ECEN689: Special Topics in High-Speed Links Circuits and Systems
Spring 2011

Lecture 3: Transmission Lines

Sam Palermo
Analog & Mixed-Signal Center
Texas A&M University

Announcements

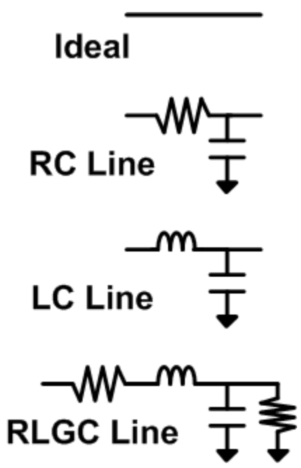
- HW1 due 1/28
  - One page summary of recent link design paper
- Lecture Reference Material
  - Dally, Chapter 3.1 – 3.4

Agenda

- Transmission Lines
  - Propagation constant
  - Characteristic impedance
  - Loss
  - Reflections
  - Termination examples
  - Differential transmission lines

Wire Models

- Model Types
  - Ideal
  - Lumped C, R, L
  - RC transmission line
  - LC transmission line
  - RLGC transmission line

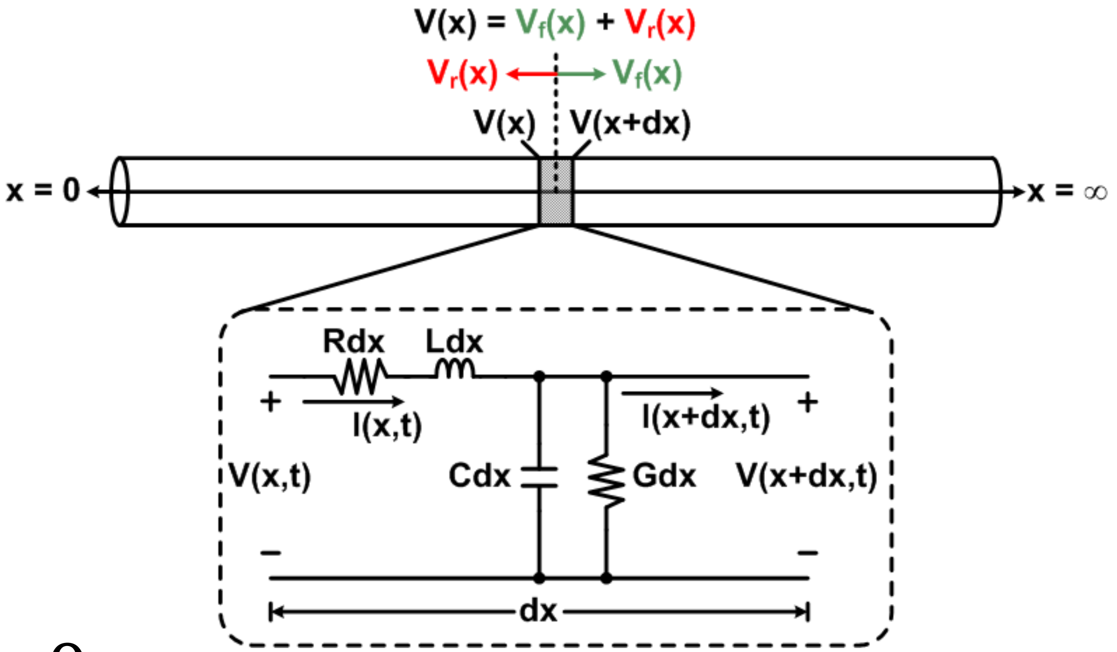


- Condition for LC or RLGC model (vs RC)

$$f_0 \geq \frac{R}{2\pi L}$$

Wire	R	L	C	>f (LC wire)
AWG24 Twisted Pair	0.08Ω/m	400nH/m	40pF/m	32kHz
PCB Trace	5Ω/m	300nH/m	100pF/m	2.7MHz
On-Chip Min. Width M6 (0.18µm CMOS node)	40kΩ/m	4µH/m	300pF/m	1.6GHz

RLGC Transmission Line Model



As dx $\rightarrow$ 0

$$\frac{\partial V(x,t)}{\partial x} = -RI(x,t) - L\frac{\partial I(x,t)}{\partial t} \quad (1)$$

$$\frac{\partial I(x,t)}{\partial x} = -GV(x,t) - C\frac{\partial V(x,t)}{\partial t} \quad (2)$$

General Transmission Line Equations

Time-Harmonic Transmission Line Eqs.

- Assuming a traveling sinusoidal wave with angular frequency, ω

$$\frac{dV(x)}{dx} = -(R + j\omega L)I(x) \quad (3)$$

$$\frac{dI(x)}{dx} = -(G + j\omega C)V(x) \quad (4)$$

- Differentiating (3) and plugging in (4) (and vice versa)

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V(x) \quad (5)$$

$$\frac{d^2I(x)}{dx^2} = \gamma^2 I(x) \quad (6)$$

Time-Harmonic Transmission Line Equations

- where γ is the **propagation constant**

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (\mathrm{m}^{-1})$$

Transmission Line Propagation Constant

- Solutions to the Time-Harmonic Line Equations:

$$V(x) = V_f(x) + V_r(x) = V_{f0}e^{-\gamma x} + V_{r0}e^{\gamma x}$$

$$I(x) = I_f(x) + I_r(x) = I_{f0}e^{-\gamma x} + I_{r0}e^{\gamma x}$$

where $$\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (\mathrm{m}^{-1})}$$

- What does the propagation constant tell us?
  - Real part (α) determines attenuation/distance (Np/m)
  - Imaginary part (β) determines phase shift/distance (rad/m)
  - **Signal phase velocity**

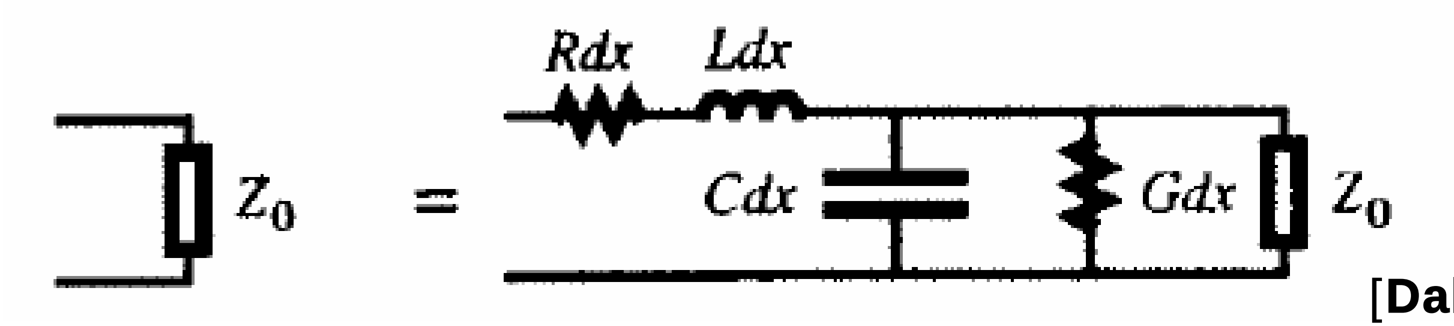
$$\boxed{\upsilon = \omega / \beta \quad (\mathrm{m/s})}$$

Transmission Line Impedance, Z_0

- For an infinitely long line, the voltage/current ratio is Z_0
- From time-harmonic transmission line eqs. (3) and (4)

$$Z_0 = \frac{V(x)}{I(x)} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (\Omega)$$

- Driving a line terminated by Z_0 is the same as driving an infinitely long line



[**Dally**]

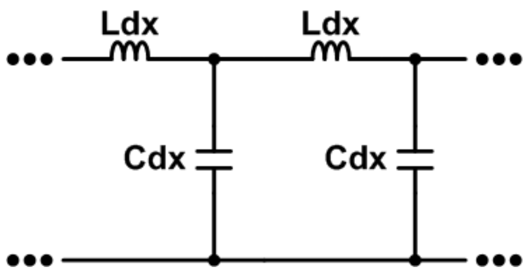
Lossless LC Transmission Lines

- If Rdx=Gdx=0

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$ → **No Loss!**

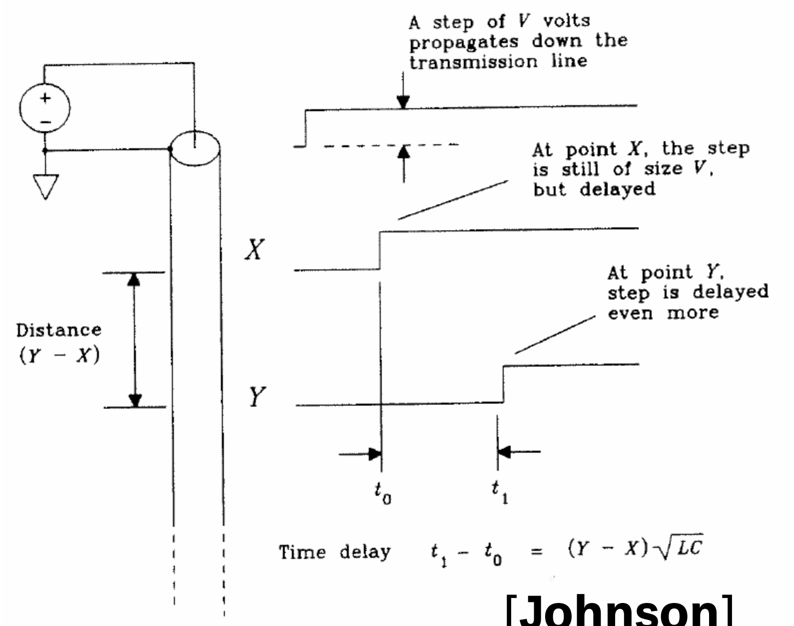
$$\beta = \omega\sqrt{LC}$$



- Waves propagate w/o distortion
  - Velocity and impedance independent of frequency
  - Impedance is purely real

$$\upsilon = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$



[Johnson]

Low-Loss LRC Transmission Lines

- If R/ωL and G/ωC << 1
- Behave similar to ideal LC transmission line, but ...
  - Experience resistive and dielectric loss
  - Frequency dependent propagation velocity results in dispersion
    - Fast step, followed by slow DC tail

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\cong j\omega\sqrt{LC}\left(1 - j\frac{RC + GL}{\omega LC}\right)^{\frac{1}{2}}$$

$$\cong \frac{R}{2Z_0} + \frac{GZ_0}{2} + j\omega\sqrt{LC}\left[1 + \frac{1}{8}\left(\frac{R}{\omega L}\right)^2 + \frac{1}{8}\left(\frac{G}{\omega C}\right)^2\right]$$

$$= \alpha_R + \alpha_D + j\beta$$

$$\alpha_R \cong \frac{R}{2Z_0}$$ **Resistive Loss**

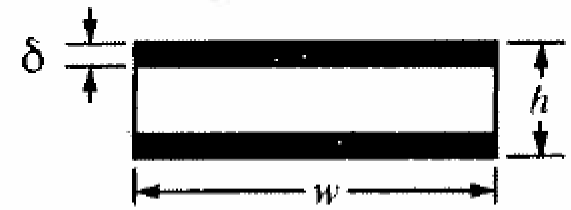
$$\alpha_D \cong \frac{GZ_0}{2}$$ **Dielectric Loss**

$$\beta \cong \omega\sqrt{LC}\left[1 + \frac{1}{8}\left(\frac{R}{\omega L}\right)^2 + \frac{1}{8}\left(\frac{G}{\omega C}\right)^2\right]$$

$$\upsilon \cong \left(\sqrt{LC}\left[1 + \frac{1}{8}\left(\frac{R}{\omega L}\right)^2 + \frac{1}{8}\left(\frac{G}{\omega C}\right)^2\right]\right)^{-1}$$

Skin Effect (Resistive Loss)

- High-frequency current density falls off exponentially from conductor surface
- Skin depth, δ, is where current falls by e^{-1} relative to full conductor
  - Decreases proportional to sqrt(frequency)
- Relevant at critical frequency f_s where skin depth equals half conductor height (or radius)
  - Above f_s resistance/loss increases proportional to sqrt(frequency)



[**Dally**]

$$J = e^{-\frac{d}{\delta}} \qquad \delta = \left(\pi f \mu \sigma\right)^{-\frac{1}{2}}$$

For rectangular conductor:

$$f_s = \frac{\rho}{\pi \mu \left(\frac{h}{2}\right)^2}$$

$$R(f) = R_{DC}\left(\frac{f}{f_s}\right)^{\frac{1}{2}}$$

$$\boxed{\alpha_R = \frac{R_{DC}}{2Z_0}\left(\frac{f}{f_s}\right)^{\frac{1}{2}}}$$

Skin Effect (Resistive Loss)

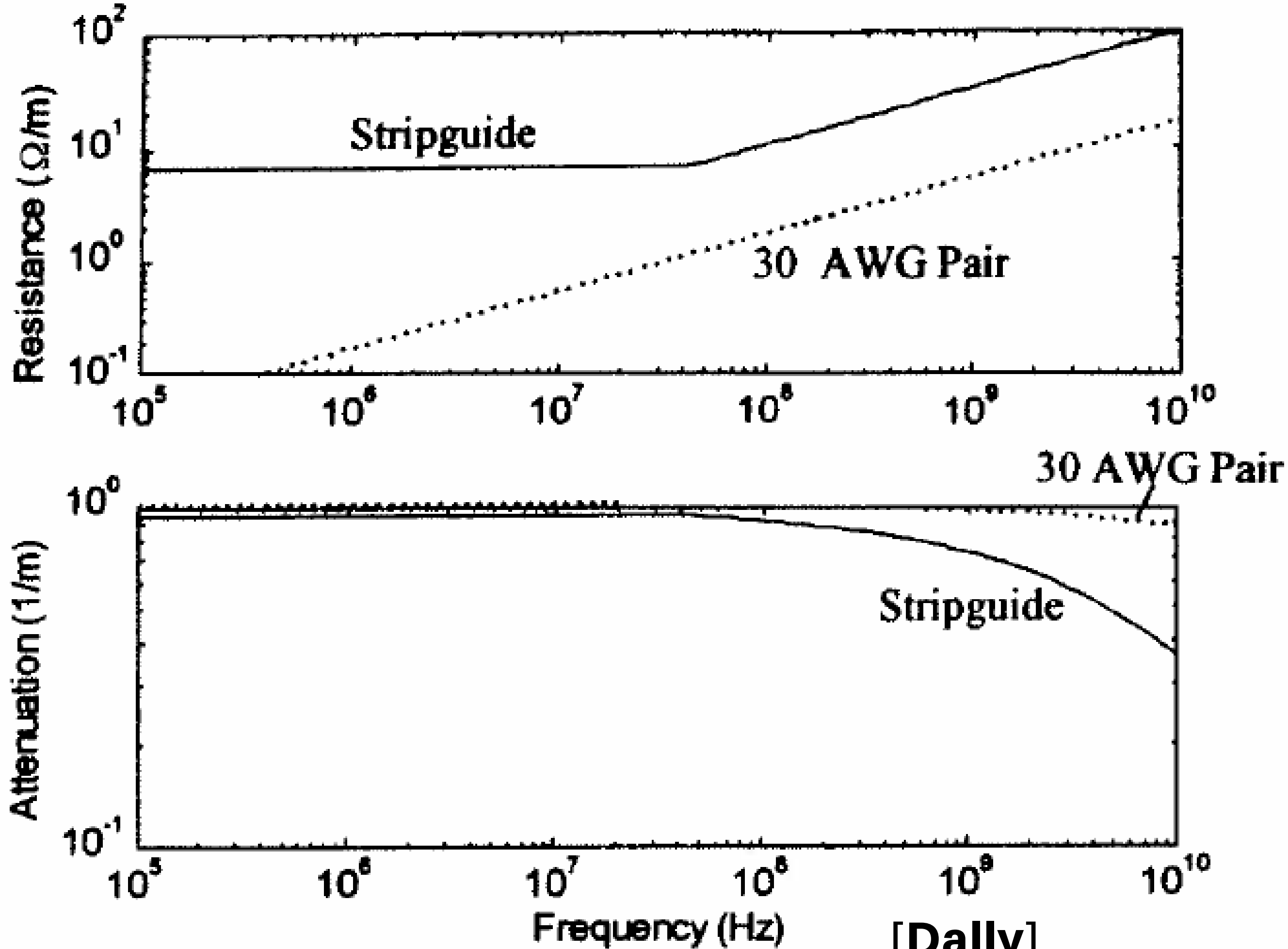
5-mil Stripguide

$$R_{DC} = 7\,\Omega/m, \;\; f_s = 43MHz$$

30 AWG Pair

$$R_{DC} = 0.08\,\Omega/m, \;\; f_s = 67kHz$$

$$\alpha_R = \frac{R_{DC}}{2Z_0}\left(\frac{f}{f_s}\right)^{\frac{1}{2}}$$



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Dielectric Absorption (Loss)

- An alternating electric field causes dielectric atoms to rotate and absorb signal energy in the form of heat
- Dielectric loss is expressed in terms of the loss tangent
- Loss increases directly proportional to frequency

$$\tan\delta_D = \frac{G}{\omega C}$$

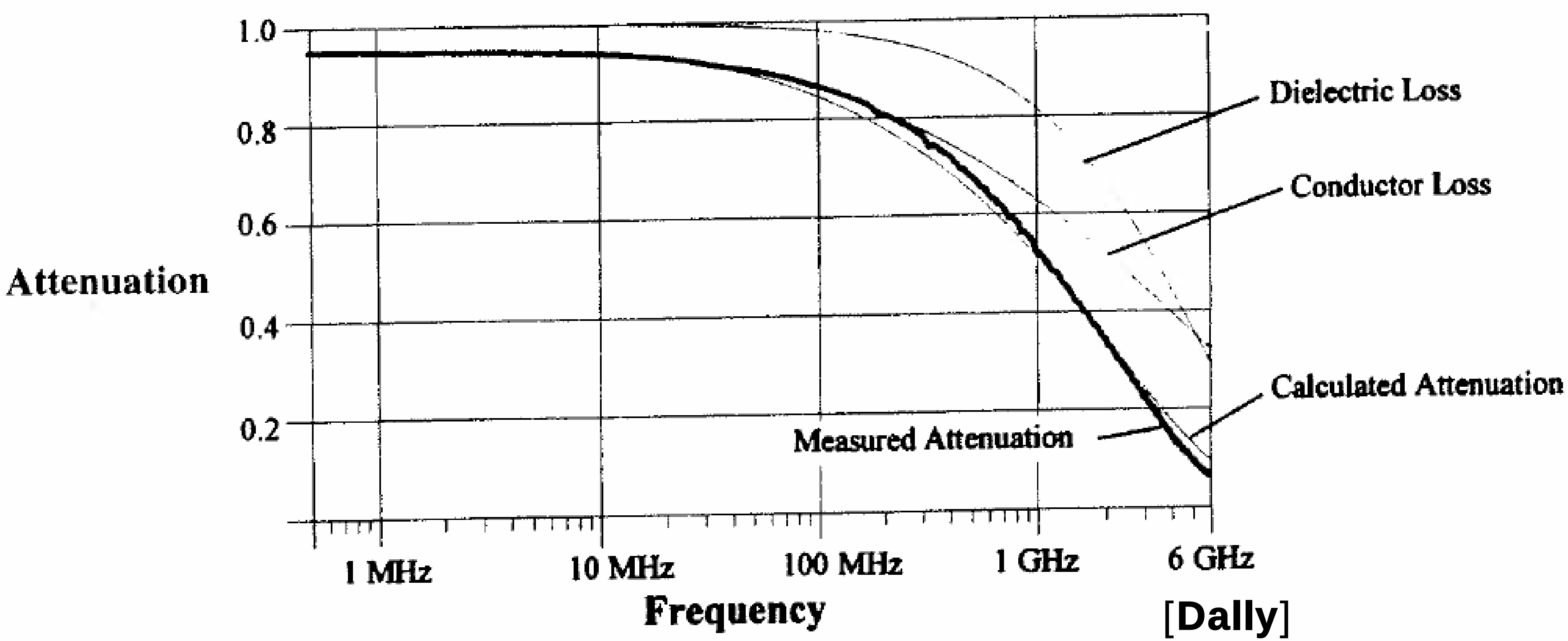
TABLE 3-4 Electrical Properties of PC Board Dielectrics

Material	ε_r	$\tan\delta_D$
Woven glass, epoxy resin ("FR-4")	4.7	0.035
Woven glass, polyimide resin	4.4	0.025
Woven glass, polyphenylene oxide resin (GETEK)	3.9	0.010
Woven glass, PTFE resin (Teflon)	2.55	0.005
Nonwoven glass, PTFE resin	2.25	0.001

[**Dally**]

$$\alpha_D = \frac{GZ_0}{2} = \frac{2\pi f C \tan\delta_D \sqrt{L/C}}{2}$$
$$= \pi f \tan\delta_D \sqrt{LC}$$

Total Wire Loss



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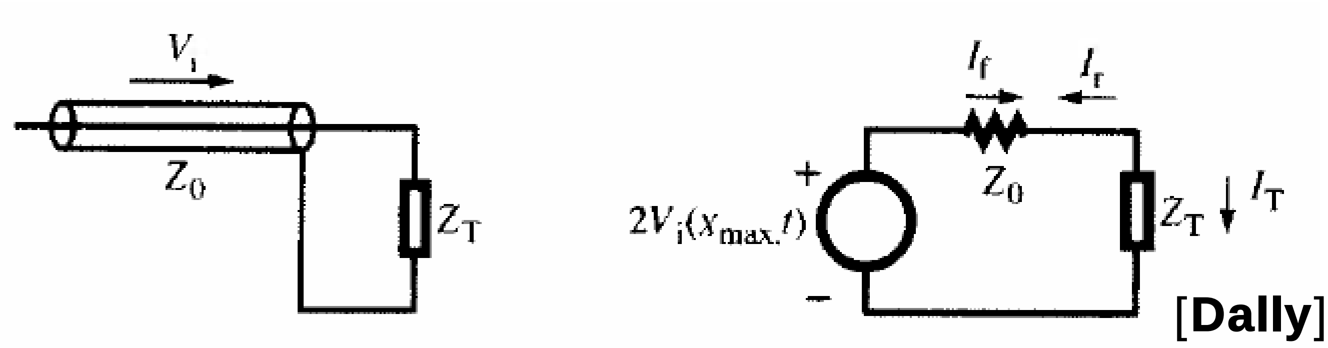
Reflections & Telegrapher's Eq.



[**Dally**]

- With a Thevenin-equivalent mode of the line:

Termination Current: $$I_T = \frac{2V_i}{Z_0 + Z_T}$$

- KCL at Termination:

$$I_r = I_f - I_T$$

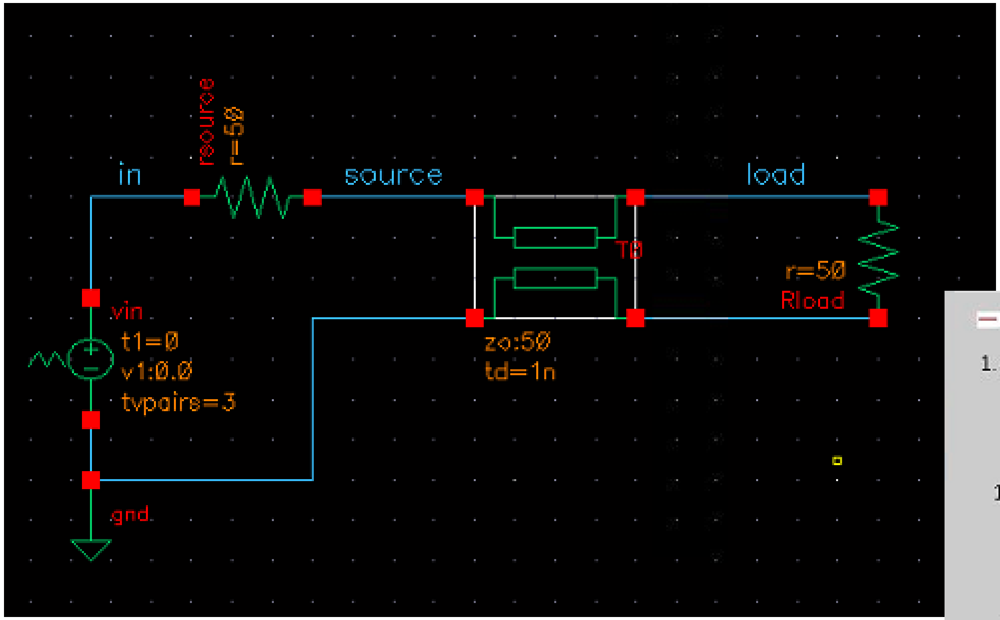
$$I_r = \frac{V_i}{Z_0} - \frac{2V_i}{Z_T + Z_0}$$

$$I_r = \frac{V_i}{Z_0}\left(\frac{Z_T - Z_0}{Z_T + Z_0}\right)$$

Telegrapher's Equation or
Reflection Coefficient

$$\boxed{k_r = \frac{I_r}{I_i} = \frac{V_r}{V_i} = \frac{Z_T - Z_0}{Z_T + Z_0}}$$

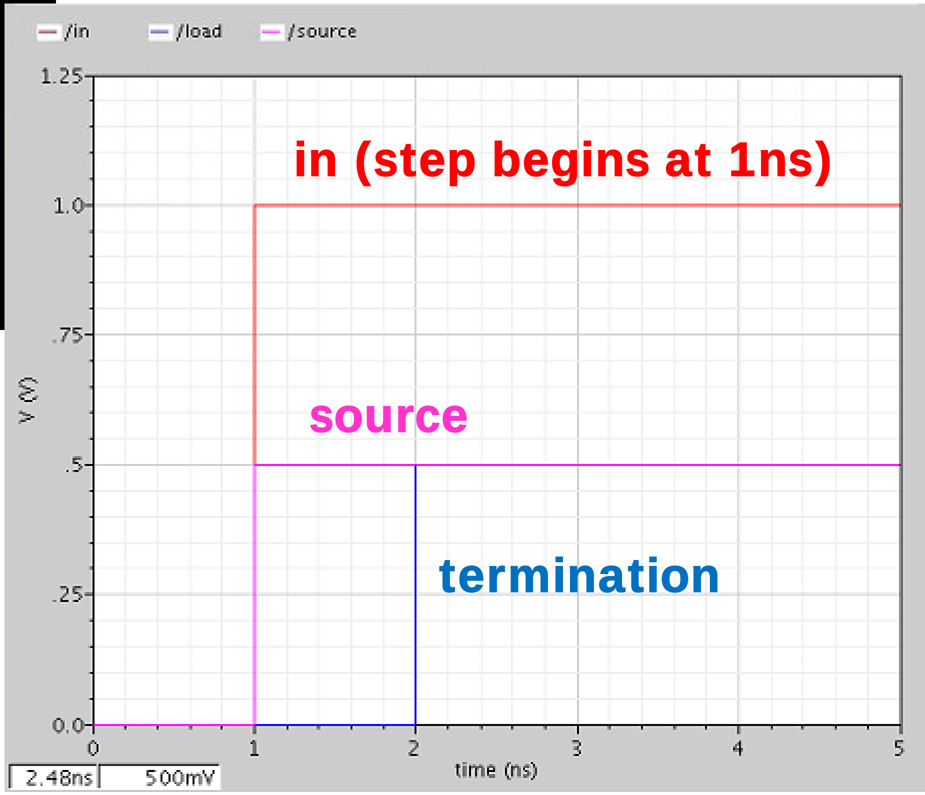
Termination Examples - Ideal



R_S = 50Ω

Z_0 = 50Ω, t_d = 1ns

R_T = 50Ω

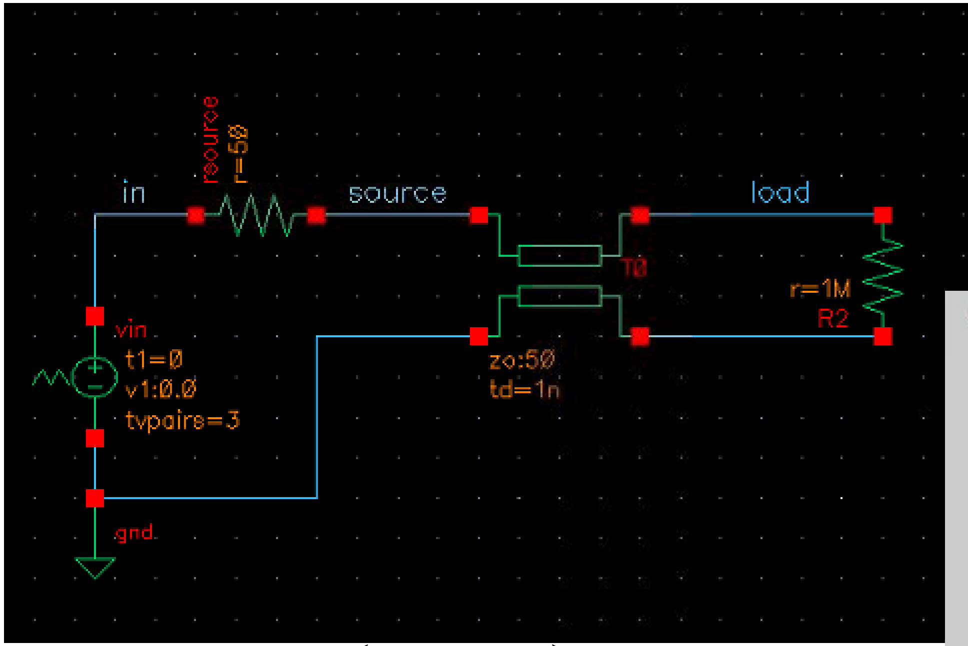


$$V_i = 1V\left(\frac{50}{50+50}\right) = 0.5V$$

$$k_{rT} = \frac{50-50}{50+50} = 0$$

$$k_{rS} = \frac{50-50}{50+50} = 0$$

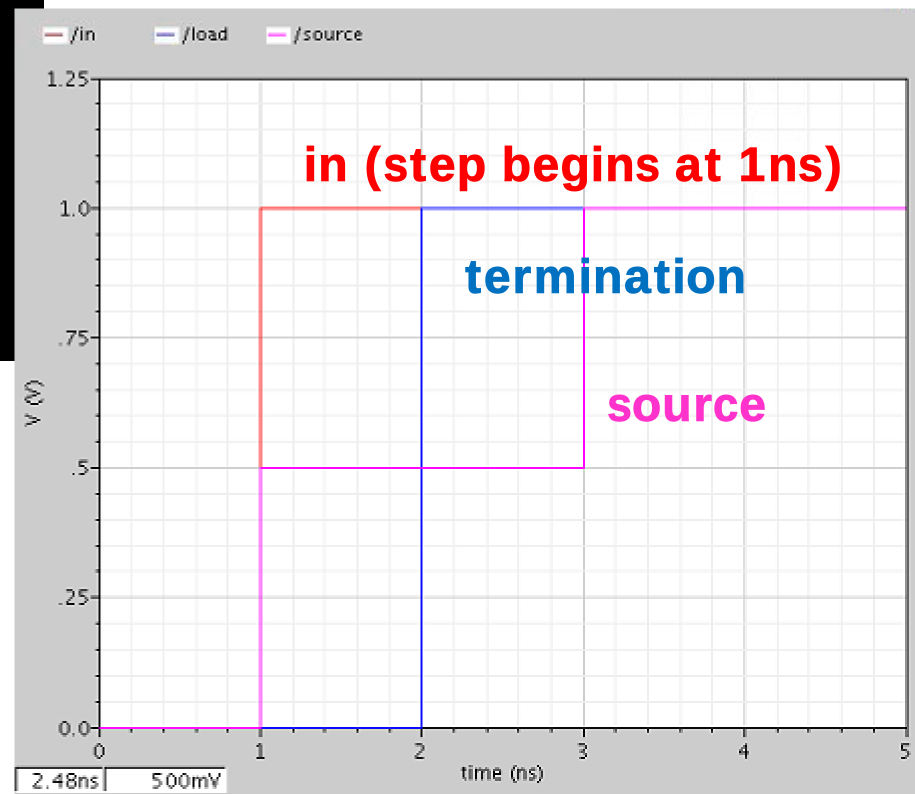
Termination Examples - Open



$R_S = 50\Omega$

$Z_0 = 50\Omega$, $t_d = 1ns$

$R_T \sim \infty$ $(1M\Omega)$

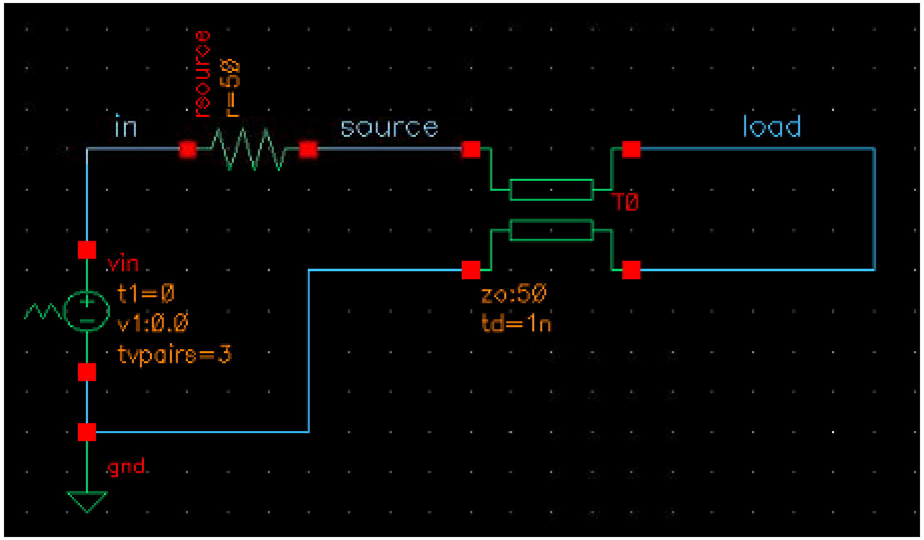


$$V_i = 1V\left(\frac{50}{50+50}\right) = 0.5V$$

$$k_{rT} = \frac{\infty - 50}{\infty + 50} = +1$$

$$k_{rS} = \frac{50-50}{50+50} = 0$$

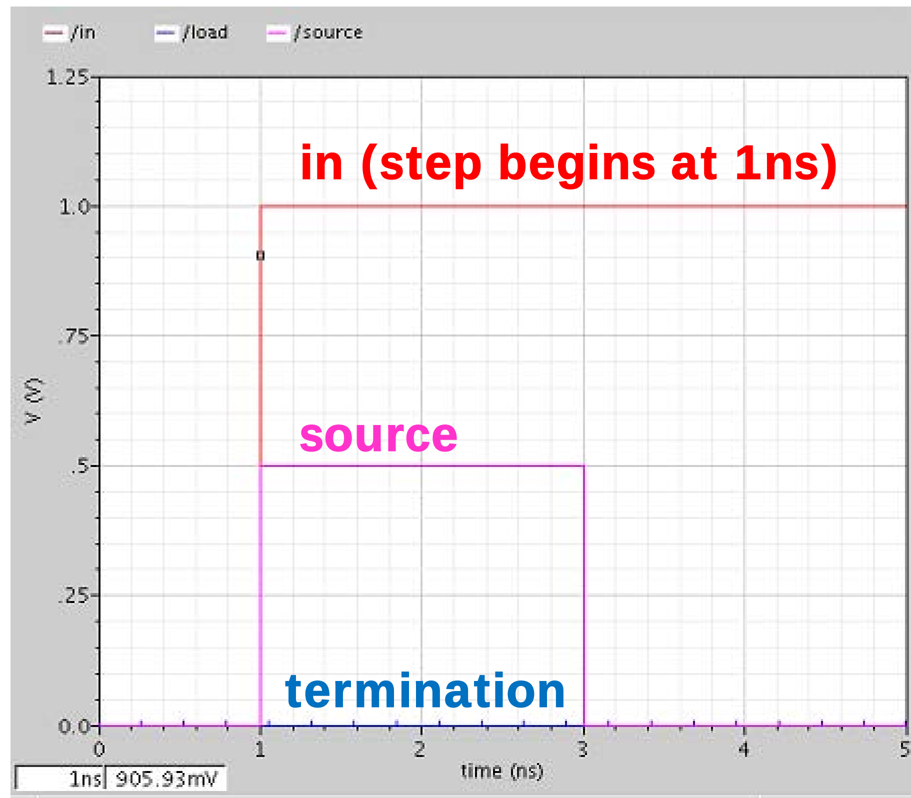
Termination Examples - Short



$R_S = 50\Omega$

$Z_0 = 50\Omega$, $t_d = 1ns$

$R_T = 0\Omega$

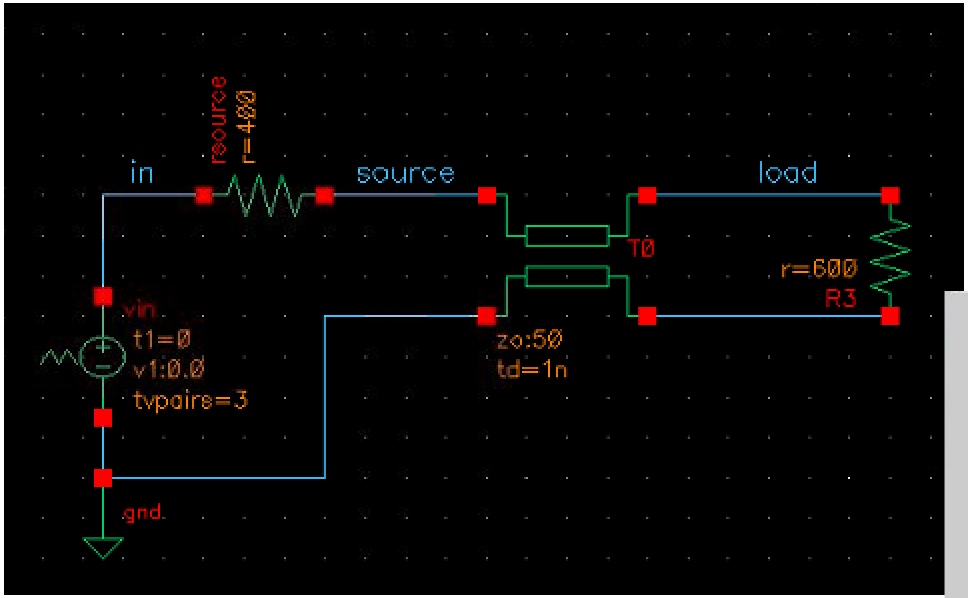


$$V_i = 1V\left(\frac{50}{50+50}\right) = 0.5V$$

$$k_{rT} = \frac{0-50}{0+50} = -1$$

$$k_{rS} = \frac{50-50}{50+50} = 0$$

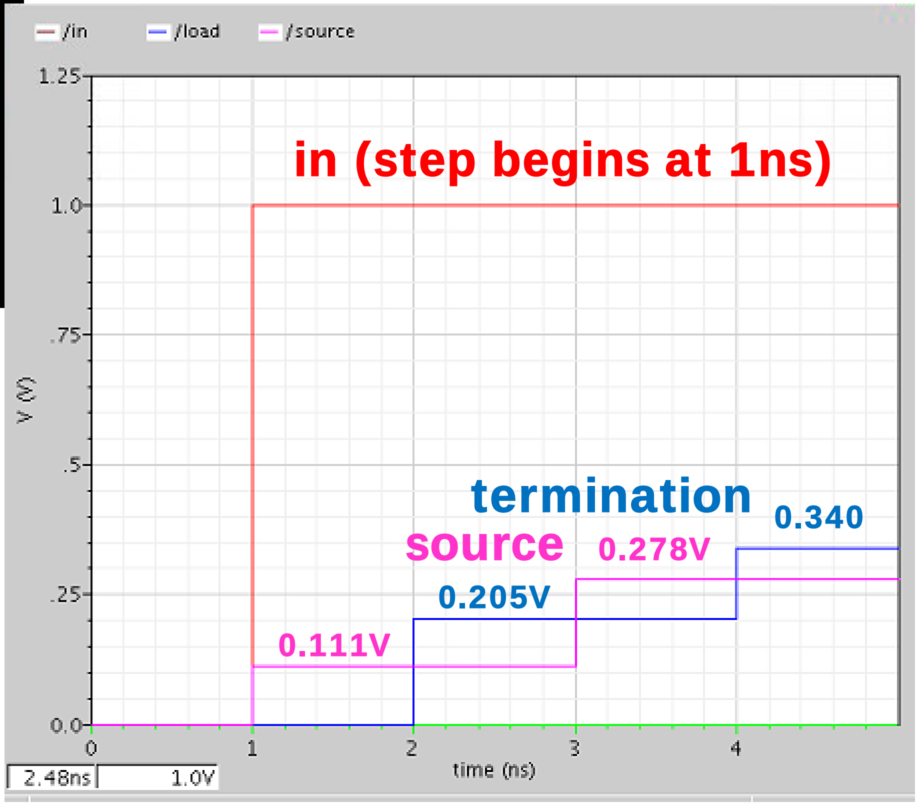
Arbitrary Termination Example



$R_S = 400\Omega$

$Z_0 = 50\Omega,\ t_d = 1\text{ns}$

$R_T = 600\Omega$



$$V_i = 1V\left(\frac{50}{400+50}\right) = 0.111V$$

$$k_{rT} = \frac{600-50}{600+50} = 0.846$$

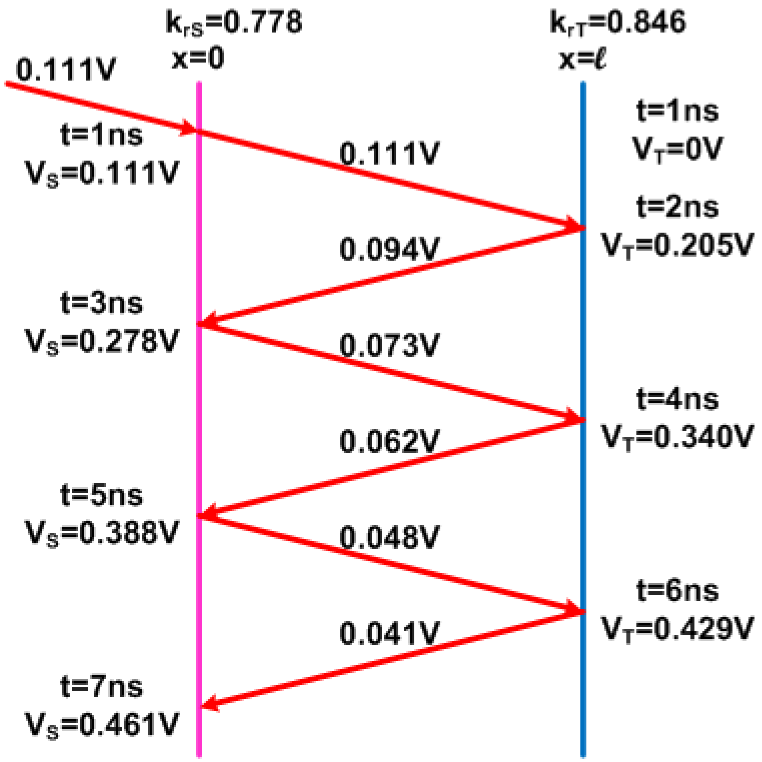
$$k_{rS} = \frac{400-50}{400+50} = 0.778$$

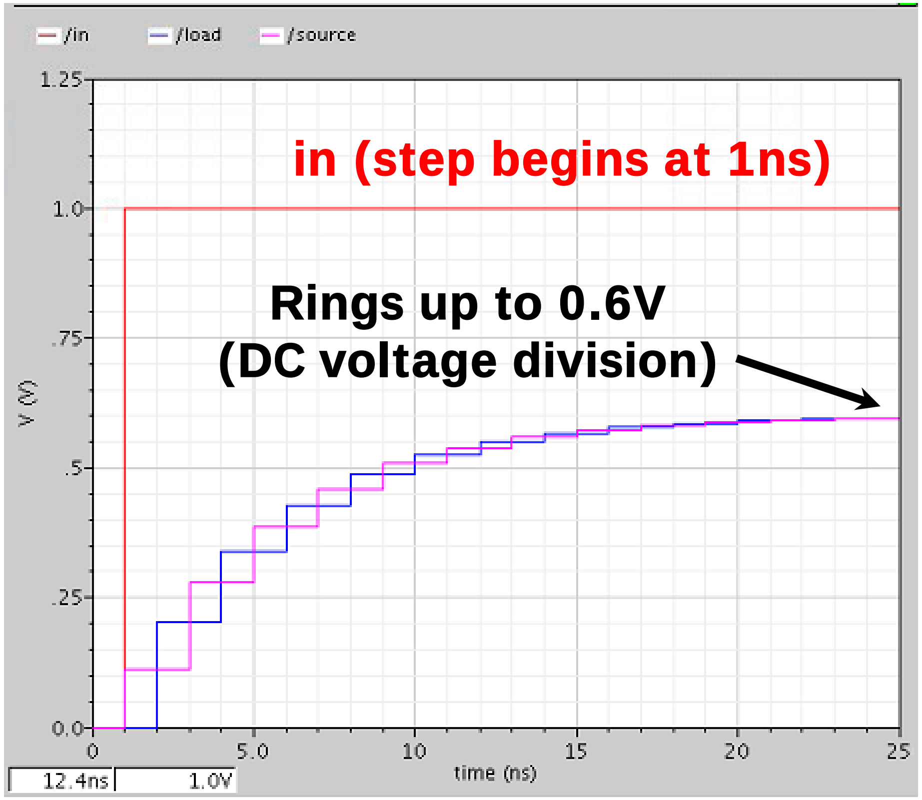
Lattice Diagram

$R_S = 400\Omega$

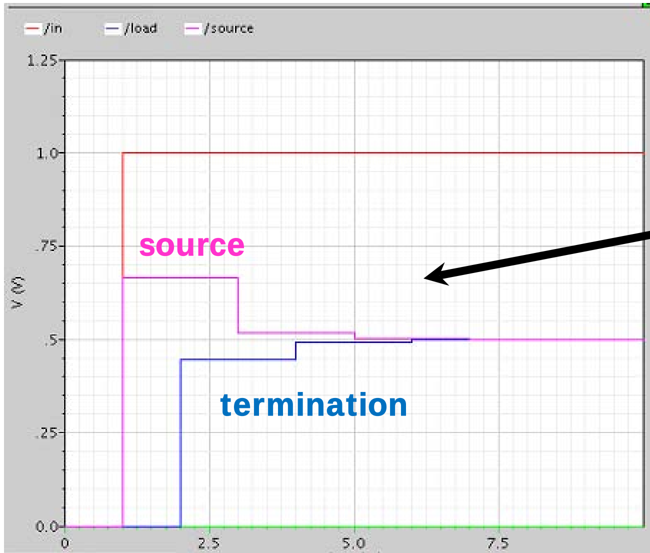
$Z_0 = 50\Omega$, $t_d = 1ns$

$R_T = 600\Omega$





Termination Reflection Patterns



R_S = 25Ω, RT = 25Ω

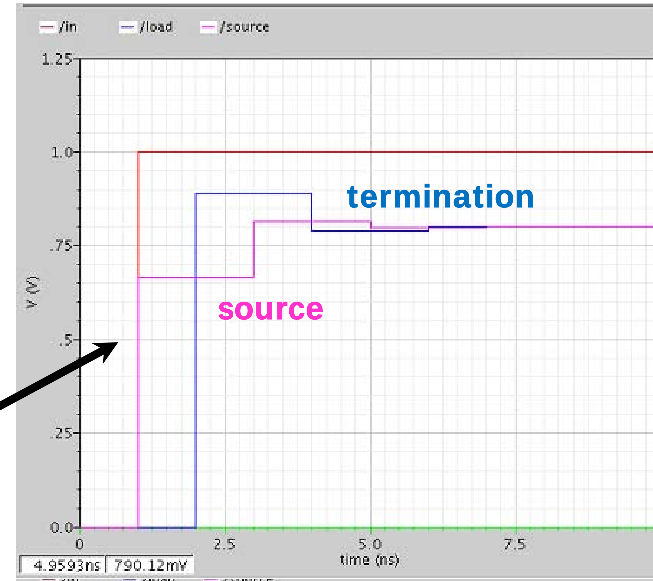
kr_S & kr_T < 0

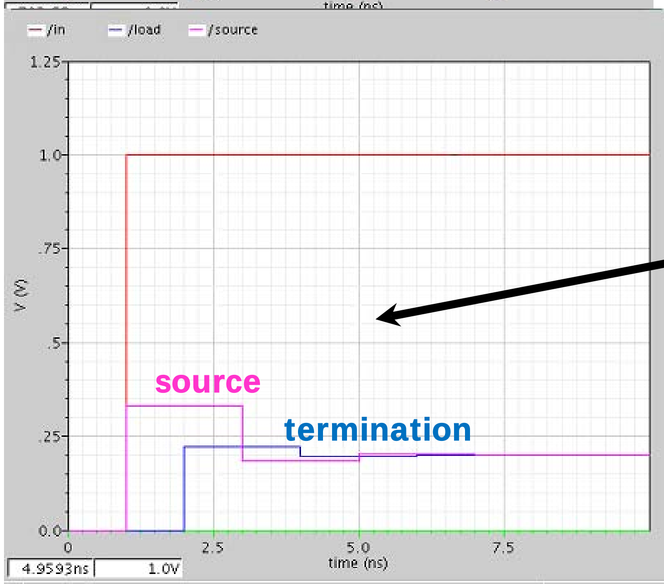
Voltages Converge

R_S = 25Ω, RT = 100Ω

kr_S < 0 & kr_T > 0

Voltages Oscillate





R_S = 100Ω, RT = 25Ω

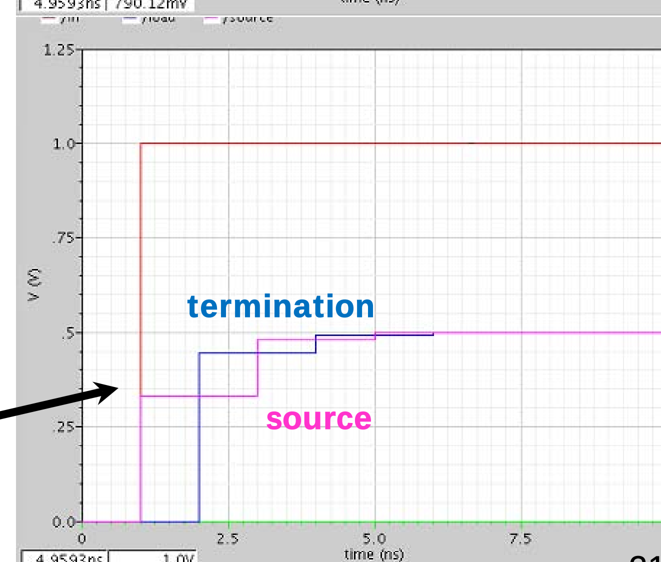
kr_S > 0 & kr_T < 0

Voltages Oscillate

R_S = 100Ω, RT = 100Ω

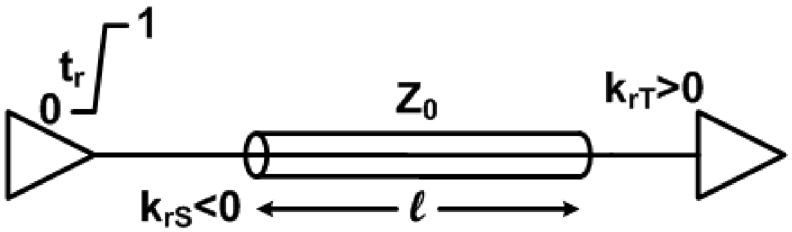
kr_S > 0 & kr_T > 0

Voltages Ring Up



Termination Schemes

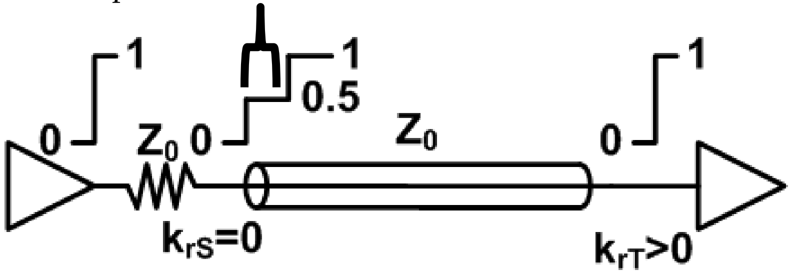
- No Termination
  - Little to absorb line energy
  - Can generate oscillating waveform
  - Line must be **very short** relative to signal transition time
    - n = 4 - 6
  - Limited off-chip use



$$t_r > nT_{round-trip} = 2nl\sqrt{LC}$$

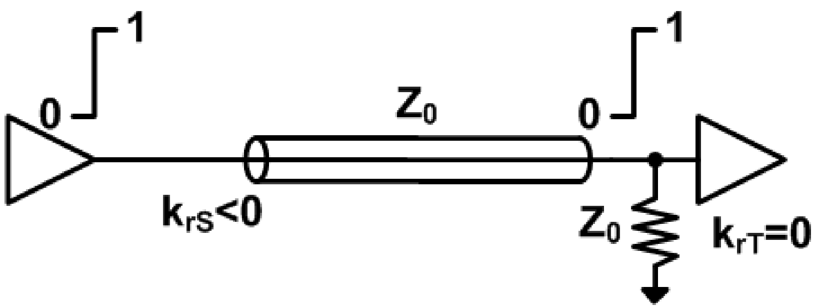
$$t_{porch} \cong 2l\sqrt{LC}$$

- Source Termination
  - Source output takes 2 steps up
  - Used in moderate speed point-to-point connections

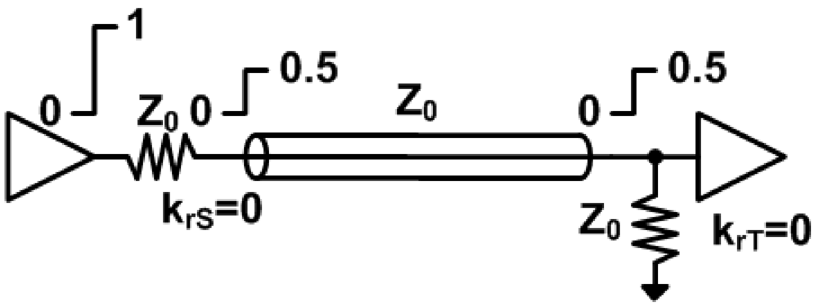


Termination Schemes

- Receiver Termination
  - No reflection from receiver
  - Watch out for intermediate impedance discontinuities
    - Little to absorb reflections at driver



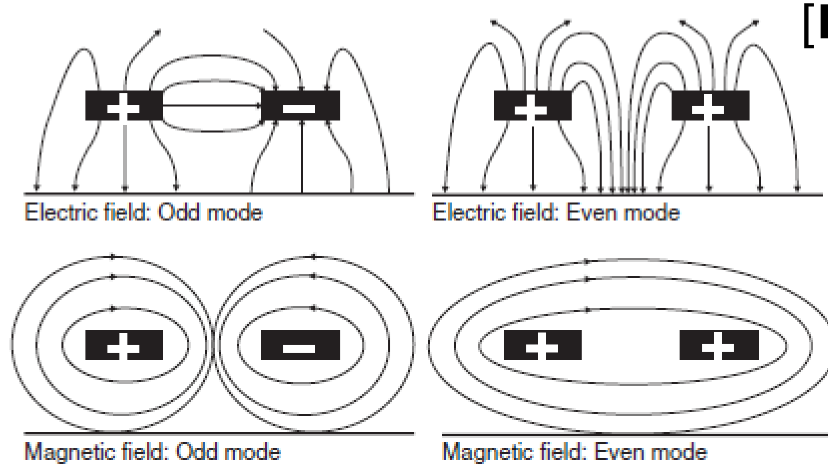
- Double Termination
  - Best configuration for min reflections
    - Reflections absorbed at both driver and receiver
  - Get half the swing relative to single termination
  - Most common termination scheme for high performance serial links



Differential Transmission Lines

- Differential signaling advantages
  - Self-referenced
  - Common-mode noise rejection
  - Increased signal swing
  - Reduced self-induced power-supply noise

[Hall]



- Requires 2x the number of signaling pins relative to single-ended signaling
  - But, smaller ratio of supply/signal (return) pins
  - Total pin overhead is typically 1.3-1.8x (vs 2x)

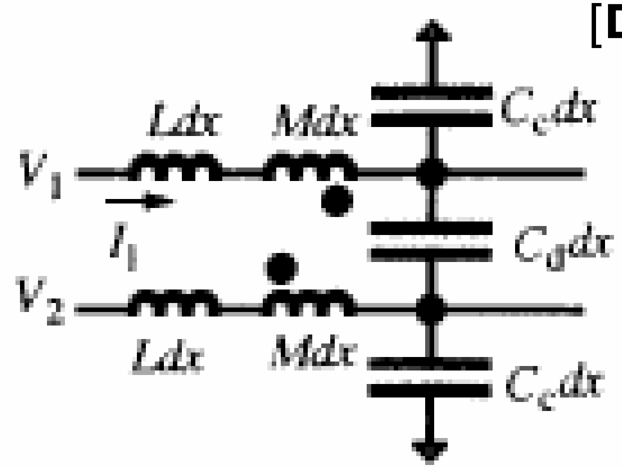
- Even mode
  - When equal voltages drive both lines, only one mode propagates called even more
- Odd mode
  - When equal in magnitude, but out of phase, voltages drive both lines, only one mode propagates called odd mode

Balanced Transmission Lines

[Dally]

- Even (common) mode excitation
  - Effective C = C_C
  - Effective L = L + M
- Odd (differential) mode excitation
  - Effective C = $C_C + 2C_d$
  - Effective L = L – M

$$Z_{DIFF} = 2Z_{odd}, \quad Z_{CM} = \frac{Z_{even}}{2}$$

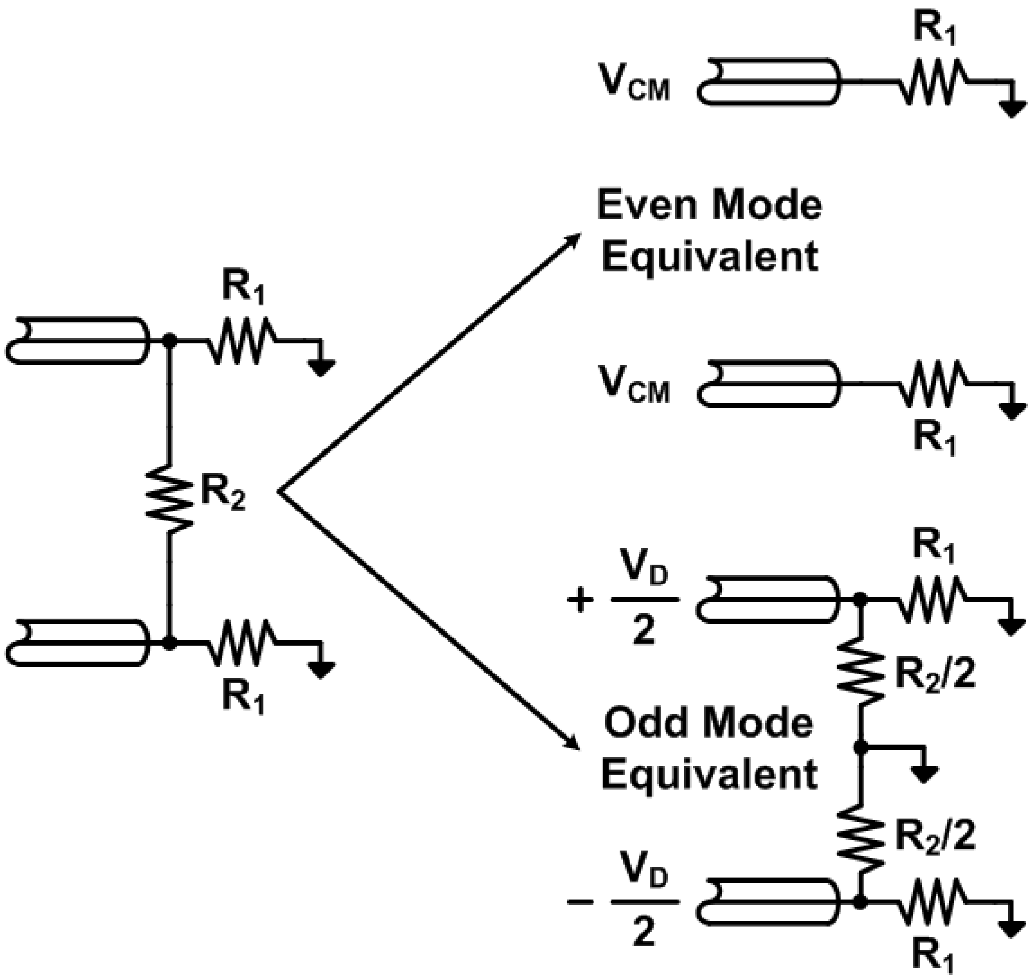


(a) Model of a Balanced Line

$$Z_{even} = \left(\frac{L+M}{C_c}\right)^{1/2}$$

$$Z_{odd} = \left(\frac{L-M}{C_c+2C_d}\right)^{1/2}$$

PI-Termination

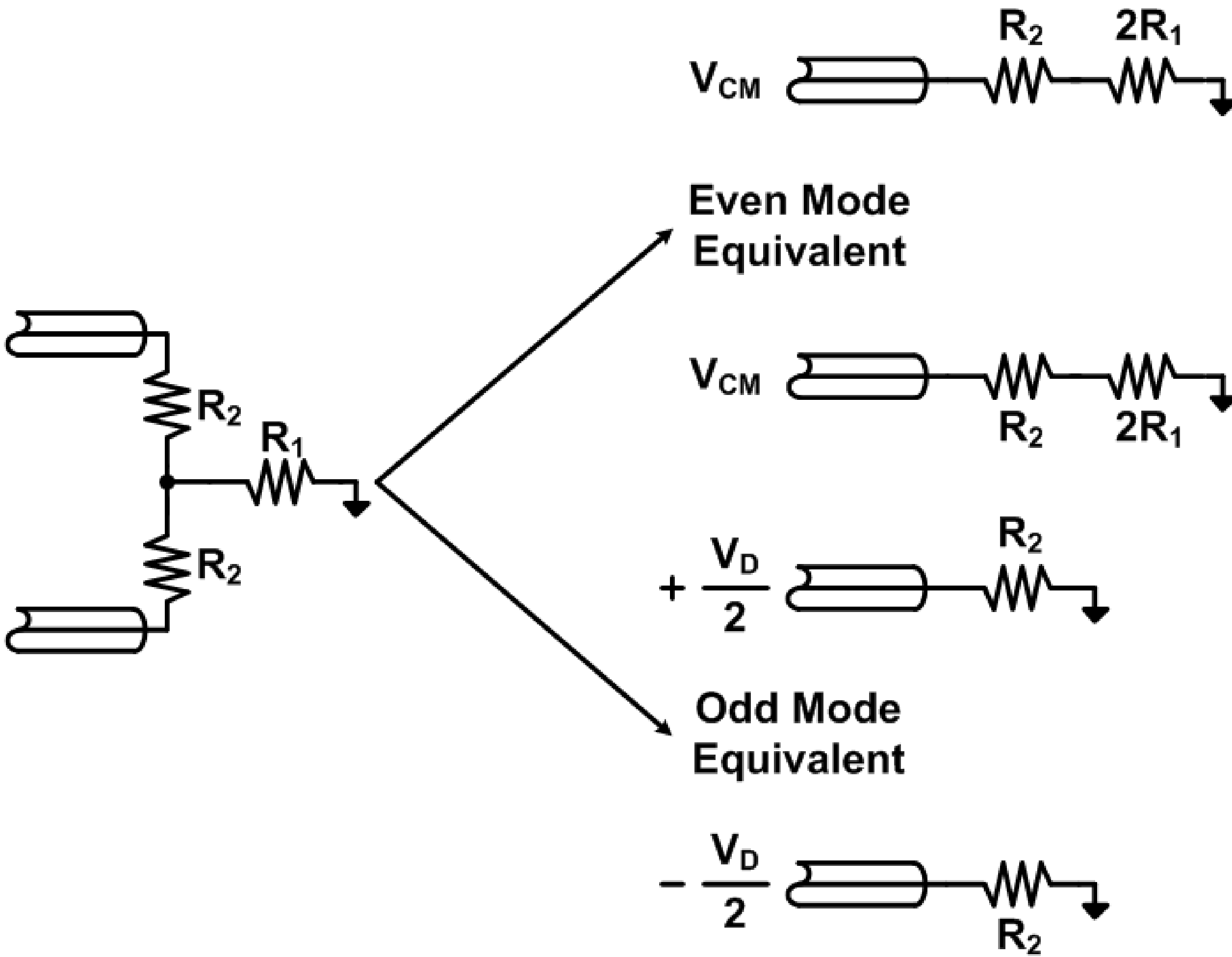


$$\boxed{Z_{even} = R_1}$$

$$Z_{odd} = R_1 \parallel R_2/2 = Z_{even} \parallel R_2/2$$

$$\boxed{R_2 = 2\left(\frac{Z_{odd} Z_{even}}{Z_{even} - Z_{odd}}\right)}$$

T-Termination



$$Z_{even} = R_2 + 2R_1$$

$$Z_{odd} = R_2$$

$$R_1 = \frac{1}{2}\left(Z_{even} - Z_{odd}\right)$$

Next Time

- Channel modeling
  - Time domain reflectometer (TDR)
  - Network analysis