ECEN689: Special Topics in High-Speed Links Circuits and Systems Spring 2010

Lecture 6: S-Parameter Channel Examples

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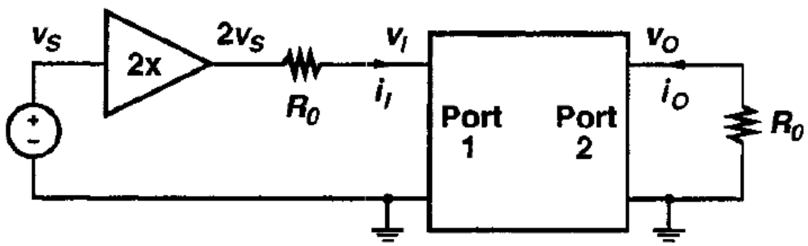
Announcements

- HW2 due 2/5
- **No class next week**
- Reading
  - Will post some material on TDR and network analyzers (S-parameters)

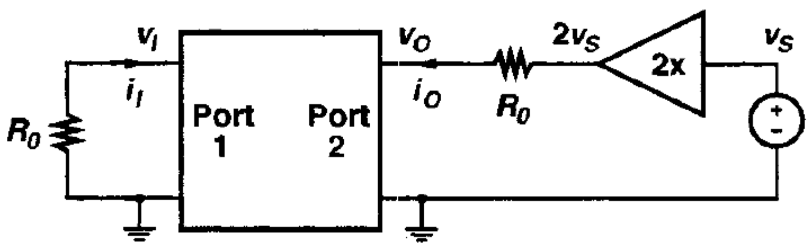
Agenda

- S-parameter examples
- Cascading S-parameter models
- Full S-parameter channel model
- Majority of today's material from Hall "Advanced Signal Integrity for High-Speed Digital Designs " Chapter 9

S-Parameter Test Circuits & Meaning



[**Sackinger**]



$$S_{11}(s) = \frac{V_{i,reflected}}{V_{i,incident}} = \frac{V_i - R_0 I_i}{V_i + R_0 I_i} = \frac{V_i - V_s}{V_s}$$

$$S_{21}(s) = \frac{V_{o,transmitted}}{V_{i,incident}} = \frac{V_o - R_0 I_o}{V_i + R_0 I_i} = \frac{V_o}{V_s}$$

$$S_{22}(s) = \frac{V_{o,reflected}}{V_{o,incident}} = \frac{V_o - R_0 I_o}{V_o + R_0 I_o} = \frac{V_o - V_s}{V_s}$$

$$S_{12}(s) = \frac{V_{i,transmitted}}{V_{o,incident}} = \frac{V_i - R_0 I_i}{V_o + R_0 I_o} = \frac{V_i}{V_s}$$

- S_{11} = Input reflection coefficient
  - $1/S_{11}$ = Input return loss
- S_{21} = Forward transmission coefficient
  - Gain w/ input matching dependency
- S_{22} = Output reflection coefficient
  - $1/S_{22}$ = Output return loss
- S_{12} = Reverse transmission coefficient (isolation)

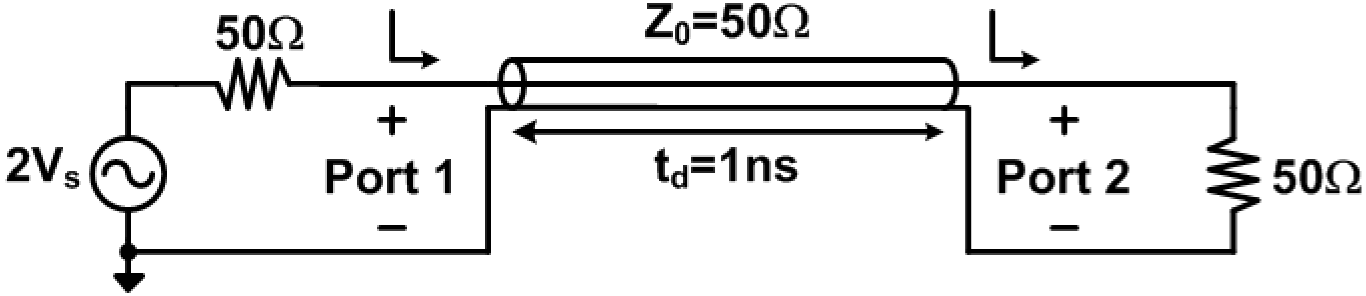
$$S_{21}(s) = [1 + S_{11}(s)]A(s)$$

where A(s) is loaded voltage gain

S-Parameter Example #1

$$k_r = \frac{Z_0 - R}{Z_0 + R} = 0$$

$$k_{rT} = 0$$



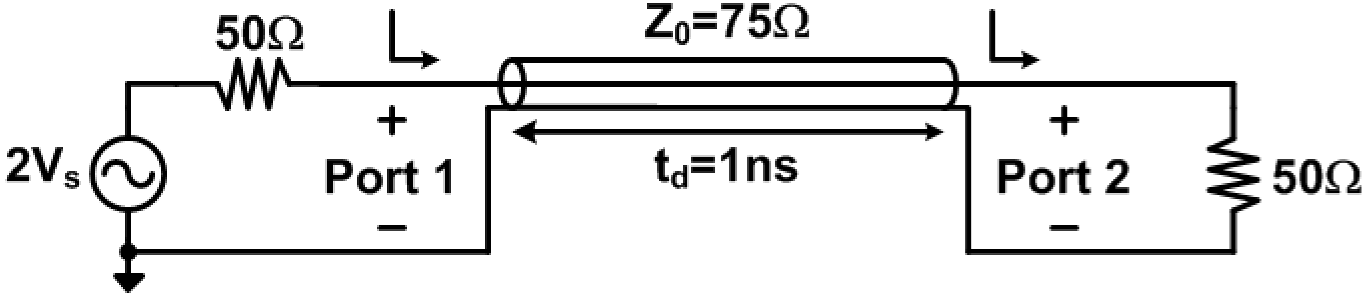
$$S_{11} = \frac{V_{1,reflected}}{V_{1,incident}} = k_r = \frac{Z_{in} - 50}{Z_{in} + 50} = \frac{50 - 50}{50 + 50} = 0$$

$$S_{21} = \frac{V_{2,transmitted}}{V_{1,incident}} = \frac{V_s}{V_s} = 1$$

S-Parameter Example #2

$$k_r(s) = \frac{Z_{in}(s) - R}{Z_{in}(s) + R}$$

$$k_{rT} = -0.2$$



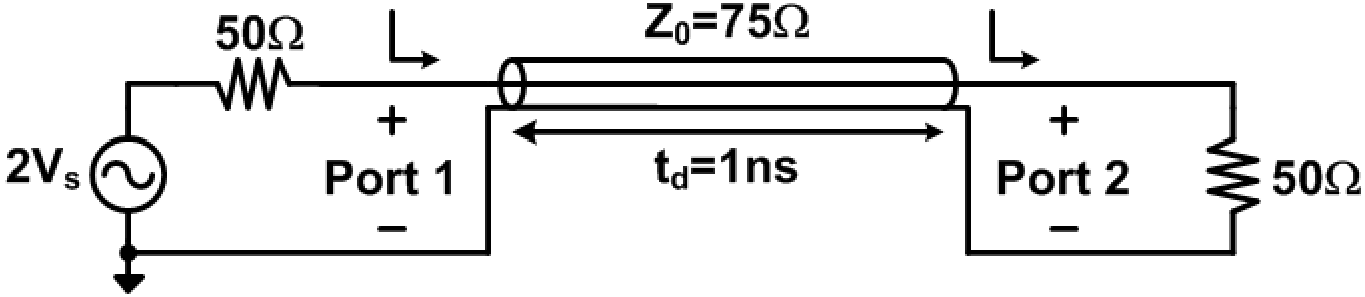
$$S_{11}(s) = \frac{V_{1,reflected}}{V_{1,incident}} = k_r(s) = \frac{Z_{in}(s) - 50}{Z_{in}(s) + 50}$$

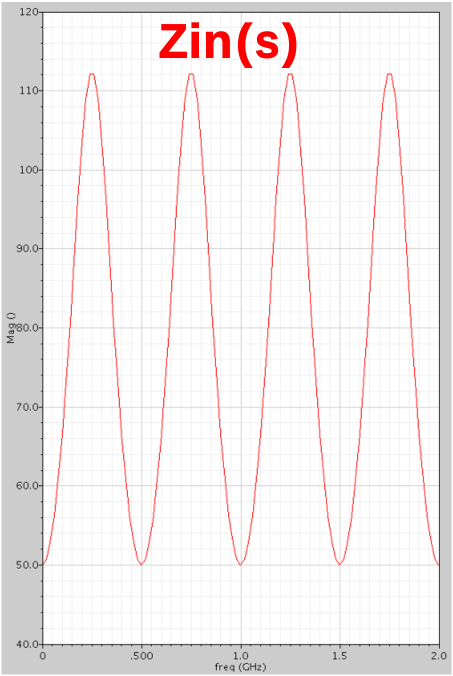
$$S_{21}(s) = \frac{V_{2,transmitted}}{V_{1,incident}} = [1 + S_{11}(s)]A(s)$$

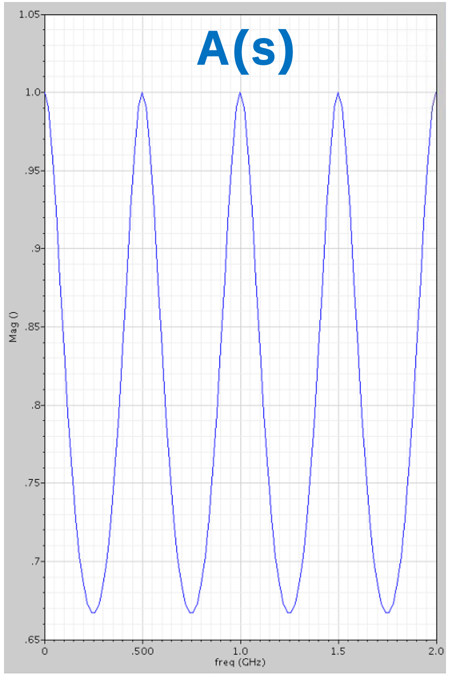
S-Parameter Example #2

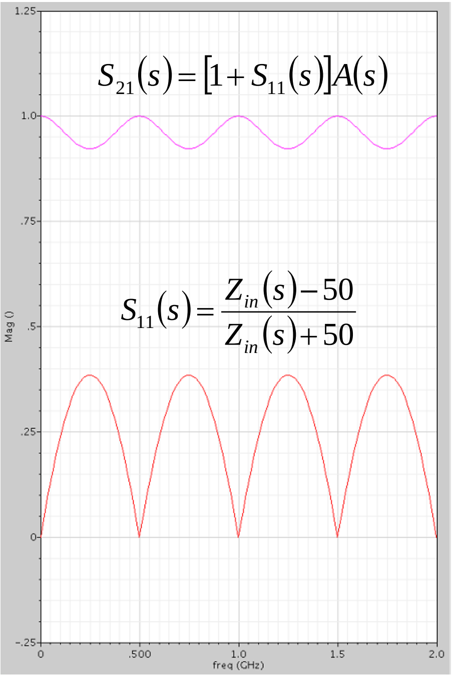
$$k_r(s) = \frac{Z_{in}(s) - R}{Z_{in}(s) + R}$$

$$k_{rT} = -0.2$$





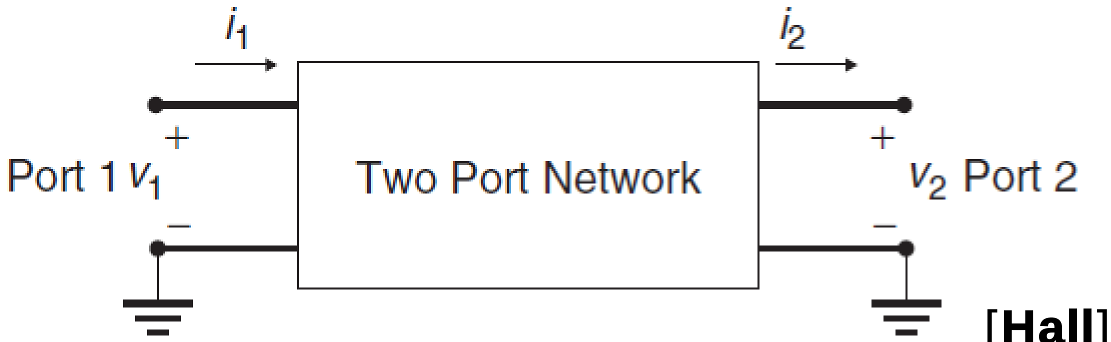




Cascading S-Parameters

- Network analysis allows cascading of independently characterized structures
- However, can’t directly cascade s-parameter matrices and multiply
- Must first convert to an ABCD matrix (or T matrix)

ABCD Parameters



[Hall]

$$A = \left.\frac{v_1}{v_2}\right|_{i_{2=0}} \quad B = \left.\frac{v_1}{i_2}\right|_{v_{2=0}} \quad C = \left.\frac{i_1}{v_2}\right|_{i_{2=0}} \quad D = \left.\frac{i_1}{i_2}\right|_{v_{2=0}}$$

$$\begin{vmatrix} v_1 \\ i_i \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \bullet \begin{vmatrix} v_2 \\ i_2 \end{vmatrix}$$

Converting Between S & ABCD Parameters

TABLE 9-3. Relationships Between Two-Port S and $ABCD$ Parameters[a]

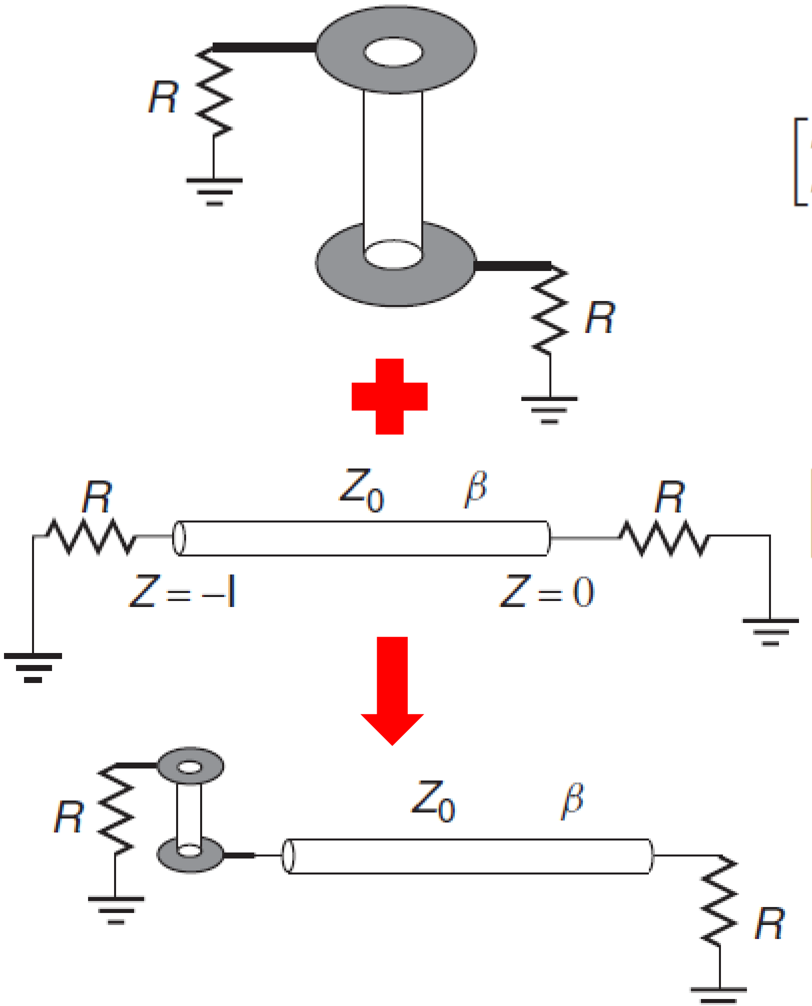
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \begin{bmatrix} \dfrac{B - Z_n(D - A + CZ_n)}{B + Z_n(D + A + CZ_n)} & \dfrac{2Z_n(AD - BC)}{B + Z_n(D + A + CZ_n)} \\ \dfrac{2Z_n}{B + Z_n(D + A + CZ_n)} & \dfrac{B - Z_n(A - D + CZ_n)}{B + Z_n(D + A + CZ_n)} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad \begin{bmatrix} \dfrac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}} & Z_n \dfrac{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}{2S_{21}} \\ \dfrac{1}{Z_n} \dfrac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}} & \dfrac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}} \end{bmatrix}$$

[a] Z_n is the termination impedance at the ports.

[**Hall**]

Example: Cascaded Via & Transmission Line



$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\text{via}} = \begin{bmatrix} -0.1235 - j0.1516 & 0.7597 - j0.6190 \\ 0.7597 - j0.6190 & -0.1235 - j0.1516 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{via}} = \begin{bmatrix} 0.790 & j22.22 \\ j0.01686 & 0.790 \end{bmatrix}$$

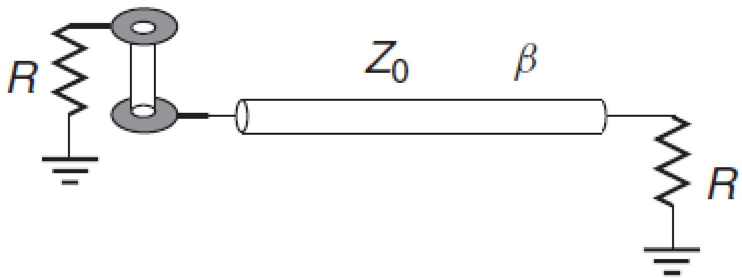
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\text{t-line}} = \begin{bmatrix} 0.00325 - j0.00323 & -1.00 - j0.003 \\ -1.00 - j0.003 & 0.00325 - j0.00323 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{t-line}} = \begin{bmatrix} -1 & j0.3228 \\ j0.000129 & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{cascade}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{via}} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{t-line}}$$

- Taken from “Advanced Signal Integrity for High-Speed Digital Designs” by Hall

Example: Cascaded Via & Transmission Line



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{cascade}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{via}} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{t-line}}$$

$$= \begin{bmatrix} 0.790 & j22.22 \\ j0.01686 & 0.790 \end{bmatrix} \cdot \begin{bmatrix} -1 & j0.3228 \\ j0.000129 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.790 & -j21.965 \\ -j0.01686 & -0.795 \end{bmatrix}$$

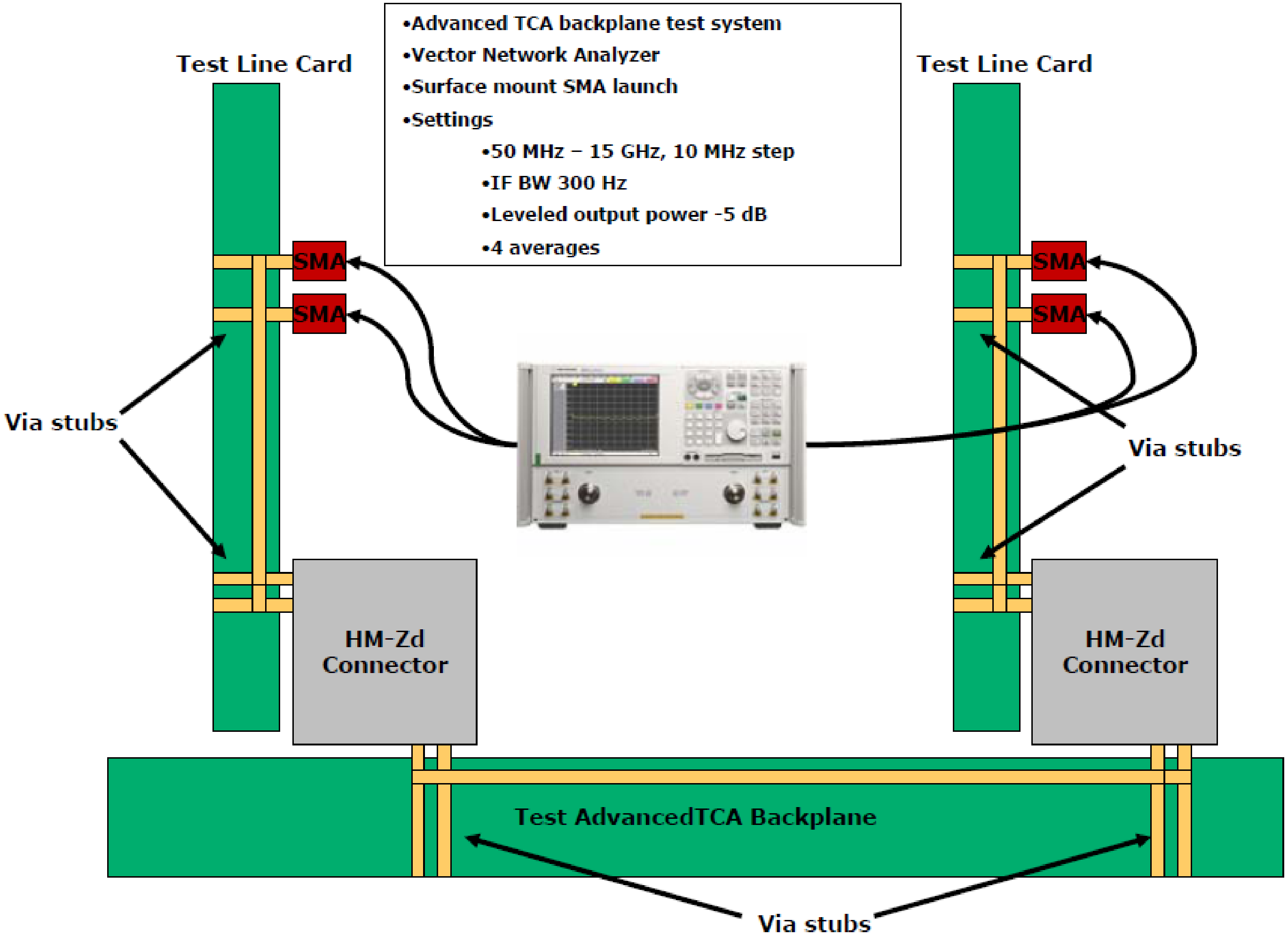
- Using conversion table:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\text{cascade}} = \begin{bmatrix} -0.1259 - j0.1553 & -0.7635 + j0.6186 \\ -0.7645 + j0.6182 & -0.1200 - j0.1565 \end{bmatrix}$$

- Can also use T matrixes to cascade

- Taken from “Advanced Signal Integrity for High-Speed Digital Designs” by Hall

S-Parameter Channel Example



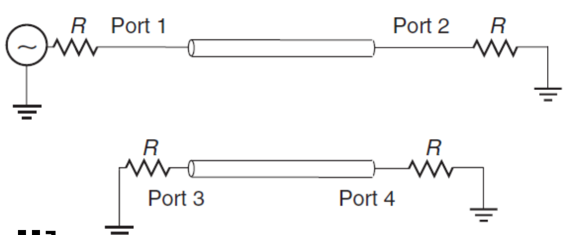
[Peters, IEEE Backplane Ethernet Task Force]

S-Parameter Channel Example (4-port differential)

```
! peters_01_0605 T20 channel thru response
#       HZ        S              RI           R       50
!
! FREQ                 S11                           S12                           S13                           S14
!                      S21                           S22                           S23                           S24
!                      S31                           S32                           S33                           S34
!                      S41                           S42                           S43                           S44
!                 REAL      IMAG                REAL      IMAG                REAL      IMAG                REAL      IMAG
5.00000000e+007  6.279266901548e-002 -5.256007502766e-002 -1.995363973143e-001 -9.018006169275e-001  7.405252014369e-002 -1.653914717779e-002  4.694410796534e-004  2.855671737566e-003
                -1.993592781969e-001 -9.017752677900e-001  6.847049395661e-002 -3.537762509466e-002  6.592975593456e-004  2.600733690373e-003  7.478976460177e-002 -1.488182269791e-002
                 7.438370524663e-002 -1.650568516548e-002  6.663957537997e-004  2.723661634513e-003  5.641343731365e-002 -5.693035832892e-002 -2.070369894915e-001 -8.986367167361e-001
                 3.380698172980e-004  2.715033111885e-003  7.497765935351e-002 -1.488546535615e-002 -2.063544808970e-001 -9.002700655374e-001  6.856095801756e-002 -3.019606086420e-002
6.00000000e+007  4.829977376755e-002 -6.288238652440e-002 -4.923832497425e-001 -7.721510464035e-001  6.298956599590e-002 -3.938489680891e-002  1.125377257145e-003  1.921732299021e-003
                -4.925547500023e-001 -7.726263821707e-001  6.163450406360e-002 -4.486265928179e-002  1.299644022342e-003  1.492436402394e-003  6.462146347807e-002 -3.736630924981e-002
                 6.308085276969e-002 -3.947655302643e-002  1.386741613180e-003  1.653454474207e-003  4.393874455850e-002 -6.448913049207e-002 -4.992743919180e-001 -7.660808533046e-001
                 1.280875740087e-003  1.936760526874e-003  6.482369657086e-002 -3.743006383763e-002 -4.995203164654e-001 -7.674804458241e-001  6.284893613667e-002 -4.132139739274e-002
```

Data from 50MHz to 15GHz in 10MHz steps

```
1.49900000e+010 -1.884123481138e-001  3.522933794755e-001  9.493645552321e-004  2.735890006358e-004  2.939002692375e-002 -8.676465491258e-003 -2.207496924854e-004  1.236065259912e-004
                 9.463443060684e-004  3.105615146344e-004 -1.742347383703e-001  4.813685271232e-002 -6.152705437030e-004  1.614752661571e-003  6.774475978813e-002  9.617239585695e-003
                 2.953403838205e-002 -8.707827389646e-003 -6.226849675423e-004  1.637610280621e-003 -1.595766021694e-001  3.757605914955e-001 -1.809501624148e-004 -7.061855554470e-004
                -2.613575703191e-004  1.368108929760e-004  6.788329666403e-002  9.551687705500e-003 -2.146293806886e-004 -7.363580847286e-004 -1.199804891859e-001  7.697336952293e-002
1.50000000e+010 -1.883176013184e-001  3.545614742110e-001  9.524680768441e-004 -5.404222971799e-005  2.935126165241e-002 -1.235086132268e-002 -1.616280086909e-004  2.347368458649e-004
                 1.039250921080e-003 -6.032017103742e-005 -1.649137634331e-001  4.966164587830e-002 -6.748937194262e-005  1.689652681670e-003  6.725041473699e-002  1.961009613152e-003
                 2.959693594806e-002 -1.251203706381e-002 -2.927441863297e-005  1.747754847916e-003 -1.531702433245e-001  3.773014940454e-001 -3.769459376261e-004 -5.671620228005e-004
                -2.089293612250e-004  2.303682313561e-004  6.740524959192e-002  1.672663579641e-003 -4.385850073691e-004 -5.810569604703e-004 -1.121319455376e-001  7.458173831411e-002
```



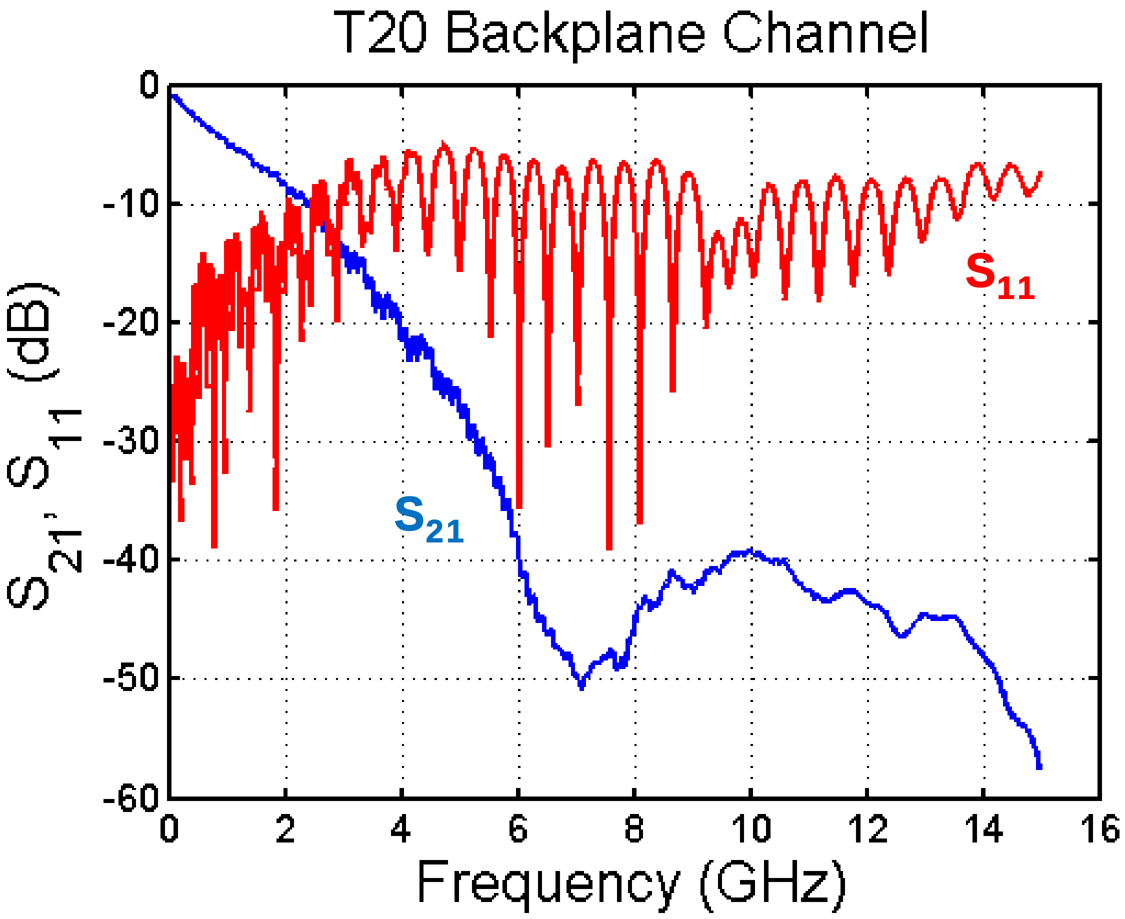
[Hall]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} v \\ 0 \\ -v \\ 0 \end{bmatrix}$$

$$S_{dd11} = \left. \frac{b_{d1}}{a_{d1}} \right|_{a_2 = a_4 = 0} = \frac{1}{2}\left(S_{11} + S_{33} - S_{13} - S_{31}\right)$$

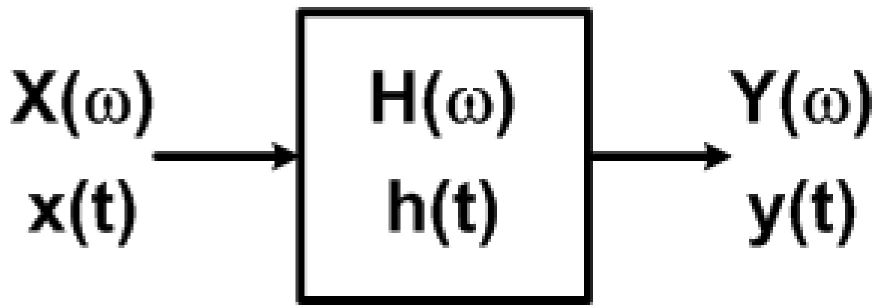
$$S_{dd21} = \left. \frac{b_{d2}}{a_{d1}} \right|_{a_2 = a_4 = 0} = \frac{1}{2}\left(S_{21} + S_{43} - S_{23} - S_{41}\right)$$

S-Parameter Channel Example



Impulse Response

- Channel impulse responses are used in
  - Time domain simulations
  - Link analysis tools



$$Y(\omega) = H(\omega)X(\omega)$$

$$y(t) = h(t) * x(t) \int_{-\infty}^{\infty} h(t-\tau)x(\tau)$$

$$h(t) = F^{-1}\{H(w)\}$$

Next Time

- Impulse response generation

- Communication techniques
  - Eye Diagram
  - Intersymbol interference
  - Modulation techniques