

# ECEN720: High-Speed Links Circuits and Systems Spring 2025

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## Lecture 4: Channel Pulse Model & Modulation Schemes



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# Announcements

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- Prelab 1 and Lab Report 1 due Feb 4
- Prelab 2, Lab Report 2, and Prelab 3 due Feb 11
- Lab Report 3 due Feb 18
- Reference material
  - Peak distortion analysis paper by Casper
  - Notes from H. Song, Arizona State
  - Papers posted on PAM-2/4 transceivers

# Agenda

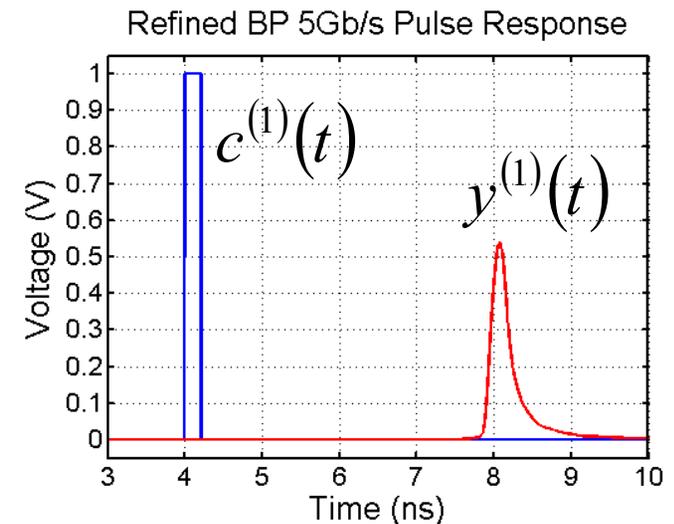
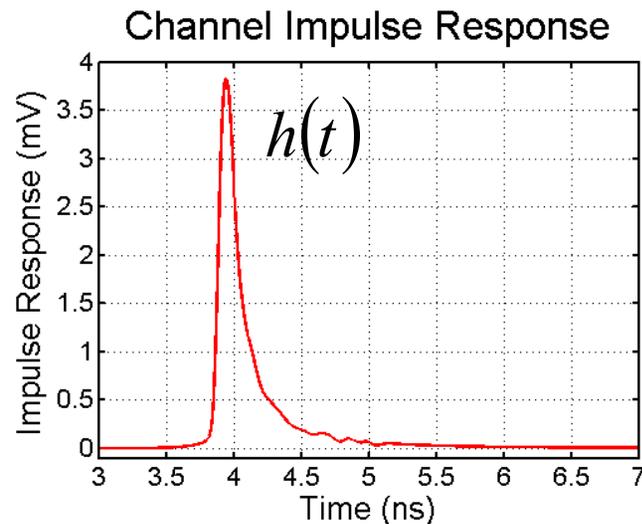
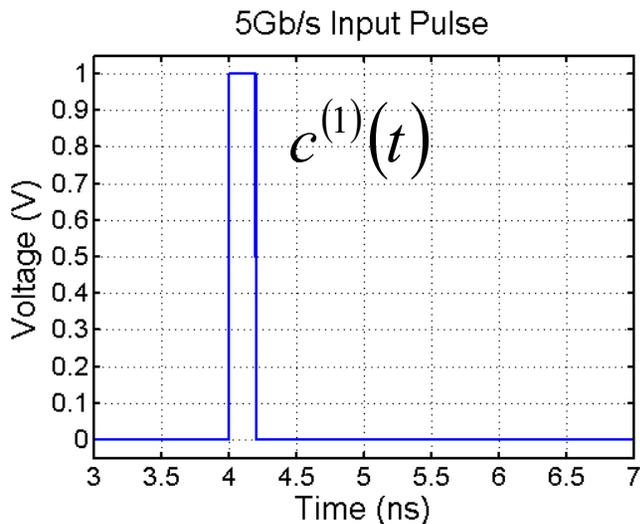
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- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation

# Inter-Symbol Interference (ISI)

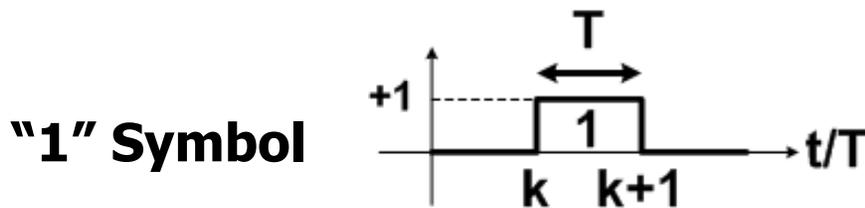
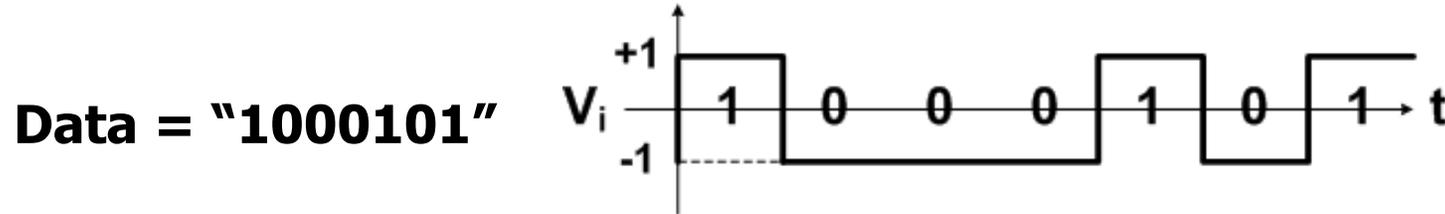
- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

$$y^{(1)}(t) = c^{(1)}(t) * h(t)$$

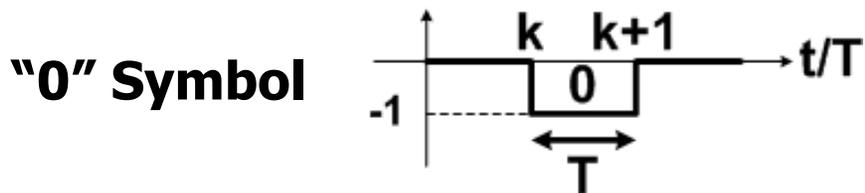


# NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s



$$c_k^{(1)}(t) \equiv u(t - kT) - u(t - (k + 1)T)$$



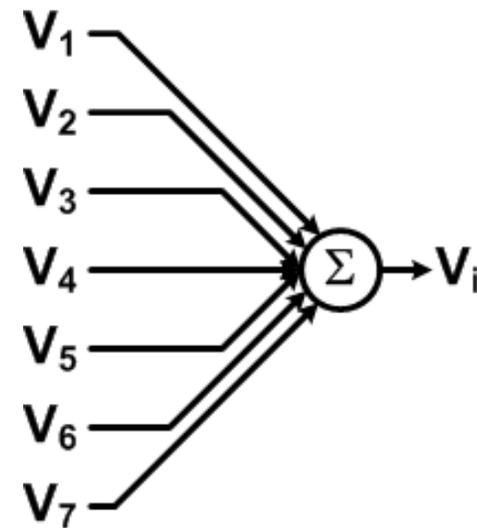
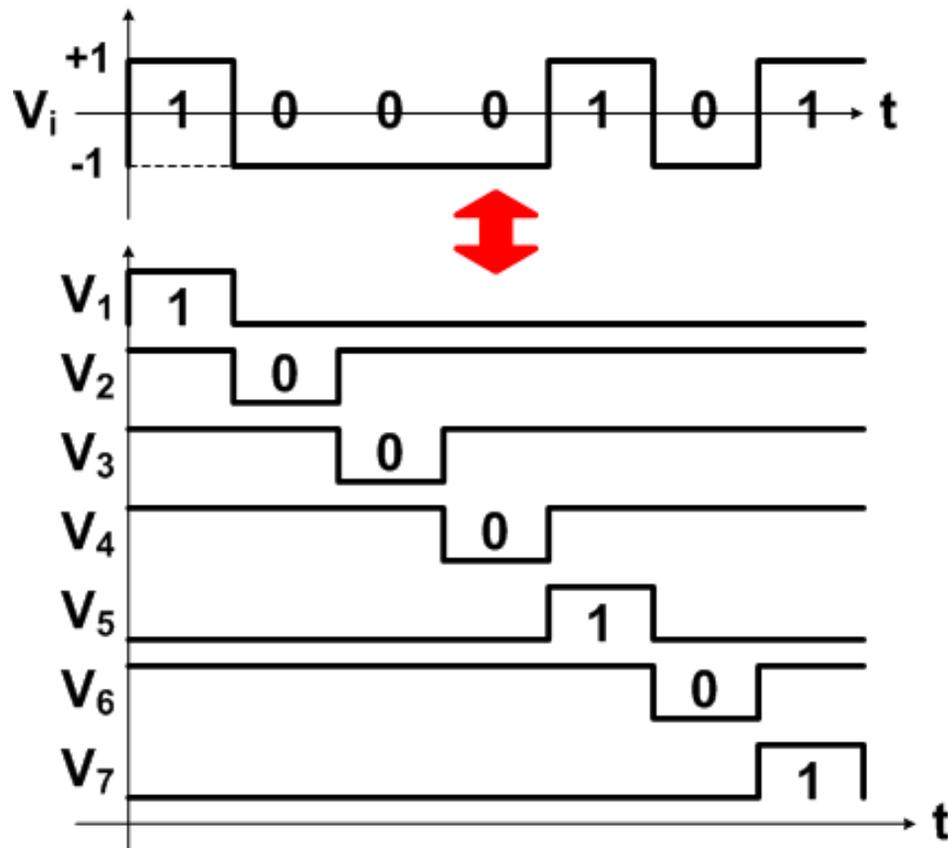
$$c_k^{(0)}(t) = -c_k^{(1)}(t)$$

[Song]

$$\text{where } u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

# NRZ Data Modeling

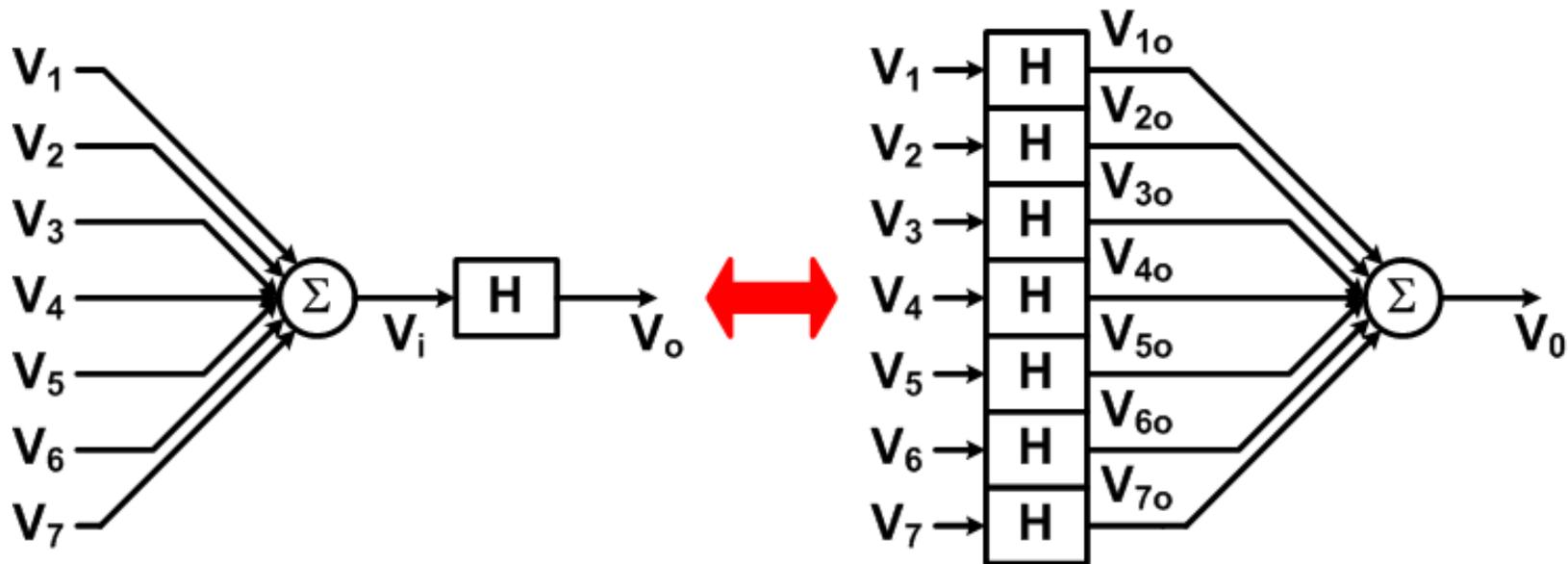
- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s



$$V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t)$$

# Channel Response to NRZ Data

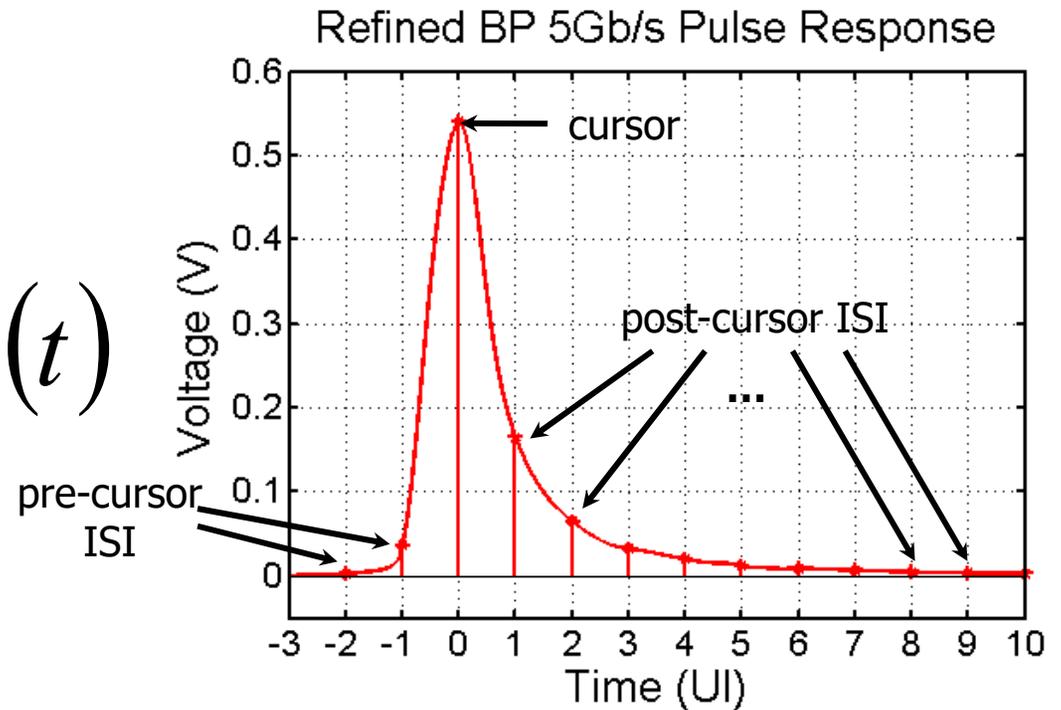
- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses



$$V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H(c_k^{(d_k)}(t)) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)$$

# Channel Pulse Response

$$y^{(d_k)}(t) = c^{(d_k)}(t) * h(t)$$



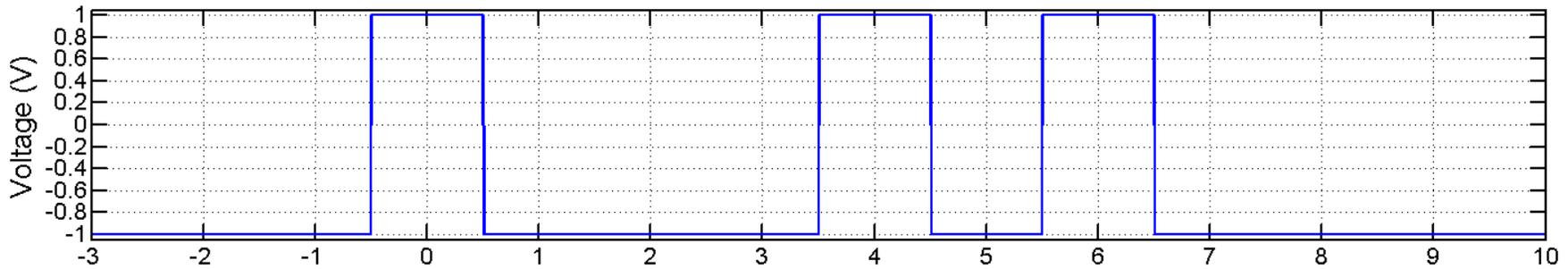
$y^{(1)}(t)$  sampled relative to pulse peak:

[... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]  
 $k = [ \dots -2 \quad 1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \dots ]$

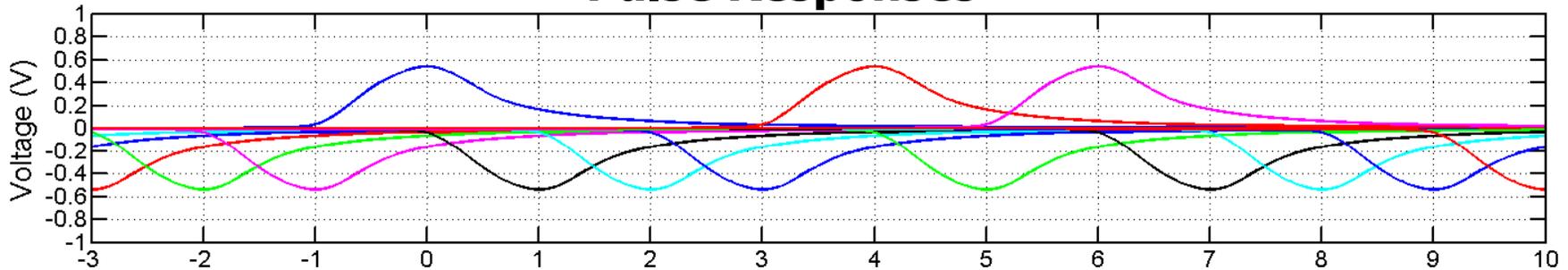
By Linearity:  $y^{(0)}(t) = -1 * y^{(1)}(t)$

# Channel Data Stream Response

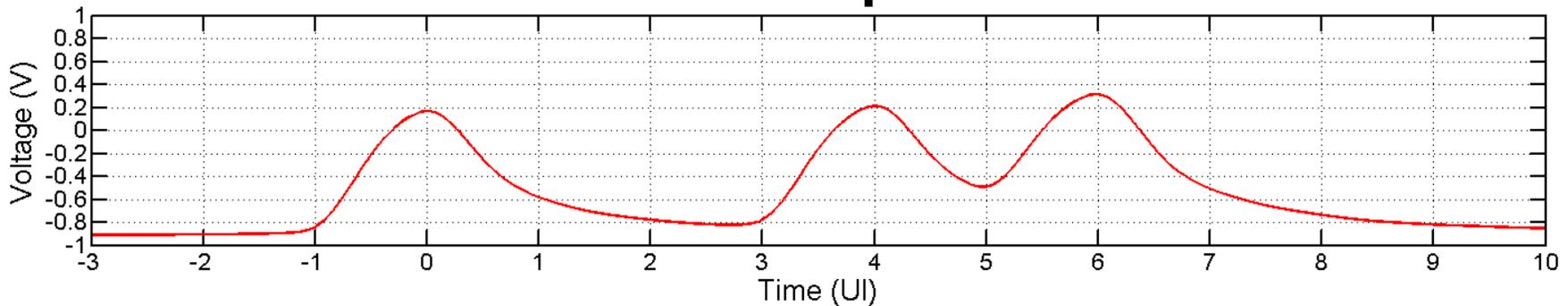
## Input Data Stream



## Pulse Responses

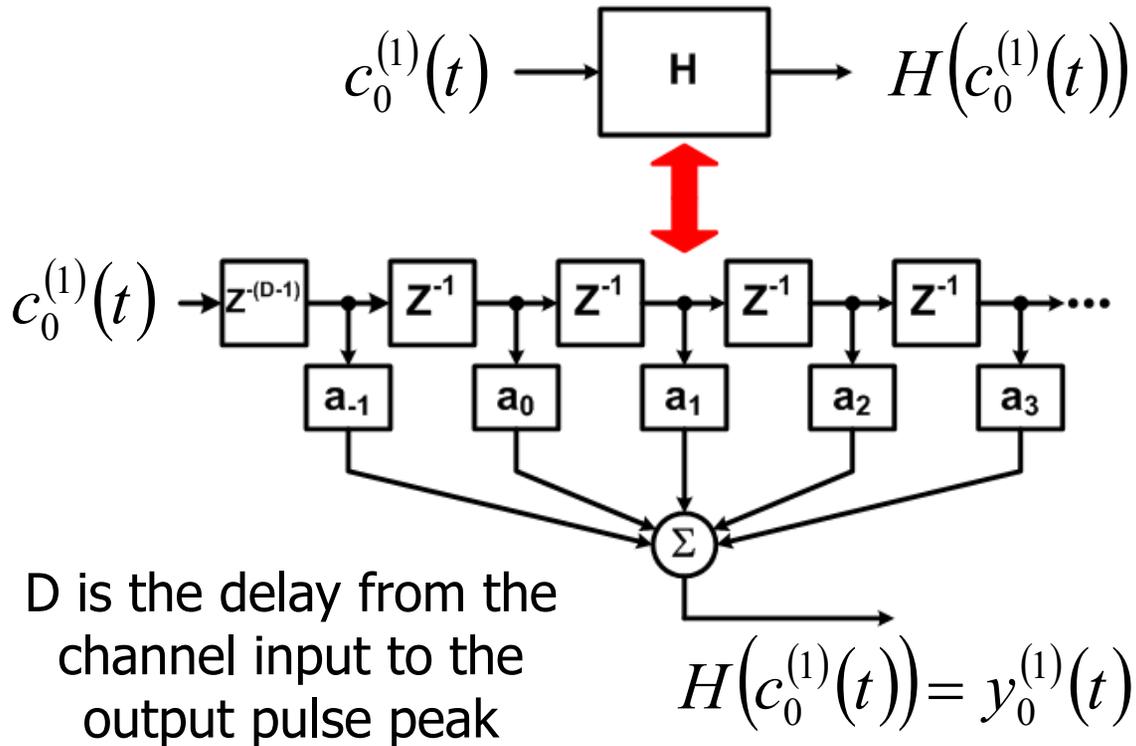


## Channel Response

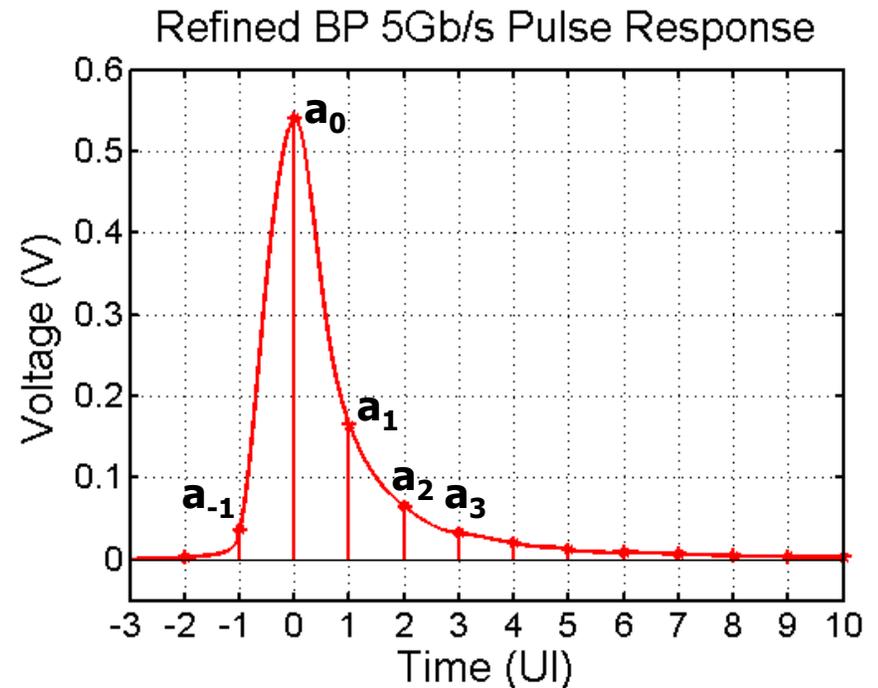


# Channel "FIR" Model

$$c_0^{(1)}(t) \rightarrow \boxed{H} \rightarrow H(c_0^{(1)}(t)) = y_0^{(1)}(t)$$



D is the delay from the channel input to the output pulse peak



$y^{(1)}(t)$  sampled relative to pulse peak:

[... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]

$a = [ \dots a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \dots ]$

# Agenda

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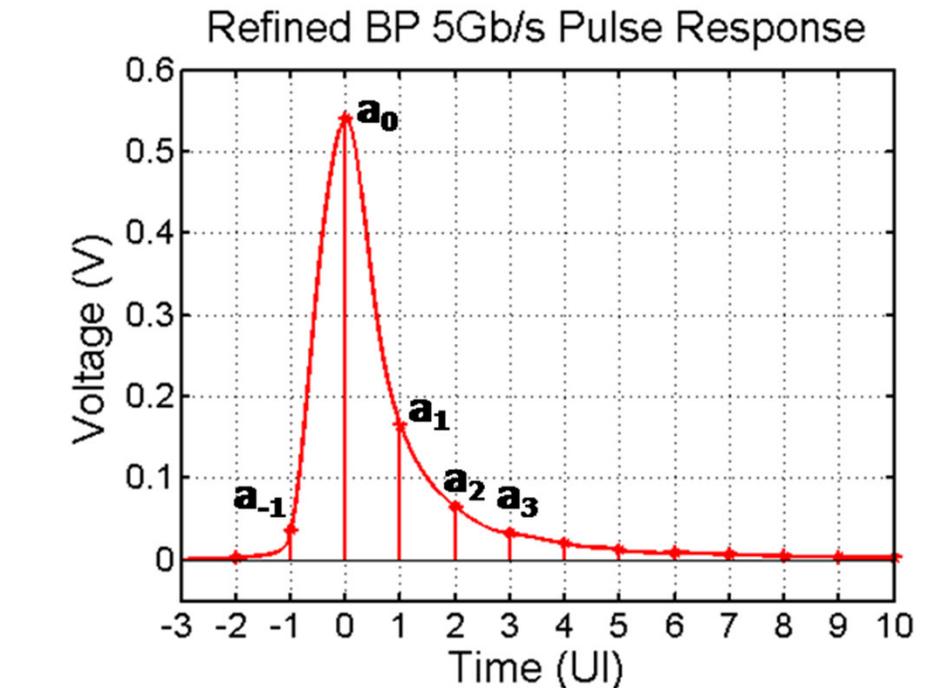
- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation

# Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case "1" is summation of a "1" pulse with all negative non  $k=0$  pulse responses

$$s_1(t) = y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t - kT) \Big|_{y(t-kT) < 0}$$

- Worst-case "0" is summation of a "0" pulse with all positive non  $k=0$  pulse responses



$$s_0(t) = y_0^{(0)}(t) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t - kT) \Big|_{y(t-kT) > 0}$$

# Peak Distortion Analysis

- Worst-case eye height is  $s_1(t) - s_0(t)$

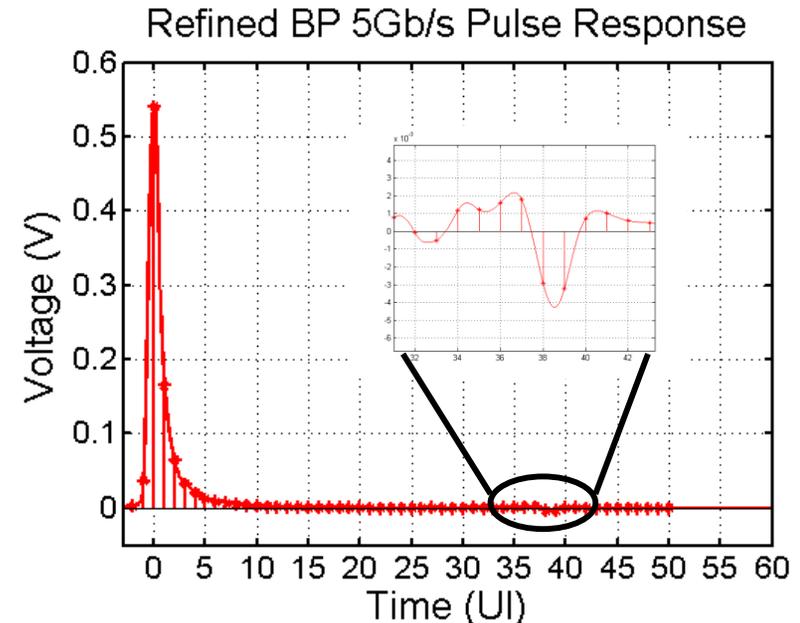
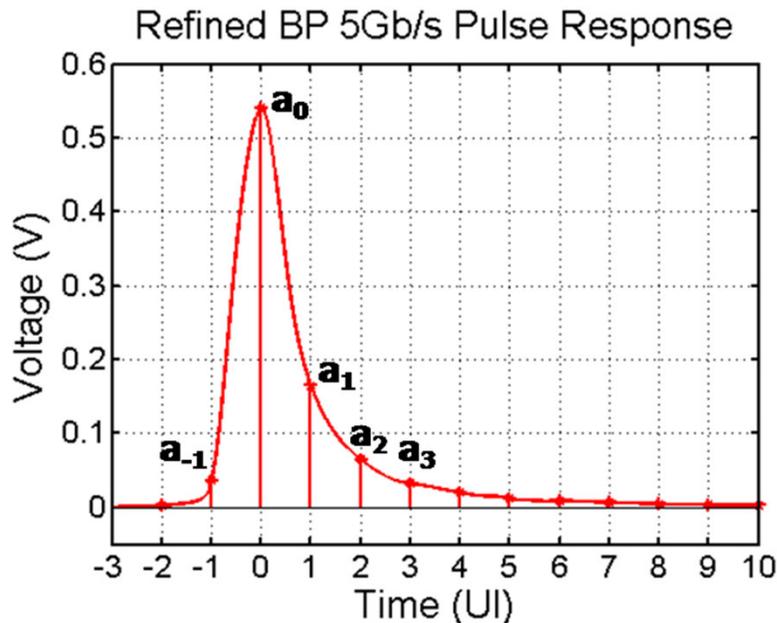
$$s(t) = s_1(t) - s_0(t) = \left( y_0^{(1)}(t) - y_0^{(0)}(t) \right) + \left( \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t - kT) \Big|_{y(t-kT) < 0} - \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t - kT) \Big|_{y(t-kT) > 0} \right)$$

Because  $y_0^{(0)}(t) = -1(y_0^{(1)}(t))$

$$s(t) = 2 \left( \underbrace{y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) < 0}}_{\text{"1" pulse worst-case "1" edge}} - \underbrace{\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) > 0}}_{\text{"1" pulse worst-case "0" edge}} \right)$$

- If symmetric "1" and "0" pulses (linearity), then only positive pulse response is needed

# Peak Distortion Analysis Example 1

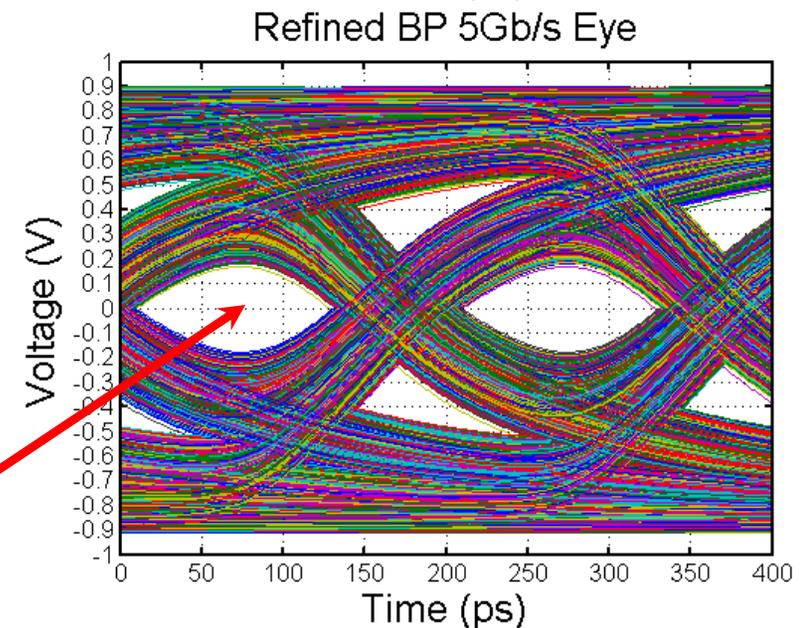


$$y_0^{(1)}(t) = 0.540$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) < 0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) > 0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



# Worst-Case Bit Pattern

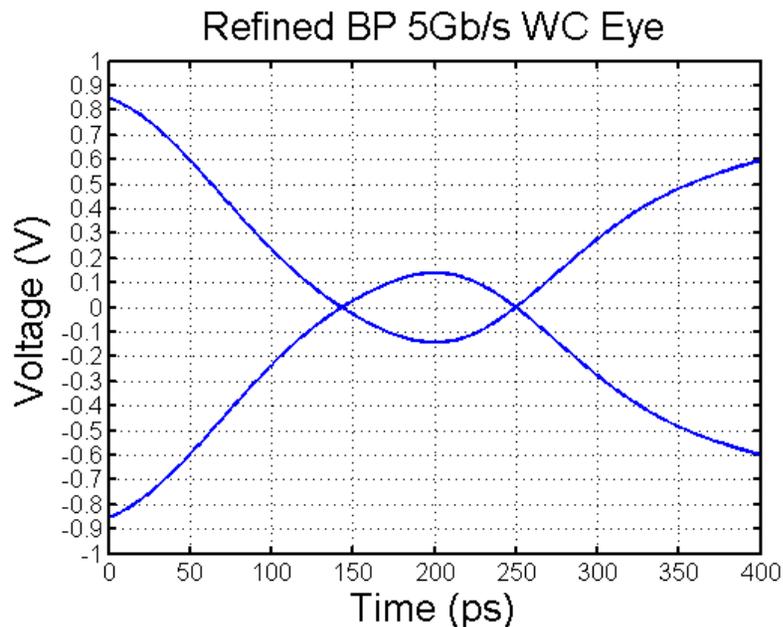
- Pulse response can be used to find the worst-case bit pattern

$$\text{Pulse } a = [\dots a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \dots]$$

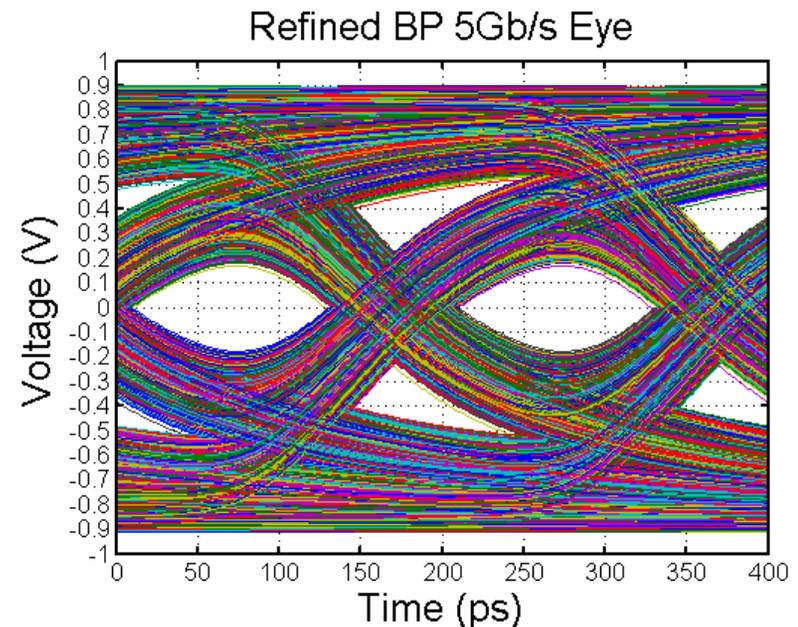
- Flip pulse matrix about cursor  $a_0$  and the bits are the inverted sign of the pulse ISI

$$[\dots -\text{sign}(a_6) \quad -\text{sign}(a_5) \quad -\text{sign}(a_4) \quad -\text{sign}(a_3) \quad -\text{sign}(a_2) \quad -\text{sign}(a_1) \quad 1 \quad -\text{sign}(a_{-1}) \quad -\text{sign}(a_{-2}) \quad \dots]$$

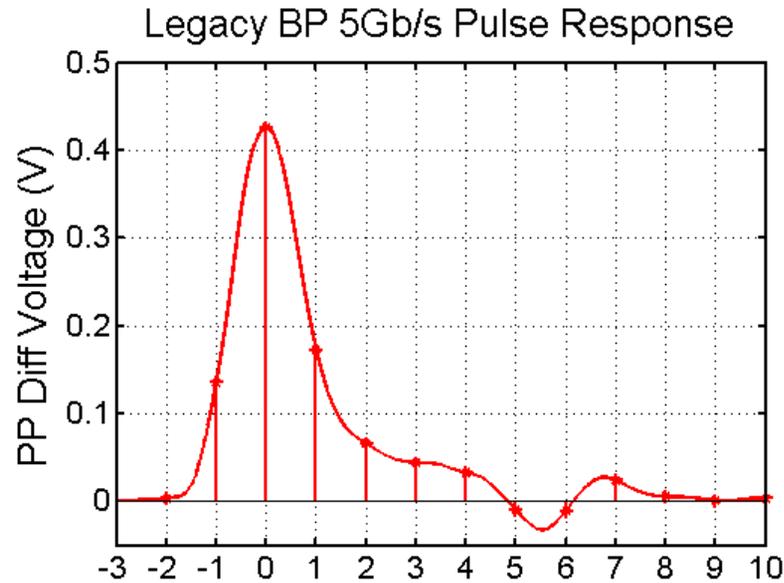
## Worst-Case Bit Pattern Eye



## 10kbits Eye



# Peak Distortion Analysis Example 2

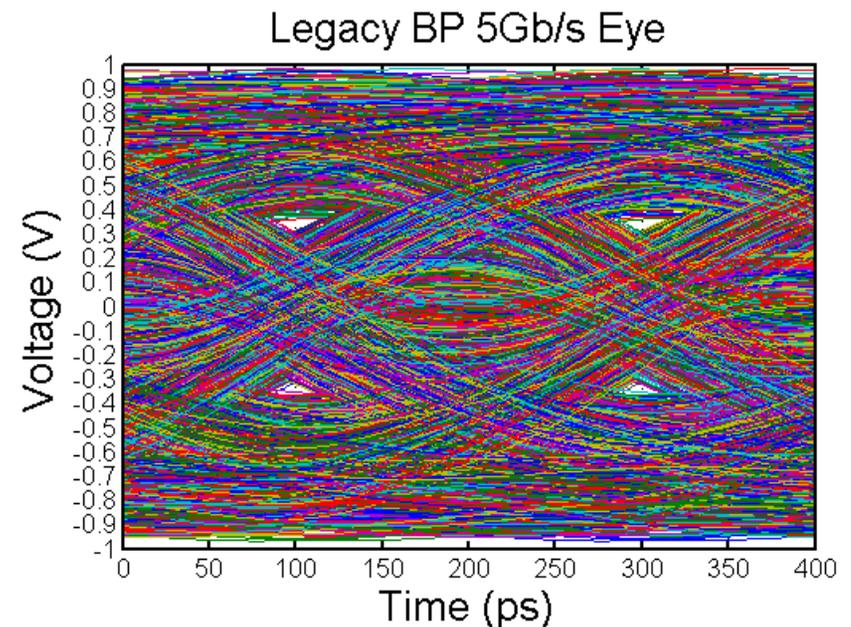
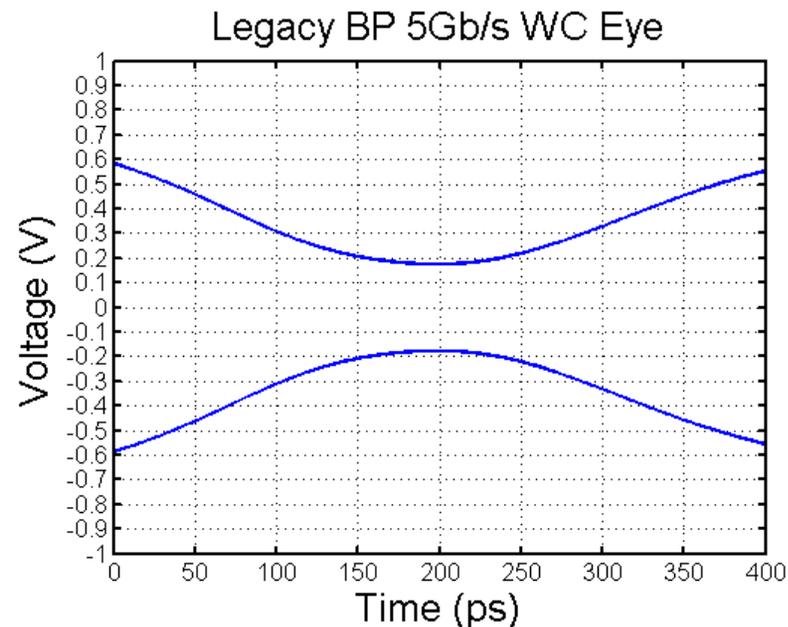


$$y_0^{(1)}(t) = 0.426$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) < 0} = -0.053$$

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT) > 0} = 0.542$$

$$s(t) = 2(0.426 - 0.053 - 0.542) = -0.338$$



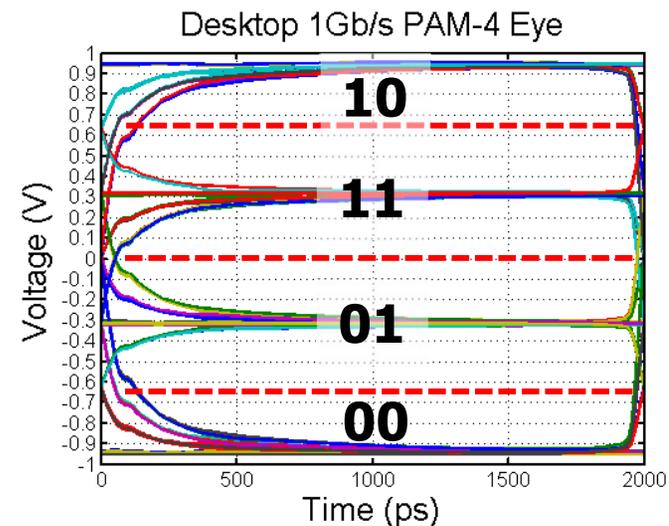
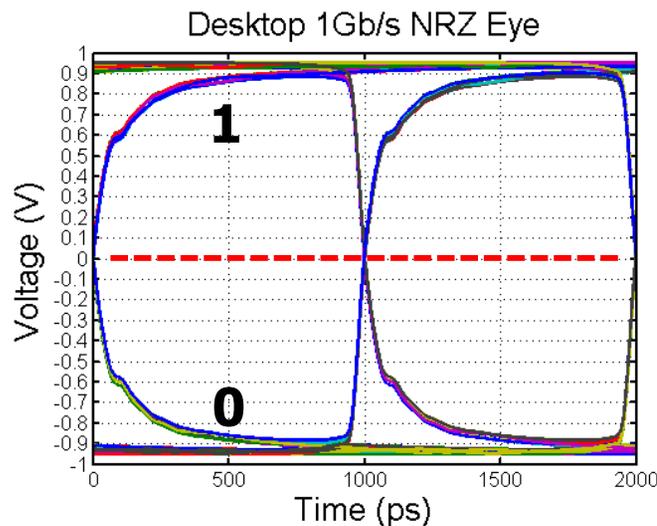
# Agenda

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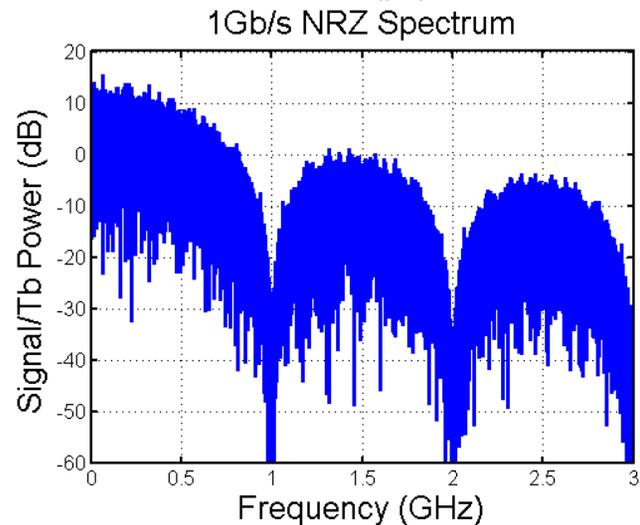
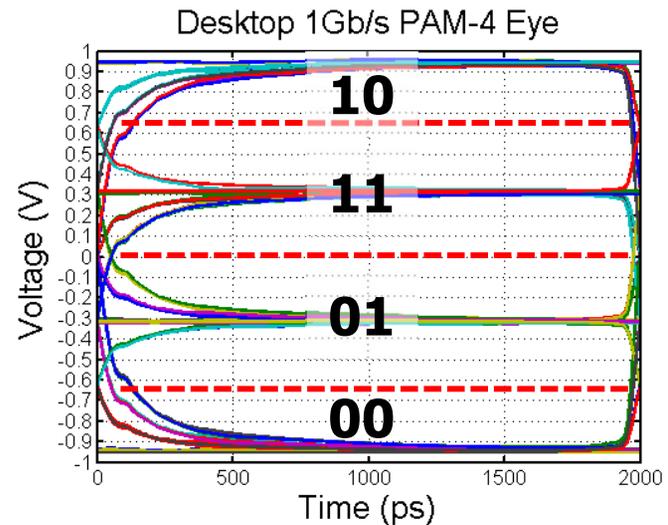
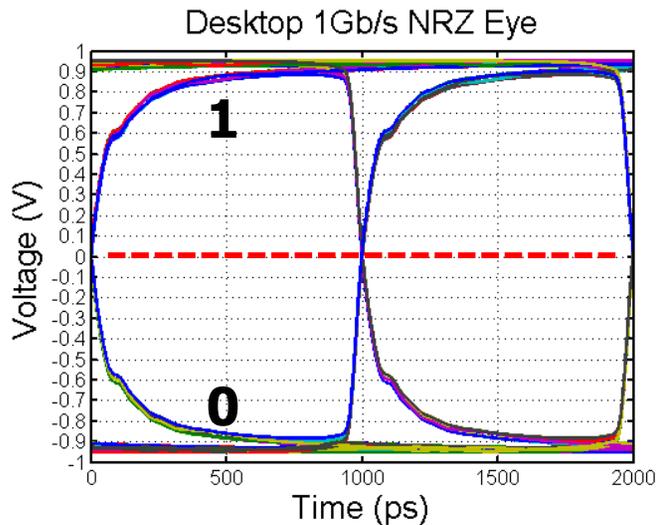
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# PAM-2 (NRZ) vs PAM-4 Modulation

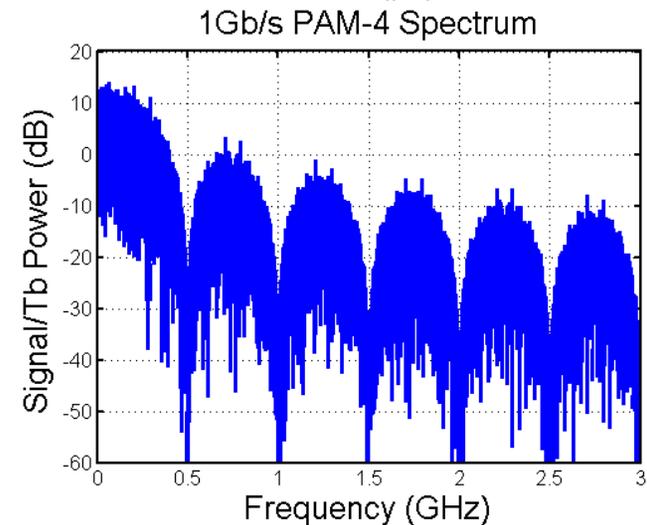
- Binary, NRZ, PAM-2
  - Simplest, most common modulation format
- PAM-4
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run  $\frac{1}{2}$  speed



# Modulation Frequency Spectrum



Majority of signal power  
in 1GHz bandwidth



Majority of signal power  
in 0.5GHz bandwidth

# Nyquist Frequency

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- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect  $R_S$  (symbols/s) without ISI is  $R_S/2$  (Hz)
  - Thus, a system with bandwidth  $W=1/2T=R_S/2$  (Hz) can support a maximum transmission rate of  $2W=1/T=R_S$  (symbols/s) without ISI

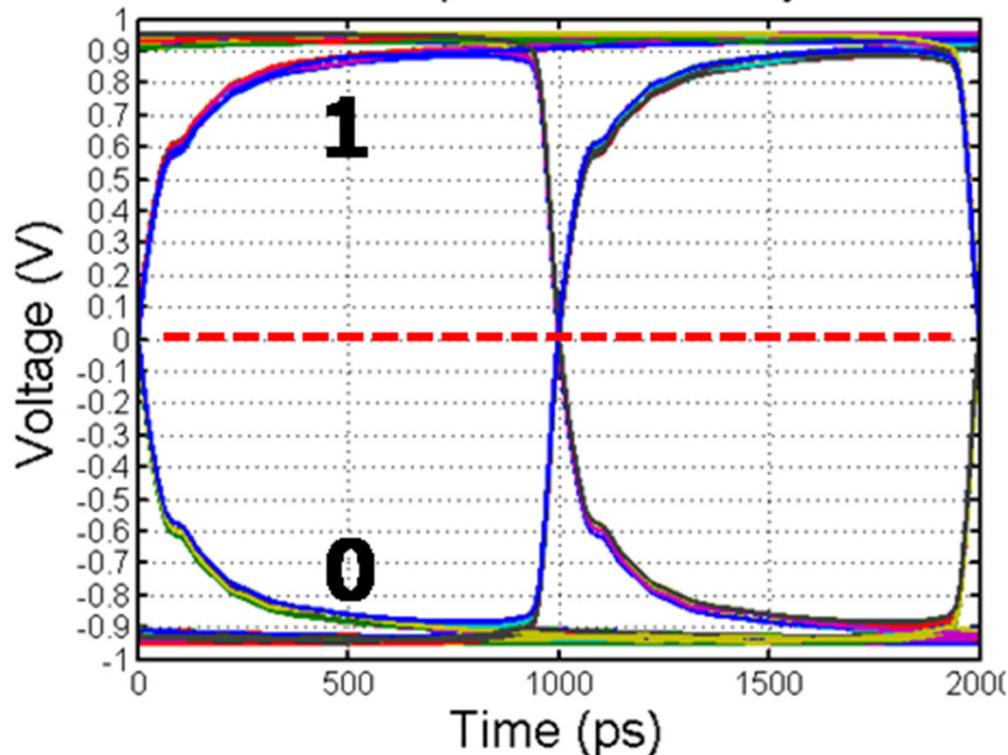
$$\frac{1}{2T} = \frac{R_S}{2} \leq W \Rightarrow \frac{R_S}{W} \leq 2 \quad (\text{symbols/s/Hz})$$

- For ideal Nyquist pulses (sinc), the required bandwidth is only  $R_S/2$  to support an  $R_S$  symbol rate

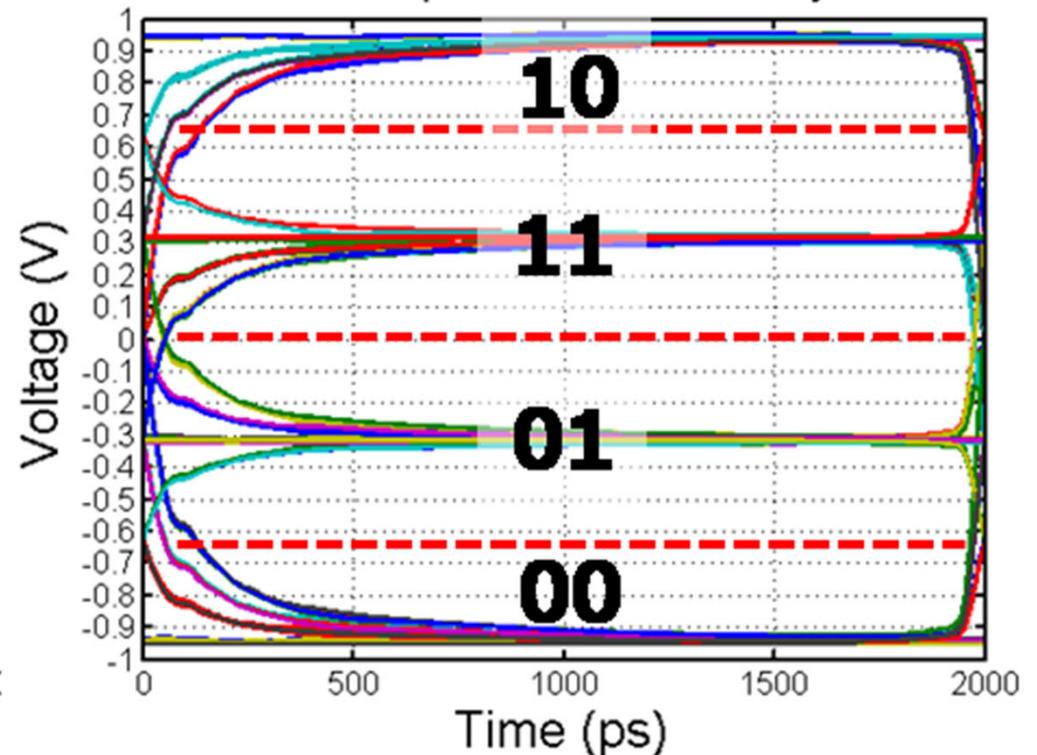
<b>Modulation</b>	<b>Bits/Symbol</b>	<b>Nyquist Frequency</b>
NRZ	1	$R_S/2=1/2T_b$
PAM-4	2	$R_S/2=1/4T_b$

# NRZ vs PAM-4

Desktop 1Gb/s NRZ Eye

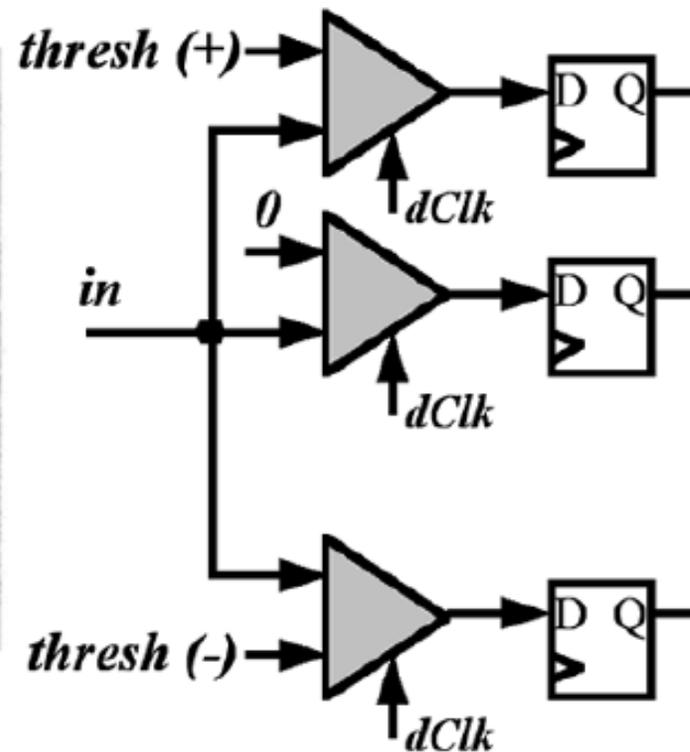
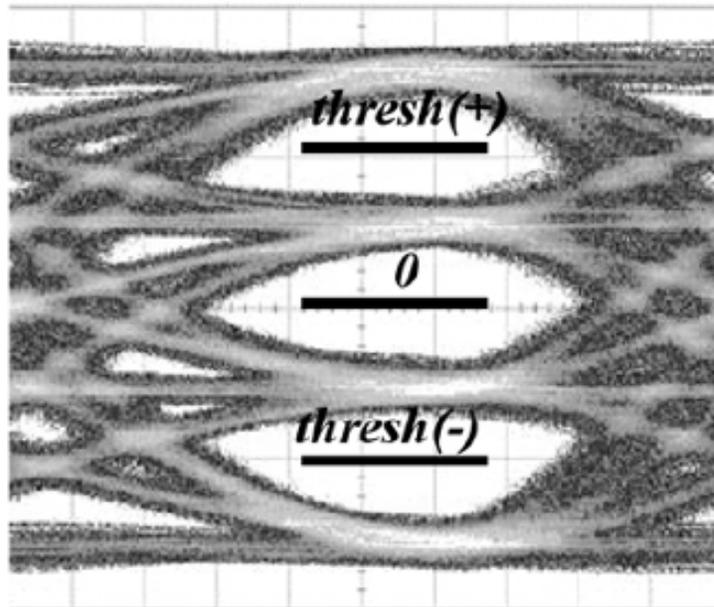


Desktop 1Gb/s PAM-4 Eye



- PAM-4 should be considered when
  - Slope of channel insertion loss ( $S_{21}$ ) exceeds reduction in PAM-4 eye height
    - Insertion loss over an octave is greater than  $20 \cdot \log_{10}(1/3) = -9.54\text{dB}$
  - On-chip clock speed limitations

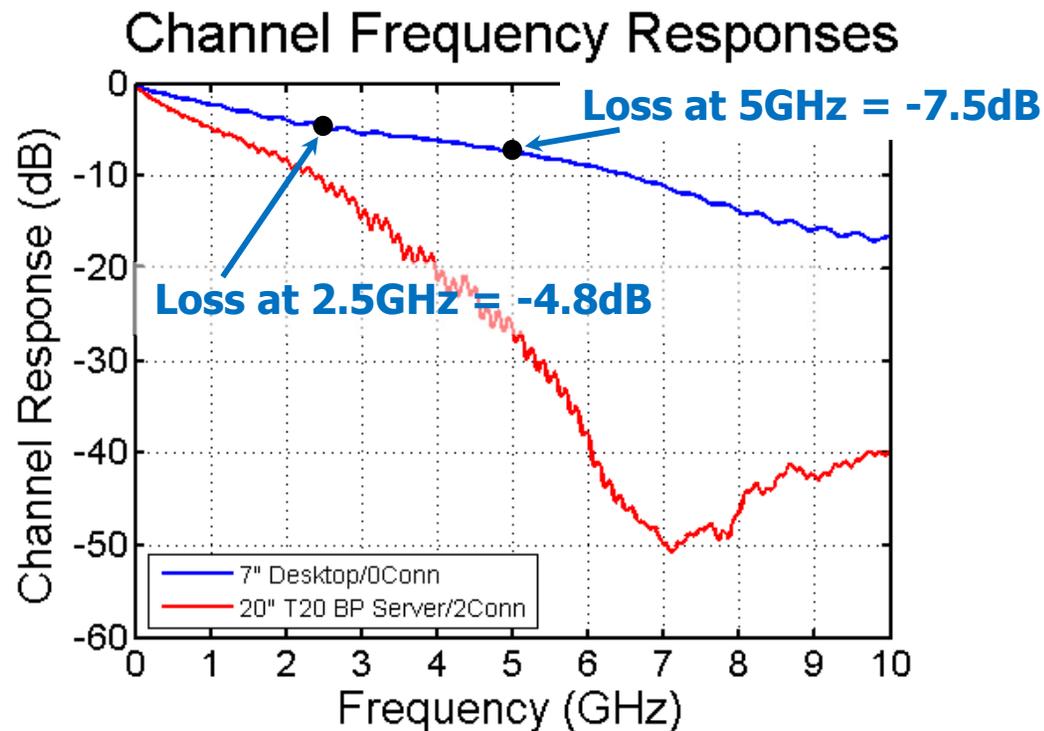
# PAM-4 Receiver



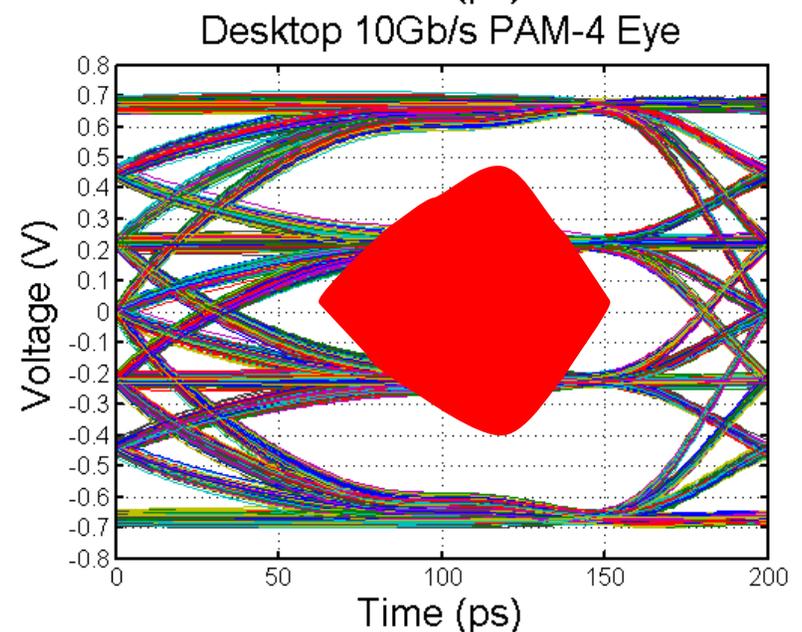
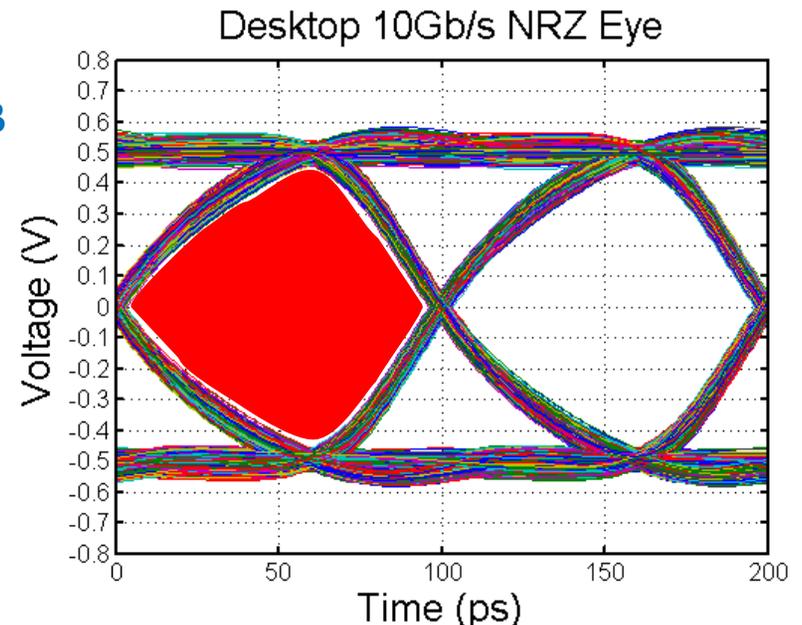
[Stojanovic JSSC 2005]

- 3x the comparators of NRZ RX

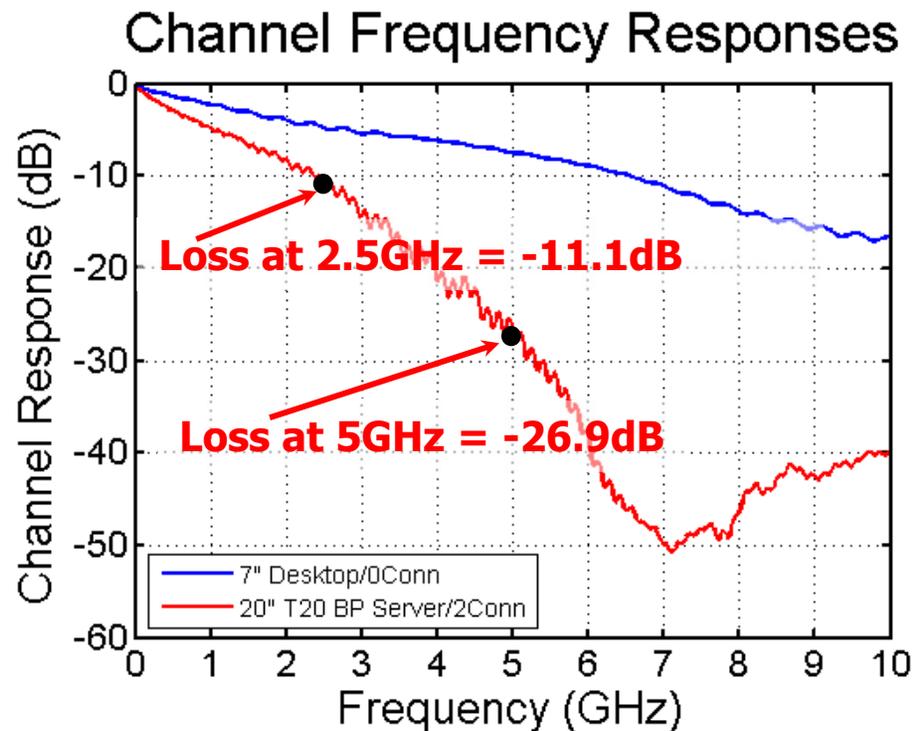
# NRZ vs PAM-4 – Desktop Channel



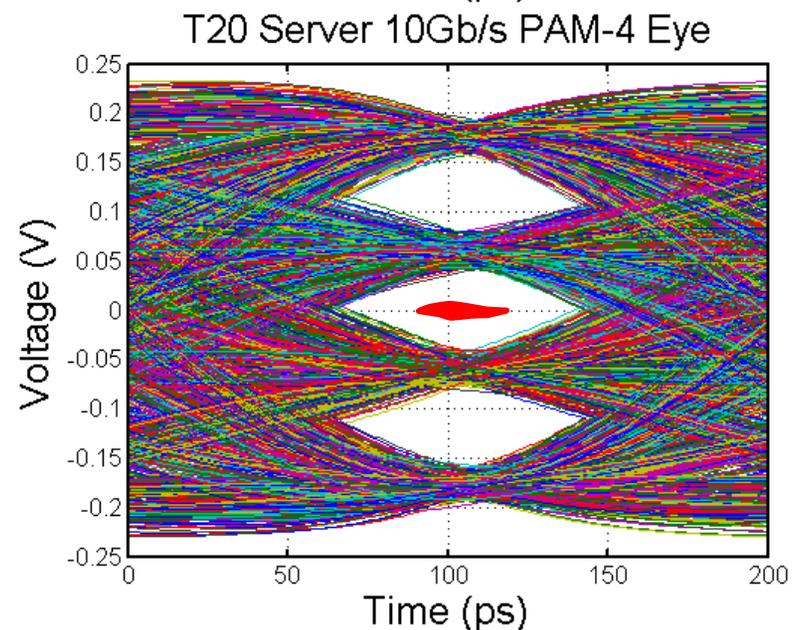
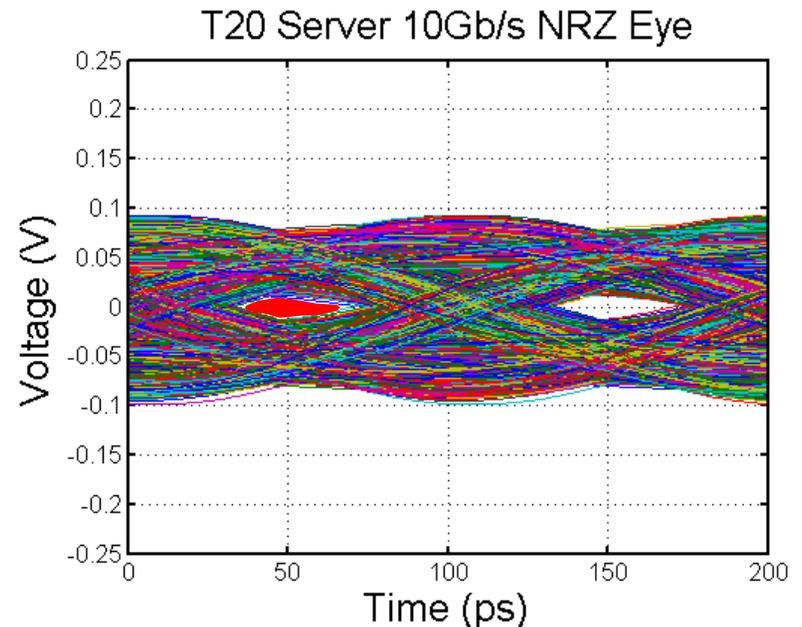
- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is only 2.7dB
  - NRZ has better voltage margin



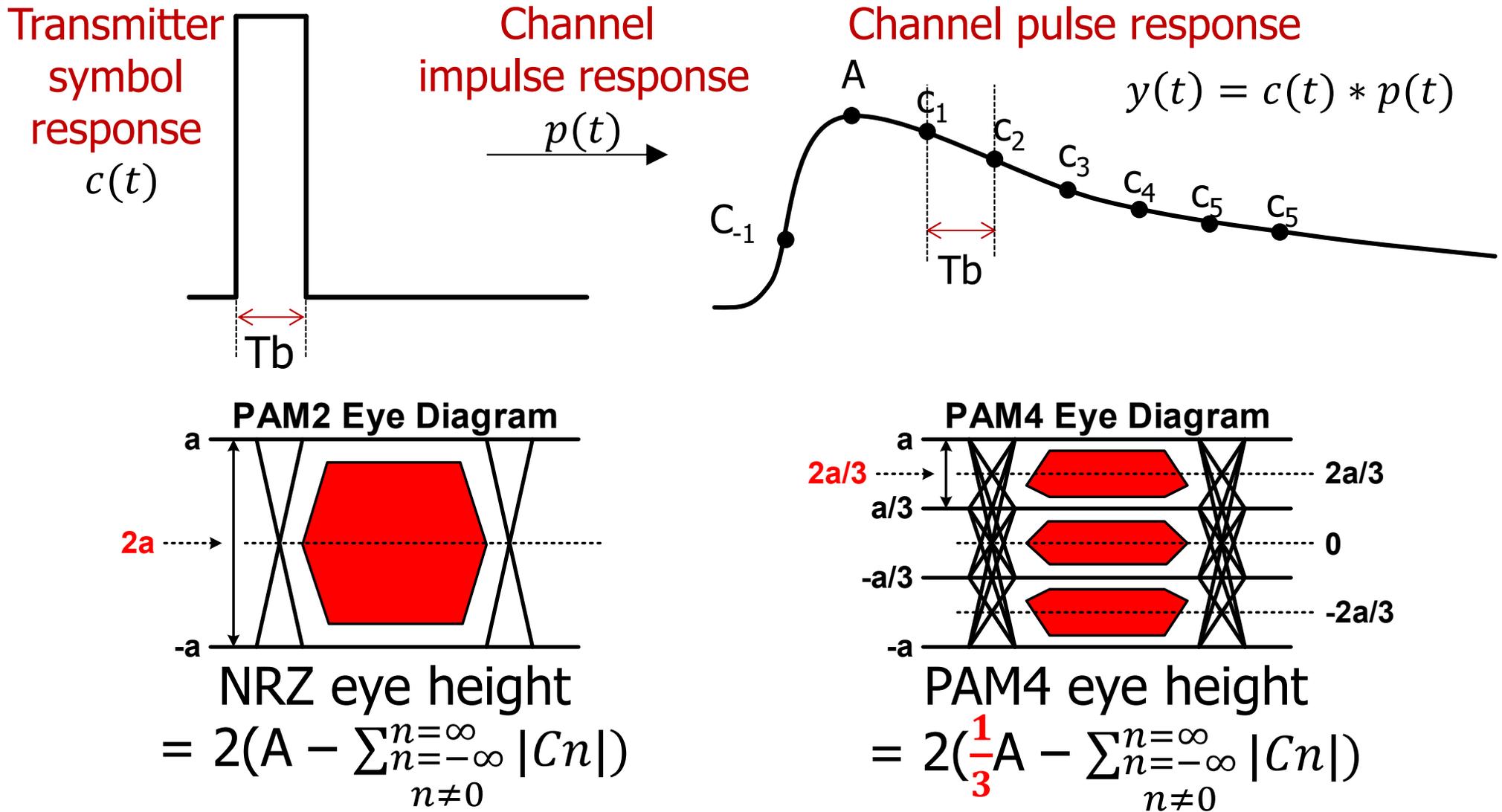
# NRZ vs PAM-4 – T20 Server Channel



- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is 15.8dB
  - PAM-4 “might” be a better choice



# PAM-4 Peak Distortion Analysis



- **PAM4 modulation is more sensitive to residual ISI**

# Multi-Level PAM Challenges

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- Receiver complexity increases considerably
  - 3x input comparators (2-bit ADC)
  - Input signal is no longer self-referenced at 0V differential
    - Need to generate reference threshold levels, which will be dependent on channel loss and TX equalization
- CDR can display extra jitter due to multiple “zero crossing” times
- Smaller eyes are more sensitive to cross-talk due to maximum transitions
- Advanced equalization (DFE) can allow NRZ signaling to have comparable (or better) performance even with >9.5dB loss per octave

# Modulation Take-Away Points

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- Loss-slope guidelines are a good place to start in consideration of alternate modulation schemes
- More advanced modulation trades-off receiver complexity versus equalization complexity
- Advanced modulation challenges
  - Peak TX power limitations
  - Setting RX comparator thresholds and controlling offsets
  - CDR complexity
  - Crosstalk sensitivity (PAM-4)
- Need link analysis tools that consider voltage, timing, and crosstalk noise to choose best modulation scheme for a given channel

# Next Time

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- Link Circuits
  - Termination structures
  - Drivers
  - Receivers