ECEN689: Special Topics in High-Speed Links Circuits and Systems
Spring 2010

Lecture 8: Channel Pulse Model

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Announcements

- Reading
  - Will post some papers of link examples with different modulation techniques

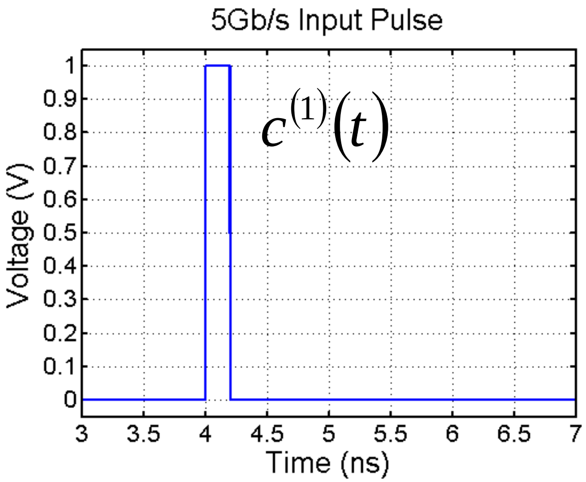
Agenda

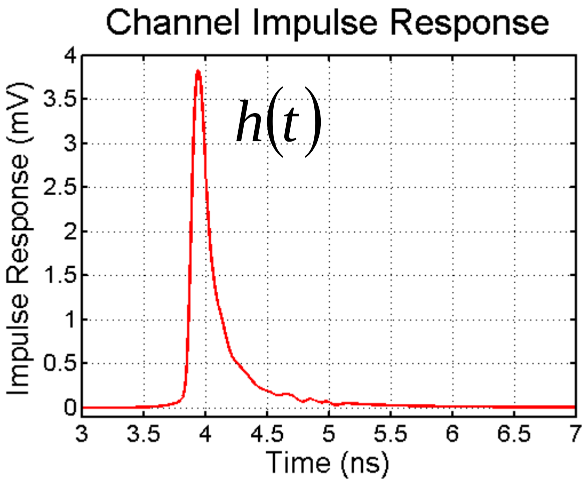
- ISI
- Channel Pulse Model
- Peak Distortion Analysis
- Modulation Schemes

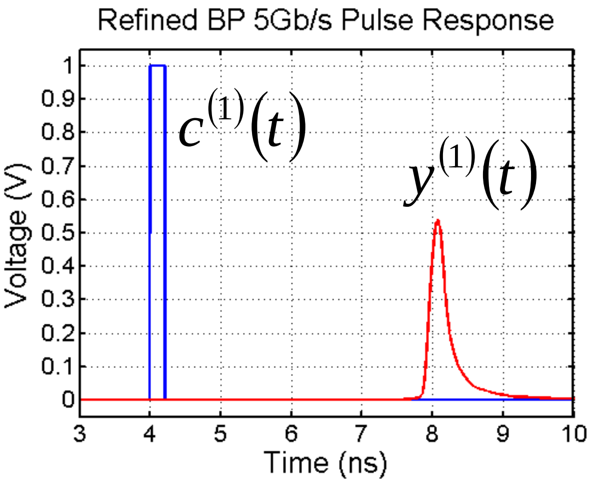
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

$$y^{(1)}(t) = c^{(1)}(t) * h(t)$$



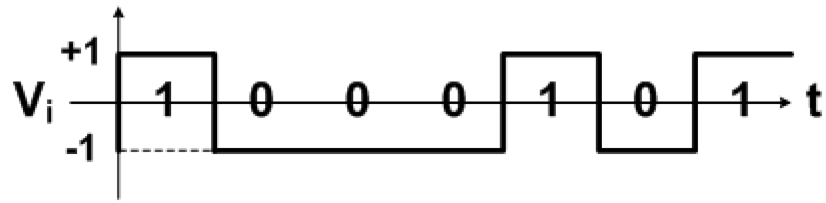




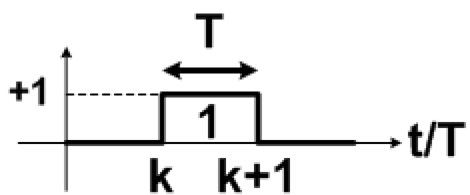
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated “1”s and “0”s

Data = “1000101”

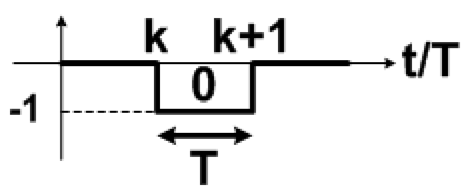


“1” Symbol



$$c_k^{(1)}(t) \equiv u(t - kT) - u(t - (k+1)T)$$

“0” Symbol

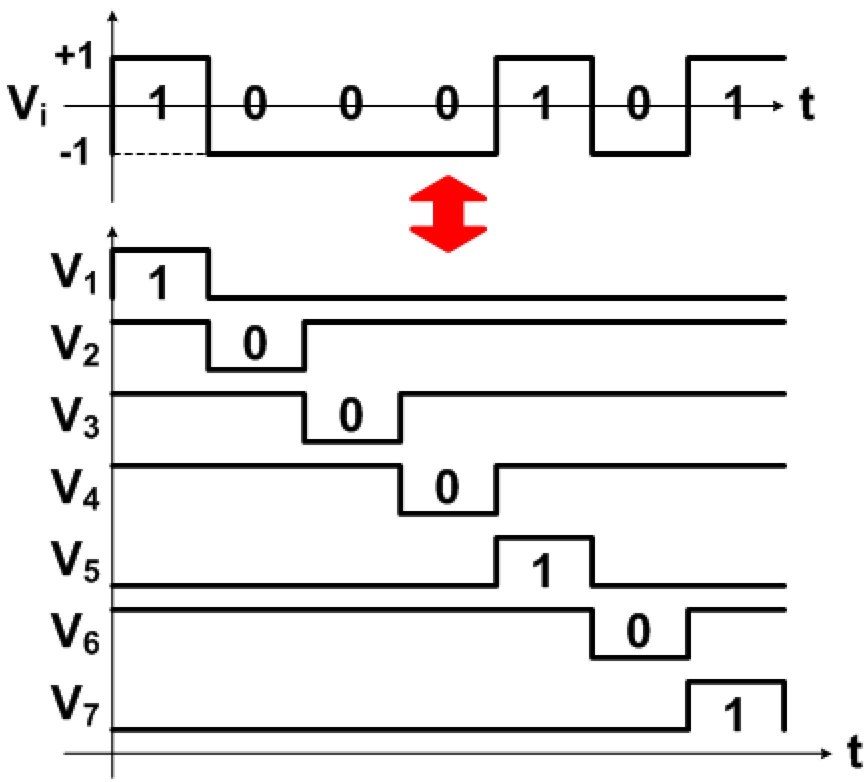


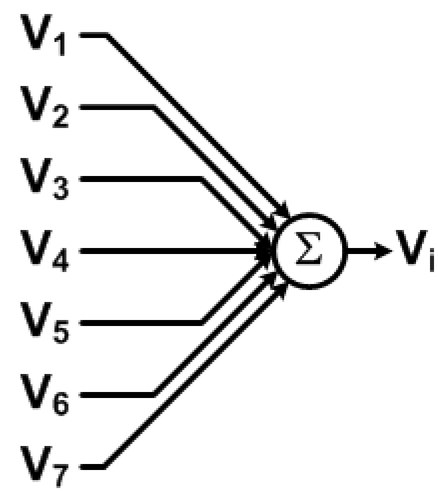
$$c_k^{(0)}(t) = -c_k^{(1)}(t)$$

$$\text{where} \quad u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated “1”s and “0”s

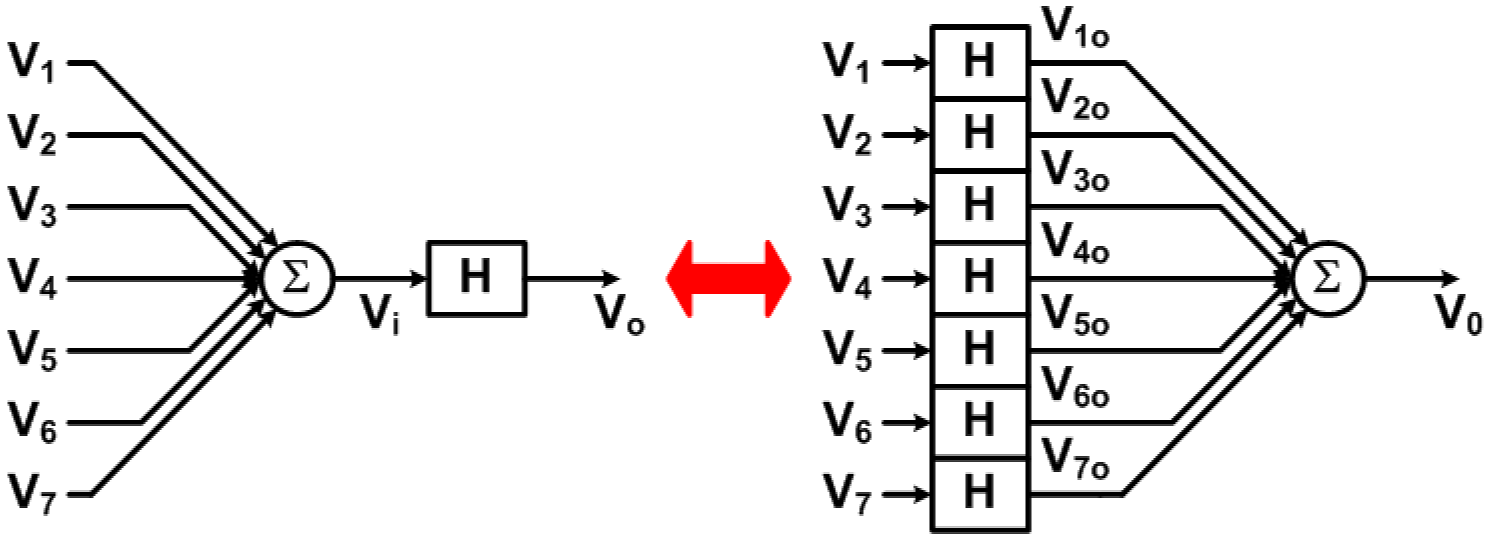




$$V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t)$$

Channel Response to NRZ Data

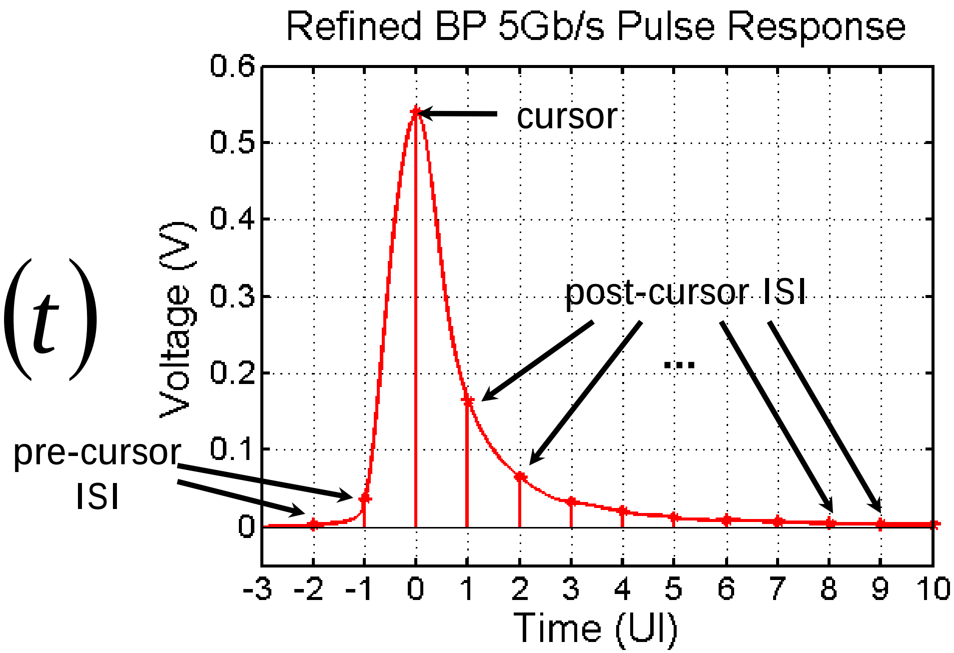
- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses



$$V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H\left(c_k^{(d_k)}(t)\right) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)$$

Channel Pulse Response

$$y^{(d_k)}(t) = c^{(d_k)}(t) * h(t)$$



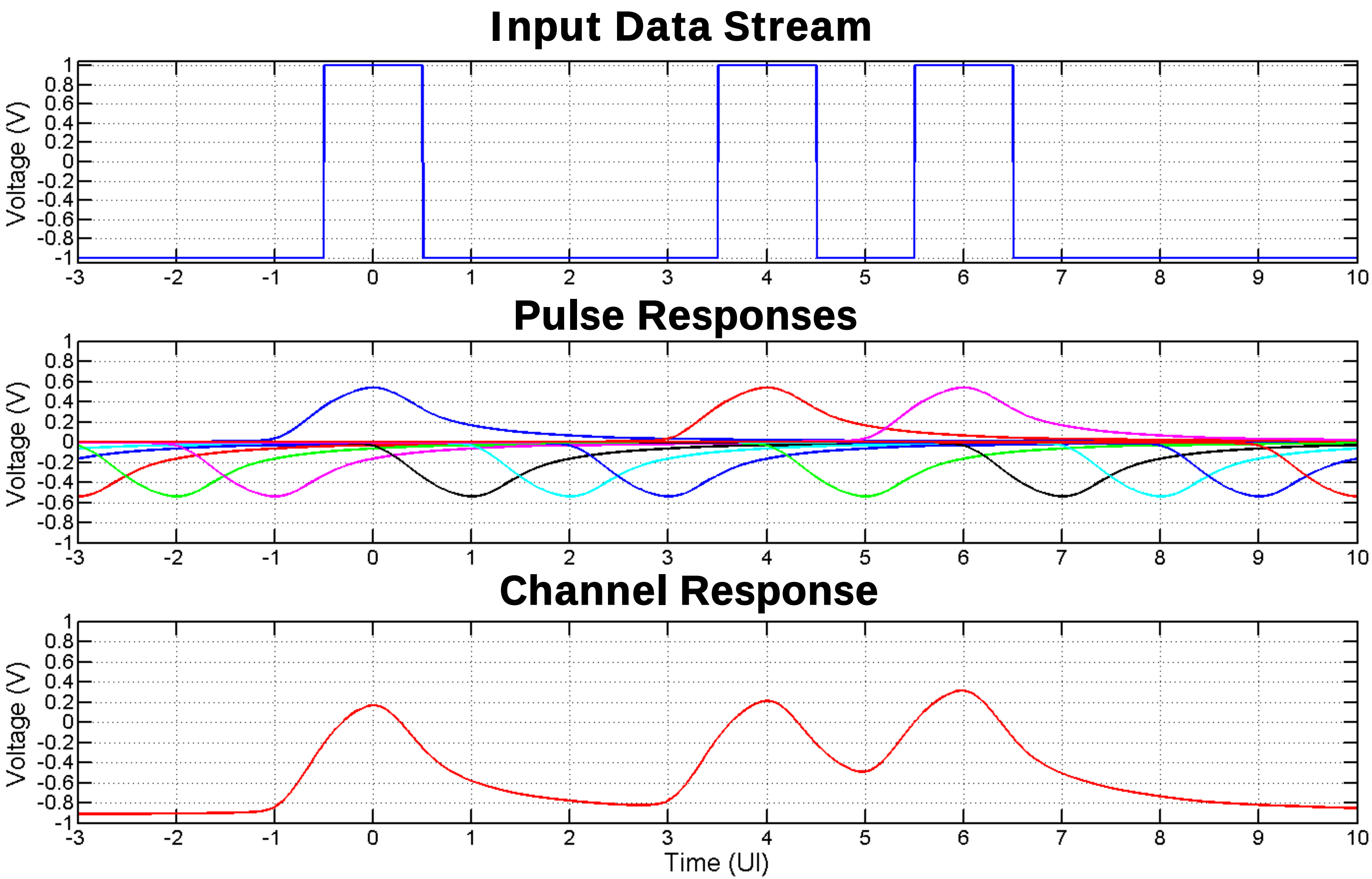
$y^{(1)}(t)$ sampled relative to pulse peak:

[... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]

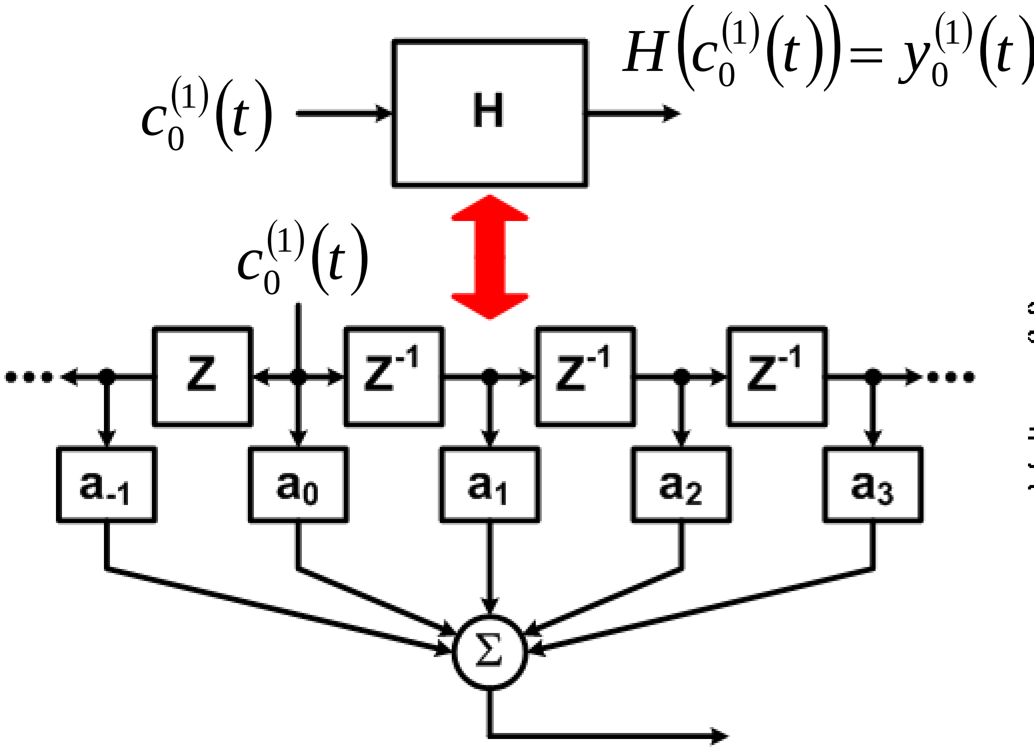
k =[... -2 1 0 1 2 3 4 5 6 ...]

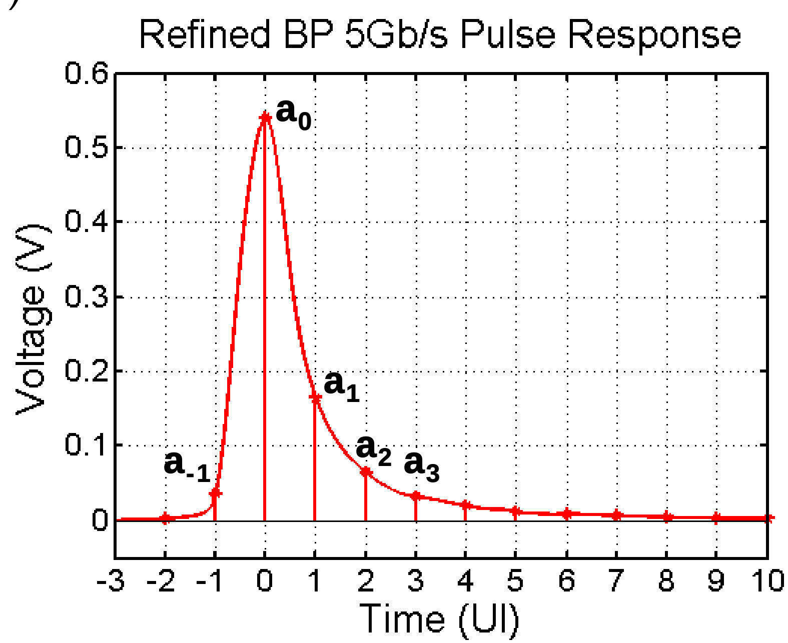
By Linearity: $y^{(0)}(t) = -1 * y^{(1)}(t)$

Channel Data Stream Response



Channel "FIR" Model





$$H\left(c_0^{(1)}(t)\right) = y_0^{(1)}(t)$$

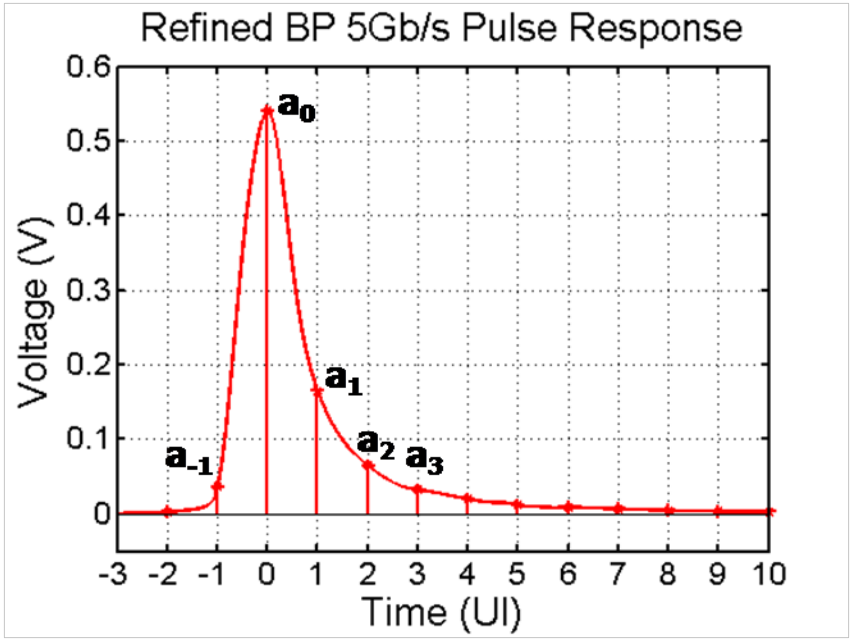
$y^{(1)}(t)$ sampled relative to pulse peak:

[... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]

$a = [\ \ldots\ a_{-2}\quad a_{-1}\quad a_0\quad a_1\quad a_2\quad a_3\quad a_4\quad a_5\quad a_6\quad \ldots]$

Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case "1" is summation of a "1" pulse with all negative non k=0 pulse responses



$$s_1(t) = y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0}$$

- Worst-case "0" is summation of a "0" pulse with all positive non k=0 pulse responses

$$s_0(t) = y_0^{(0)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0}$$

Peak Distortion Analysis

- Worst-case eye height is $s_1(t)$-$s_0(t)$

$$s(t) = s_1(t) - s_0(t) = \left(y_0^{(1)}(t) - y_0^{(0)}(t)\right) + \left(\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0} - \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0} \right)$$

$$\text{Because } y_0^{(0)}(t) = -1\left(y_0^{(1)}(t)\right)$$

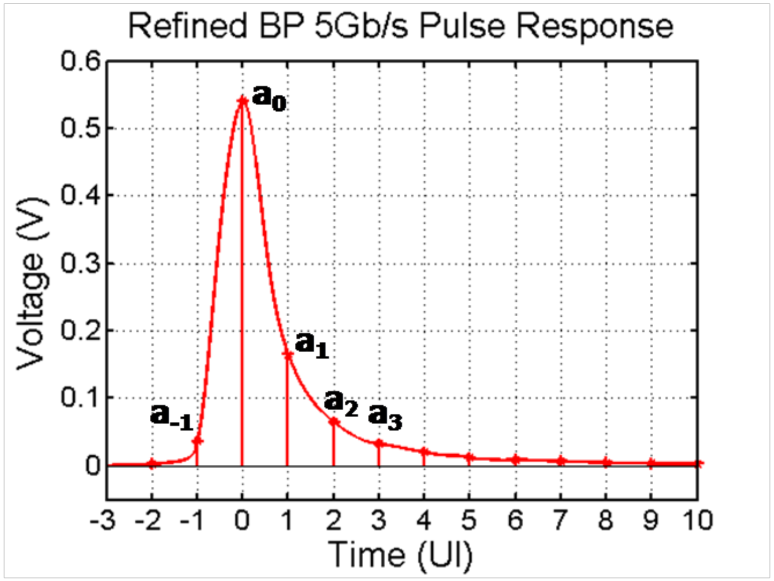
$$s(t) = 2\left(\underbrace{y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0}}_{\text{“1” pulse worst-case “1” edge}} - \underbrace{\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0}}_{\text{“1” pulse worst-case “0” edge}} \right)$$

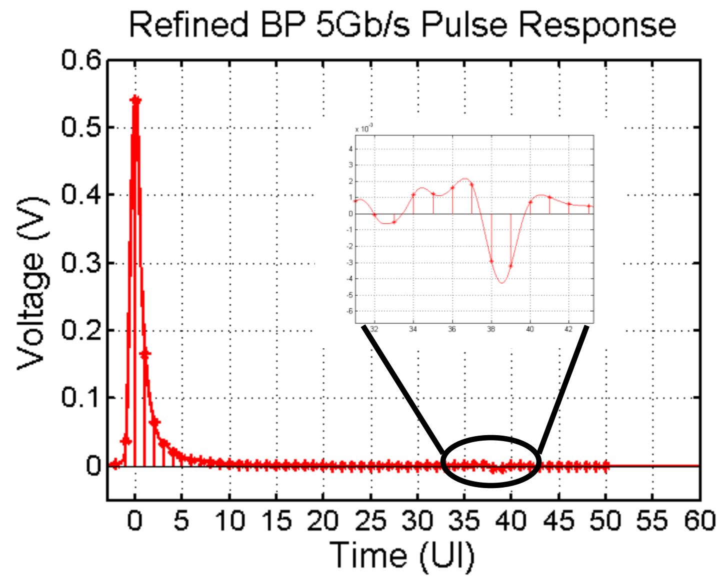
“1” pulse worst-case “1” edge

“1” pulse worst-case “0” edge

- If symmetric “1” and “0” pulses (linearity), then only positive pulse response is needed

Peak Distortion Analysis Example 1



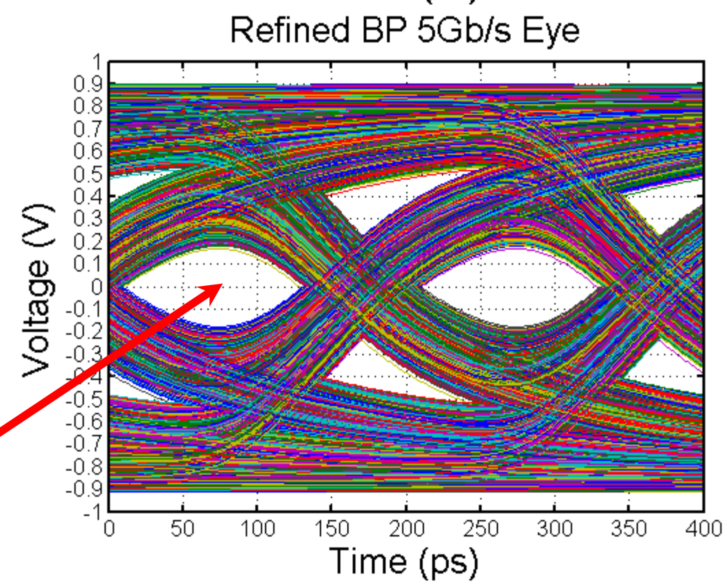


$$y_0^{(1)}(t) = 0.540$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



Worst-Case Bit Pattern

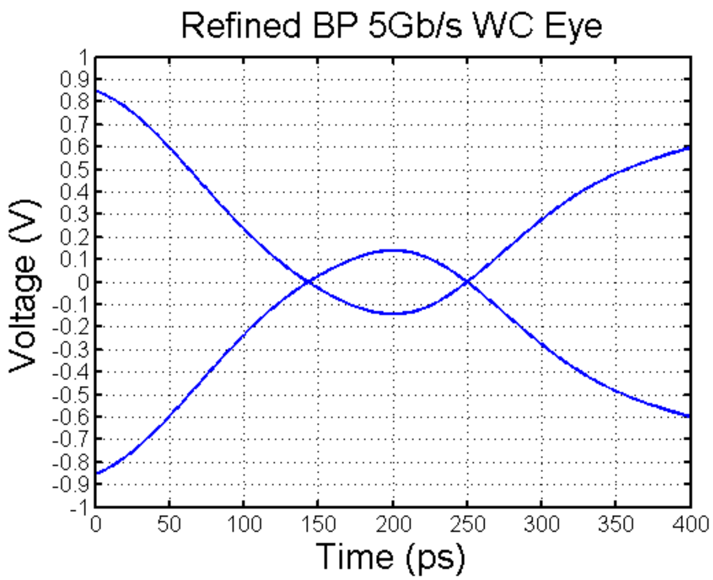
- Pulse response can be used to find the worst-case bit pattern

$$\text{Pulse} \quad \mathrm{a} = [\ldots \; a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \ldots]$$

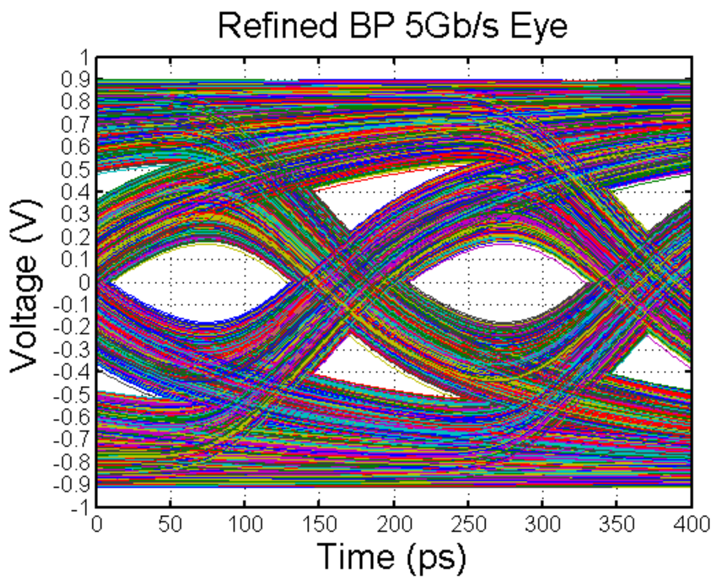
- Flip pulse matrix about cursor a_0 and the bits are the inverted sign of the pulse ISI

$$[\ldots \; -sign(a_6) \quad -sign(a_5) \quad -sign(a_4) \quad -sign(a_3) \quad -sign(a_2) \quad -sign(a_1) \quad 1 \quad -sign(a_{-1}) \quad -sign(a_{-2}) \quad \ldots]$$

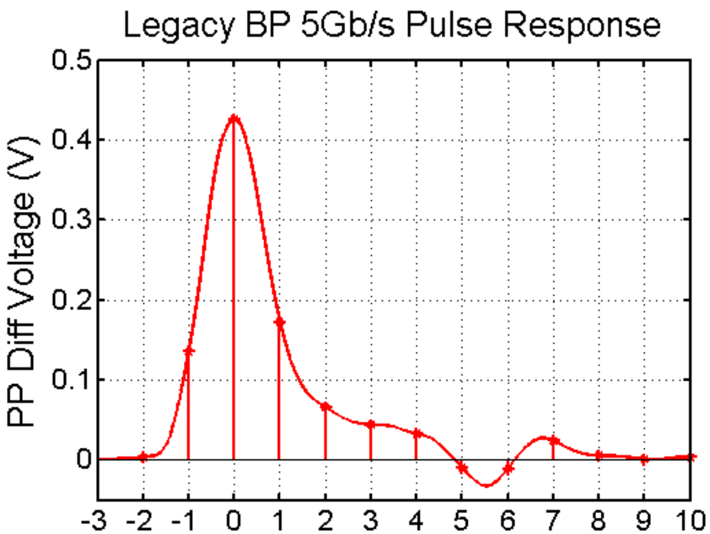
Worst-Case Bit Pattern Eye



10kbits Eye



Peak Distortion Analysis Example 2

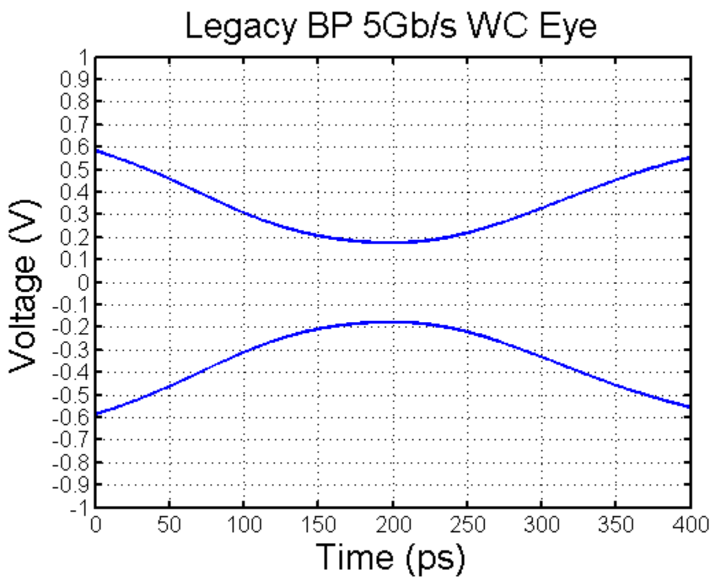


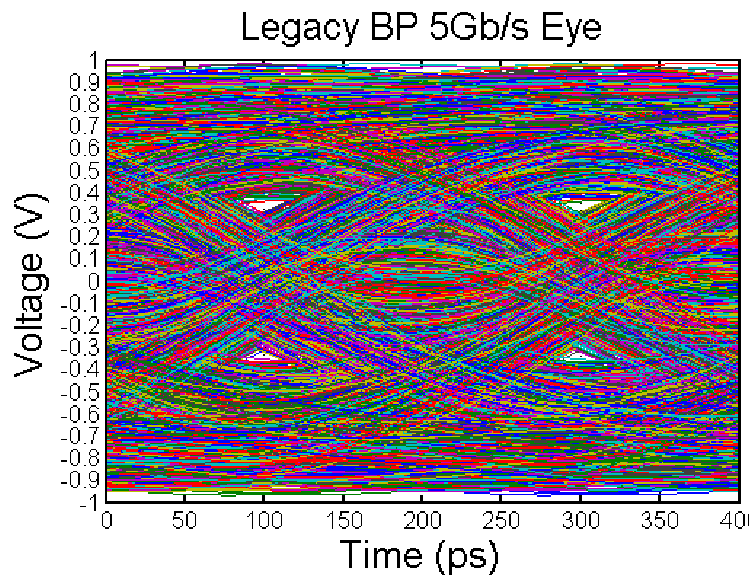
$$y_0^{(1)}(t) = 0.426$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.053$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.542$$

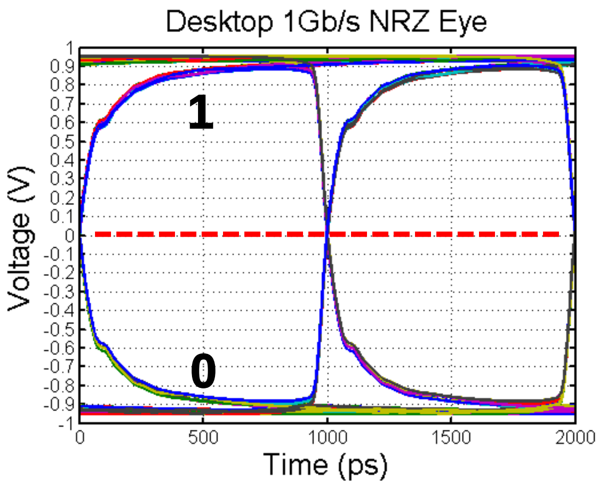
$$s(t) = 2(0.426 - 0.053 - 0.542) = -0.338$$

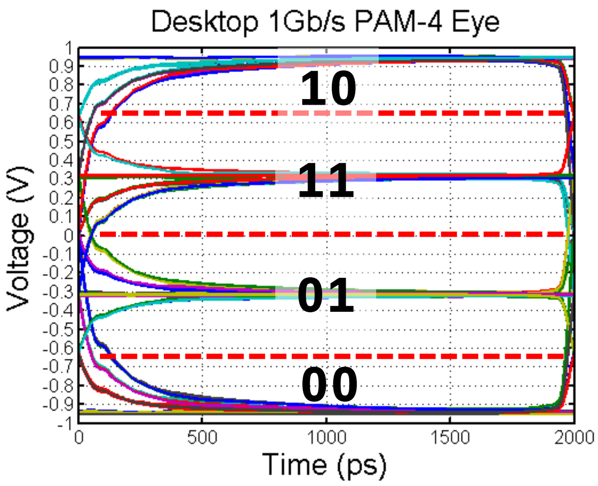




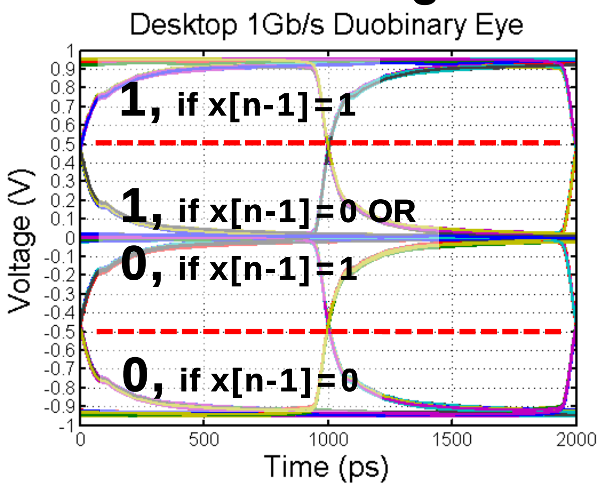
Modulation Schemes

- Binary, NRZ, PAM-2
  - Simplest, most common modulation format
- PAM-4
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run ½ speed
- Duo-binary $w[n] = x[n] + x[n-1]$
  - Allows for controlled ISI, symbol at RX is current bit plus preceding bit
  - Results in less channel equalization

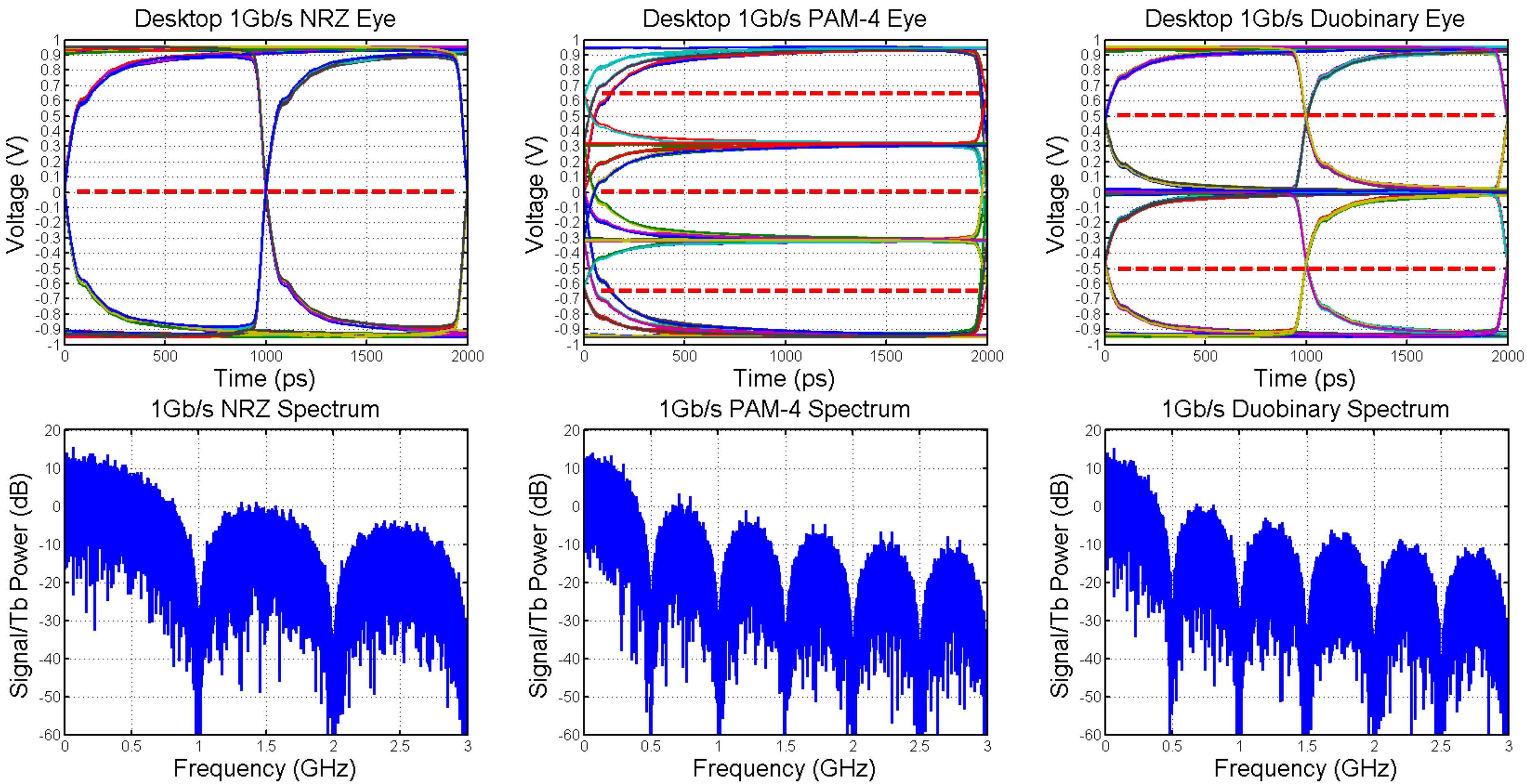




No Pre-Coding Case



Modulation Frequency Spectrum



Majority of signal power in 1GHz bandwidth

Majority of signal power in 0.5GHz bandwidth

Majority of signal power in 0.5GHz bandwidth

Nyquist Frequency

- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect R_S (symbols/s) without ISI is $R_S/2$ (Hz)
  - Thus, a system with bandwidth $W=1/2T=R_S/2$ (Hz) can support a maximum transmission rate of $2W=1/T=R_S$ (symbols/s) without ISI

$$\frac{1}{2T}=\frac{R_S}{2}\leq W \Rightarrow \frac{R_S}{W}\leq 2 \quad \text{(symbols/s/Hz)}$$

  - For ideal Nyquist pulses (sinc), the required bandwidth is only $R_S/2$ to support an R_S symbol rate

Modulation	**Bits/Symbol**	**Nyquist Frequency**
NRZ	1	$R_s/2=1/2T_b$
PAM-4	2	$R_s/2=1/4T_b$
Duobinary	1 (or 1.5) ??	$R_s/2=1/2T_b$ (or $1/3T_b$) ??

Next Time

- Modulation scheme comparison
- Link Circuits
  - Termination structures
  - Drivers
  - Receivers