Texas A&M University
Department of Electrical and Computer Engineering

ECEN 620 – Network Theory (Broadband Circuit Design)

Fall 2024

Exam #1

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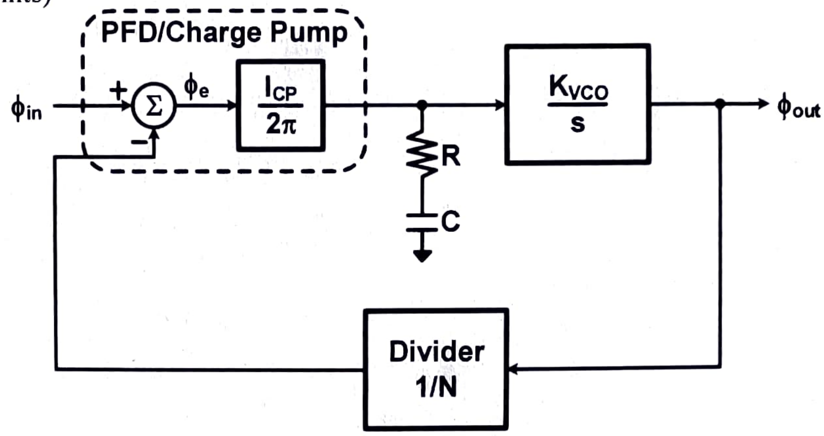
- Please write your name in the space provided below
- Please verify that there are **7** pages in your exam
- You may use one double-sided page of notes and equations for the exam
- Good Luck!

Problem	Score	Max Score
1		50
2		50
Total		**100**

Name:____

UIN:____

Problem 1 (50 points)



a) For the PLL block diagram shown above, find the expressions for the product of the forward path gain and feedback factor, $\frac{G(s)}{N}$, the closed-loop transfer function $H(s)=\phi_{out}(s)/\phi_{in}(s)$, and the phase error transfer function, $E(s)=\phi_e(s)/\phi_{in}(s)$.

$$F(s) = R + \frac{1}{sC}$$

$$\frac{G(s)}{N} = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R \left(s + \frac{1}{RC}\right)}{N s^2}$$

$$H(s) = \frac{G(s)}{1 + \frac{G(s)}{N}} = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R \left(s + \frac{1}{RC}\right)}{s^2 + \frac{\frac{I_{CP}}{2\pi} K_{VCO} R}{N} s + \frac{\frac{I_{CP}}{2\pi} K_{VCO}}{NC}}$$

$$E(s) = \frac{1}{1 + \frac{G(s)}{N}} = \frac{s^2}{s^2 + \frac{\frac{I_{CP}}{2\pi} K_{VCO} R}{N} s + \frac{\frac{I_{CP}}{2\pi} K_{VCO}}{NC}}$$

$\frac{G(s)}{N} =$

$H(s)=\phi_{out}(s)/\phi_{in}(s)=$

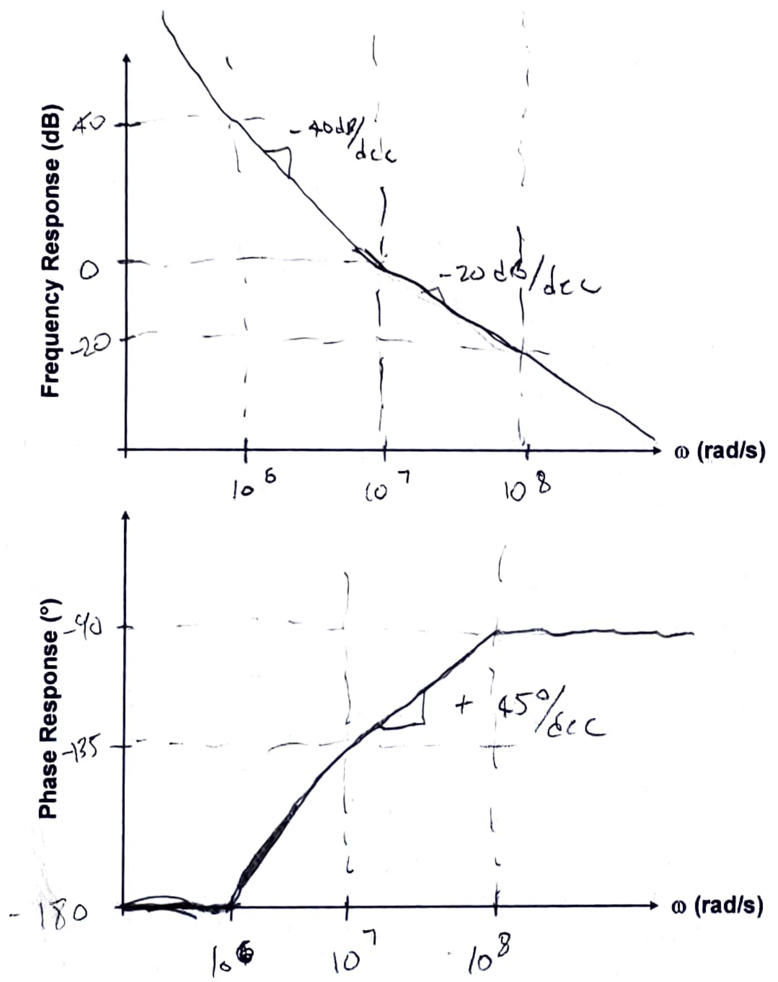
$E(s)=\phi_e(s)/\phi_{in}(s)=$

b) Assume that I_{CP}=100μA, C=100pF, K_{VCO}=2π*(10GHz/V), and N=100. What is the R value required for the closed-loop transfer function zero, ω_z, to equal 10Mrad/s? Sketch the $\frac{G(s)}{N}$ Bode Plot (magnitude and phase). What is the phase margin?

$$\omega_z = \frac{1}{RC} \Rightarrow R = \frac{1}{\omega_z C} = \frac{1}{(10^7\,\text{rad/s})(100pF)} = 1k\Omega$$

$$\frac{G(s)}{N} = \frac{\left(\frac{100\mu A}{2\pi}\right)\left(2\pi\cdot 10^{10}\,\text{Hz/V}\right)(1k\Omega)(s+10^7)}{100\,s^2} = \frac{10^9(s+10^7)}{100\,s^2}$$

At $\omega = 10^6$ $\left|\frac{G(j10^6)}{N}\right| \simeq 100$



R = 1kΩ

Phase Margin = 45°

or 51.9°
(more exact)

c) Using the same numerical values from part(b), what value should the charge pump current I_{CP} be changed to achieve a phase margin of 60°?

$$PM = 180° + \angle \frac{G(j\omega_u)}{N} \qquad \frac{G(s)}{N} = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R \left(s + \frac{1}{RC}\right)}{N s^2}$$

$$= 180° + \tan^{-1}(\omega_u RC) - 180° = \tan^{-1}(\omega_u RC)$$

$$\tan^{-1}(\omega_u RC) = 60°$$

$$\omega_u = \frac{\sqrt{3}}{RC} = \sqrt{3}\,\omega_z = \sqrt{3} \cdot 10^7 \text{ rad/s}$$

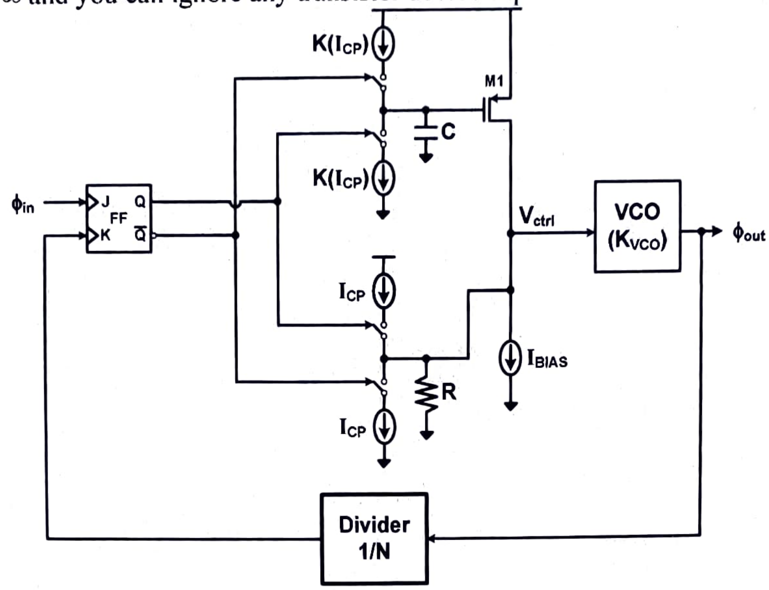
$$\left| \frac{G(j\omega_u)}{N} \right| = 1$$

$$\frac{\frac{I_{CP}}{2\pi}\left(2\pi \cdot 10^{10} \text{ Hz/V}\right)(1k\Omega)\sqrt{(\sqrt{3}\cdot 10^7)^2 + (10^7)^2}}{100\left(\sqrt{3}\cdot 10^7\right)^2} = 1$$

$$I_{CP} = 150\mu A$$

I_{CP} for 60° Phase Margin = $150\mu A$

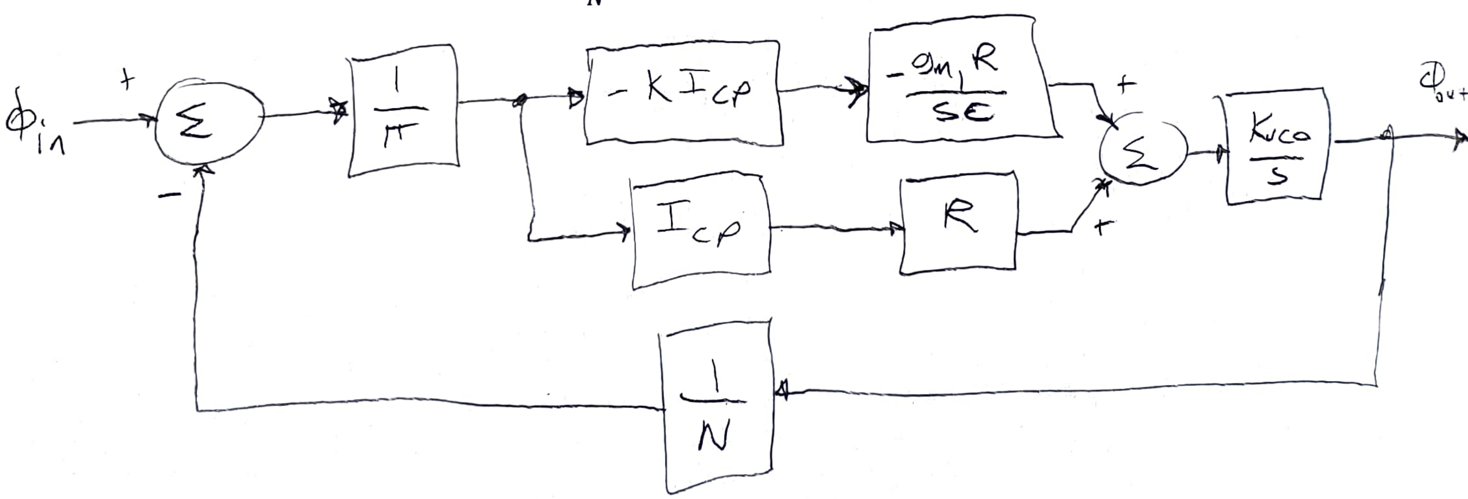
Problem 2 (50 points)
For the PLL shown below, assume that the VCO gain is K_{VCO} is positive, all transistors are operating in saturation with $r_o=\infty$ and you can ignore any transistor device capacitors.



a) Draw the phase domain small signal model of the loop.

b) Find the expressions for the product of the forward path gain and feedback factor, $\frac{G(s)}{N}$, and determine the pole-zero locations of $\frac{G(s)}{N}$.

a:



b. $$\frac{G(s)}{N} = \frac{1}{\pi}\left[(-KI_{cp})\left(-\frac{g_{m1}R}{sC}\right) + I_{cp}R\right]\left(\frac{K_{VCO}}{s}\right)\left(\frac{1}{N}\right)$$

$$\boxed{\frac{G(s)}{N} = \frac{\left(\frac{I_{cp}RK_{VCO}}{\pi}\right)\left[s + \frac{Kg_{m1}}{C}\right]}{Ns^2}}$$

2 poles at ϕ
1 zero at $-\frac{Kg_{m1}}{C}$

c) Assume that I_{CP}=100μA, K=0.1, C=100pF, g_{m1}=100μA/V, R=100Ω, and K_{VCO}=2π*(100MHz/V). Calculate the maximum loop division factor, N, for a minimum phase margin of 60°.

$$\frac{G(s)}{N} = \frac{\left(\frac{I_{cp} R K_{vco}}{\pi}\right)\left[s + \frac{K g_{m1}}{C}\right]}{N s^2}$$

$$PM = 180^\circ + \angle \frac{G(j\omega_u)}{N} = 180^\circ + \tan^{-1}\left(\frac{\omega_u C}{K g_{m1}}\right) - 180^\circ = \tan^{-1}\left(\frac{\omega_u C}{K g_{m1}}\right)$$

$$\tan^{-1}\left(\frac{\omega_u C}{K g_{m1}}\right) = 60^\circ$$

$$\omega_u = \frac{\sqrt{3}\, K g_{m1}}{C} = \frac{\sqrt{3}(0.1)(100\mu)}{100p} = \sqrt{3}\cdot 10^5$$

$$\left|\frac{G(j\omega_u)}{N}\right| = 1$$

$$\frac{\left[\frac{(100\mu)(100)(2\pi\cdot 100M)}{\pi}\right]\sqrt{(\sqrt{3}\cdot 10^5)^2 + (10^5)^2}}{N(\sqrt{3}\cdot 10^5)^2} = 1$$

$$N = 13.3$$

Max. N for 60° Phase Margin = 13.3 ⇒ 13 (assuming integer)

d) What is the phase relationship between ϕ_{in} and ϕ_{out} when is PLL locked?

Due to the JK FF Phase Detector, the loop will lock with a 180° phase difference.

$\phi_{in} - \phi_{out} = 180^\circ$