ECEN474/704: (Analog) VLSI Circuit Design
Spring 2018

Lecture 3: MOS Transistor Modeling

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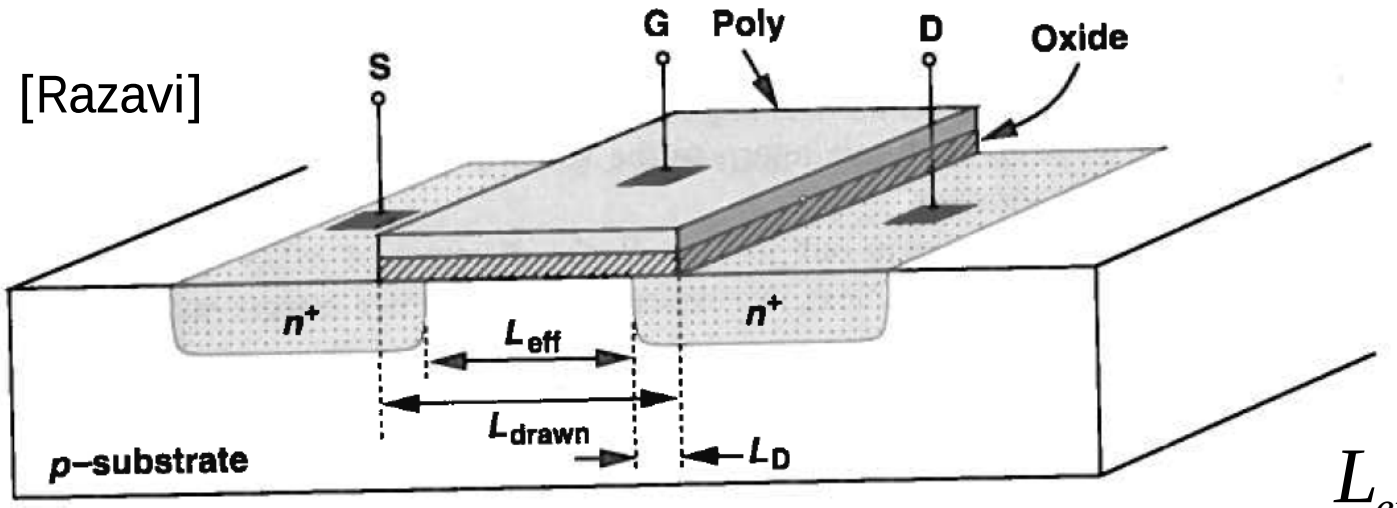
Agenda

- MOS Transistor Modeling
  - Large-Signal “DC” Model
  - Small-Signal “AC” Model
  - MOS Capacitors
- Current Reading
  - Razavi Chapters 2 & 17

Drawn & Effective Channel Lengths

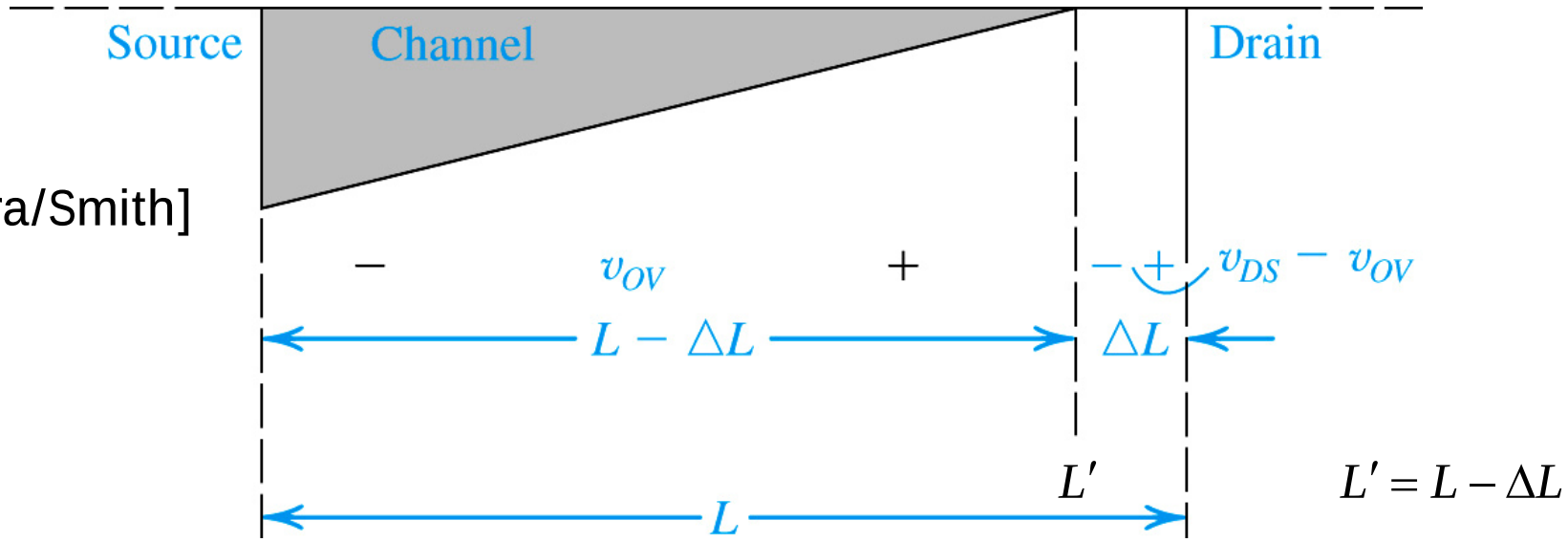
[Razavi]



$$L_{eff} = L_{drawn} - 2L_D$$

- The transistor gate overlaps both the source and drain region by a length of L_D due to side diffusion in the fabrication process
- This results in the effective transistor gate length, L_{eff}, being shorter than the drawn length, L_{drawn}
- Throughout the remainder of the course, L will generally refer to L_{eff}

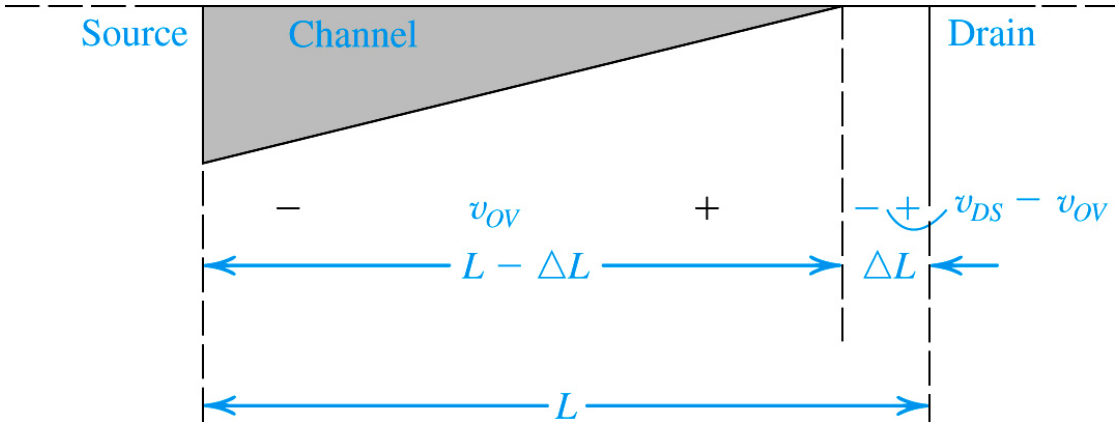
Finite Output Resistance in Saturation



- In saturation, as V_{DS} is increased the channel pinch-off point moves slightly towards the source

- This phenomenon is called channel-length modulation and is characterized by a parameter λ

Finite Output Resistance in Saturation



[Sedra/Smith]

- The current will increase slightly with V_{DS} in saturation, resulting in a finite incremental output resistance
- Note, the channel-length modulation parameter λ is inversely proportional to L

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L-\Delta L}(V_{GS}-V_{Tn})^2$$

$$= \frac{\mu_n C_{ox}}{2}\frac{W}{L}\frac{1}{1-\frac{\Delta L}{L}}(V_{GS}-V_{Tn})^2$$

$$\approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{Tn})^2\left(1+\frac{\Delta L}{L}\right)$$

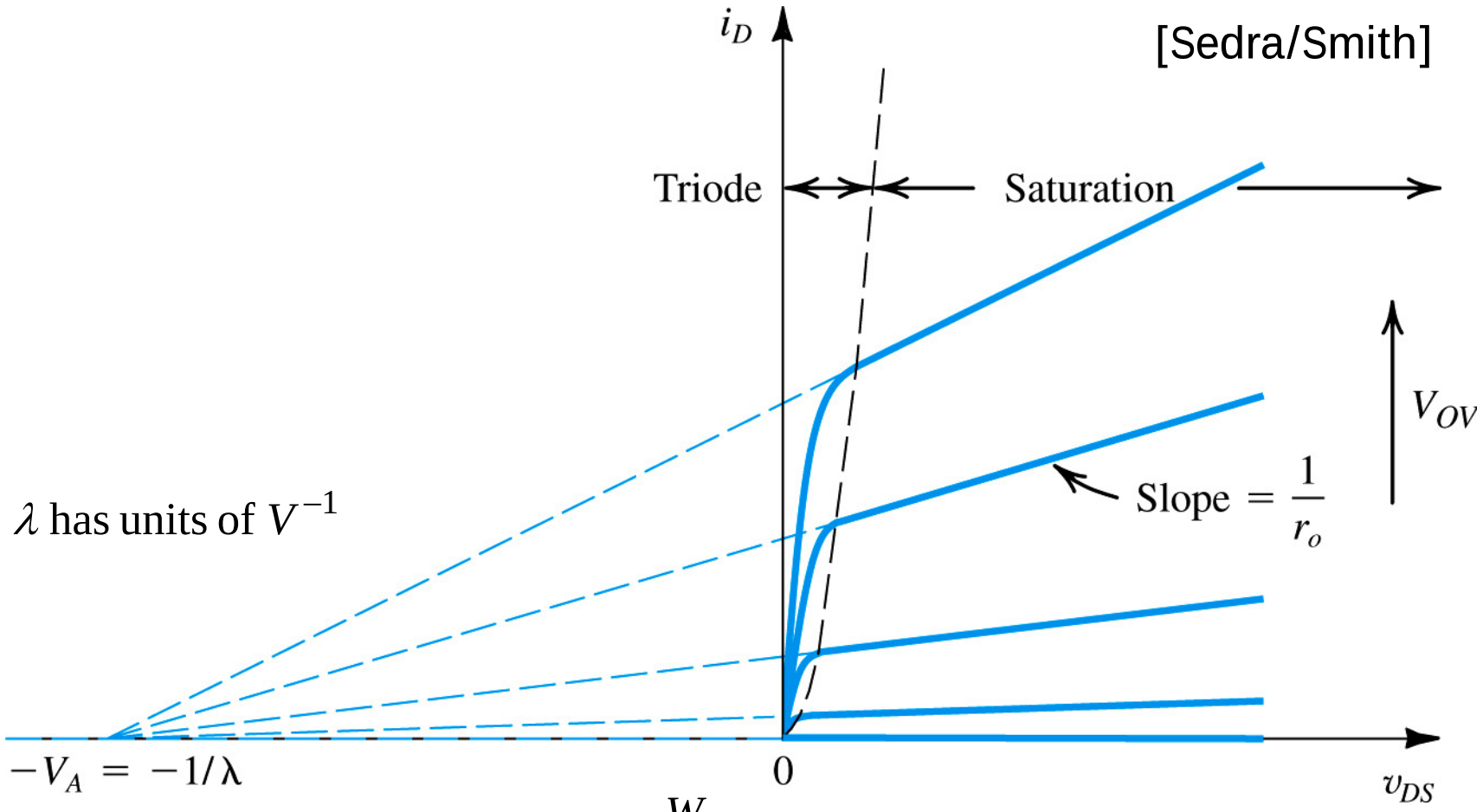
$$\Delta L = \lambda' V_{DS}$$

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{Tn})^2\left(1+\frac{\lambda' V_{DS}}{L}\right)$$

$$\lambda = \frac{\lambda'}{L}$$

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{Tn})^2(1+\lambda V_{DS})$$

MOS "Large-Signal" Output Characteristic with Finite Output Resistance

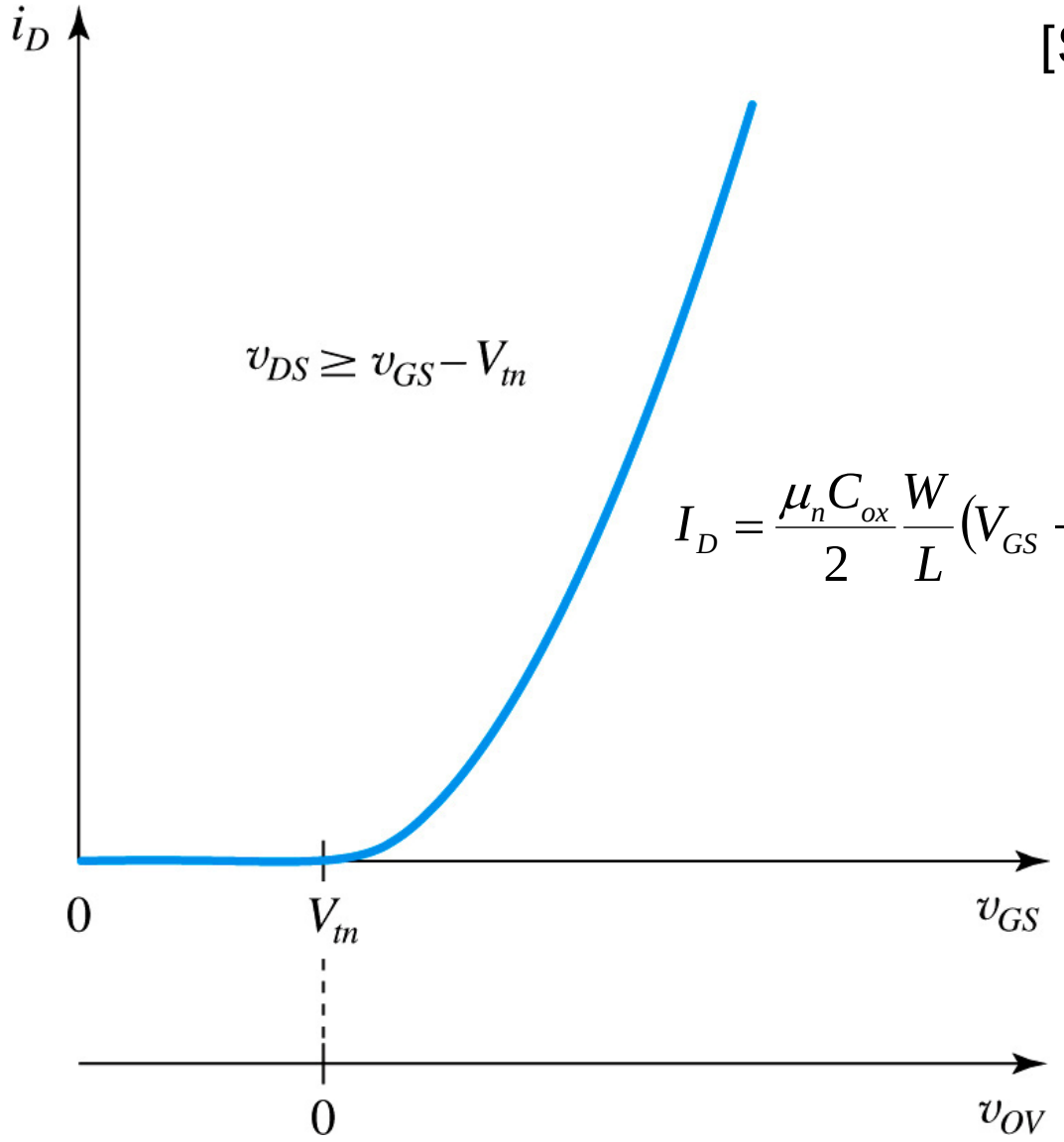


$$\text{Triode}: I_D = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_{Tn} - 0.5V_{DS}\right)V_{DS}$$

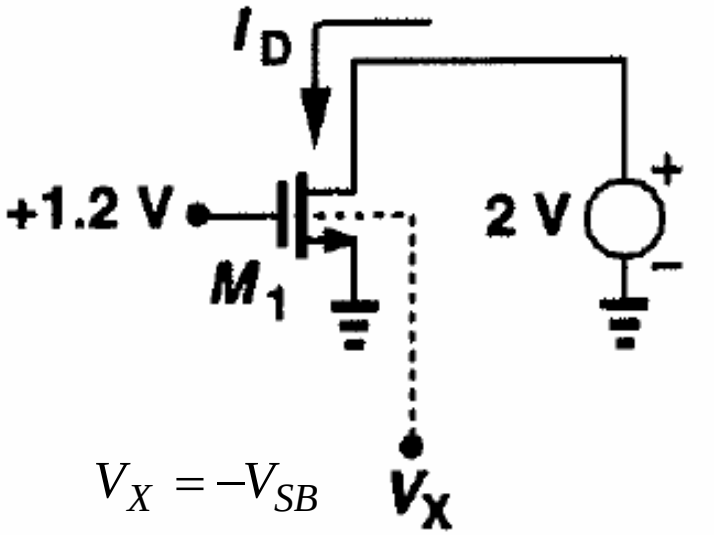
$$\text{Saturation}: I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\left(V_{GS} - V_{Tn}\right)^2\left(1 + \lambda V_{DS}\right)$$

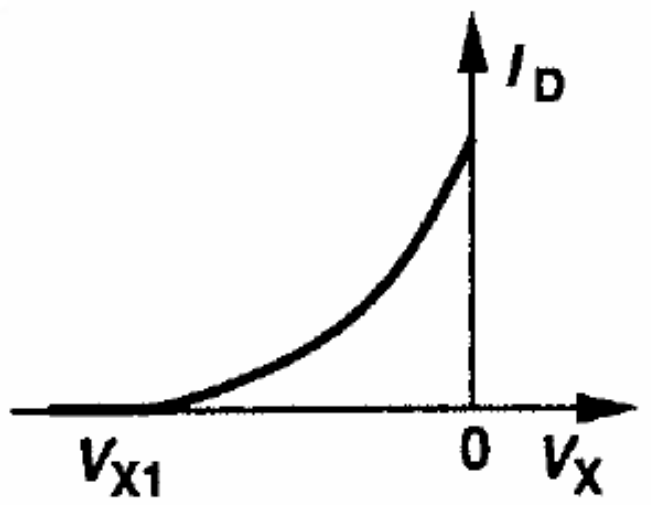
MOS “Large-Signal” Transfer Characteristic

[Sedra/Smith]



Impact of Bulk Voltage





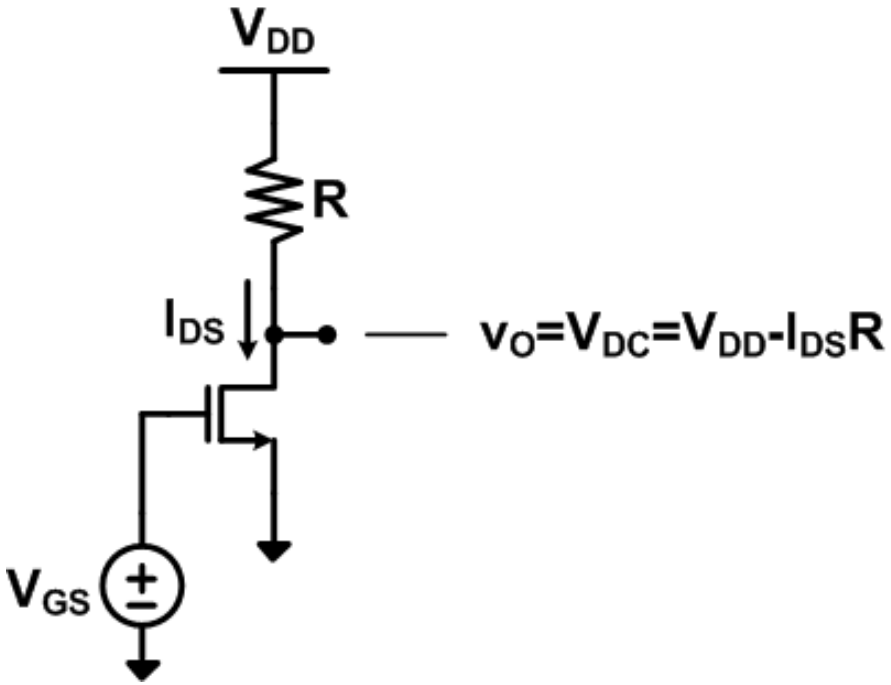
[Razavi]

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{Tn})^2(1 + \lambda V_{DS})$$

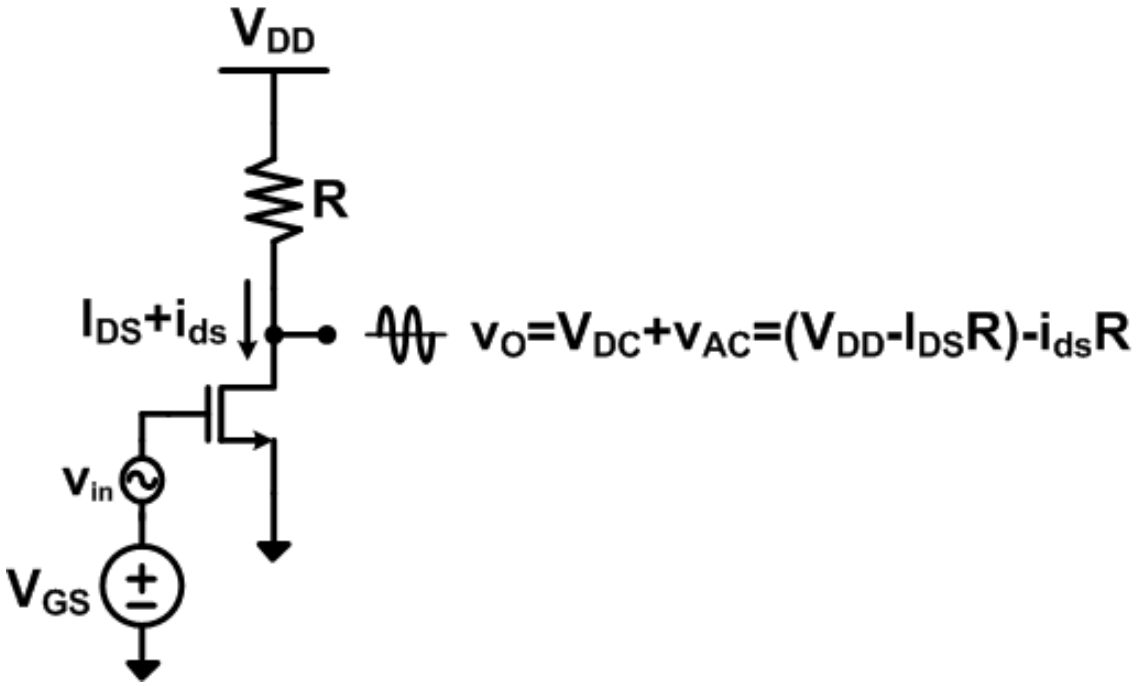
$$V_T = V_{T0} + \gamma\left[\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}\right] \Rightarrow V_{T0}\big|_{V_{SB}=0}$$

- The current decreases as V_{SB} increases due to an increased threshold voltage

Large-Signal “DC” Response



Large-Signal “DC” + Small-Signal “AC” Response

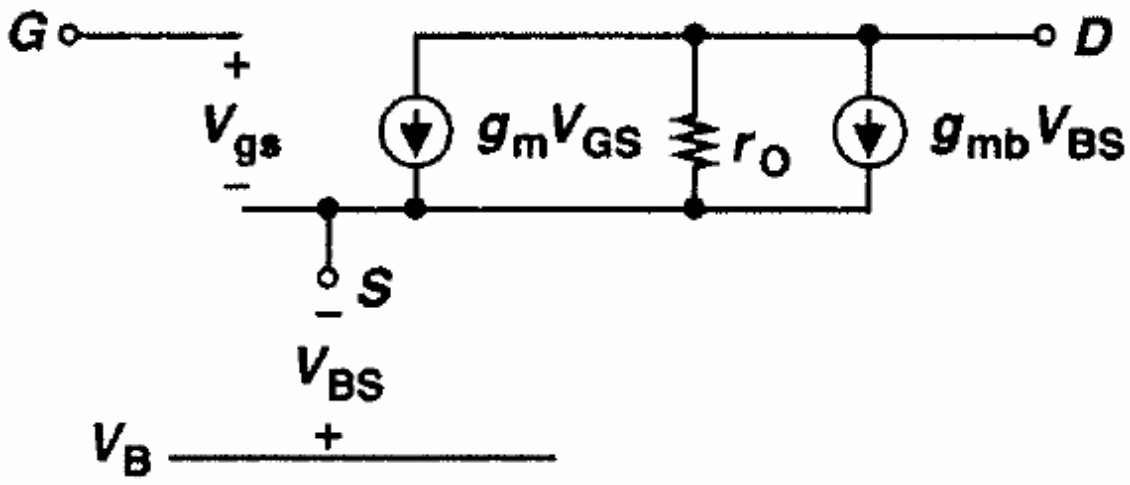


- For small-signal analysis, we “linearize” the response about the DC operating point
- If the signal is small enough, linearity holds and the complete response is the summation of the large-signal “DC” response and the small-signal “AC” response

Low-Frequency Small-Signal Model

- The linearized small-signal model is formed by computing an effective voltage-to-current transformation factor "conductance" by differentiating the large-signal response at the DC operating point

[Razavi]

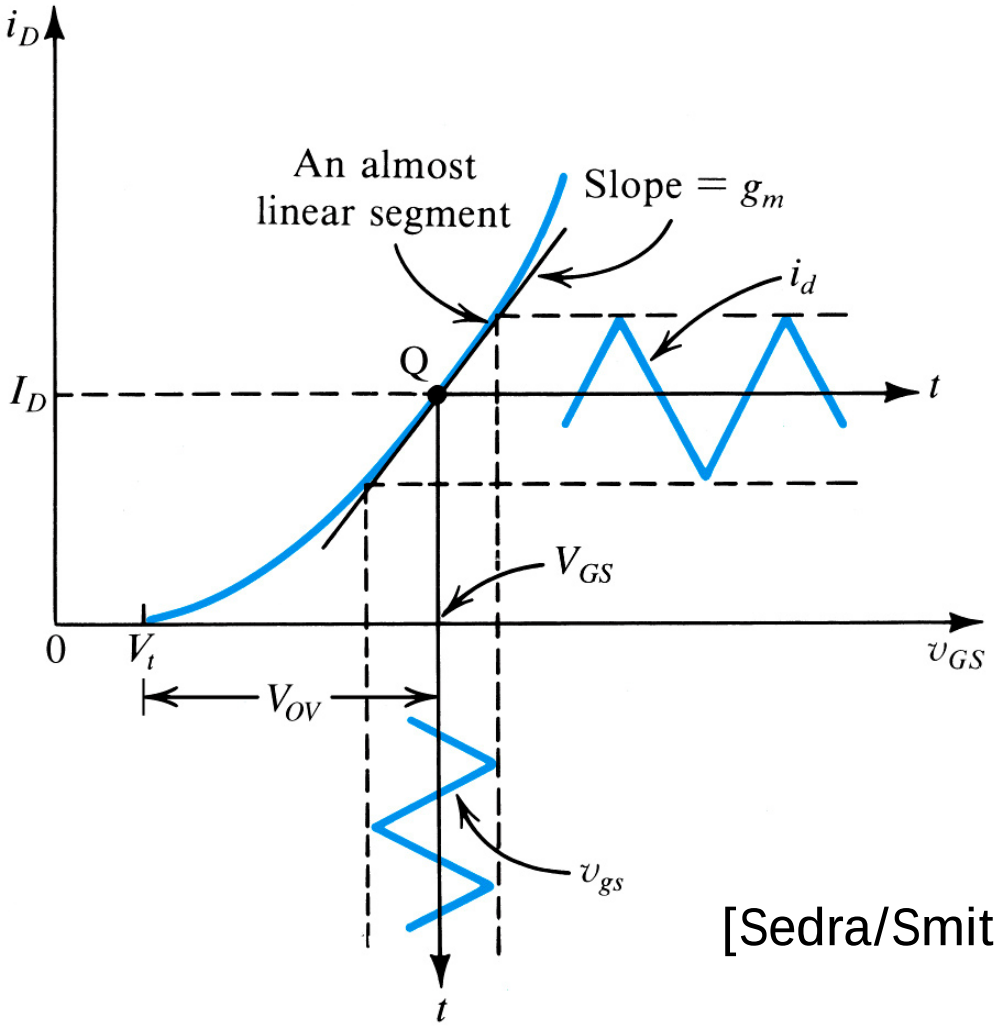


$$i_{ds} = \frac{\partial I_{ds}}{\partial V_{gs}} v_{gs} + \frac{\partial I_{ds}}{\partial V_{bs}} v_{bs} + \frac{\partial I_{ds}}{\partial V_{ds}} v_{ds}$$

$$i_{ds} = g_m V_{gs} + g_{mb} V_{bs} + g_{ds} V_{ds}$$

Transconductance, g_m

- Transistor transfer characteristic is used to extract transconductance, g_m



[Sedra/Smith]

In Saturation (Neglecting λ Effects)

$$I_D = \frac{\mu C_{OX}}{2}\frac{W}{L_{eff}}(V_{GS} - V_T)^2$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{gs}}\right|_Q = \mu C_{OX}\frac{W}{L_{eff}}\left.(V_{GS} - V_T)\right|_Q$$

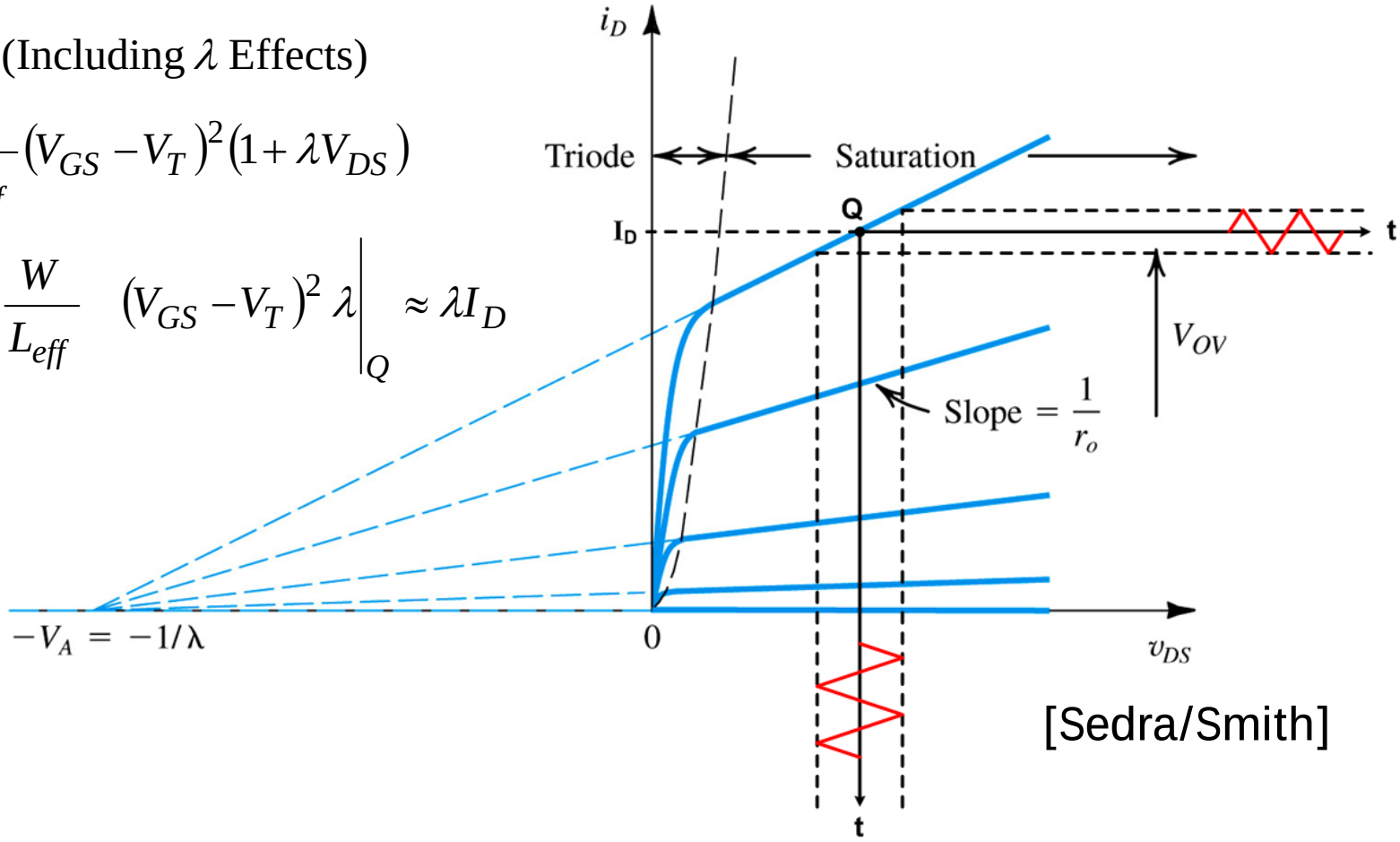
Output Conductance, g_o

- Transistor output characteristic is used to extract output conductance, g_o

In Saturation (Including λ Effects)

$$I_D = \frac{\mu C_{OX}}{2}\frac{W}{L_{eff}}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

$$g_o = \left.\frac{\partial i_D}{\partial v_{ds}}\right|_Q = \frac{\mu C_{OX}}{2}\frac{W}{L_{eff}}\left.(V_{GS}-V_T)^2\lambda\right|_Q \approx \lambda I_D$$

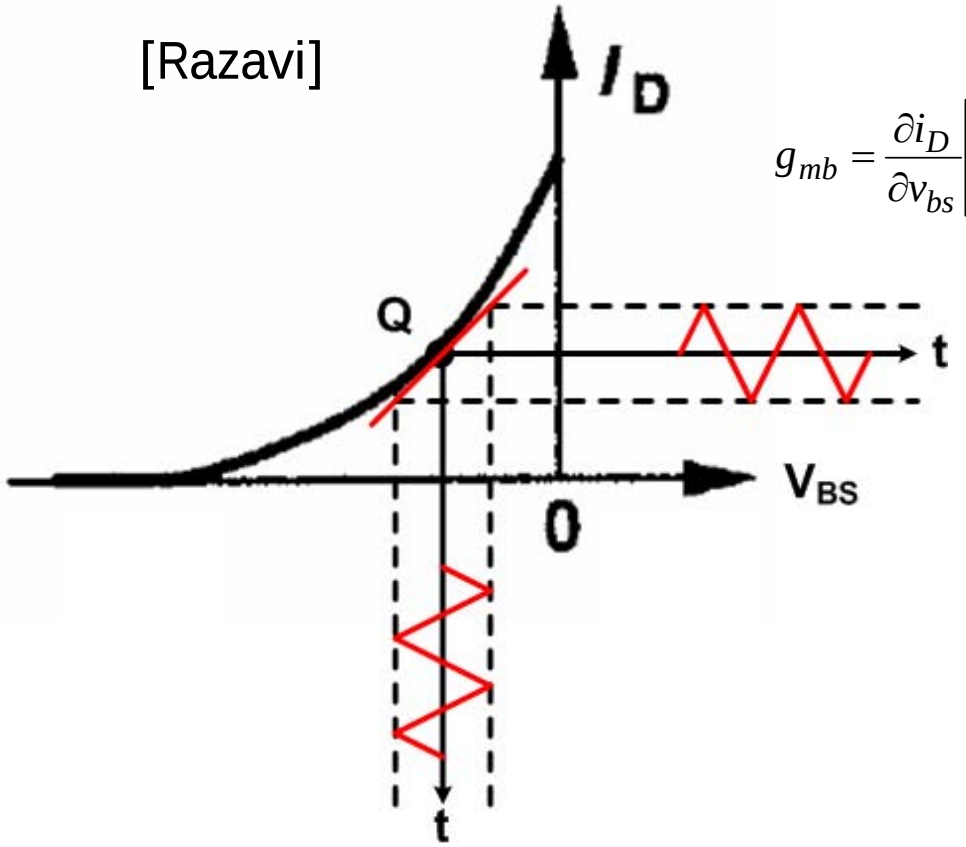


[Sedra/Smith]

Body Transconductance, g_{mb}

- The small-signal drain current changes with V_{BS} modulation due to changes in V_T

[Razavi]

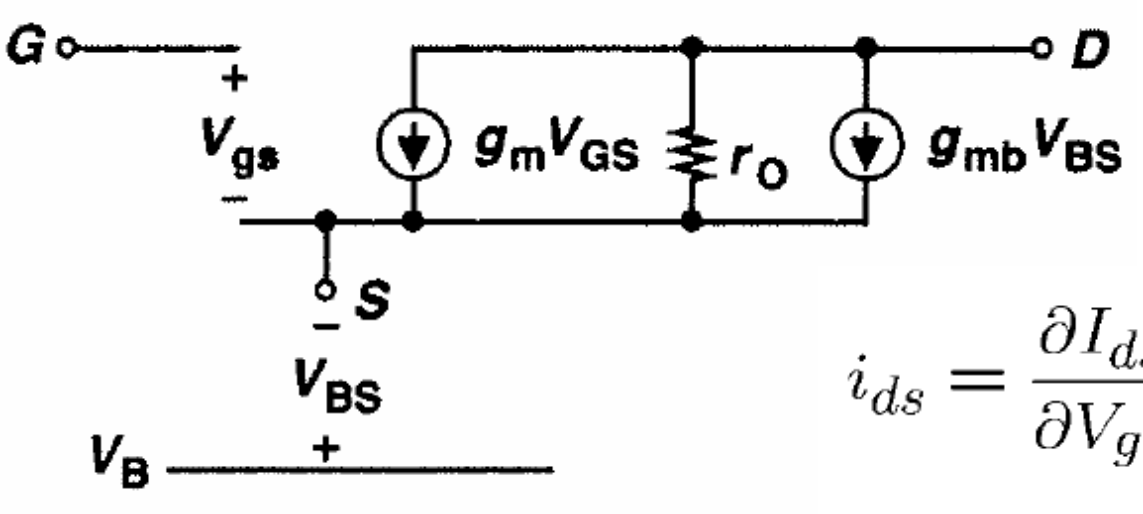


In Saturation (Neglecting λ Effects)

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{bs}}\right|_Q = \mu C_{OX} \left.\frac{W}{L_{eff}}(V_{GS} - V_T)\right|_Q * \left(-\left.\frac{\partial V_T}{\partial v_{bs}}\right|_Q\right) = \frac{\gamma g_m}{2\sqrt{2\phi_F + V_{SB}}}$$

Low-Frequency Small-Signal Model

[Razavi]



$$i_{ds} = \frac{\partial I_{ds}}{\partial V_{gs}} v_{gs} + \frac{\partial I_{ds}}{\partial V_{bs}} v_{bs} + \frac{\partial I_{ds}}{\partial V_{ds}} v_{ds}$$

$$i_{ds} = g_m V_{gs} + g_{mb} V_{bs} + g_{ds} V_{ds}$$

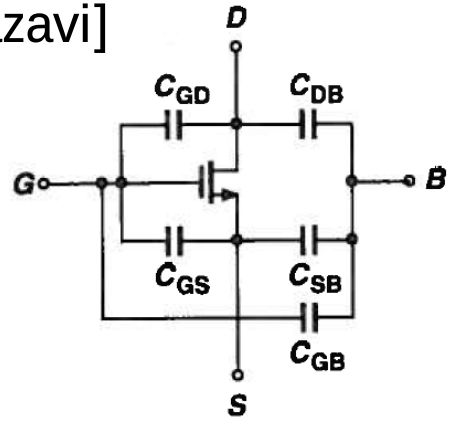
$$g_m = \left.\frac{\partial i_D}{\partial v_{gs}}\right|_Q = \left. \mu C_{OX} \frac{W}{L_{eff}} \quad (V_{GS} - V_T) \right|_Q$$

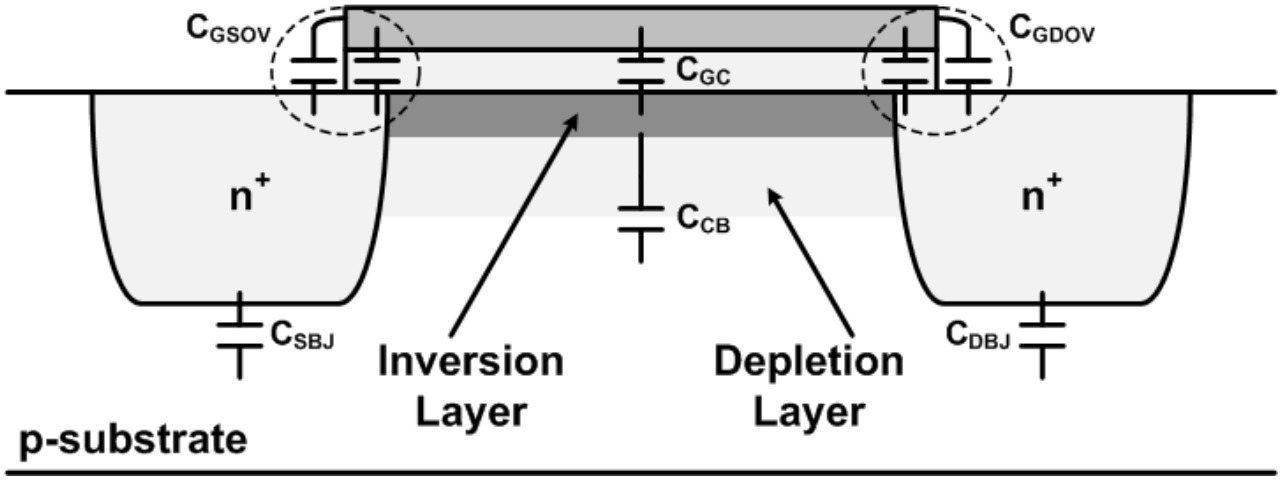
$$g_0 = \left.\frac{\partial i_D}{\partial v_{ds}}\right|_Q = \left(\frac{\mu C_{OX}}{2} \right) \left(\left. \frac{W}{L_{eff}} (V_{GS} - V_T)^2 \right|_Q \right) \lambda$$

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{bs}}\right|_Q = \mu C_{OX} \quad \left. \frac{W}{L_{eff}} (V_{GS} - V_T) \right|_Q * \left(- \left.\frac{\partial V_T}{\partial v_{bs}}\right|_Q \right) = \frac{\gamma g_m}{2\sqrt{2\phi_F + V_{SB}}}$$

MOS Transistor Capacitances

[Razavi]





$$\text{Gate - Channel Cap} = C_{GC} = WL_{eff}C_{ox}$$

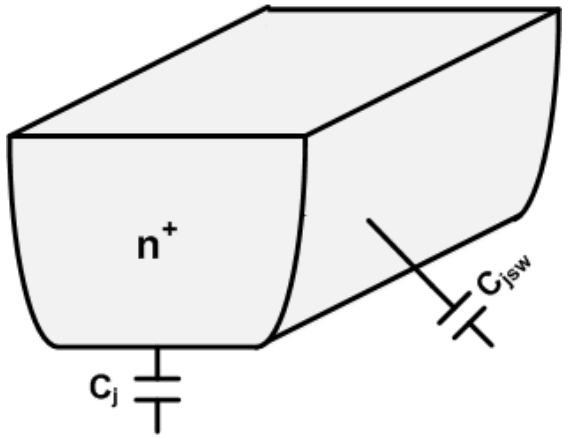
$$\text{Channel - Bulk Cap} = C_{CB} = WL_{eff}\sqrt{\frac{q\varepsilon_{Si}N_{sub}}{4\Phi_F}}$$

$$\text{Gate - Source Overlap (Fringing) Cap} = C_{GSov} = WC_{ov} \quad \text{Note, } C_{ov} \neq C_{ox}L_D$$

$$\text{Gate - Drain Overlap (Fringing) Cap} = C_{GDov} = WC_{ov}$$

$$\text{Source - Bulk Junction Cap} = C_{SBJ} = A_S C_j + P_S C_{jsw}$$

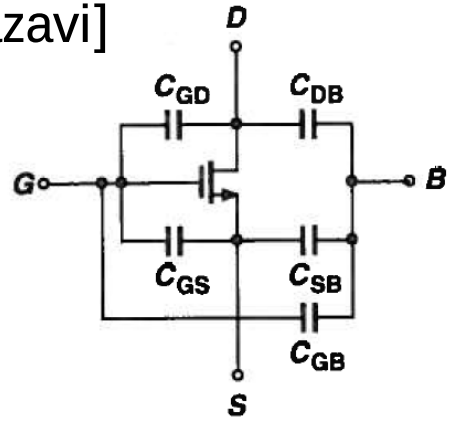
$$\text{Drain - Bulk Junction Cap} = C_{DBJ} = A_D C_j + P_D C_{jsw}$$

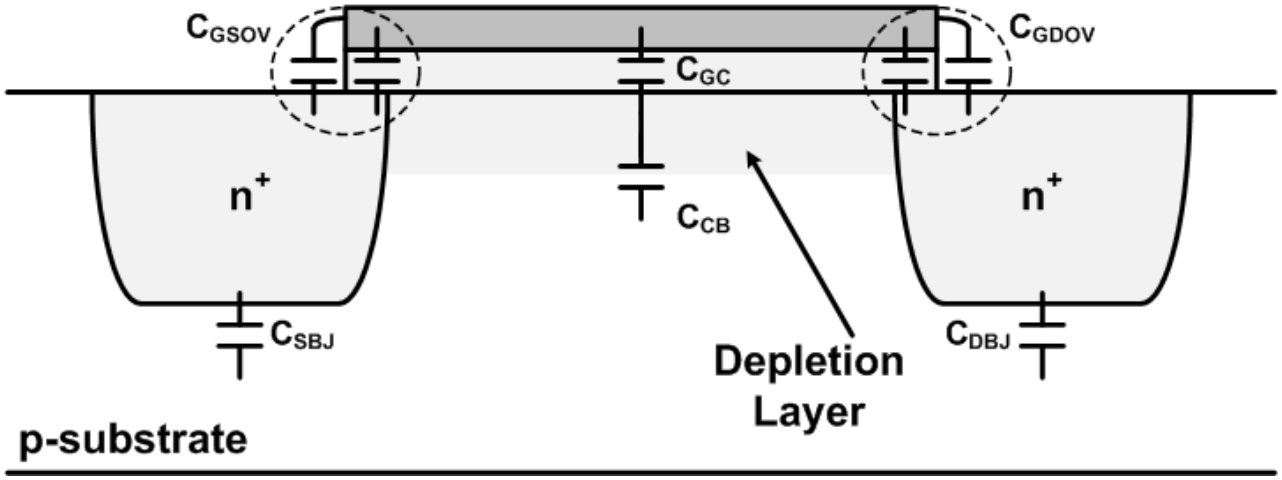


$$C_j = \frac{C_{j0}}{\left(1+\frac{V_{BX}}{\Phi_B}\right)^m} \qquad C_{jsw} = \frac{C_{jsw0}}{\left(1+\frac{V_{BX}}{\Phi_B}\right)^{msw}}$$

MOS Transistor Capacitances (Off)

[Razavi]





$$\text{Gate - Drain Cap} = C_{GD} = C_{GDov} = WC_{ov}$$

$$\text{Gate - Source Cap} = C_{GS} = C_{GDov} = WC_{ov}$$

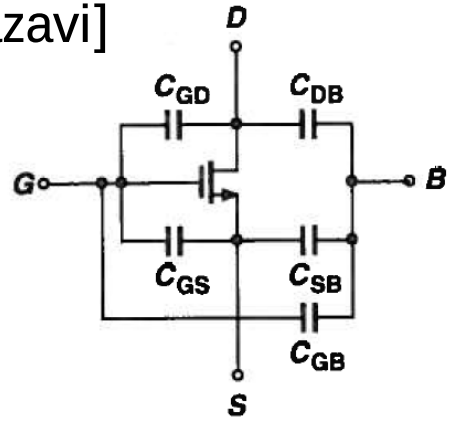
$$\text{Gate - Bulk Cap} = C_{GB} = \frac{C_{GC}C_{CB}}{C_{GC} + C_{CB}}$$

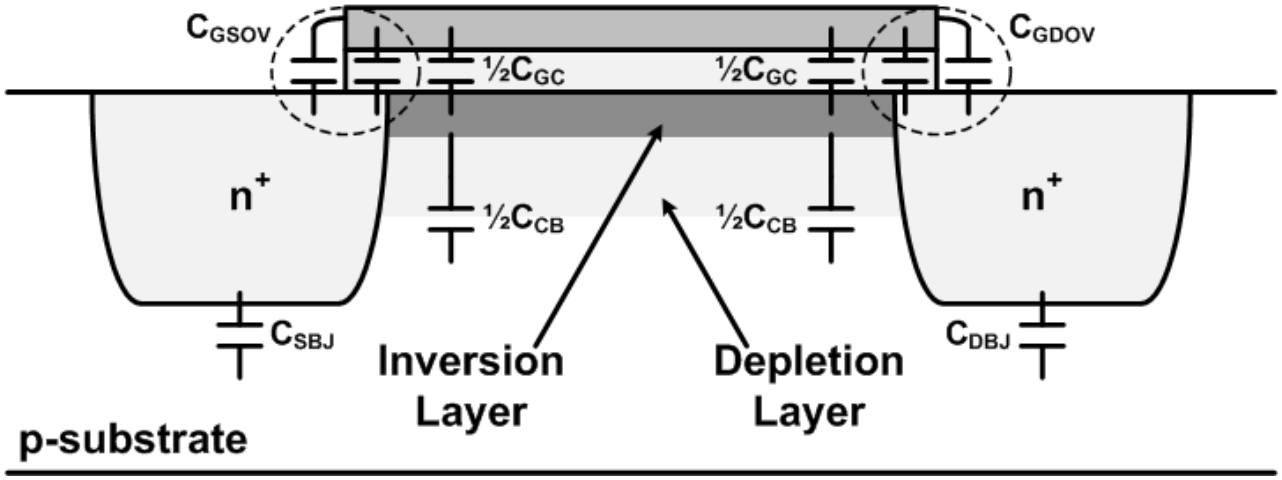
$$\text{Drain - Bulk Cap} = C_{DB} = C_{DBJ}$$

$$\text{Source - Bulk Cap} = C_{SB} = C_{SBJ}$$

MOS Transistor Capacitances (Triode)

[Razavi]





$$\text{Gate - Drain Cap} = C_{GD} = C_{GDov} + \frac{1}{2}C_{GC}$$

$$\text{Gate - Source Cap} = C_{GS} = C_{GSov} + \frac{1}{2}C_{GC}$$

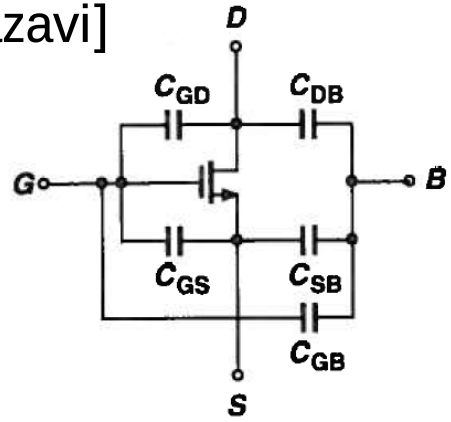
$$\text{Gate - Bulk Cap} = C_{GB} \approx 0$$

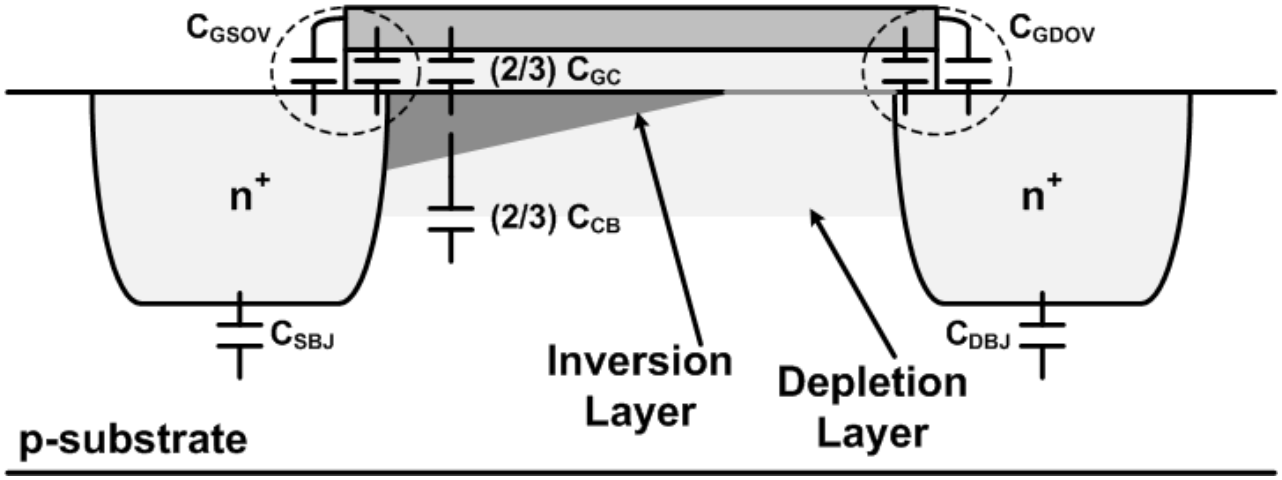
$$\text{Drain - Bulk Cap} = C_{DB} = C_{DBJ} + \frac{1}{2}C_{CB}$$

$$\text{Source - Bulk Cap} = C_{SB} = C_{SBJ} + \frac{1}{2}C_{CB}$$

MOS Transistor Capacitances (Saturation)

[Razavi]





$$\text{Gate - Drain Cap} = C_{GD} = C_{GDov}$$

$$\text{Gate - Source Cap} = C_{GS} = C_{GSov} + \frac{2}{3}C_{GC}$$

$$\text{Gate - Bulk Cap} = C_{GB} \approx 0$$

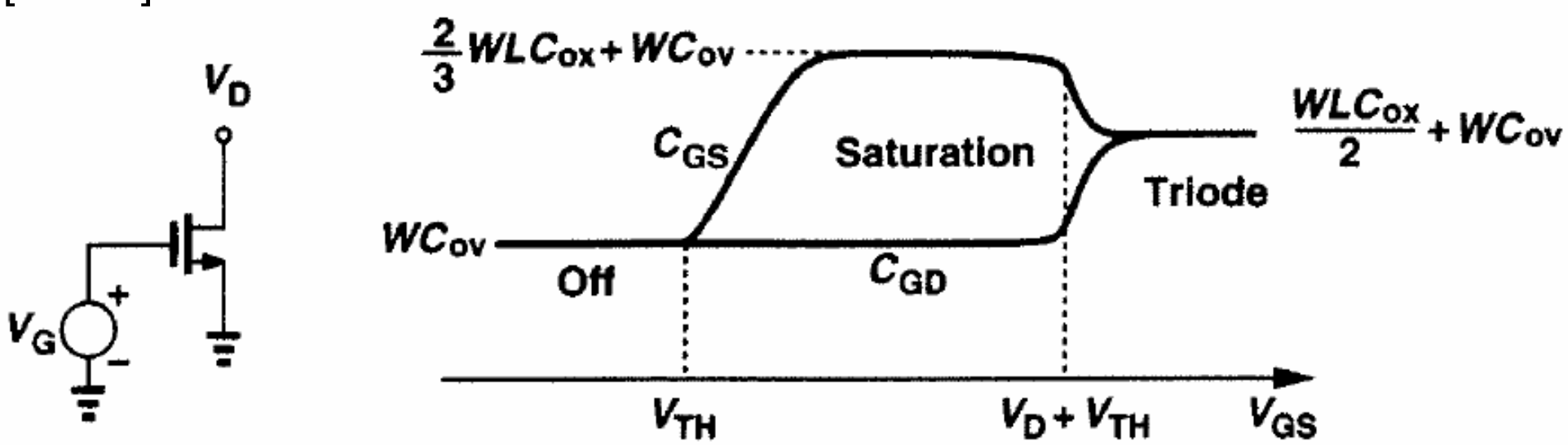
$$\text{Drain - Bulk Cap} = C_{DB} = C_{DBJ}$$

$$\text{Source - Bulk Cap} = C_{SB} = C_{SBJ} + \frac{2}{3}C_{CB}$$

MOS Gate Capacitors Response

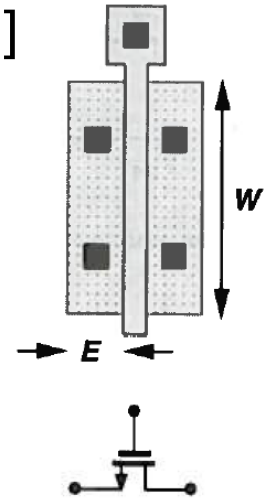
[Razavi]

Note, $C_{ov} \neq C_{ox} L_D$



MOS Source & Drain Junction Capacitors

[Razavi]



$$A_S = A_D = WE$$

$$P_S = P_D = 2(W + E)$$

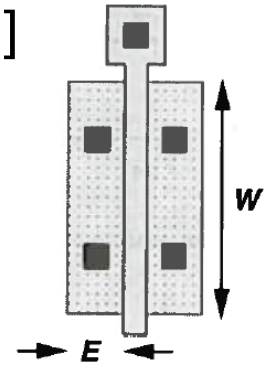
$$\text{Junction } C_{SB} = C_{DB} = AC_j + PC_{jsw} = WEC_j + 2(W + E)C_{jsw}$$

Source/Drain Junction Perimeter Caps Disclaimer

- Note, there is some ambiguity on how to model the source/drain junction perimeter (sidewall) capacitance on the side of the gate
  - This is due to the channel occupying a portion of the sidewall area
  - Different textbooks present different approaches
- The Razavi text conservatively assumes that the sidewall perimeter capacitance is the same on all sides
- The Johns/Martin text (sometimes previously used) optimistically sets the sidewall perimeter cap to zero under the gate
- The correct answer is somewhere in the middle
- We will follow the Razavi method and assume that the sidewall perimeter capacitance is the same on all sides (even under the gate)
  - I will try to make it clear on any problem description

MOS Source & Drain Junction Capacitors

[Razavi]



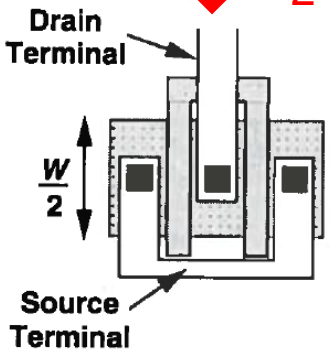
$$A_S = A_D = WE$$

$$P_S = P_D = 2(W + E)$$

$$\text{Junction } C_{SB} = C_{DB} = AC_j + PC_{jsw} = WEC_j + 2(W + E)C_{jsw}$$

Folding into 2 “fingers” & sharing drain

Folding into 2 “fingers”



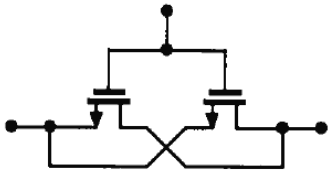
$$A_S = 2\left(\frac{W}{2}E\right) = WE$$

$$A_D = \frac{W}{2}E$$

$$P_S = 2\left(2\left(\frac{W}{2} + E\right)\right)$$

$$P_D = 2\left(\frac{W}{2} + E\right)$$

Folding the transistor allows for approximately half the drain junction capacitance with a small increase in source junction capacitance

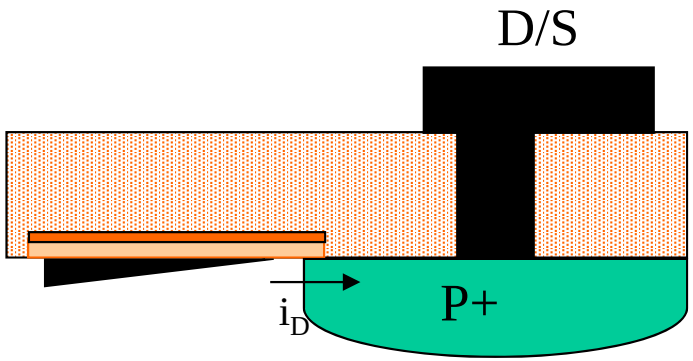


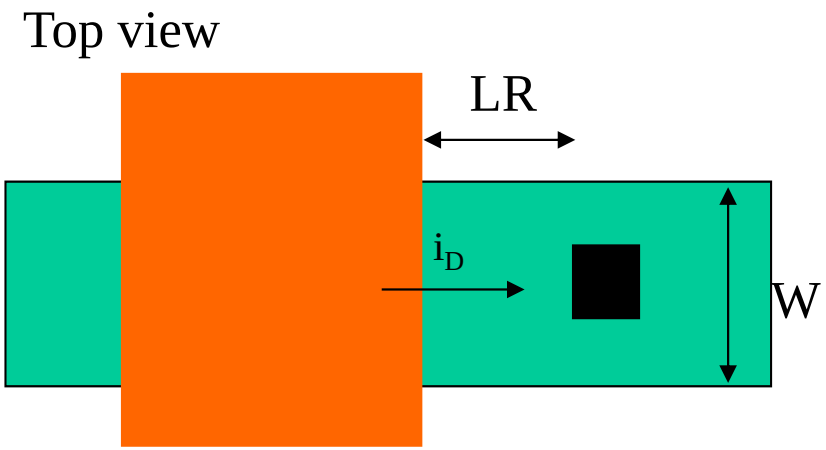
$$\text{Junction } C_{SB} = A_S C_j + P_S C_{jsw} = WEC_j + 2(W + 2E)C_{jsw}$$

$$\text{Junction } C_{DB} = A_D C_j + P_D C_{jsw} = \frac{W}{2}EC_j + (W + 2E)C_{jsw}$$

Other resistors: Source/Drain

Drain/Source Resistance





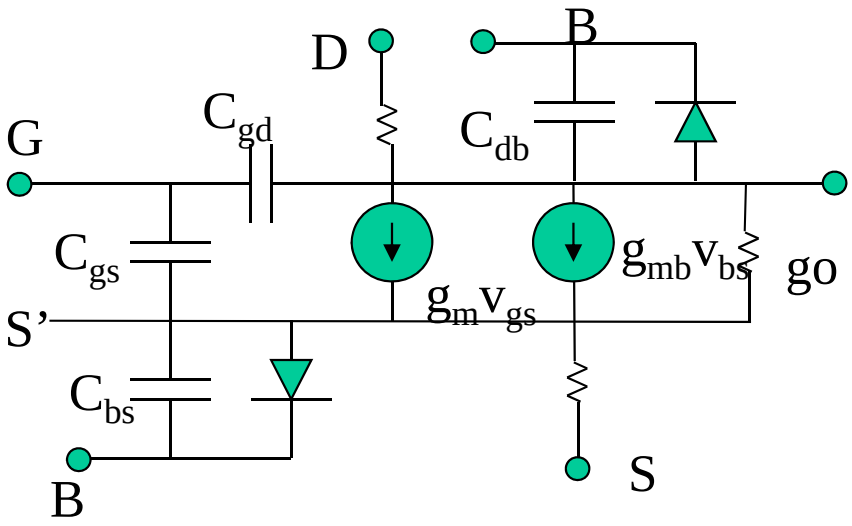
- **Drain/Source Resistance**

In addition to the contact resistance , the diffusion resistance has to be considered.

$$R_{series} = \frac{L_R}{W}\left(R_{[]}\right)$$

- **In SPICE, $R_{[]}$ is defined as RSH**

Small Signal Model (Saturation region)



Small signal model (saturation region)

$$I_D = \frac{\mu C_{OX}}{2}\frac{W}{L_{eff}}[V_{GS}-V_T]^2[1+\lambda V_{DS}]$$

$$V_T = V_{T0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

$$i_D \approx \frac{\mu C_{OX}}{2}\frac{W}{L_{eff}}[V_{GS}-V_T]^2[1+\lambda V_{DS}]\Big|_Q + \frac{\partial i_D}{\partial v_{gs}}\Big|_Q v_{gs} + \frac{\partial i_D}{\partial v_{ds}}\Big|_Q v_{ds} + \frac{\partial i_D}{\partial v_{bs}}\Big|_Q v_{bs}$$

$$g_m = \frac{\partial i_D}{\partial v_{gs}}\Big|_Q = \mu C_{OX}\frac{W}{L_{eff}}(V_{GS}-V_T)\Big|_Q$$

$$g_0 = \frac{\partial i_D}{\partial v_{ds}}\Big|_Q = \left(\frac{\mu C_{OX}}{2}\right)\left(\frac{W}{L_{eff}}(V_{GS}-V_T)^2\right)\Big|_Q \lambda$$

$$i_D \cong I_D + g_m v_{gs} + g_0 v_{ds} + g_{mb} v_{bs}$$

$$g_{mb} = \frac{\partial i_D}{\partial v_{bs}}\Big|_Q = \mu C_{OX}\frac{W}{L_{eff}}[V_{GS}-V_T]\Big|_Q * \left(-\frac{\partial V_T}{\partial v_{bs}}\Big|_Q\right) \cong \frac{\gamma g_m}{2\sqrt{2\phi_F + V_{SB}}}$$

Next Time

- MOS Transistor Modeling
  - High-Field “Short-Channel” Effects
  - Spice Models