ECEN720: High-Speed Links Circuits and Systems Spring 2023

Lecture 4: Channel Pulse Model & Modulation Schemes

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Announcements

- Lab 2 report and Prelab 3 due Feb 13

- Reference material
  - Peak distortion analysis paper by Casper
  - Notes from H. Song, Arizona State
  - Papers posted on PAM-2/4 transceivers

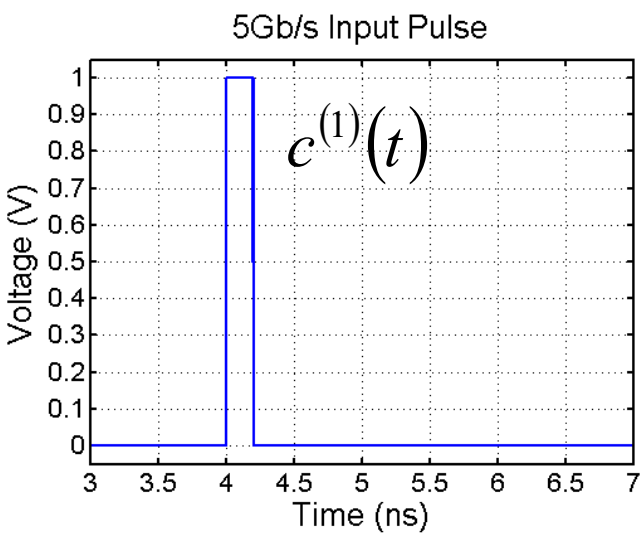
Agenda

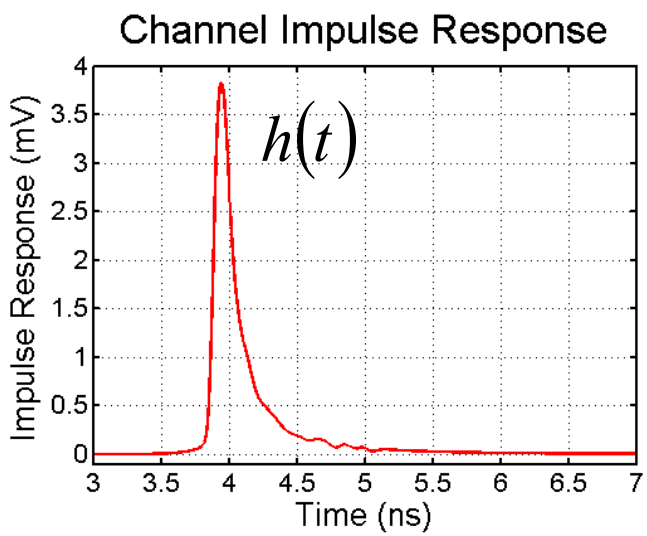
- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation

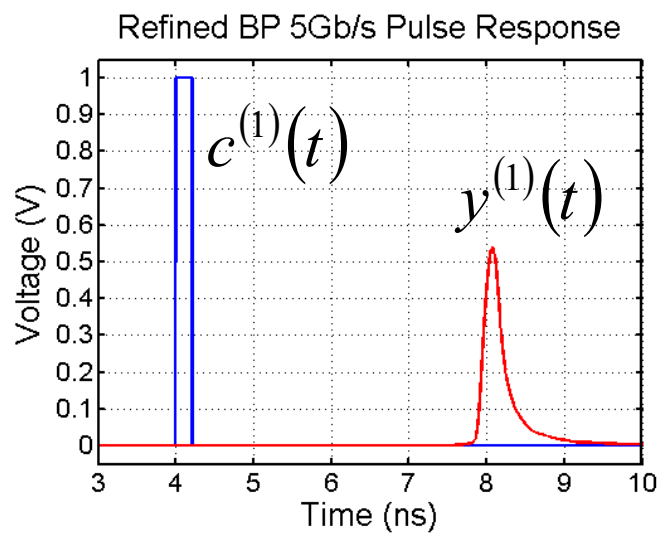
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

$$y^{(1)}(t) = c^{(1)}(t) * h(t)$$



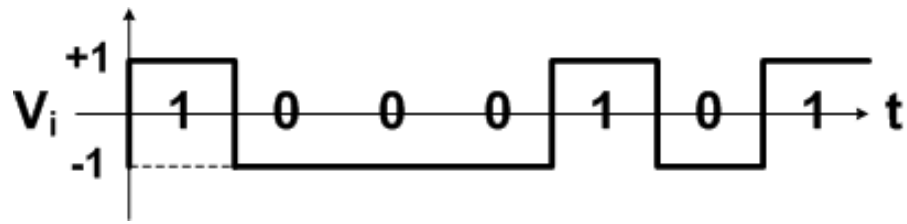




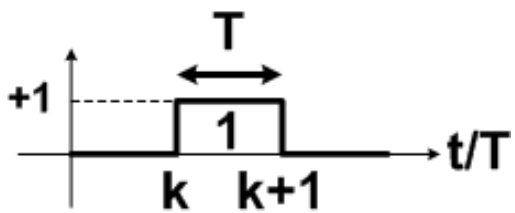
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s

Data = "1000101"

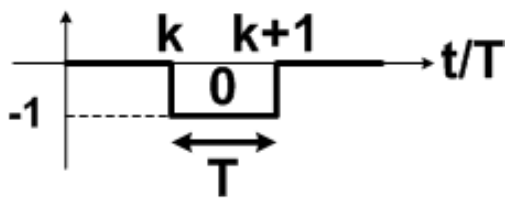


"1" Symbol



$$c_k^{(1)}(t) \equiv u(t - kT) - u(t - (k+1)T)$$

"0" Symbol



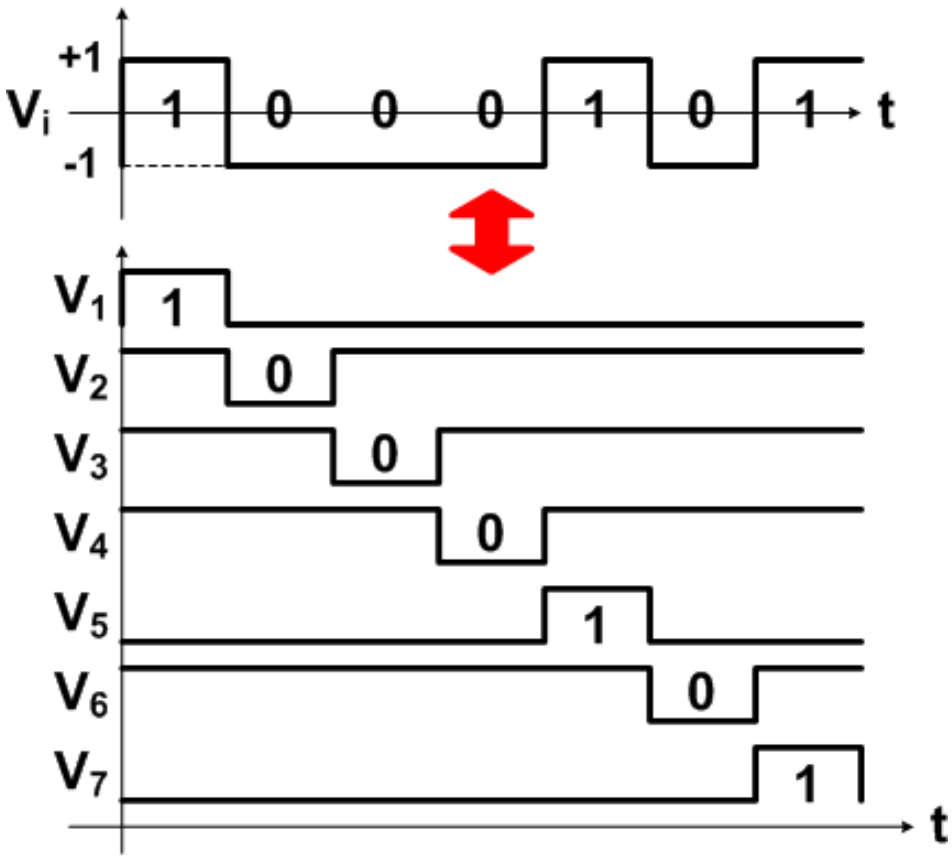
$$c_k^{(0)}(t) = -c_k^{(1)}(t)$$

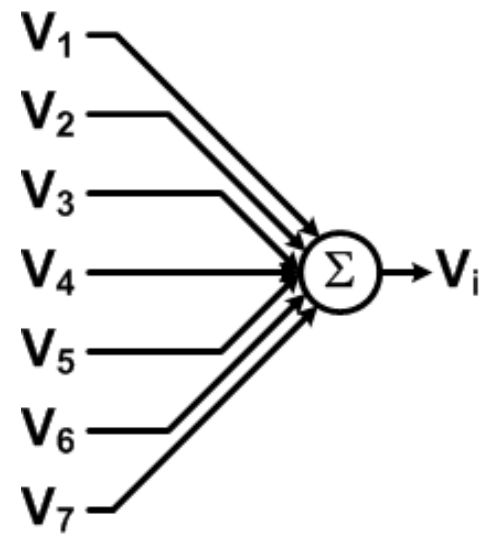
[**Song**]

$$\text{where} \quad u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s



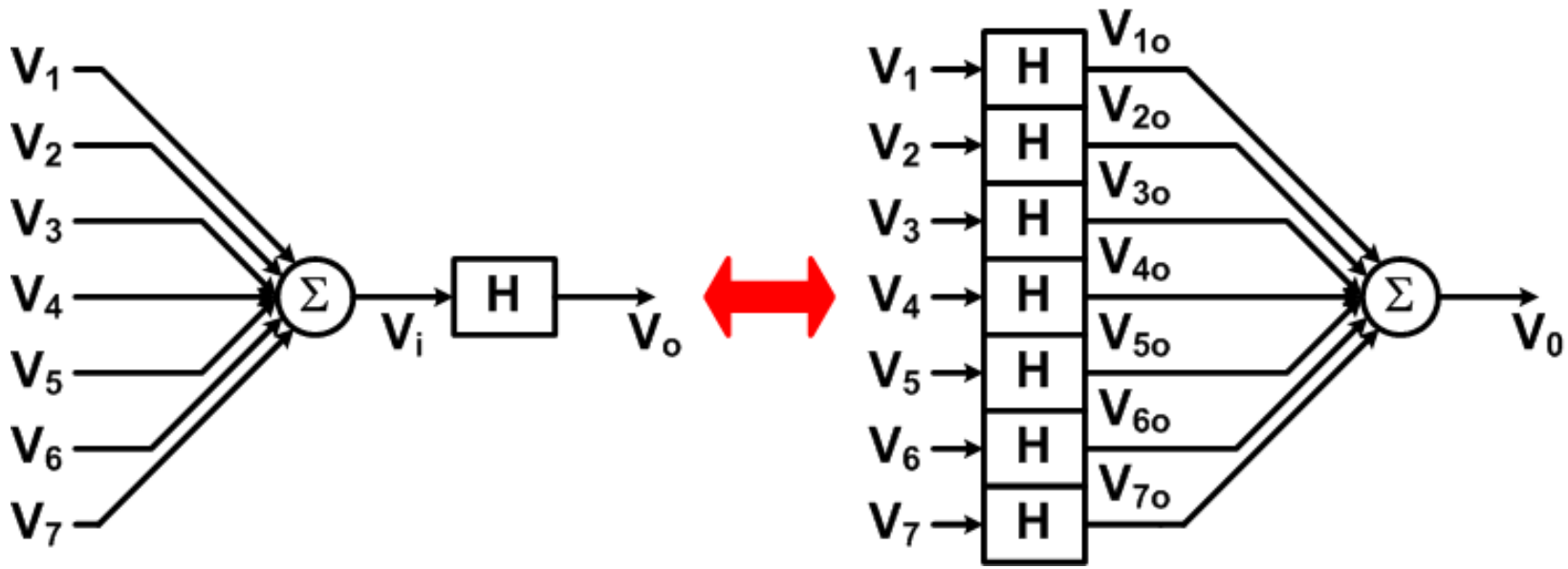


$$V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t)$$

[**Song**]

Channel Response to NRZ Data

- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses

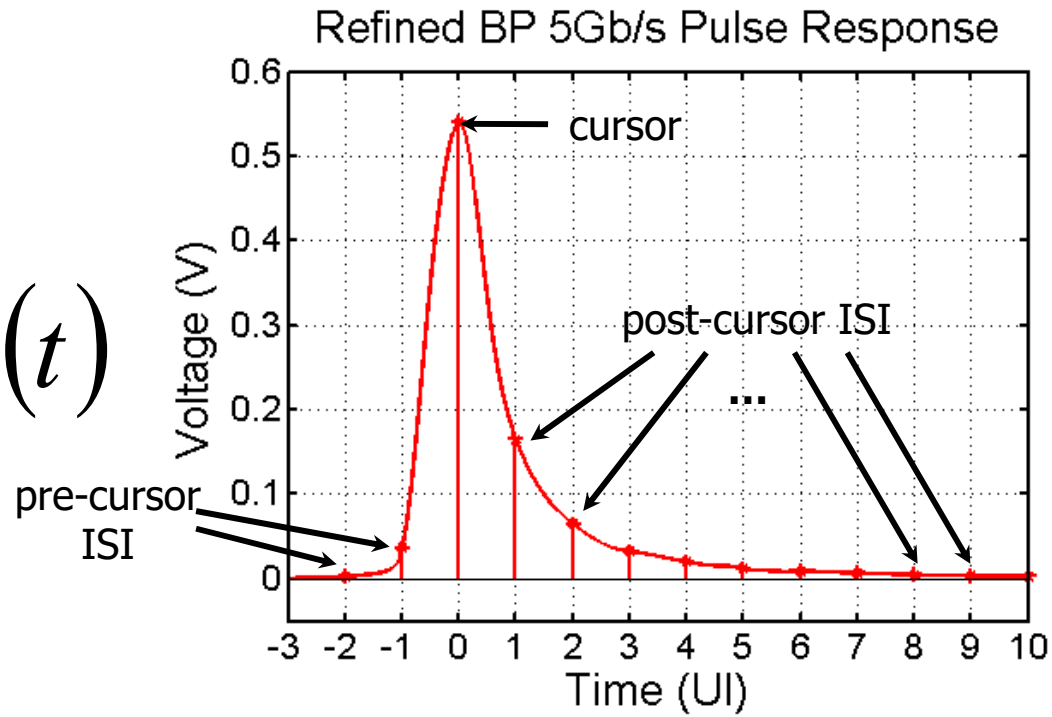


$$V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H\left(c_k^{(d_k)}(t)\right) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)$$

[**Song**]

Channel Pulse Response

$$y^{(d_k)}(t) = c^{(d_k)}(t) * h(t)$$



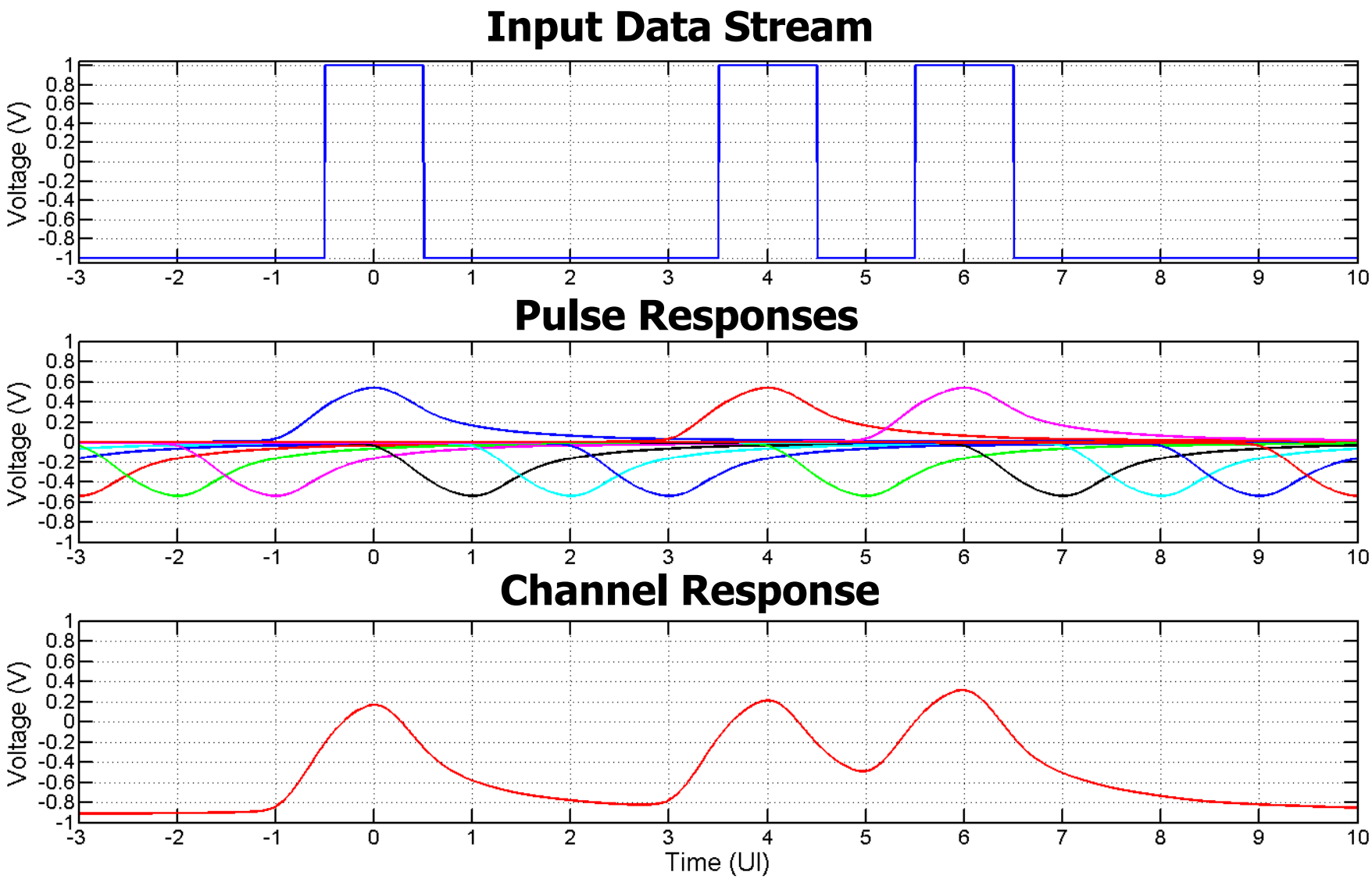
$y^{(1)}(t)$ sampled relative to pulse peak:

[… 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 …]

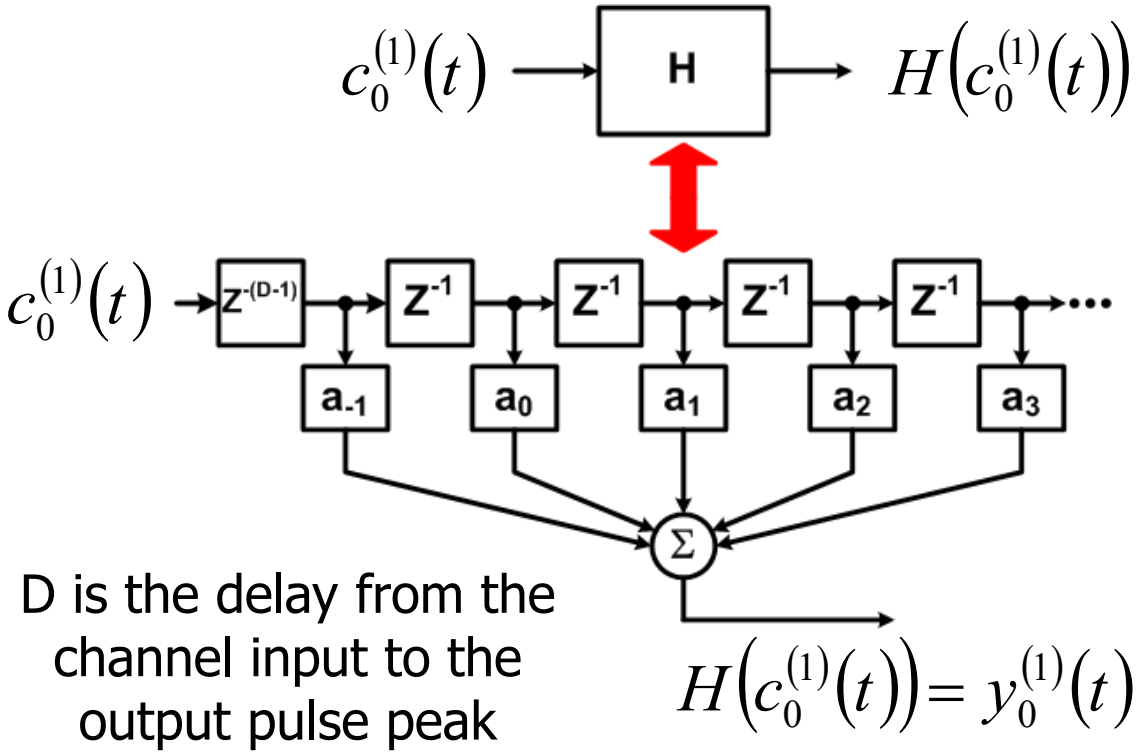
k =[… -2 1 0 1 2 3 4 5 6 …]

By Linearity: $y^{(0)}(t) = -1 * y^{(1)}(t)$

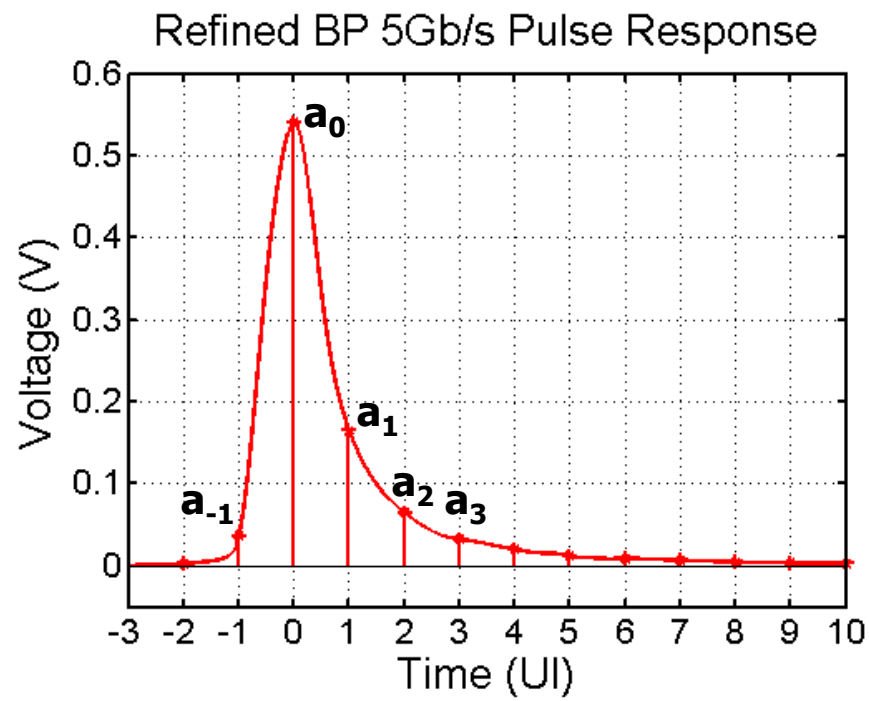
Channel Data Stream Response



Channel "FIR" Model



D is the delay from the channel input to the output pulse peak



$y^{(1)}(t)$ sampled relative to pulse peak:

[… 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 …]

a =[… a_{-2} a_{-1} a_0 a_1 a_2 a_3 a_4 a_5 a_6 …]

Agenda

- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation

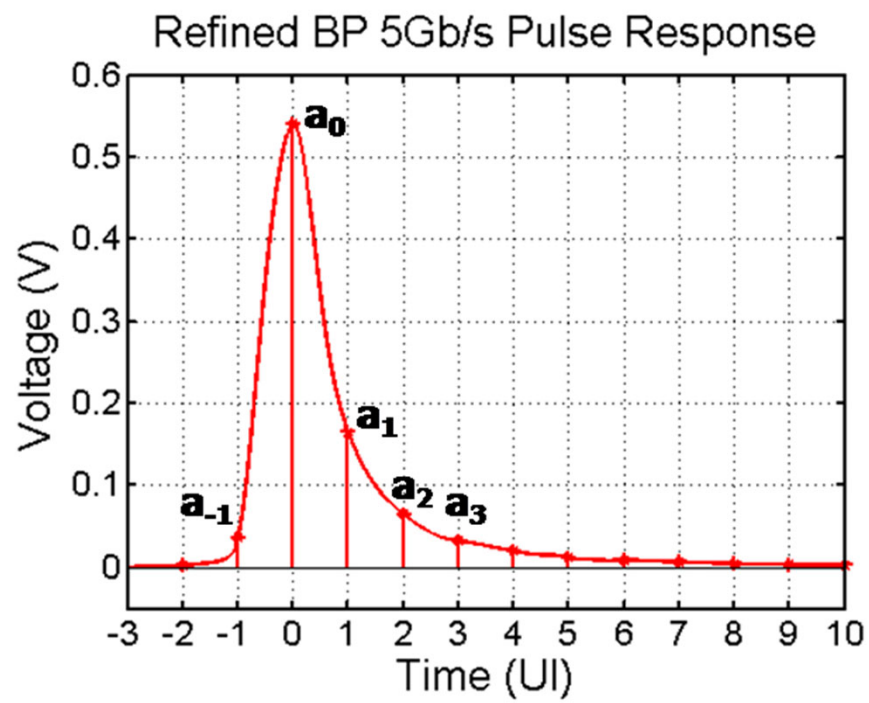
Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case "1" is summation of a "1" pulse with all negative non k=0 pulse responses

$$s_1(t) = y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0}$$

- Worst-case "0" is summation of a "0" pulse with all positive non k=0 pulse responses

$$s_0(t) = y_0^{(0)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0}$$



Peak Distortion Analysis

- Worst-case eye height is $s_1(t)$-$s_0(t)$

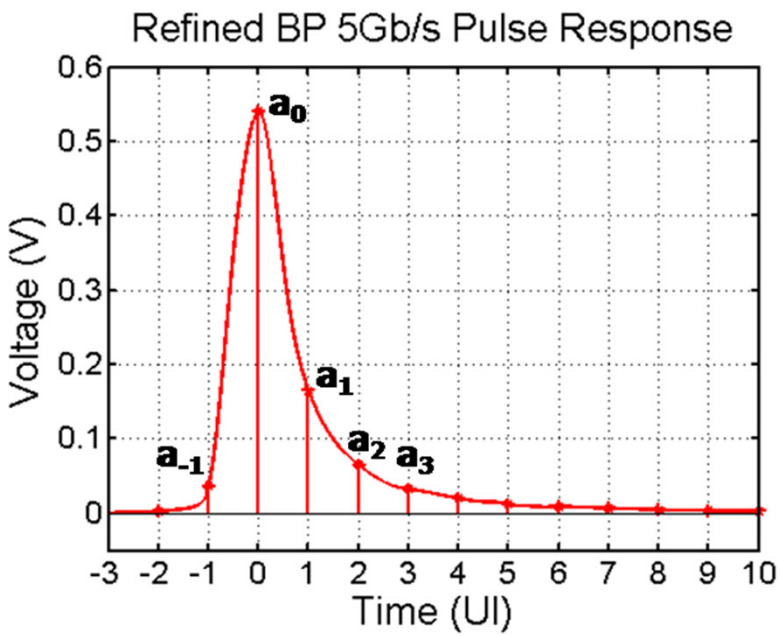
$$s(t)=s_1(t)-s_0(t)=\left(y_0^{(1)}(t)-y_0^{(0)}(t)\right)+\left(\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0}-\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0}\right)$$

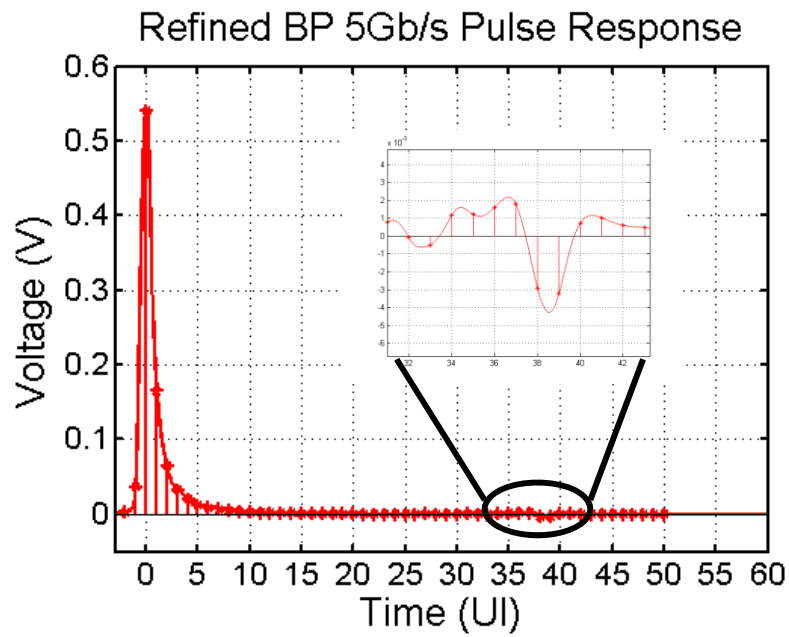
$$\text{Because } y_0^{(0)}(t)=-1\left(y_0^{(1)}(t)\right)$$

$$s(t)=2\left(\underbrace{y_0^{(1)}(t)+\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0}}_{\text{“1” pulse worst-case “1” edge}}-\underbrace{\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0}}_{\text{“1” pulse worst-case “0” edge}}\right)$$

- If symmetric “1” and “0” pulses (linearity), then only positive pulse response is needed

Peak Distortion Analysis Example 1



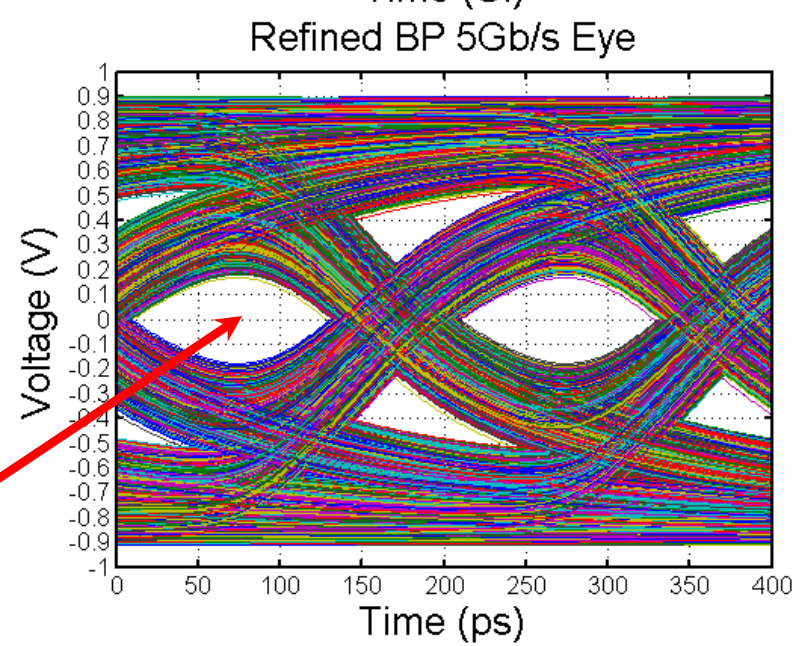


$$y_0^{(1)}(t) = 0.540$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



Worst-Case Bit Pattern

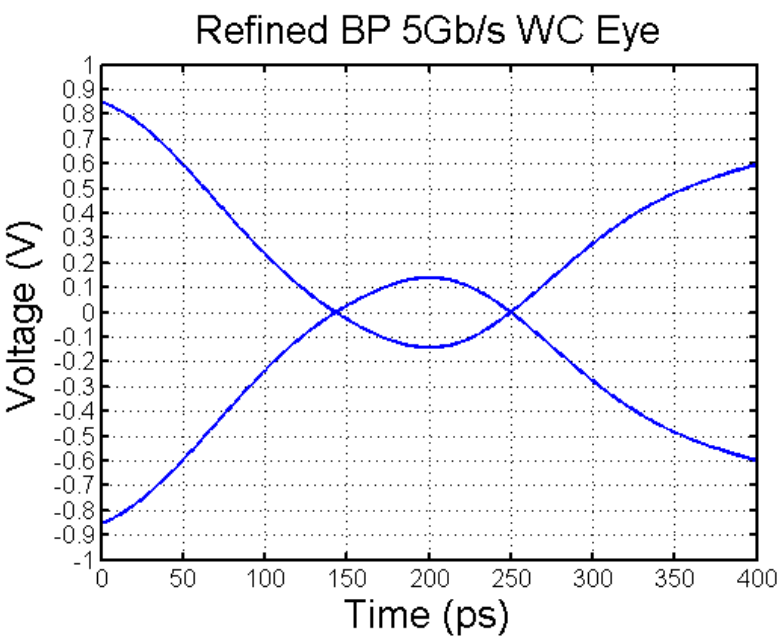
- Pulse response can be used to find the worst-case bit pattern

$$\text{Pulse} \quad \mathrm{a} = [\ldots \; a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \ldots]$$

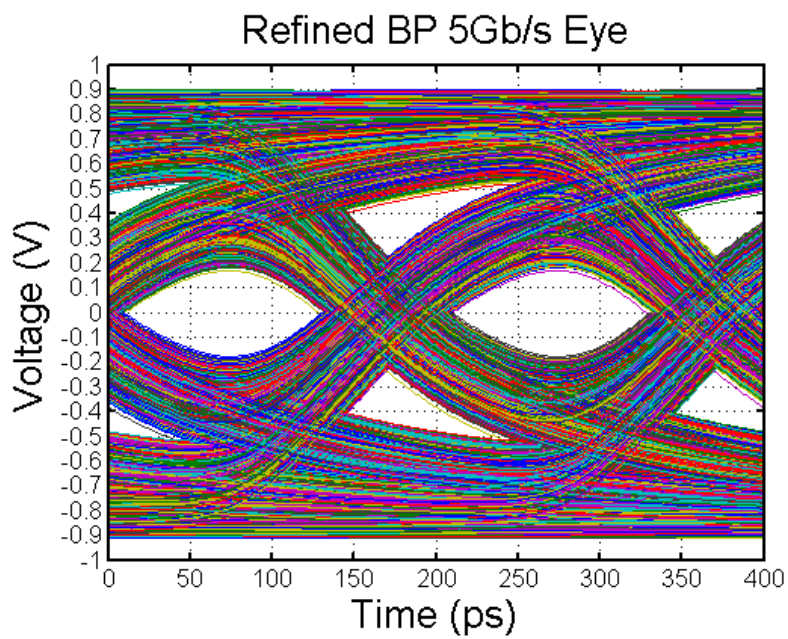
- Flip pulse matrix about cursor a_0 and the bits are the inverted sign of the pulse ISI

$$[\ldots \; -sign(a_6) \quad -sign(a_5) \quad -sign(a_4) \quad -sign(a_3) \quad -sign(a_2) \quad -sign(a_1) \quad 1 \quad -sign(a_{-1}) \quad -sign(a_{-2}) \quad \ldots]$$

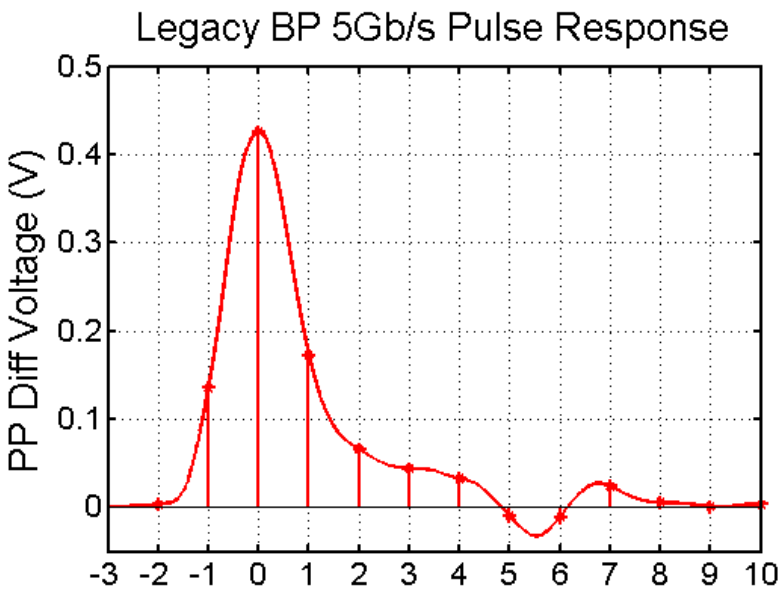
Worst-Case Bit Pattern Eye



10kbits Eye



Peak Distortion Analysis Example 2

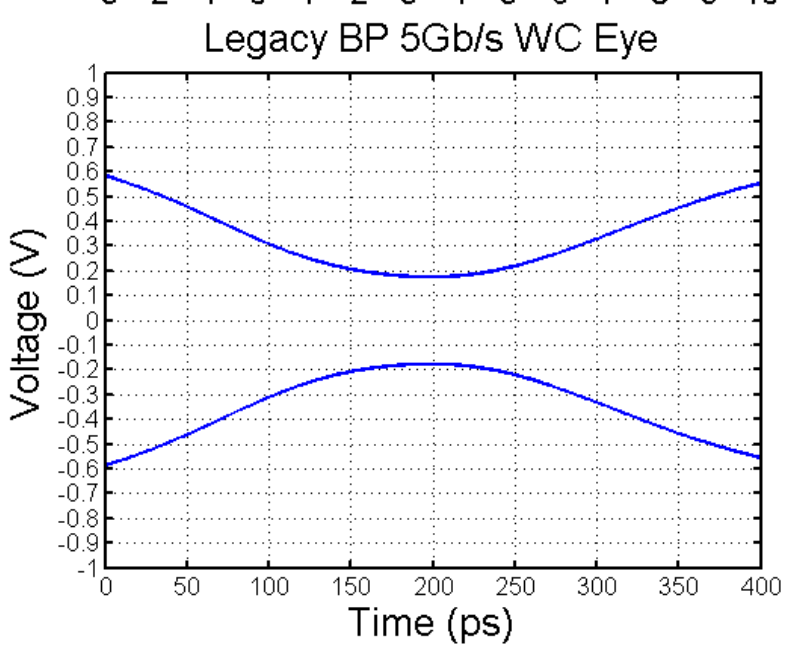


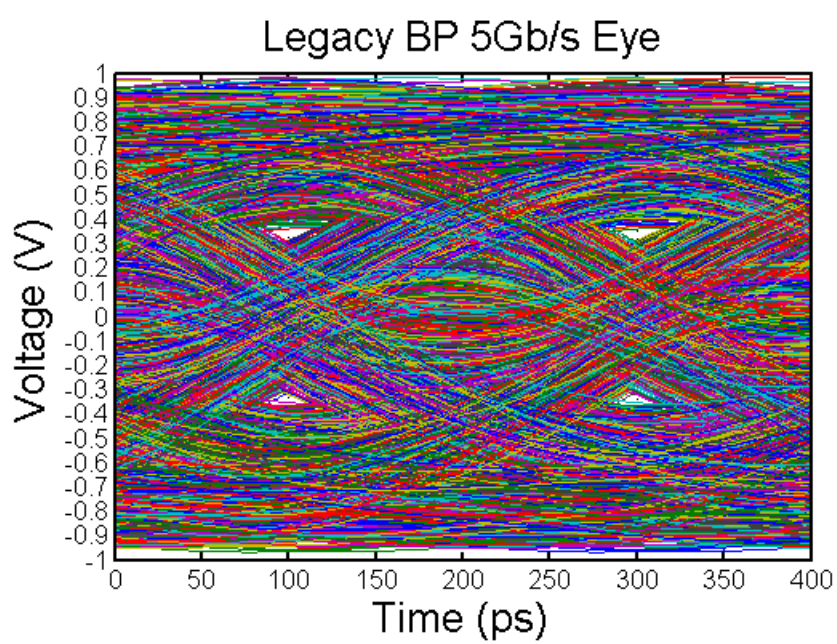
$$y_0^{(1)}(t) = 0.426$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.053$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.542$$

$$s(t) = 2(0.426 - 0.053 - 0.542) = -0.338$$



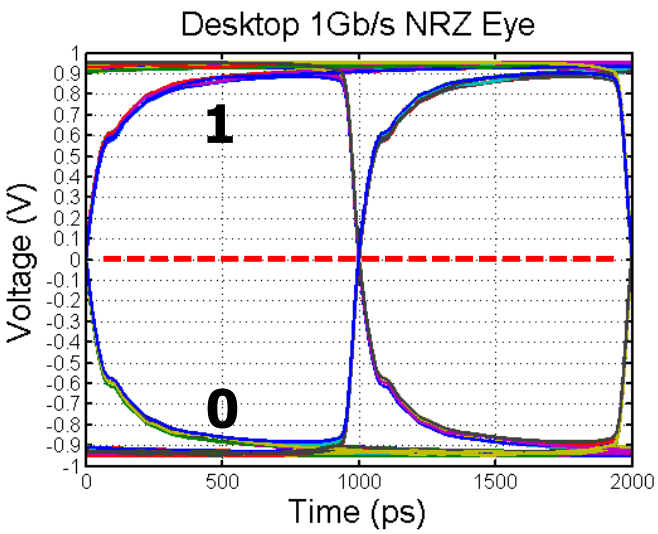


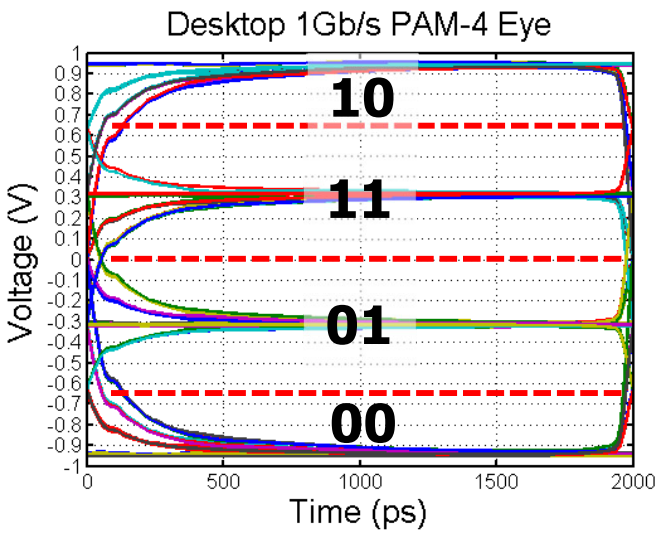
Agenda

- ISI and channel pulse model
- Peak distortion analysis
- Compare NRZ (PAM-2) and PAM-4 modulation

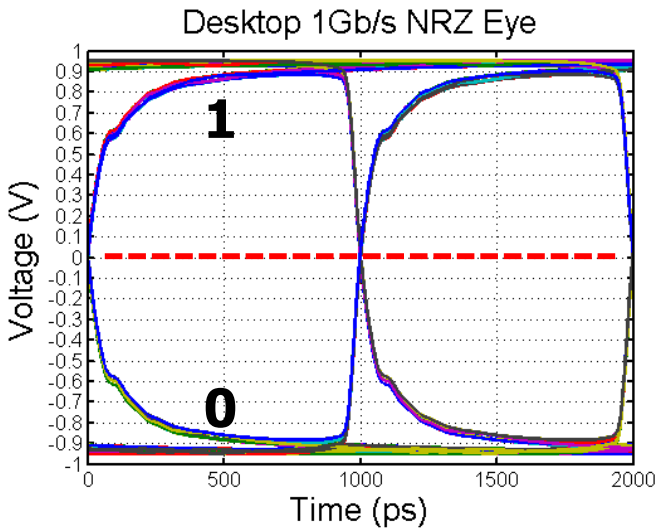
PAM-2 (NRZ) vs PAM-4 Modulation

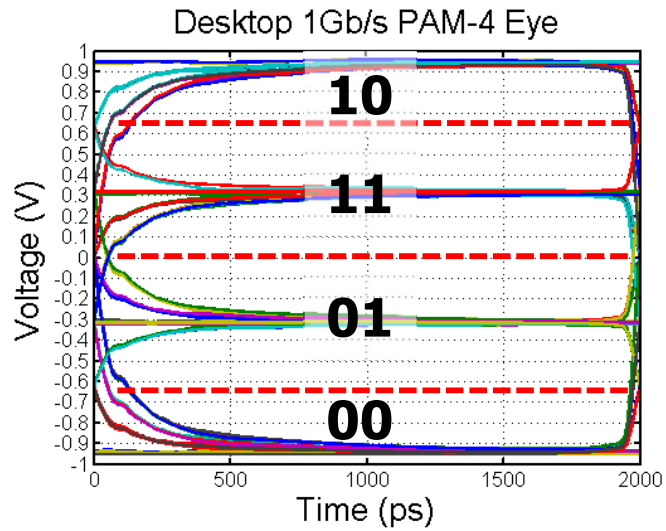
- Binary, NRZ, PAM-2
  - Simplest, most common modulation format
- PAM-4
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run ½ speed

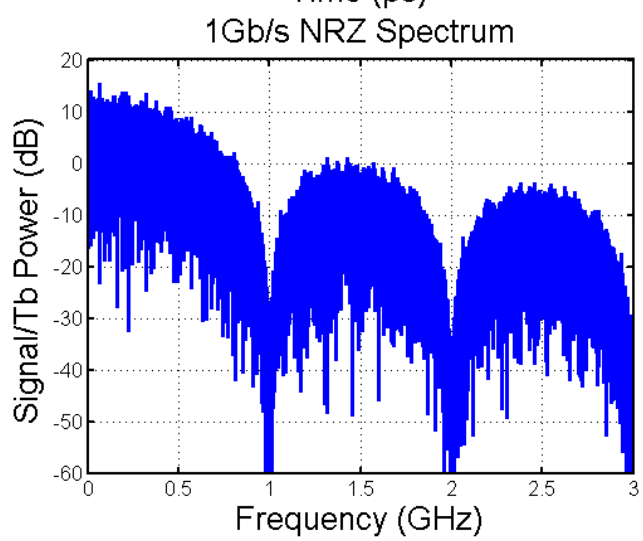


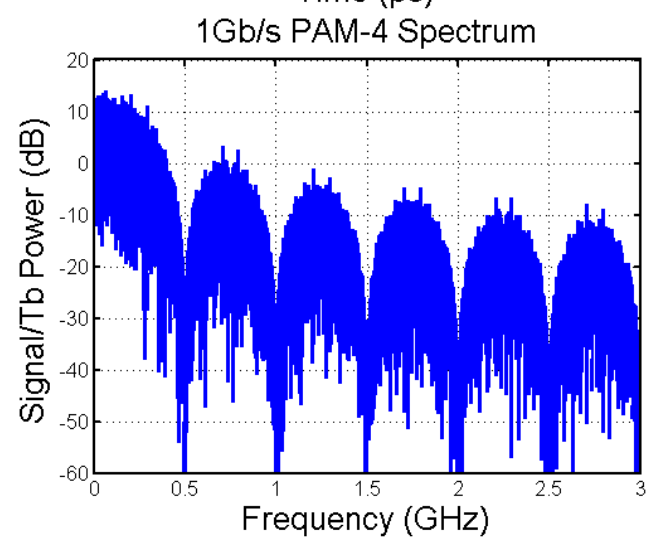


Modulation Frequency Spectrum









Majority of signal power in 1GHz bandwidth

Majority of signal power in 0.5GHz bandwidth

Nyquist Frequency

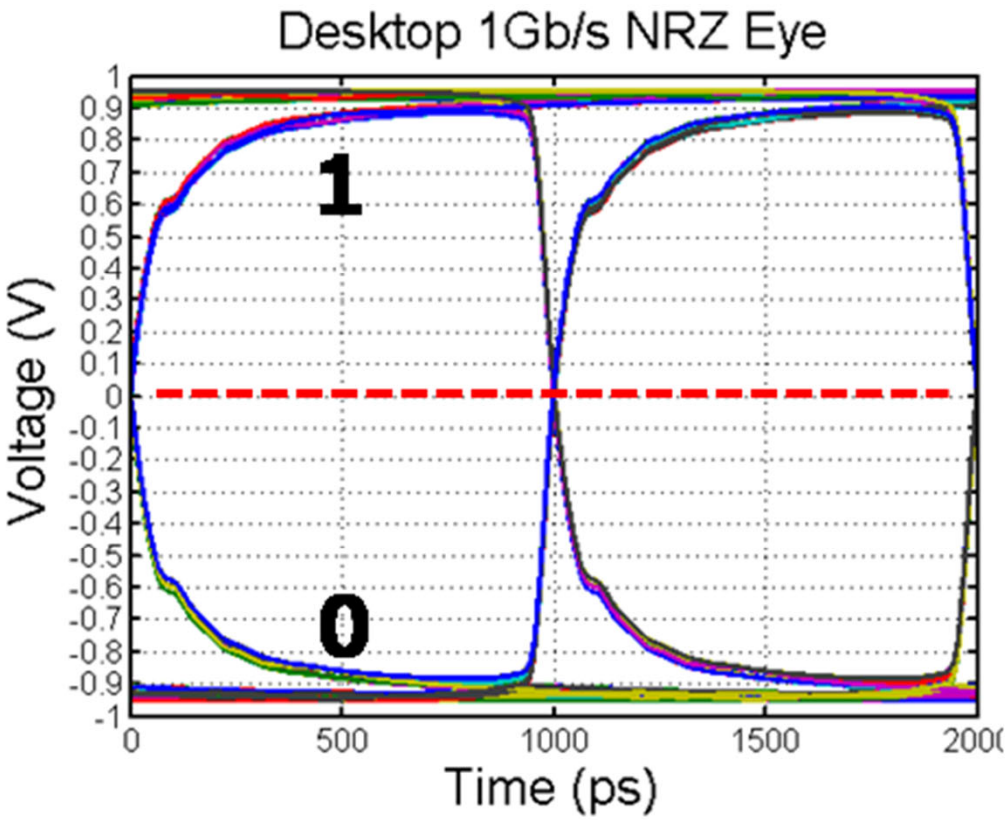
- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect R_S (symbols/s) without ISI is $R_S/2$ (Hz)
  - Thus, a system with bandwidth $W=1/2T=R_S/2$ (Hz) can support a maximum transmission rate of $2W=1/T=R_S$ (symbols/s) without ISI

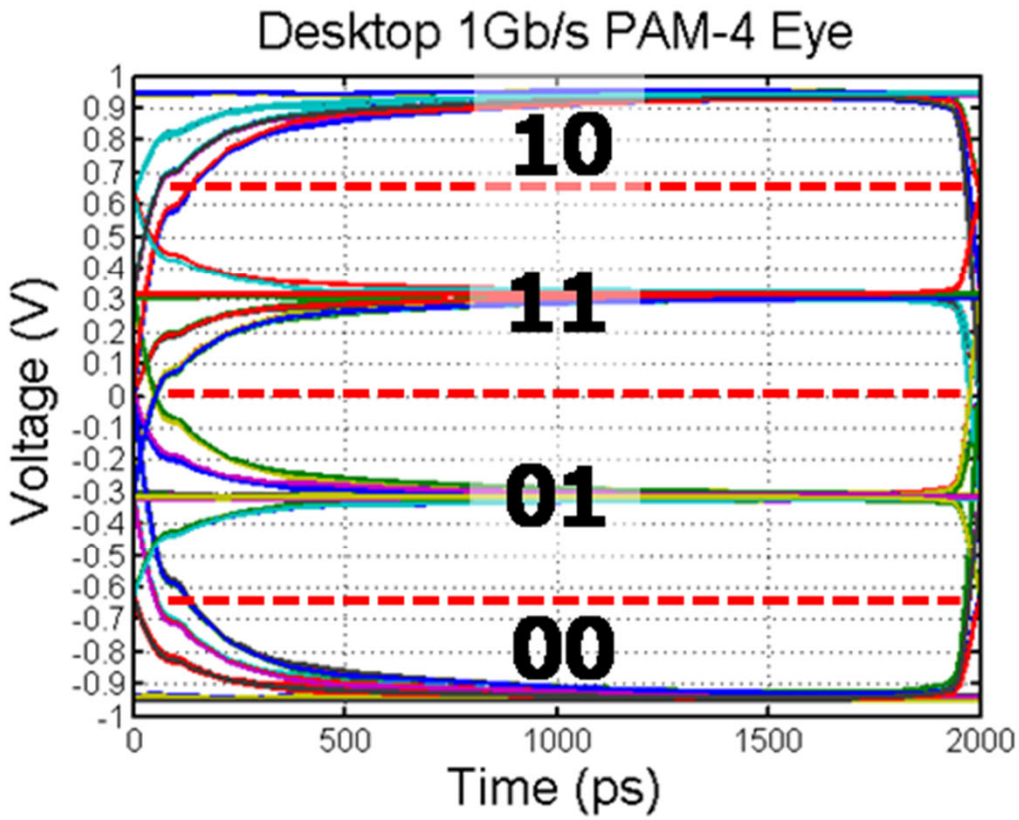
$$\frac{1}{2T} = \frac{R_S}{2} \le W \Rightarrow \frac{R_S}{W} \le 2 \quad \text{(symbols/s/Hz)}$$

  - For ideal Nyquist pulses (sinc), the required bandwidth is only $R_S/2$ to support an R_S symbol rate

Modulation	Bits/Symbol	Nyquist Frequency
NRZ	1	$R_s/2=1/2T_b$
PAM-4	2	$R_s/2=1/4T_b$

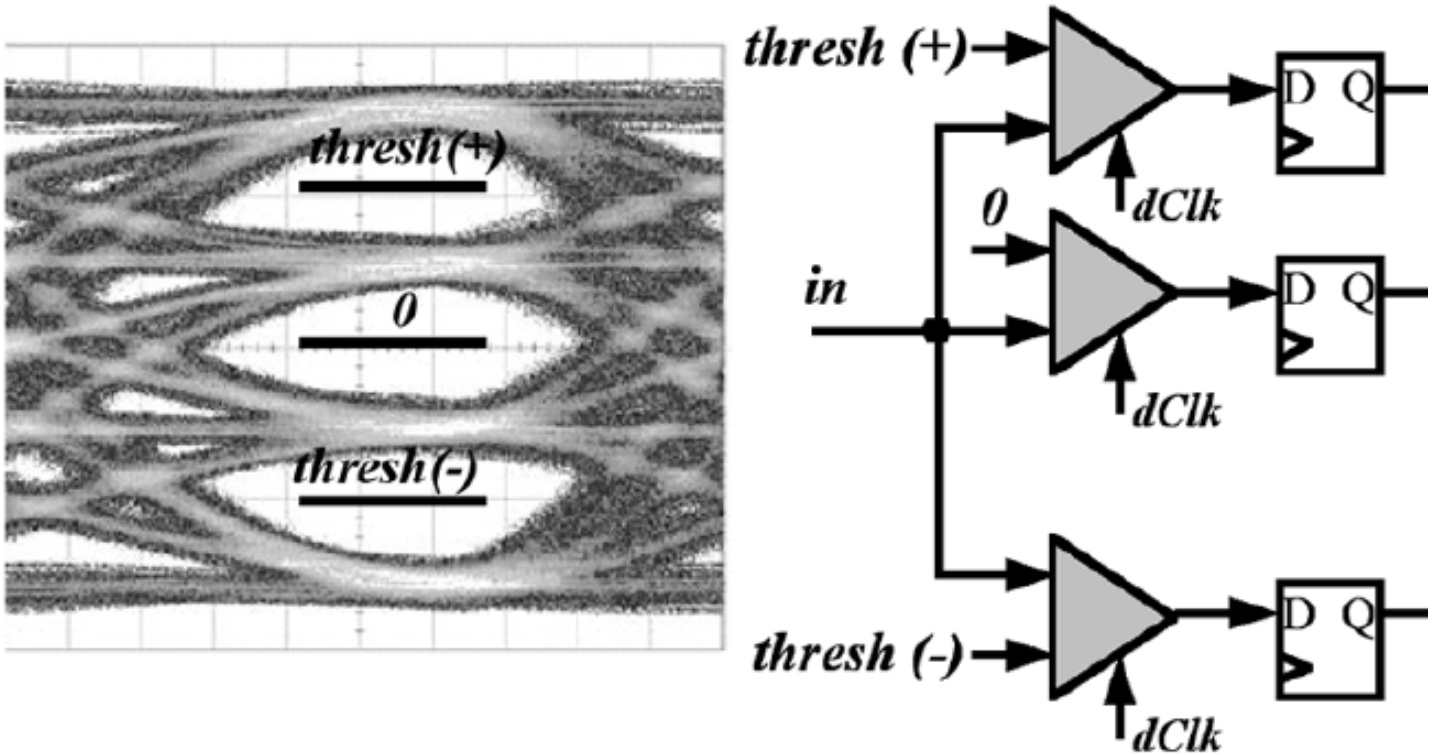
NRZ vs PAM-4





- PAM-4 should be considered when
  - Slope of channel insertion loss (S_{21}) exceeds reduction in PAM-4 eye height
    - Insertion loss over an octave is greater than 20*log10(1/3)=-9.54dB
  - On-chip clock speed limitations

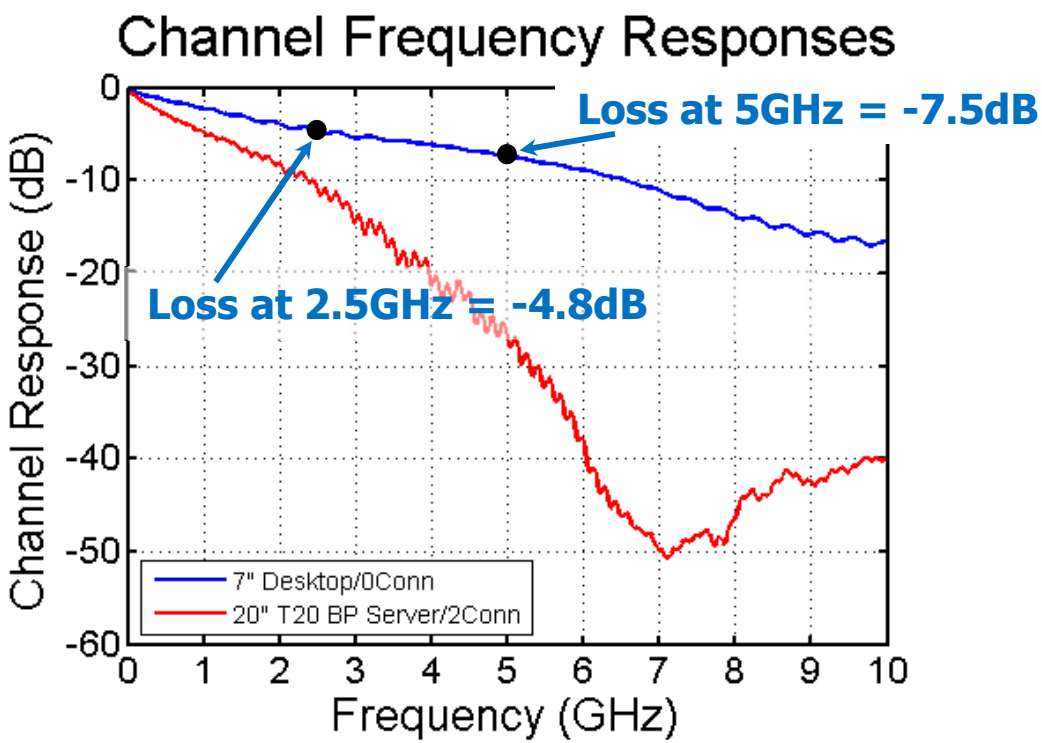
PAM-4 Receiver

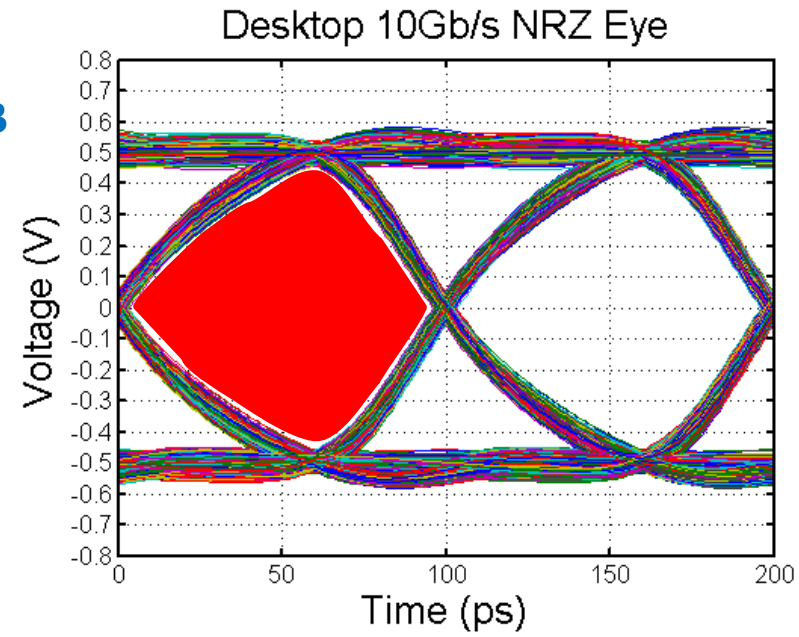


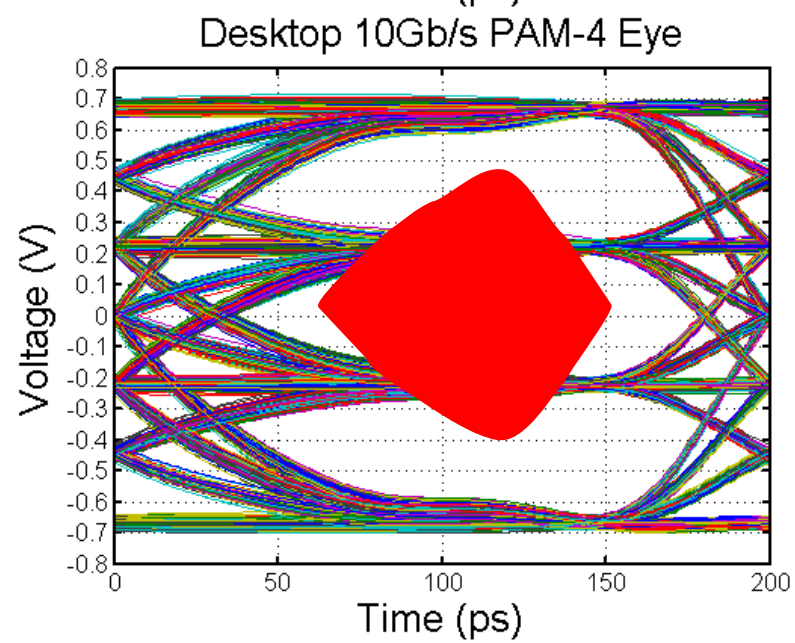
[**Stojanovic JSSC 2005**]

- 3x the comparators of NRZ RX

NRZ vs PAM-4 – Desktop Channel

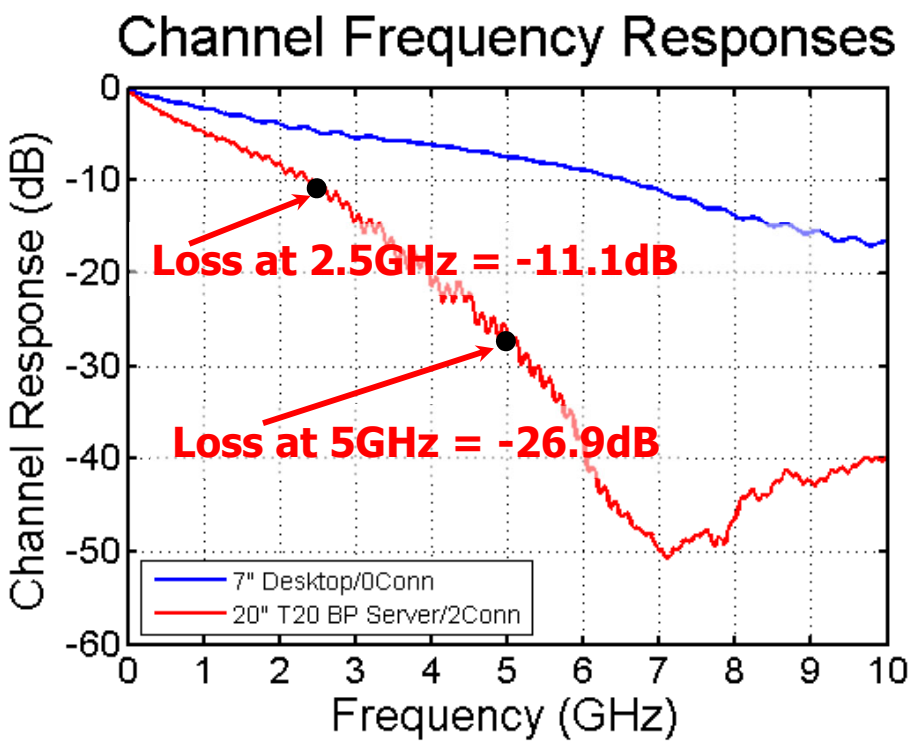


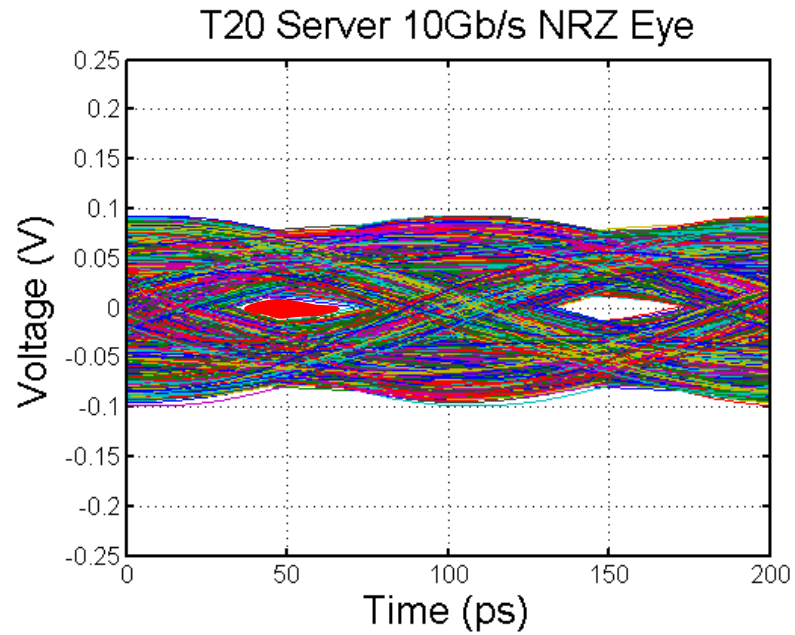


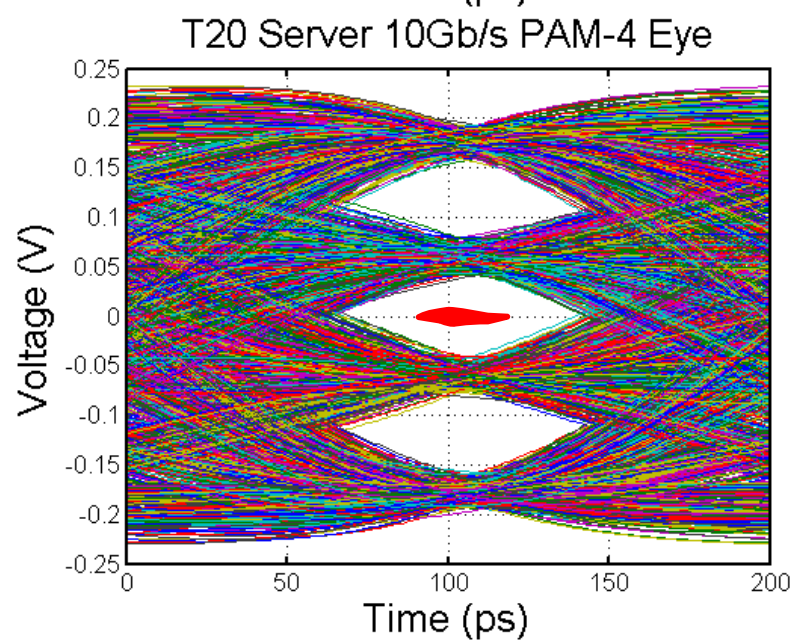


- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is only 2.7dB
  - NRZ has better voltage margin

NRZ vs PAM-4 – T20 Server Channel



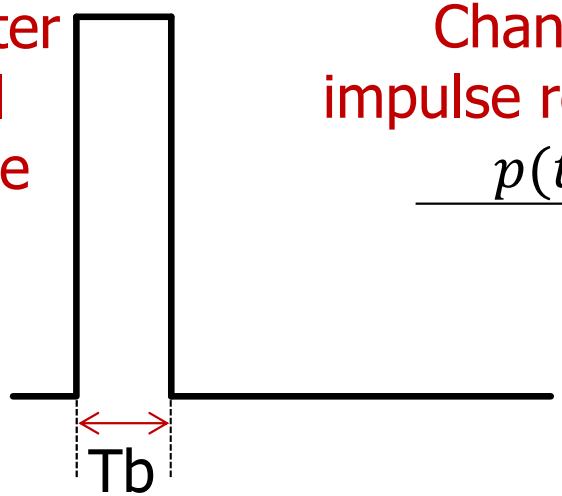




- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is 15.8dB
  - PAM-4 "might" be a better choice

PAM-4 Peak Distortion Analysis

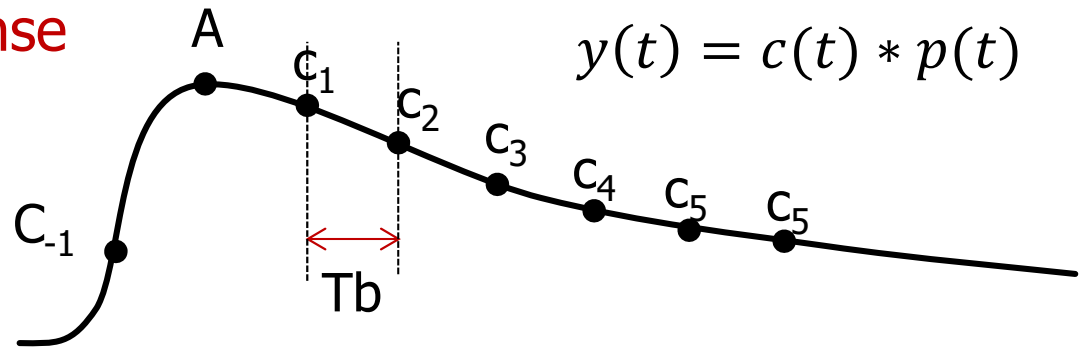
Transmitter symbol response $c(t)$

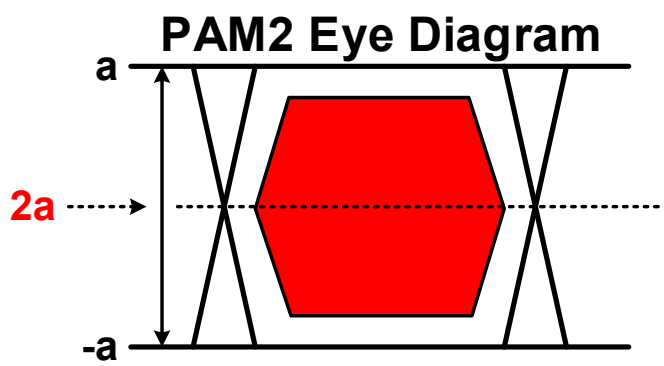


Channel impulse response $p(t)$

Channel pulse response

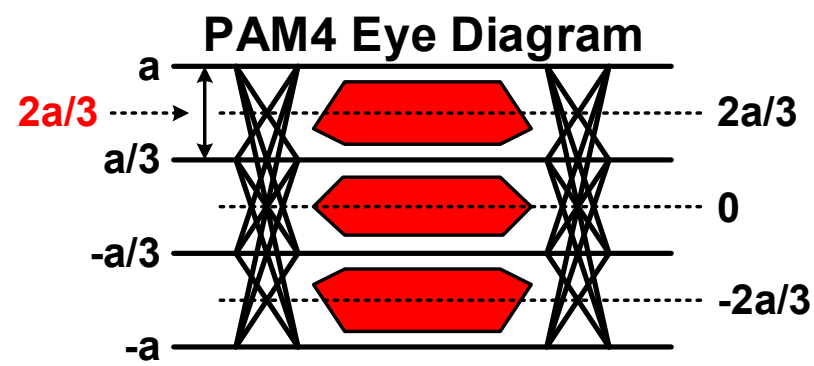
$$y(t) = c(t) * p(t)$$





NRZ eye height

$$= 2(A - \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} |Cn|)$$



PAM4 eye height

$$= 2(\frac{1}{3}A - \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} |Cn|)$$

- **PAM4 modulation is more sensitive to residual ISI**

Multi-Level PAM Challenges

- Receiver complexity increases considerably
  - 3x input comparators (2-bit ADC)
  - Input signal is no longer self-referenced at 0V differential
    - Need to generate reference threshold levels, which will be dependent on channel loss and TX equalization
- CDR can display extra jitter due to multiple "zero crossing" times
- Smaller eyes are more sensitive to cross-talk due to maximum transitions
- Advanced equalization (DFE) can allow NRZ signaling to have comparable (or better) performance even with >9.5dB loss per octave

Modulation Take-Away Points

- Loss-slope guidelines are a good place to start in consideration of alternate modulation schemes
- More advanced modulation trades-off receiver complexity versus equalization complexity
- Advanced modulation challenges
  - Peak TX power limitations
  - Setting RX comparator thresholds and controlling offsets
  - CDR complexity
  - Crosstalk sensitivity (PAM-4)
- Need link analysis tools that consider voltage, timing, and crosstalk noise to choose best modulation scheme for a given channel

Next Time

- Link Circuits
  - Termination structures
  - Drivers
  - Receivers