ECEN474/704: (Analog) VLSI Circuit Design
Fall 2018

Lecture 10: Simple OTA

Sam Palermo
Analog & Mixed-Signal Center
Texas A&M University

Announcements

- HW2 is due today
- No class on Thursday Mar 8

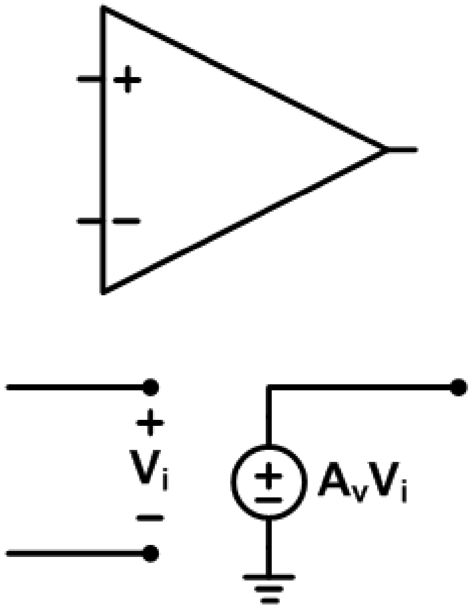
Announcements & Agenda

- Simple OTA Parameters
- OTA w/ Cascode Load & Tail Current Source

- References
  - Razavi Chapters 4, 6, 9.8-9.11
  - Sedra/Smith Section 9.5.5

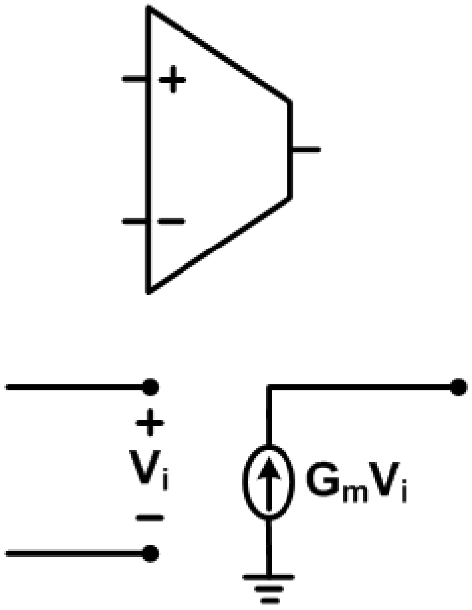
OpAmps and OTAs

OpAmp



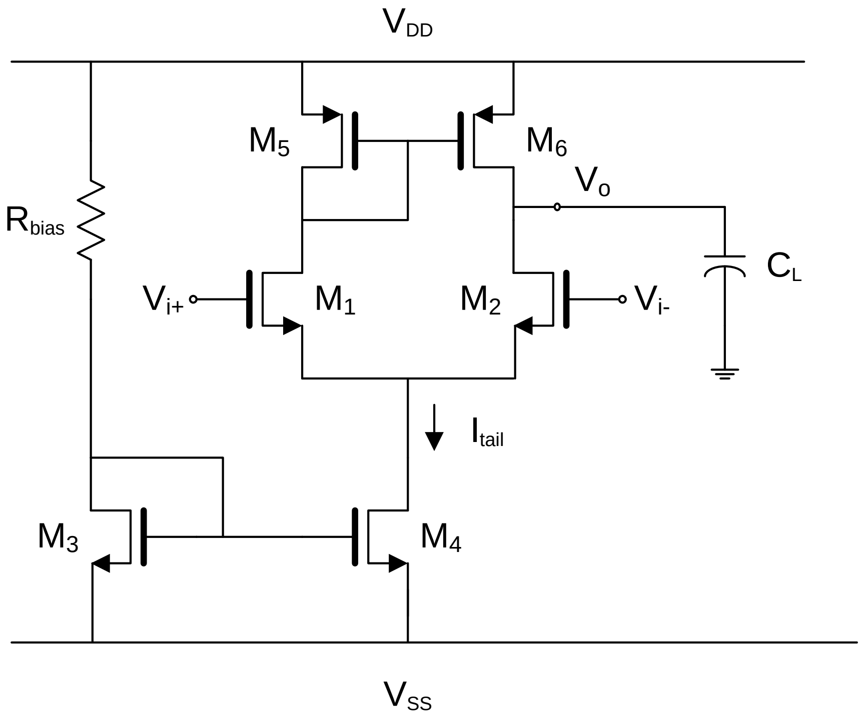
- High voltage gain
- High input impedance
- Voltage source output (low impedance)

OTA



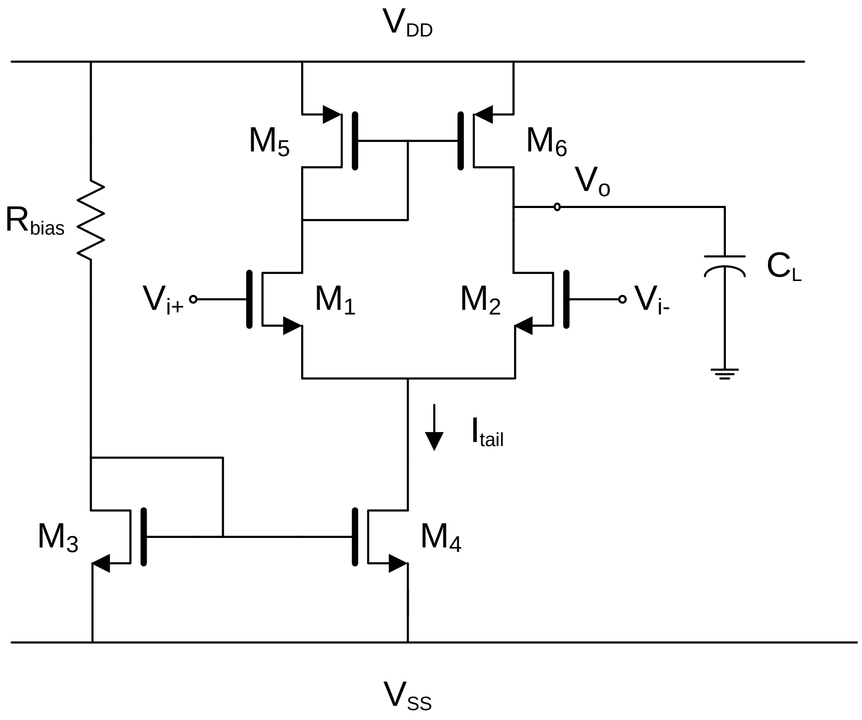
- High “voltage” gain
- High input impedance
- Current source output (high impedance)

Operational Transconductance Amplifier



- Important Parameters
  - Differential Gain
  - Gain-Bandwidth Product
  - Common-Mode Input Range
  - Common-Mode Gain
  - Common-Mode Rejection Ratio (CMRR)
  - Power-Supply Rejection Ratio (PSRR)
  - Slew Rate

OTA Differential Gain



$$\text{Let } V_{i+} = \frac{v_{id}}{2} \text{ and } V_{i-} = -\frac{v_{id}}{2}$$

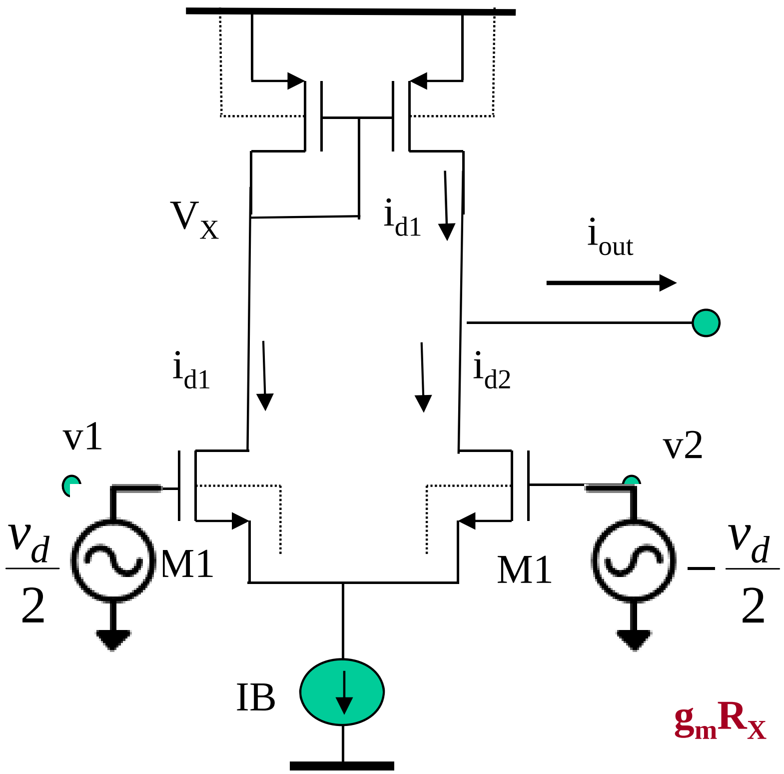
$$v_o = -g_{m2} r_{out}\left(-\frac{v_{id}}{2}\right) - \frac{g_{m1}}{g_{m5}}\left(-g_{m6} r_{out}\right)\left(\frac{v_{id}}{2}\right)$$

$$\text{By design } g_{m1} = g_{m2} \text{ and } g_{m5} = g_{m6}$$

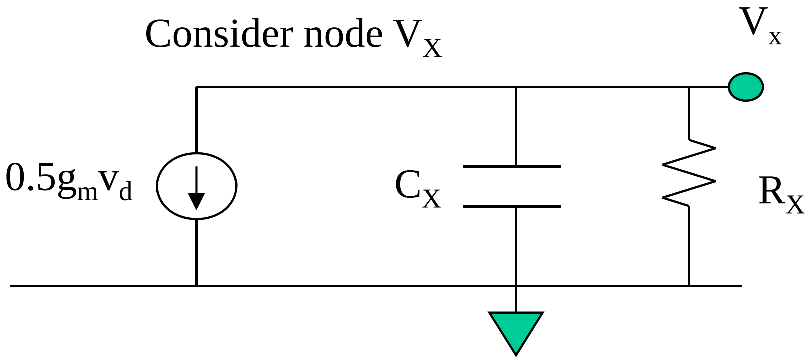
$$v_o = g_{m1} r_{out} v_{id}$$

$$\boxed{A_{DM} = \frac{v_o}{v_{id}} = g_{m1} r_{out} = \frac{g_{m1}}{g_{o6} + g_{o2}}}$$

Frequency Response



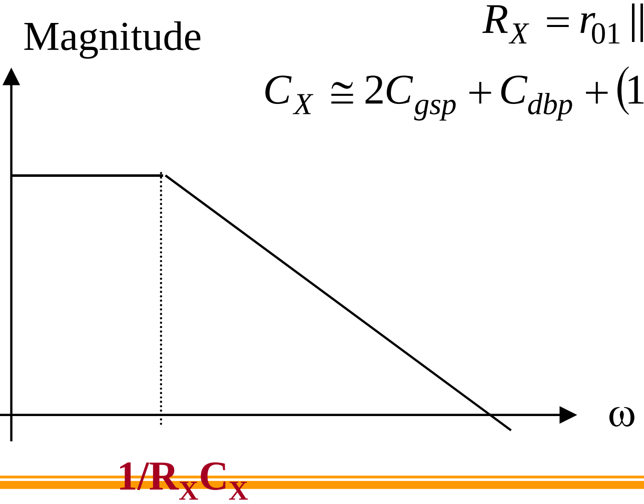
Consider node V_X



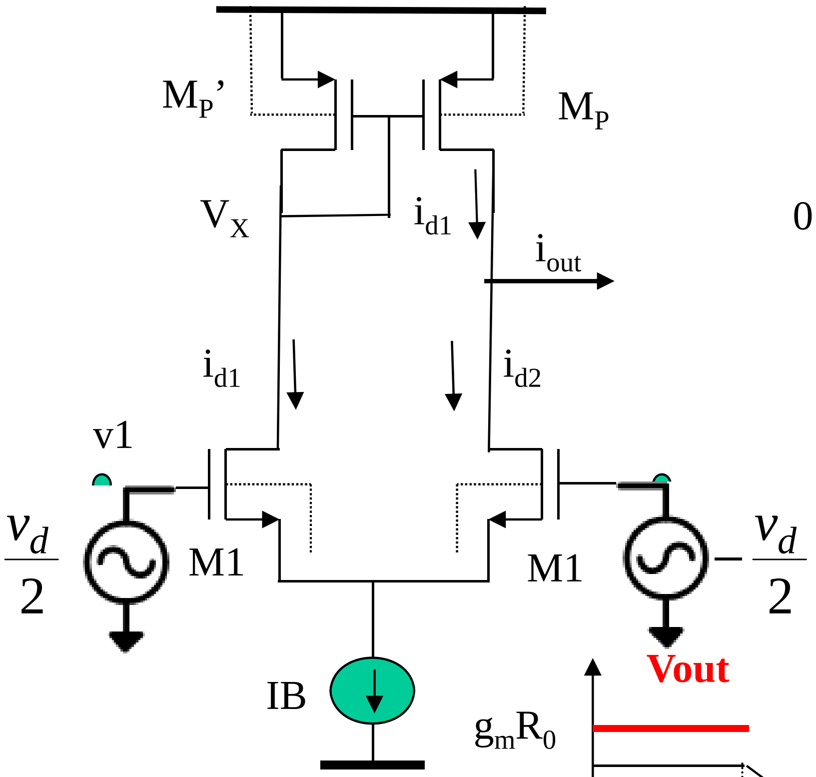
$$V_X = -\frac{0.5 g_m R_X}{1+SR_XC_X}V_d$$

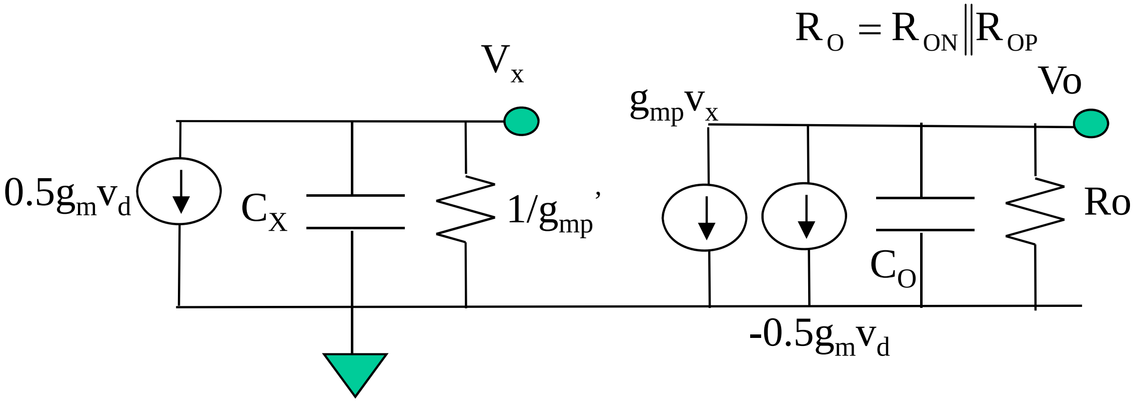
$$R_X = r_{01} \| r_{0P} \| \left(1/g_{mp}\right) \approx 1/g_{mp}$$

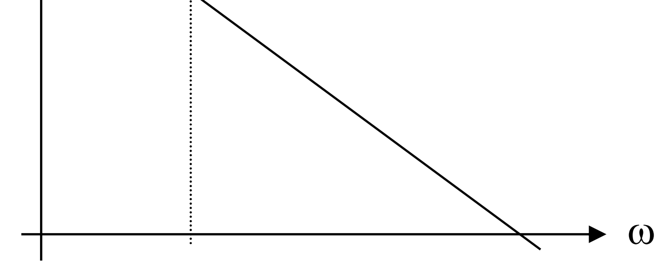
$$C_X \cong 2C_{gsp} + C_{dbp} + (1+miller\ factor)C_{dgp} + C_{dbn} + C_{gdn}$$



Low Frequency Response





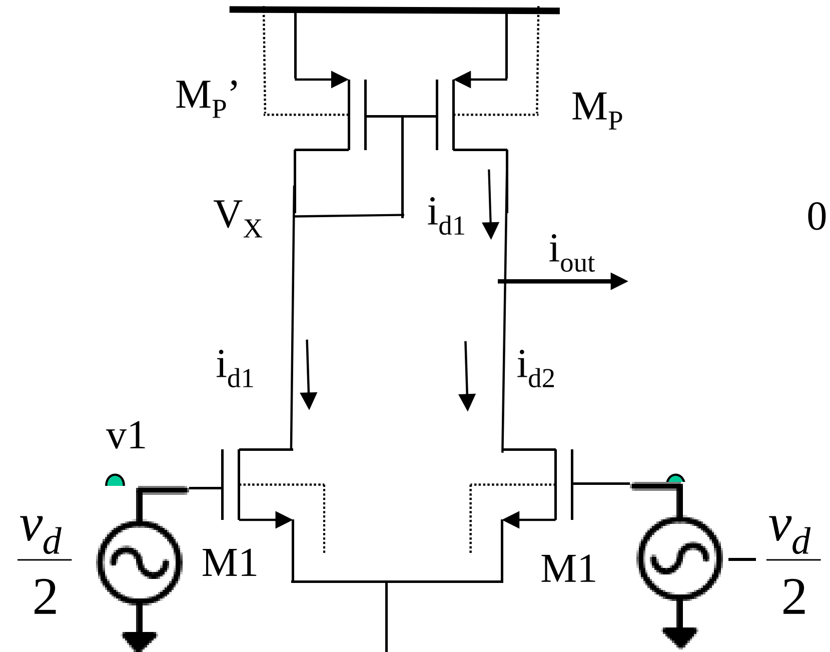


$$v_o = -g_m\left(-\frac{v_d}{2}\right)R_o - g_m\left(\frac{v_d}{2}\right)R_x\left(-g_{mp}R_o\right)$$

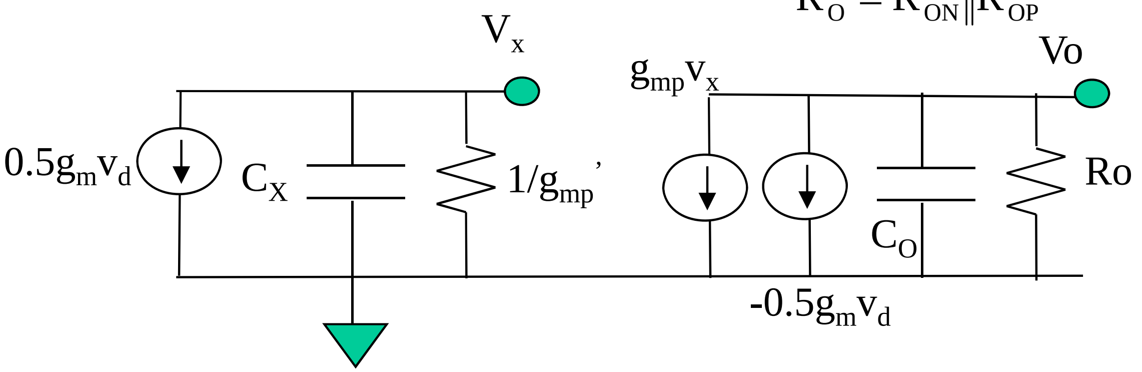
$$R_x \approx \frac{1}{g_{mp}}$$

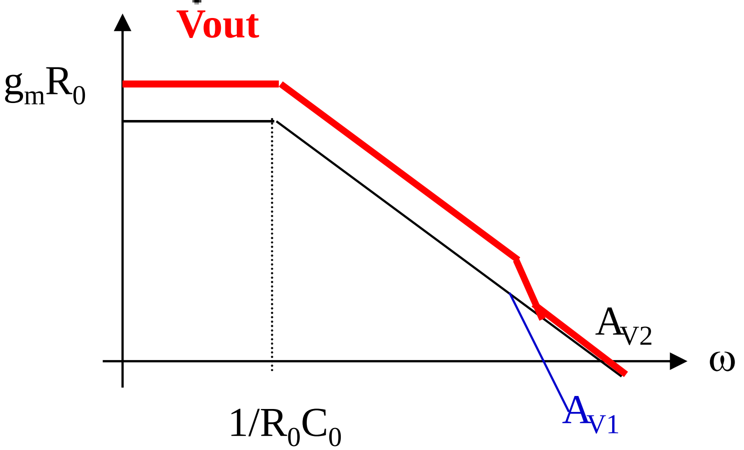
$$v_o = g_m v_d R_o$$

Frequency Response



$$R_O = R_{ON} \| R_{OP}$$



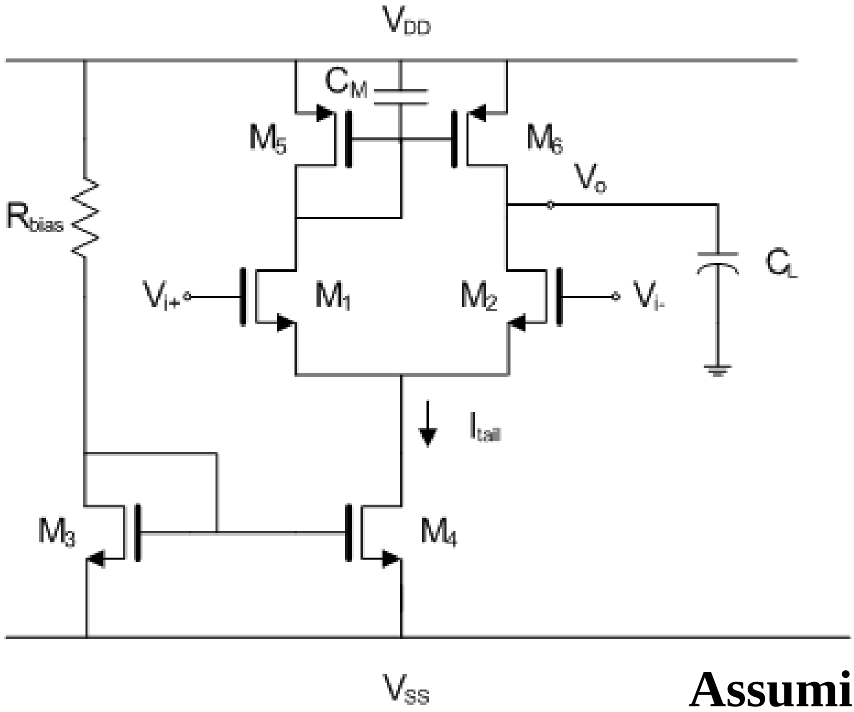


$$v_o = -g_m\left(-\frac{v_d}{2}\right)\frac{R_o}{1+sR_oC_o} - g_m\left(\frac{v_d}{2}\right)\left(\frac{\frac{1}{g_{mp}}}{1+s\frac{C_x}{g_{mp}}}\right)\left(-\frac{g_{mp}R_o}{1+sR_oC_o}\right)$$

$$v_o = g_mR_o\left(\frac{v_d}{2}\right)\frac{1}{1+sR_oC_o}\left[1+\frac{1}{1+s\frac{C_x}{g_{mp}}}\right]$$

$$v_o = g_mR_o\left(\frac{v_d}{2}\right)\frac{1}{1+sR_oC_o}\left[\frac{2+s\frac{C_x}{g_{mp}}}{1+s\frac{C_x}{g_{mp}}}\right]$$

OTA Gain-Bandwidth Product (GBW)



$$A_{DM} = \frac{g_{m1}}{g_{o6} + g_{o2}}$$

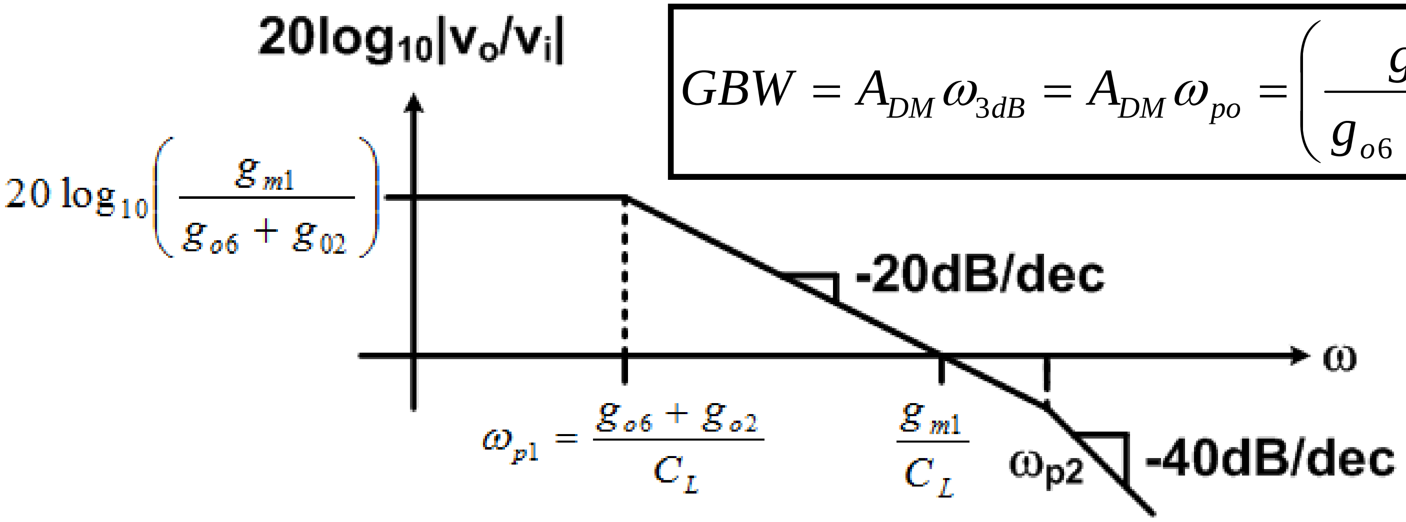
The circuit will have 2 poles

ω_{po} **at the output node and** ω_{pm} **at the "mirror" node**

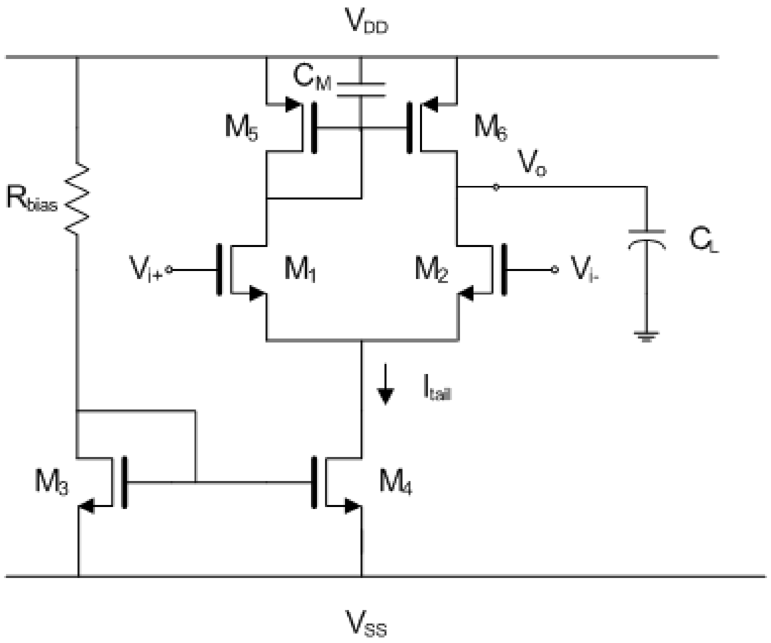
$$\omega_{po} \approx \frac{g_{o6} + g_{o2}}{C_L}, \qquad \omega_{pm} \approx \frac{g_{m5}}{C_M}$$

Assuming the poles are widely spaced and ω_{po} **dominates**

$$\boxed{GBW = A_{DM}\,\omega_{3dB} = A_{DM}\,\omega_{po} = \left(\frac{g_{m1}}{g_{o6} + g_{o2}}\right)\left(\frac{g_{o6} + g_{o2}}{C_L}\right) = \frac{g_{m1}}{C_L}}$$



OTA Common-Mode Input Range



- Common-mode input range set by transistor saturation conditions
  - Low-end set by tail current source saturation

$$V_{icm} \ge V_{SS} + V_{DSAT4} + V_{GS1} = V_{SS} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_4}} + \sqrt{\frac{I_{tail}}{\mu_n C_{ox} \frac{W}{L}_1}} + V_{Tn1}$$

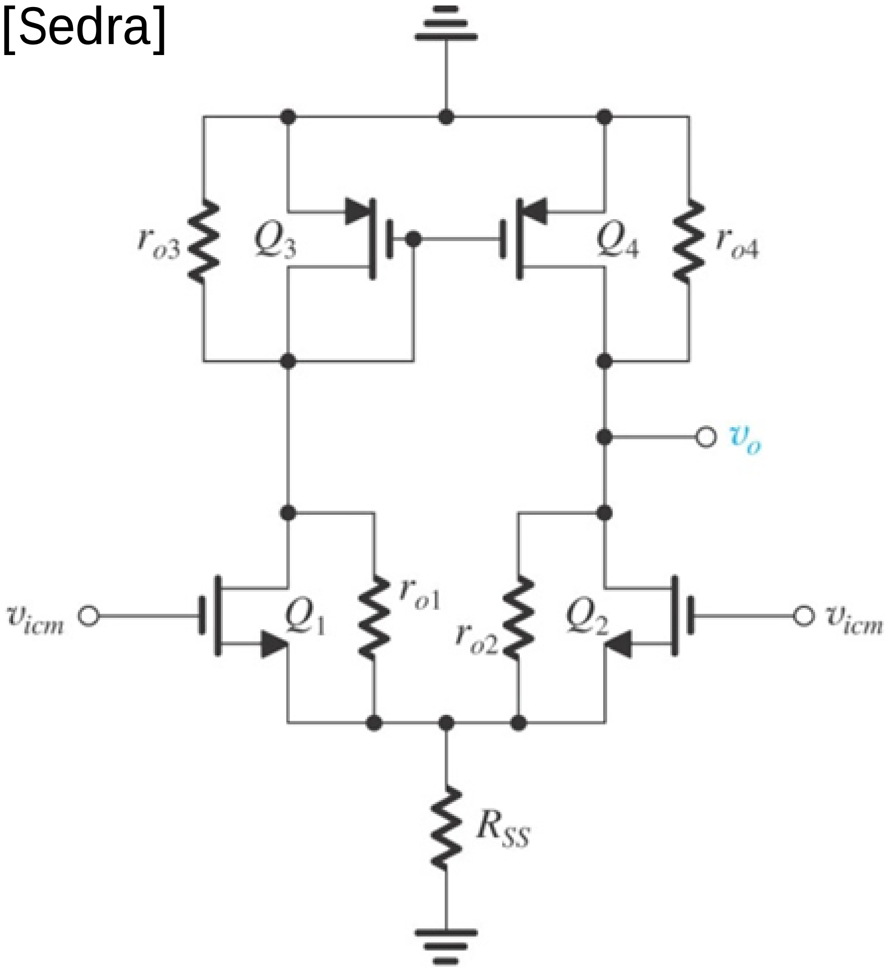
- High-end set by differential pair saturation

$$V_{icm} \le V_{ocm} + V_{T1} = V_{DD} - V_{GS5} + V_{T1} = V_{DD} - \left(\sqrt{\frac{I_{tail}}{\mu_p C_{ox} \frac{W}{L}_5}} + \left|V_{Tp5}\right| \right) + V_{Tn1}$$

$$\boxed{V_{SS} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_4}} + \sqrt{\frac{I_{tail}}{\mu_n C_{ox} \frac{W}{L}_1}} + V_{Tn1} \le V_{icm} \le V_{DD} - \left(\sqrt{\frac{I_{tail}}{\mu_p C_{ox} \frac{W}{L}_5}} + \left|V_{Tp5}\right| \right) + V_{Tn1}}$$

OTA Common-Mode Gain

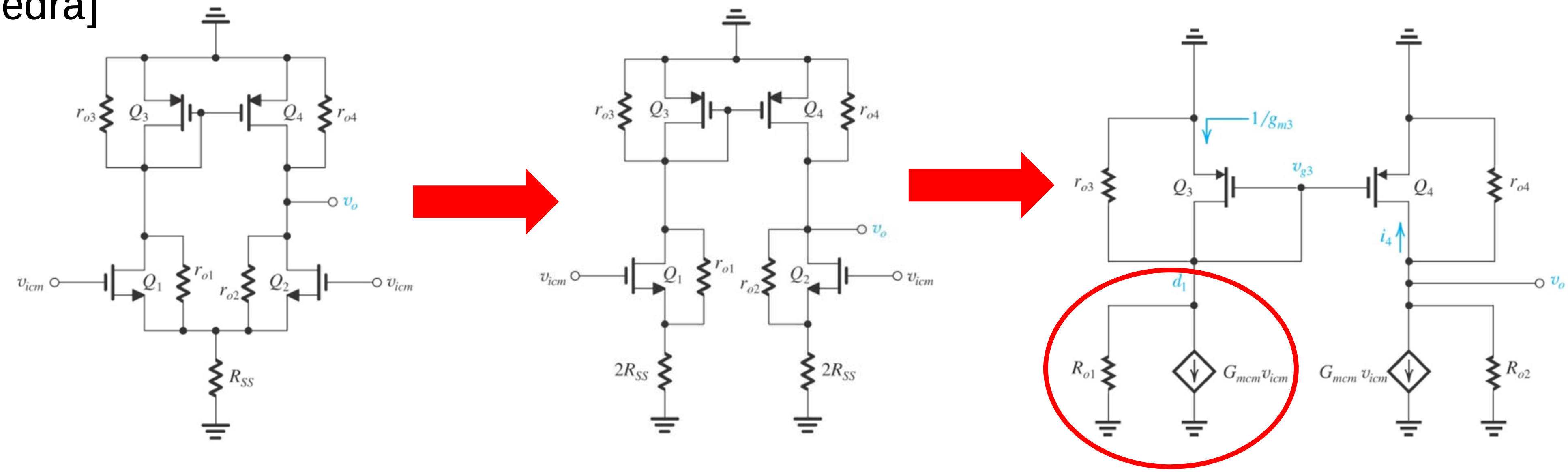
[Sedra]



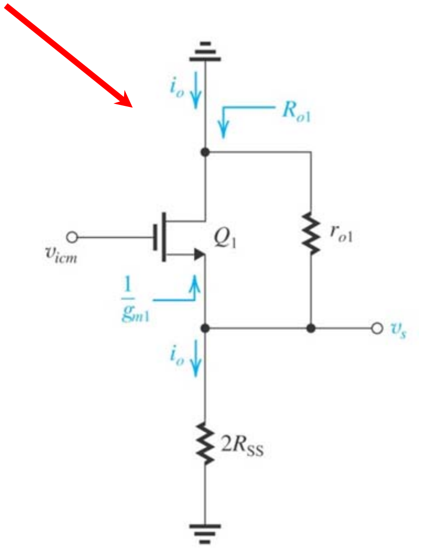
- Ideally, common-mode perturbations are suppressed by the differential amplifier, i.e. $A_{cm} = 0$
- Finite common-mode gain exists due to amplifier asymmetries and finite tail current source impedance
- Note transistor numbers are different from previous slides, as I borrow figures from Sedra/Smith text

OTA Common-Mode Gain

[Sedra]



- Modeling the input transistors as a transconductance current source with a parallel output resistance



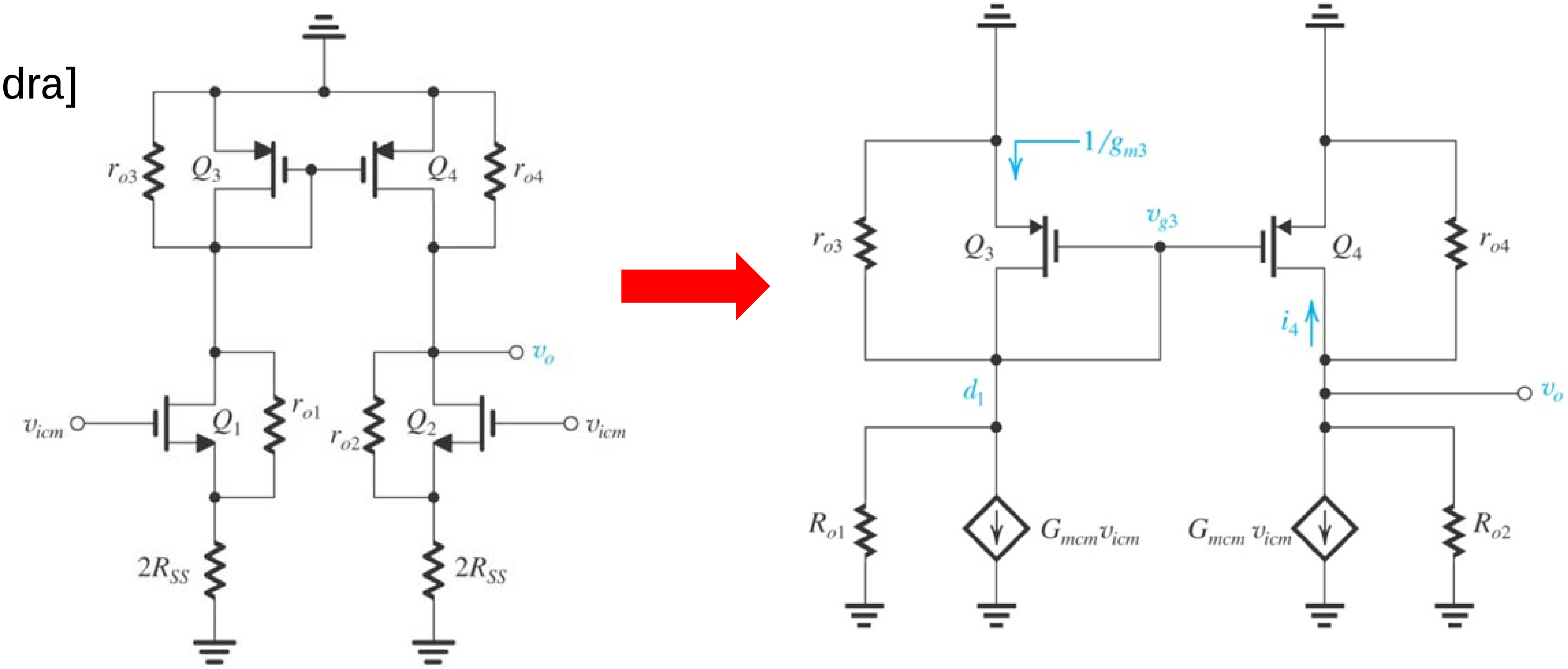
$$v_s = v_{icm}\frac{(2R_{SS}\|r_{o1})}{(2R_{SS}\|r_{o1}) + \frac{1}{g_{m1}}} \approx v_{icm}$$

$$i_o = \frac{v_{icm}}{2R_{SS}}$$

$$G_{mcm} = \frac{i_o}{v_{icm}} = \frac{1}{2R_{SS}}$$

OTA Common-Mode Gain

[Sedra]



$$G_{mcm} = \frac{i_o}{v_{icm}} = \frac{1}{2R_{SS}}$$

$$R_{o1} = 2R_{SS} + r_{o1} + g_{m1}(2R_{SS})r_{o1}$$

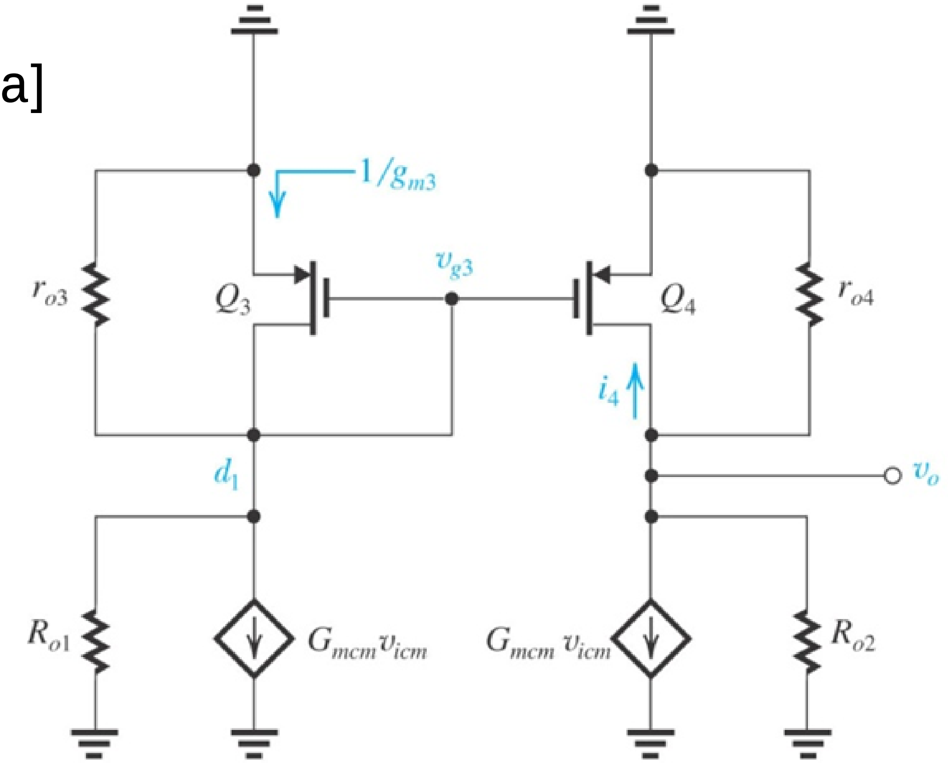
$$R_{o2} = 2R_{SS} + r_{o2} + g_{m2}(2R_{SS})r_{o2}$$

$$v_{g3} = -G_{mcm}v_{icm}\left(R_{o1}\|r_{o3}\|\frac{1}{g_{m3}}\right)$$

$$i_4 = -g_{m4}G_{mcm}v_{icm}\left(R_{o1}\|r_{o3}\|\frac{1}{g_{m3}}\right)$$

OTA Common-Mode Gain

[Sedra]



$$i_4 = -g_{m4} G_{mcm} v_{icm} \left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}} \right)$$

KCL at v_o

$$G_{mcm} v_{icm} + i_4 + \frac{v_o}{R_{o2}} + \frac{v_o}{r_{o2}} = 0$$

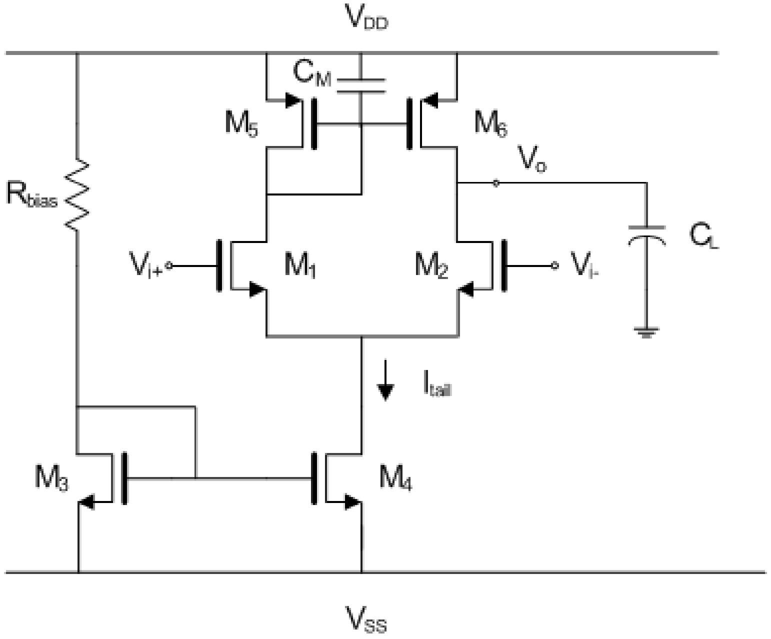
$$v_o = -v_{icm} \frac{r_{o4} \| R_{o2}}{2R_{SS}} \left[1 - g_{m4} \left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}} \right) \right]$$

Since $R_{o2} >> r_{o4}$ and $R_{o1} >> r_{o3}$ and $g_{m3} = g_{m4}$

$$A_{CM} = \frac{v_o}{v_{icm}} \approx -\frac{r_{o4}}{2R_{SS}} \frac{1}{1 + g_{m3} r_{o3}}$$

$$A_{CM} \approx -\frac{1}{2 g_{m3} R_{SS}}$$

OTA Common-Mode Gain & Rejection Ration (CMRR)



$$\boxed{A_{cm} = \frac{v_o}{v_{CM}} \approx -\frac{1}{2 g_{m6} r_{o4}}}$$

- To improve (lower) common-mode gain, we need a high tail current impedance

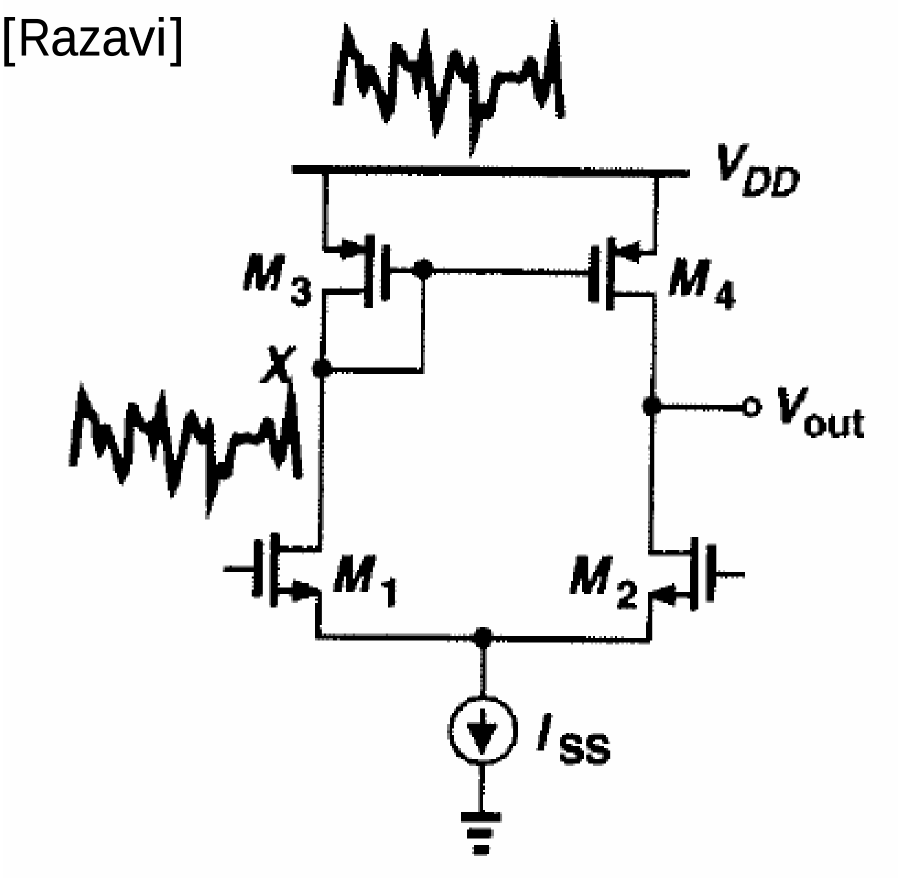
- An amplifier figure-of-merit is the common-mode rejection ratio (CMRR)

$$\boxed{CMRR = 20\log_{10}\left|\frac{A_{dm}}{A_{cm}}\right|}$$

$$CMRR = 20\log_{10}\left|\frac{A_{dm}}{A_{cm}}\right| = 20\log_{10}\left|\left(\frac{g_{m1}}{g_{o6}+g_{o2}}\right)\left(-2 g_{m6} r_{o4}\right)\right|$$

OTA Power-Supply Rejection Ration (PSRR)

[Razavi]



$$\boxed{\begin{aligned} PSRR^{+} &= 20\cdot\log_{10}\left(\left|\frac{A_{dm}}{v_o/v_{DD}}\right|\right) \\ PSRR^{-} &= 20\cdot\log_{10}\left(\left|\frac{A_{dm}}{v_o/v_{SS}}\right|\right) \end{aligned}}$$

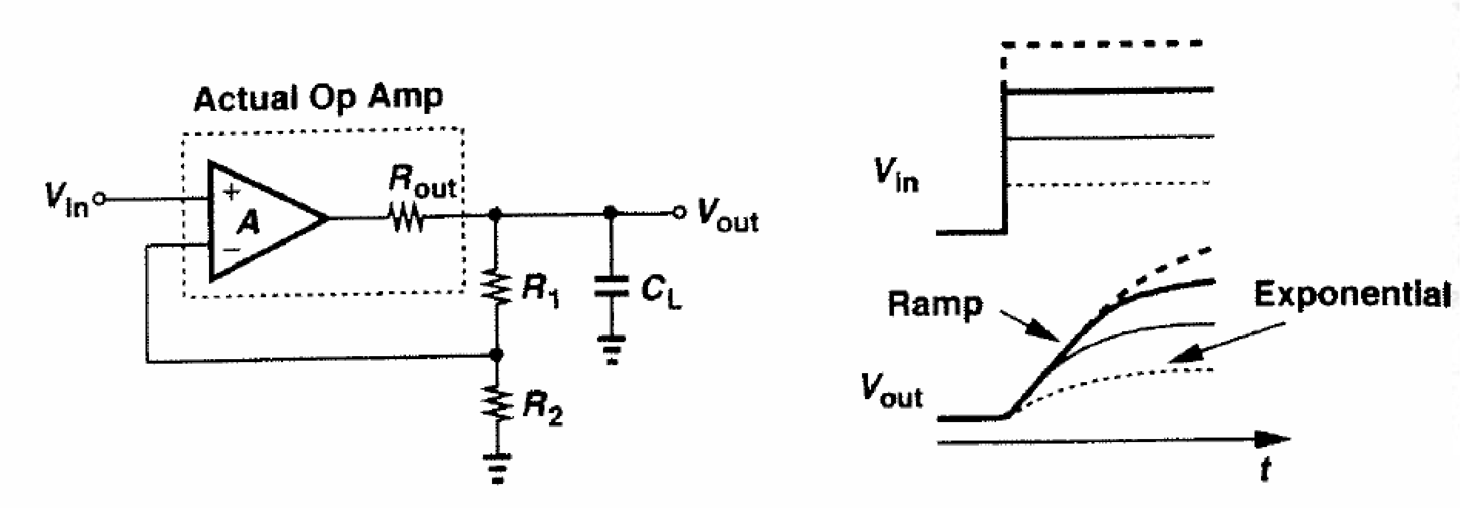
$$A_{dm} = \frac{g_{m1}}{g_{o2}+g_{o4}}$$

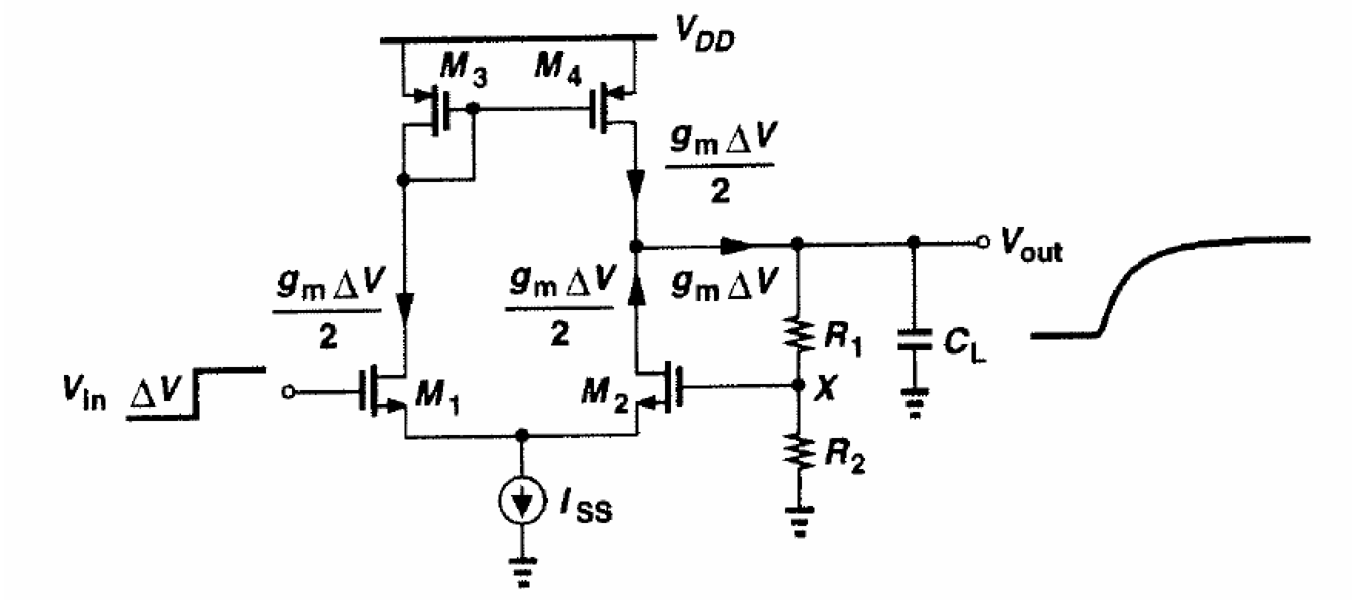
$$A_{vdd} \approx 1$$

$$PSRR^{+} \approx \frac{g_{m1}}{g_{o2}+g_{o4}}$$

OTA Slew Rate

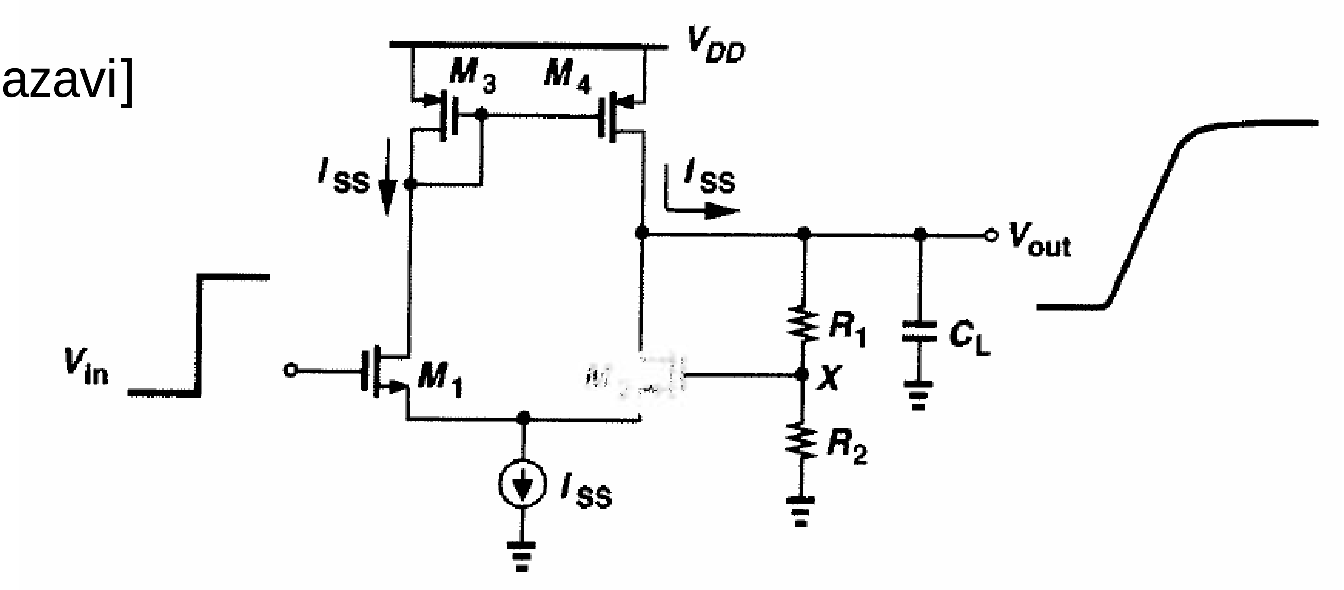
[Razavi]

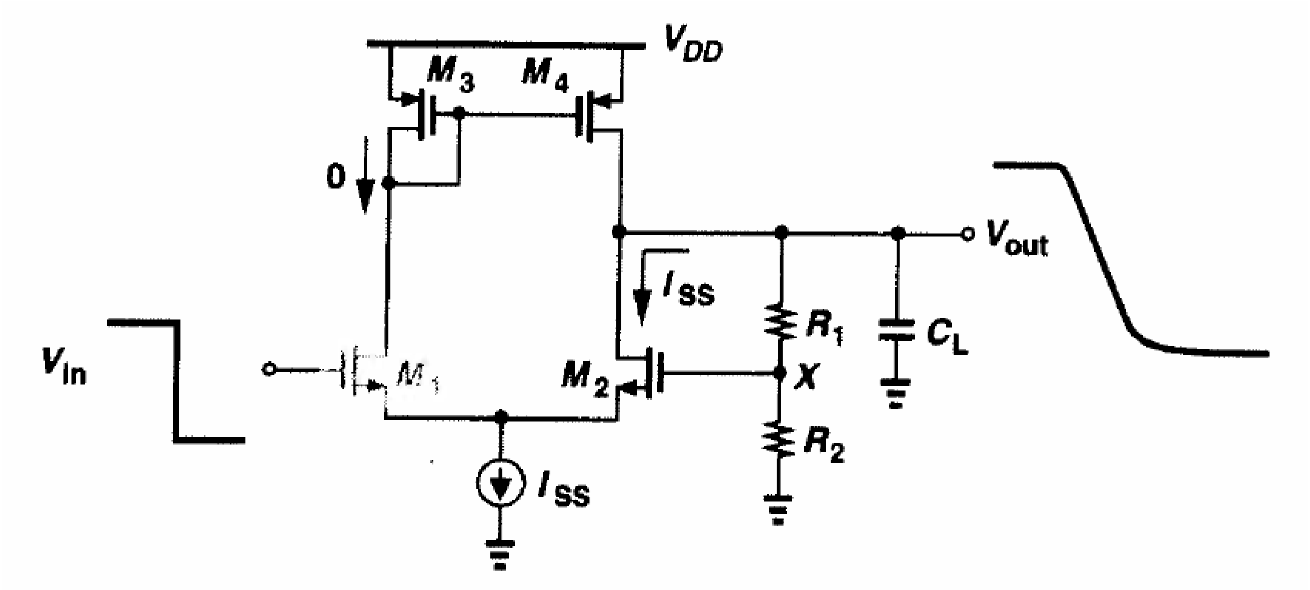




OTA Slew Rate

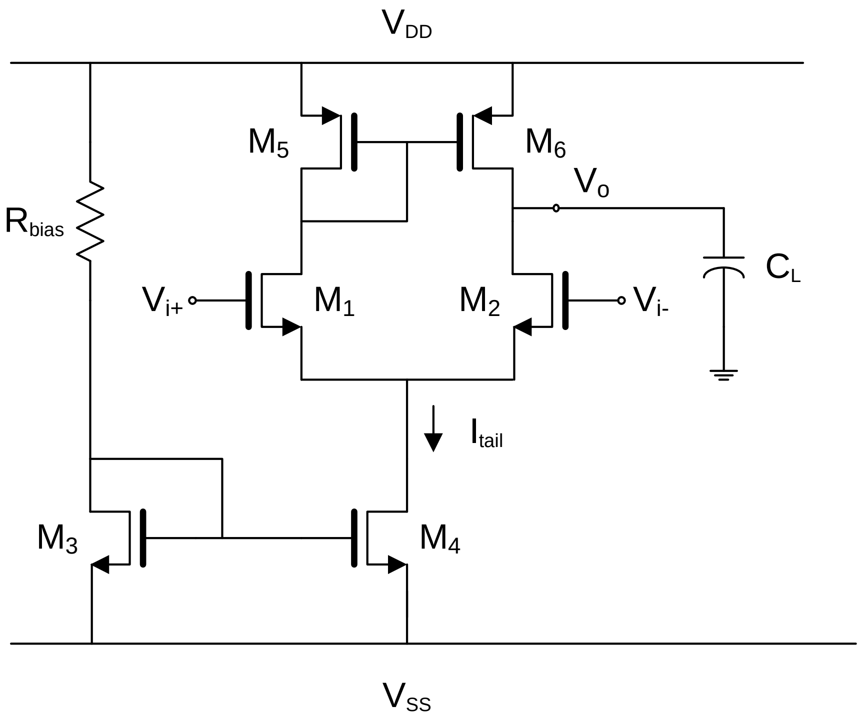
[Razavi]





$$\boxed{\textbf{Slew Rate} = \frac{I_{SS}}{C_L}}$$

Simple OTA Summary



Transconductance $G_m = g_{m1} = \sqrt{KP_n \frac{W}{L}_1 I_{TAIL}}$

Output Conductance $g_{out} = g_{o2} + g_{o6} = \frac{I_{TAIL}}{2}\left(\lambda_n + \lambda_p\right)$

DC Gain $A_v = G_m R_{out} = \frac{g_{m1}}{g_{o2} + g_{o6}} = \frac{2\sqrt{\frac{KP_n}{I_{TAIL}} \frac{W}{L}_1}}{\lambda_n + \lambda_p}$

Dominant Pole $\omega_{\mathbf{p1}} = \frac{g_{o2} + g_{o6}}{C_L}$

Non-Dominant Pole $\omega_{p2} = \frac{g_{m6}}{C_M} \approx \frac{g_{mg}}{2C_{gs6}}$

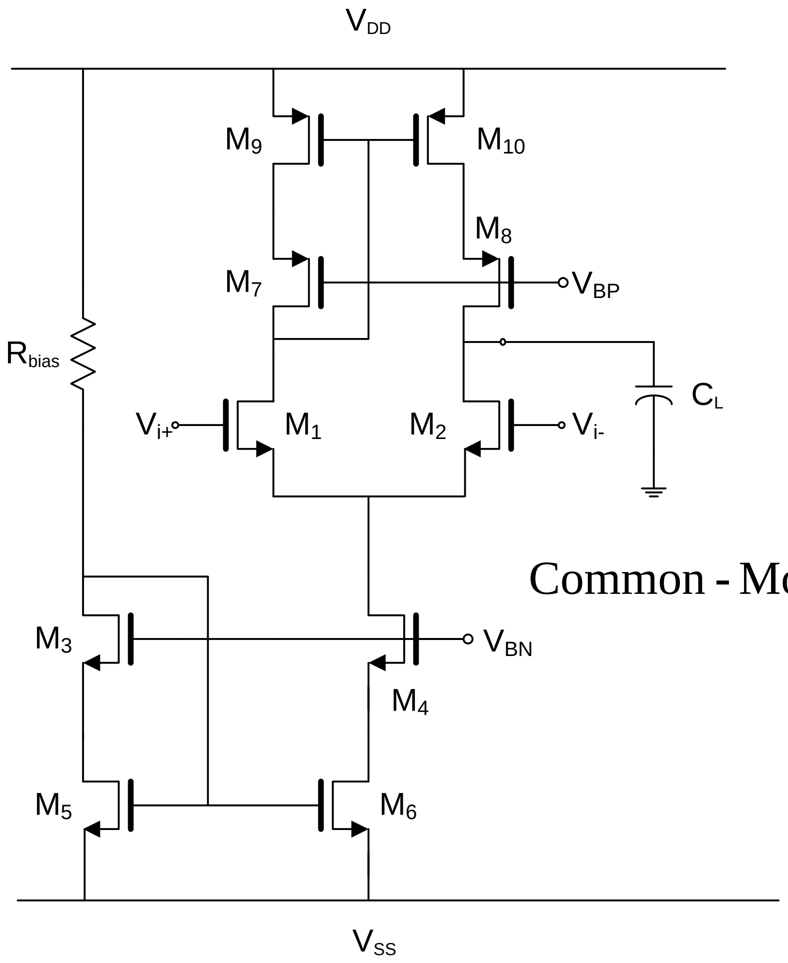
Output Noise Current $i_{on}^2 = 2\left(\frac{8}{3}kT\right)\left(g_{m1} + g_{m6}\right)$

$GBW = \frac{G_m}{C_L} = \frac{\sqrt{KP_n \frac{W}{L}_1 I_{TAIL}}}{C_L}$

Input Noise Voltage $v_{in}^2 = 2\left(\frac{8}{3}kT\right)\left(\frac{1}{g_{m1}}\right)\left(1 + \frac{g_{m6}}{g_{m1}}\right)$

Slew Rate $SR = \frac{I_{tail}}{C_L}$

OTA w/ Cascode Load and Tail Current Source



DC Gain $A_{DM} = g_{m1}\left(r_{o2} \| \left(r_{o8} + r_{o10} + g_{m8} r_{o10} r_{o8}\right)\right) \approx g_{m1} r_{o2}$

Dominant Pole $\omega_{p1} = \dfrac{1}{\left(r_{o2} \| \left(r_{o8} + r_{o10} + g_{m8} r_{o10} r_{o8}\right)\right) C_L} \approx \dfrac{1}{r_{o2} C_L}$

$$GBW = \frac{g_{m1} r_{o2}}{r_{o2} C_L} = \frac{g_{m1}}{C_L}$$

Slew Rate $SR = \dfrac{I_{tail}}{C_L}$

Common-Mode Gain $A_{CM} \approx -\dfrac{1}{2 g_{m9} R_{SS}}$ where $R_{SS} = r_{o4} + r_{o6} + g_{m4} r_{o6} r_{o4}$

$$A_{CM} \approx -\frac{1}{2 g_{m9} g_{m4} r_{o6} r_{o4}}$$

CMRR $= -2 g_{m1} r_{o2} g_{m9} g_{m4} r_{o6} r_{o4}$

Input Common-Mode Range

$$V_{SS} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_6}} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_4}} + \sqrt{\frac{I_{tail}}{\mu_n C_{ox} \frac{W}{L}_1}} + V_{Tn1} \le V_{icm} \le V_{DD} - \left(\sqrt{\frac{I_{tail}}{\mu_p C_{ox} \frac{W}{L}_9}} + \left|V_{Tp9}\right|\right) + V_{Tn1}$$

Next Time

- Noise