ECEN620: Network Theory
Broadband Circuit Design
Fall 2012

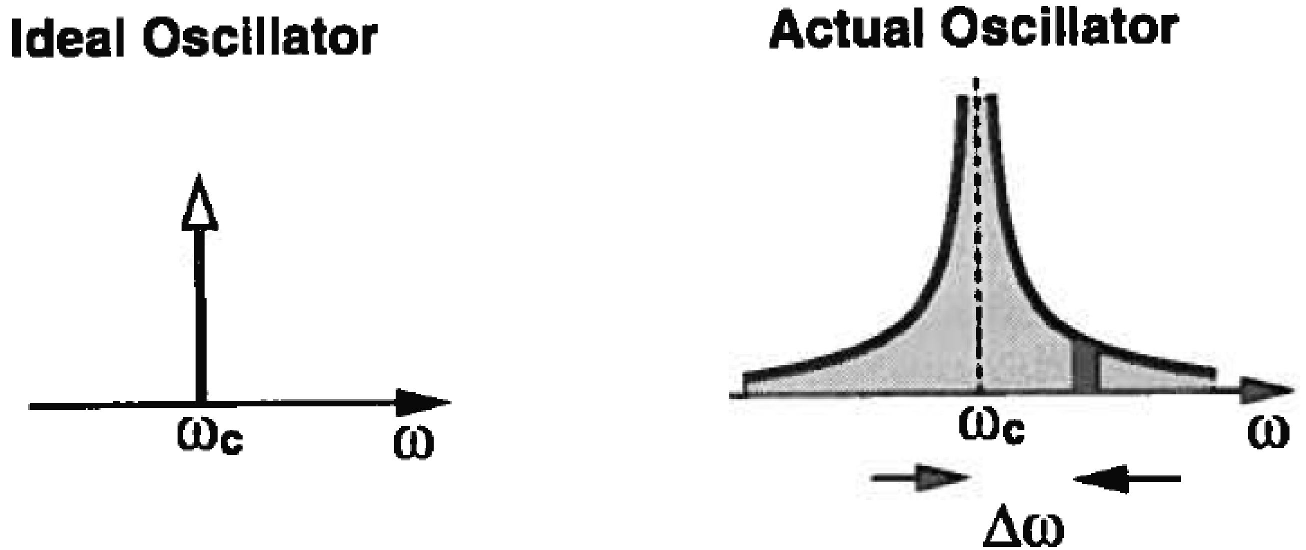
Lecture 16: VCO Phase Noise

Sam Palermo
Analog & Mixed-Signal Center
Texas A&M University

Agenda

- Phase Noise Definition and Impact
- Ideal Oscillator Phase Noise
- Leeson Model
- Hajimiri Model

Phase Noise Definition

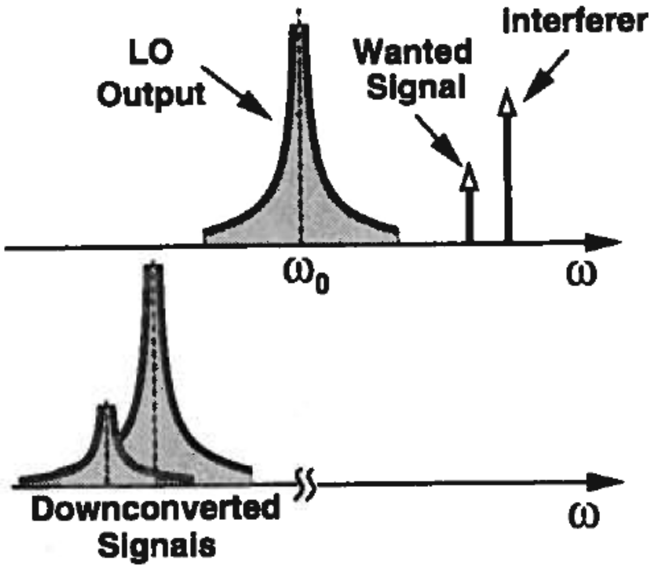


- An ideal oscillator has an impulse shape in the frequency domain
- A real oscillator has phase noise "skirts" centered at the carrier frequency
- Phase noise is quantified as the normalized noise power in a 1Hz bandwidth at a frequency offset $\Delta\omega$ from the carrier

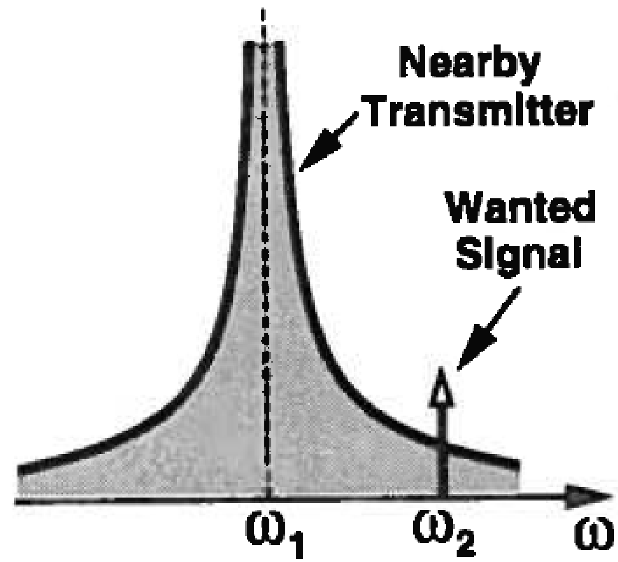
$$L(\Delta\omega) = 10\log\left(\frac{P_{\text{sideband}}(\omega_o + \Delta\omega, 1\text{Hz})}{P_{\text{carrier}}}\right) \ (\text{dBc/Hz})$$

Phase Noise Impact in RF Communication

RX Reciprocal Mixing

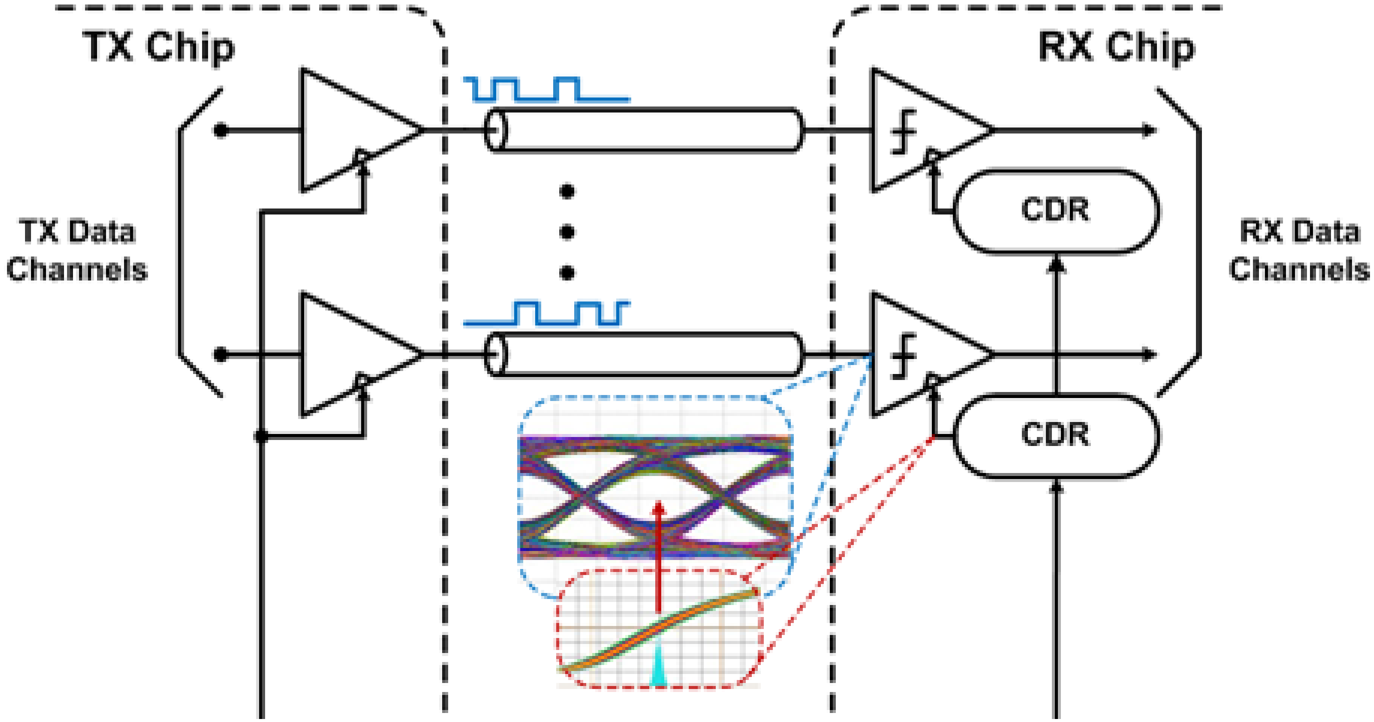


Strong Noisy TX Interfering with RX



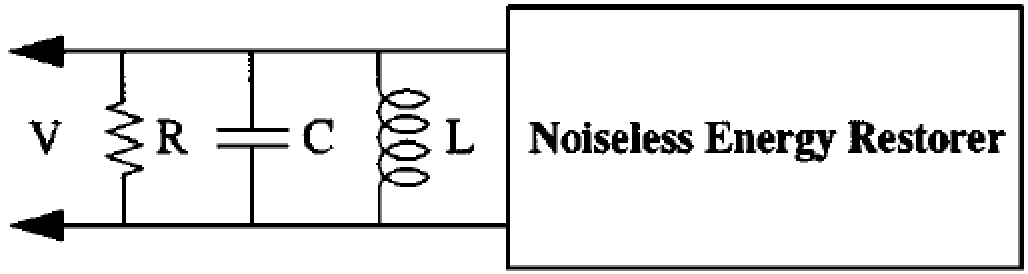
- At the RX, a large interferer can degrade the SNR of the wanted signal due to "reciprocal mixing" caused by the LO phase noise
- Having large phase noise at the TX can degrade the performance of a nearby RX

Jitter Impact in HS Links



- RX sample clock jitter reduces the timing margin of the system for a given bit-error-rate
- TX jitter also reduces timing margin, and can be amplified by low-pass channels

Ideal Oscillator Phase Noise



The tank resistance will introduce thermal noise

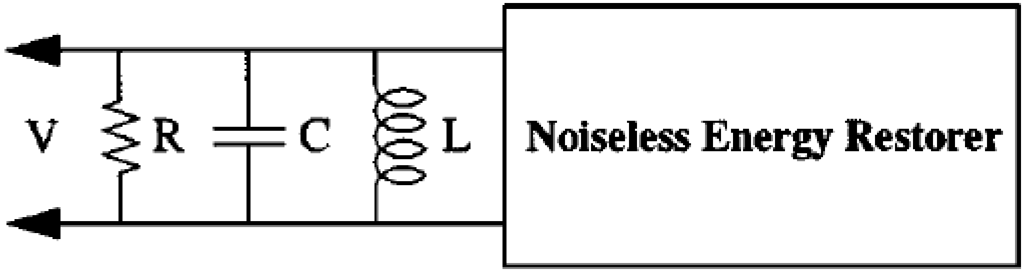
$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$$

The spectral density of the mean-squared noise voltage is

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}\left|Z_{\text{tank}}\right|^2$$

Tank Impedance Near Resonance

$$Z_{\text{tank}}(\omega) = \frac{1}{j\omega C} \Big\| j\omega L = \frac{j\omega L}{1-\omega^2 LC}$$



$$\text{Resonance Frequency: } \omega_o = \frac{1}{\sqrt{LC}}$$

Consider frequencies close to resonance $\omega = \omega_o + \Delta\omega$

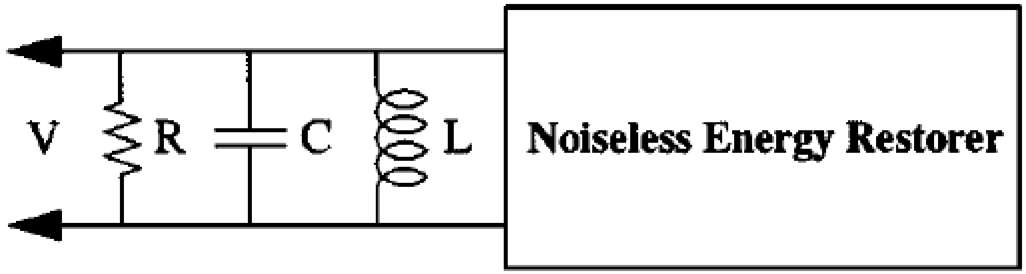
$$Z_{\text{tank}}(\Delta\omega) = \frac{j(\omega_o + \Delta\omega)L}{1-\omega_o^2 LC - 2\omega_o \Delta\omega LC - \Delta\omega^2 LC} \approx -\frac{j\omega_o L}{-2\omega_o \Delta\omega LC} = -\frac{j}{2}\frac{1}{\omega_o C}\left(\frac{\omega_o}{\Delta\omega}\right)$$

$$\text{Tank } Q = R\omega_o C \;\Rightarrow\; \frac{1}{\omega_o C} = \frac{R}{Q}$$

$$Z_{\text{tank}}(\Delta\omega) \approx -\frac{j}{2}\frac{R}{Q}\left(\frac{\omega_o}{\Delta\omega}\right)$$

$$\left|Z_{\text{tank}}(\Delta\omega)\right|^2 = \left(\frac{R\omega_o}{2Q\Delta\omega}\right)^2$$

Ideal Oscillator Phase Noise



$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}\left|Z_{\text{tank}}\right|^2 = \left(\frac{4kT}{R}\right)\left(\frac{R\omega_o}{2Q\Delta\omega}\right)^2 = 4kTR\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2$$

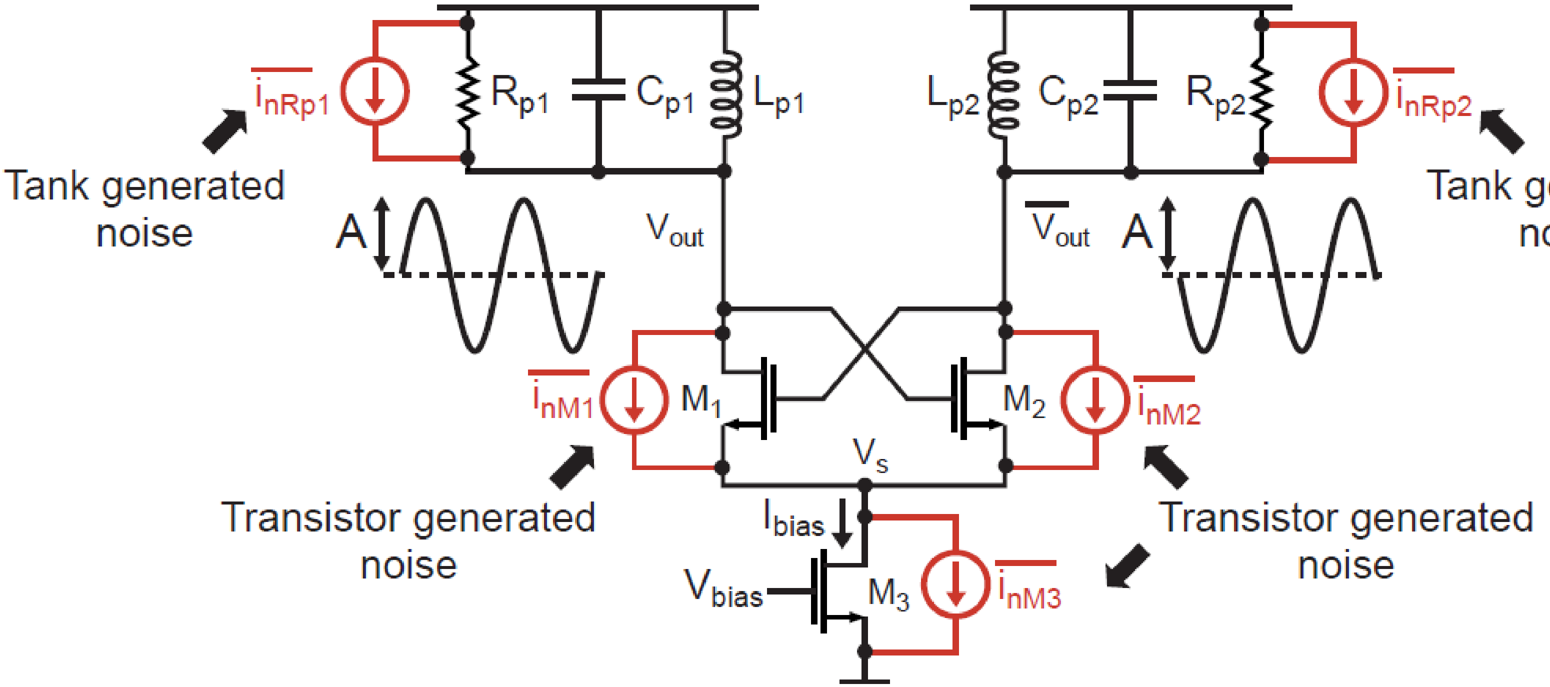
The Equipartition Theorem [Lee JSSC 2000] states that, in equilibrium, amplitude and phase-noise power are equal. Therefore, this noise power is split evenly $\left(\frac{1}{2}\right)$ between amplitude and phase.

$$L\{\Delta\omega\} = 10\log\left[\frac{2kT}{P_{sig}}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right] \quad (\text{dBc/Hz})$$

Phase noise due to thermal noise will display a -20dB/dec slope away from the carrier

Other Phase Noise Sources

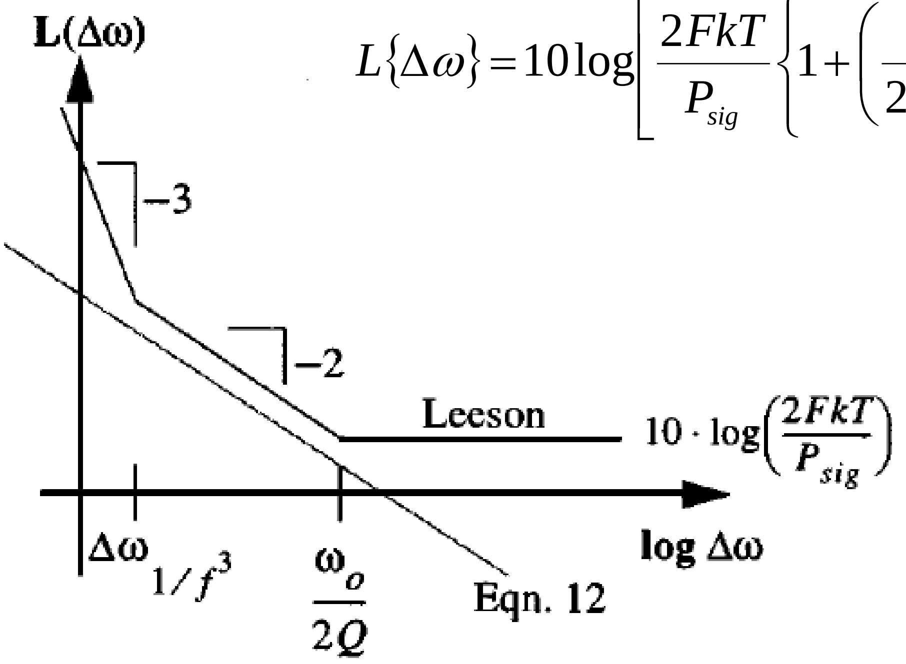
[Perrott]



- Tank thermal noise is only one piece of the phase noise puzzle
- Oscillator transistors introduce their own thermal noise and also flicker (1/f) noise

Leeson Phase Noise Model

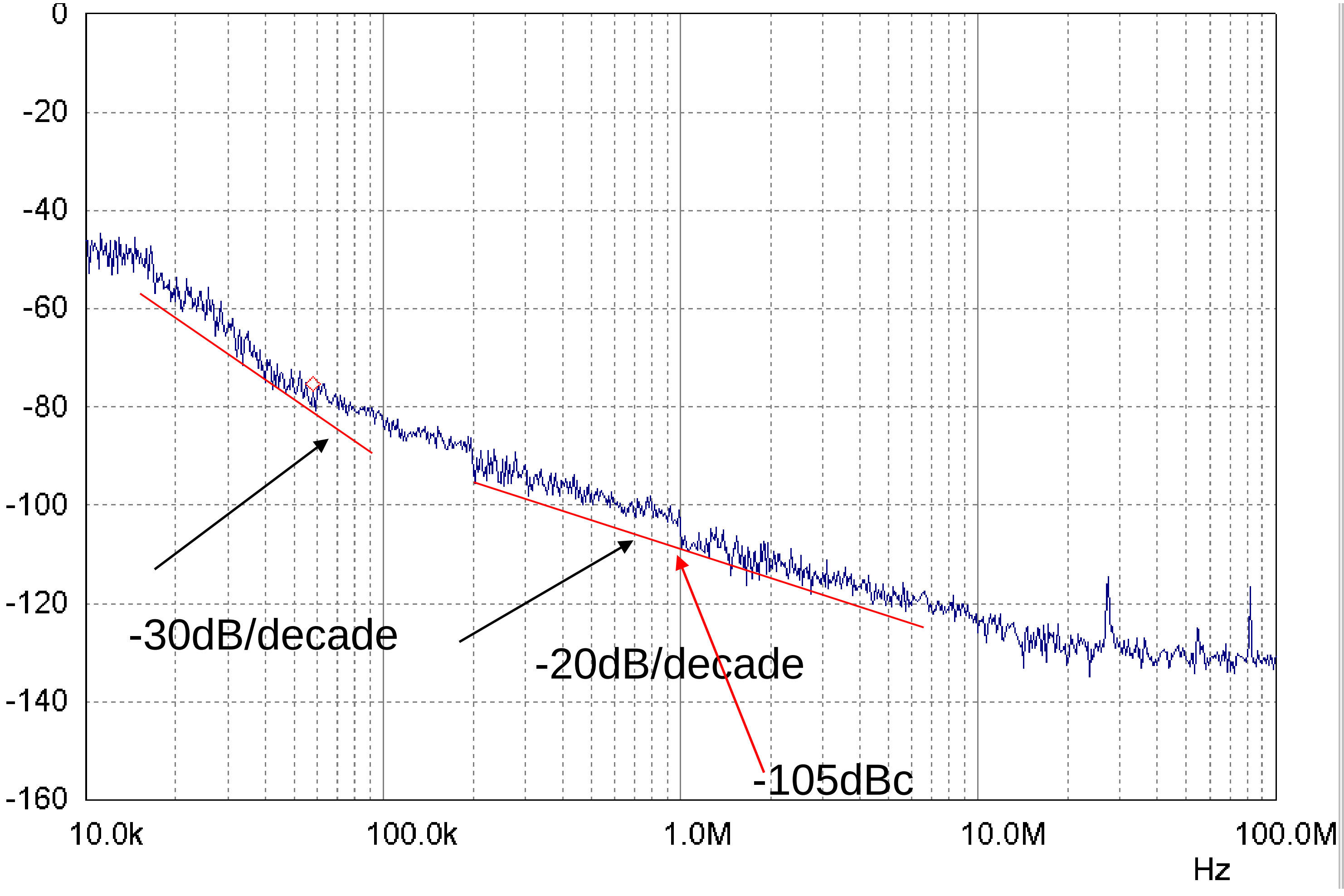
$$L\{\Delta\omega\} = 10\log\left[\frac{2FkT}{P_{sig}}\left\{1+\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right\}\left(1+\frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)\right] \quad \text{(dBc/Hz)}$$



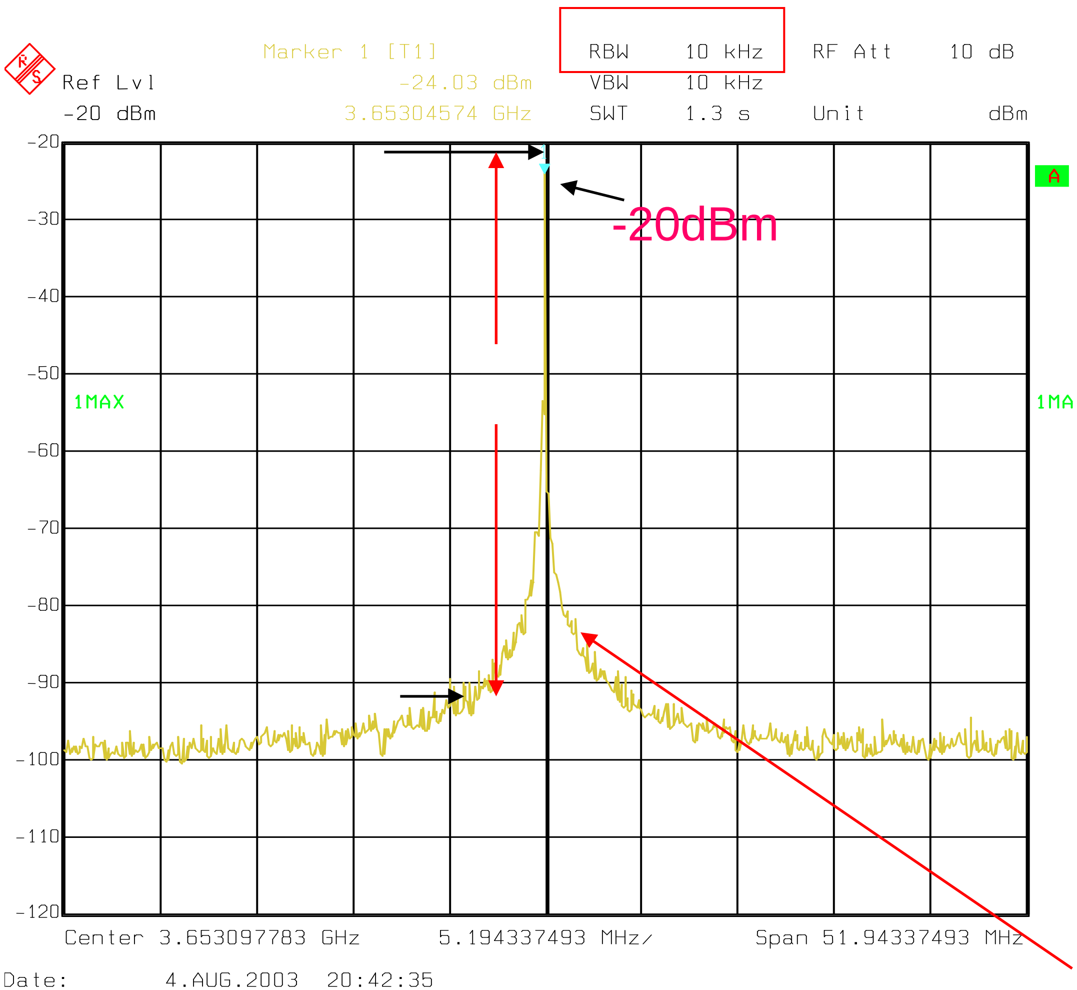
- Leeson's model modifies the previously derived expression to account for the high frequency noise floor and 1/f noise upconversion
- A empirical fitting parameter F is introduced to account for increased thermal noise
- Model predicts that the $(1/\Delta\omega)^3$ region boundary is equal to the 1/f corner of device noise and the oscillator noise flattens at half the resonator bandwidth

A 3.5GHz LC tank VCO Phase Noise

Measure Phase noise



VCO Output Spectrum Example



-85dBm

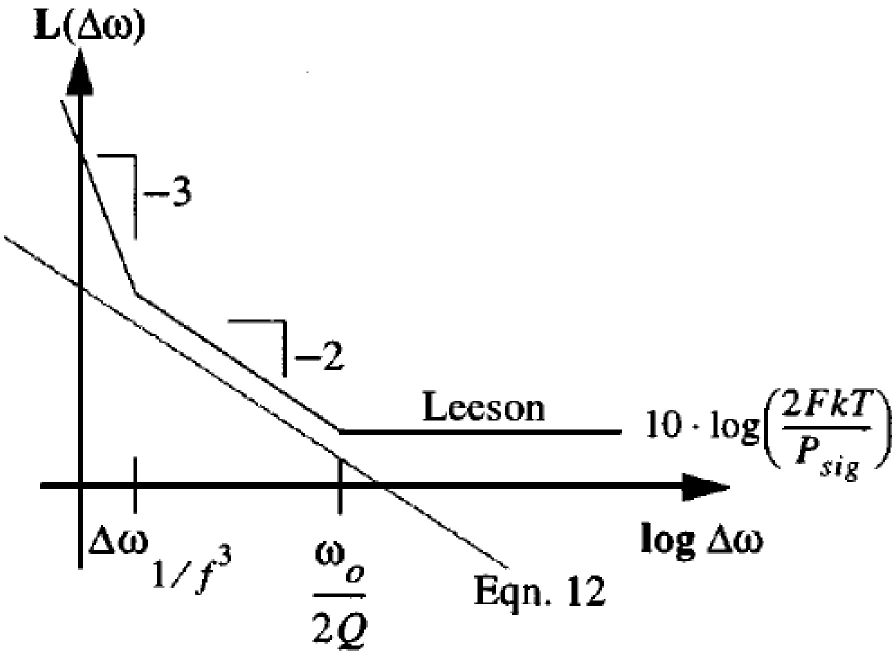
RBW=10K

PN=-85dBm-(-20dBm)-10log$_{10}$(10e3)

=-105dBc

dBc---in dB with respect to carrier

Leeson Model Issues

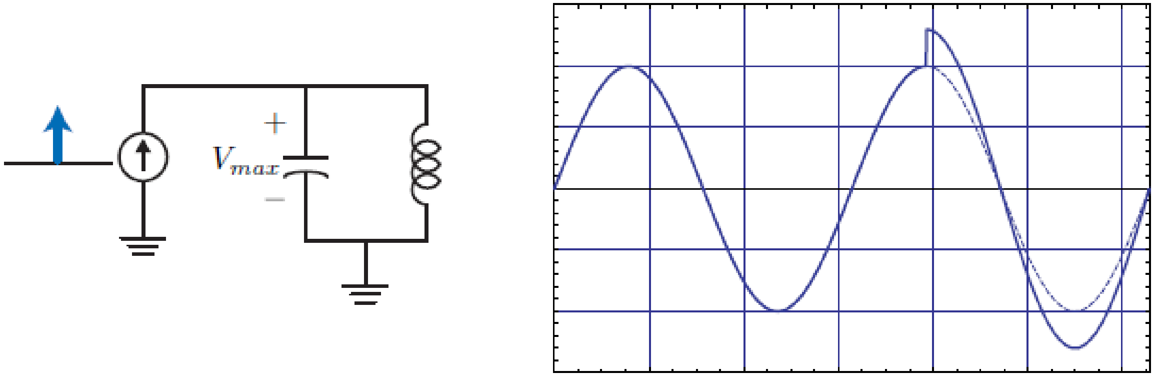


- The empirical fitting parameter F is not known in advance and can vary with different process technologies and oscillator topologies

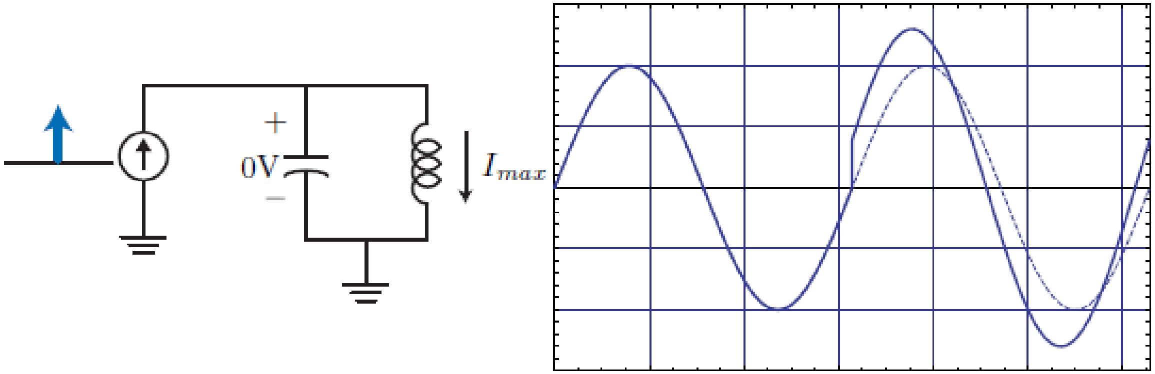
- The actual transition frequencies predicted by the Leeson model does not always match measured data

Harjimiri's Model (T. H. Lee)

- Injection at Peak (amplitude noise only)



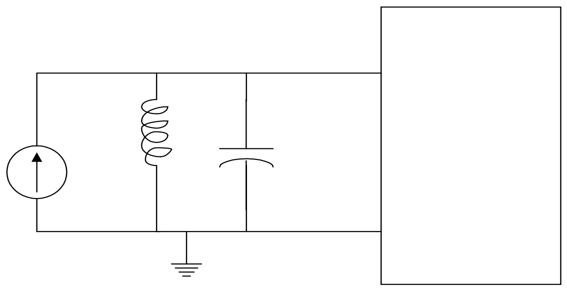
- Injection at Zero Crossing (maximum phase noise)

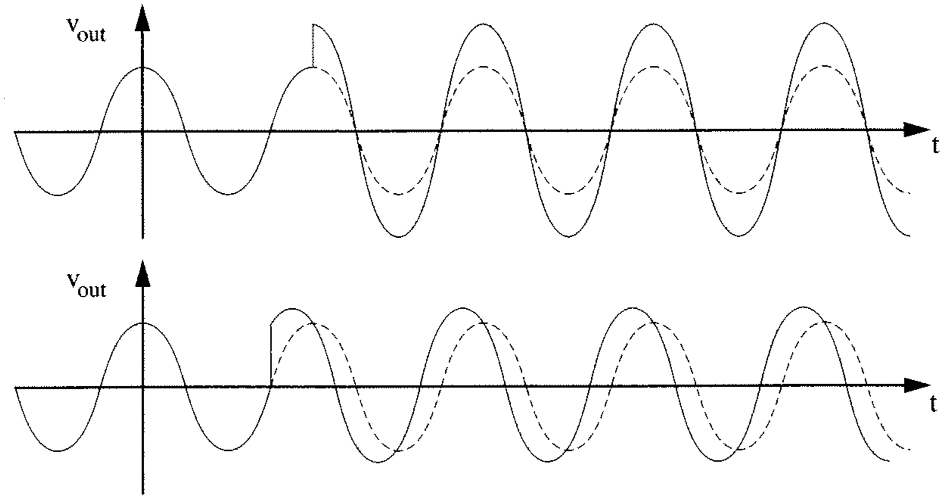


A time-Varying Phase Noise model: Hajimiri-Lee model

Impulse applied to the tank to measure its sensitivity function

$$\frac{\overline{i_n^2}}{\Delta f}$$





The impulse response for the phase variation can be represented as

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0 \tau)}{q_{max}} u(t-\tau),$$

Γ is the impulse sensitivity function ISF

qmax, the maximum charge displacement across the capacitor, is a normalizing factor

ISF Model

❑ The phase variation due to injecting noise can be modeled as:

$$\Delta\phi = \Gamma(\omega_0\tau)\frac{\Delta V}{V_{max}} = \Gamma(\omega_0\tau)\frac{\Delta q}{q_{max}} \qquad \Delta q \ll q_{max}$$

❑ The function, Γ(x), is the time-varying proportionality factor and called the "*impulse sensitivity function*".

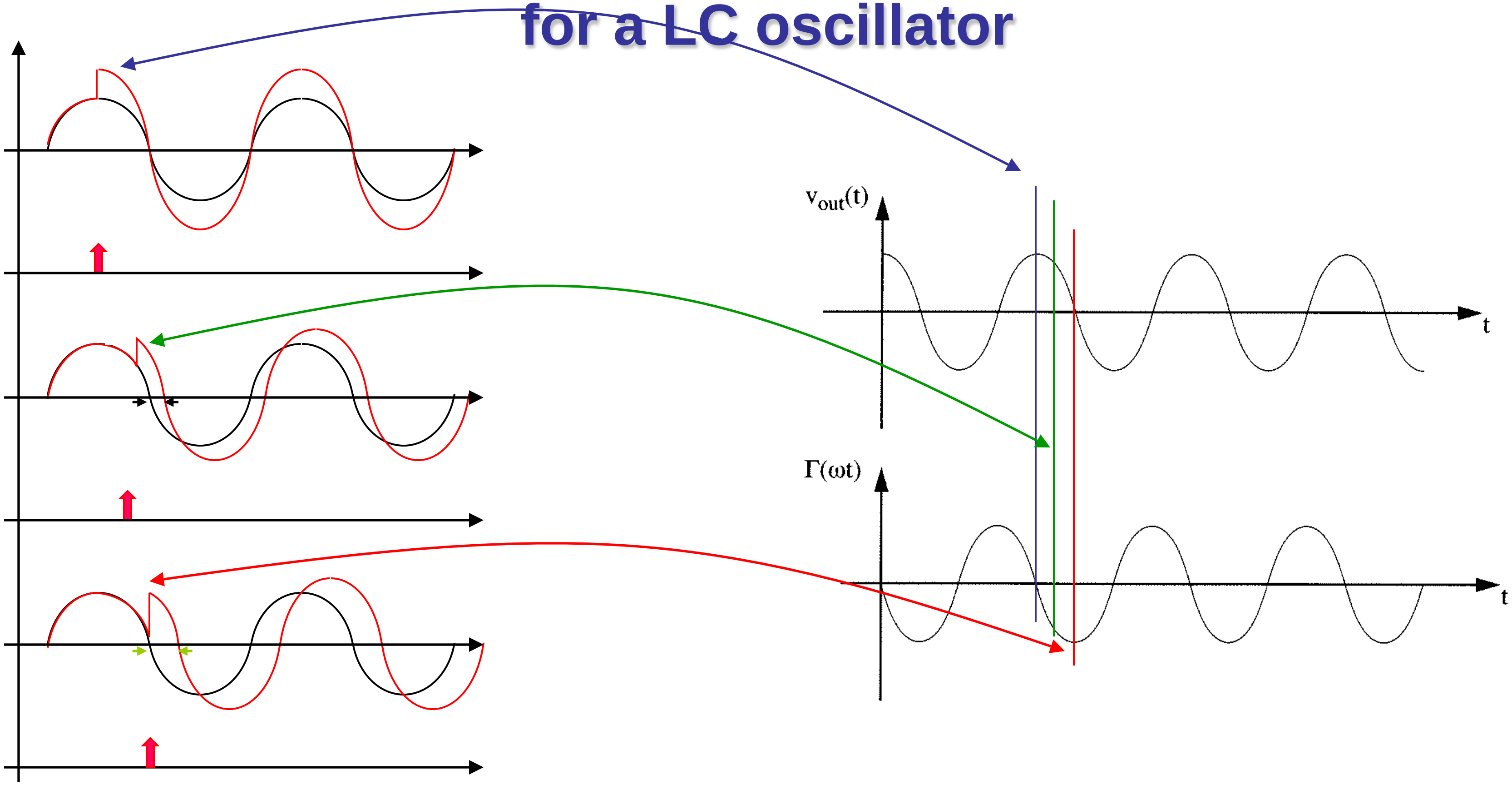
❑ The phase shift is assumed linear to injection charge.

❑ ISF has the same oscillation period T of the oscillator itself.

❑ The unity phase impulse response can be written as:

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0\tau)}{q_{max}}u(t-\tau)$$

How to obtain the Impulse sensitivity function for a LC oscillator

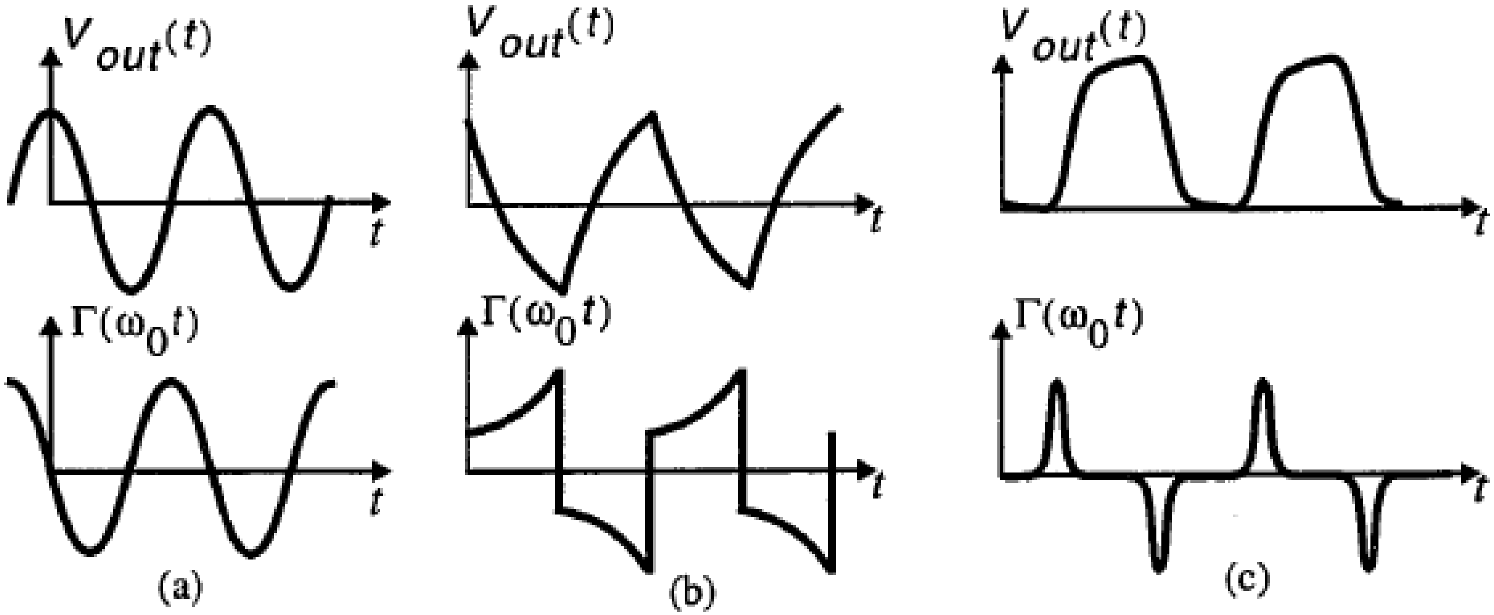


$\Gamma(\omega\tau)$ can be obtained using Cadence

Consider the effect on phase noise of each noise source

Typical ISF Example

- The ISF can be estimated analytically or calculated from simulation.
- The ISF reaches peak during zero crossing and zero at peak.



Typical ISF for (a) LC, (b) Bose and (c) ring oscillators.

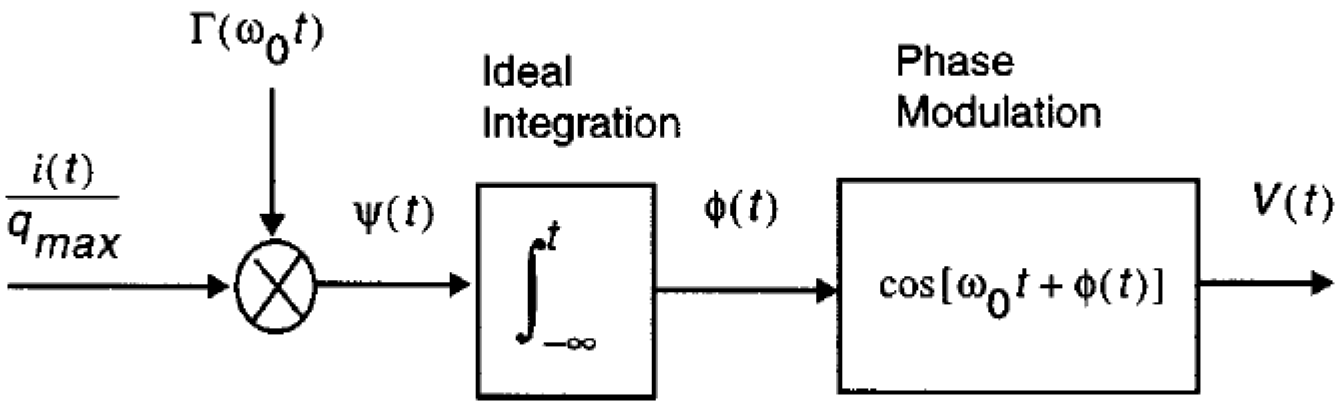
Phase Noise Computation

The impulse sensitivity function is used to obtain the phase noise impulse function

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_o \tau)}{q_{\max}} u(t-\tau)$$

The phase noise can then be computed by the superposition (convolution) integral of the any arbitrary noise current with the phase noise impulse function

$$\phi(\mathrm{t}) = \int_{-\infty}^{\infty} h_\phi(t,\tau) i(\tau) d\tau = \frac{1}{q_{\max}} \int_{-\infty}^{t} \Gamma(\omega_o \tau) i(\tau) d\tau$$



ISF Decomposition w/ Fourier Series

In order to gain further insight, and because the ISF is periodic, it may be expressed as a Fourier series

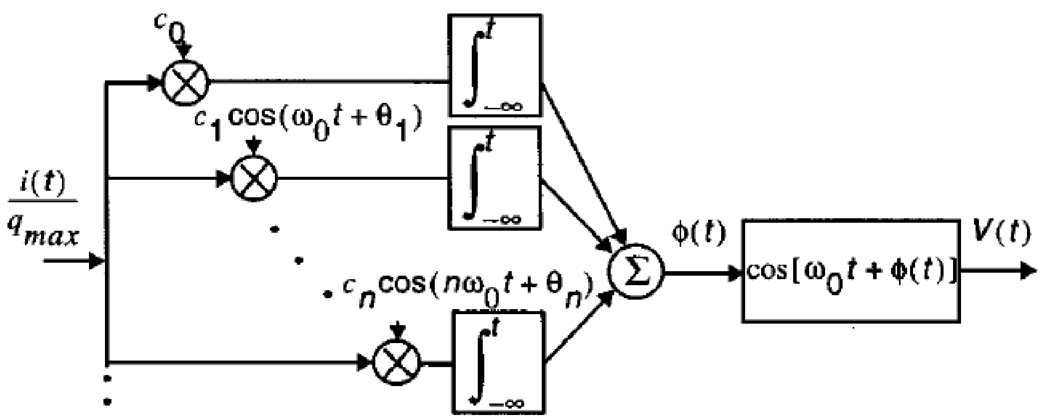
$$\Gamma(\omega_o\tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o\tau + \theta_n)$$

where the coefficients c_n are real and θ_n is the phase of the *n*th ISF harmonic.

The phase noise can then be computed by

$$\phi(\mathrm{t}) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_o\tau)d\tau\right]$$

This allows the excess phase from an arbitrary noise source to be computed once the ISF Fourier coefficients are deteremined. Essentially, the current noise is mixed down from different frequency bands and scaled according to the ISF coefficients.



Phase Noise Frequency Conversion

First consider a simple case where we have a sinusoidal noise current whose frequency is near an

integer multiple m of the oscillation frequency

$$i(t) = I_m \cos[(m\omega_o + \Delta\omega)t]$$

When performing the phase noise computation integral, there will be a negligible contribution from all terms

other than $n = m$

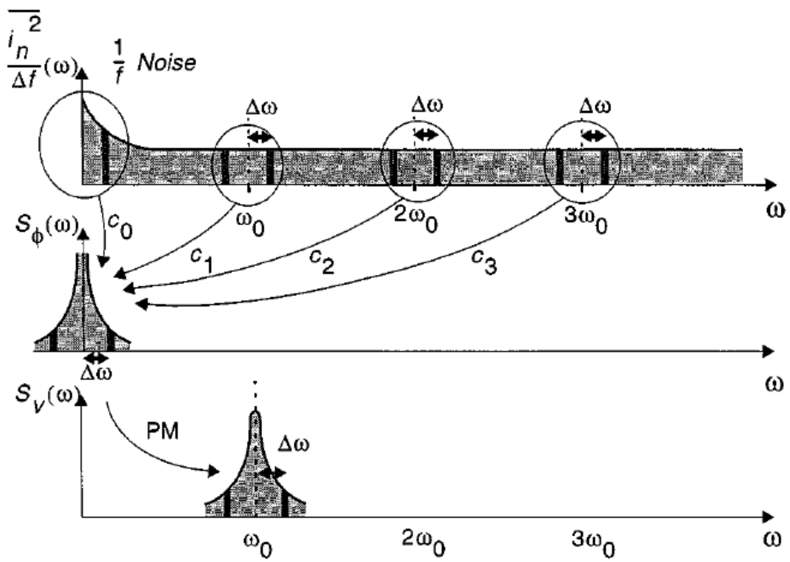
$$\varphi(t) \approx \frac{I_m c_m \sin(\Delta\omega t)}{2q_{\max}\Delta\omega}$$

The resulting frequency spectrum will show two equal sidebands at $\pm\Delta\omega$. Assuming a sinusoidal waveform

$v_{out}(t) = \cos[\omega_o t + \phi(t)]$, there will be two equally weighted sidebands symmetric about the carrier with power

$$P_{SBC}(\Delta\omega) \approx 10\log\left(\frac{I_m c_m}{4q_{\max}\Delta\omega}\right)^2$$

Note that this power is proportional to $\left(\frac{1}{\Delta\omega}\right)^2$.



Phase Noise Due to White & 1/f Sources

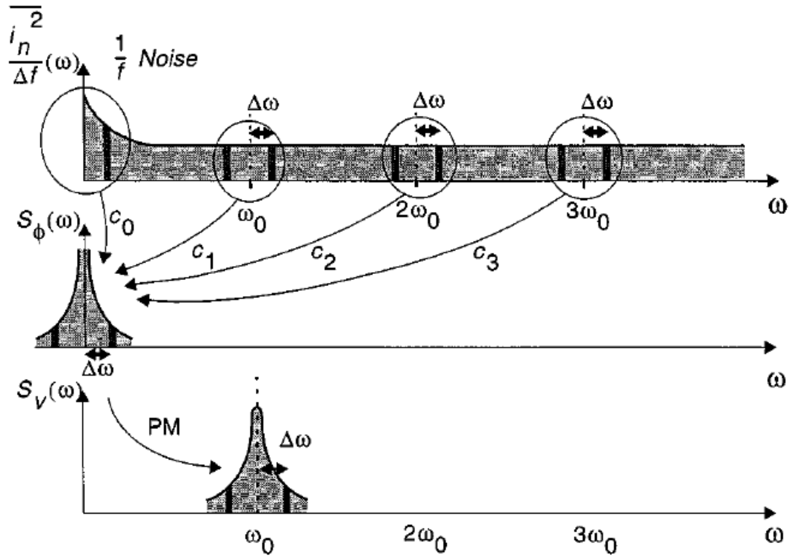
Extending the previous analysis to the general case of a white noise source results in

$$P_{SBC}(\Delta\omega) \approx 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}\sum_{m=0}^{\infty} c_m^2}{4q_{\max}^2\Delta\omega^2}\right)$$

Here noise components near integer multiples of the carrier frequency all fold near the carrier itself and are weighted by $\left(\frac{1}{\Delta\omega}\right)^2$.

Noise near dc gets upconverted, weighted by coefficient c_0, so $1/f$ noise becomes $1/f^3$ noise near the carrier.

Noise near the carrier stays there. White noise near higher integer multiples of the carrier gets downconverted and weighted by $1/f^2$.



How to Minimize Phase Noise?

In order to minimize phase noise, the ISF coefficients c_n should be minimized. Using Parseval's theorem

$$\sum_{m=0}^{\infty} c_m^2 = \frac{1}{\pi} \int_0^{2\pi} \left|\Gamma(x)\right|^2 dx = 2\Gamma_{rms}^2$$

The spectrum in the $1/f^2$ region can be expressed as

$$L(\Delta\omega) = 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}\Gamma_{rms}^2}{2q_{\max}^2 \Delta\omega^2} \right)$$

Thus, reducing Γ_{rms} will reduce the phase noise at all frequencies.

1/f Corner Frequency

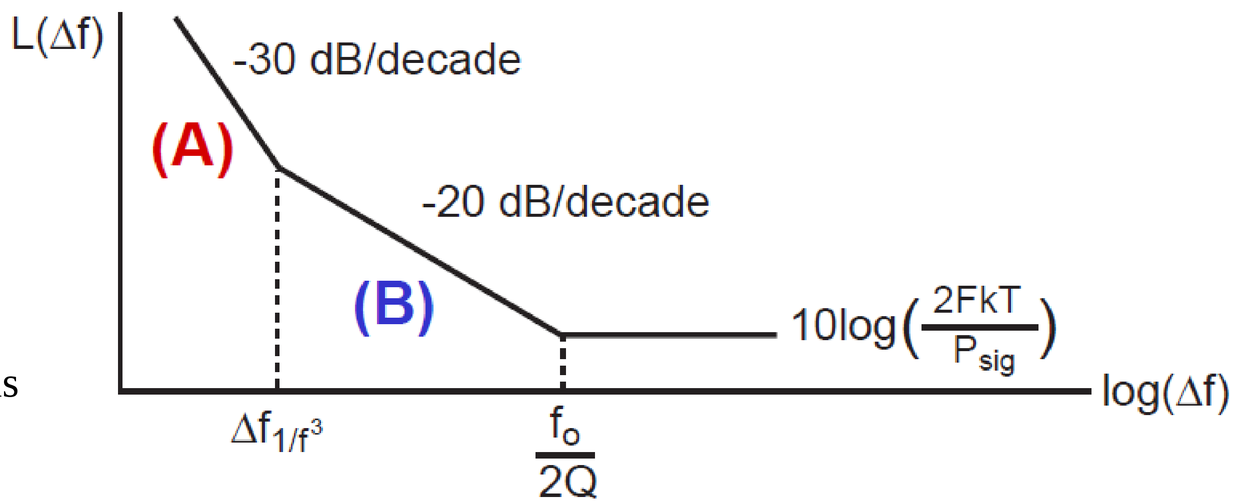
Consider current noise which includes $1/f$ content

$$\overline{i_{n,1/f}^2} = \overline{i_n^2}\frac{\omega_{1/f}}{\Delta\omega}$$

where $\omega_{1/f}$ is the $1/f$ corner frequency

From the previous slide

$$L(\Delta\omega) = 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}c_0^2}{8q_{\max}^2\Delta\omega^2}\frac{\omega_{1/f}}{\Delta\omega}\right)$$



Thus, the $1/f^3$ corner frequency is

$$\Delta\omega_{1/f^3} = \omega_{1/f}\frac{c_0^2}{4\Gamma_{rms}^2} = \omega_{1/f}\left(\frac{\Gamma_{dc}}{\Gamma_{rms}}\right)^2$$

This is generally lower than the $1/f$ device/circuit noise corner. If Γ_{dc} is minimized through rise- and fall-time symmetry, then there is the potential for dramatic reductions in $1/f$ noise.

Cyclostationary Noise Treatment

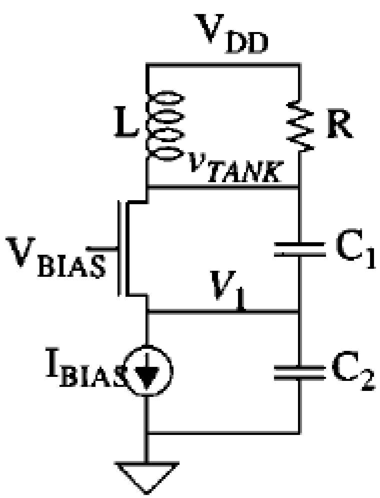
Transistor drain current, and thus noise, can change dramatically over an oscillator cycle.

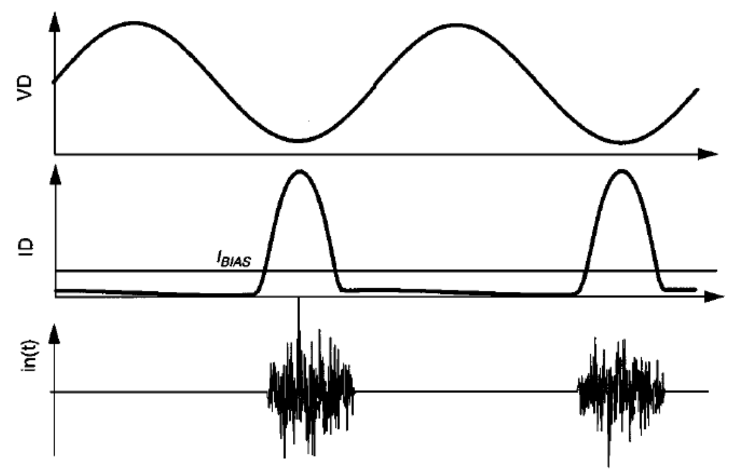
The LTV model can easily handle this by treating it as the product of stationary white noise and a periodic function.

$$i_n(t) = i_{n0}(t)\alpha(\omega_0 t)$$

Here i_{n0} is a stationary white noise source whose peak value is equal to that of the cyclostationary noise source, and $\alpha(x)$ is a periodic unitless function with a peak value of unity. Using this, we can formulate an effective ISF

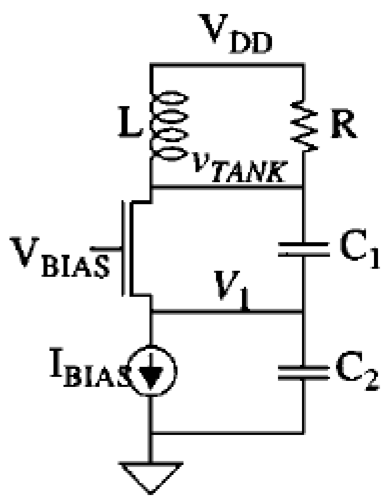
$$\Gamma_{\text{eff}}(x) = \Gamma(x)\alpha(x)$$

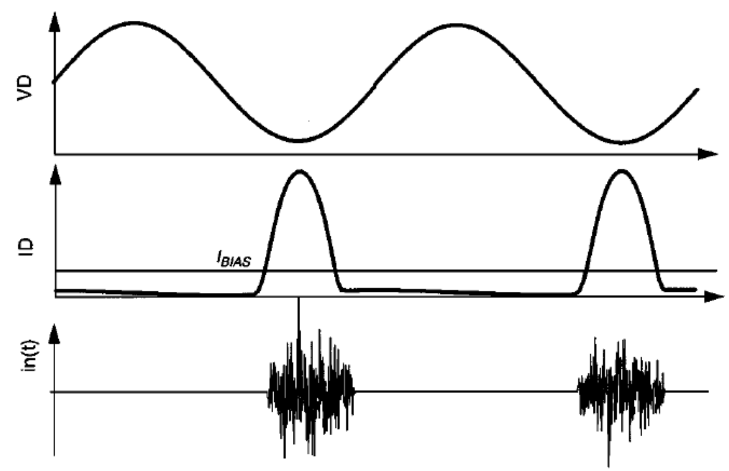




Key Oscillator Design Points

- As the LTI model predicts, oscillator signal power and Q should be maximized
- Ideally, the energy returned to the tank should be delivered all at once when the ISF is minimum
- Oscillators with symmetry properties that have small Γ_{dc} will provide minimum 1/f noise upconversion

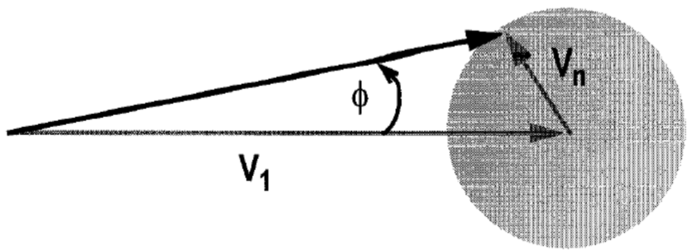




Phasor-Based Phase Noise Analysis

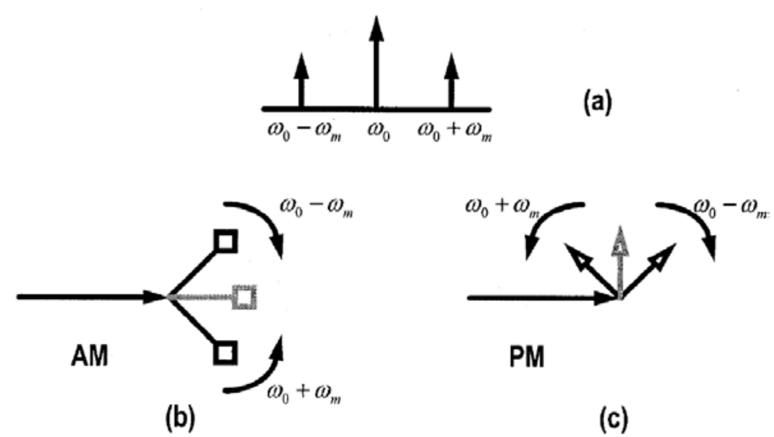
Physical Processes of Phase Noise in Differential LC Oscillators

J. J. Rael and A. A. Abidi



$$\begin{aligned}V_{out} &= V_1\cos(\omega_0 t)\\ &+\phi_1\left[\cos(\omega_l t)-\cos(\omega_u t)\right]+\alpha_1\left[\cos(\omega_l t)+\cos(\omega_u t)\right]\\ &+\phi_2\left[\sin(\omega_l t)+\sin(\omega_u t)\right]+\alpha_2\left[\sin(\omega_l t)-\sin(\omega_u t)\right]\end{aligned}$$

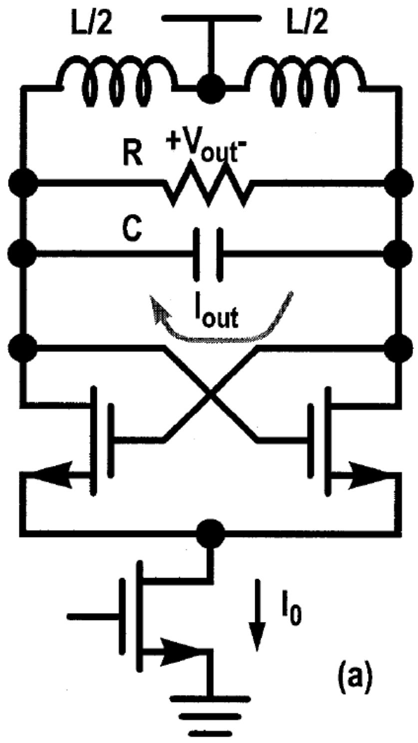
- Models noise at 2 sideband frequencies with modulation terms
- The α_1 and α_2 terms sum co-linear with the carrier phasor and produce amplitude modulation (AM)
- The ϕ_1 and ϕ_2 terms sum orthogonal with the carrier phasor and produce phase modulation (PM)



Phasor-Based LC Oscillator Analysis

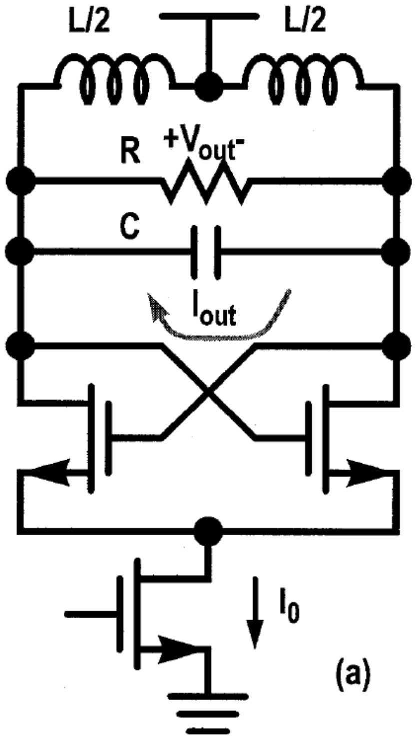
- This phasor-based approach can be used to find closed-form expressions for LC oscillator phase noise that provide design insight

- In particular, an accurate expression for the Leeson model F parameter is obtained



$$L\{\Delta\omega\} = 10\log\left[\frac{2FkT}{P_{sig}}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right] \quad (\mathrm{dBc/Hz})$$

LC Oscillator F Parameter



$$L\{\Delta\omega\} = 10\log\left[\frac{2FkT}{P_{sig}}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right] \quad (\text{dBc/Hz})$$

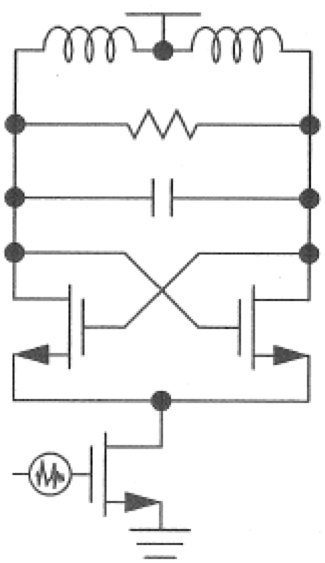
$$F = 1 + \frac{4\gamma IR}{\pi V_0} + \gamma\frac{4}{9}g_{m\text{bias}}R$$

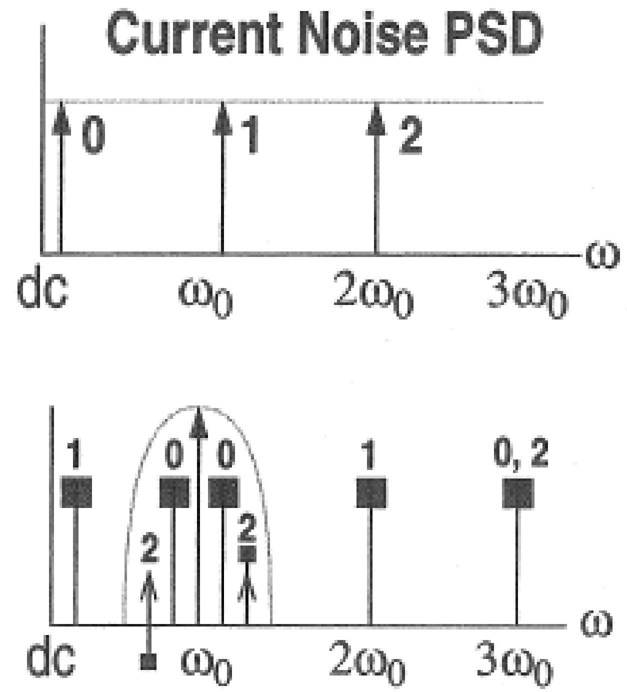
- 1st Term = Tank Resistance Noise
- 2nd Term = Cross-Coupled Pair Noise
- 3rd Term = Tail Current Source Noise

- The above expression gives us insight on how to optimize the oscillator to reduce phase noise
- The tail current source is often a significant contributor to total noise

Tail Current Noise

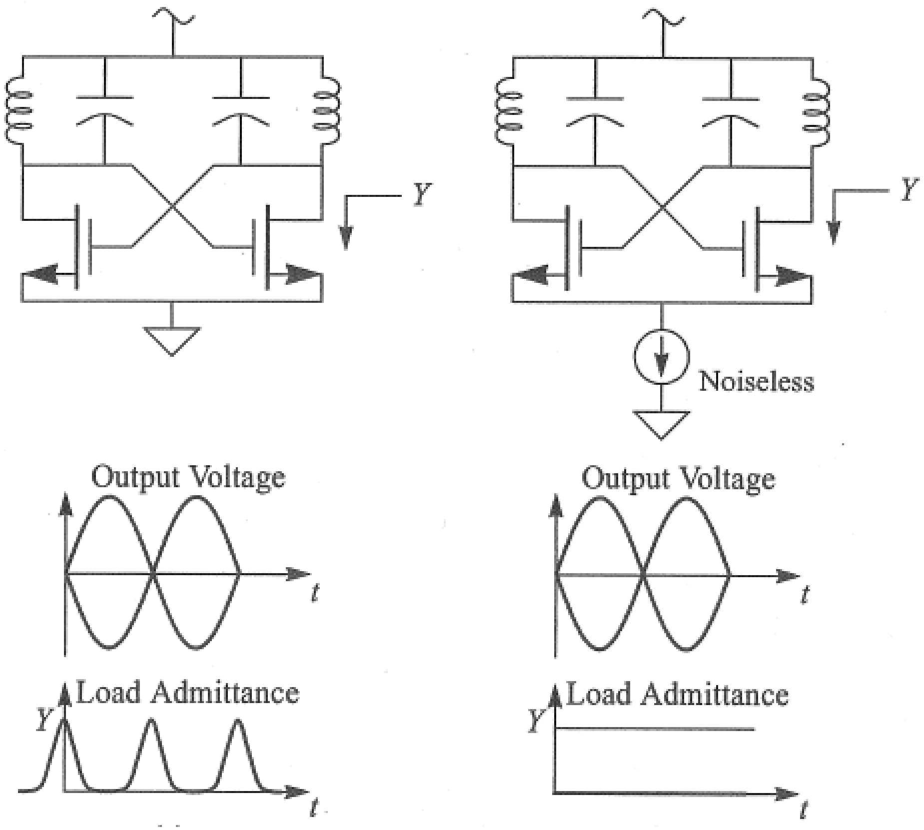
- ❑The switching differential pair can be modeled as a mixer for noise in the current source
- ❑Only the noise located at even harmonics will produce phase noise.





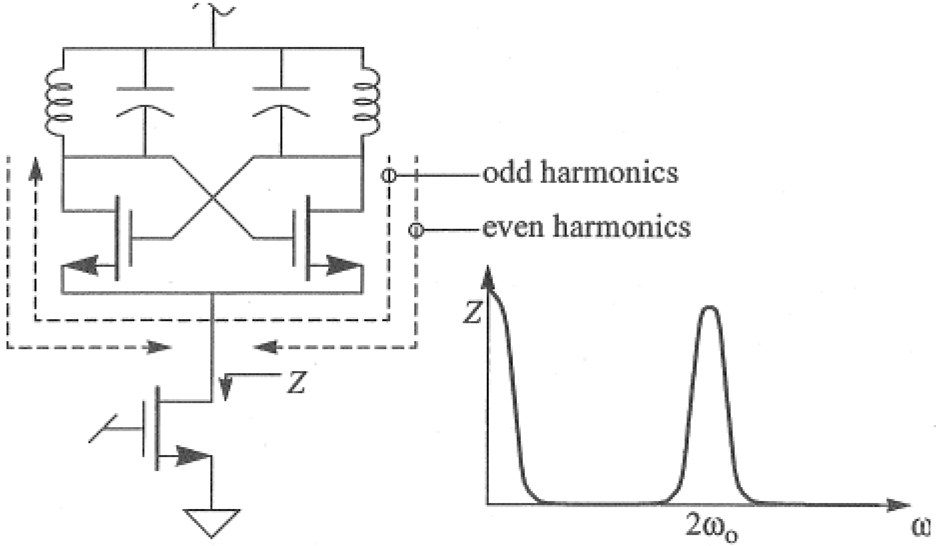
Loading in Current-Biased Oscillator

- The current source plays 2 roles
  - It sets the oscillator bias current
  - Provides a high impedance in series with the switching transistors to prevent resonator loading



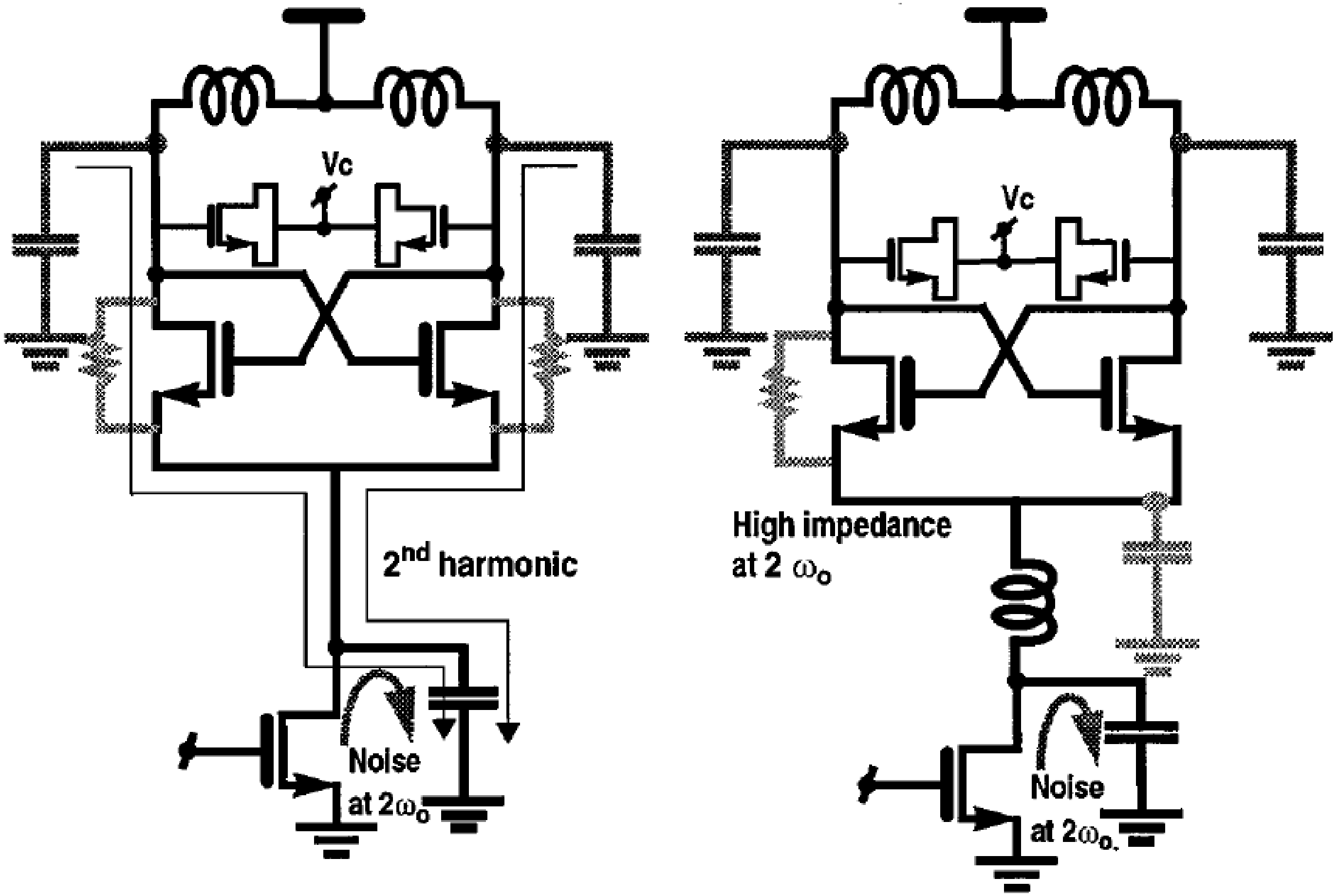
Noise Filtering in Oscillator

- Only thermal noise in the current source transistor around 2nd harmonic of the oscillation causes phase noise.
- In balanced circuits, odd harmonics circulate in a differential path, while even harmonics flow in a common-mode path
- A high impedance at the tail is only required at the 2nd harmonic to stop the differential pair FETs in triode from loading the resonator.

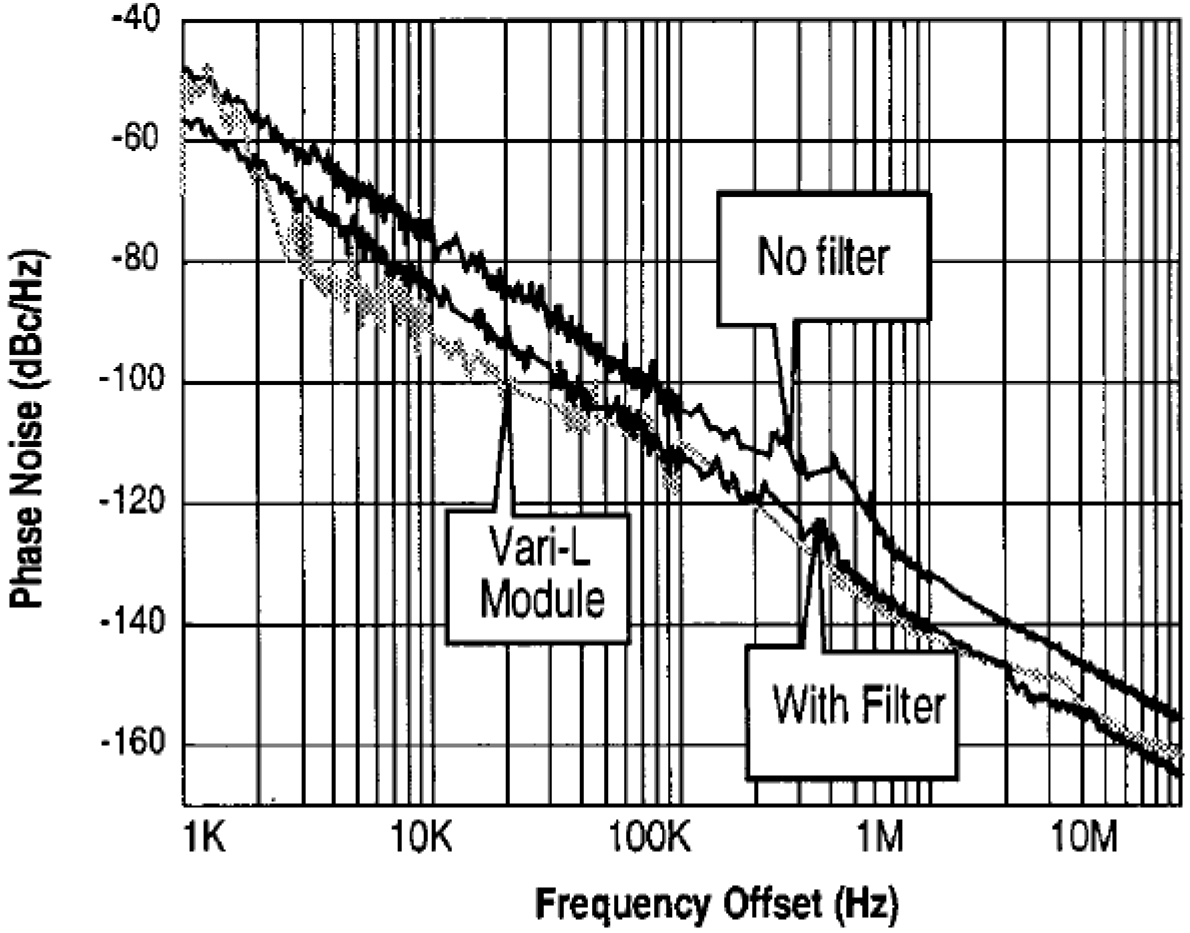


Noise Filtering in Oscillator

- Tail-biased VCO with noise filtering.



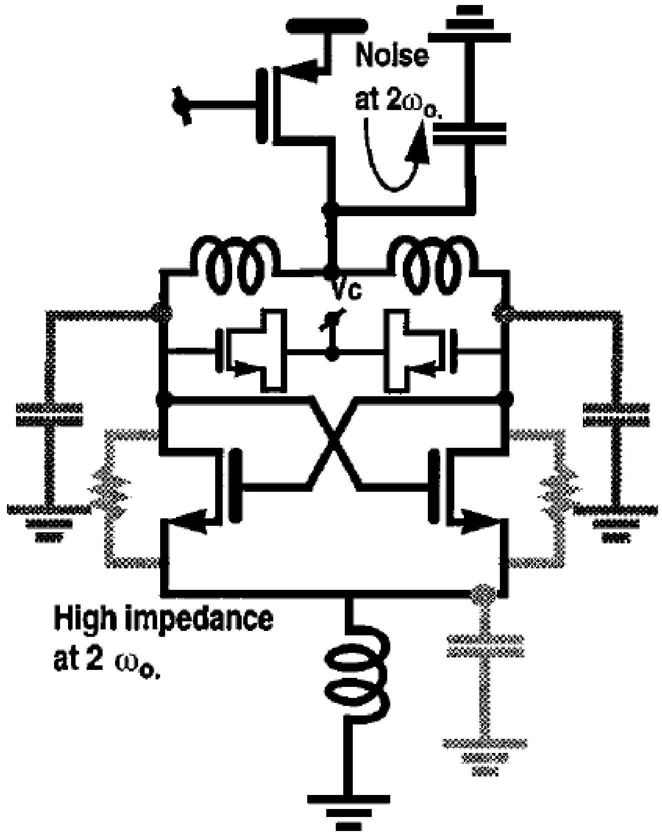
Phase Noise w/ Tail Current Filtering

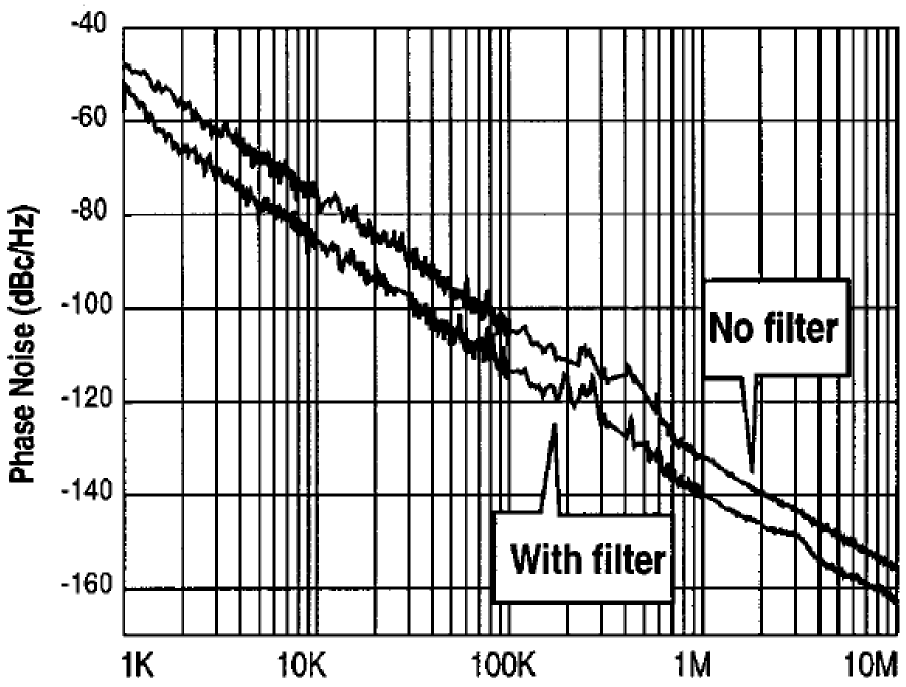


- Tail current noise filtering provides near 7dB improvement

Noise Filtering in Oscillator

❑A top-biased VCO often provides improved substrate noise rejection and reduced flicker noise





Next Time

- Divider Circuits

In general the impulse sensitivity function is periodic with a fundamental frequency equal to the oscillating frequency

The input sensitivity function can be characterized as a Fourier Series:

$$\Gamma(\omega_0\tau) = \frac{C_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n),$$

An the phase noise is then

$$\phi(t) = \int_{-\infty}^{\infty} h_\phi(t,\tau) i(\tau)\, d\tau = \frac{1}{q_{\max}} \int_{-\infty}^{t} \Gamma(\omega_0\tau) i(\tau)\, d\tau.$$

Therefore:

$$\phi(t) = \frac{1}{q_{\max}} \left[\frac{c_0}{2} \int_{-\infty}^{t} i(\tau)\, d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau) \cos(n\omega_0\tau)\, d\tau \right].$$

If the input noise is represented as $i(t) = I_m \cos[(m\omega_0 + \Delta\omega)t]$,

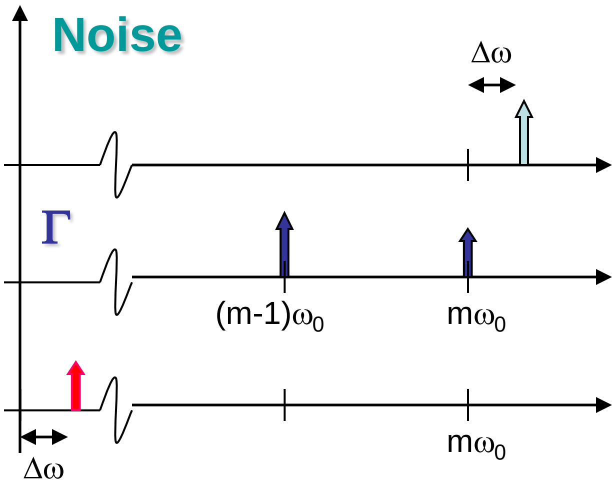
$$\Gamma(\omega_0\tau) = \frac{C_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n),$$

$$\phi(t) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} \{I_m \cos[(m\omega_0 + \Delta\omega)t]\}\cos(n\omega_0\tau)\,d\tau\right]$$

Due to the periodicity of the terms, the series converge to (n=m term only):

Phase noise

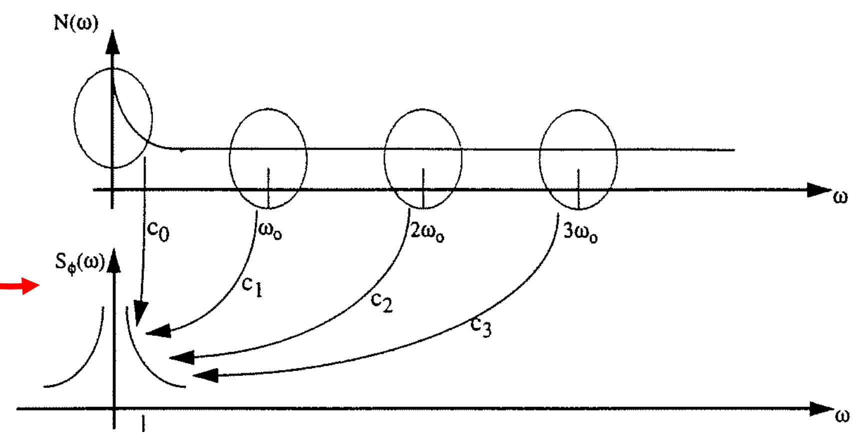
$$\phi(t) \approx \frac{I_m c_m \sin(\Delta\omega t)}{2q_{\max}\Delta\omega}.$$



If the input noise is represented by its power distribution function

$$\phi(t) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)\,d\tau\right].$$

$$\phi(t) \approx \frac{I_m c_m \sin(\Delta\omega t)}{2 q_{\max}\Delta\omega}.$$



For small noise level

$$V_{out} = V_0\cos(\omega_o t + \phi(t))$$

$$V_{out} \cong V_0\cos(\omega_o t) - V_0\sin(\omega_o t)\sin(\phi(t$$

$$V_{out} \cong V_0\cos(\omega_o t) - V_0(\sin(\omega_o t))\phi(t)$$