Lecture 4: Channel Pulse Model & Modulation Schemes
Announcements

• Lab 2 report and Prelab 3 due Feb 13

• Reference material
  • Peak distortion analysis paper by Casper
  • Notes from H. Song, Arizona State
  • Papers posted on PAM-2/4 transceivers
Agenda

- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

\[ y^{(1)}(t) = c^{(1)}(t) * h(t) \]
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s

Data = “1000101”

"1" Symbol

\[ c_{k}^{(1)}(t) = u(t - kT) - u(t - (k + 1)T) \]

where \( u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \)

"0" Symbol

\[ c_{k}^{(0)}(t) = -c_{k}^{(1)}(t) \]

[Song]
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1″s and "0″s

\[ V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t) \]
Channel Response to NRZ Data

- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses.

\[
V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H(c^{(d_k)}(t)) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)
\]
Channel Pulse Response

\[ y^{(d_k)}(t) = c^{(d_k)}(t) * h(t) \]

by linearity:
\[ y^{(0)}(t) = -1 * y^{(1)}(t) \]

\[ y^{(1)}(t) \text{ sampled relative to pulse peak:} \]
\[ [... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...] \]
\[ k = [... -2 1 0 1 2 3 4 5 6 ...] \]
Channel Data Stream Response

Input Data Stream

Pulse Responses

Channel Response
Channel “FIR” Model

\[ c_0^{(1)}(t) \xrightarrow{\mathbf{H}} H(c_0^{(1)}(t)) = y_0^{(1)}(t) \]

\[ H(c_0^{(1)}(t)) = y_0^{(1)}(t) \]

\( D \) is the delay from the channel input to the output pulse peak

\[ y^{(1)}(t) \text{ sampled relative to pulse peak:} \]

\[ [... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]\n
\[ a = [... a_{-2} a_{-1} a_0 a_1 a_2 a_3 a_4 a_5 a_6 ...] \]
Agenda

• ISI and channel pulse model

• Peak distortion analysis

• Compare NRZ (PAM-2) and PAM-4 modulation
Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response

- Worst-case “1” is summation of a “1” pulse with all negative non k=0 pulse responses

\[ s_1(t) = y_0^{(1)}(t) + \sum_{k=\infty}^{\infty} y^{(d_k)}(t - kT) \bigg|_{y(t-kT)<0} \]

- Worst-case “0” is summation of a “0” pulse with all positive non k=0 pulse responses

\[ s_0(t) = y_0^{(0)}(t) + \sum_{k=\infty}^{\infty} y^{(d_k)}(t - kT) \bigg|_{y(t-kT)>0} \]
Peak Distortion Analysis

- Worst-case eye height is $s_1(t) - s_0(t)$

$$s(t) = s_1(t) - s_0(t) = (y_0^{(1)}(t) - y_0^{(0)}(t)) + \left( \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)|_{y(t-kT)<0} - \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)|_{y(t-kT)>0} \right)$$

Because $y_0^{(0)}(t) = -1(y_0^{(1)}(t))$

$$s(t) = 2 \left( y_0^{(1)}(t) + \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT)|_{y(t-kT)<0} - \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT)|_{y(t-kT)>0} \right)$$

- If symmetric "1" and "0" pulses (linearity), then only positive pulse response is needed
Peak Distortion Analysis Example 1

\[ y_0^{(1)}(t) = 0.540 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT)<0} = -0.007 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT)>0} = 0.389 \]

\[ s(t) = 2(0.540 - 0.007 - 0.389) = 0.288 \]
Worst-Case Bit Pattern

- Pulse response can be used to find the worst-case bit pattern

\[ \text{Pulse } a = [\ldots a_{-2} \ a_{-1} \ a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ \ldots] \]

- Flip pulse matrix about cursor \( a_0 \) and the bits are the inverted sign of the pulse ISI

\[ [\ldots -\text{sign}(a_6) \ -\text{sign}(a_5) \ -\text{sign}(a_4) \ -\text{sign}(a_3) \ -\text{sign}(a_2) \ -\text{sign}(a_1) \ 1 \ -\text{sign}(a_{-1}) \ -\text{sign}(a_{-2}) \ \ldots] \]
Peak Distortion Analysis Example 2

\[ y_0^{(1)}(t) = 0.426 \]

\[
\sum_{k=-\infty}^{\infty} y^{(1)}(t - kT)\bigg|_{y(t-kT)<0} = -0.053
\]

\[
\sum_{k=-\infty}^{\infty} y^{(1)}(t - kT)\bigg|_{y(t-kT)>0} = 0.542
\]

\[ s(t) = 2(0.426 - 0.053 - 0.542) = -0.338 \]
Agenda

- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation
PAM-2 (NRZ) vs PAM-4 Modulation

- Binary, NRZ, PAM-2
  - Simplest, most common modulation format
- PAM-4
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run $\frac{1}{2}$ speed
Modulation Frequency Spectrum

**Desktop 1Gb/s NRZ Eye**

Majority of signal power in 1GHz bandwidth

**Desktop 1Gb/s PAM-4 Eye**

Majority of signal power in 0.5GHz bandwidth
Nyquist Frequency

- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect $R_S$ (symbols/s) without ISI is $R_S/2$ (Hz)
  - Thus, a system with bandwidth $W=1/2T=R_S/2$ (Hz) can support a maximum transmission rate of $2W=1/T=R_S$ (symbols/s) without ISI

$$\frac{1}{2T} = \frac{R_S}{2} \leq W \Rightarrow \frac{R_S}{W} \leq 2 \text{ (symbols/s/Hz)}$$

- For ideal Nyquist pulses (sinc), the required bandwidth is only $R_S/2$ to support an $R_S$ symbol rate

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Bits/Symbol</th>
<th>Nyquist Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ</td>
<td>1</td>
<td>$R_S/2=1/2T_b$</td>
</tr>
<tr>
<td>PAM-4</td>
<td>2</td>
<td>$R_S/2=1/4T_b$</td>
</tr>
</tbody>
</table>
NRZ vs PAM-4

- PAM-4 should be considered when
  - Slope of channel insertion loss ($S_{21}$) exceeds reduction in PAM-4 eye height
    - Insertion loss over an octave is greater than $20 \times \log_{10}(1/3) = -9.54$ dB
  - On-chip clock speed limitations
PAM-4 Receiver

- 3x the comparators of NRZ RX

[Stojanovic JSSC 2005]
NRZ vs PAM-4 – Desktop Channel

- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is only 2.7dB
  - NRZ has better voltage margin

Loss at 5GHz = -7.5dB
Loss at 2.5GHz = -4.8dB

Loss at 2.5GHz = -4.8dB
Loss at 5GHz = -7.5dB
NRZ vs PAM-4 – T20 Server Channel

- Eyes are produced with 4-tap TX FIR equalization

- Loss in the octave between 2.5 and 5GHz is 15.8dB
  - PAM-4 “might” be a better choice
PAM-4 Peak Distortion Analysis

Transmitter symbol response \( c(t) \)

Channel impulse response \( p(t) \)

Channel pulse response

\[ y(t) = c(t) \ast p(t) \]

\[
\text{NRZ eye height} = 2(A - \sum_{n=-\infty}^{n=\infty} |C_n|)
\]

\[
\text{PAM2 eye height} = 2a
\]

\[
\text{PAM4 eye height} = 2\left(\frac{1}{3}A - \sum_{n=-\infty}^{n=\infty} |C_n|\right)
\]

- PAM4 modulation is more sensitive to residual ISI
Multi-Level PAM Challenges

- Receiver complexity increases considerably
  - 3x input comparators (2-bit ADC)
  - Input signal is no longer self-referenced at 0V differential
    - Need to generate reference threshold levels, which will be dependent on channel loss and TX equalization

- CDR can display extra jitter due to multiple “zero crossing” times

- Smaller eyes are more sensitive to cross-talk due to maximum transitions

- Advanced equalization (DFE) can allow NRZ signaling to have comparable (or better) performance even with >9.5dB loss per octave
Modulation Take-Away Points

- Loss-slope guidelines are a good place to start in consideration of alternate modulation schemes

- More advanced modulation trades-off receiver complexity versus equalization complexity

- Advanced modulation challenges
  - Peak TX power limitations
  - Setting RX comparator thresholds and controlling offsets
  - CDR complexity
  - Crosstalk sensitivity (PAM-4)

- Need link analysis tools that consider voltage, timing, and crosstalk noise to choose best modulation scheme for a given channel
Next Time

- Link Circuits
  - Termination structures
  - Drivers
  - Receivers