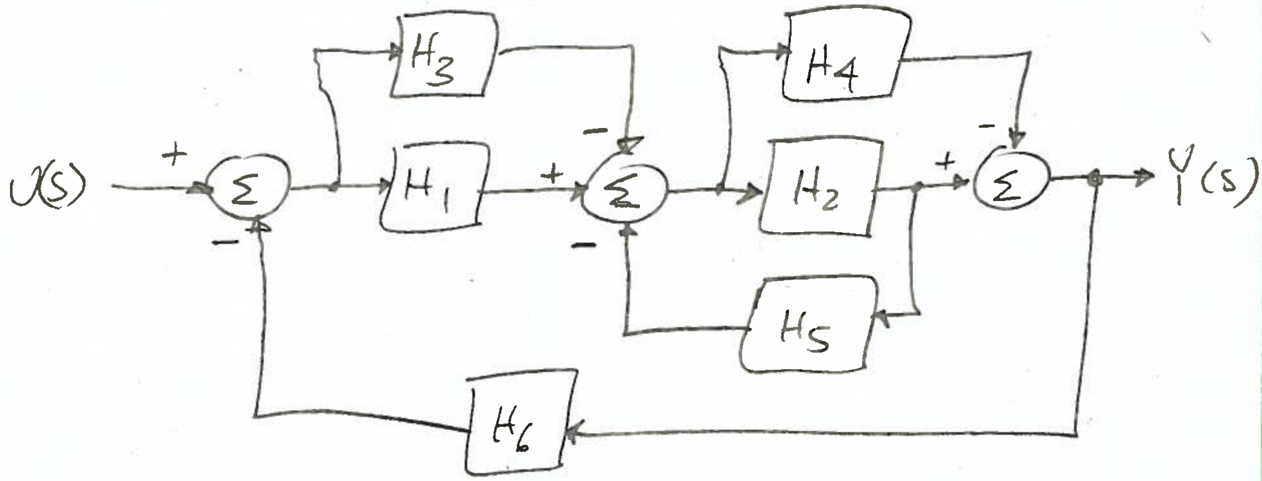
1.



Forward Path Gains

$G_1 = H_1 H_2$

$G_2 = -H_1 H_4$

$G_3 = -H_3 H_2$

$G_4 = H_3 H_4$

Loop Gains

$\ell_1 = -H_2 H_5$

$\ell_2 = -H_1 H_2 H_6$

$\ell_3 = H_3 H_2 H_6$

$\ell_4 = H_1 H_4 H_6$

$\ell_5 = -H_3 H_4 H_6$

System Determinant (all loops touch)

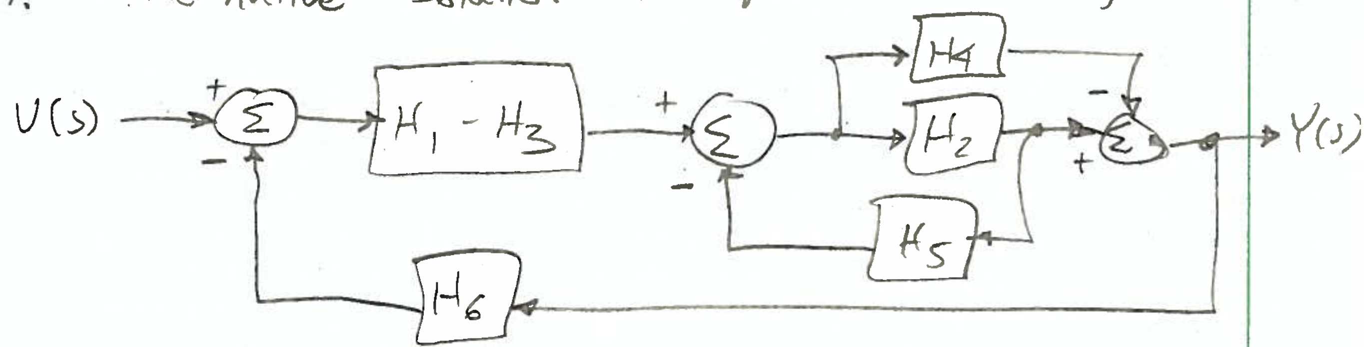
$\Delta = 1 + H_2 H_5 + H_1 H_2 H_6 - H_3 H_2 H_6 - H_1 H_4 H_6 + H_3 H_4 H_6$

Forward Path Determinants (all loops touch forward paths)

$\Delta_{1-4} = 1$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{H_1 H_2 - H_1 H_4 - H_3 H_2 + H_3 H_4}{1 + H_2 H_5 + H_1 H_2 H_6 - H_3 H_2 H_6 - H_1 H_4 H_6 + H_3 H_4 H_6}$$

1. Alternative Solution w/ Simplified Block Diagram



Forward Path Gains

$G_1 = (H_1 - H_3)\, H_2$

$G_2 = -(H_1 - H_3)\, H_4$

Loop Gains

$l_1 = -H_2 H_5$

$l_2 = -(H_1 - H_3) H_2 H_6$

$l_3 = (H_1 - H_3) H_4 H_6$

System Determinant (all loops touch)

$\Delta = 1 + H_2 H_5 + (H_1 - H_3) H_2 H_6 - (H_1 - H_3) H_4 H_6$

Forward Path Determinants (all loops touch forward paths)

$\Delta_{1-2} = 1$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(H_1 - H_3) H_2 - (H_1 - H_3) H_4}{1 + H_2 H_5 + (H_1 - H_3) H_2 H_6 - (H_1 - H_3) H_4 H_6}$$

* This simplifies to the same value

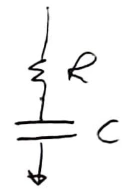
2. For a 2nd-order Type-2 charge-pump PLL

$$H(s) = \frac{K_{PD}K_{VCO}\left(s+\frac{1}{RC}\right)}{s^2+\left(\frac{K_{PD}K_{VCO}R}{N}\right)s+\frac{K_{PD}K_{VCO}}{NC}} = \frac{N2\zeta\omega_n\left(s+\frac{\omega_n}{2\zeta}\right)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

where $\omega_n = \sqrt{\frac{K_{PD}K_{VCO}}{NC}}$, $\zeta = \frac{\omega_n}{2}RC$

$$E(s) = \frac{s^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Loop Filter:



For $\omega_n = 2\pi(10\,MHz)$

$$C = \frac{K_{PD}K_{VCO}}{N\omega_n^2} = \frac{\left(\frac{100\,\mu A}{2\pi}\right)\left(2\pi\cdot 1\,GHz/V\right)}{32\left(2\pi\cdot 10\,MHz\right)^2} = 792\,fF$$

$$R = \frac{2\zeta}{\omega_n} = \frac{2\zeta}{2\pi(10\,MHz)(792\,fF)}$$

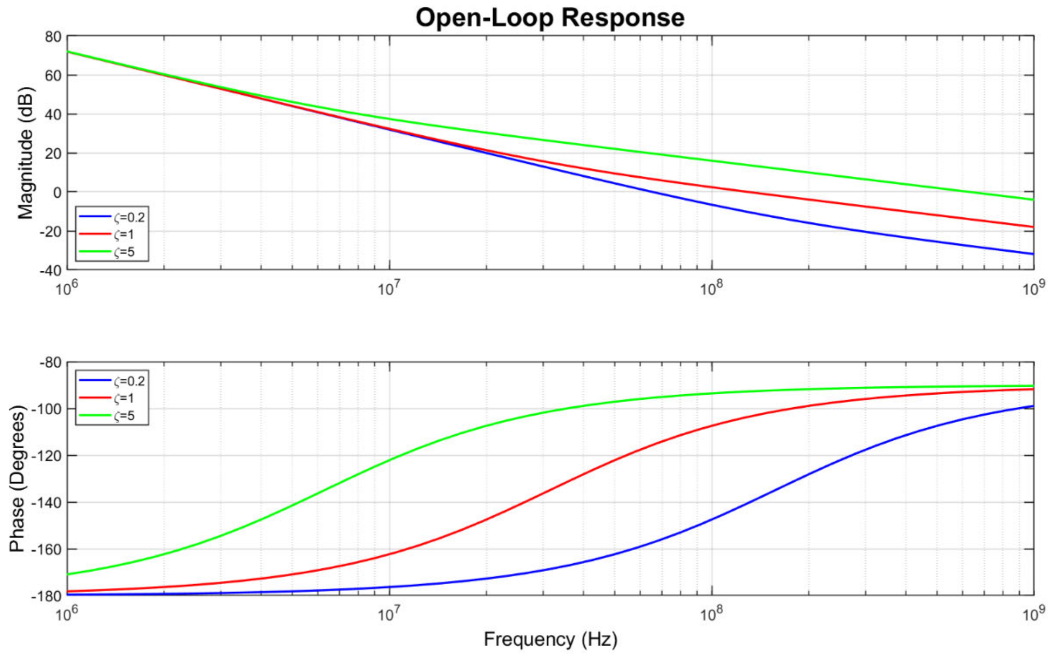
$$\boxed{C = 792\,fF}$$

ζ	R
0.2	8.04 kΩ
1	40.2 kΩ
5	201 kΩ

* In order to find phase margin, we need to plot

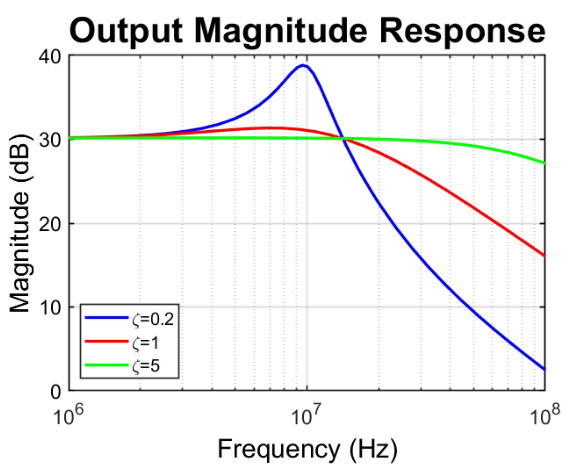
$$\frac{G(s)}{N} = \frac{K_{PD}K_{VCO}R\left(s+\frac{1}{RC}\right)}{Ns^2} = \frac{2\zeta\omega_n\left(s+\frac{\omega_n}{2\zeta}\right)}{s^2}$$

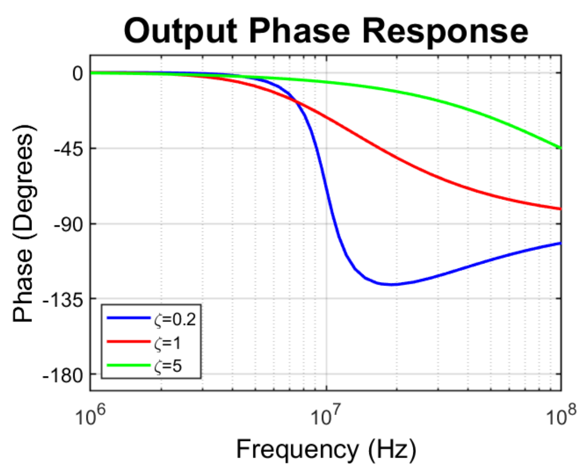
Ideal 2nd-Order System



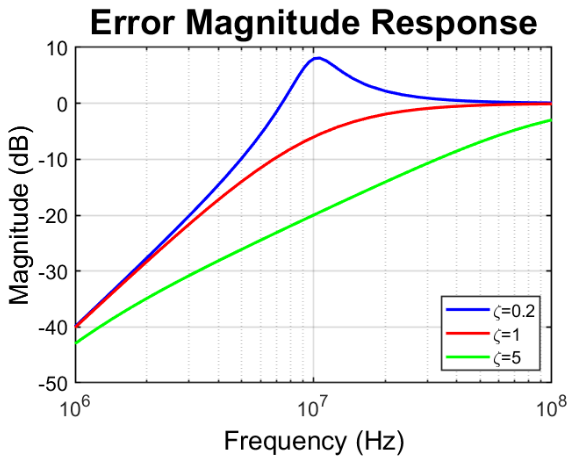
ζ	**Phase Margin (°)**	**Closed-Loop Poles (rad/s)**
0.2	22.6	-1.26e7 ± j6.15e7
1	76.3	-6.28e7 ± j0.055e7
5	89.4	-6.22e8, -6.35e6

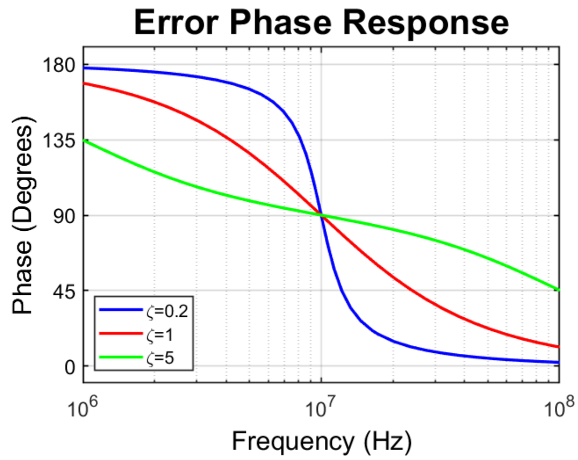
Output Bode Plots



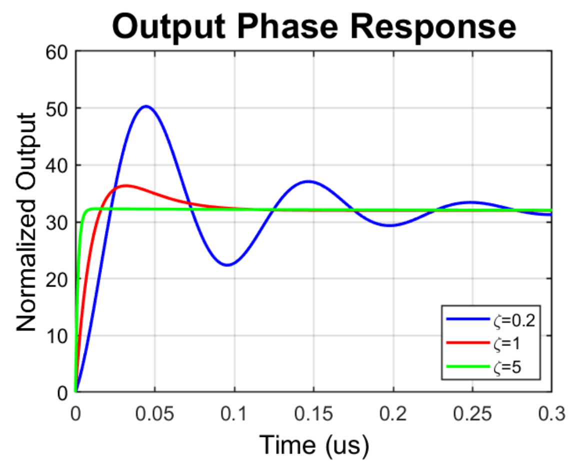


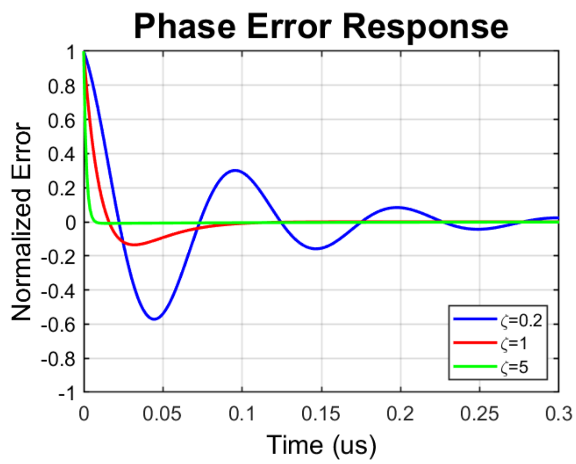
Error Bode Plots





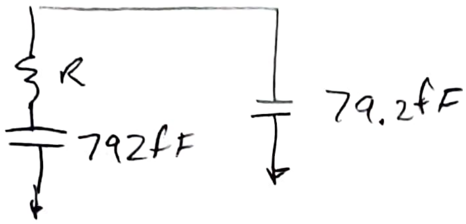
Unit Step Response





For this ideal second-order system, system stability improves and the bandwidth increases as ζ increases. Regarding system stability, peaking is observed in the frequency-domain magnitude plots with low ζ. This is reflected in the time-domain plots with excessive ringing and settling time observed with low ζ.

3. New Loop Filter w/ additional filtering $C_2 = 0.1C_1$



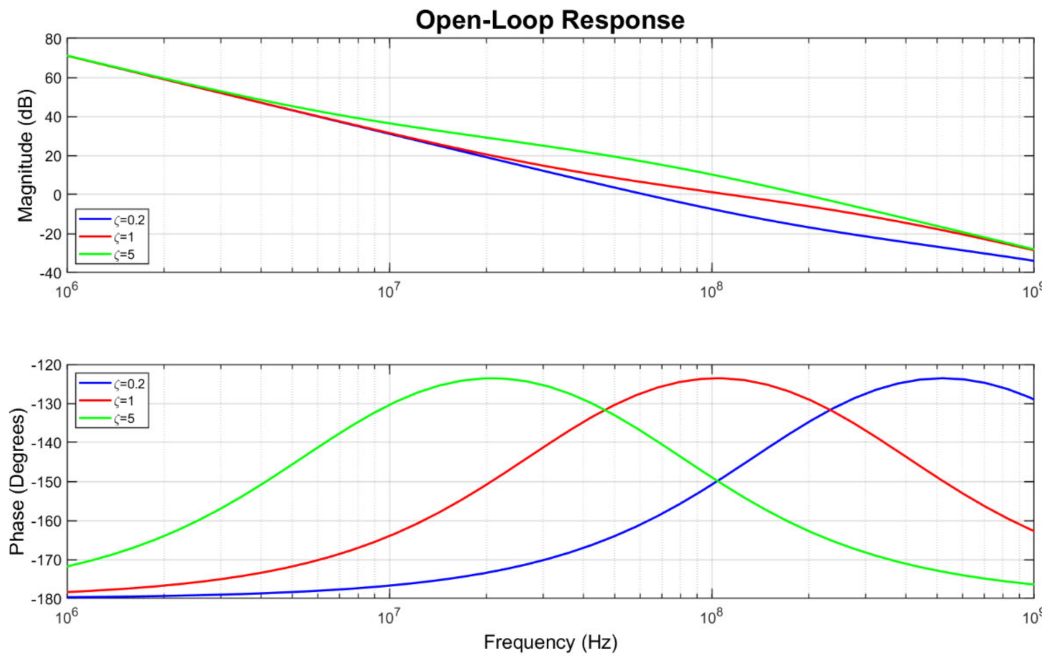
Now we have a 3rd-order system

$$H(s) = \frac{\frac{K_{PD}K_{VCO}}{C_2}\left(s + \frac{1}{RC_1}\right)}{s^2 + \frac{C_1+C_2}{RC_1C_2}s^2 + \frac{K_{PD}K_{VCO}}{NC_2}s + \frac{K_{PD}K_{VCO}}{NRC_1C_2}}$$

$$E(s) = \frac{s^2\left(s + \frac{C_1+C_2}{RC_1C_2}\right)}{s^3 + \frac{C_1+C_2}{RC_1C_2}s^2 + \frac{K_{PD}K_{VCO}}{NC_2}s + \frac{K_{PD}K_{VCO}}{NRC_1C_2}}$$

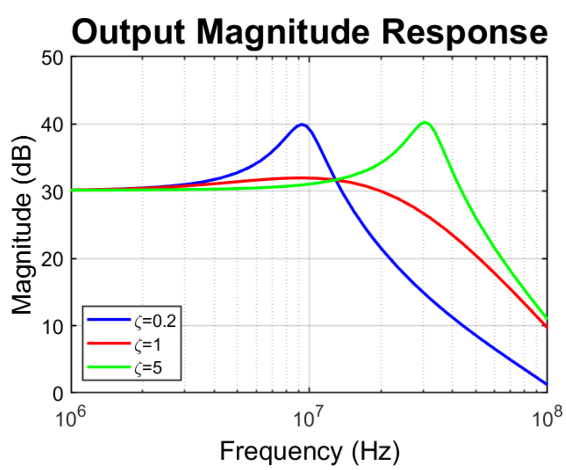
$$\frac{G(s)}{N} = \frac{\frac{K_{PD}K_{VCO}}{NC_2}\left(s + \frac{1}{RC_1}\right)}{s^2\left(s + \frac{C_1+C_2}{RC_1C_2}\right)}$$

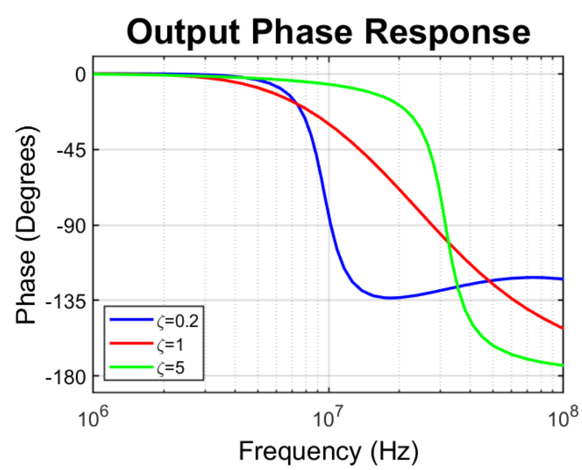
3rd-Order System w/ C2



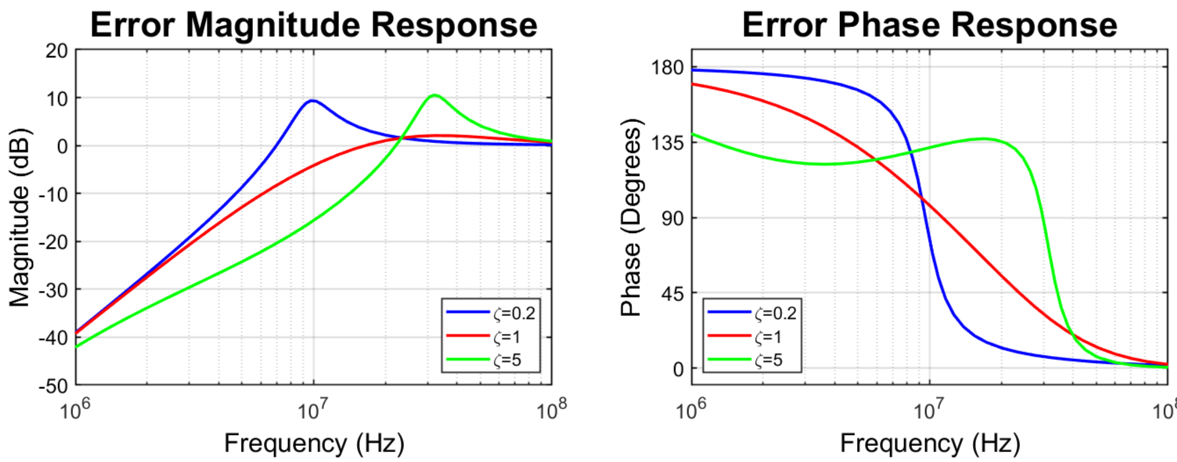
Ideal ζ	Phase Margin (°)	Closed-Loop Poles (rad/s)
0.2	19.5	-1.05e7 ± j5.93e7, -1.71e9
1	56.4	-5.04e7, -1.48e8 ± j0.53e8
5	17.9	-6.35e6, -3.14e7 ± j19.5e7

Output Bode Plots

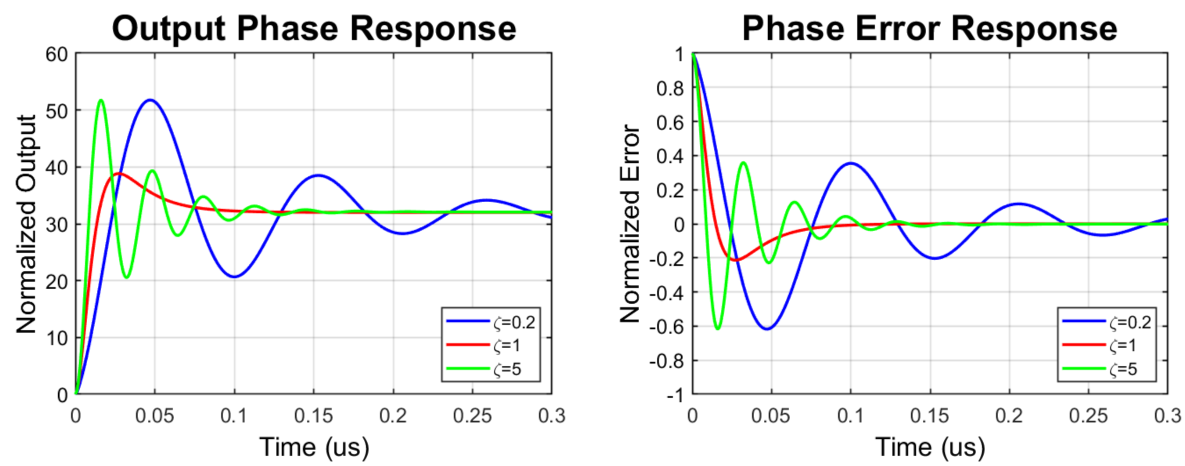




Error Bode Plots



Unit Step Response



For this more realistic third-order system, system stability improves and the bandwidth increases as ideal ζ is increased from 0.2 to 1. However, if the resistor value is increased further to have an ideal $\zeta = 5$, then the third pole becomes small enough to degrade the phase margin. This results in excessive peaking in the frequency-domain magnitude plots with ζ=5. This is reflected in the time-domain plots with excessive ringing observed with ζ=5. Thus, the impact of additional poles should be carefully considered when a high damping factor is desired.