Texas A&M University
Department of Electrical and Computer Engineering

ECEN 620 – Network Theory (Broadband Circuit Design)

Fall 2021

Exam #1

Instructor: Sam Palermo

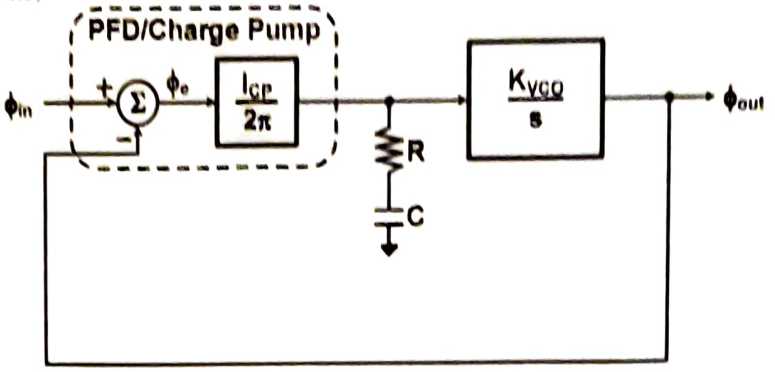
- Please write your name in the space provided below
- Please verify that there are **6** pages in your exam
- You may use <u>one</u> double-sided page of notes and equations for the exam
- Good Luck!

Problem	Score	Max Score
1		55
2		45
Total		**100**

Name: SAM PALERMO

UIN: _____

Problem 1 (55 points)



a) For the PLL block diagram shown above, find the expressions for the forward path gain G(s) and the closed-loop transfer function $H(s)=\phi_{out}(s)/\phi_{in}(s)$. (10 points)

$$F(s) = R + \frac{1}{sC}$$

$$G(s) = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R\left(s + \frac{1}{RC}\right)}{s^2}$$

G(s) =

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R\left(s + \frac{1}{RC}\right)}{s^2 + s\frac{I_{CP}}{2\pi} K_{VCO} R + \frac{\frac{I_{CP}}{2\pi} K_{VCO}}{C}}$$

$H(s)=\phi_{out}(s)/\phi_{in}(s)=$

$I_{CP}/2\pi$

b) Assume that ~~I_{CP}~~=10μA/(2π), K_{VCO}=2π*(100MHz/V), and that R=0. What is the value of C required for a ~~loop~~ forward path gain unity-gain crossover frequency of 1Mrad/s? What is the phase margin? (10 points)

$$R=0 \Rightarrow G(s) = \frac{\frac{I_{CP}}{2\pi} K_{VCO}}{s^2 C} = \frac{\frac{10\mu A}{2\pi}\left(2\pi \cdot 100MHz/V\right)}{s^2 C}$$

$$|G(j10^6)| = \frac{\left(\frac{10\mu A}{2\pi}\right)\left(2\pi \cdot 100MHz/V\right)}{(10^6 rad/s)^2 C} = 1$$

$C = 1nF$

$PM = 180^\circ + \angle G(j10^6)$

$= 180^\circ - 180^\circ = 0^\circ$

$C = 1nF$

Phase Margin = 0°

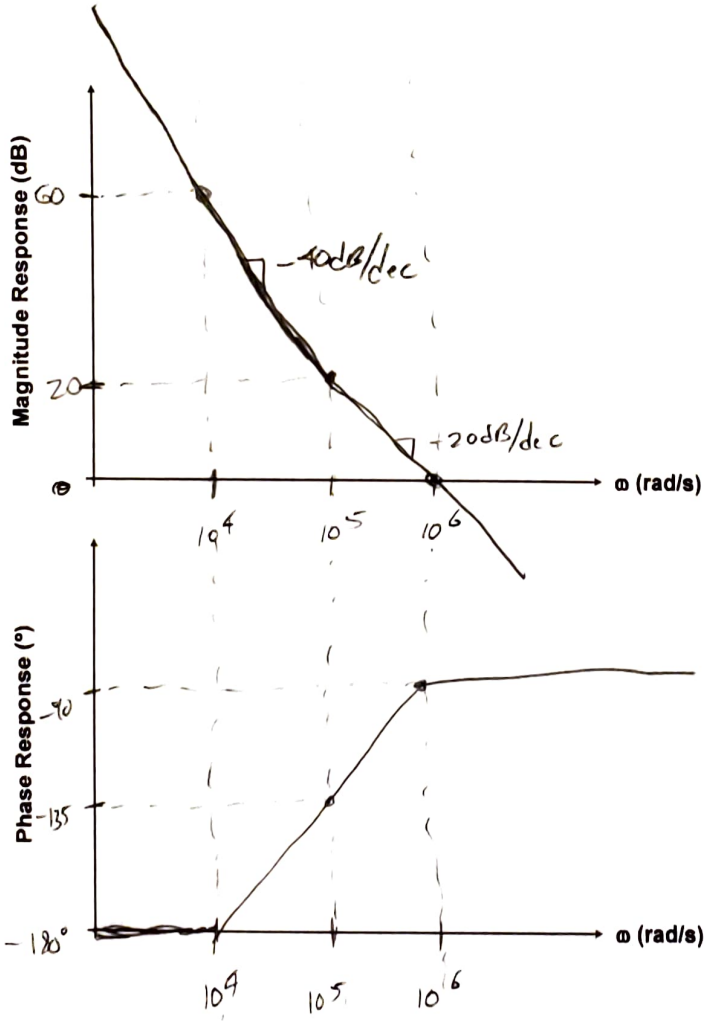
c) Now assume that **C=10nF**. Note, this is not the answer to (b). What is the R value required for the closed-loop transfer function zero, ω_z, to equal 100krad/s? Sketch the G(s) Bode Plot (magnitude and phase). What is the phase margin now? (20 points)

$$\omega_z = \frac{1}{RC} \Rightarrow R = \frac{1}{\omega_z C} = \frac{1}{(10^5\ rad/s)(10nF)} = 1k\Omega$$

$$G(s) = \frac{\frac{10\mu A}{2\pi}(2\pi 10^8)(1k)(s+10^5)}{s^2} = \frac{10^6(s+10^5)}{s^2}$$

At $\omega = 10^4$ $|G| \approx \frac{10^6(10^5)}{10^8}$

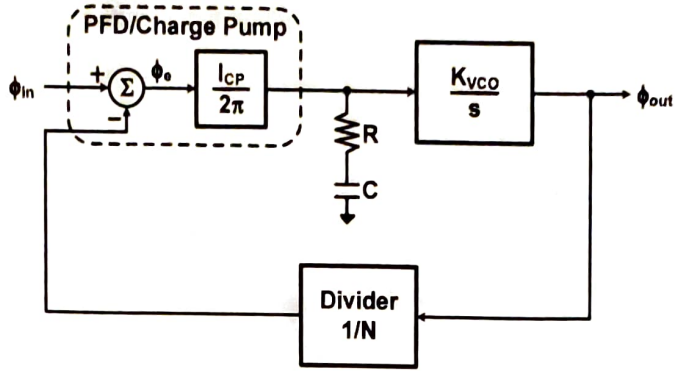
$= 10^3$



R = 1kΩ

Phase Margin = ~90°

d) Now assume that the PLL is modified to include a loop divider. Using the same numerical values from part(c), what is the maximum loop division factor, N, for a minimum phase margin of 30°? (15 points)



$$\frac{G(s)}{N} = \frac{\frac{I_{CP}}{2\pi} K_{VCO} R \left(s + \frac{1}{RC}\right)}{N s^2}$$

$$PM = 180° + \angle \frac{G(j\omega_u)}{N} = 180° + \tan^{-1}(\omega_u RC) - 180°$$

$$= \tan^{-1}(\omega_u RC) = 30°$$

$$\omega_u = \frac{1}{\sqrt{3} RC} = \frac{\omega_z}{\sqrt{3}} = \frac{10^5}{\sqrt{3}}$$

$$\left| \frac{G(j\omega_u)}{N} \right| = 1$$

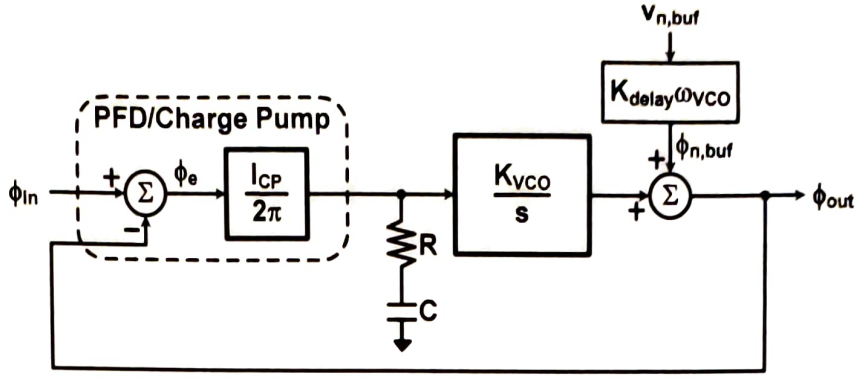
$$\frac{\left(\frac{10\,\mu A}{2\pi}\right)\left(2\pi \cdot 10^8\right)(1k)\sqrt{\left(\frac{10^5}{\sqrt{3}}\right)^2 + (10^5)^2}}{N\left(\frac{10^5}{\sqrt{3}}\right)^2} = 1$$

$$N = 34.6$$

Max. N for 30° Phase Margin = 34.6 $\Rightarrow$ 34 (assuming only integer)

Problem 2 (45 points)

This problem investigates how voltage noise on an inside-the-loop post-VCO buffer impacts the output phase noise. Assume that the post-VCO buffer noise is modeled as an additive phase noise term, as shown in the model below.



a) Find the expression for the post-VCO buffer voltage noise induced phase noise transfer function, $T(s)=\phi_{out}(s)/v_{n,buf}(s)$. (15 points)

$$\frac{\phi_{out}(s)}{V_{n,buf}(s)} = \frac{s^2 K_{delay}\omega_{VCO}}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

where $\omega_n = \sqrt{\frac{I_{CP}K_{VCO}}{2\pi C}}$ and $\zeta = \frac{\omega_n}{2}RC$

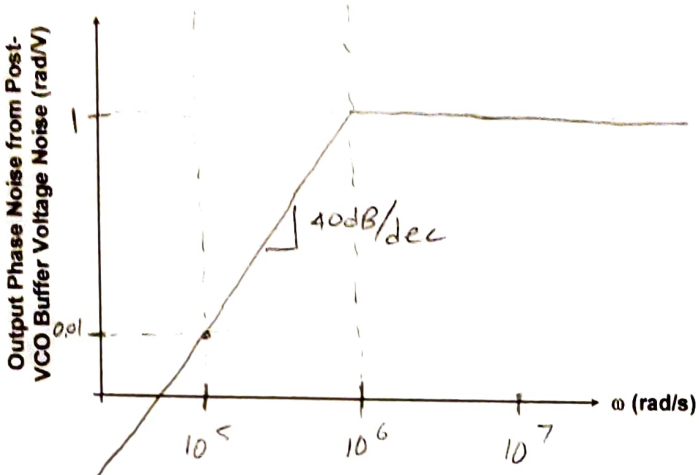
$T(s)=\phi_{out}(s)/v_{n,buf}(s)$ =

b) Assume that the PLL has been designed for ω_n=1Mrad/s and ζ=1. Also, assume that ω_{VCO}=10Grad/s and K_{delay} = -100ps/V. Sketch the post-VCO buffer voltage noise induced phase noise **magnitude versus frequency**. Label the high frequency value and the key frequencies and slopes in the plot. **Use a log scale for both the x and y-axis.** (10 points)

$$\frac{\Phi_{out}(s)}{V_{n,buf}(s)} = \frac{s^2 K_{delay}\omega_{VCO}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow \text{High Frequency Magnitude} = |K_{delay}\omega_{VCO}|$$

$$= |(-100ps/V)(10Grad/s)|$$

$$= 1\ rad/V$$



c) Assume that the post-VCO buffer voltage noise has sinusoidal modulation.

$v_{n,buf}(t) = 0.1\sin(10^7 t)$ (V)

What is the output jitter amplitude in ps? (10 points)

10^7 is well above the 2-poles, in the high-pass region

Output jitter amplitude = $(1\ rad/V)(100mV) = 0.1\ rad$

At 10Grad/s $\Rightarrow$ $0.1\ rad\left(\frac{2\pi(100ps)}{2\pi\ rad}\right) = 10ps$

Output Jitter Amplitude (ps) = 10ps

d) Assuming 10^5 rad/s sinusoidal noise, what is the maximum voltage amplitude for an output jitter amplitude of 0.5ps? (10 points)

At $10^5\ rad/s \Rightarrow 0.01\ rad/V$

Output jitter amplitude = $V_{amp}(0.01\ rad/V)\left(\frac{2\pi(100ps)}{2\pi\ rad}\right) = 0.5ps$

$V_{amp} = 0.5V$

Max Voltage Amplitude = 500mV

Scratch Paper