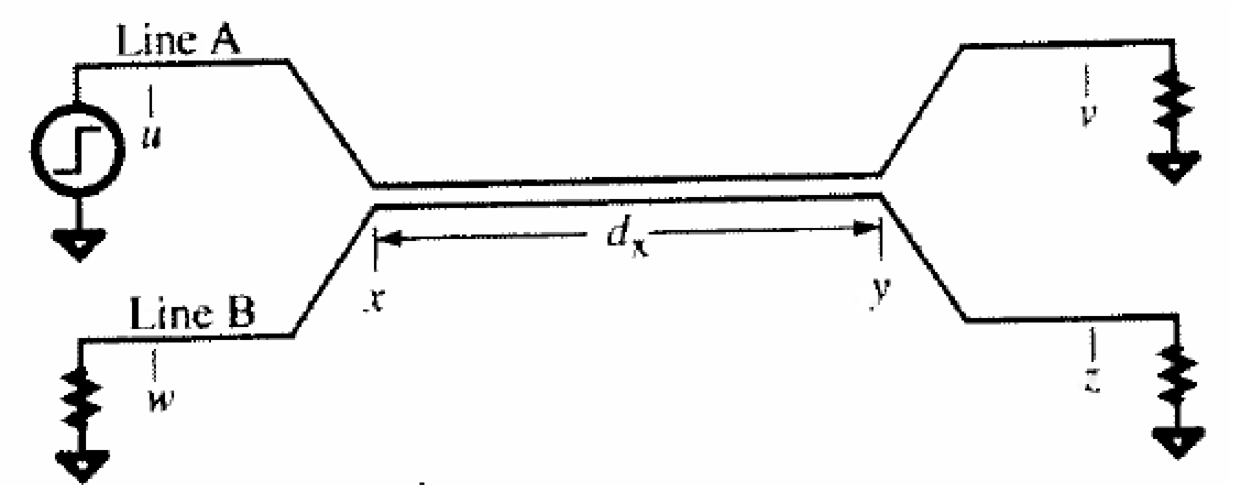


Figure 4-22 Summary of propagation of forward- and backward-coupled noise: (a) initial wave launch; (b) halfway down the line; (c) one full trip down the line; (d) round trip.



- Near-end crosstalk (NEXT) is immediately observed starting at the aggressor transition time and continuing for a round-trip delay
- Due to the capacitive and inductive coupling terms having the same polarity, the NEXT signal will have the same polarity as the aggressor
- Far-end crosstalk (FEXT) propagates along the victim channel with the incident signal and is only observed once
- Due to the capacitive and inductive coupling terms having the opposite polarity, the FEXT signal can have either polarity, and in a homogeneous medium (stripline) cancel out

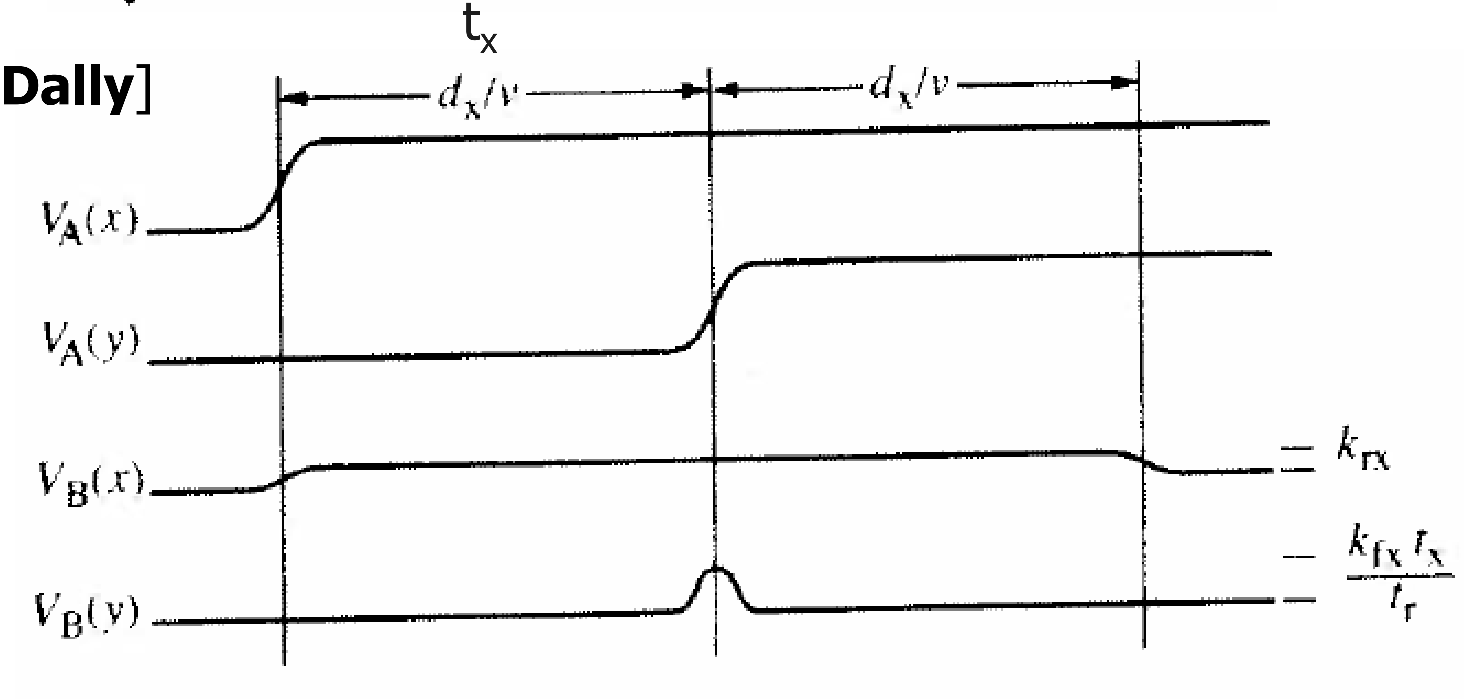
Near- and Far-End Crosstalk



Reverse Coupling Coefficient
k_{rx} (NEXT)

Forward Coupling Coefficient
k_{fx} (FEXT)

[**Dally**]

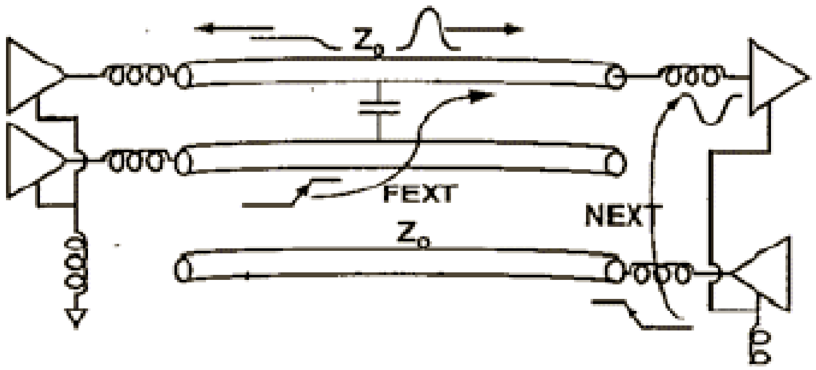


$$\boxed{\begin{aligned} k_{rx} &= \frac{(k_{cx} + k_{lx})}{4} \\ k_{fx} &= \frac{(k_{cx} - k_{lx})}{2} \end{aligned}}$$

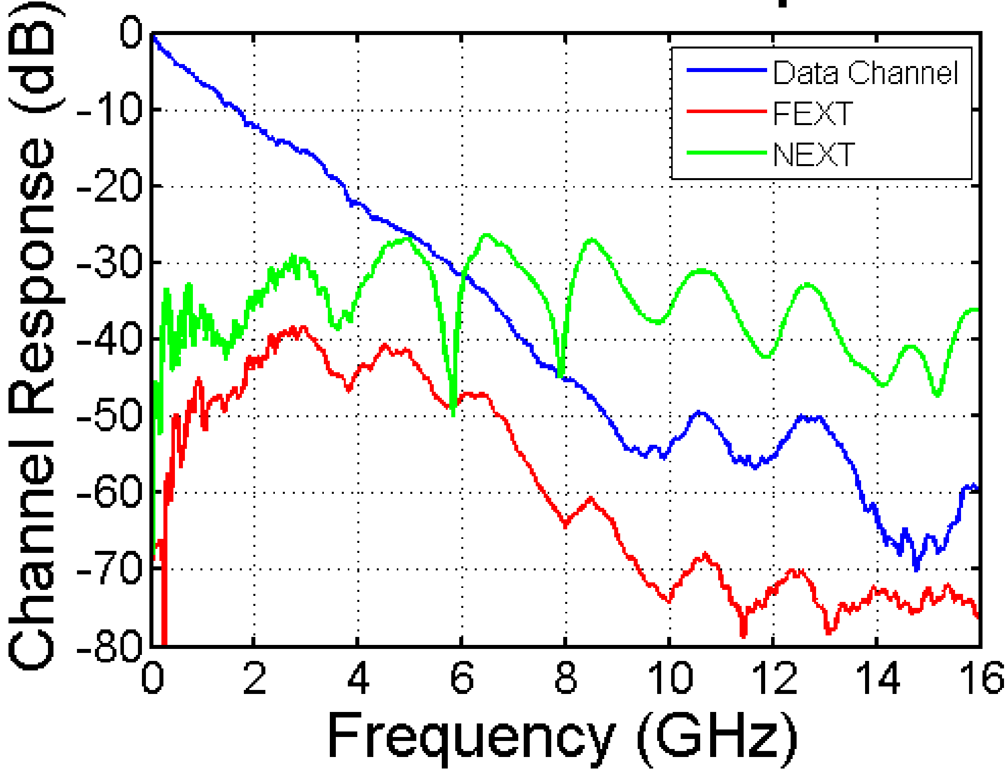
For derivation of k_{rx} and k_{fx}, see Dally 6.3.2.3

Off-Chip Crosstalk

- Occurs mostly in package and board-to-board connectors
- FEXT is attenuated by channel response and has band-pass characteristic
- NEXT directly couples into victim and has high-pass characteristic

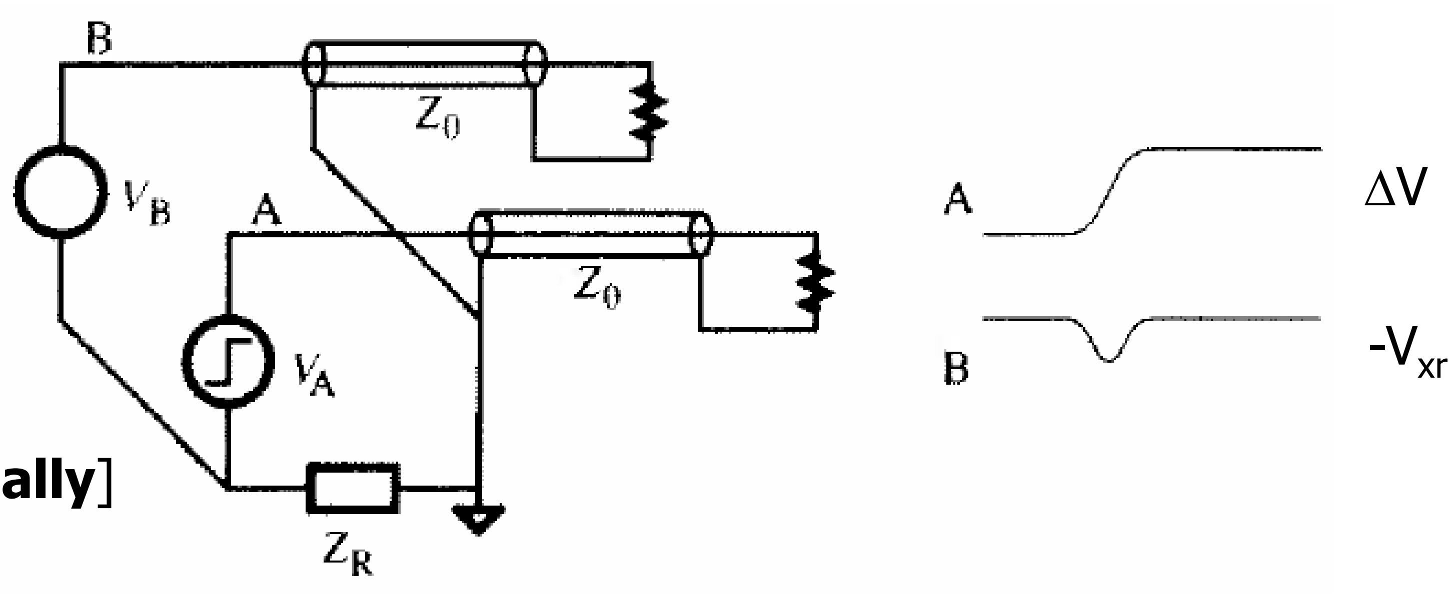


17" Server Channel Response



Signal Return Crosstalk

- Shared return path with finite impedance
- Return currents induce crosstalk occurs among signals



[**Dally**]

$$\text{Return Crosstalk Voltage}: V_{xr} = \Delta V \frac{Z_R}{Z_0} = k_{xr} \Delta V$$

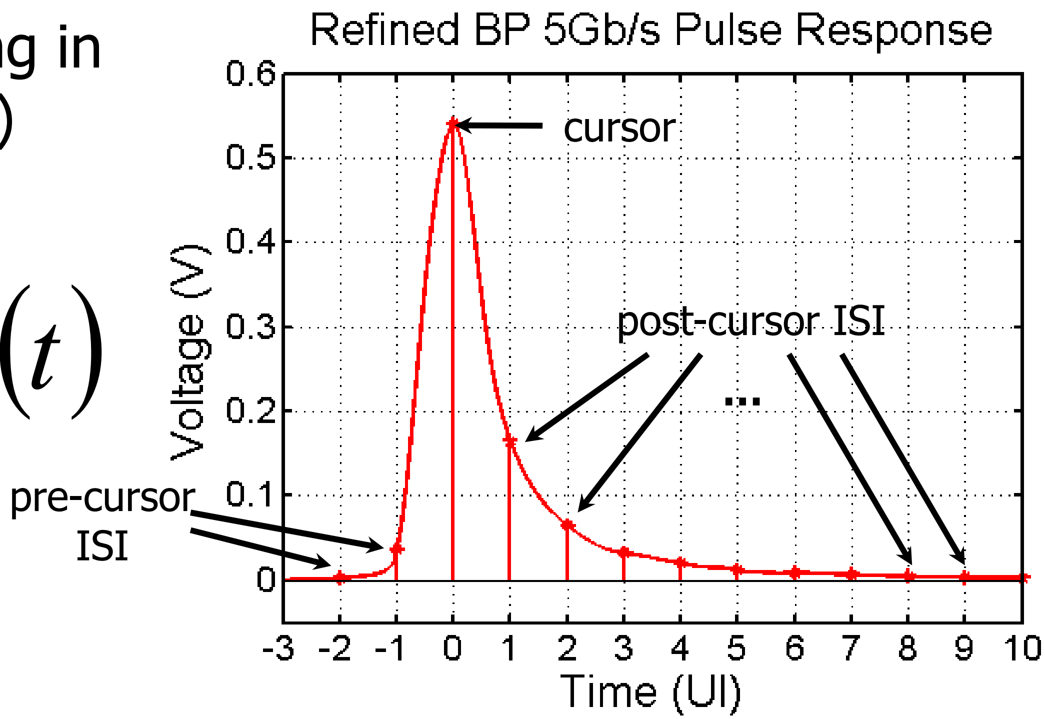
Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)

$$y^{(d_k)}(t)=c^{(d_k)}(t)*h(t)$$



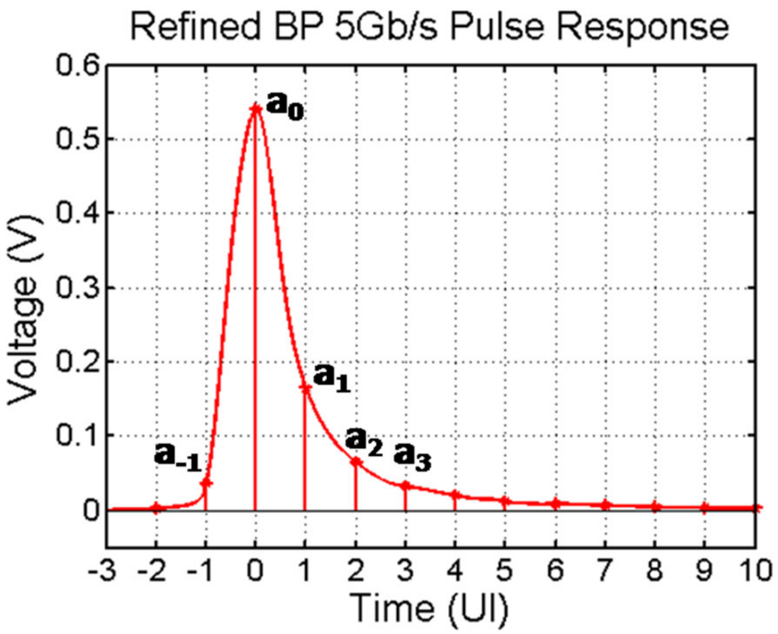
$y^{(1)}(t)$ sampled relative to pulse peak:

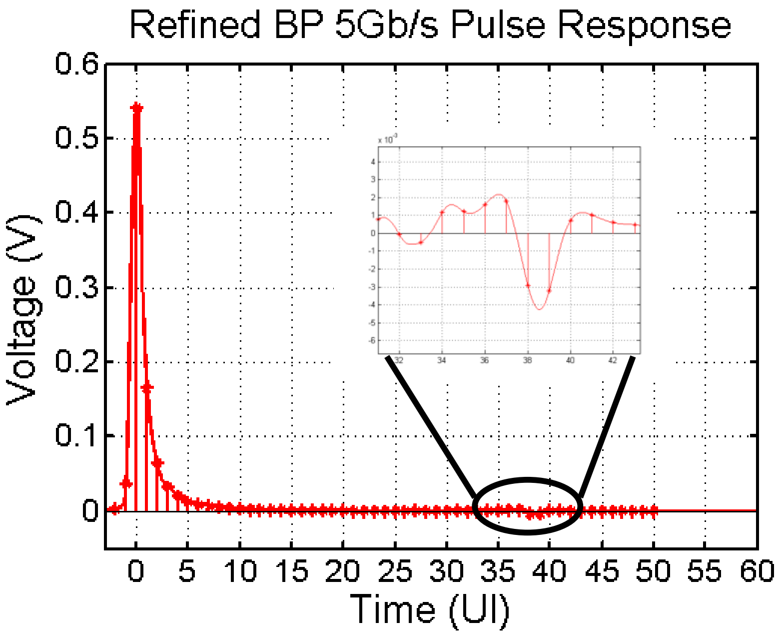
[… 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 …]

k =[… -2 1 0 1 2 3 4 5 6 …]

By Linearity: $y^{(0)}(t) = -1*y^{(1)}(t)$

Peak Distortion Analysis Example



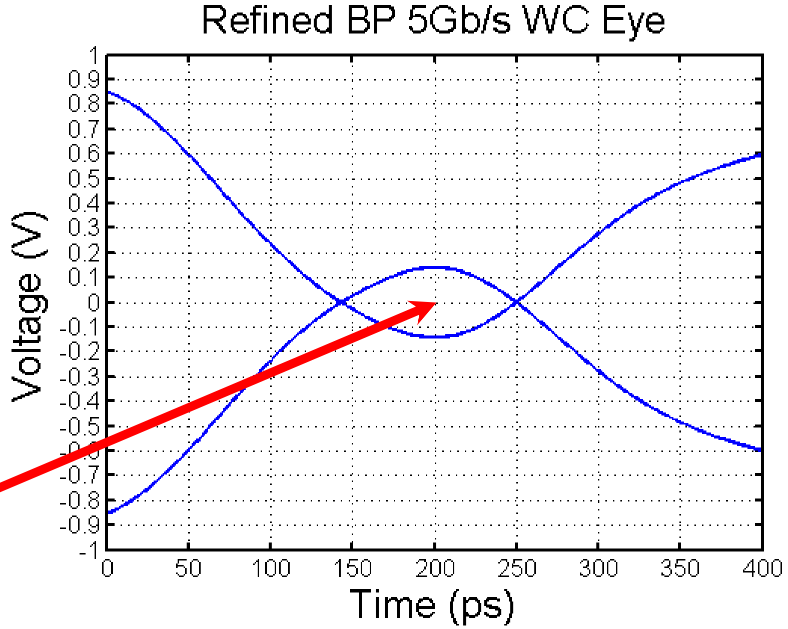


$$y_0^{(1)}(t) = 0.540$$

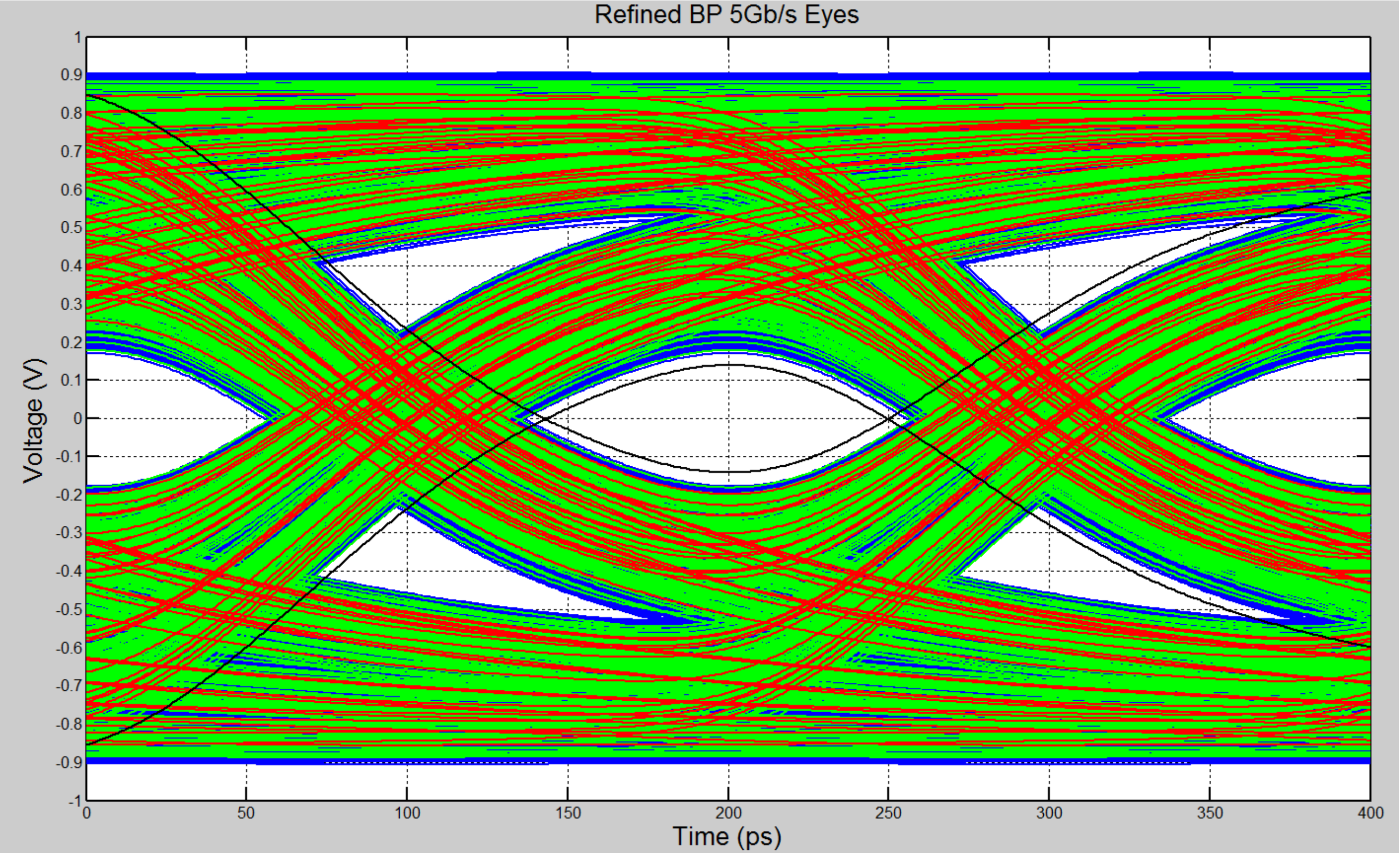
$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



Worst-Case Eye vs Random Data Eye

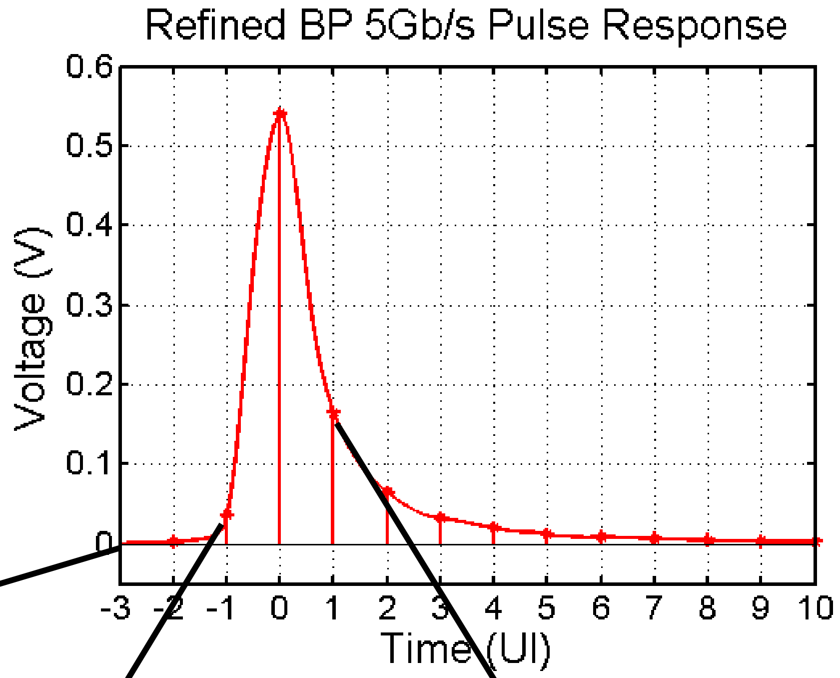


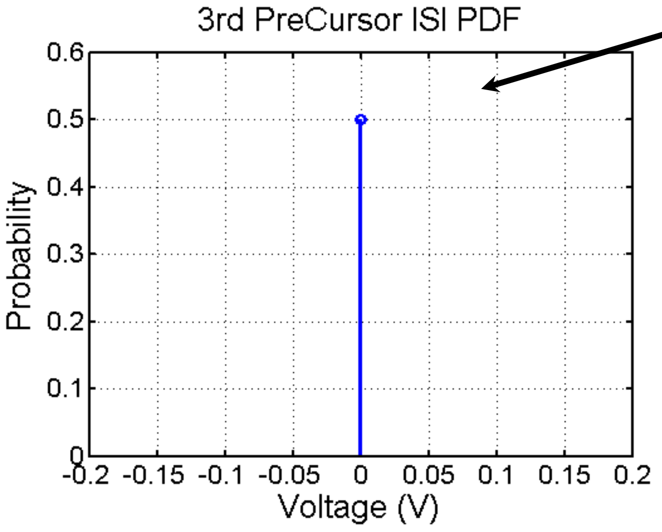
Worst-Case Eye
100 Random Bits
1000 Random Bits
1e4 Random Bits

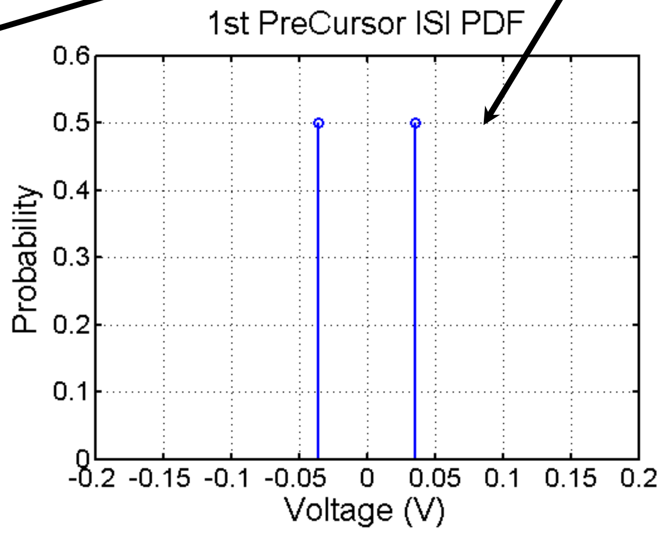
- Worst-case data pattern can occur at very low probability!
- Considering worst-case is too pessimistic

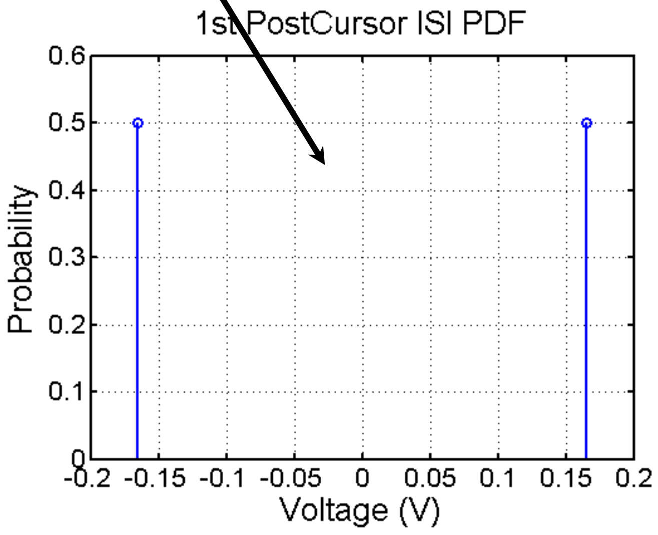
Constructing ISI Probability Density Function (PDF)

- Using ISI probability density function will yield a more accurate BER performance estimate
- In order to construct the total ISI PDF, need to convolve all of the individual ISI term PDFs together
    - 50% probability of "1" symbol ISI and "-1" symbol ISI

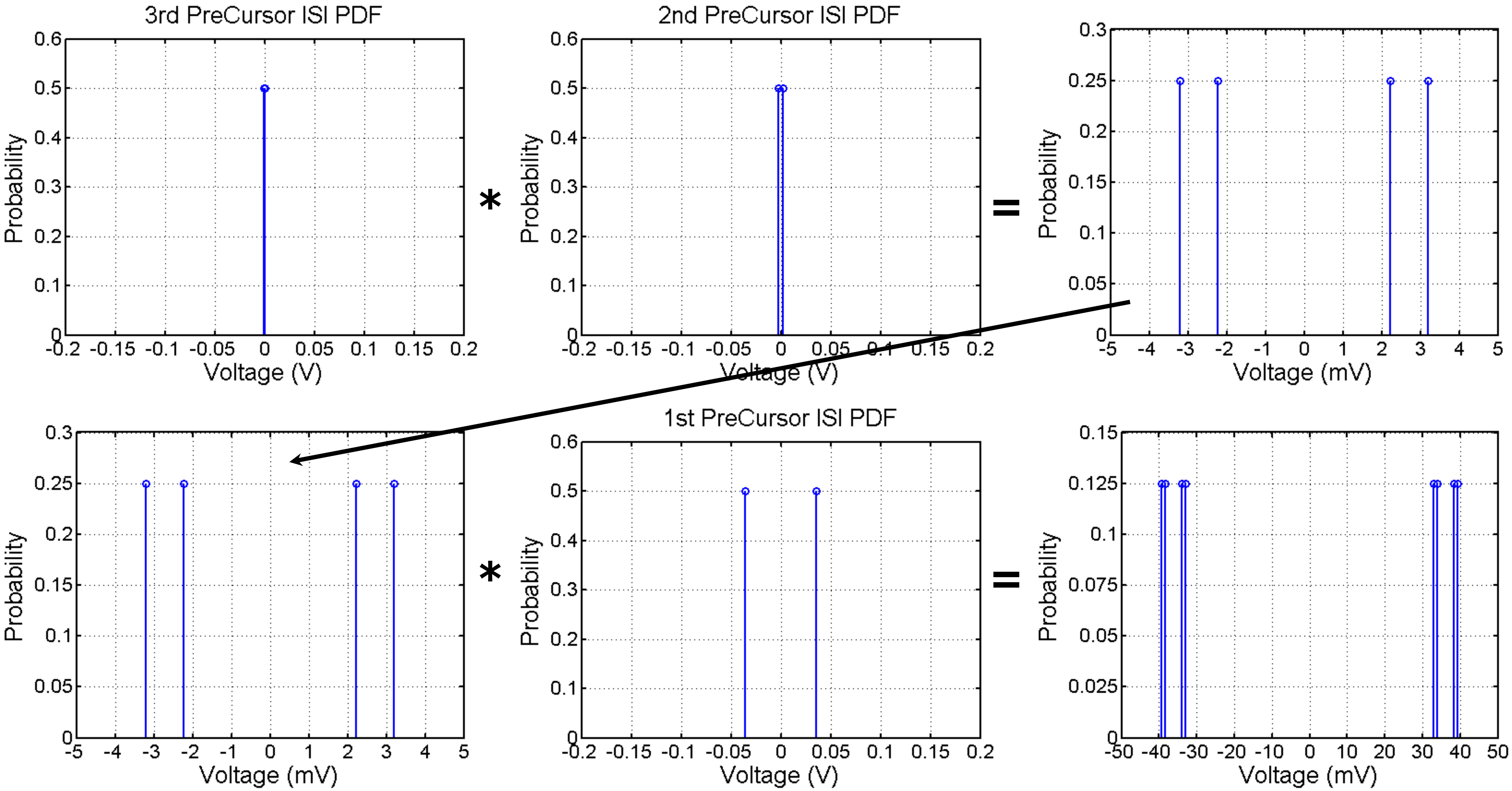






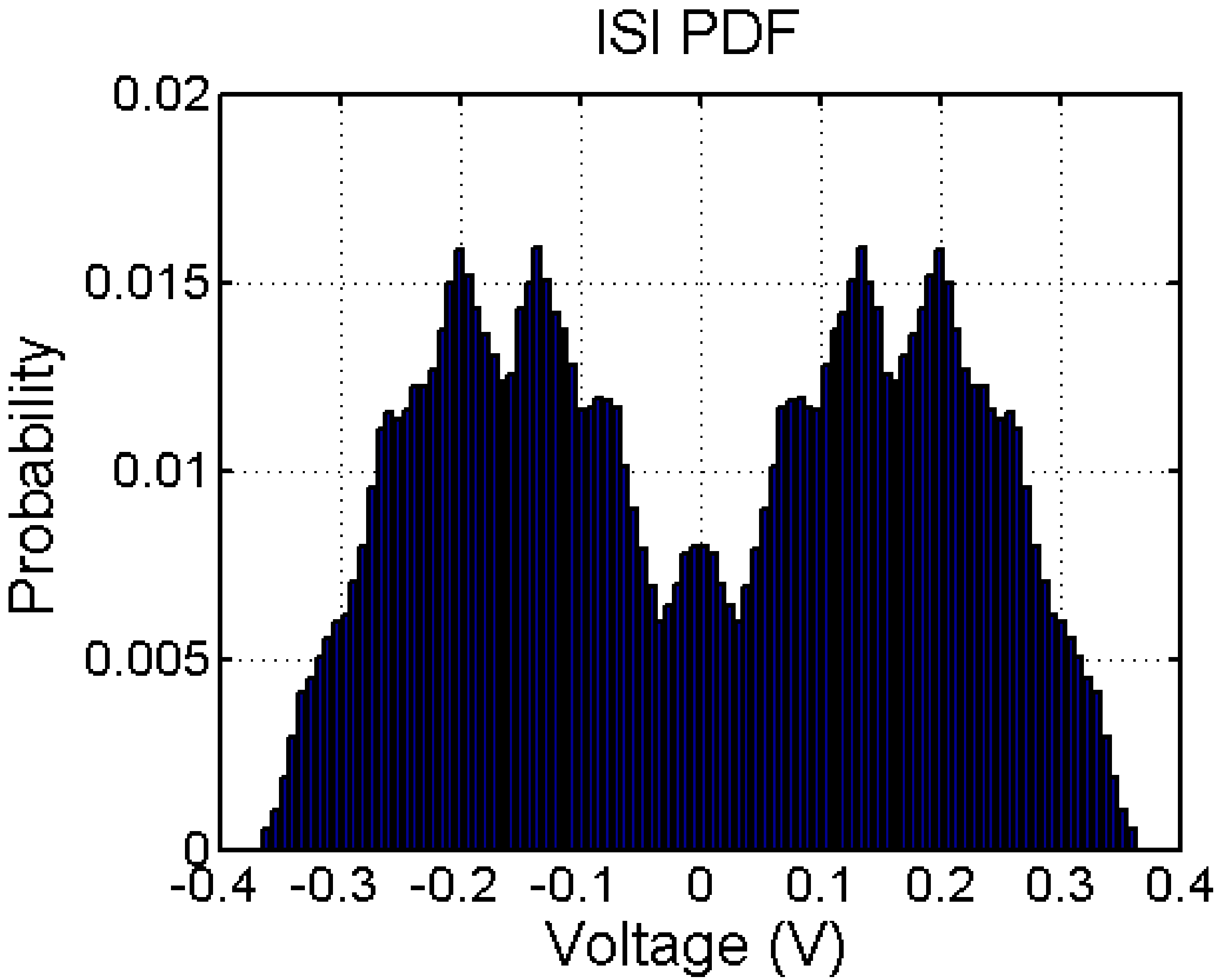


Convolving Individual ISI PDFs Together

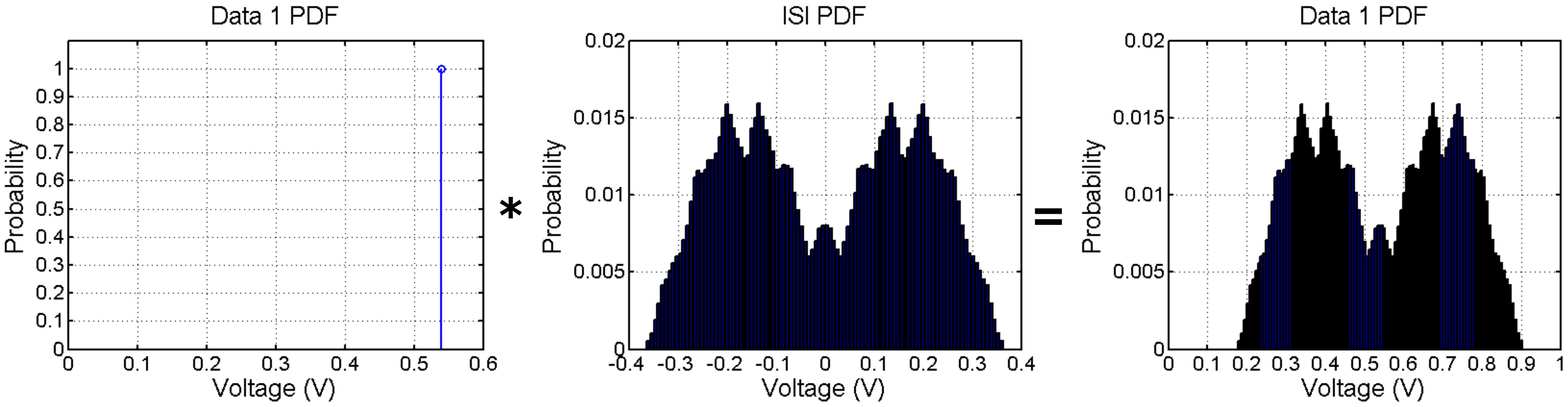


- Keep going until all individual PDFs convolved together

Complete ISI PDF



Cursor PDF – Data 1

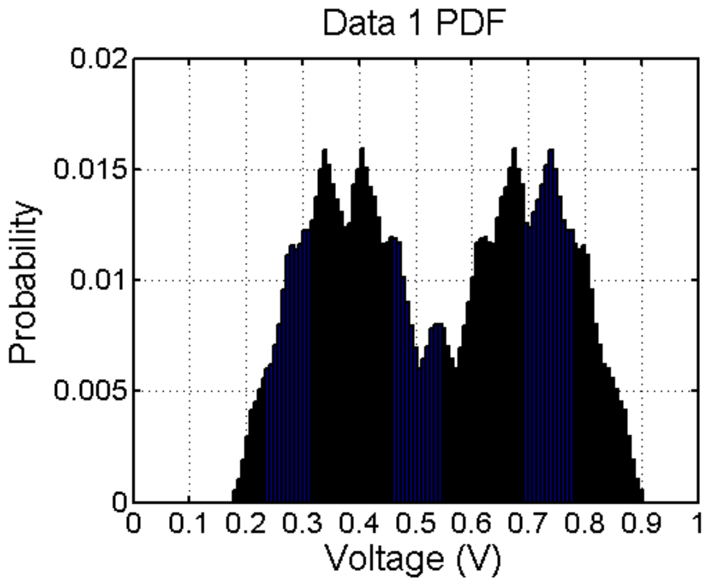


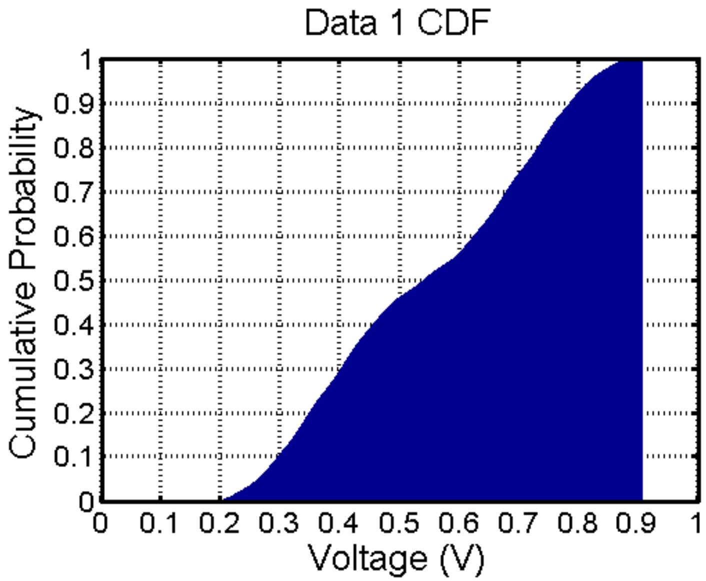
- Data 1 PDF is centered about the cursor value and varies from a maximum positive value to the worst-case value predicted by PDA
  - This worst-case value occurs at a low probability!

Cursor Cumulative Distribution Function (CDF)

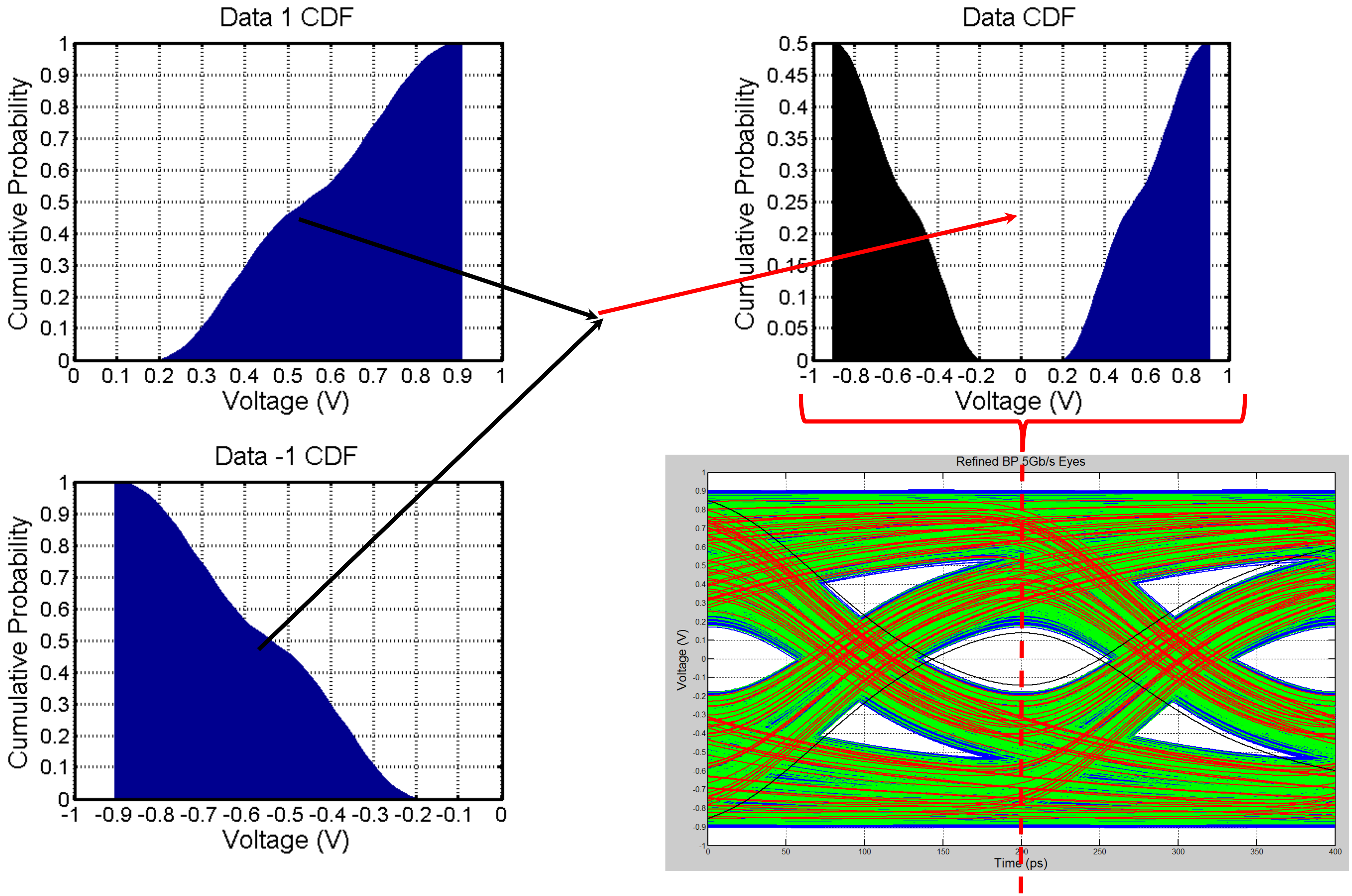
- For a given offset, what is the probability of a Data 1 error?
  - Data 1 error probability for a given offset is equal to the Data 1 CDF

$$BER(X)=\int_{-\infty}^{X}(PDF)\,dx$$



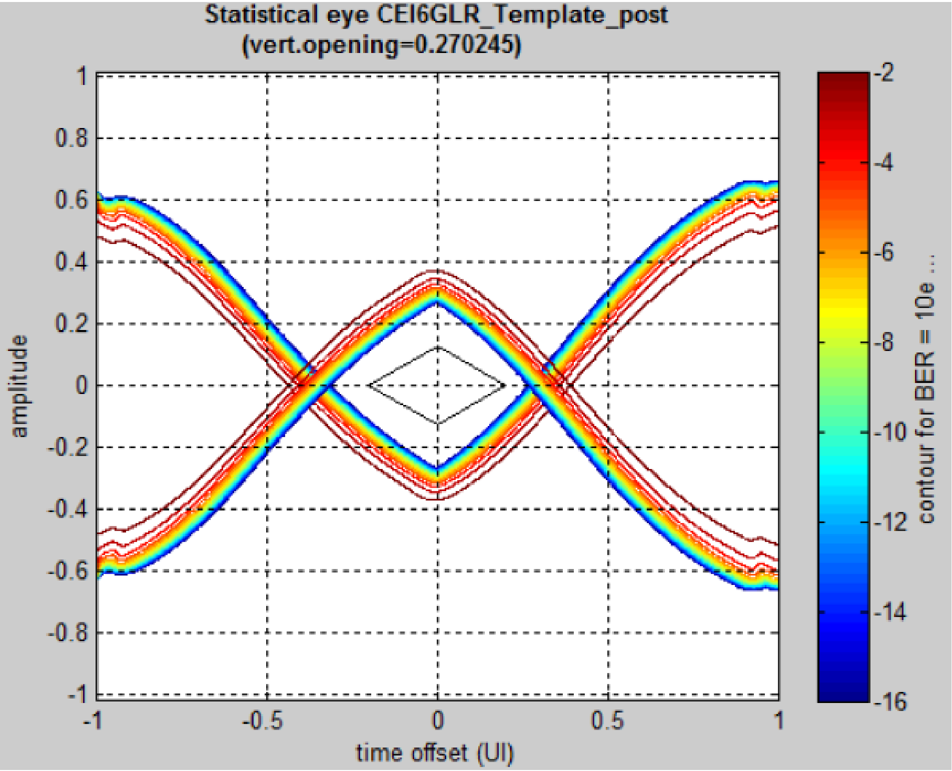


Combining Cursor CDFs



Bit-Error-Rate (BER) Distribution Eye

- Statistical BER analysis tools use this technique to account for ISI distribution and also other noise sources
  - Example from Stateye
    - Note: Different channel & data rate from previous slides



Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

Random Noise

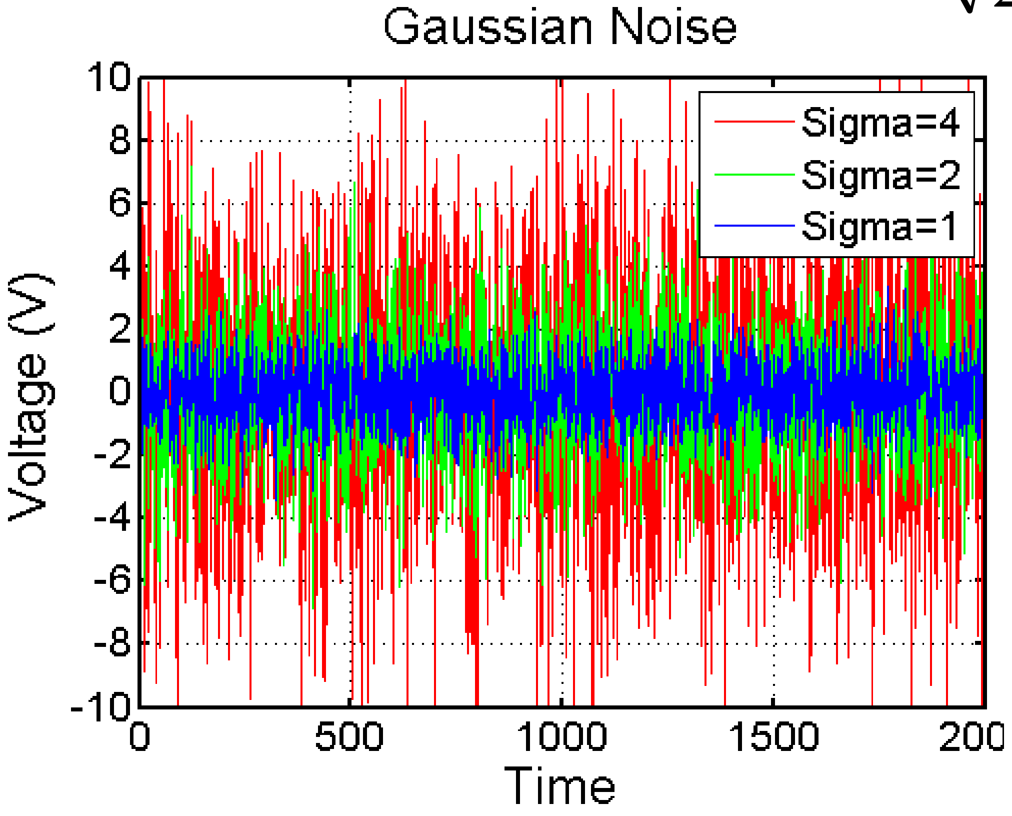
- Random noise is unbounded and modeled statistically
  - Example: Circuit thermal and shot noise
- Modeled as a continuous random variable described by
  - Probability density function (PDF)
  - Mean, μ
  - Standard deviation, σ

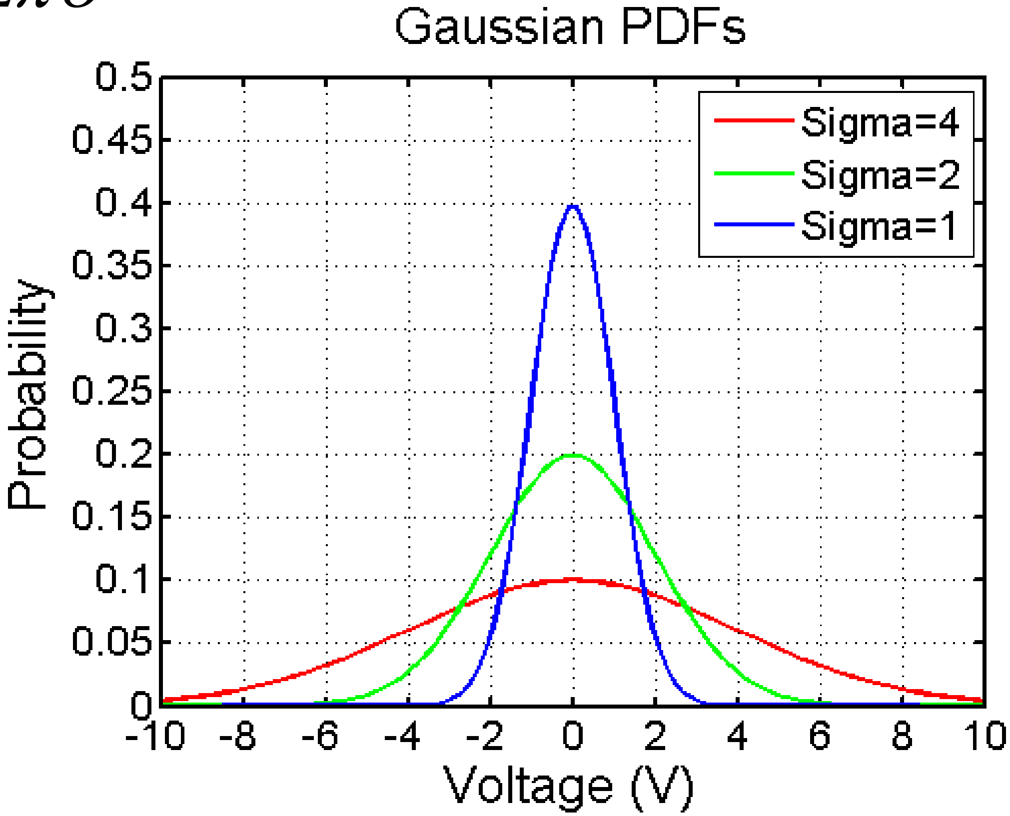
$$PDF = P_n(x), \quad \mu_n = \int_{-\infty}^{\infty} x P_n(x) dx, \quad \sigma_n^2 = \int_{-\infty}^{\infty} (x - \mu_n)^2 P_n(x) dx$$

Gaussian Distribution

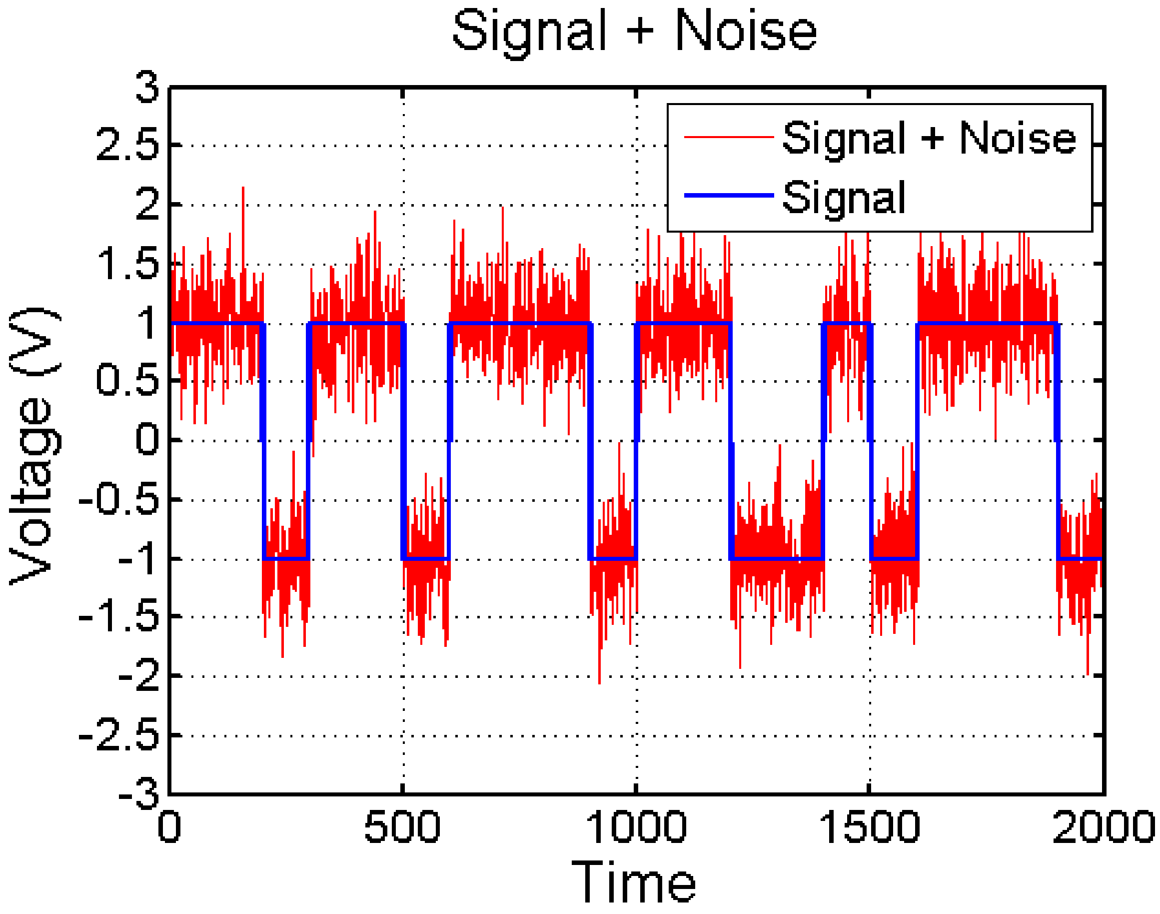
- Gaussian distribution is normally assumed for random noise
  - Larger sigma value results in increased distribution spread

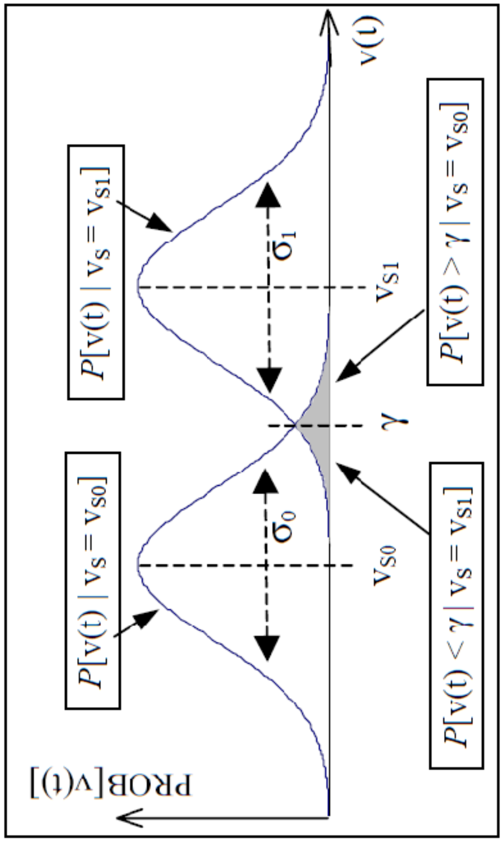
$$P_n(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu_n)^2}{2\sigma^2}}$$





Signal with Added Gaussian Noise

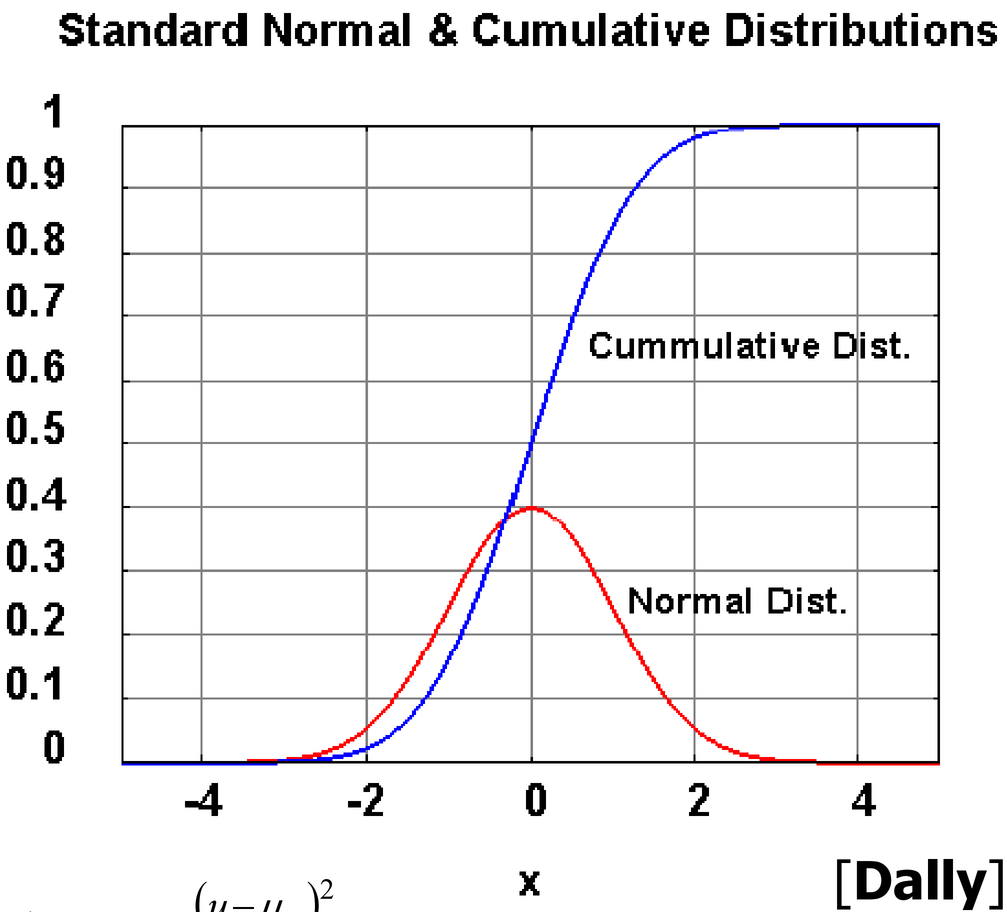




- Finite probability of noise pushing signal past threshold to yield an error

Cumulative Distribution Function (CDF)

- The CDF tells what is the probability that the noise signal **is less than or equal to** a certain value



[Dally]

$$\Phi_n(x) = \int_{u=-\infty}^{x} P_n(u)du = \int_{u=-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u-\mu_n)^2}{2\sigma^2}} du$$

Error and Complimentary Error Functions

- Error Function:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{u=0}^{x} \exp(-u^2)du$$

- Relationship between normal CDF (0,1) and Error Function:

$$\Phi(x) = \frac{1}{2}\left[1 + erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

- The complementary error function gives the probability that the noise will exceed a given value

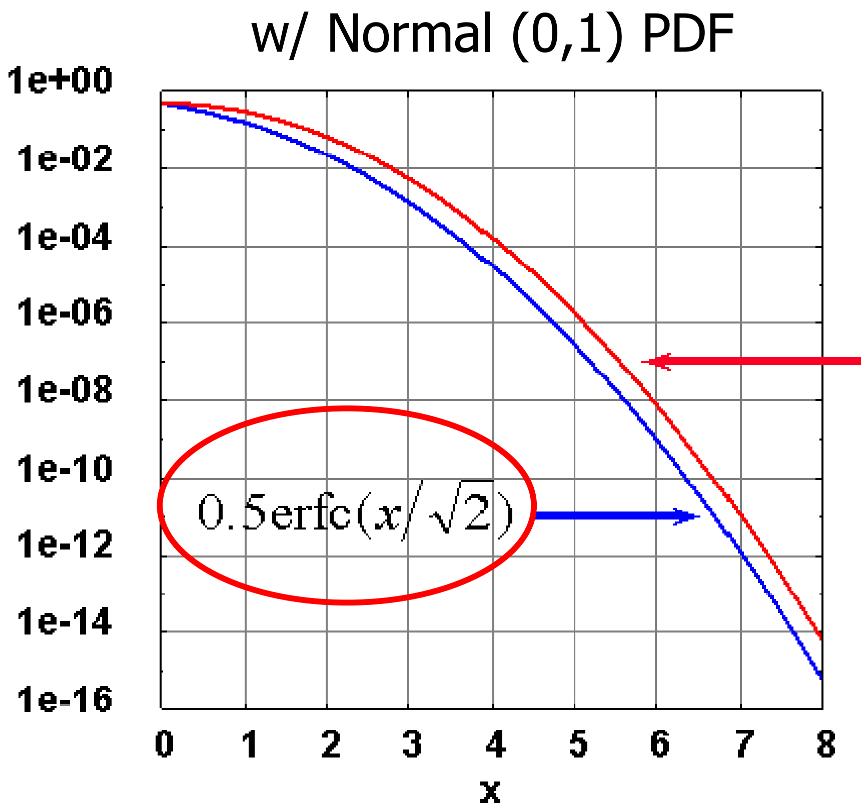
$$Q(x) = 1 - \Phi(x) = \frac{1}{2}\left[1 - erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$$

$$Q_{\mu\sigma}(x) == \frac{1}{2} erfc\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)$$

Bit Error Rate (BER)

- Using erfc to predict BER:



$0.5\,\mathrm{erfc}(x/\sqrt{2})$

Conservative Upper-Bound Approximation

$0.5\exp\left(-x^2/2\right)$

[**Dally**]

- Need a symbol of about 7σ for BER=10^{-12}
  - Peak-to-peak value will be 2x this

Noise Source Classifications

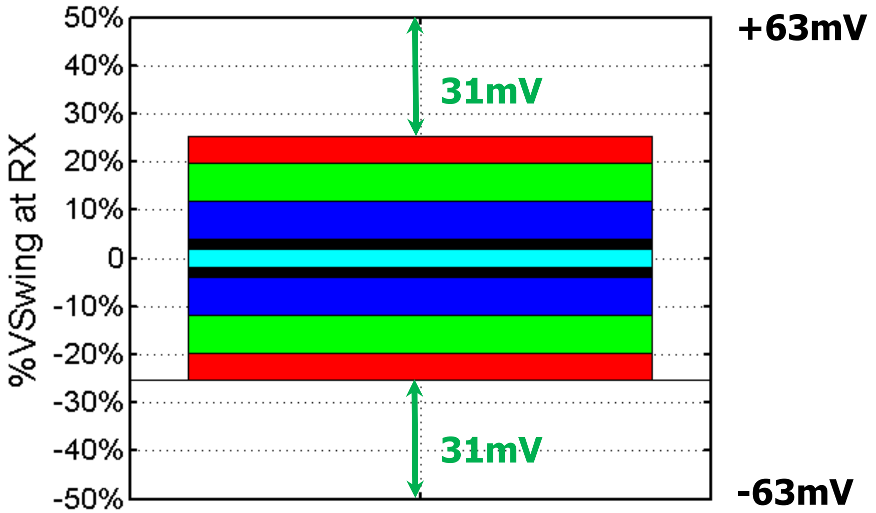
- Determining whether noise source is
  - Proportional vs Independent
  - Bounded vs Statistical
- is important in noise budgeting

	Proportional	Independent
Bounded	Residual ISI Crosstalk	RX Offset RX Sensitivity Power Supply Noise
Statistical	Large-Channel Crosstalk	Random Noise

Noise Budget Example

- Peak TX differential swing of 400mV$_{ppd}$ equalized down 10dB
  - ±200mV → ±63mV

Parameter	**K_n**	**RMS**	**Value (BER=10^{-12})**
Peak Differential Swing			0.4V
RX Offset + Sensitivity			5mV
Power Supply Noise			5mV
Residual ISI	0.05		20mV
Crosstalk	0.05		20mV
Random Noise		1mV	14mV
Attenuation	10dB = 0.684		0.274V
Total Noise			0.338V
Differential Eye Height Margin			**62mV**



- Conservative analysis
  - Assumes all distributions combine at worst-case
- Better technique is to use statistical BER link simulators

Next Time

- Timing Noise
- BER Analysis Techniques