ECEN326: Electronic Circuits
Spring 2022

Lecture 7: Feedback

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Announcements

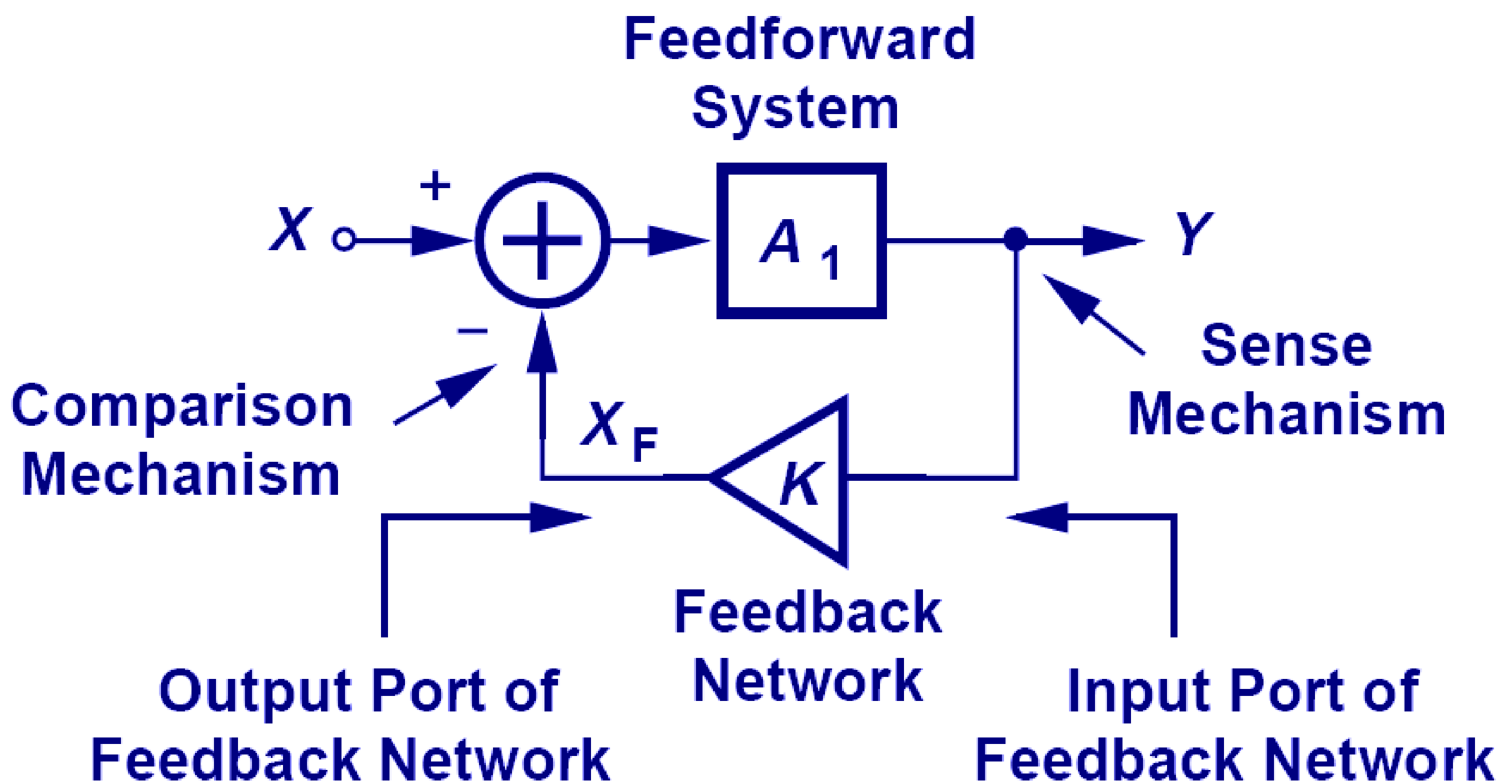
- Homework 7 due Apr 21

- Reading
  - Razavi Chapter 12

Agenda

- Feedback Overview
- Feedback Properties
- Amplifier Types
- Sense and Return Techniques
- Feedback Polarity
- Feedback Topologies
- Effect of Nonideal I/O Impedances
- Stability
- Two-Stage Miller OTA

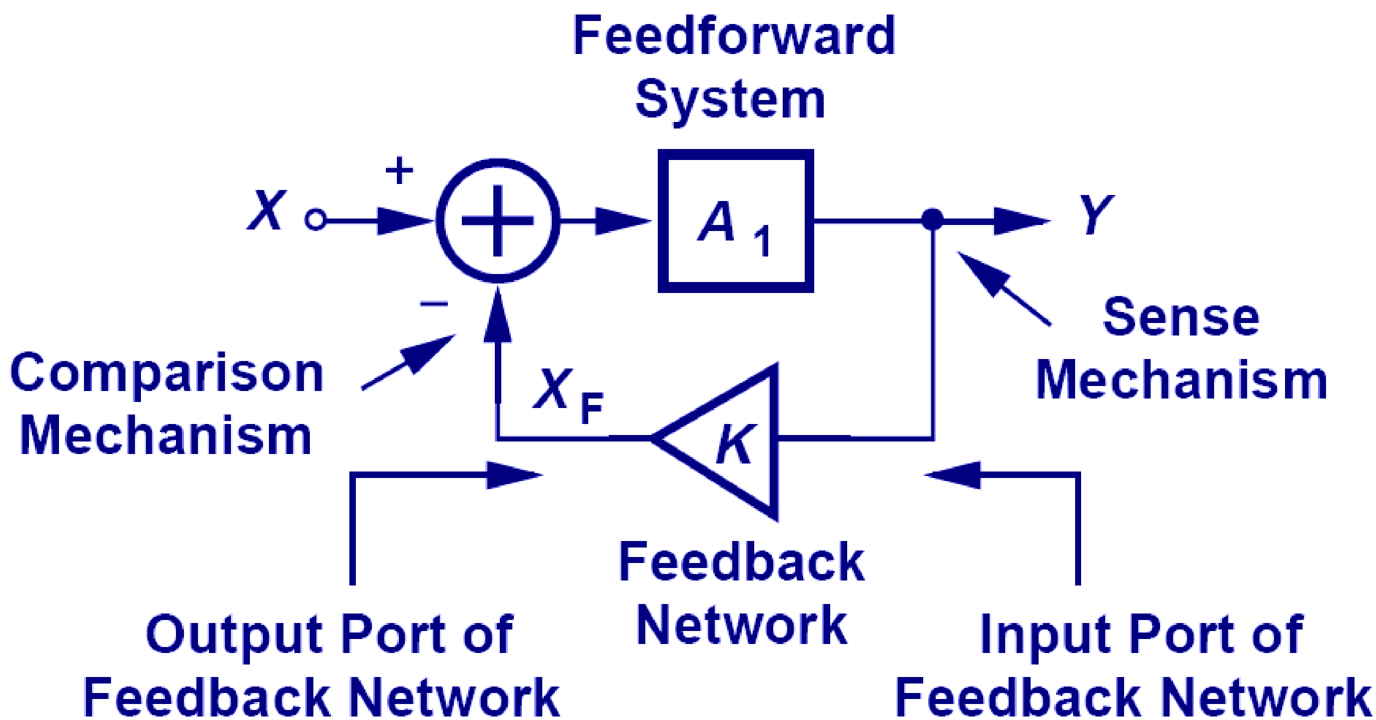
Negative Feedback System



$$\text{Feedback Factor } K = \frac{X_F}{Y}$$

- **A negative feedback system consists of four components: 1) feedforward system, 2) sense mechanism, 3) feedback network, and 4) comparison mechanism.**

Close-loop Transfer Function

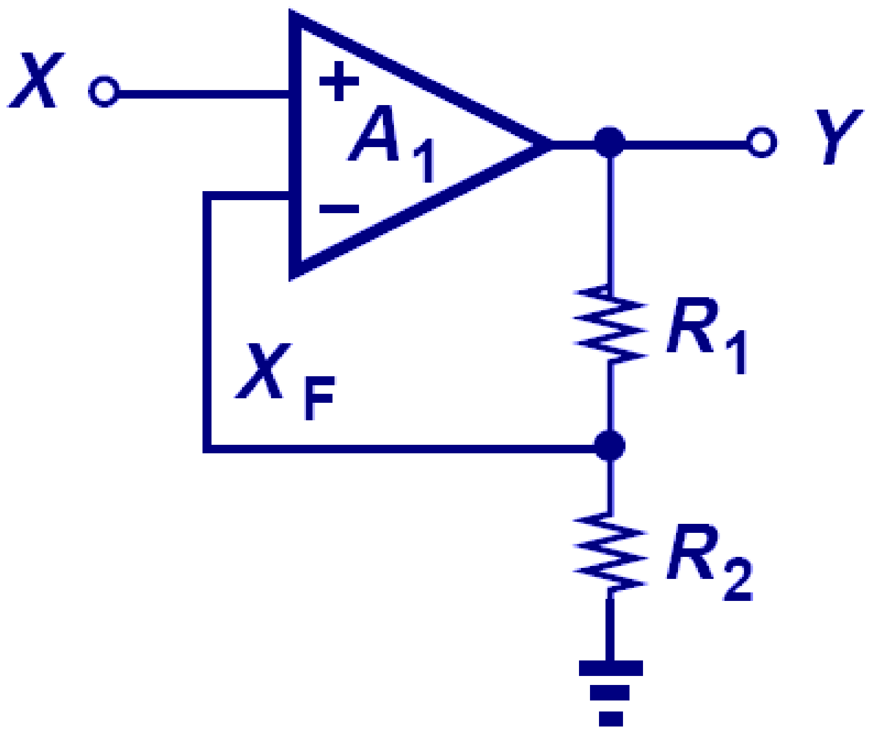


$$Y = (X - X_F)A_1 = (X - YK)A_1$$
$$Y(1 + KA_1) = XA_1$$

Closed-Loop Gain

$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$

Feedback Example

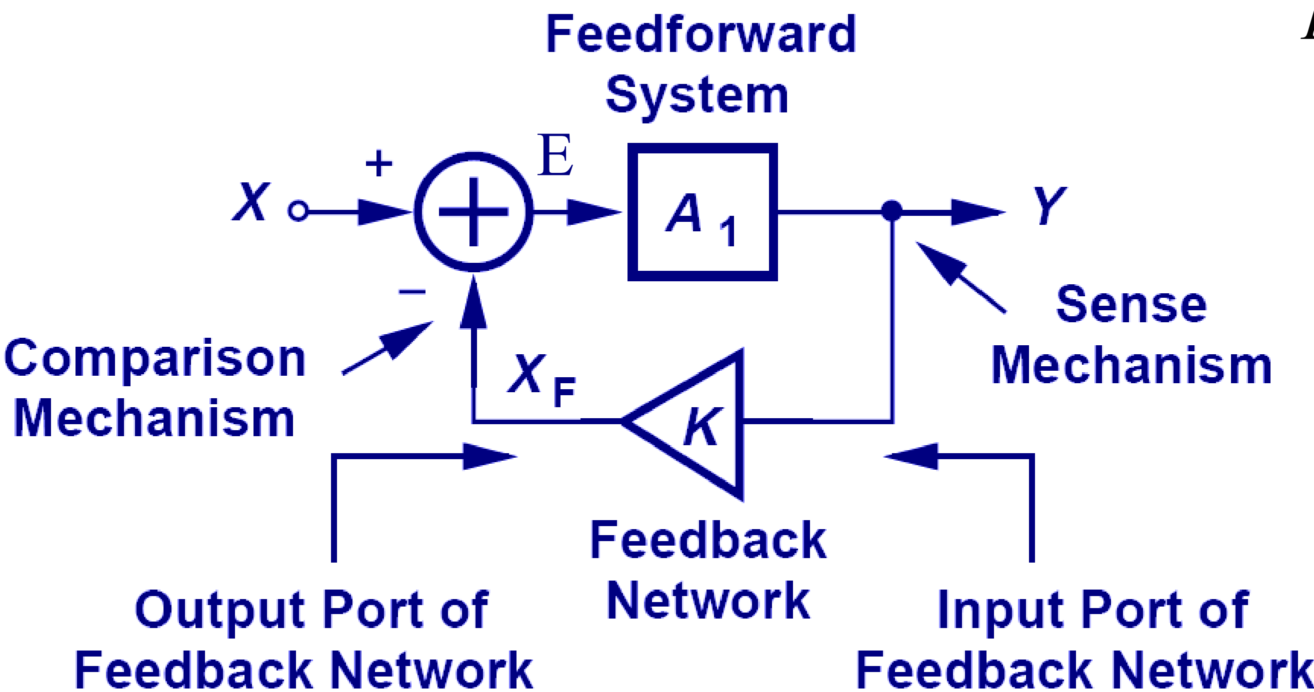


$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{Y}{X} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

- **A_1 is the feedforward network, R_1 and R_2 provide the sensing and feedback capabilities, and comparison is provided by differential input of A_1.**

Comparison Error



$$E = X - X_F = X - EA_1K$$

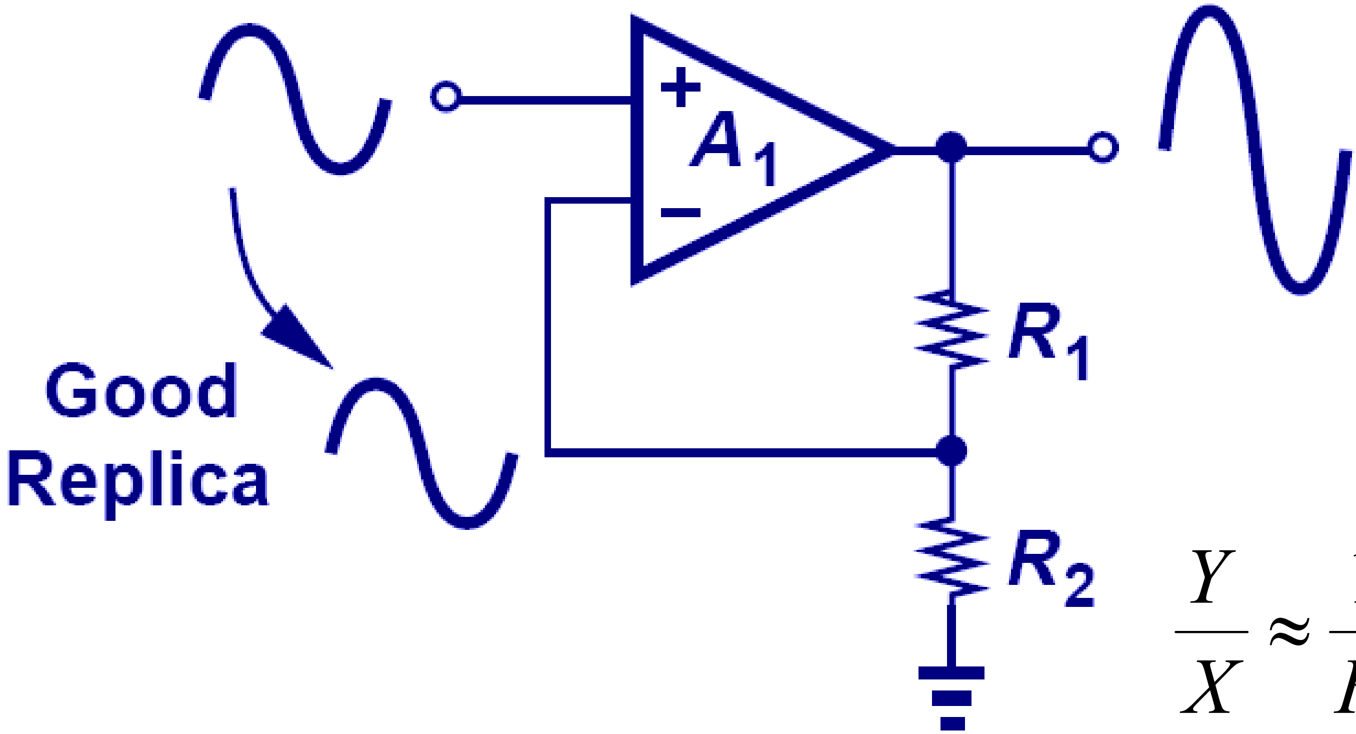
$$E(1 + A_1K) = X$$

$$\frac{E}{X} = \frac{1}{1 + A_1K}$$

- **As A_1K increases, the error between the input and fed back signal decreases. Or the fed back signal approaches a good replica of the input.**

Comparison Error

- What happens to the closed-loop and error transfer function as A1 $\rightarrow \infty$?

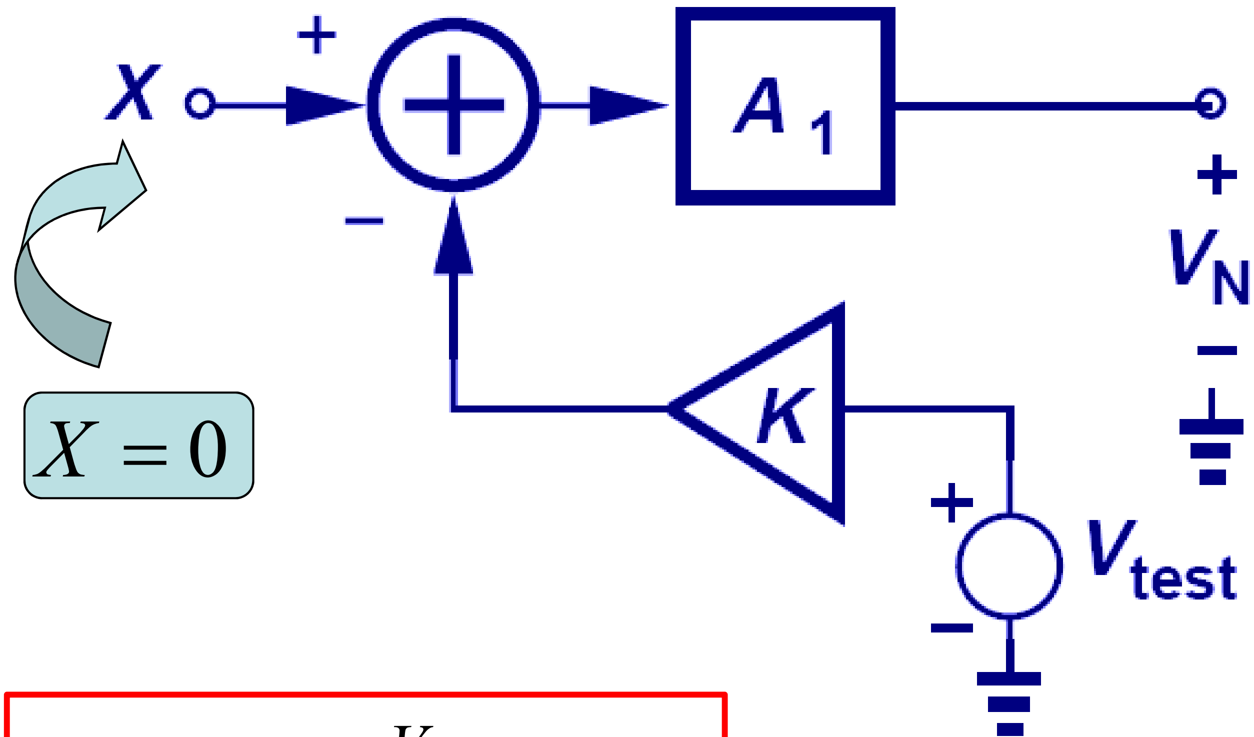


$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$

$$\frac{Y}{X} \approx \frac{1}{K} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$\frac{E}{X} = \frac{1}{1 + KA_1} \approx 0$$

Loop Gain



$X = 0$

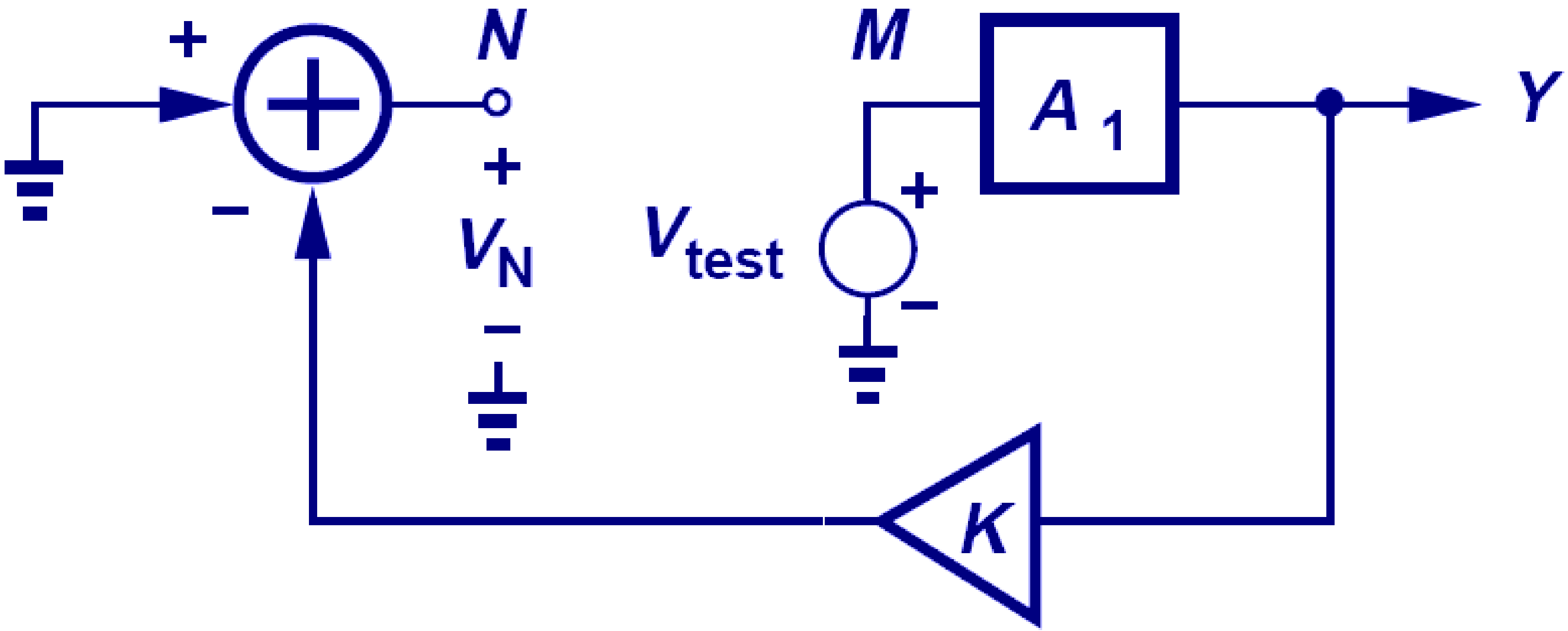
$$V_N = V_{test}(K)(-1)(A_1)$$

$$KA_1 = -\frac{V_N}{V_{test}}$$

$$\text{Loop Gain} = \frac{V_N}{V_{test}} = -KA_1$$

- **When the input is grounded, and the loop is broken at an arbitrary location, the loop gain is measured to be -KA_1.**

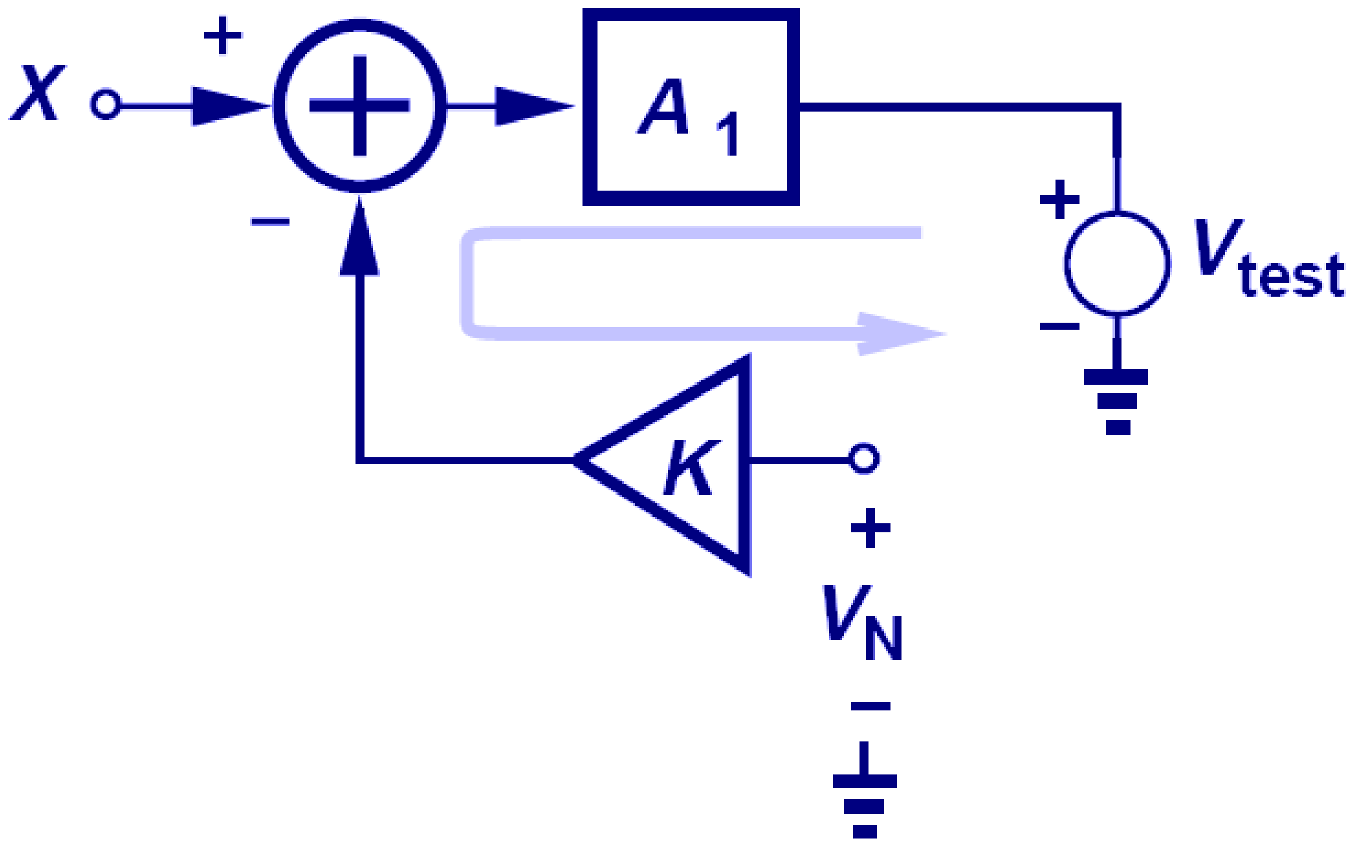
Example: Alternative Loop Gain Measurement



$$V_N = -KA_1V_{test}$$

- Result should be the same wherever we break the loop as long as we analyze the loop in the proper signal direction

Incorrect Calculation of Loop Gain



- Signal naturally flows from the input to the output of a feedforward/feedback system. If we apply the input the other way around, the “output” signal we get is not a result of the loop gain, but due to poor isolation.

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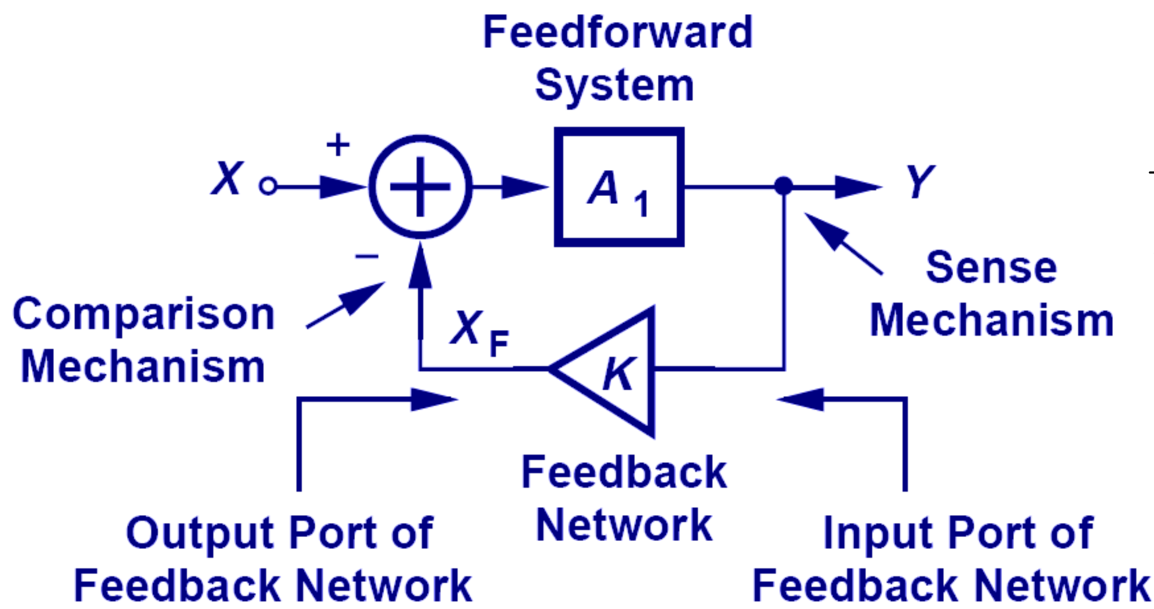
Gain Desensitization

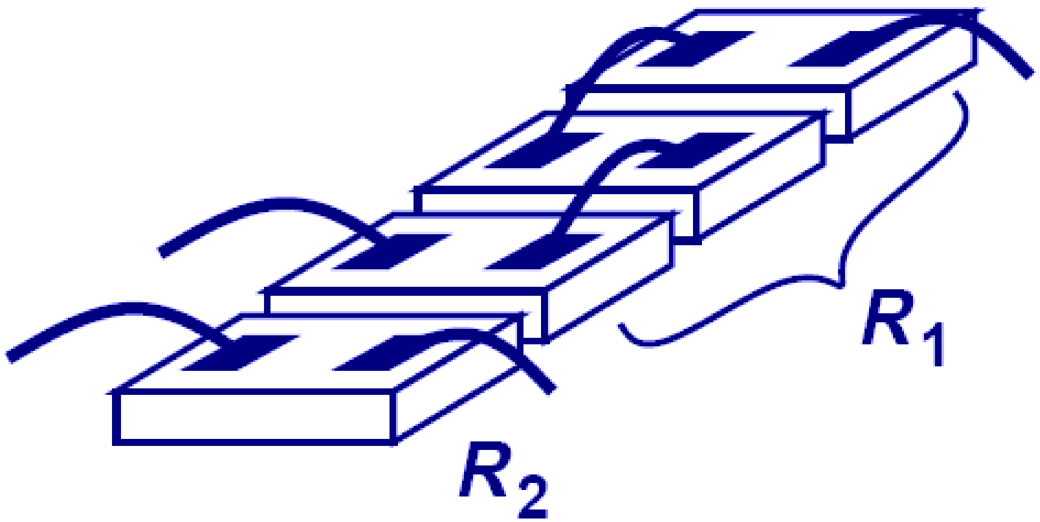


$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$

$$A_1 K >> 1 \Rightarrow \frac{Y}{X} \approx \frac{1}{K}$$

- A large loop gain is needed to create a precise gain, one that does not depend on A_1, which can vary by ±20% with process and temperature variations.
- Can we make a feedback factor K with low variations?

Ratio of Resistors



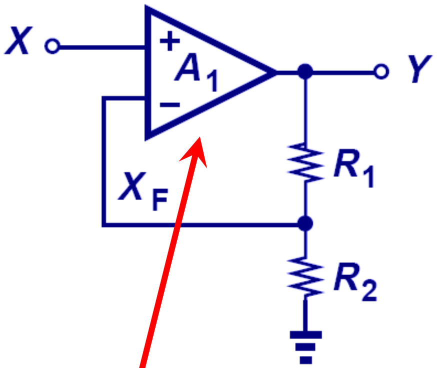
- **When two resistors are composed of the same unit resistor, their ratio is very accurate. Since when they vary, they will vary together and maintain a constant ratio.**
- **Consider the previous circuit**

$$\frac{1}{K}=\frac{R_2+R_1}{R_2}\xrightarrow{\text{w/ variations }(\alpha)}\frac{\alpha R_2+\alpha R_1}{\alpha R_2}=\frac{R_2+R_1}{R_2}\quad\text{(ideally not changed)}$$

Merits of Negative Feedback

- Bandwidth enhancement
- Modification of I/O impedances
  - Reduced sensitivity to load impedance
- Linearization

Bandwidth Enhancement



Closed Loop

$$\frac{Y}{X}(s)=\frac{\dfrac{A_0}{1+KA_0}}{1+\dfrac{s}{(1+KA_0)\omega_0}}$$

Open Loop

$$A_1(s)=\frac{A_0}{1+\dfrac{s}{\omega_0}}$$

Negative Feedback

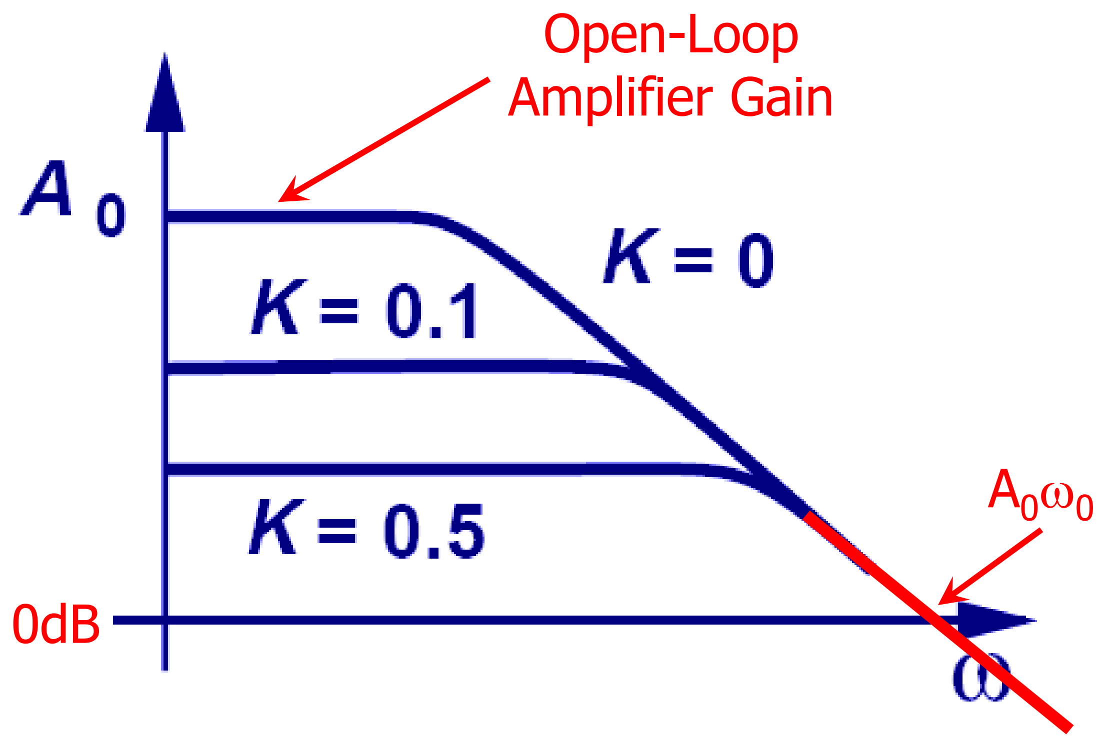
$$\text{Closed-Loop "DC" Gain}=\frac{A_0}{1+KA_0}$$

$$\text{Closed-Loop Bandwidth}=(1+KA_0)\omega_0$$

$$\text{Constant Gain-Bandwith Product (GBW)}=A_0\omega_0$$

- **Although negative feedback lowers the gain by (1+KA$_0$), it also extends the bandwidth by the same amount.**

Bandwidth Extension Example



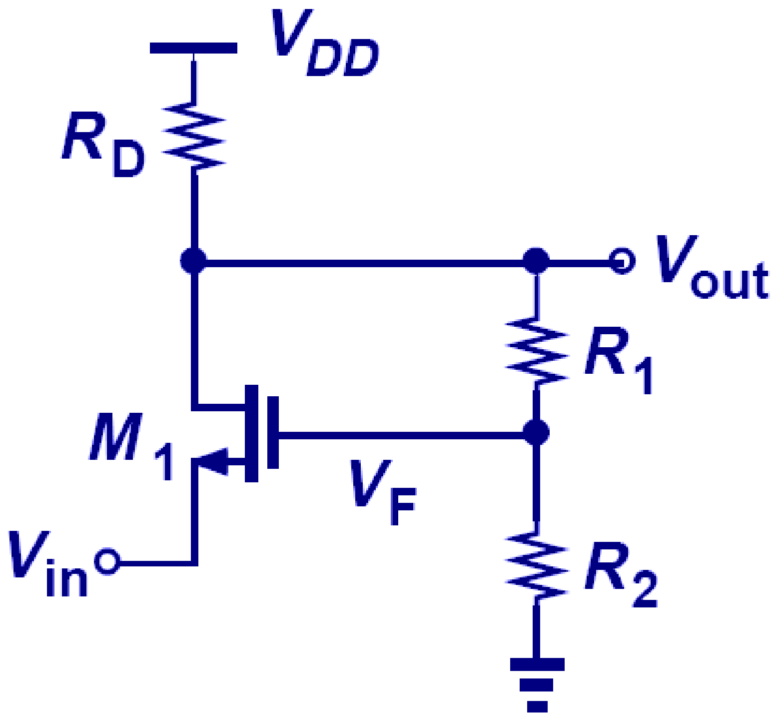
- **As the loop gain increases, we can see the decrease of the overall gain and the extension of the bandwidth.**

Example: Open Loop Parameters

Assume:

$\lambda = 0$

$R_1 + R_2 >> R_D$

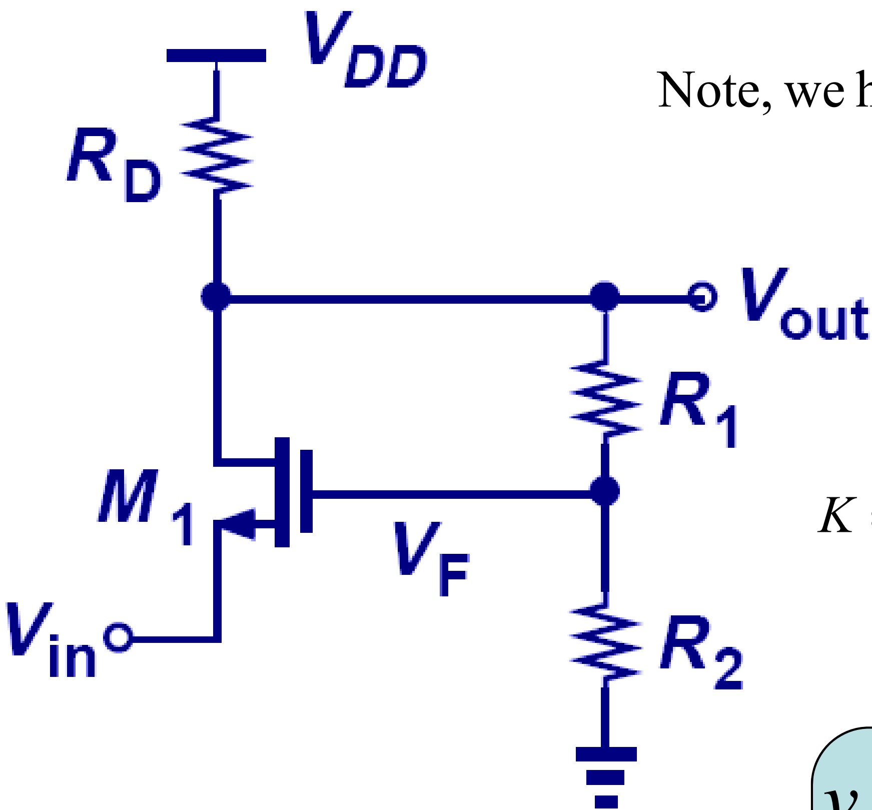


$$A_0 \approx g_m R_D$$

$$R_{in} = \frac{1}{g_m}$$

$$R_{out} = R_D$$

Example: Closed Loop Voltage Gain



Note, we have negative feedback due to

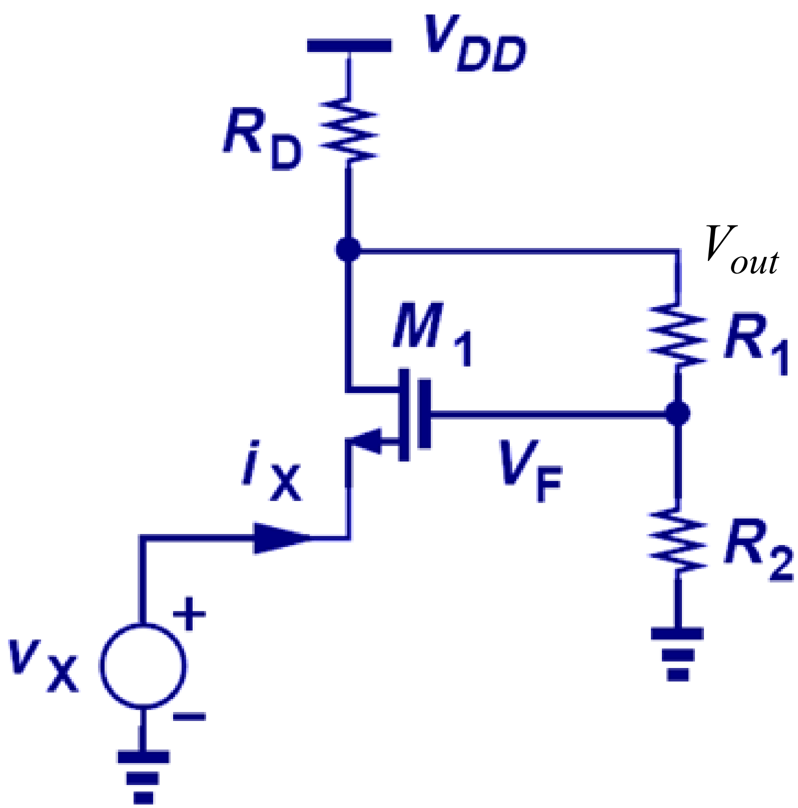
$$i_D = g_m(v_F - v_{in})$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1 + KA_0} = \frac{g_m R_D}{1 + \frac{R_2}{R_1 + R_2} g_m R_D}$$

- Closed-loop gain decreases by $1+KA_0$ factor

Example: Closed Loop I/O Impedance – Input Resistance



Assuming that $R_1 + R_2 >> R_D$

$$v_{out} = i_x R_D$$

$$v_F = \frac{i_x R_D R_2}{R_1 + R_2}$$

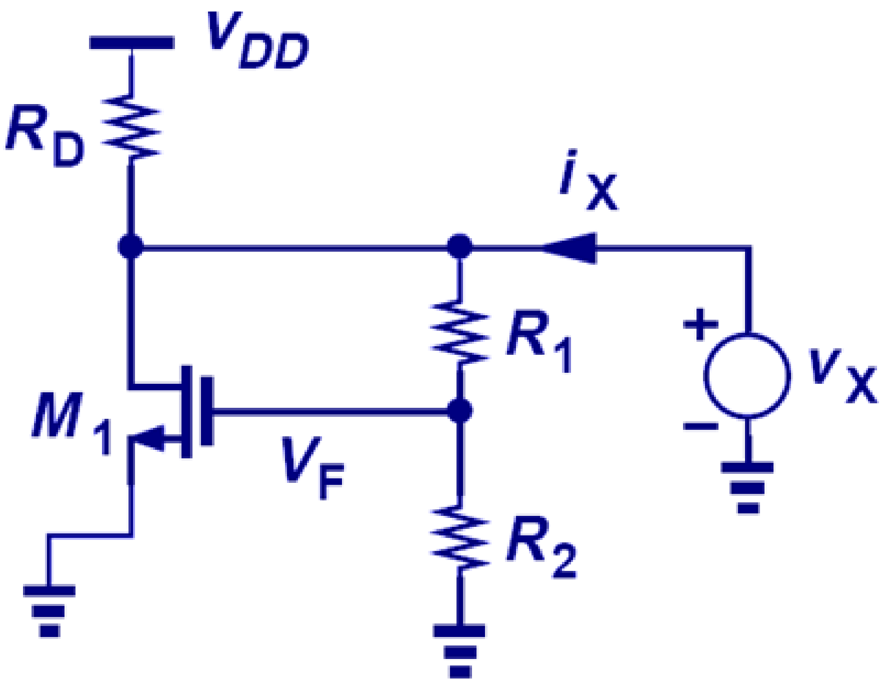
$$i_X = -g_{m1}(v_F - v_x) = g_{m1}\left(v_x - \frac{i_x R_D R_2}{R_1 + R_2}\right)$$

$$i_X\left(1 + \frac{g_{m1} R_D R_2}{R_1 + R_2}\right) = g_{m1} v_x$$

- Input resistance increases by $1+KA_0$ factor
  - Same factor as the closed-loop gain decreases

$$R_{in} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right)$$

Example: Closed Loop I/O Impedance – Output Resistance



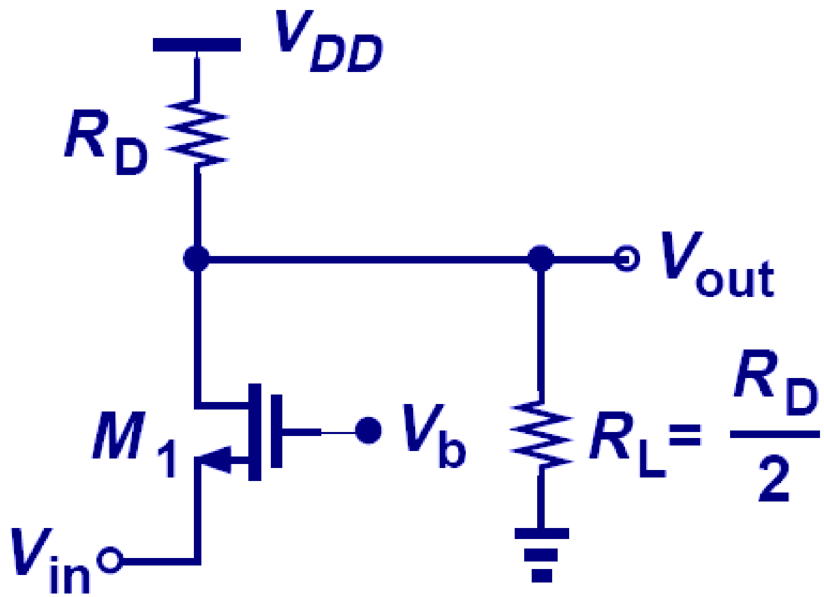
Assuming that $R_1 + R_2 >> R_D$

$$i_x = g_{m1} v_F + \frac{v_X}{R_D} = g_{m1}\left(\frac{v_X R_2}{R_1 + R_2}\right) + \frac{v_X}{R_D}$$

$$R_{out} = \frac{R_D}{1 + \dfrac{R_2}{R_1 + R_2} g_m R_D}$$

- Output resistance decreases by $1+KA_0$ factor
  - Same factor as the closed-loop gain decreases

Example: Load Desensitization



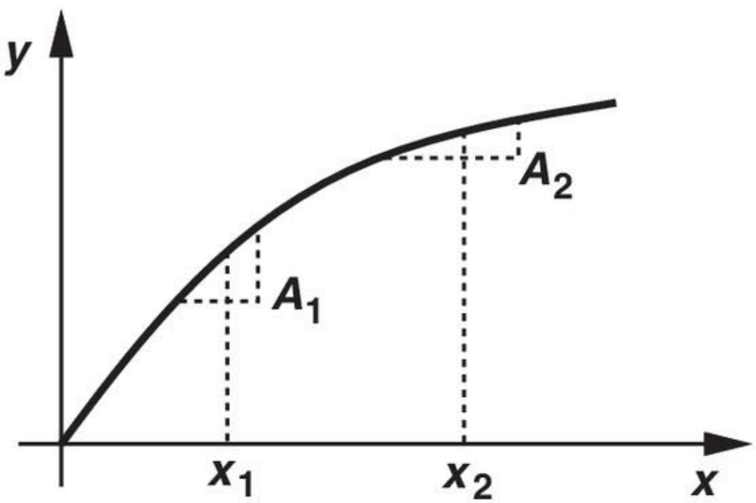
W/O Feedback
Large Difference

$$g_m R_D \rightarrow g_m R_D / 3$$

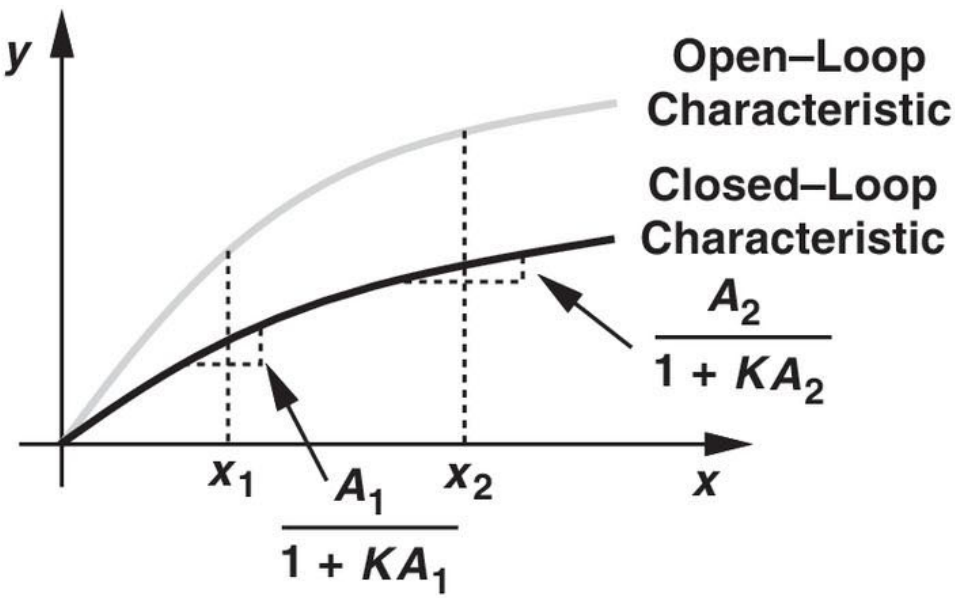
With Feedback
Small Difference

$$\frac{g_m R_D}{1 + \frac{R_2}{R_1 + R_2} g_m R_D} \rightarrow \frac{g_m R_D}{3 + \frac{R_2}{R_1 + R_2} g_m R_D}$$

Linearization



(a)



(b)

- Significant distortion with large input signal
  - $A_2 << A_1$

$$\text{Gain at } x_1 = \frac{A_1}{1+KA_1} \approx \frac{1}{K}\left(1-\frac{1}{KA_1}\right)$$

$$\text{Gain at } x_2 = \frac{A_2}{1+KA_2} \approx \frac{1}{K}\left(1-\frac{1}{KA_2}\right)$$

- If KA_1 and KA_2 remain large, overall gain is ~ 1/K

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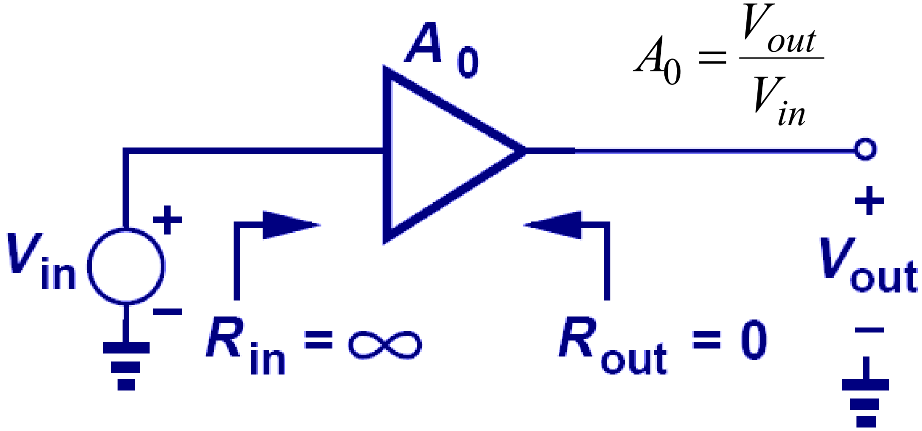
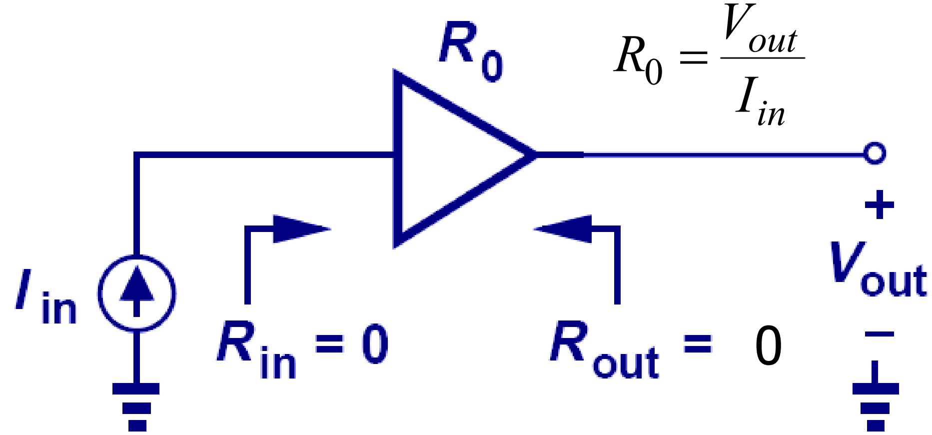
Four Types of Amplifiers

Voltage Amplifier



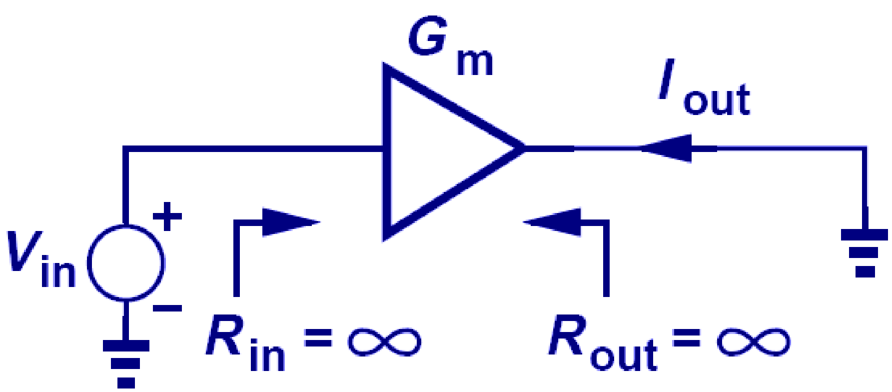
$$A_0 = \frac{V_{out}}{V_{in}}$$

Transimpedance Amplifier



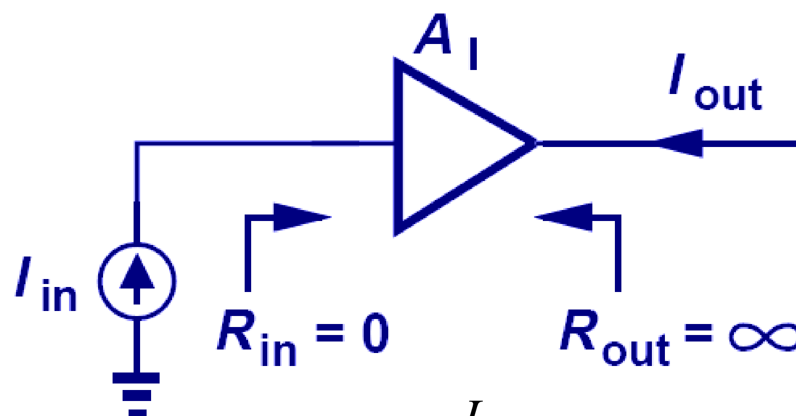
$$R_0 = \frac{V_{out}}{I_{in}}$$

Transconductance Amplifier



$$G_m = \frac{I_{out}}{V_{in}}$$

Current Amplifier

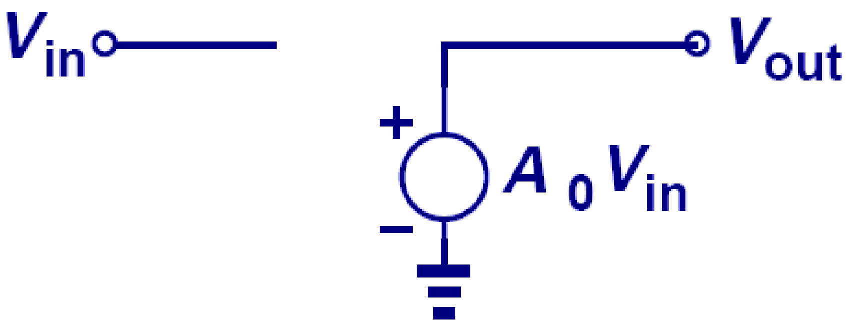


$$A_I = \frac{I_{out}}{I_{in}}$$

Ideal Models of the Four Amplifier Types

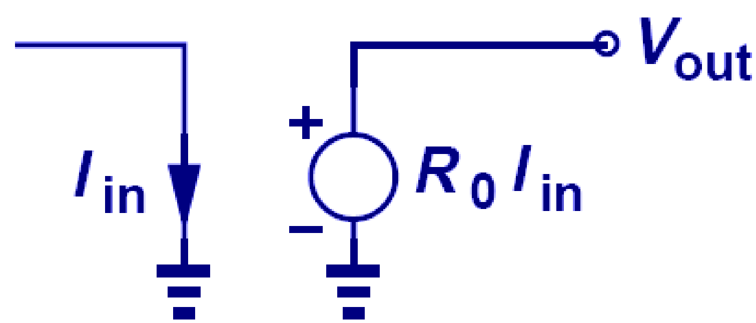
Voltage Amplifier

$$A_0 = \frac{V_{out}}{V_{in}}$$

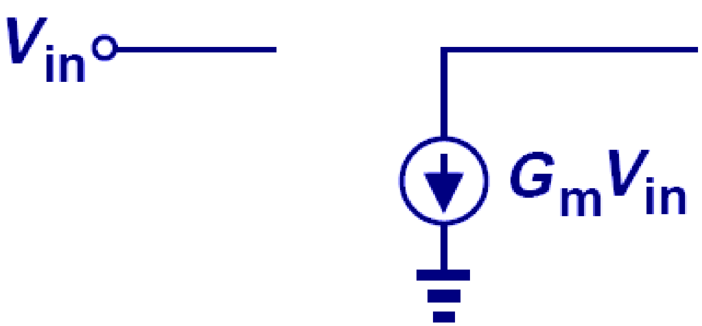


Transimpedance Amplifier

$$R_0 = \frac{V_{out}}{I_{in}}$$

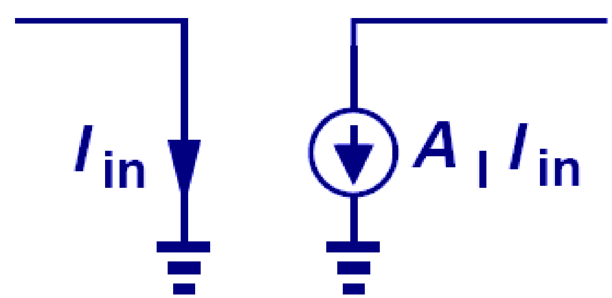


Transconductance Amplifier



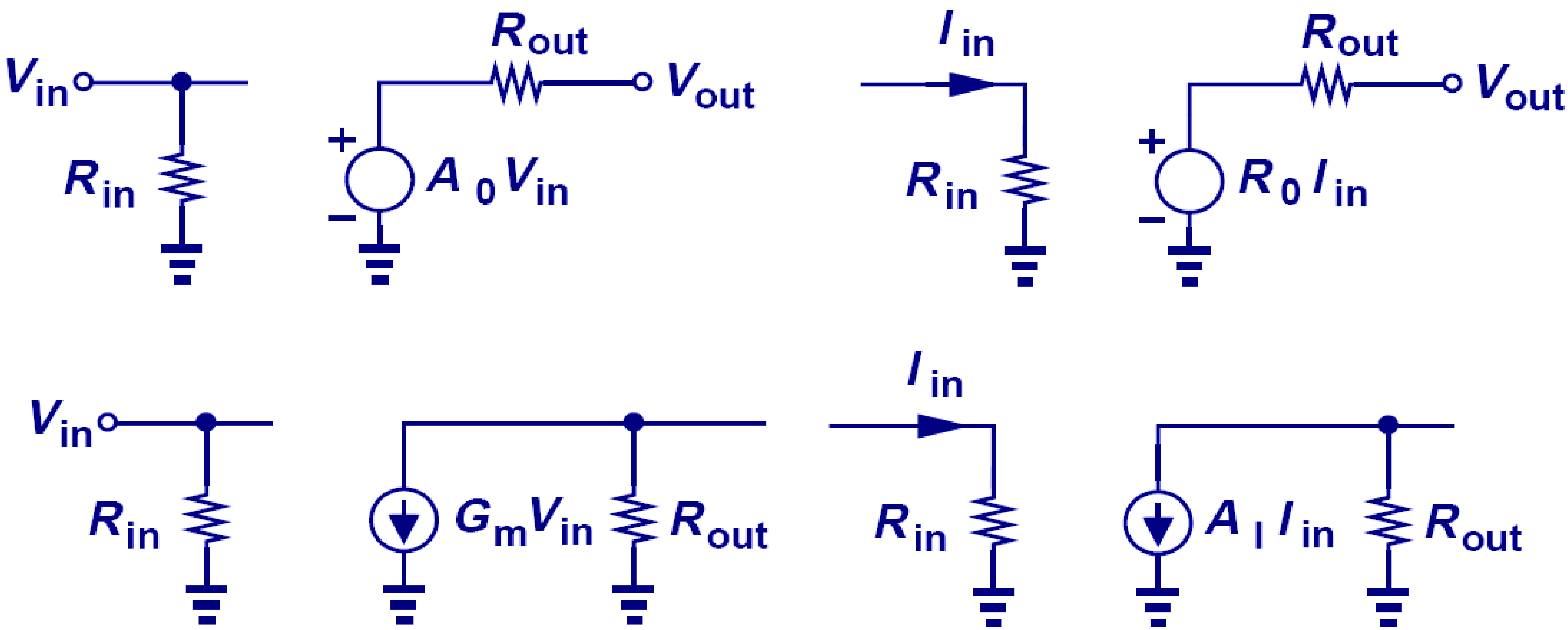
$$G_m = \frac{I_{out}}{V_{in}}$$

Current Amplifier



$$A_I = \frac{I_{out}}{I_{in}}$$

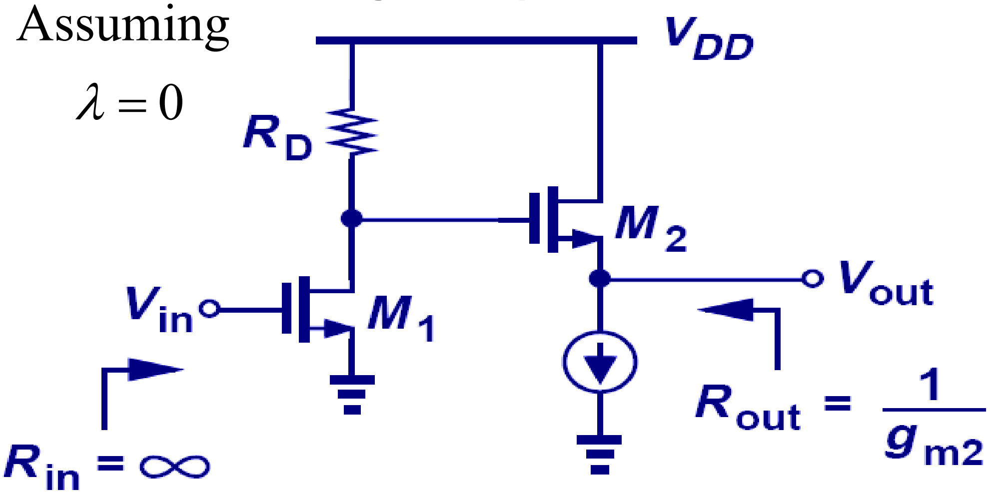
Realistic Models of the Four Amplifier Types



Examples of the Four Amplifier Types

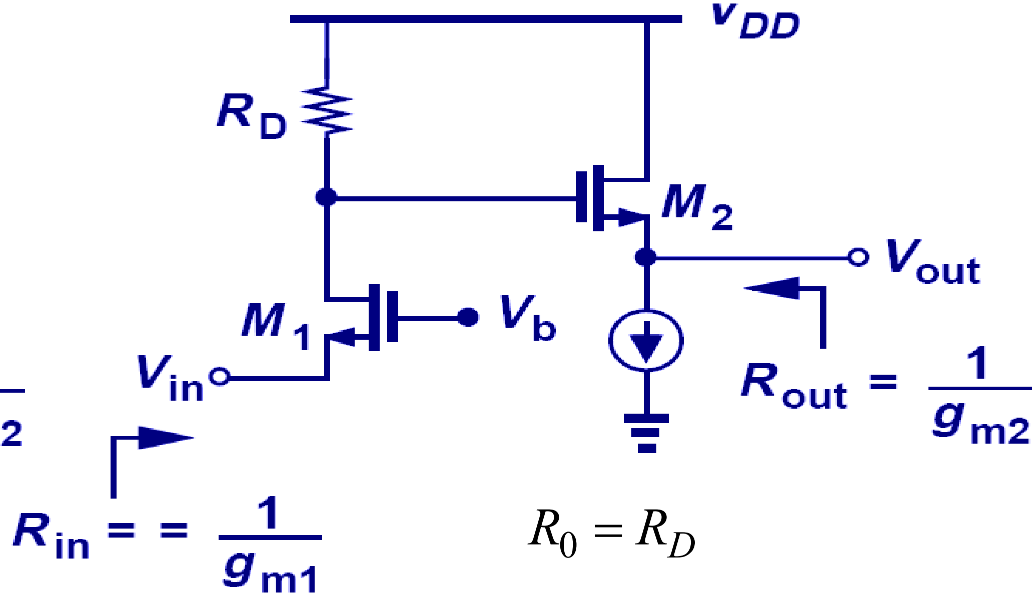
Voltage Amplifier

Assuming $\lambda = 0$



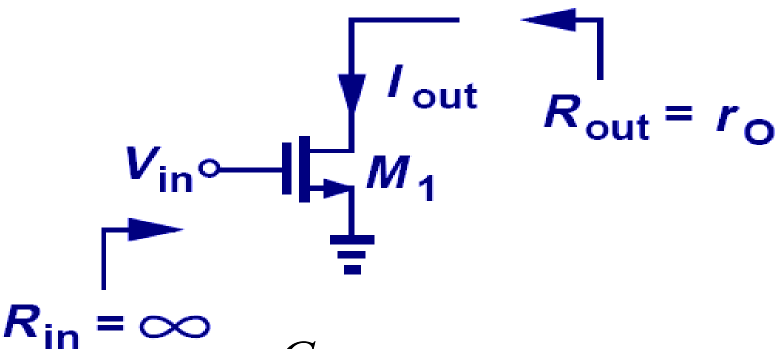
$A_0 = -g_{m1} R_D$

Transimpedance Amplifier



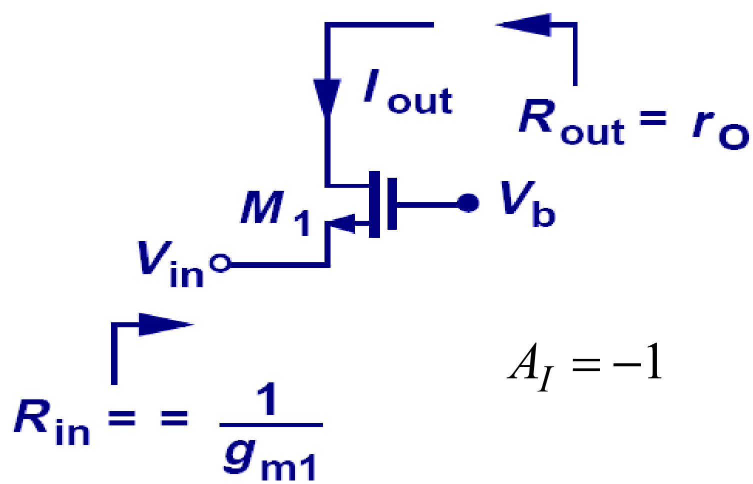
$R_0 = R_D$

Transconductance Amplifier



$G_m = g_{m1}$

Current Amplifier/Buffer



$A_I = -1$

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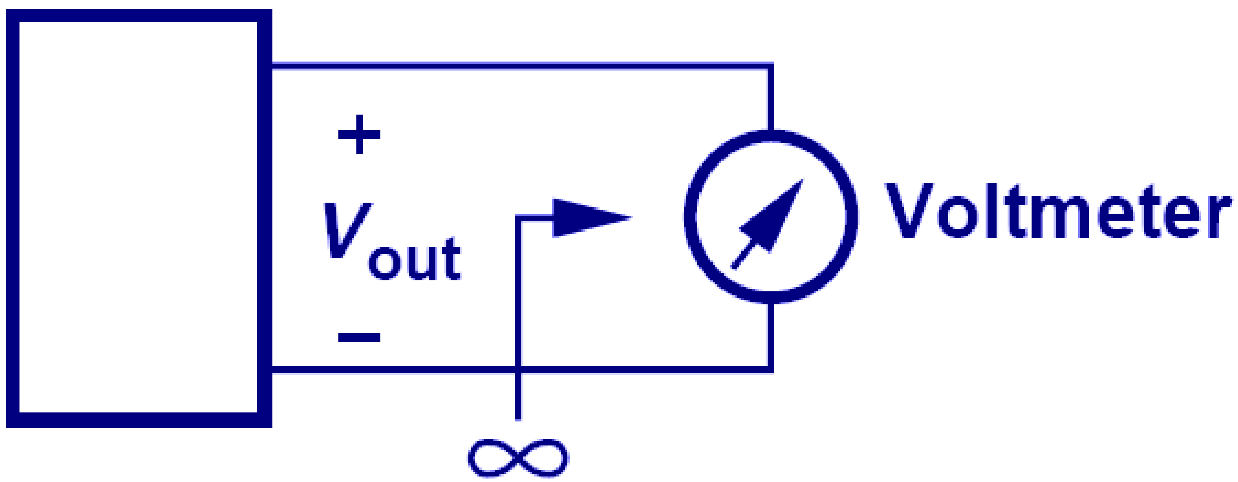
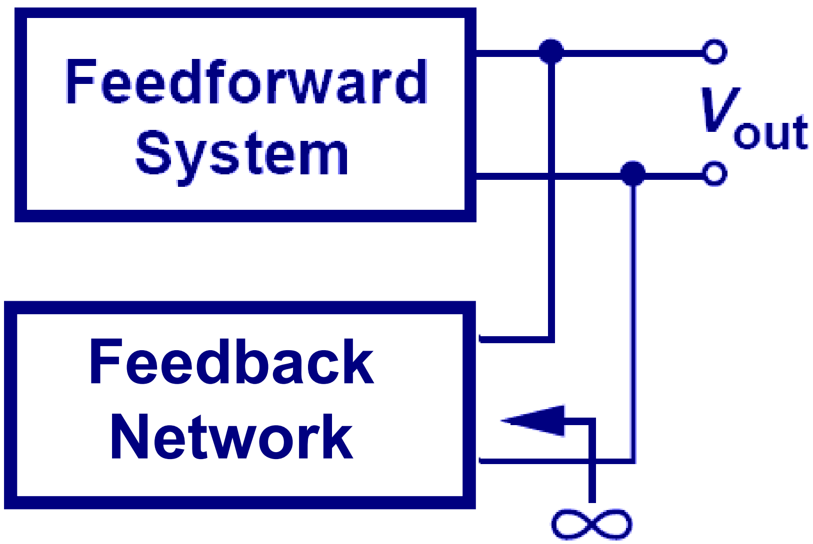
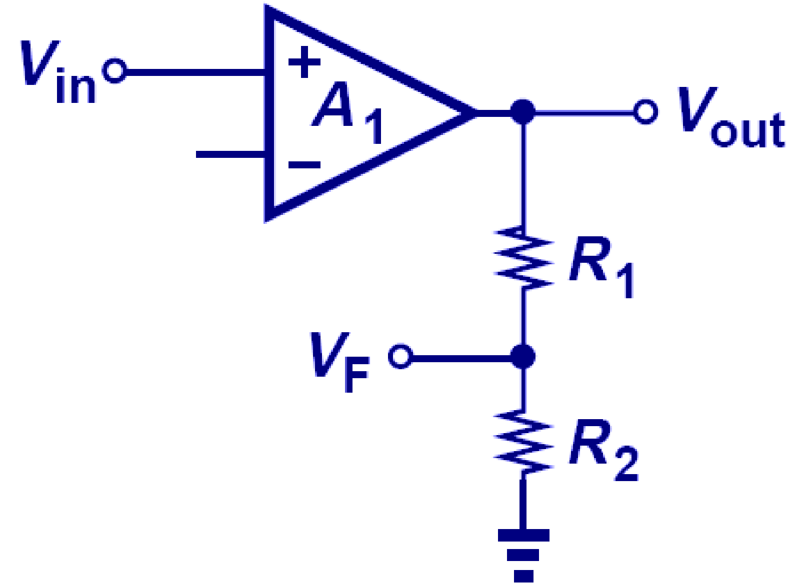
Sensing a Voltage



- In order to sense a voltage across two terminals, a voltmeter with **ideally infinite impedance** is used.

Sensing and Returning a Voltage

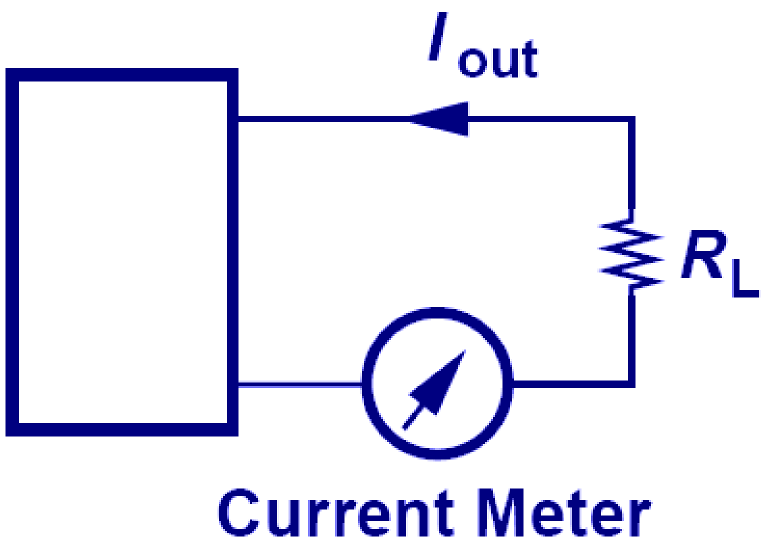


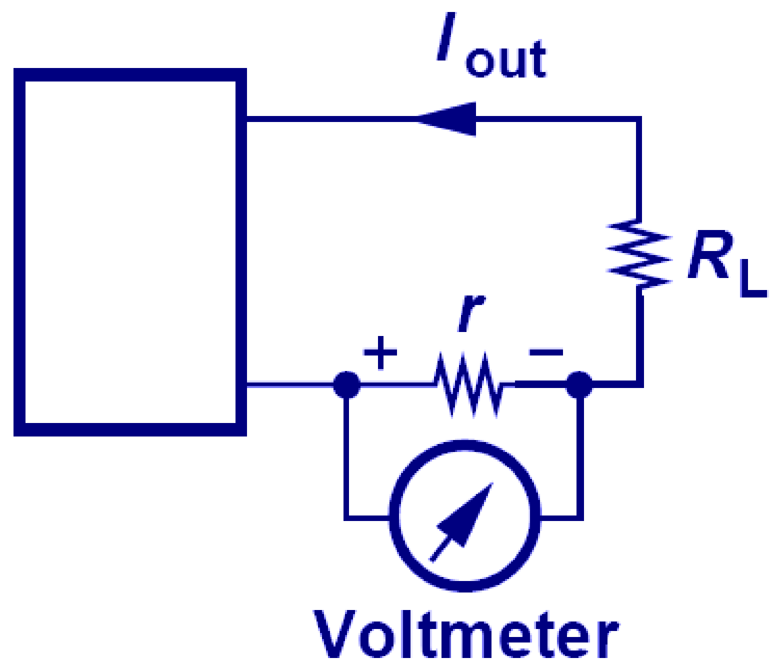


$$R_1 + R_2 \approx \infty$$

- **Similarly, for a feedback network to correctly sense the output voltage, its input impedance needs to be large.**
- **R_1 and R_2 also provide a mean to return the voltage.**

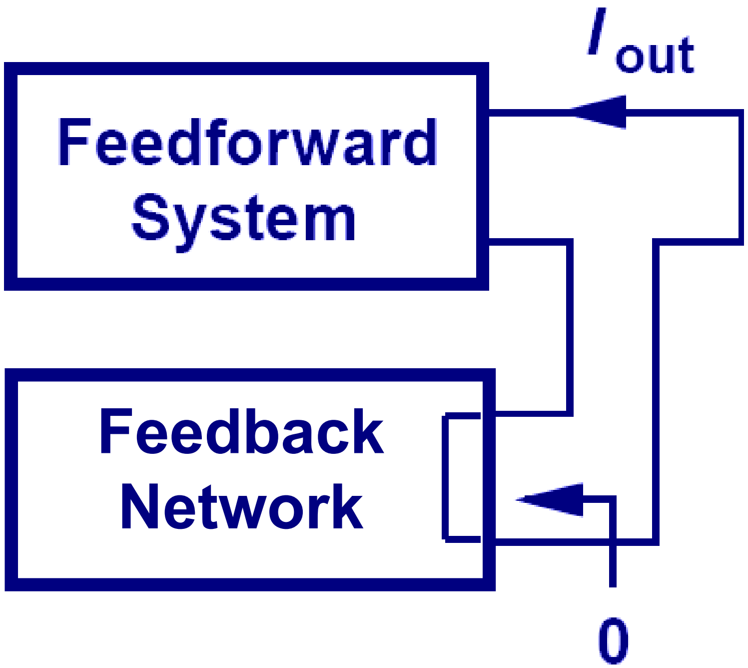
Sensing a Current

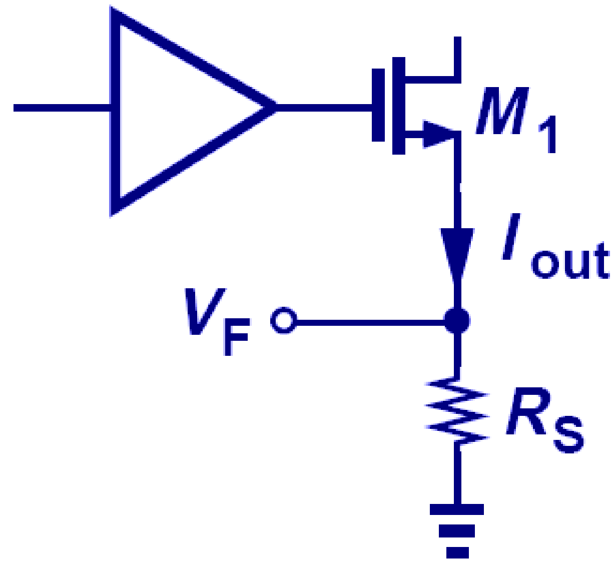




- A current is measured by inserting a current meter with **ideally zero impedance** in series with the conduction path.
- The current meter is composed of a small resistance r in parallel with a voltmeter.

Sensing and Returning a Current

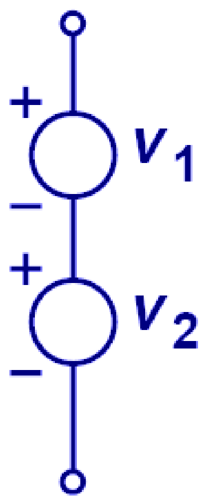




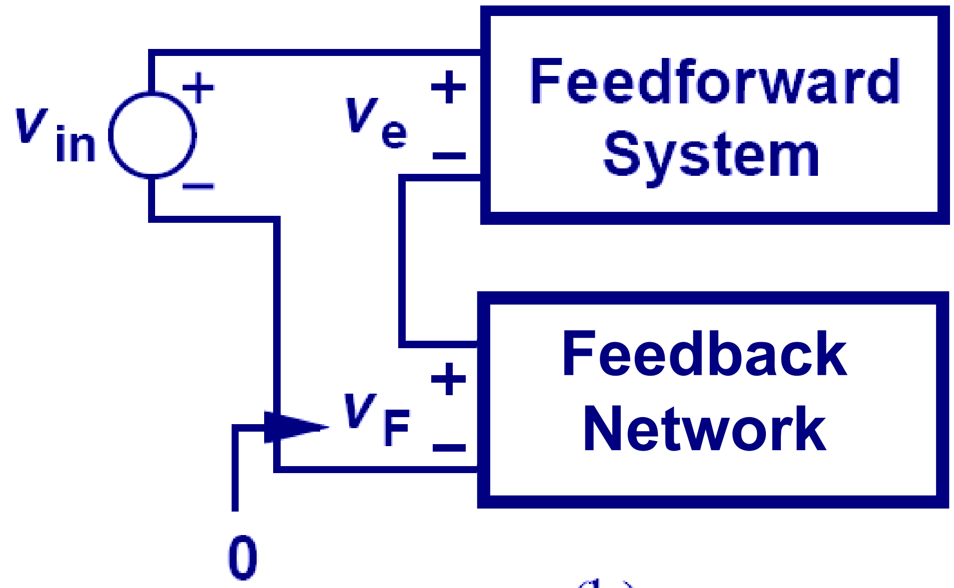
$$R_S \approx 0$$

- Similarly for a feedback network to correctly sense the current, its input impedance has to be small.
- R_S has to be small so that its voltage drop will not change I_{out}.

Addition (Subtraction) of Two Voltage Sources



(a)

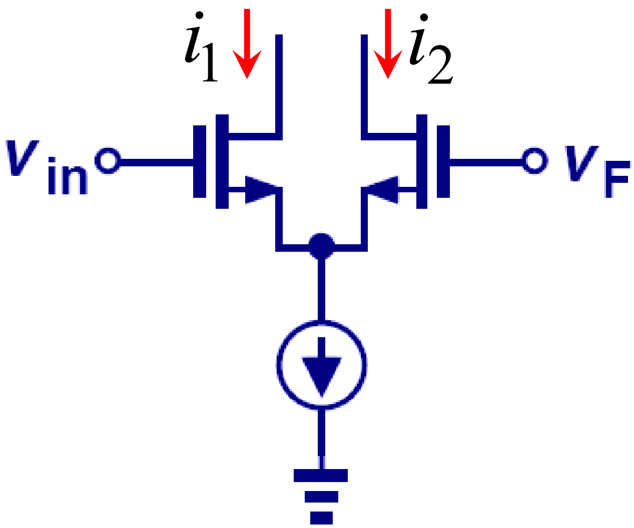


(b)

$$v_e = v_{in} - v_F$$

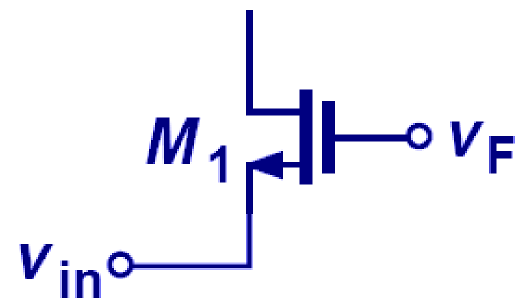
- **In order to add or subtract two voltage sources, we place them in series. So the feedback network is placed in series with the input source.**

Practical Circuits to Subtract Two Voltage Sources



(c)

$$i_1 - i_2 = g_m(v_{in} - v_F)$$

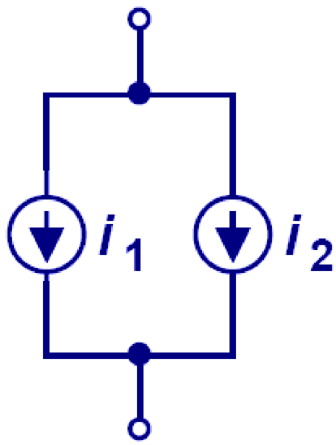


(d)

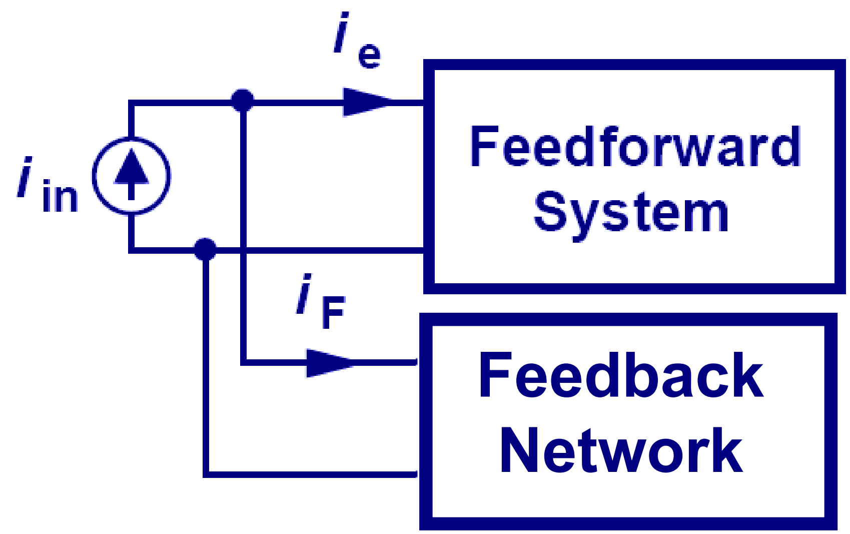
$$i_1 = g_m(v_F - v_{in})$$

- Although not directly in series, V_{in} and V_F are being subtracted since the resultant currents, differential and single-ended, are proportional to the difference of V_{in} and V_F.

Addition (Subtraction) of Two Current Sources



(a)

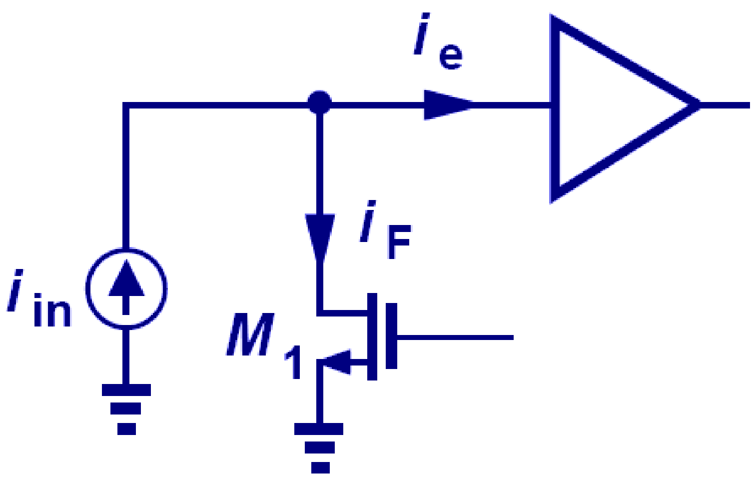


(b)

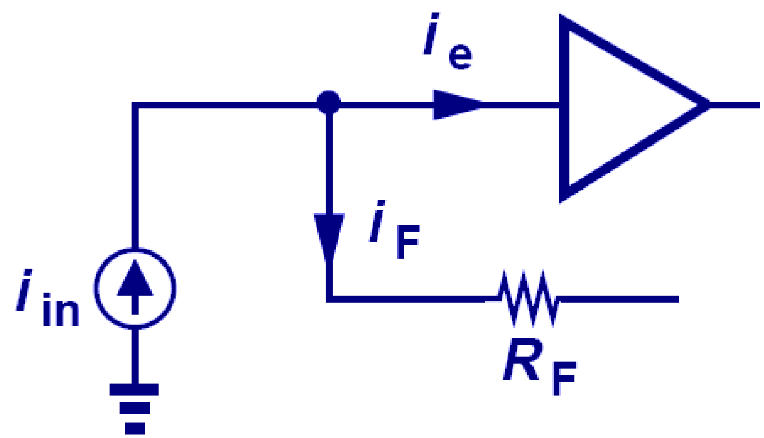
$$i_e = i_{in} - i_F$$

- In order to add two current sources, we place them in parallel. So the feedback network is placed in parallel with the input signal.

Practical Circuits to Subtract Two Current Sources



(c)

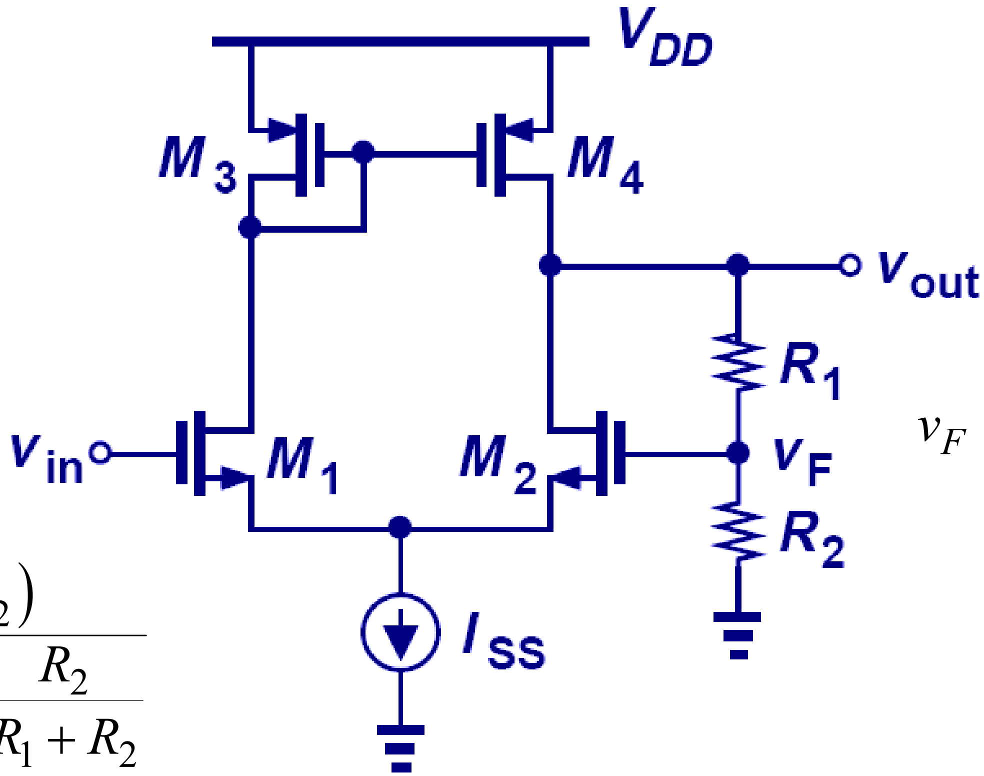


(d)

$$i_e = i_{in} - i_F$$

- Since M_1 and R_F are in parallel with the input current source, their respective currents are being subtracted. Note, R_F has to be large enough to approximate a current source.

Example: Sense a Voltage and Return a Voltage



If $R_1 + R_2$ is large

$$A_0 \approx g_{m1}(r_{o4} \| r_{o2})$$

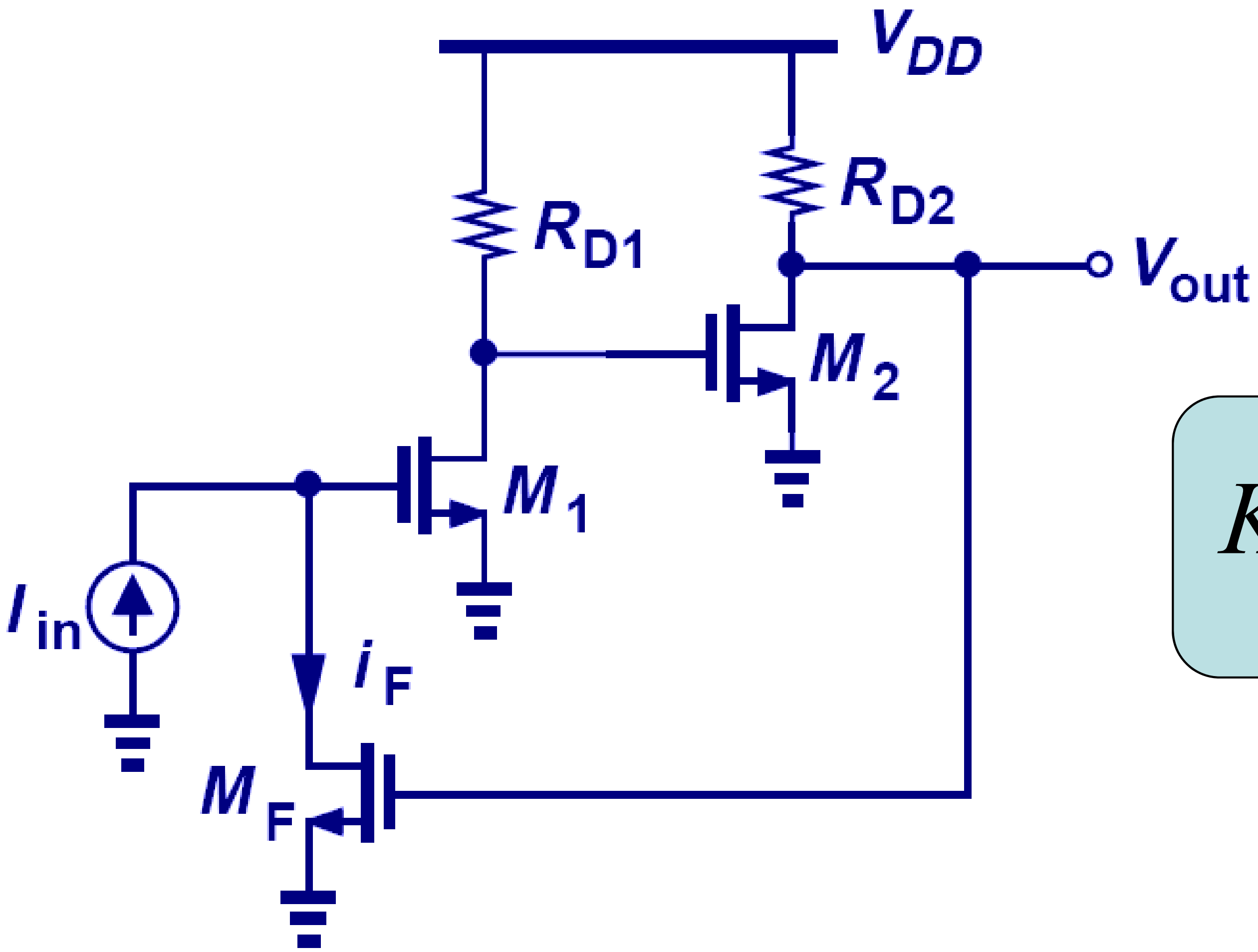
$$K = \frac{R_2}{R_1 + R_2}$$

$$v_F = \frac{R_2}{R_1 + R_2} v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1 + A_0 K} = \frac{g_{m1}(r_{o4} \| r_{o2})}{1 + g_{m1}(r_{o4} \| r_{o2}) \frac{R_2}{R_1 + R_2}}$$

- R_1 and R_2 sense and return the output voltage to feedforward network consisting of M_1- M_4.
- M_1 and M_2 also act as a voltage subtractor.

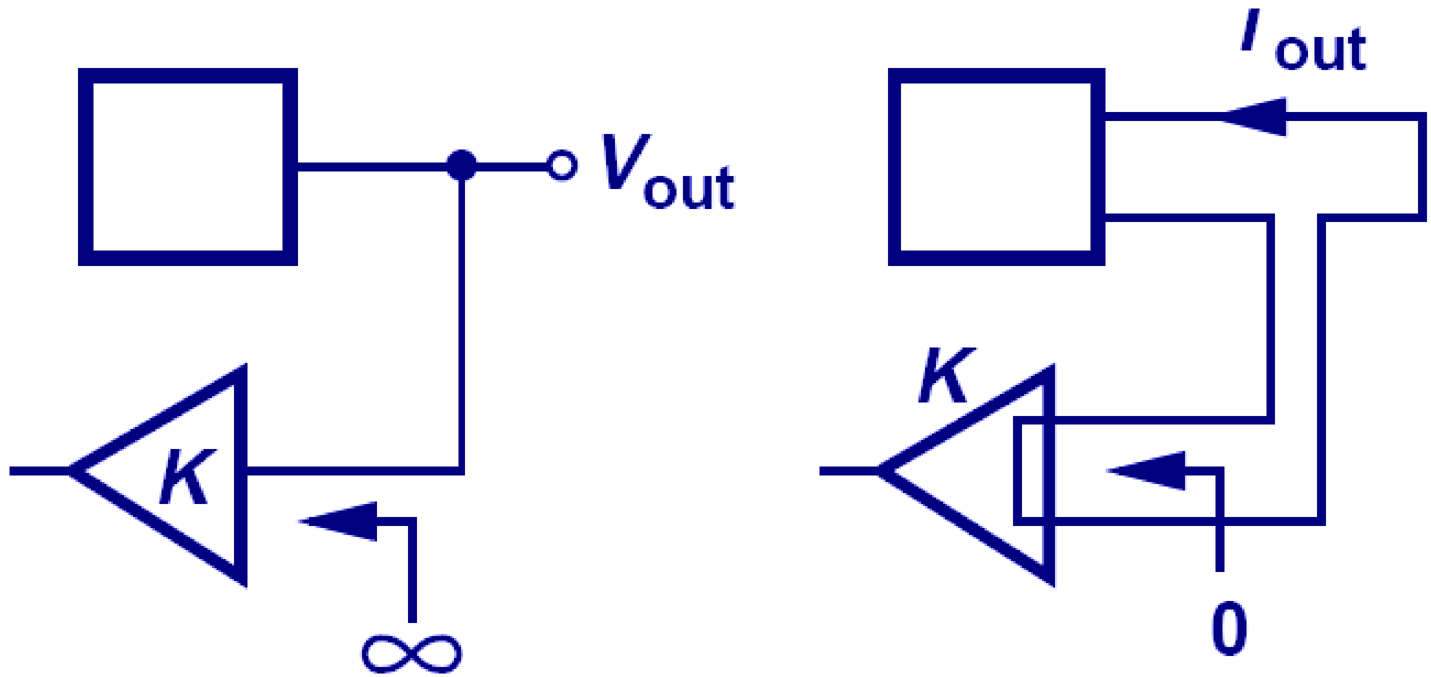
Example: Feedback Factor



$$K = \frac{i_F}{v_{out}} = g_{mF}$$

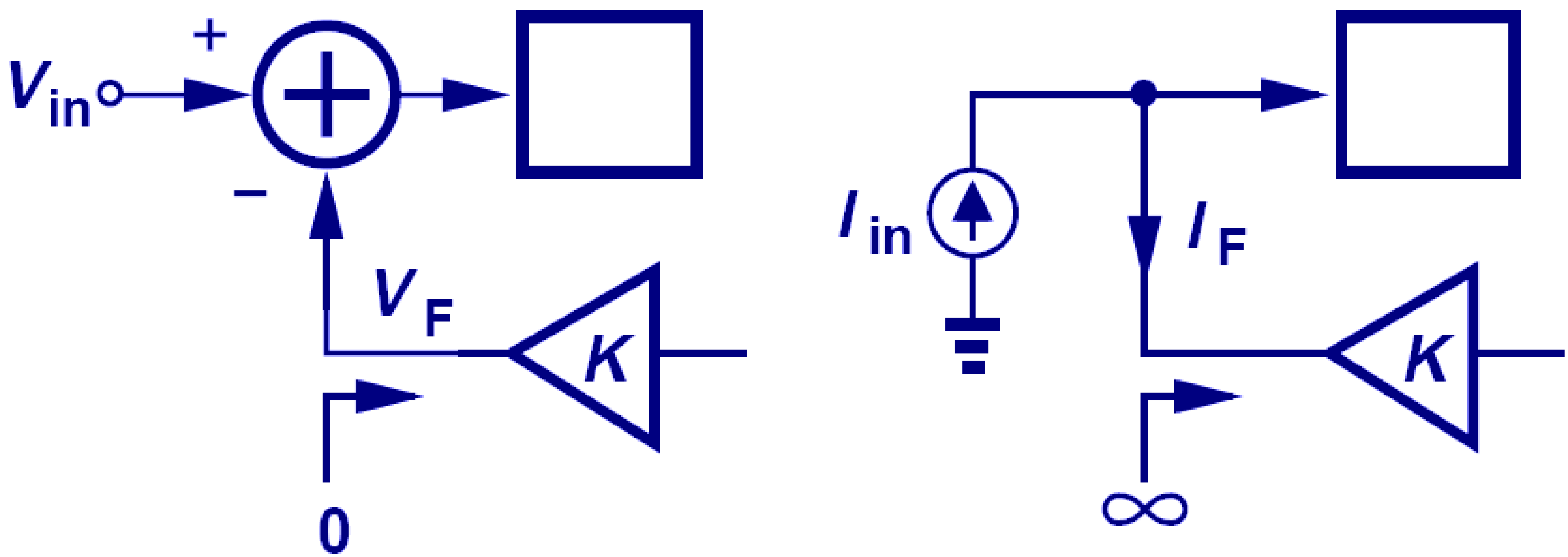
- This circuit senses a voltage and returns a current

Input Impedance of an Ideal Feedback Network



- To sense a voltage, the input impedance of an ideal feedback network must be infinite.
- To sense a current, the input impedance of an ideal feedback network must be zero.

Output Impedance of an Ideal Feedback Network



- To return a voltage, the output impedance of an ideal feedback network must be zero.
- To return a current, the output impedance of an ideal feedback network must be infinite.

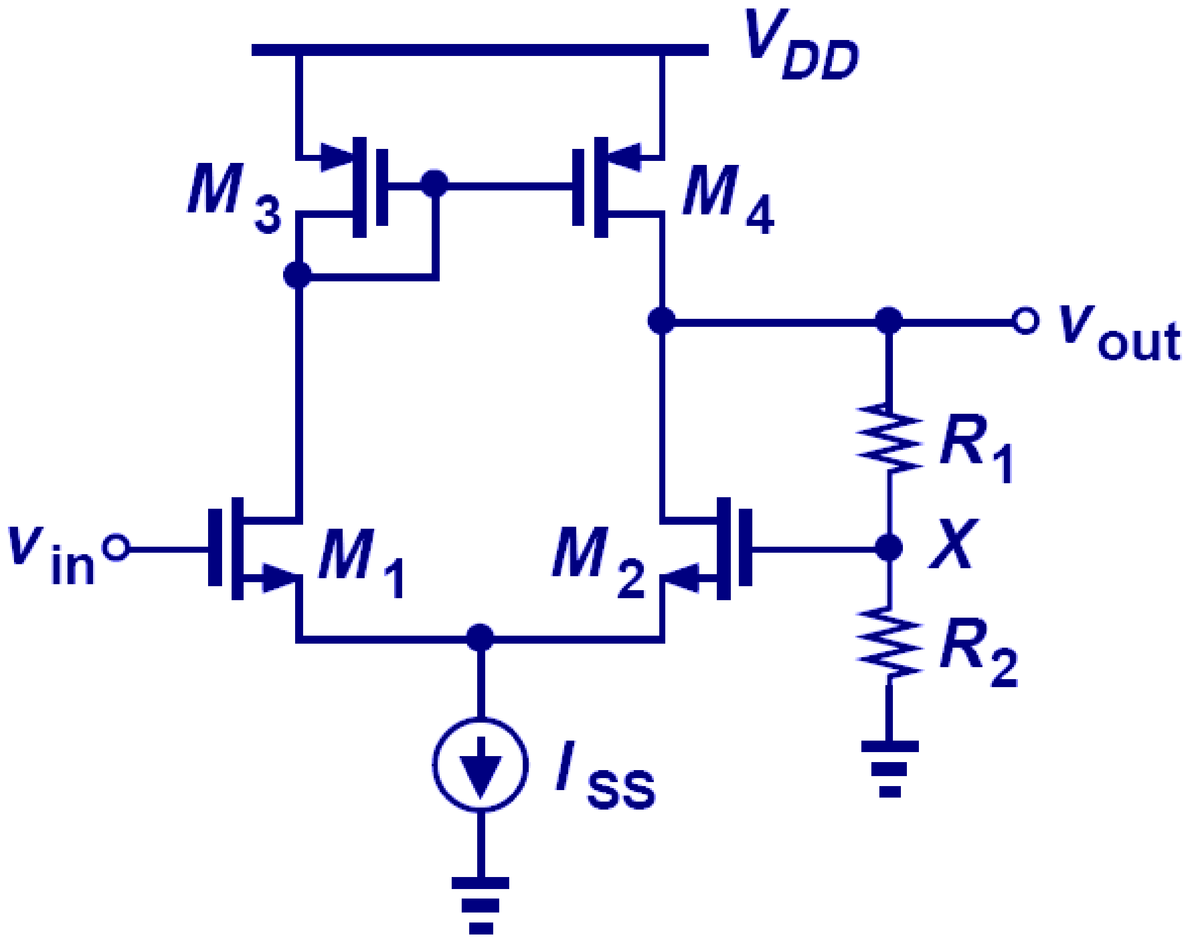
Agenda

- Feedback Overview
- Feedback Properties
- Amplifier Types
- Sense and Return Techniques
- Feedback Polarity
- Feedback Topologies
- Effect of Nonideal I/O Impedances
- Stability
- Two-Stage Miller OTA

Determining the Polarity of Feedback

- 1) Assume the input goes either up or down.
- 2) Follow the signal through the loop.
- 3) Determine whether the returned quantity enhances or opposes the original change.

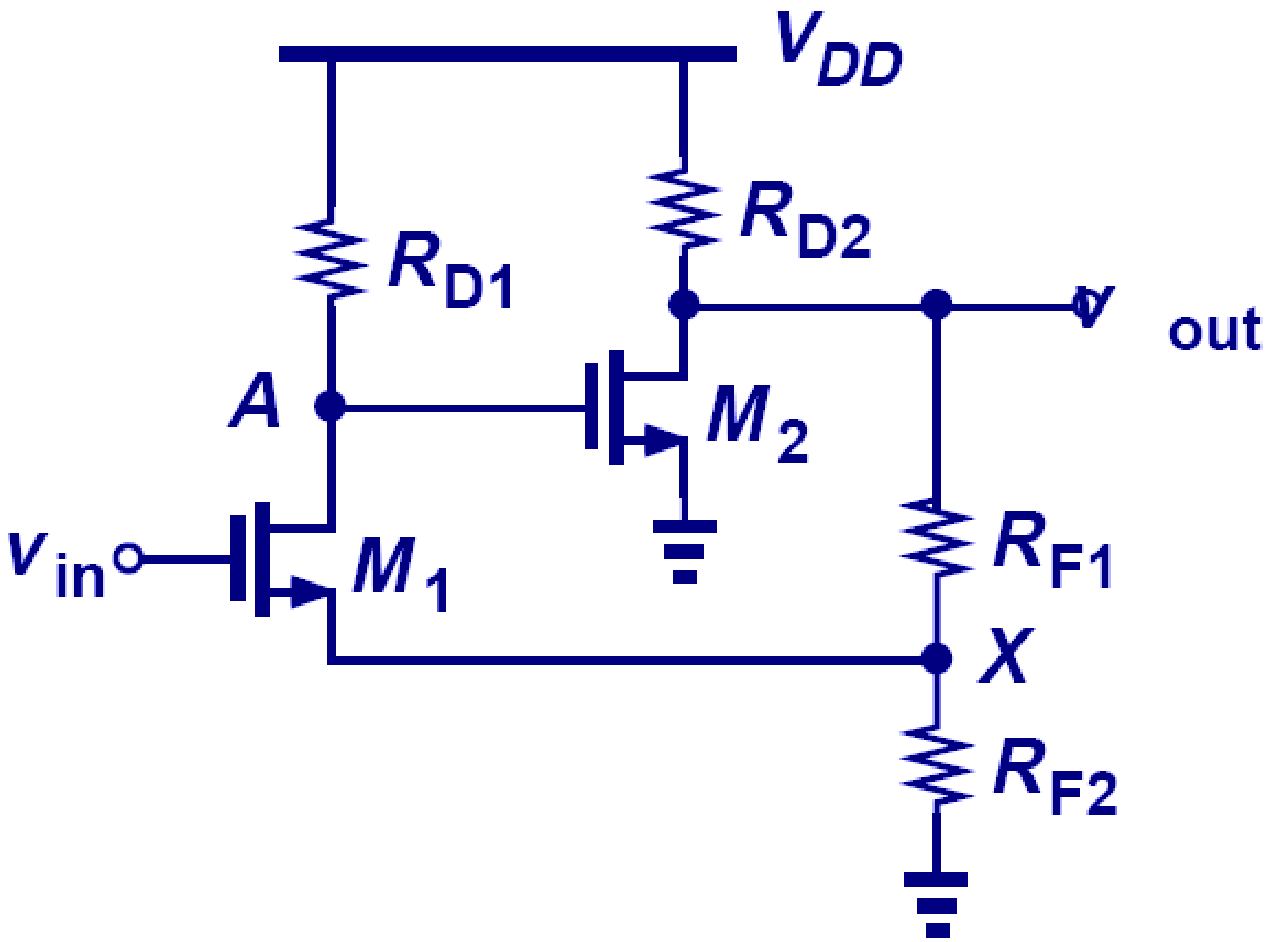
Polarity of Feedback Example I



$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, I_{D2}\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D2}\uparrow, I_{D1}\downarrow$$

Negative Feedback

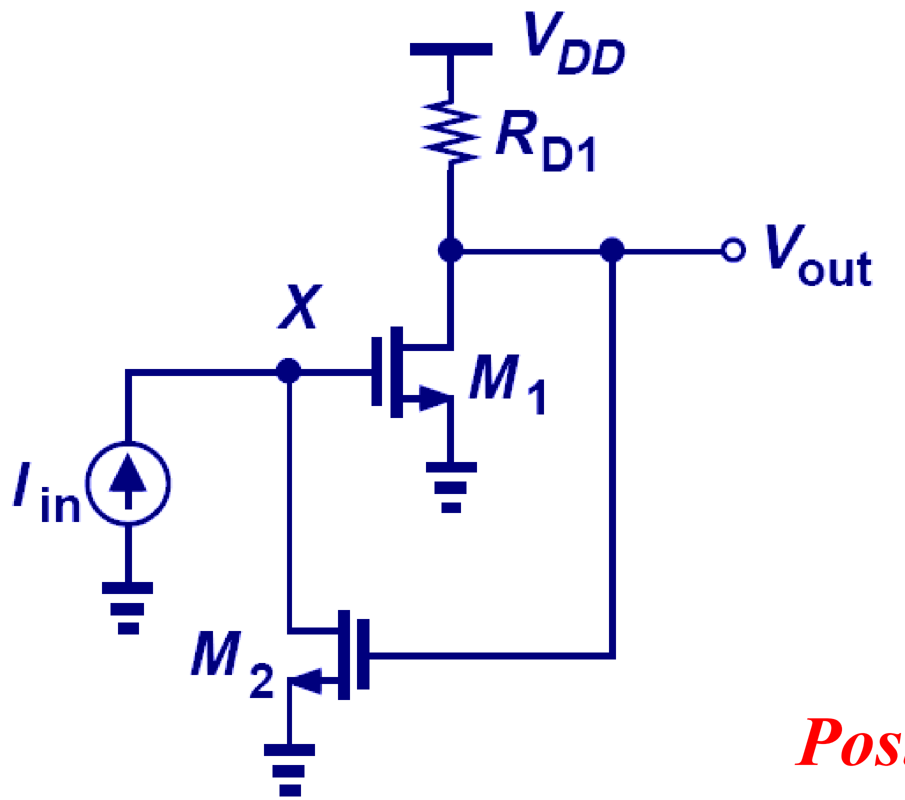
Polarity of Feedback Example II



$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_A\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D1}\downarrow, V_A\uparrow$$

Negative Feedback

Polarity of Feedback Example III



Positive Feedback

$$I_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_X\uparrow \Rightarrow V_{out}\downarrow, I_{D2}\downarrow \Rightarrow I_{D1}\uparrow, V_X\uparrow$$

- If we are trying to build a linear amplifier, positive feedback is bad
- Circuit can latch up or oscillate

Agenda

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- Feedback Properties
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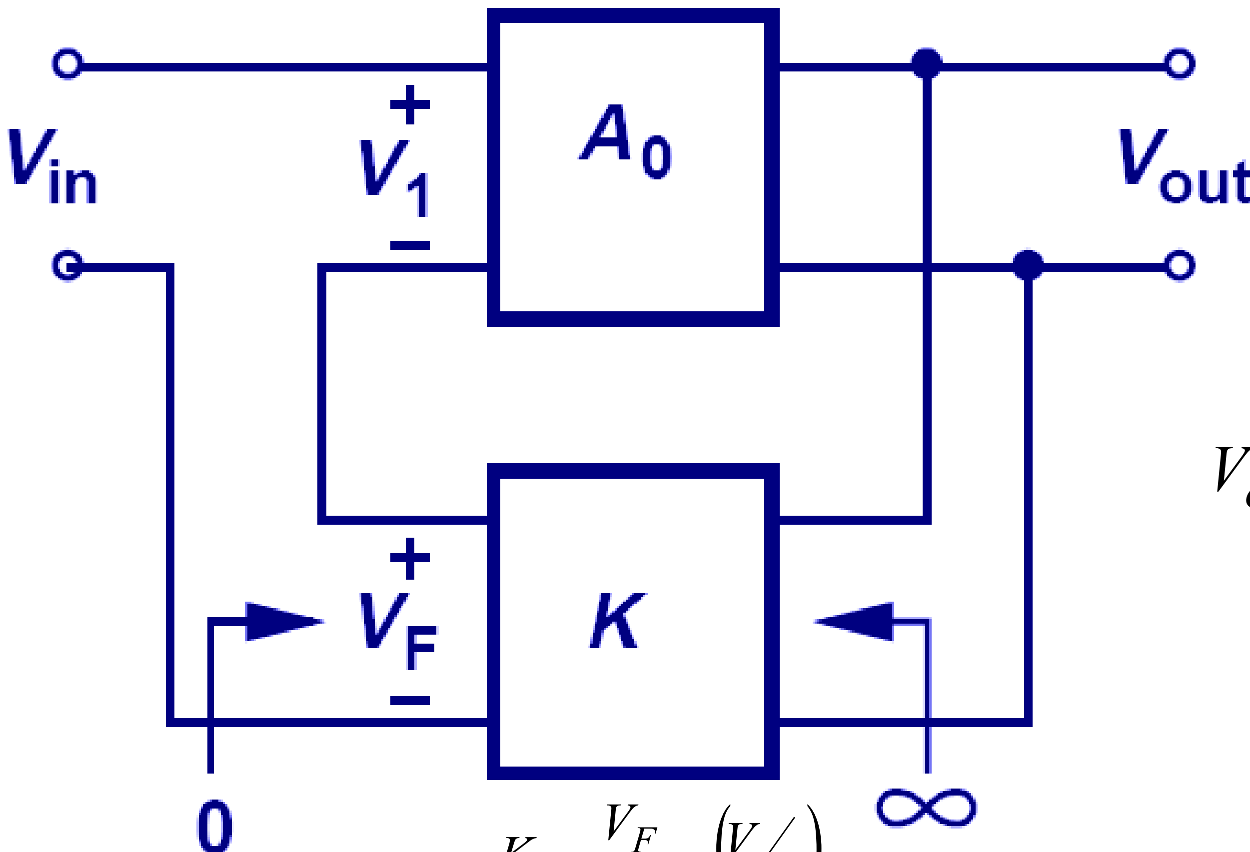
Voltage-Voltage Feedback

- A voltage amplifier requires sensing of the output voltage to produce a feedback voltage
- Output voltage is sensed in parallel and feedback voltage applied in series with the input



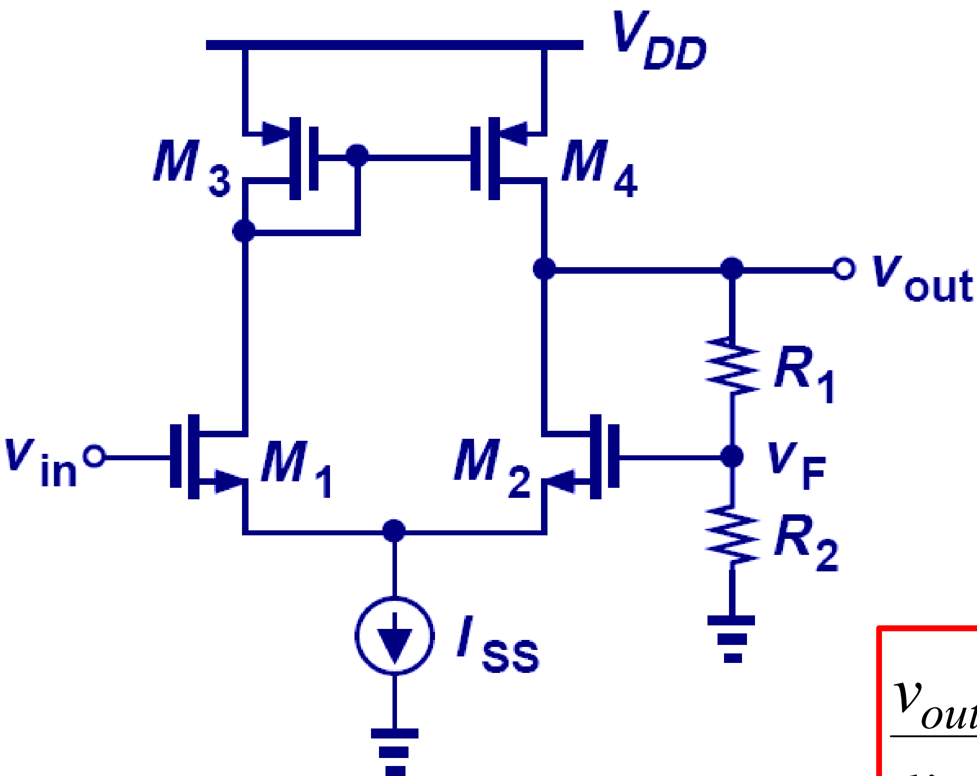
$$V_1 = V_{in} - V_F = V_{in} - KV_{out}$$

$$V_{out} = A_0 V_1 = A_0 \left(V_{in} - KV_{out}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + KA_0}$$

$$K = \frac{V_F}{V_{out}} \quad \left(V/V\right)$$

Example: Voltage-Voltage Feedback



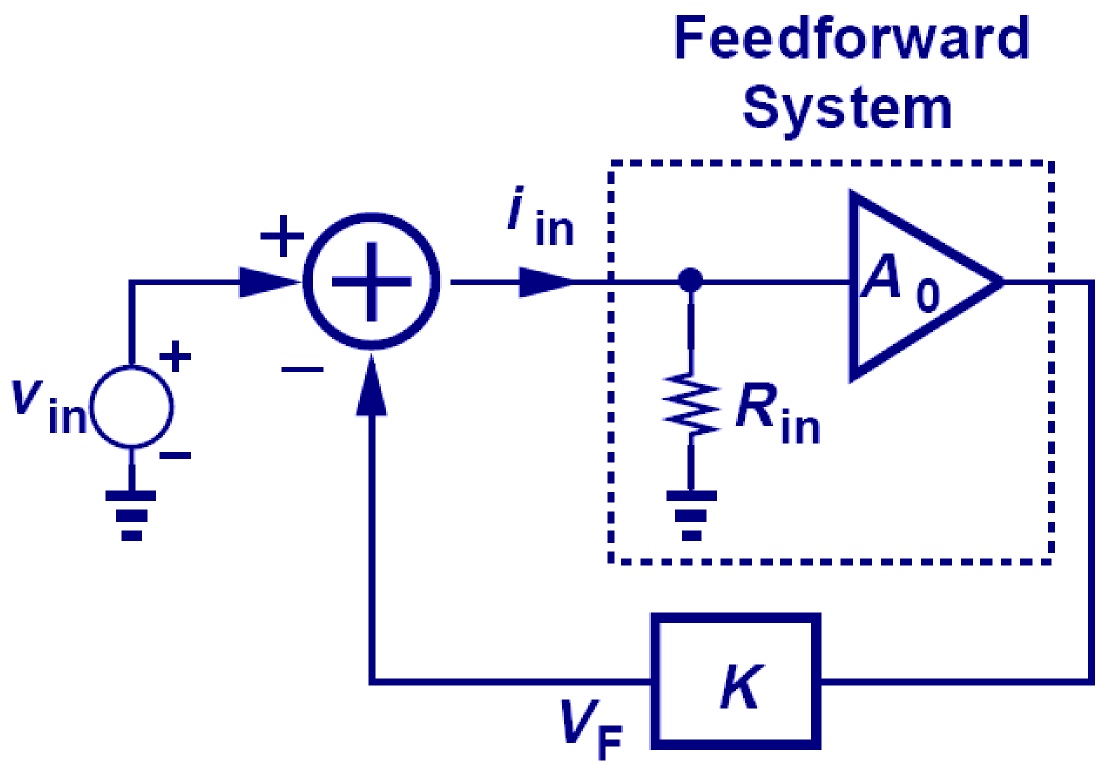
$$\text{If } R_1 + R_2 \text{ is large } (>> r_{o4} \| r_{o2})$$

$$A_0 \approx g_{m1}\left(r_{o4} \| r_{o2}\right)$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1 + A_0 K} = \frac{g_{m1}\left(r_{o4} \| r_{o2}\right)}{1 + g_{m1}\left(r_{o4} \| r_{o2}\right)\frac{R_2}{R_1 + R_2}}$$

Input Impedance of a V-V Feedback

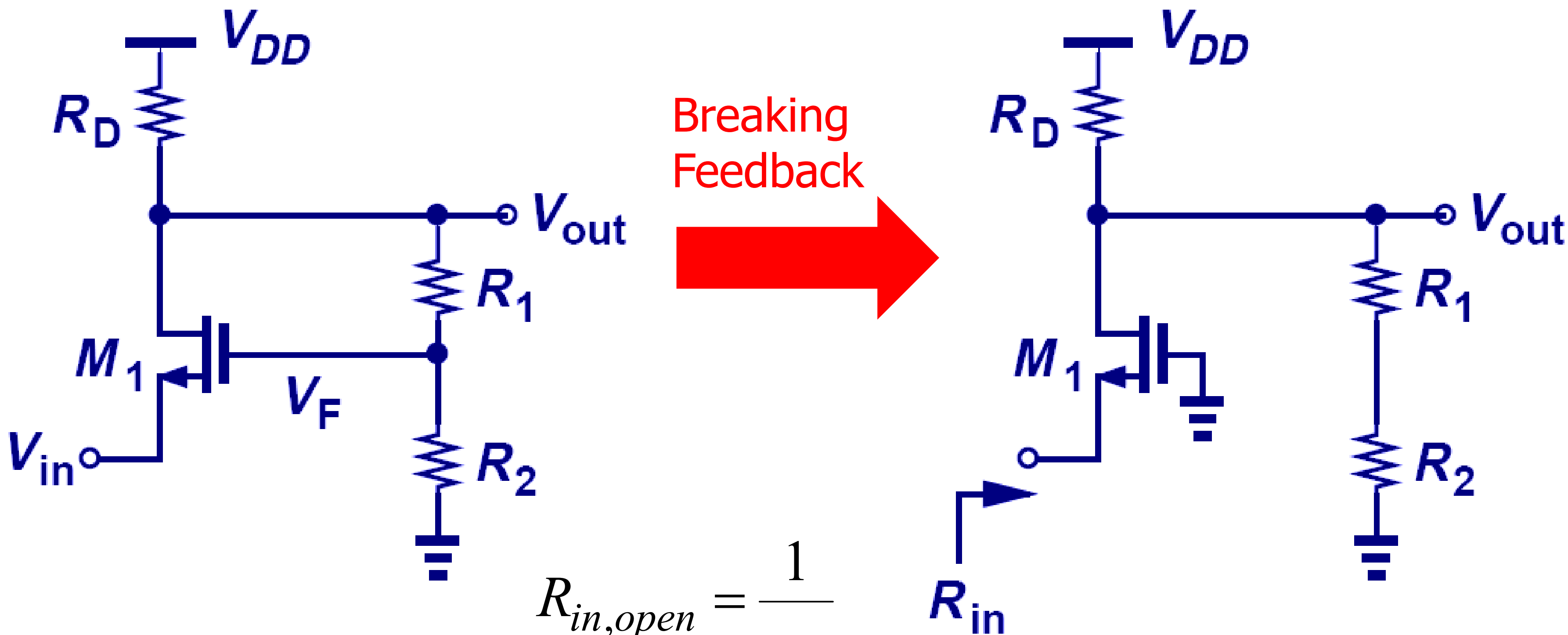


$$I_{in}R_{in} = V_{in} - V_F = V_{in} - (I_{in}R_{in})A_0K$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + A_0K)$$

- **A better input voltage sensor, as the input impedance increases by $1+A_0K$**

Example: V-V Feedback Input Impedance



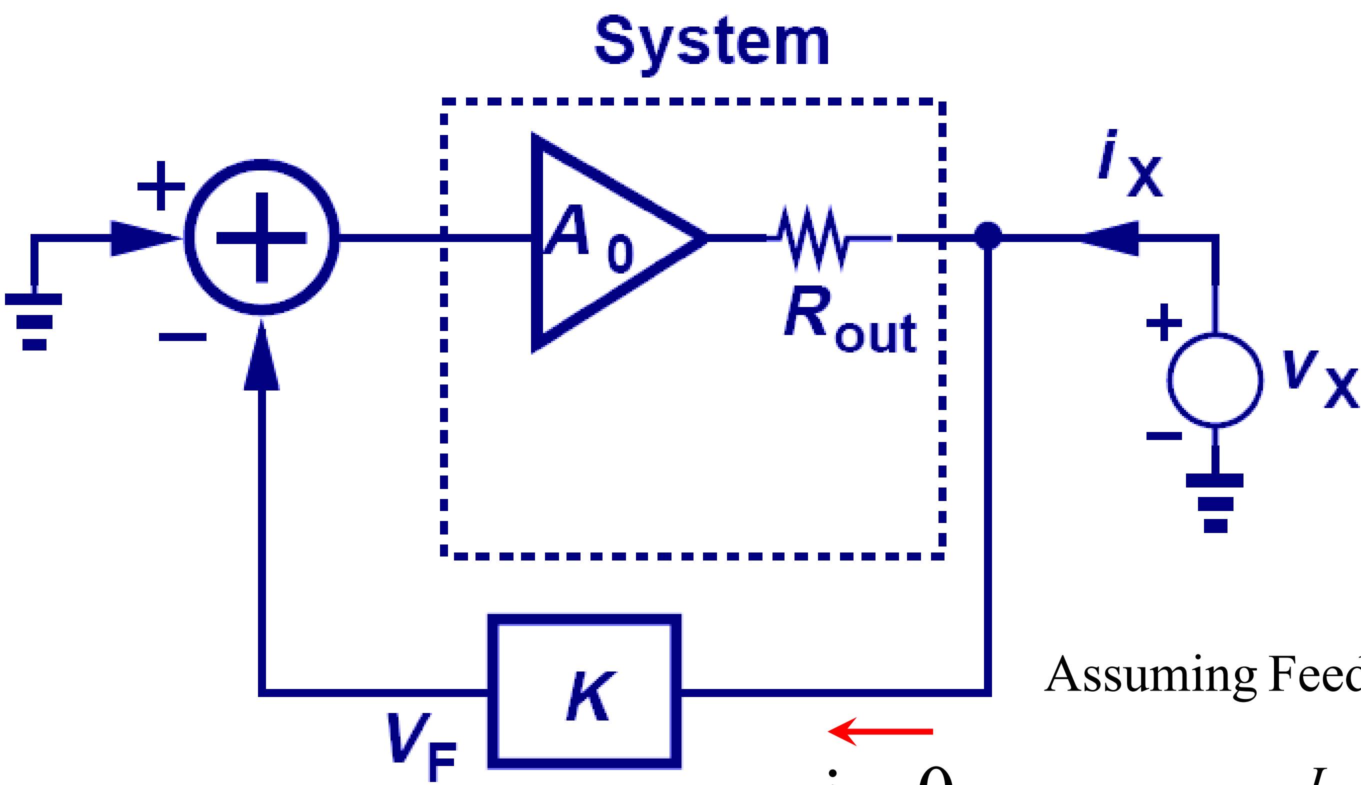
$$R_{in,open} = \frac{1}{g_m}$$

$$A_0 = g_m R_D \quad \left(\text{Assuming } R_1 + R_2 >> R_D\right)$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$R_{in,closed} = \frac{V_{in}}{I_{in}} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right)$$

Output Impedance of a V-V Feedback



$$i = 0$$

$$V_F = KV_X$$

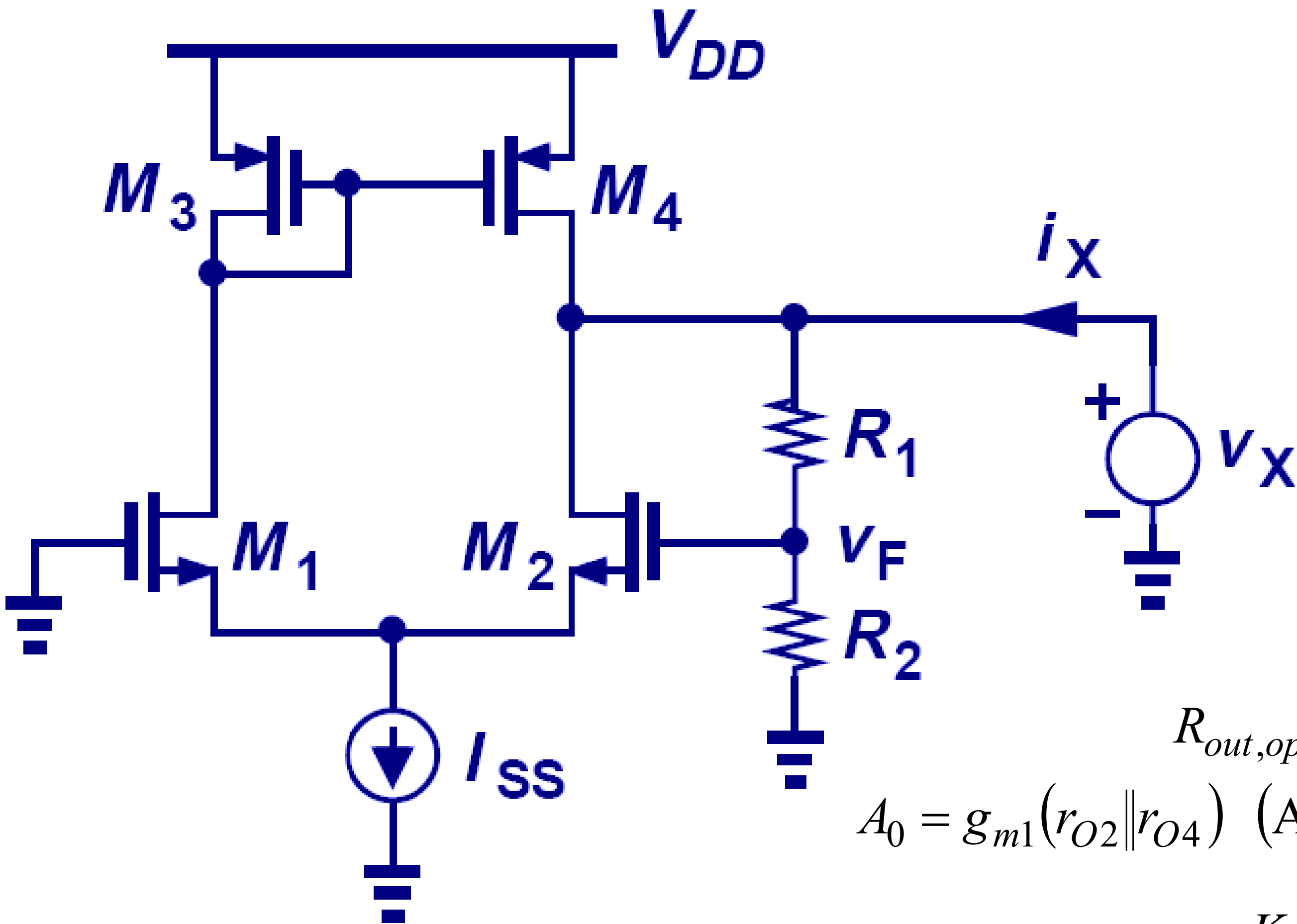
Assuming Feedback has large input impedance

$$I_X = \frac{V_X - (-A_0 K V_X)}{R_{out}}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + A_0 K}$$

- **A better output voltage source, as R_{out} has been reduced by $(1+A_0K)^{-1}$**

Example: V-V Feedback Output Impedance



$$R_{out,open} = (r_{O2} \| r_{O4})$$

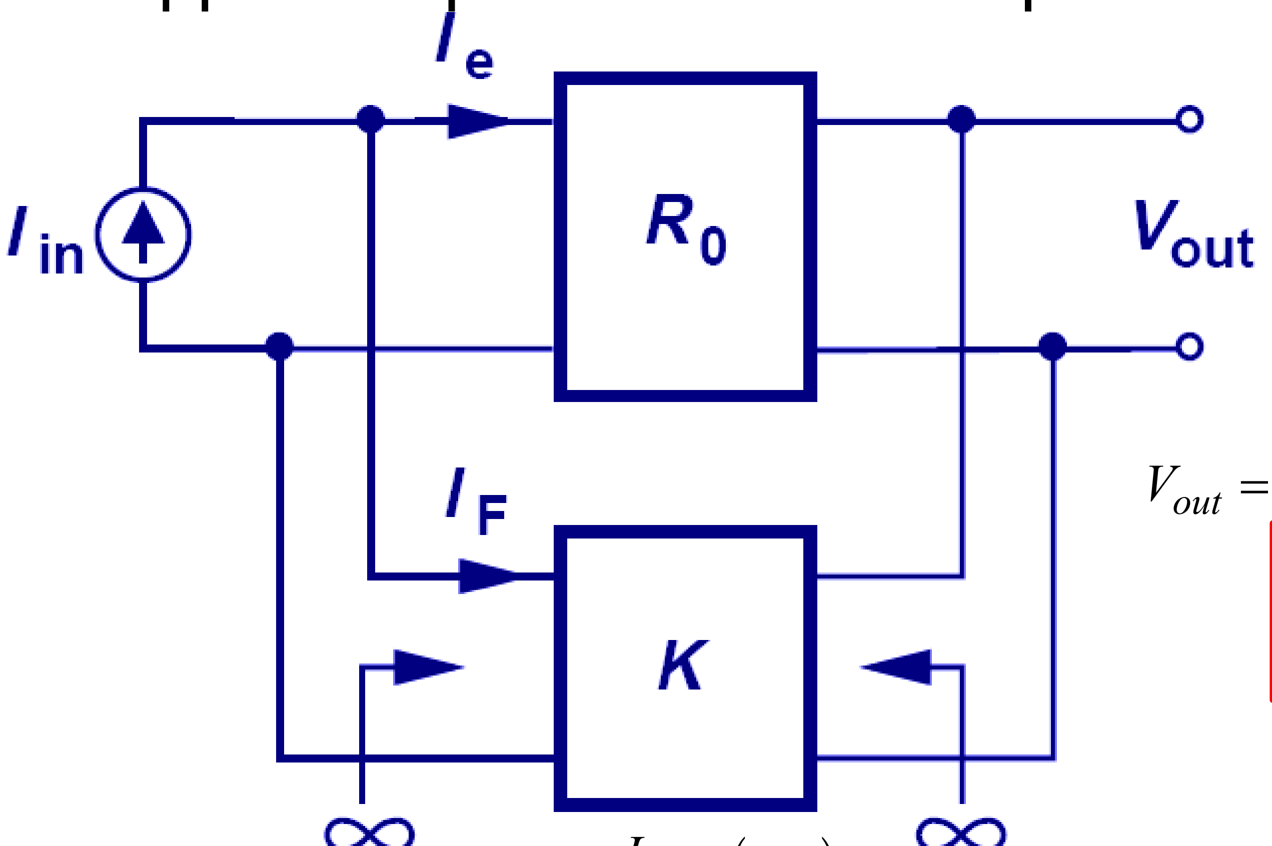
$$A_0 = g_{m1}(r_{O2} \| r_{O4}) \quad (\text{Assuming } R_1 + R_2 >> r_{O2} \| r_{O4})$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$R_{out,closed} = \frac{V_X}{I_X} = \frac{r_{O2} \| r_{O4}}{1 + \frac{R_2}{R_1 + R_2} g_{m1}(r_{O2} \| r_{O4})} \approx \left(1 + \frac{R_1}{R_2}\right)\frac{1}{g_{m1}}$$

Voltage-Current Feedback

- A transimpedance amplifier requires sensing of the output voltage to produce a feedback current
- Output voltage is sensed in parallel and feedback current applied in parallel with the input

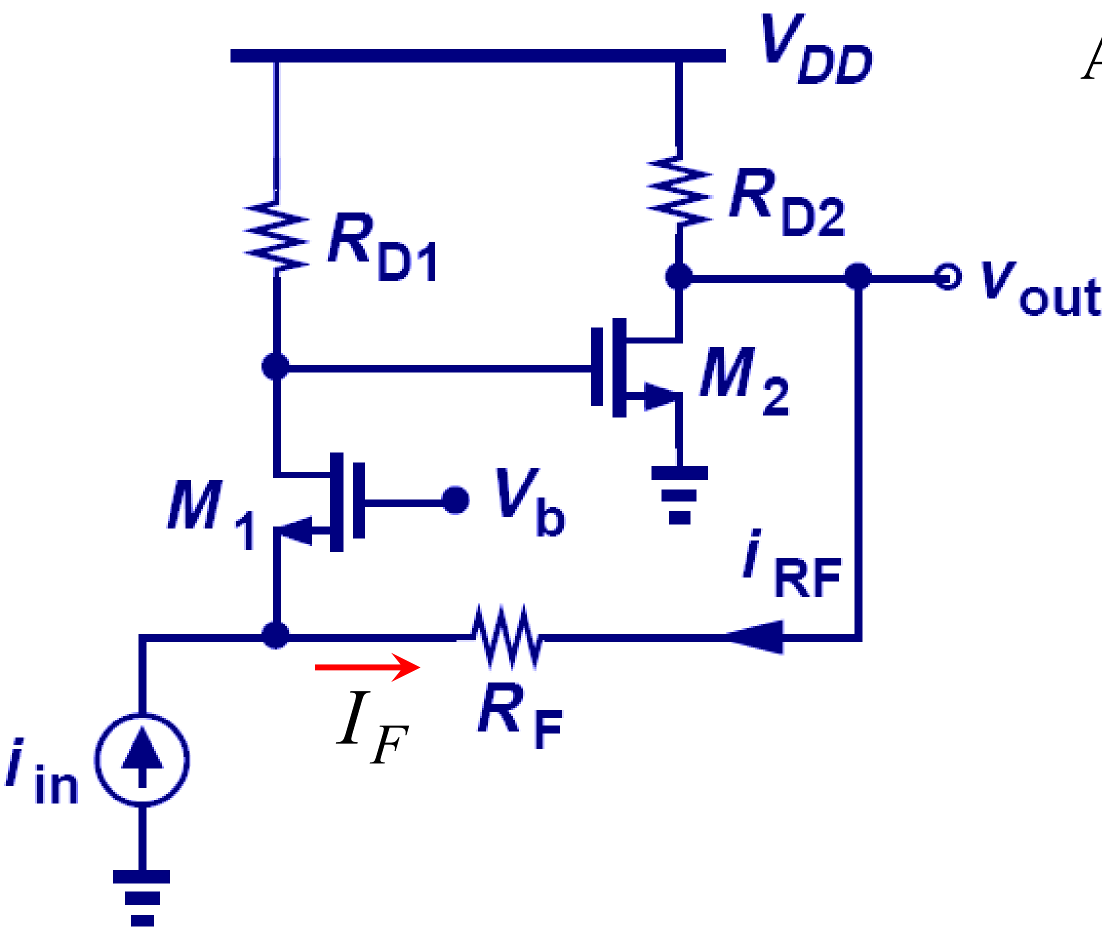


$$V_{out} = (I_{in} - I_F)R_0 = (I_{in} - KV_{out})R_0$$

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + KR_0}$$

$$K = \frac{I_F}{V_{out}} \quad (A/V)$$

Example: Voltage-Current Feedback



Assume R_F is large $(R_F >> R_{D2})$

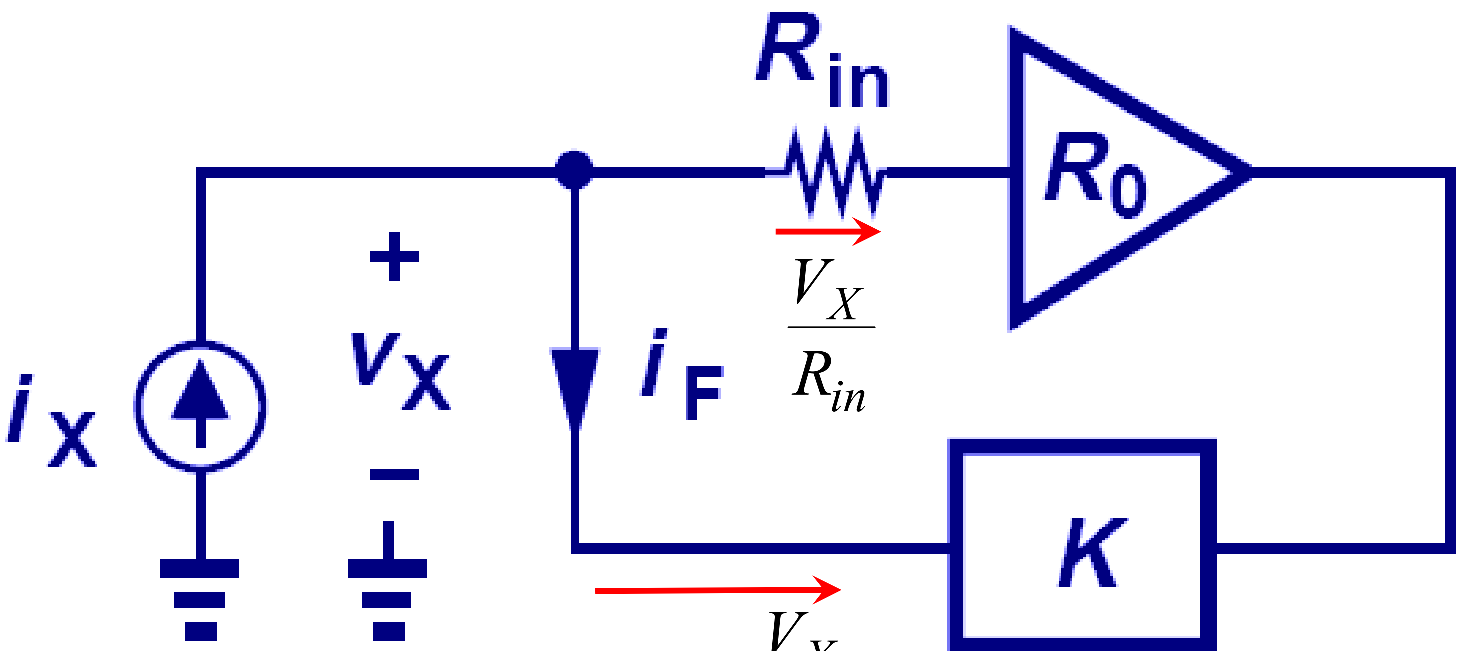
$$R_0 = R_{D1}(-g_{m2}R_{D2})$$

$$I_{RF} = \frac{V_{out}}{R_F + \frac{1}{g_{m1}}} \approx \frac{V_{out}}{R_F}$$

$$K = \frac{I_F}{V_{out}} = \frac{-I_{RF}}{V_{out}} = -\frac{1}{R_F}$$

$$\frac{V_{out}}{I_{in}} = \frac{-g_{m2}R_{D2}R_{D1}}{1 + \frac{g_{m2}R_{D2}R_{D1}}{R_F}}$$

Input Impedance of a V-C Feedback



$$V_{out} = \left(\frac{V_X}{R_{in}}\right) R_0$$

$$I_F = K \frac{V_X}{R_{in}} R_0$$

KCL at input node

$$I_X - K \frac{V_X}{R_{in}} R_0 = \frac{V_X}{R_{in}}$$

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + KR_0}$$

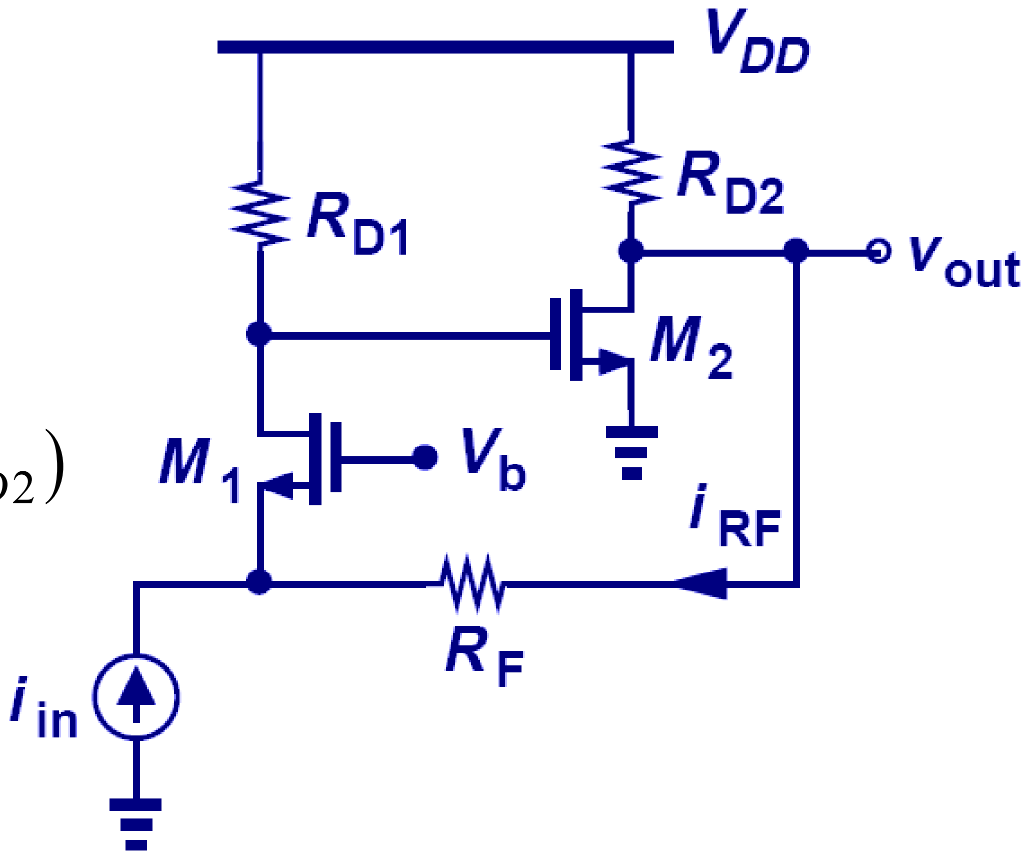
- **A better input current sensor, as R_{in} has been reduced by $(1+KR_0)^{-1}$**

Example: V-C Feedback Input Impedance

$$R_{in,open} = \frac{1}{g_{m1}}$$

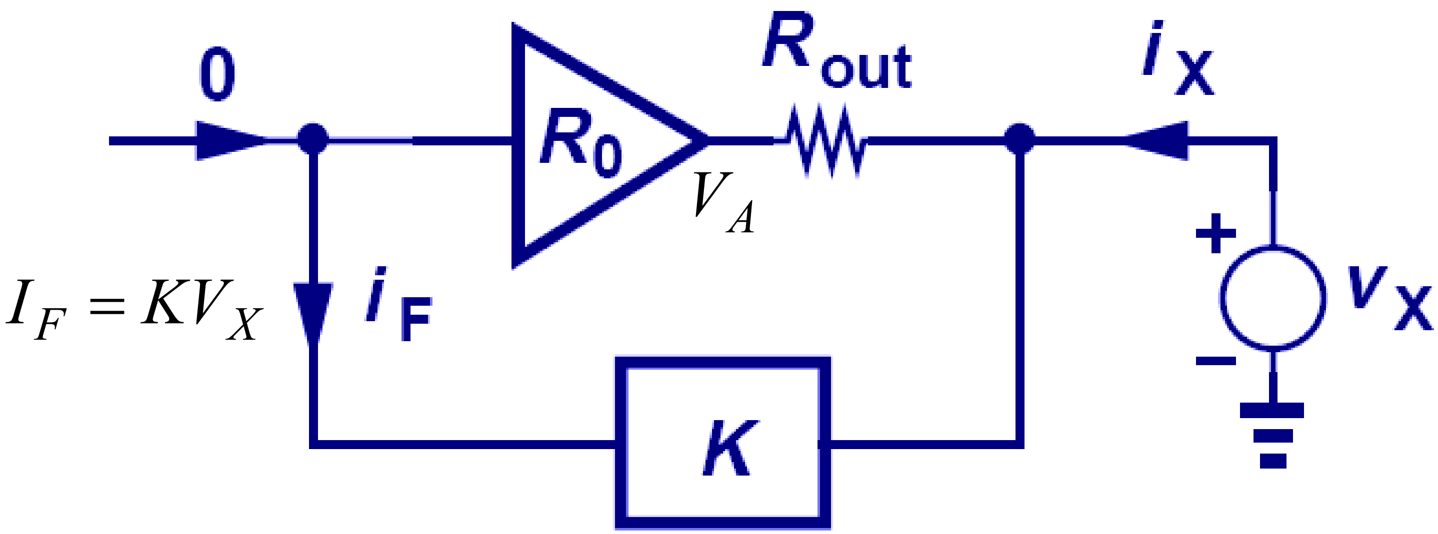
$$K = -\frac{1}{R_F}$$

$$R_0 = R_{D1}\left(-g_{m2}R_{D2}\right)$$



$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + \frac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

Output Impedance of a V-C Feedback



$$V_A = -I_F R_0 = -KV_X R_0$$

Neglecting the small feedback current

$$I_X = \frac{V_X - V_A}{R_{out}} = \frac{V_X + KV_X R_0}{R_{out}}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + R_0 K}$$

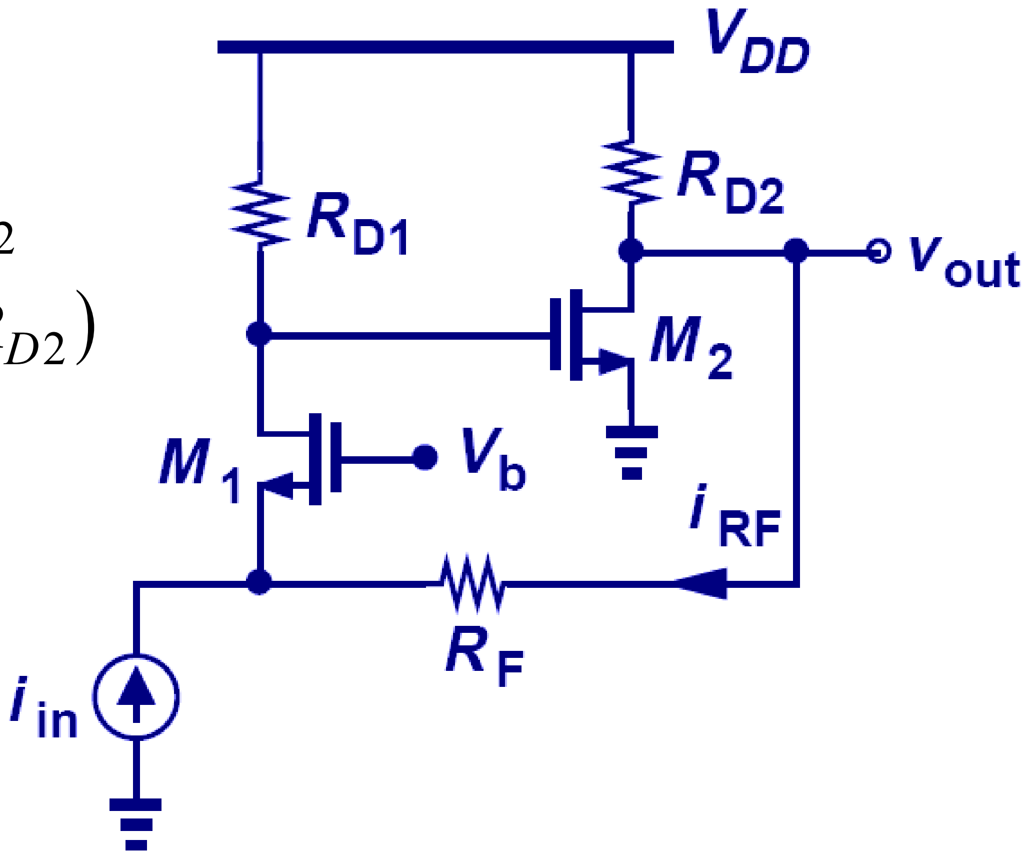
- A better output voltage source, as R_{out} has been reduced by $(1+KR_0)^{-1}$

Example: V-C Feedback Output Impedance

$$R_{out,open} = R_{D2}$$

$$R_0 = R_{D1}(-g_{m2}R_{D2})$$

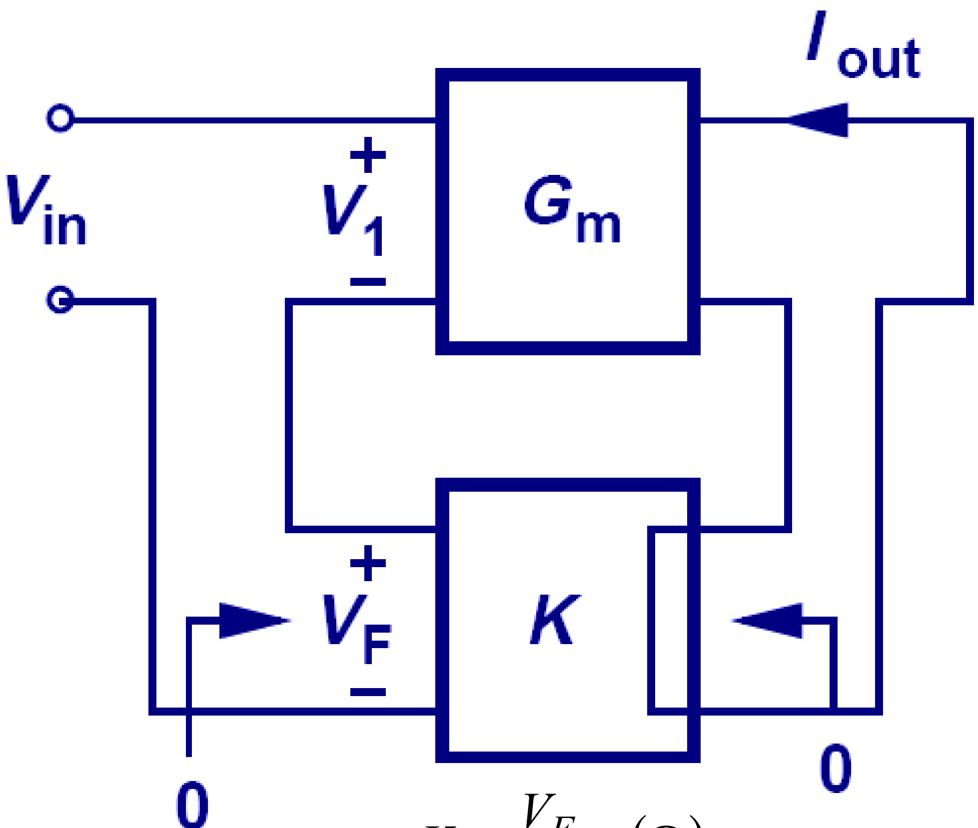
$$K = -\frac{1}{R_F}$$



$$R_{out,closed} = \frac{R_{D2}}{1 + \frac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

Current-Voltage Feedback

- A transconductance amplifier requires sensing of the output current to produce a feedback voltage
- Output current is sensed in series and feedback voltage applied in series with the input



$$I_{out} = G_m(V_{in} - V_F) = G_m(V_{in} - KI_{out})$$

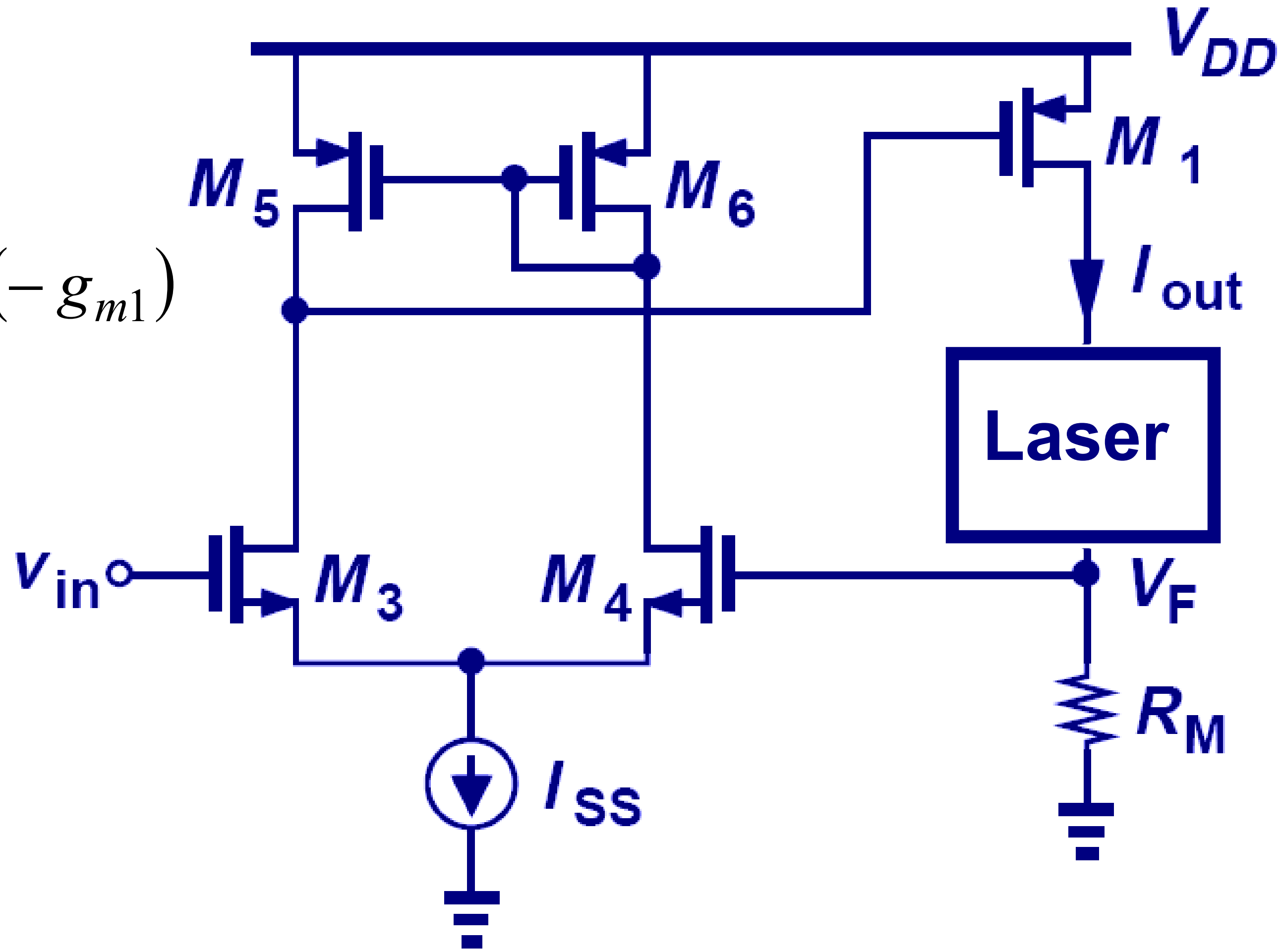
$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + KG_m}$$

$$K = \frac{V_F}{I_{out}} \quad (\Omega)$$

Example: Current-Voltage Feedback

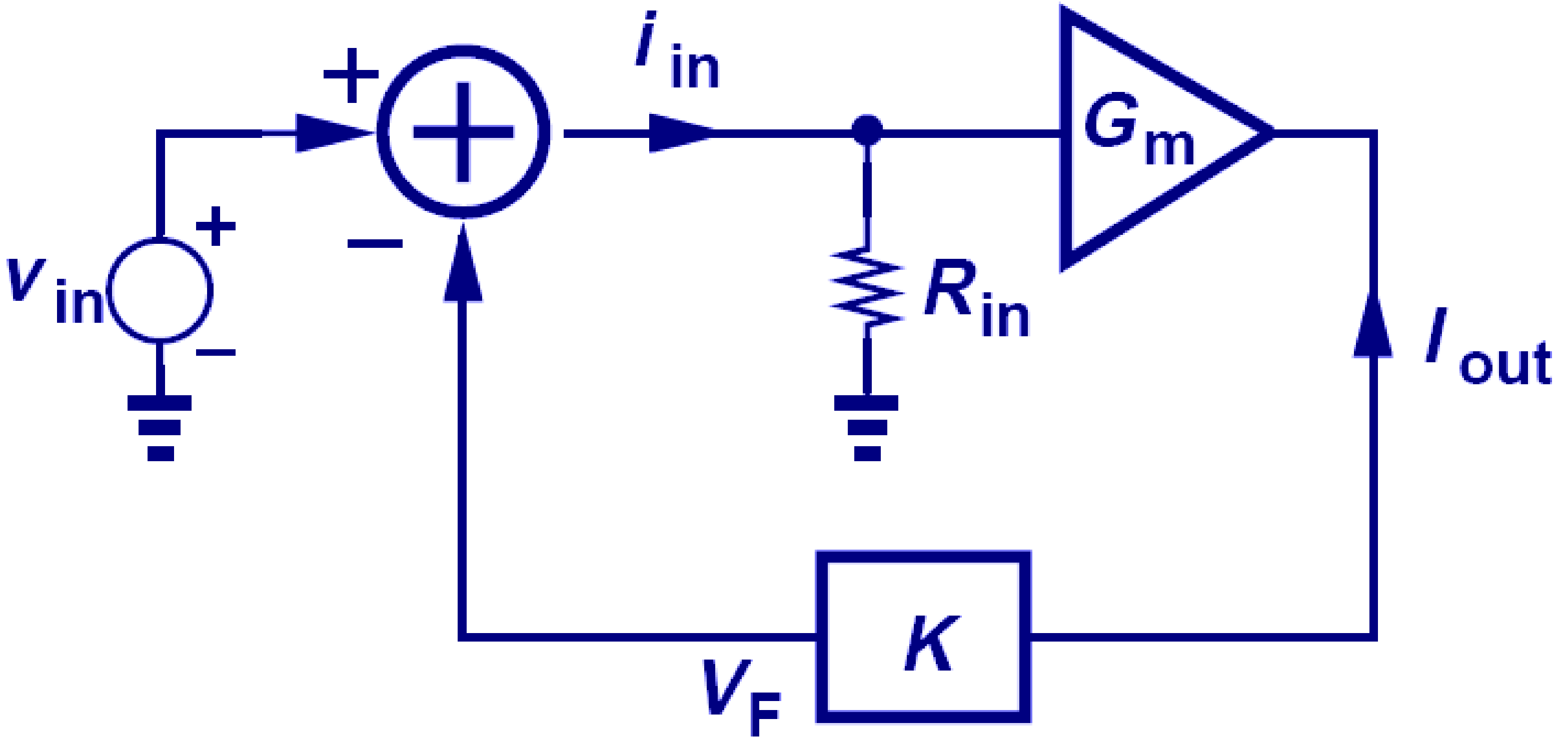
$$G_m = -g_{m3}(r_{O3} \| r_{O5})(-g_{m1})$$

$$K = R_M$$



$$\frac{I_{out}}{V_{in}}\Big|_{closed} = \frac{g_{m1}g_{m3}(r_{O3} \| r_{O5})}{1 + g_{m1}g_{m3}(r_{O3} \| r_{O5})R_M} \approx \frac{1}{R_M}$$

Input Impedance of a C-V Feedback

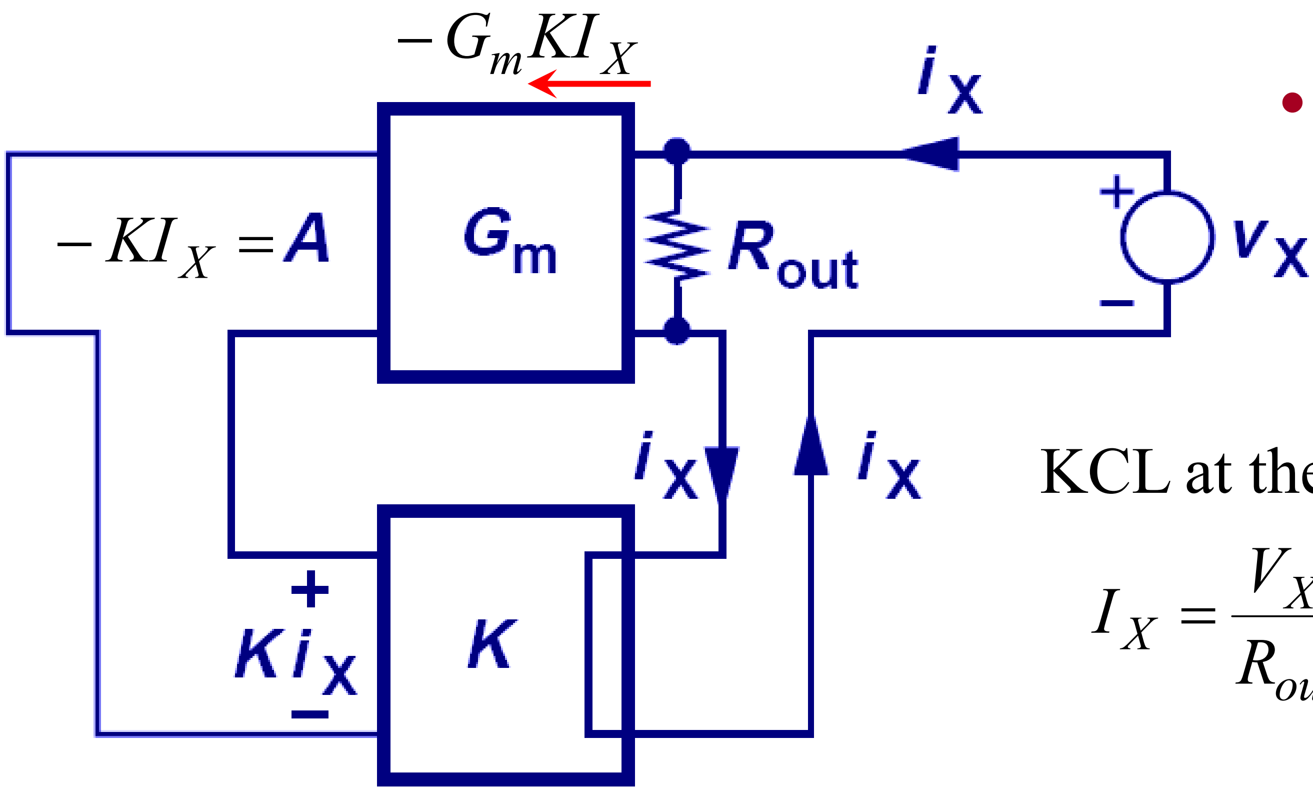


$$I_{in}R_{in} = V_{in} - V_F = V_{in} - (I_{in}R_{in})G_m K$$
$$I_{in}R_{in}(1 + G_m K) = V_{in}$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + KG_m)$$

- **A better input voltage sensor, as R_{in} increases by $1+KG_m$**

Output Impedance of a C-V Feedback



- For correct measurement of a feedback current system's R_{out}, V_X must be inserted in series

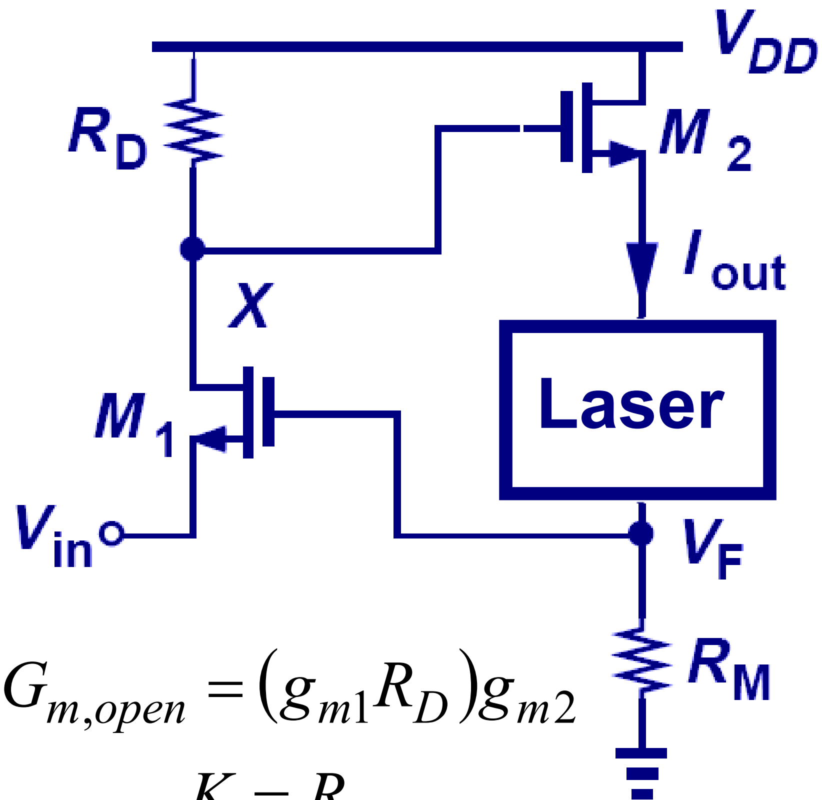
KCL at the output node

$$I_X = \frac{V_X}{R_{out}} - G_mKI_X$$

$$\frac{V_X}{I_X} = R_{out}(1 + KG_m)$$

➢ **A better output current source, as R_{out} increases by $1+KG_m$**

Example: Current-Voltage Feedback



$$G_{m,open} = (g_{m1}R_D)g_{m2}$$

$$K = R_M$$

$$\frac{I_{out}}{V_{in}}\bigg|_{closed} = \frac{g_{m1}g_{m2}R_D}{1+g_{m1}g_{m2}R_DR_M}$$

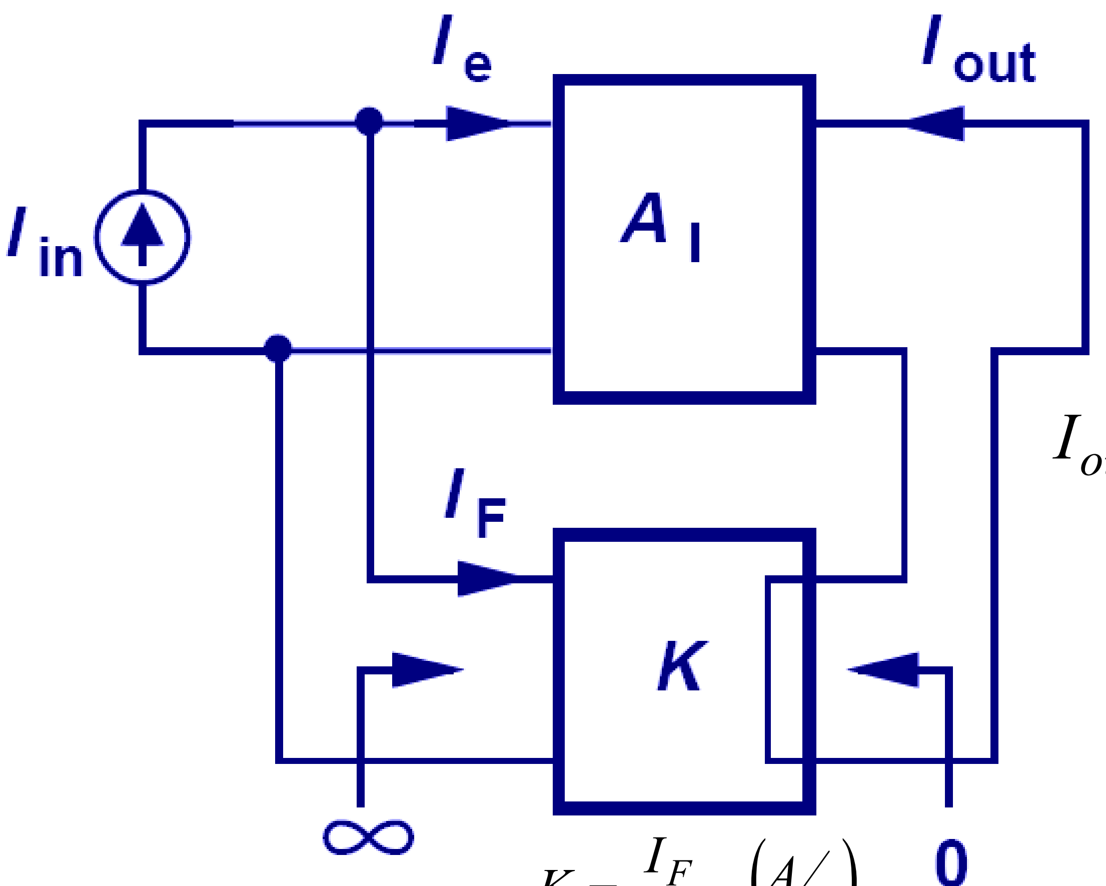
$$R_{in}\big|_{closed} = \frac{1}{g_{m1}}(1+g_{m1}g_{m2}R_DR_M)$$

$$R_{out}\big|_{closed} = \frac{1}{g_{m2}}(1+g_{m1}g_{m2}R_DR_M)$$

R_{out} expression assumes that R_M is small

Current-Current Feedback

- A current amplifier requires sensing of the output current to produce a feedback current
- Output current is sensed in series and feedback current applied in parallel with the input

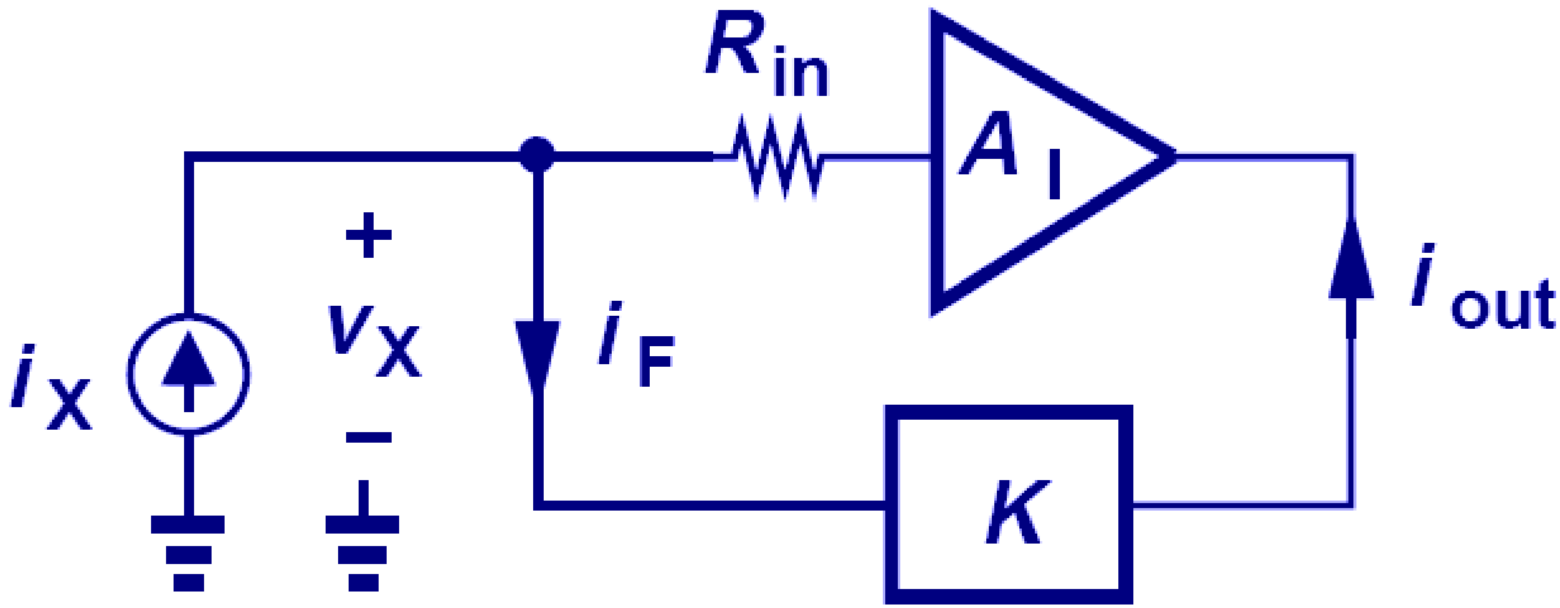


$$K = \frac{I_F}{I_{out}} \quad \left(A/A\right)$$

$$I_{out} = A_I\left(I_{in} - I_F\right) = A_I\left(I_{in} - KI_{out}\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{A_I}{1 + KA_I}$$

Input Impedance of C-C Feedback

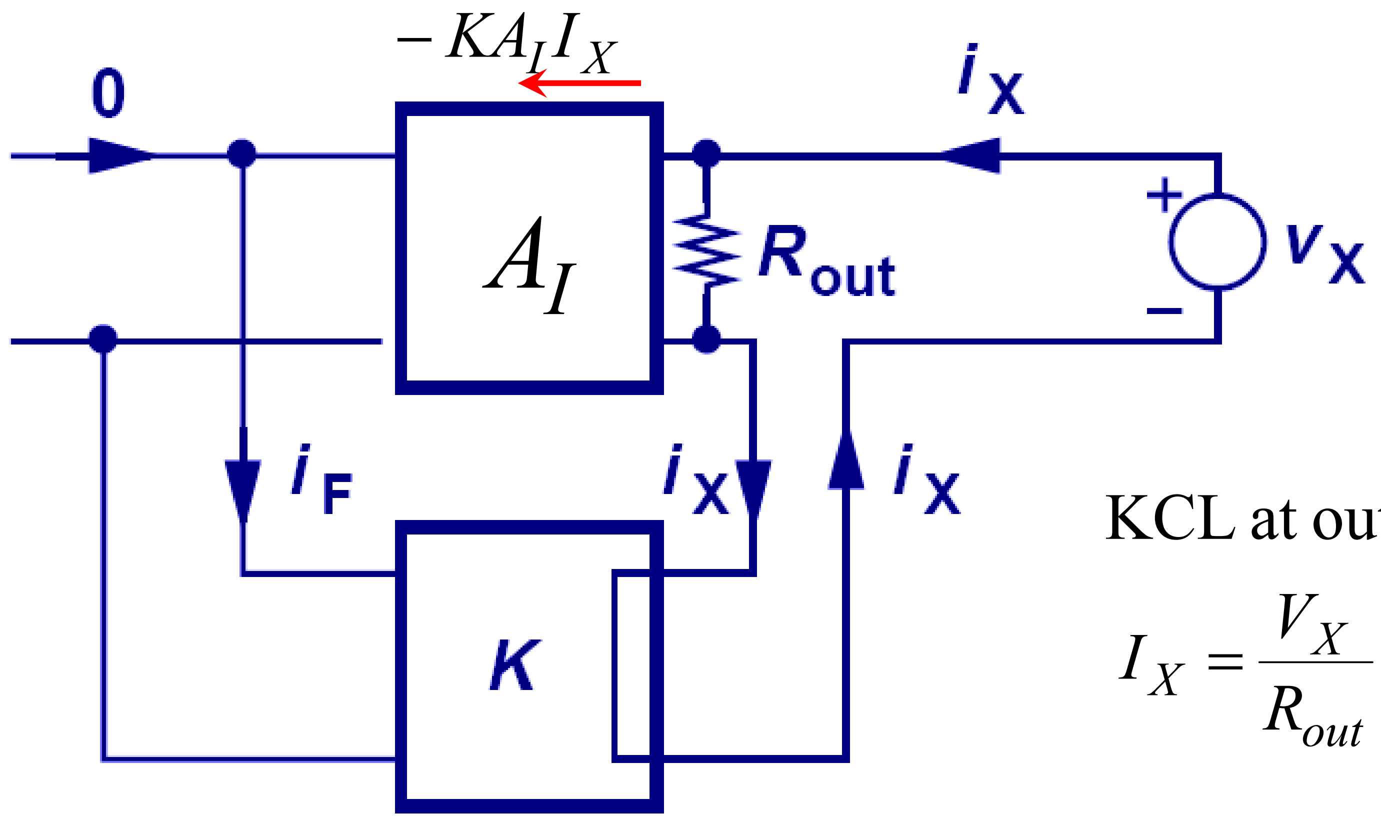


$$I_X = \frac{V_X}{R_{in}} + I_F = \frac{V_X}{R_{in}} + KA_I \frac{V_X}{R_{in}}$$

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + KA_I}$$

- A better input current sensor, as R_{in} decreases by $(1+KA_I)^{-1}$

Output Impedance of C-C Feedback



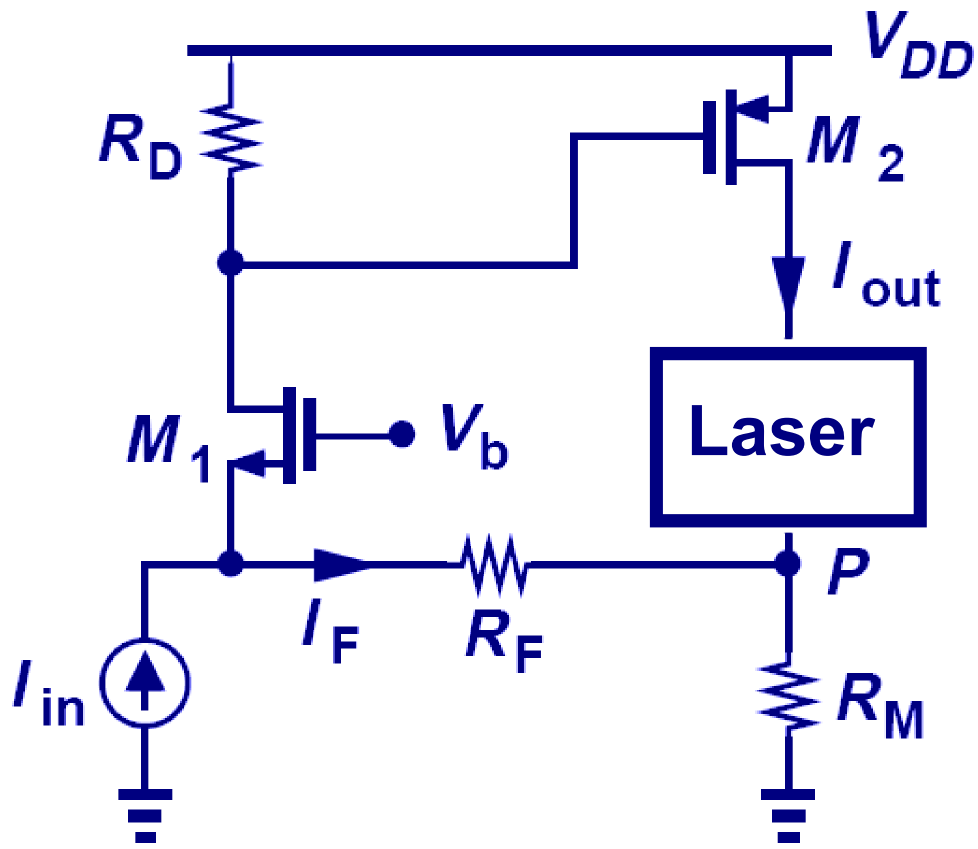
KCL at output node

$$I_X = \frac{V_X}{R_{out}} - KA_I I_X$$

$$\frac{V_X}{I_X} = R_{out}(1 + KA_I)$$

- A better output current source, as R_{out} increases by $(1+KA_I)$

Example: Test of Negative Feedback



$$I_{in}\uparrow \Rightarrow V_{D1}\uparrow, I_{out}\downarrow \Rightarrow V_P\downarrow, I_F\uparrow \Rightarrow V_{D1}\downarrow, I_{out}\uparrow$$

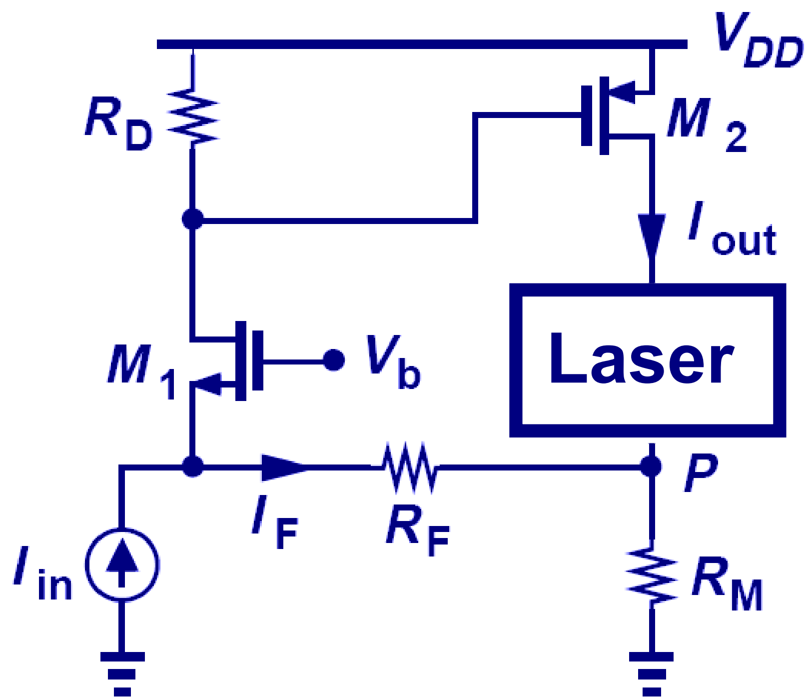
Negative Feedback

Example: C-C Negative Feedback

$$A_{I,open} = R_D(-g_{m2})$$

Assuming $R_F >> R_M$

$$K = -\frac{R_M}{R_F + R_M} \approx -\frac{R_M}{R_F}$$



$$A_I\,|_{closed} = \frac{-g_{m2}R_D}{1 + g_{m2}R_D(R_M/R_F)}$$

$$R_{in}\,|_{closed} = \frac{1}{g_{m1}}\cdot\frac{1}{1 + g_{m2}R_D(R_M/R_F)}$$

$$R_{out}\,|_{closed} = r_{O2}[1 + g_{m2}R_D(R_M/R_F)]$$

R_{out} expression assumes that R_M is small

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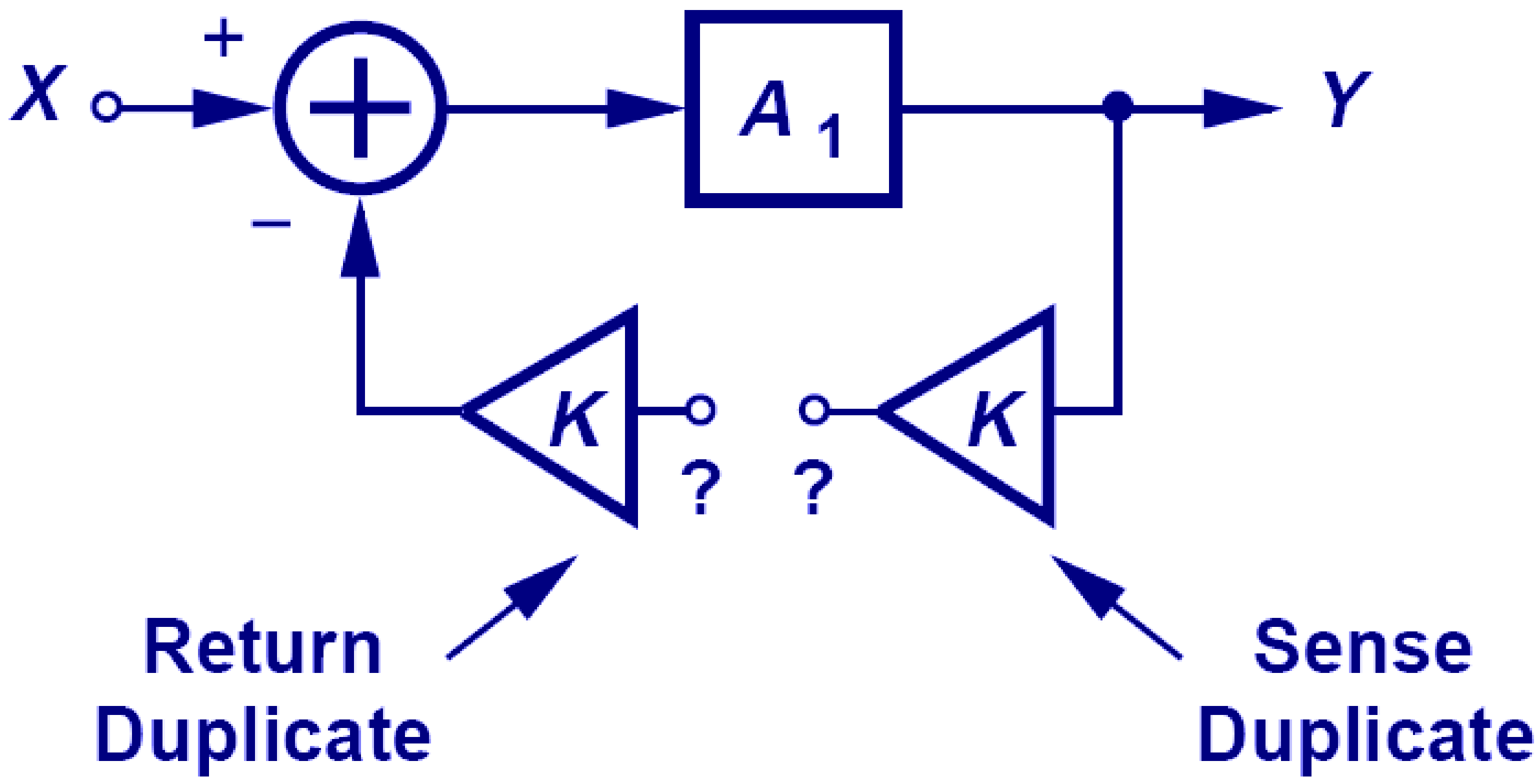
Feedback Network Loading

- In the previous examples, we made a lot of simplifying assumptions that neglect the loading the feedback network has on the amplifiers I/O ports
- However, the finite feedback network impedance may alter the overall circuit's performance
- In order to include the feedback network loading effects on the I/O impedances, the following methodology can be employed

Feedback Analysis Methodology with I/O Loading

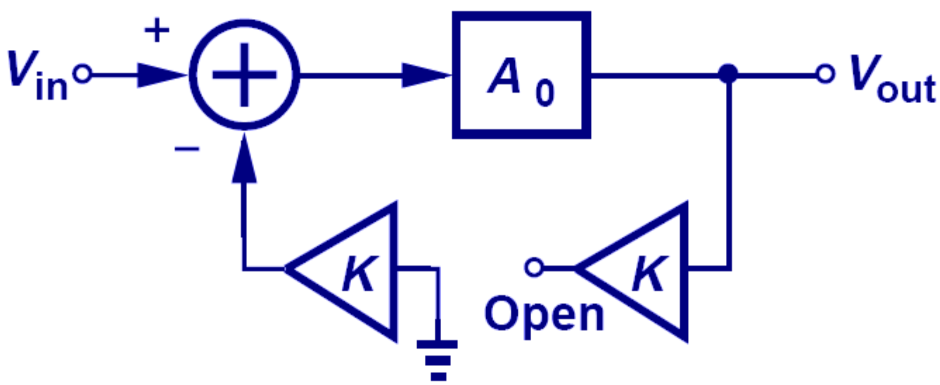
1. Identify the forward amplifier
2. Identify the feedback network
3. Break the feedback network correctly
4. Calculate the open-loop parameters
5. Determine the feedback factor correctly
6. Calculate the closed-loop parameters

How to Break a Loop

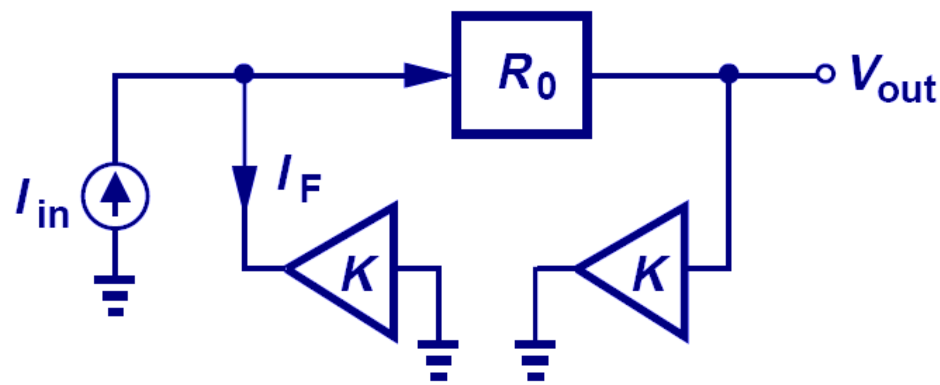


- The correct way of breaking a loop is such that the loop does not know it has been broken. Therefore, we need to present the feedback network to both the input and the output of the feedforward amplifier.

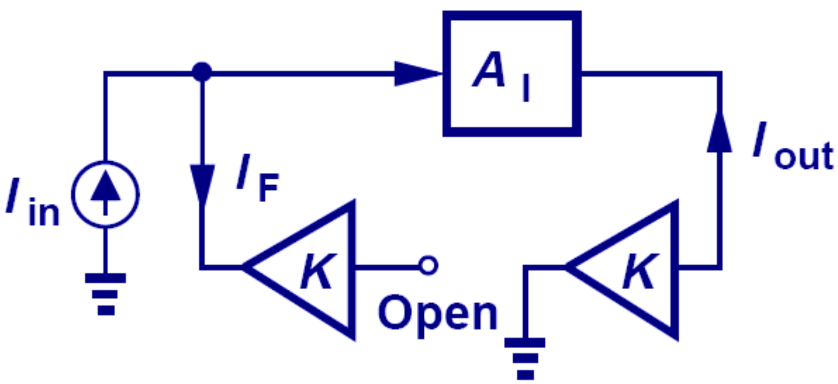
Rules for Breaking the Loop of Amplifier Types

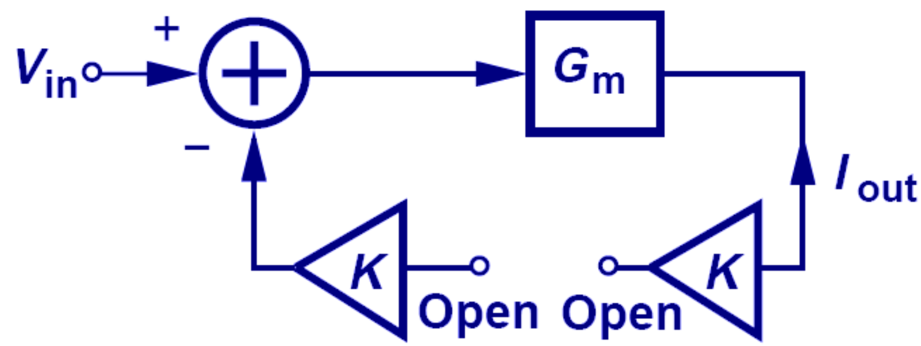


(a)



(b)

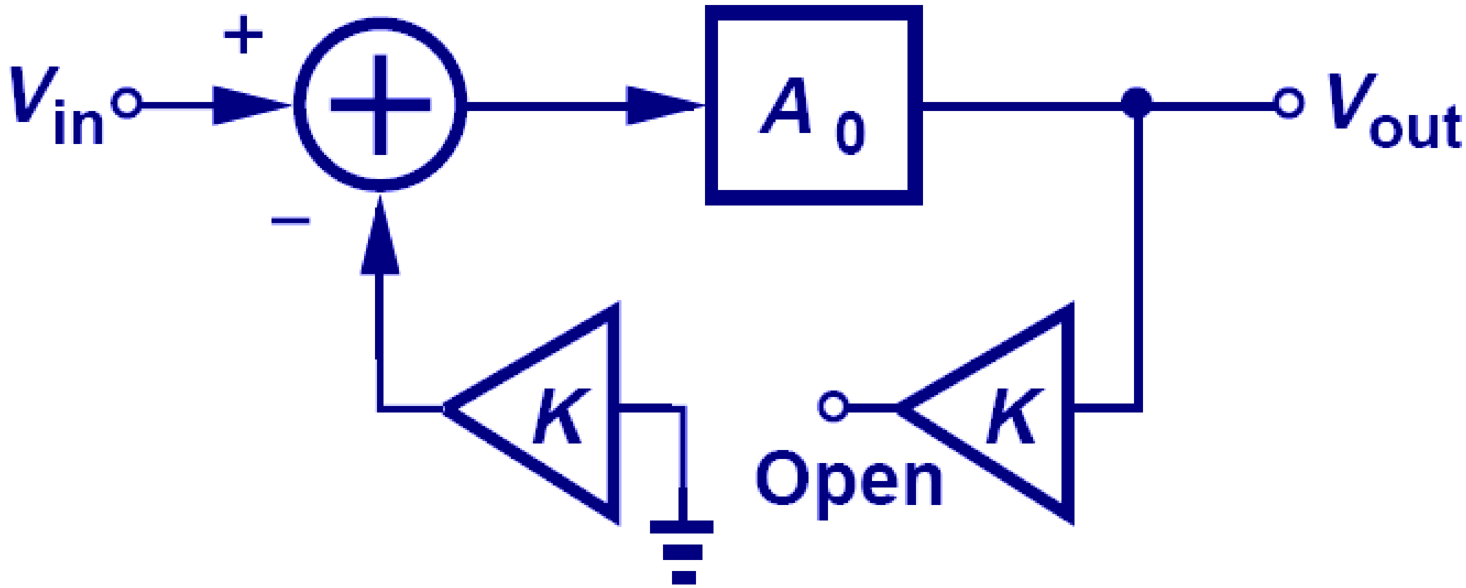




- Sense duplicate output is loaded with the ideal input impedance of the forward amplifier
- Return duplicate input is set with the ideal output impedance of the forward amplifier

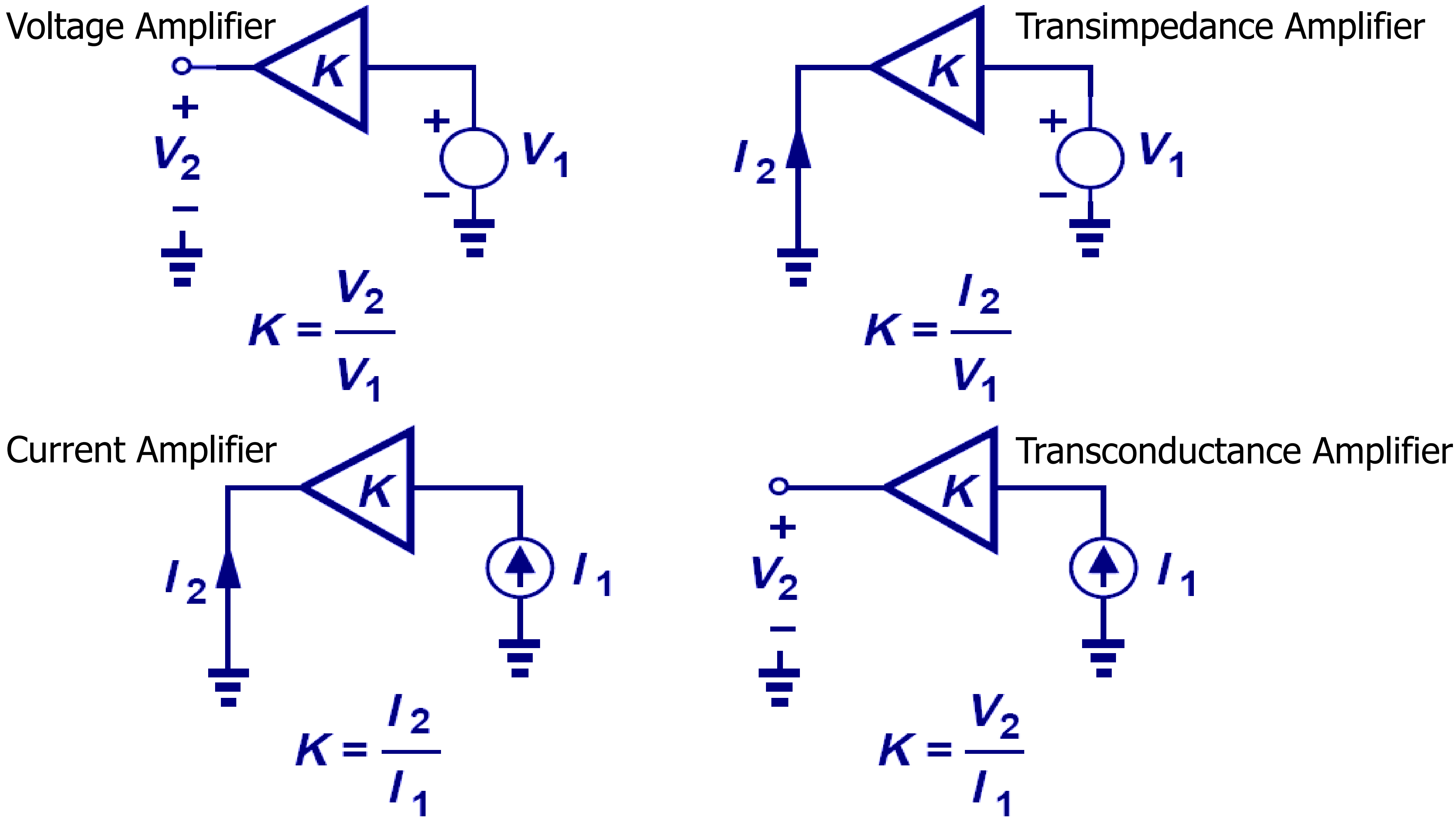
Intuitive Understanding of these Rules

Voltage-Voltage Feedback



- Since ideally, the input of the feedback network sees zero impedance (Z_{out} of an ideal voltage source), the return replicate needs to be grounded. Similarly, the output of the feedback network sees an infinite impedance (Z_{in} of an ideal voltage sensor), the sense replicate needs to be open.
- Similar ideas apply to the other types.

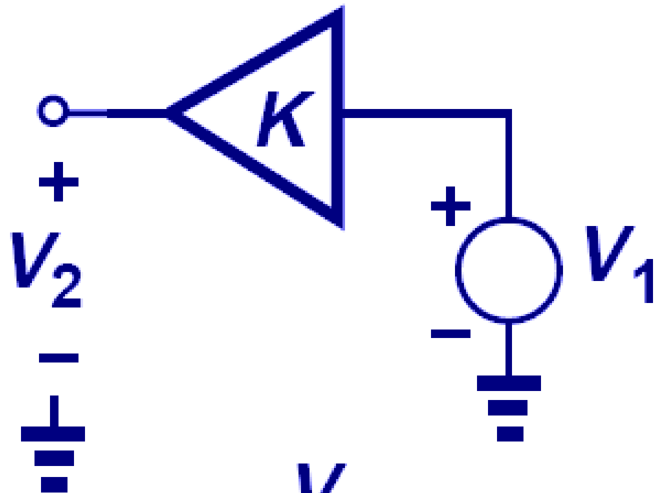
Rules for Calculating Feedback Factor



- Voltage feedback: feedback network output is opened
- Current feedback: feedback network output is shorted

Intuitive Understanding of these Rules

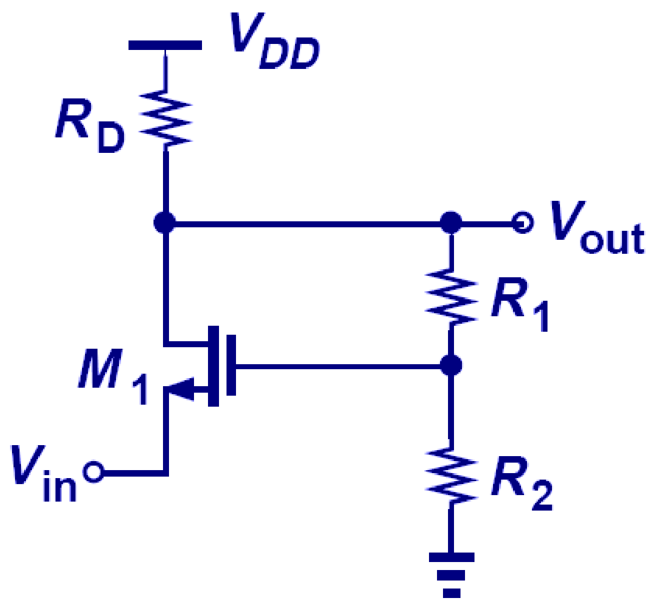
Voltage-Voltage Feedback

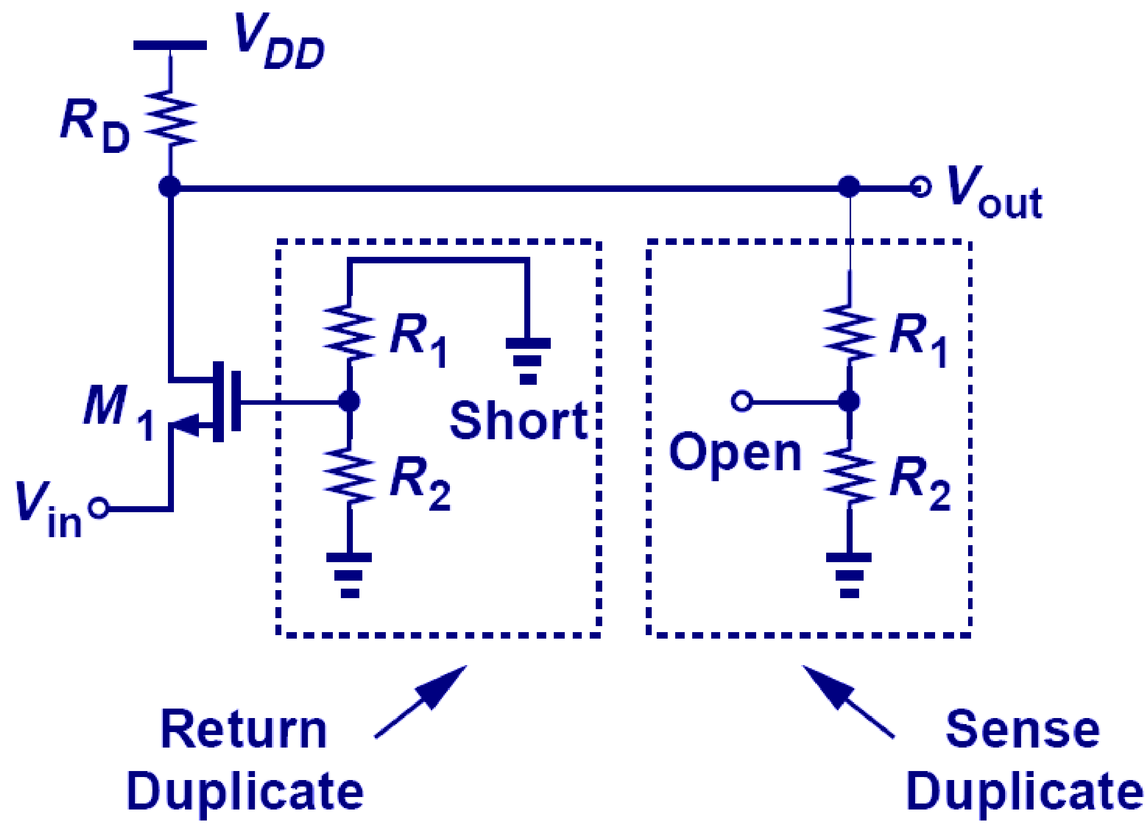


$$K = \frac{V_2}{V_1}$$

- Since the feedback senses voltage, the input of the feedback is a voltage source. Moreover, since the return quantity is also voltage, the output of the feedback is left open (a short means the output is always zero).
- Similar ideas apply to the other types.

Breaking the Loop Example I



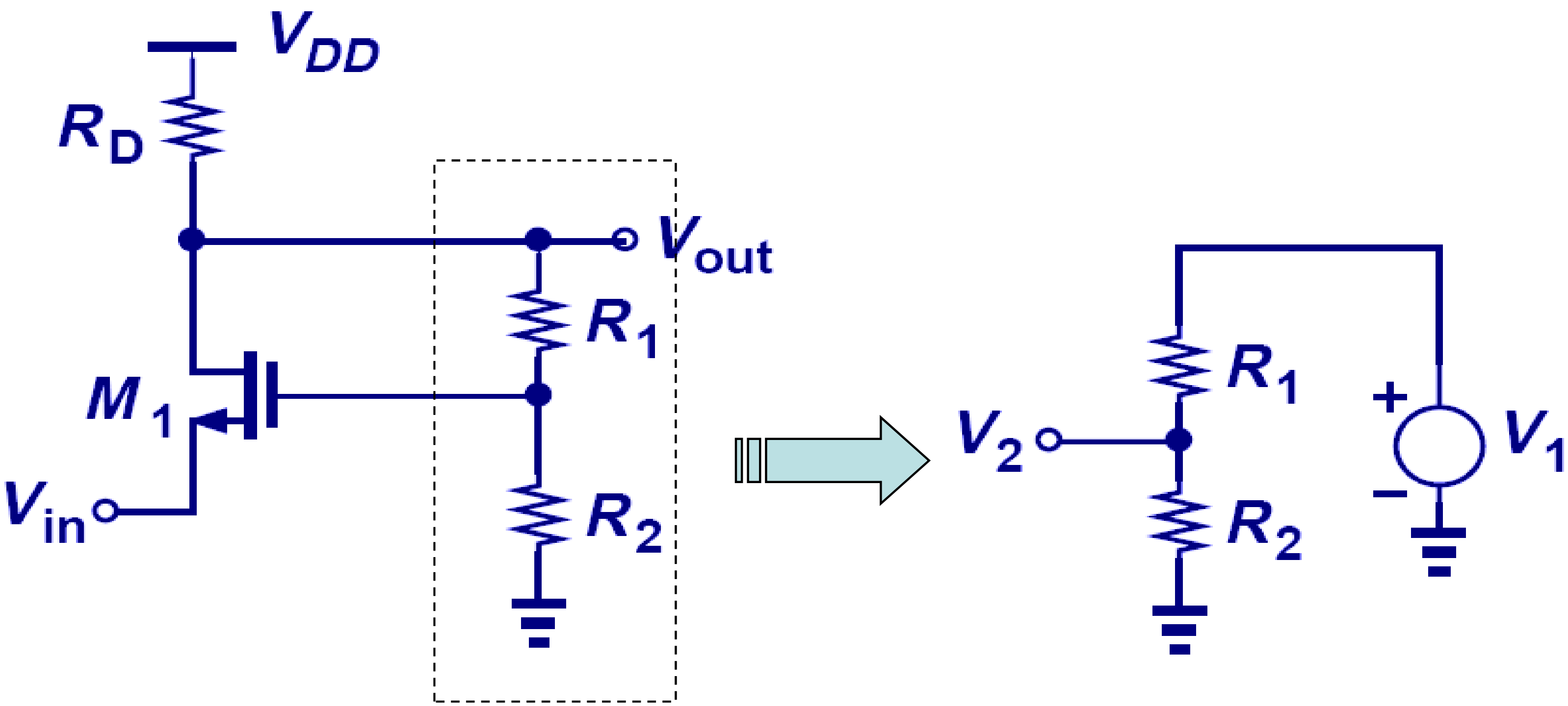


$$A_{v,open} = g_{m1}\left[R_D \parallel (R_1 + R_2)\right]$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = R_D \parallel (R_1 + R_2)$$

Feedback Factor Example I



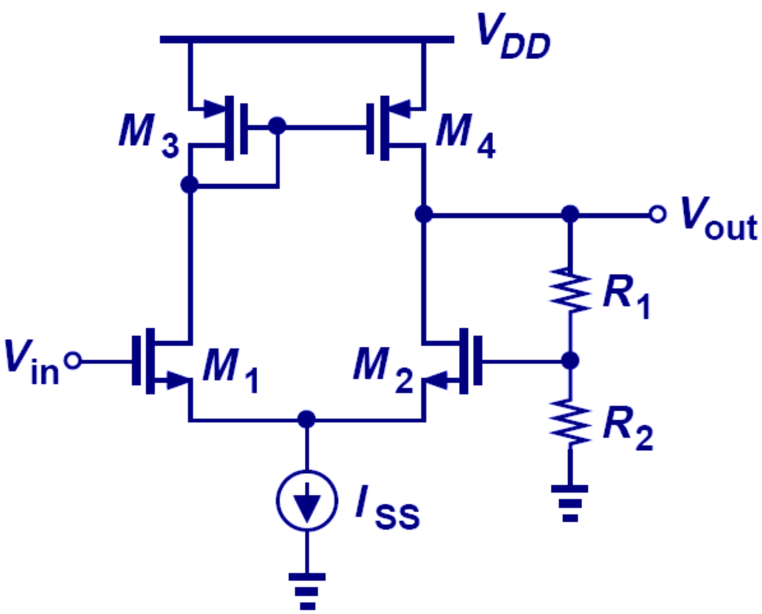
$$K = R_2/(R_1 + R_2)$$

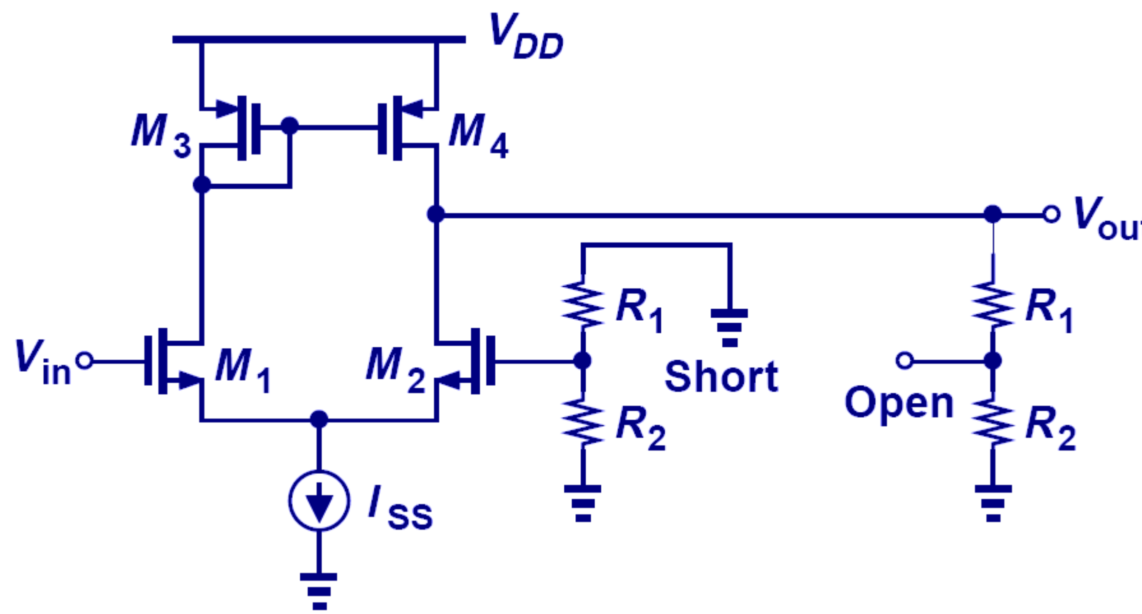
$$A_{v,closed} = A_{v,open}/(1 + KA_{v,open})$$

$$R_{in,closed} = R_{in,open}(1 + KA_{v,open})$$

$$R_{out,closed} = R_{out,open}/(1 + KA_{v,open})$$

Breaking the Loop Example II



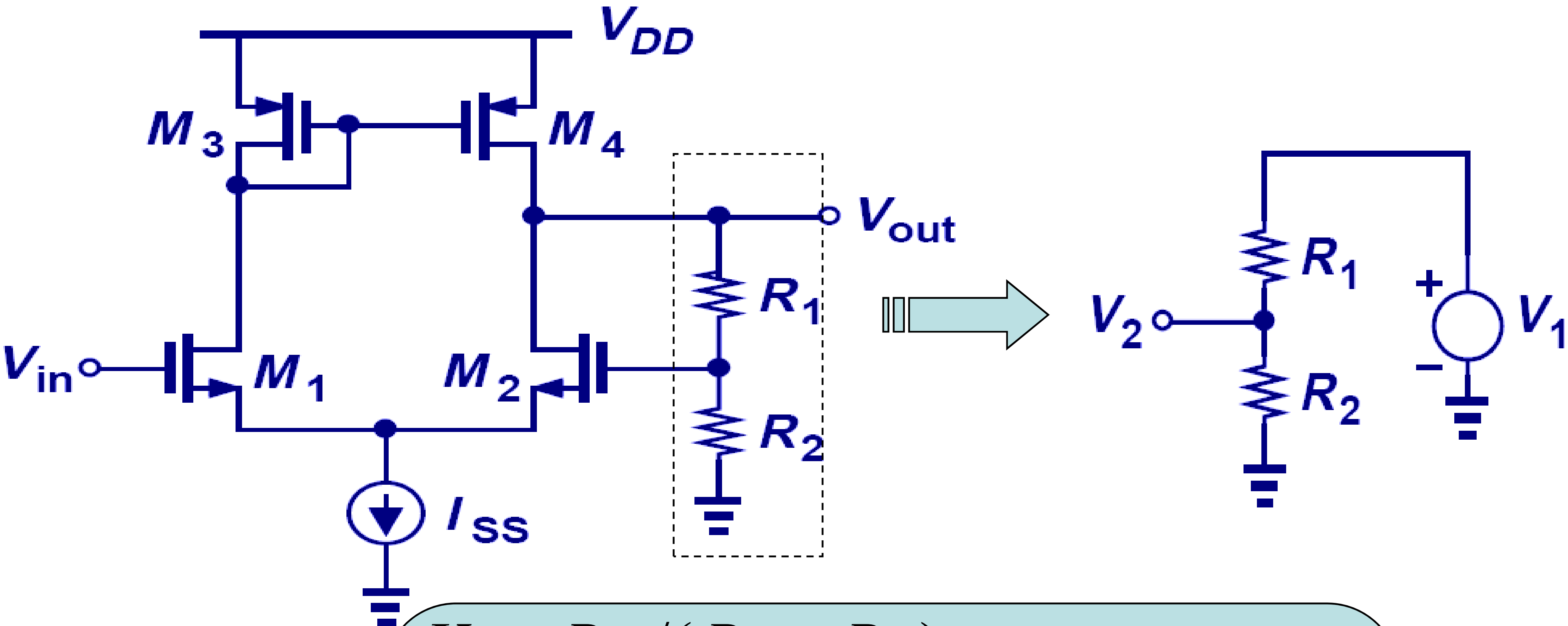


$$A_{v,open} = g_{mN}\left[r_{ON} \| r_{OP} \| \left(R_1 + R_2\right)\right]$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{ON} \| r_{OP} \| \left(R_1 + R_2\right)$$

Feedback Factor Example II



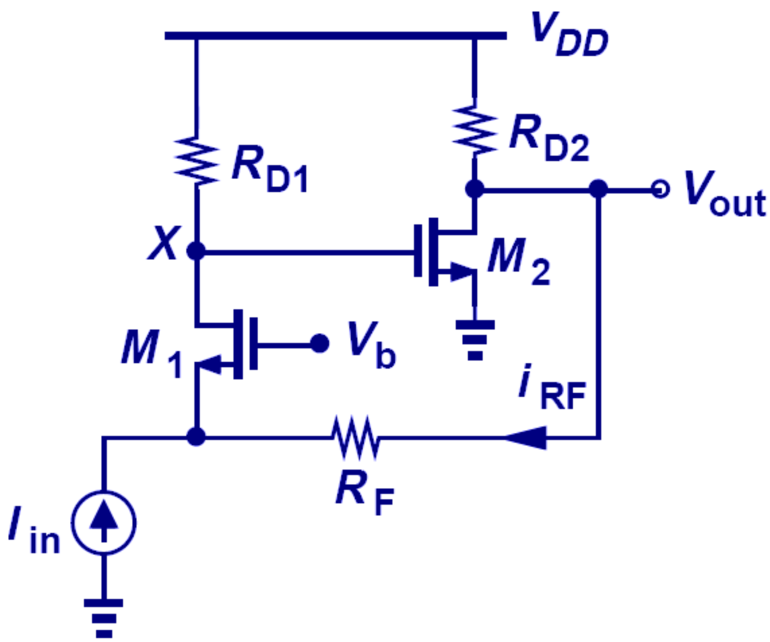
$$K = R_2 / (R_1 + R_2)$$

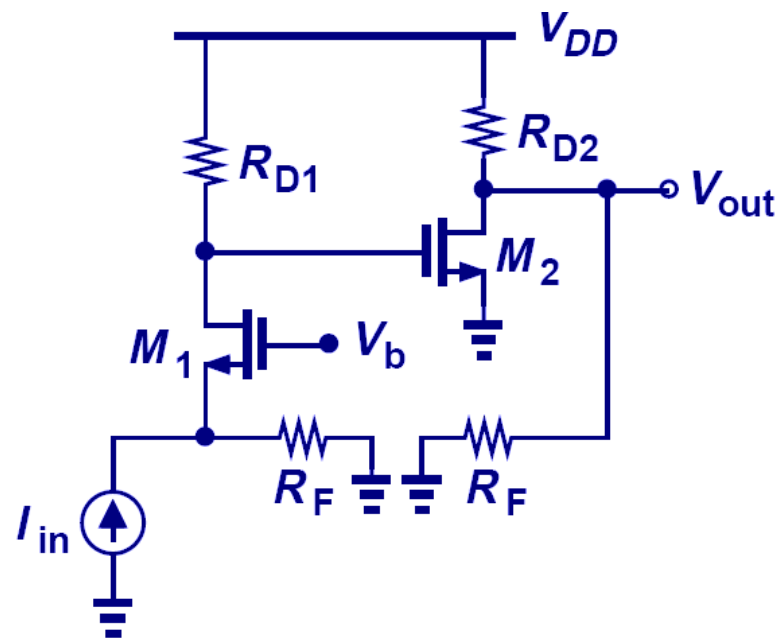
$$A_{v,closed} = A_{v,open} / (1 + KA_{v,open})$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open} / (1 + KA_{v,open})$$

Breaking the Loop Example IV



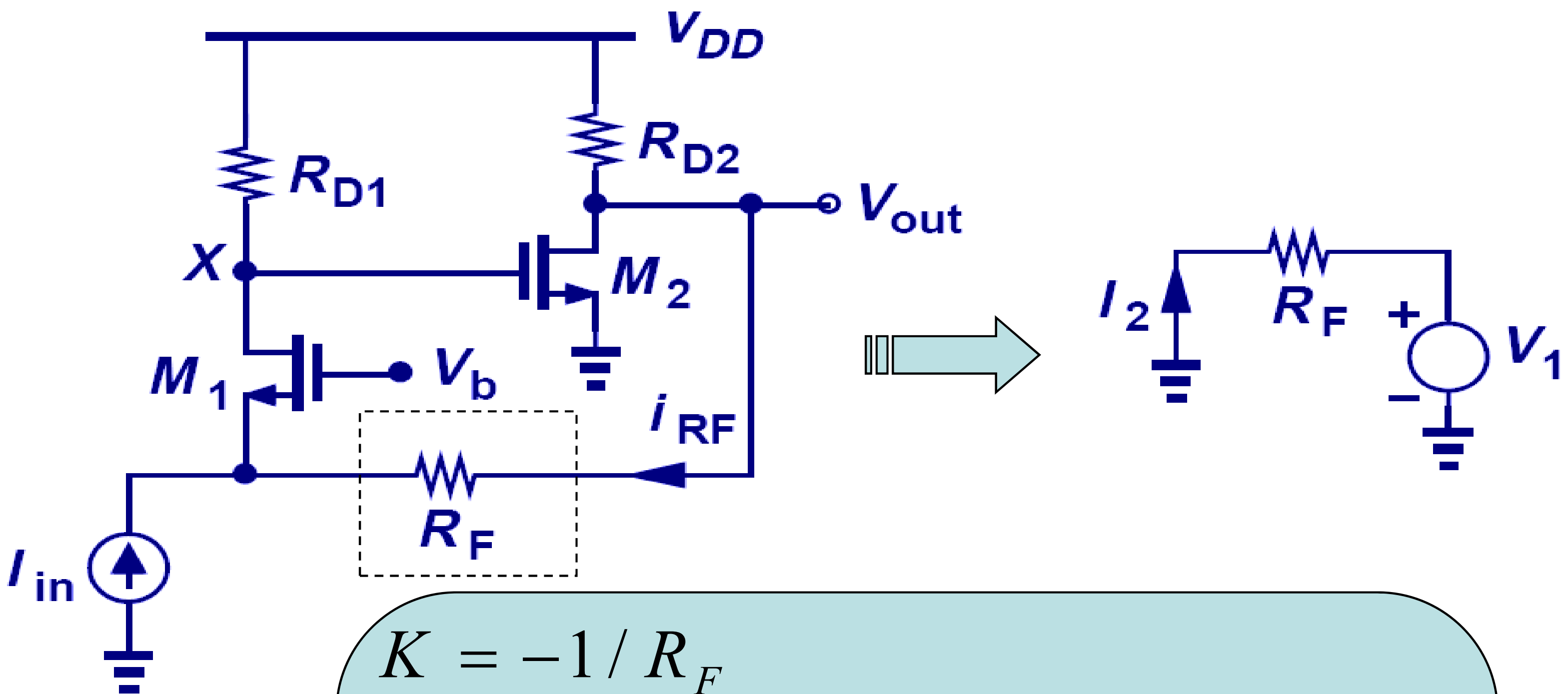


$$\frac{V_{out}}{I_{in}}\bigg|_{open} = \frac{R_F R_{D1}}{R_F + \frac{1}{g_{m1}}} . [-g_{m2}(R_{D2} \| R_F)]$$

$$R_{in,open} = \frac{1}{g_{m1}} \| R_F$$

$$R_{out,open} = R_{D2} \| R_F$$

Feedback Factor Example IV



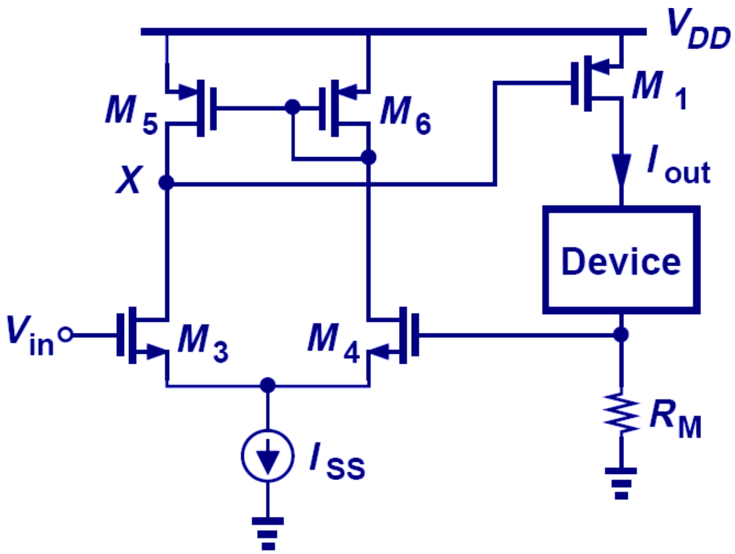
$$K = -1/R_F$$

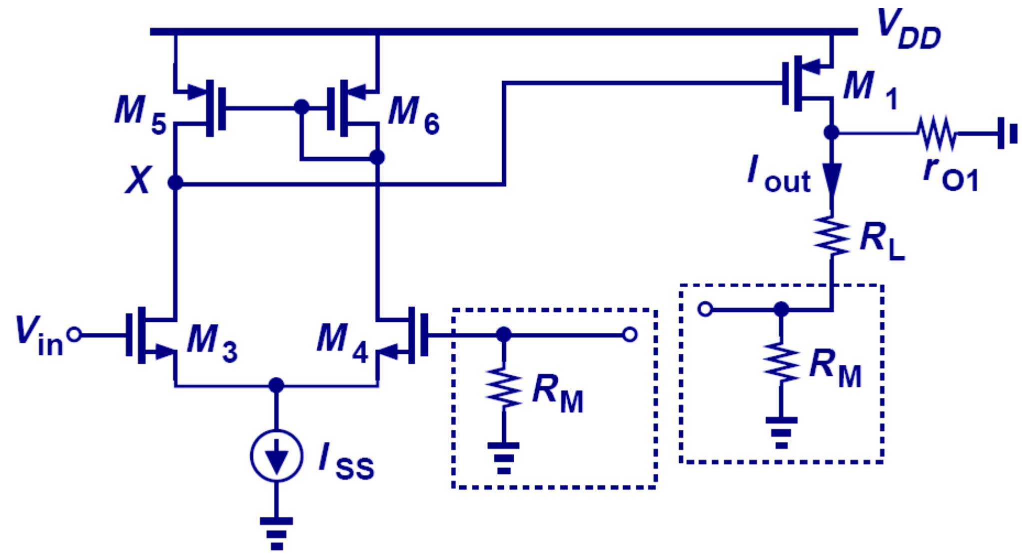
$$\frac{V_{out}}{I_{in}}\Big|_{closed} = \frac{V_{out}}{I_{in}}\Big|_{open} \Big/\left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

$$R_{in,closed} = R_{in,open} \Big/\left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

$$R_{out,closed} = R_{out,open} \Big/\left(1 + K\frac{V_{out}}{I_{in}}\Big|_{open}\right)$$

Breaking the Loop Example V



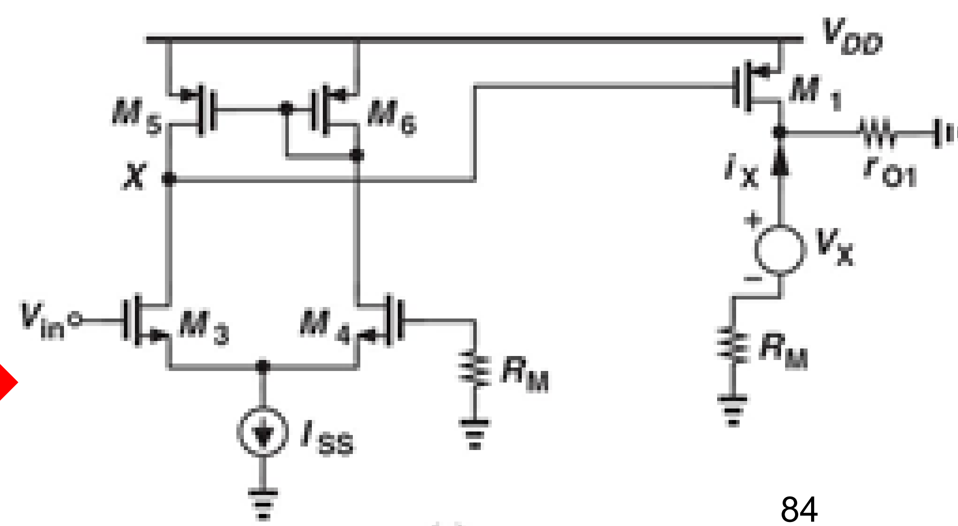


$$\frac{I_{out}}{V_{in}}\bigg|_{open} = \frac{g_{m3}(r_{O3} \| r_{O5})g_{m1}r_{O1}}{r_{O1} + R_L + R_M}$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{O1} + R_M$$

For a current output with feedback, the output resistance must be obtained with the test stimulus in series



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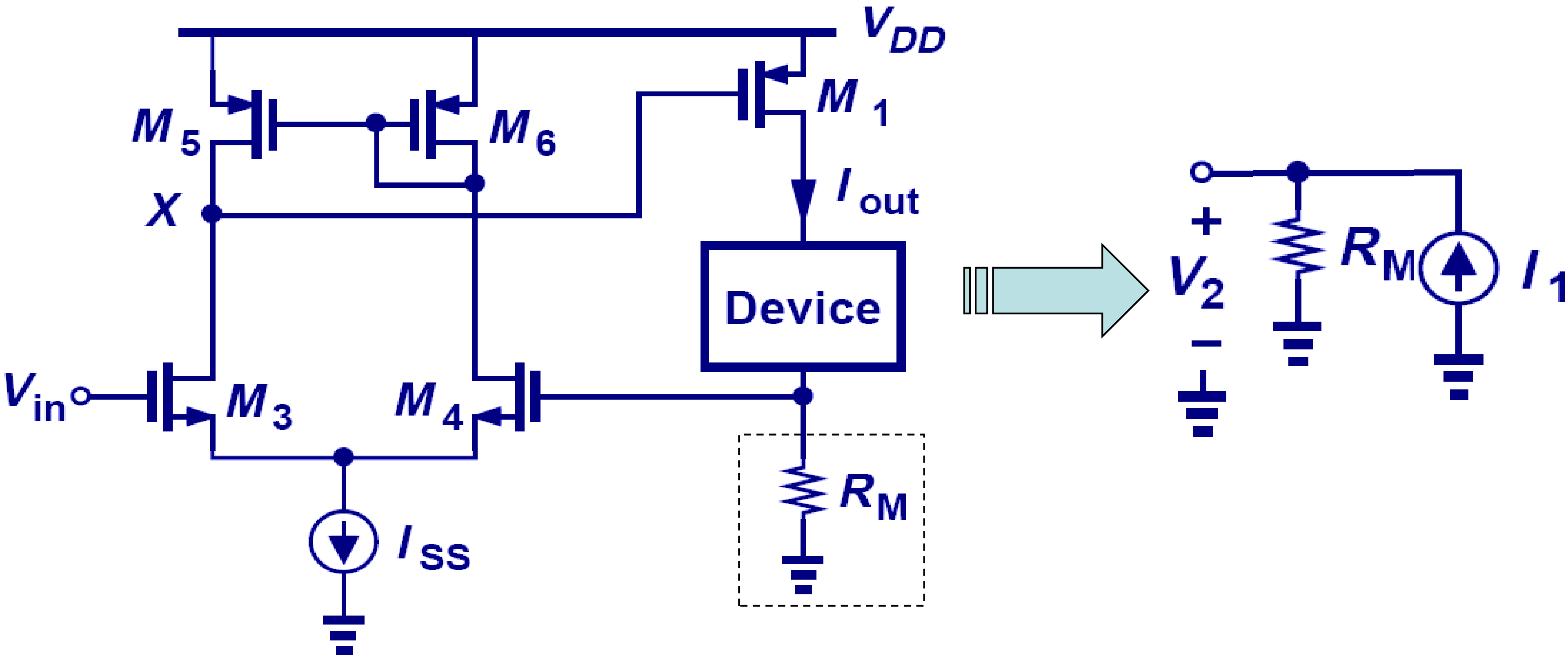
Feedback Factor Example V



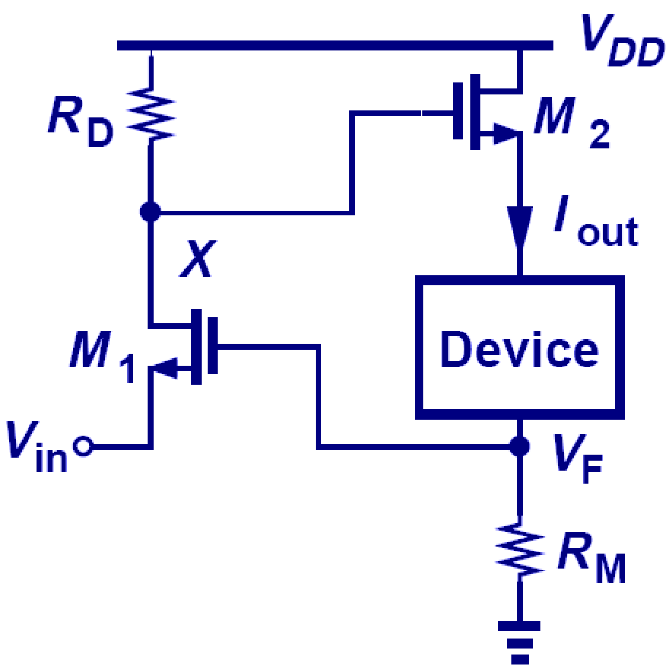
$$K = R_M$$

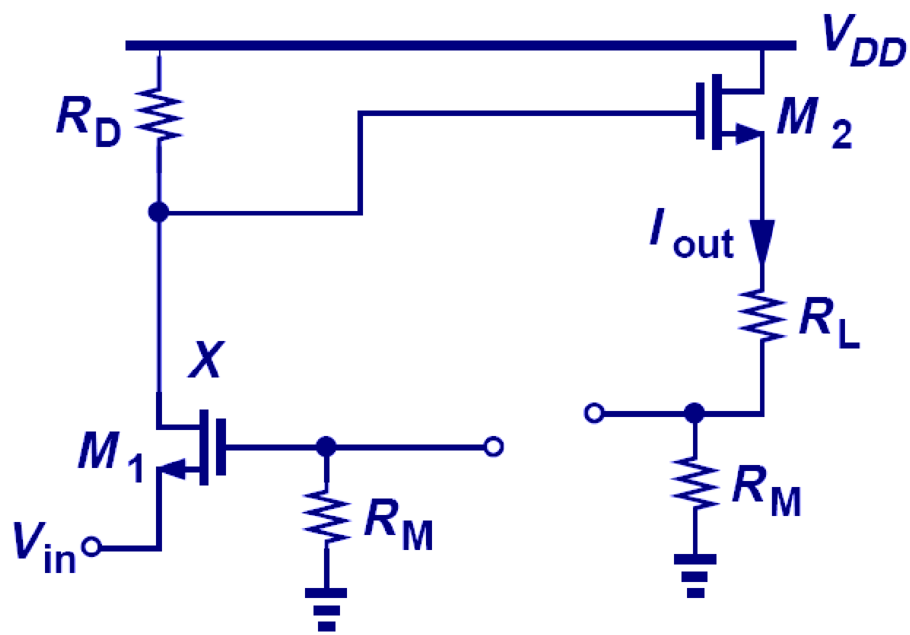
$$(I_{out}/V_{in}|_{closed}) = (I_{out}/V_{in}|_{open})/[1+K(I_{out}/V_{in})|_{open}]$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open}[1+K(I_{out}/V_{in})|_{open}]$$

Breaking the Loop Example VI

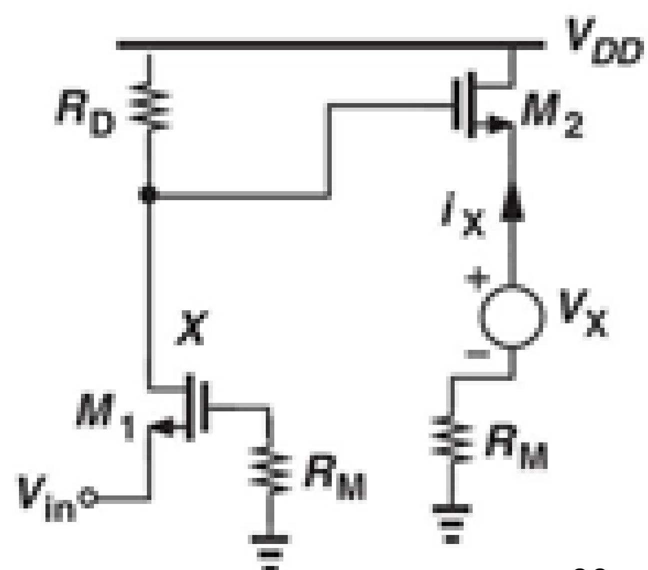




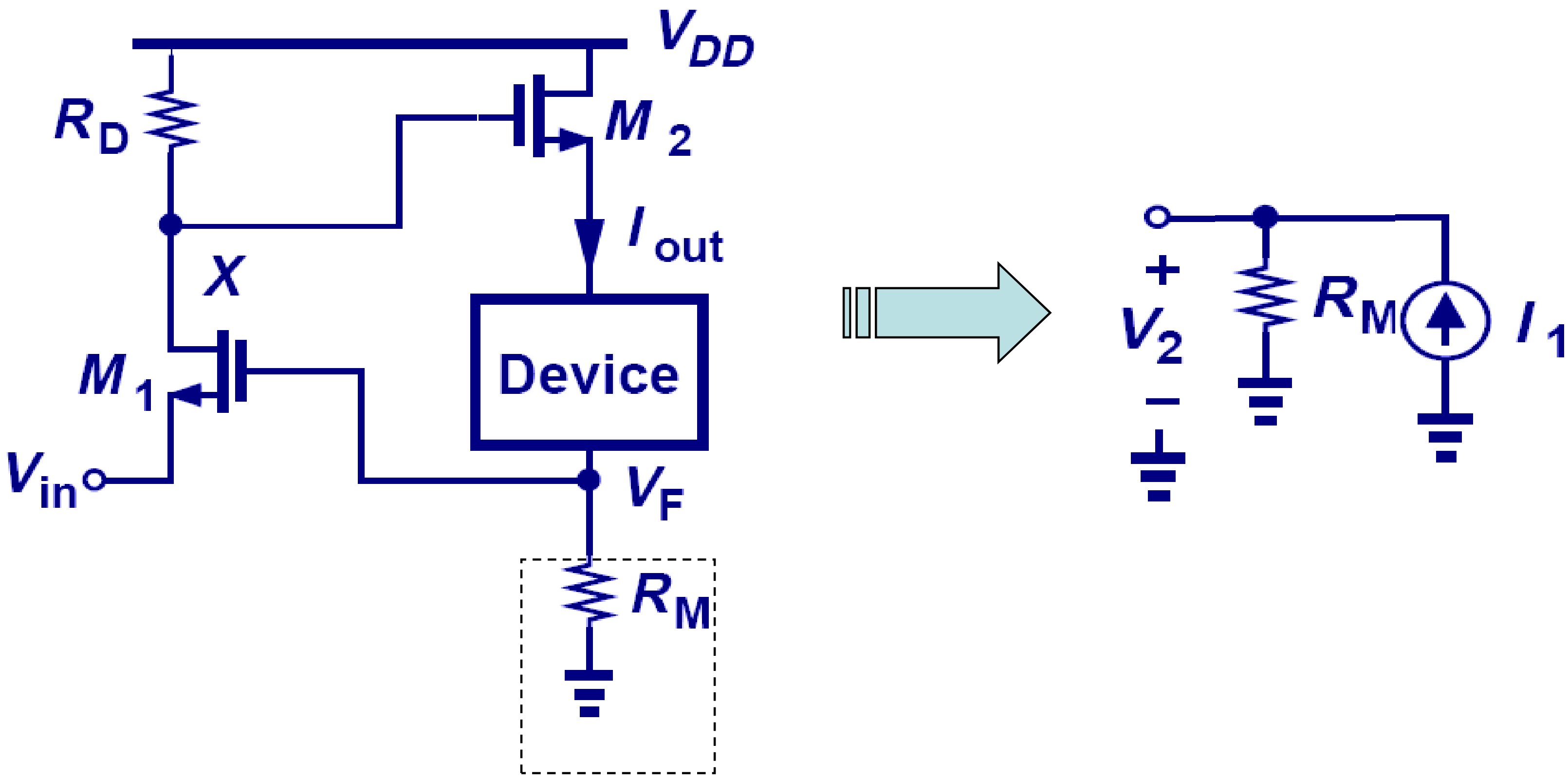
$$\frac{I_{out}}{V_{in}}\Big|_{open} = \frac{g_{m1}R_D}{R_L + R_M + 1/g_{m2}}$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = (1/g_{m2}) + R_M$$



Feedback Factor Example VI



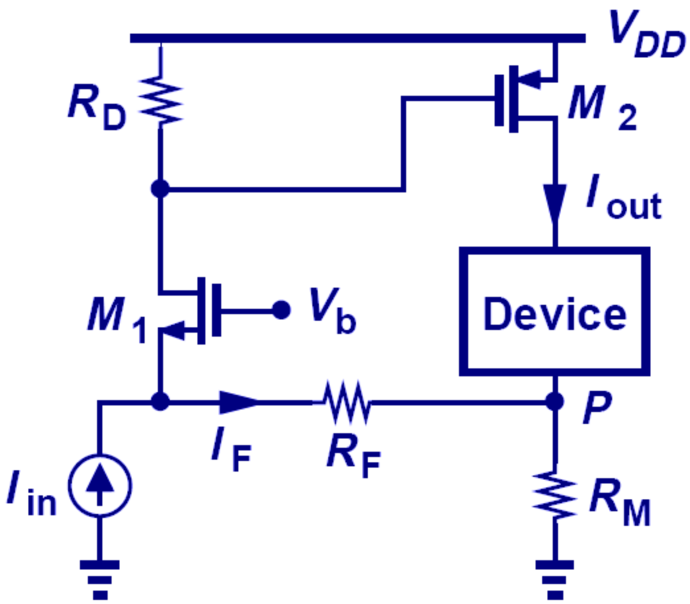
$$K = R_M$$

$$(I_{out}/V_{in}|_{closed}) = (I_{out}/V_{in}|_{open})/[1 + K(I_{out}/V_{in})|_{open}]$$

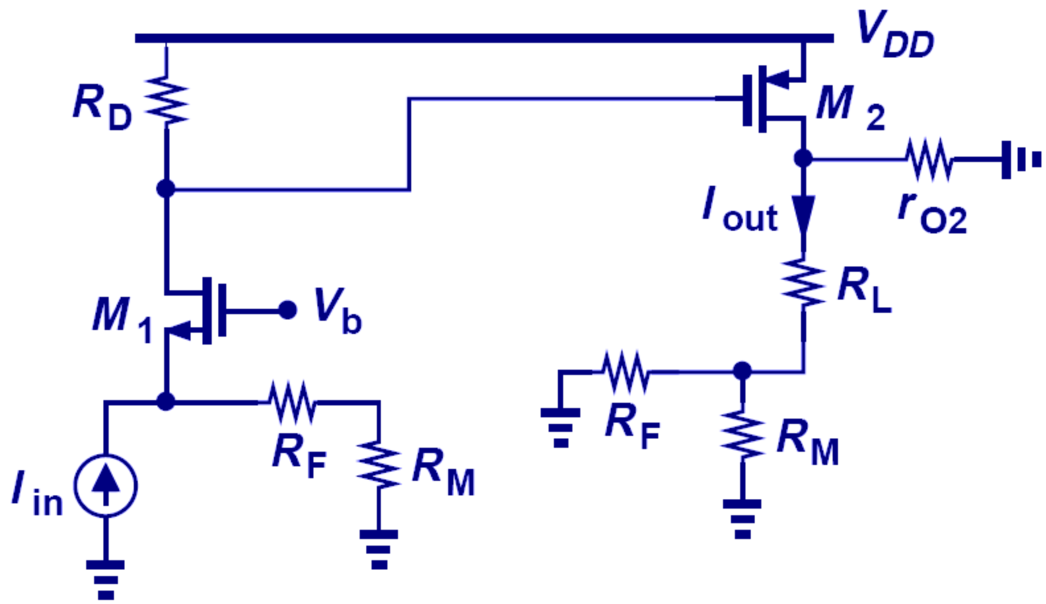
$$R_{in,closed} = R_{in,open}[1 + K(I_{out}/V_{in})|_{open}]$$

$$R_{out,closed} = R_{out,open}[1 + K(I_{out}/V_{in})|_{open}]$$

Breaking the Loop Example VII



$$\lambda_1 = 0$$
$$\lambda_2 > 0$$

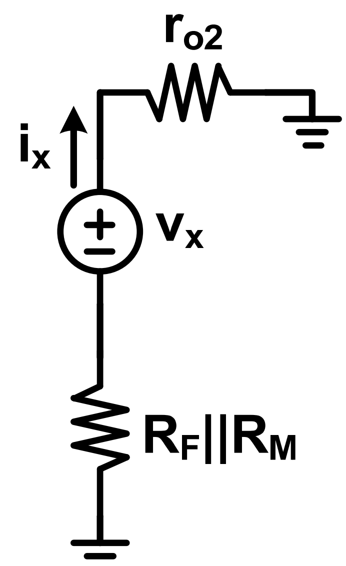


$$A_{I,open} = \frac{(R_F + R_M)R_D}{R_F + R_M + \frac{1}{g_{m1}}} \cdot \frac{-g_{m2} r_{O2}}{r_{O2} + R_L + R_M \parallel R_F}$$

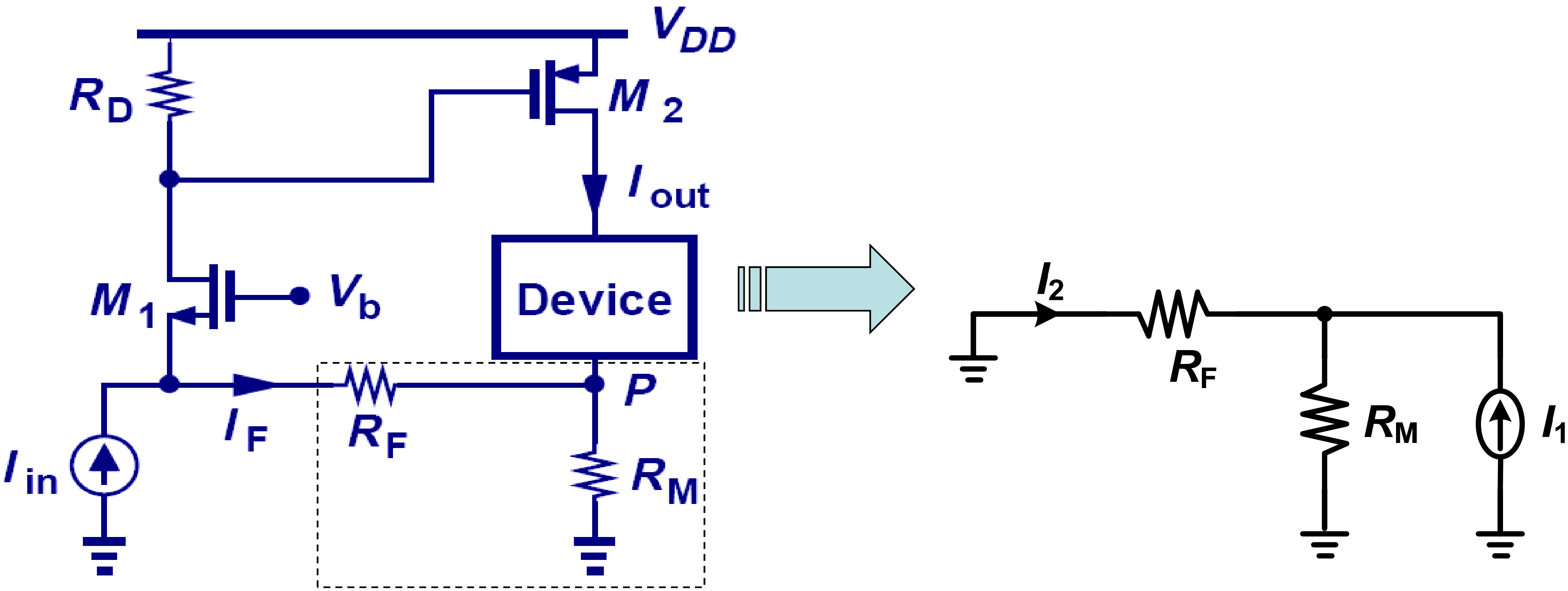
$$R_{in,open} = \frac{1}{g_{m1}} \parallel (R_F + R_M)$$

$$R_{out,open} = r_{O2} + R_F \parallel R_M$$

Rout Eq. Circuit



Feedback Factor Example VII



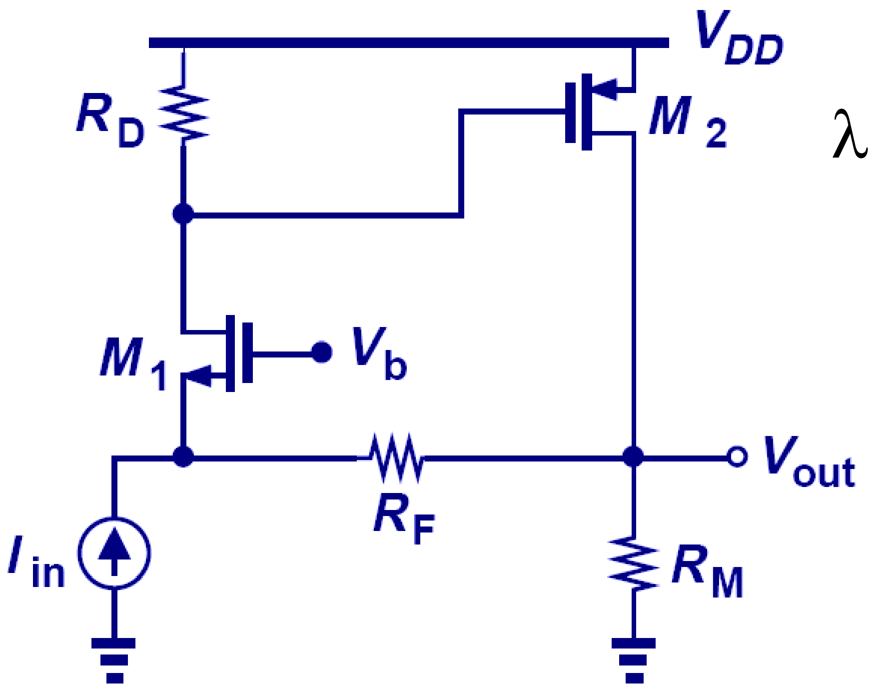
$$K = -R_M / (R_F + R_M)$$

$$A_{I,closed} = A_{I,open} / (1 + K A_{I,open})$$

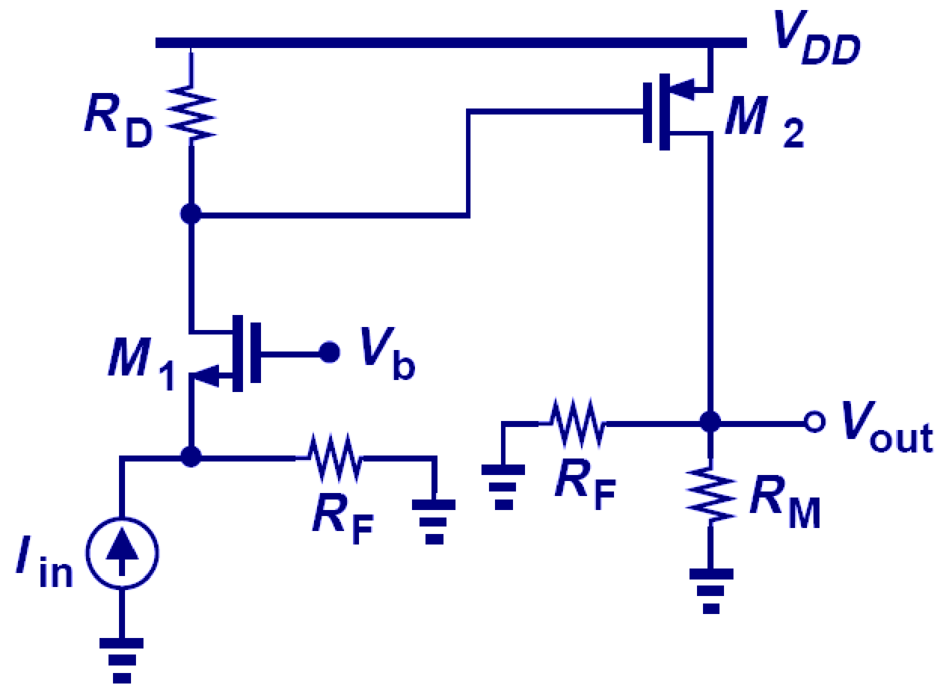
$$R_{in,closed} = R_{in,open} / (1 + K A_{I,open})$$

$$R_{out,closed} = R_{out,open} (1 + K A_{I,open})$$

Breaking the Loop Example VIII



$$\lambda = 0$$

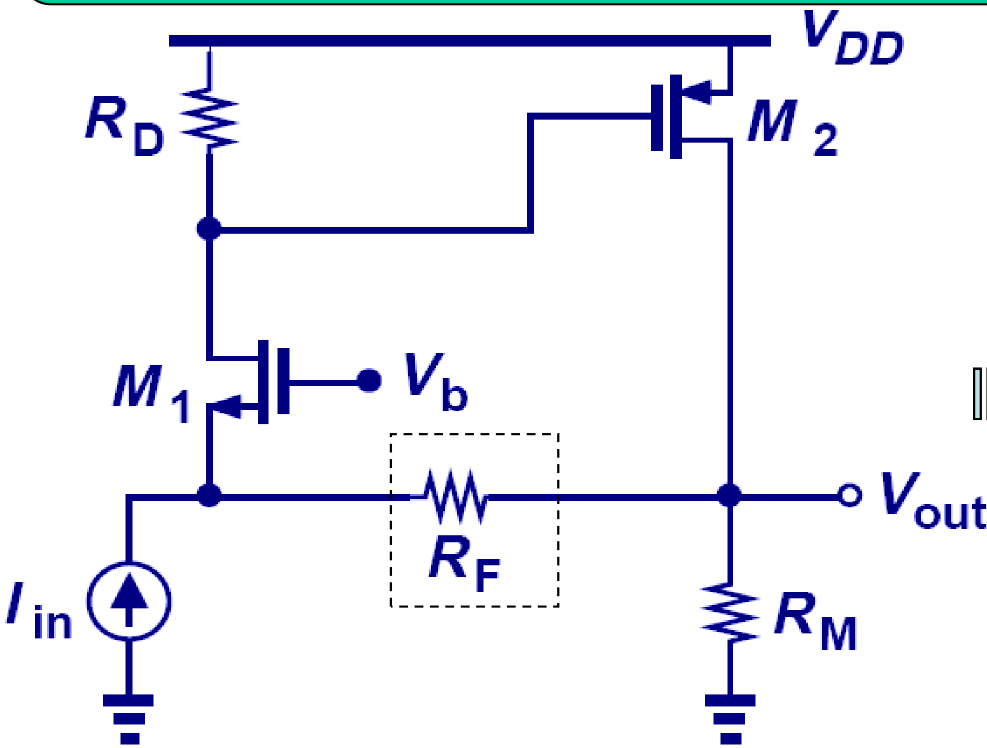


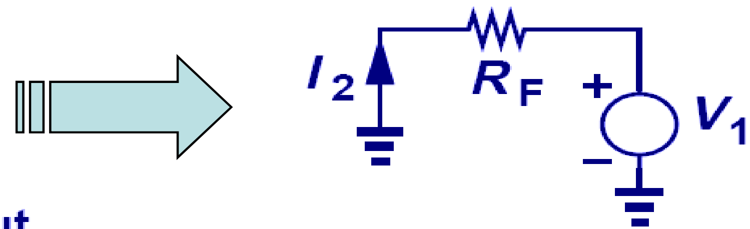
$$\frac{V_{out}}{I_{in}}\bigg|_{open} = \frac{R_F R_D}{R_F + 1/g_{m1}}\left[-g_{m2}(R_F \parallel R_M)\right]$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel R_F$$

$$R_{out,open} = R_F \parallel R_M$$

Feedback Factor Example VIII





$$K = -1 / R_F$$

$$(V_{out} / I_{in})|_{closed} = (V_{out} / I_{in})|_{open} / [1 + K(V_{out} / I_{in})|_{open}]$$

$$R_{in,closed} = R_{in,open} / [1 + K(V_{out} / I_{in})|_{open}]$$

$$R_{out,closed} = R_{out,open} / [1 + K(V_{out} / I_{in})|_{open}]$$

Agenda

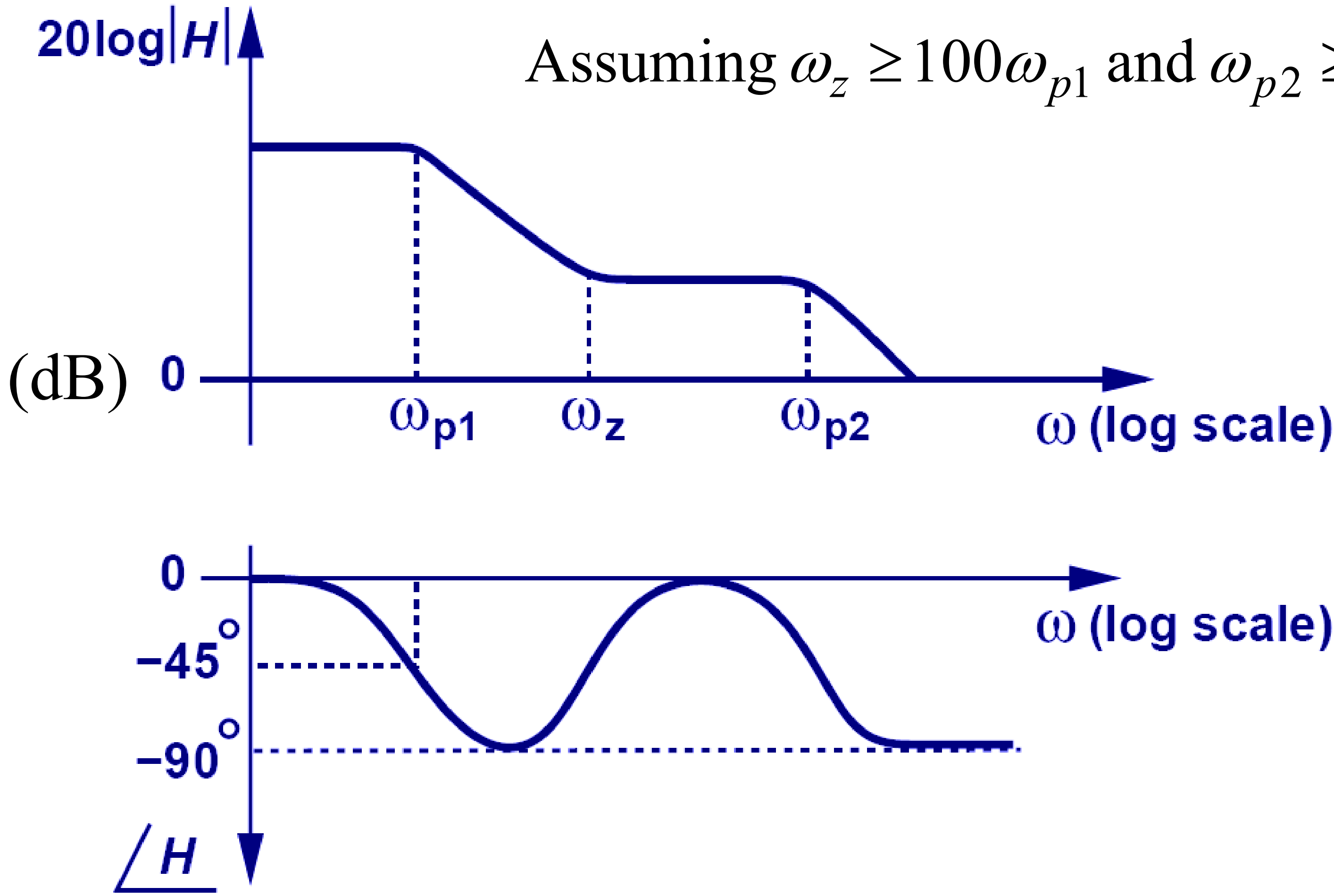
- Feedback Overview
- Feedback Properties
- Amplifier Types
- Sense and Return Techniques
- Feedback Polarity
- Feedback Topologies
- Effect of Nonideal I/O Impedances
- Stability
- Two-Stage Miller OTA

Bode Plot Algorithm - Phase

1. Calculate low frequency value of Phase(H($j\omega$))
   a. An negative sign introduces -180° phase shift
   b. A DC pole introduces -90° phase shift
   c. A DC zero introduces +90° phase shift
2. Where are the poles and zeros?
   a. For negative poles: 1 dec. before the pole freq., the phase will decrease with a slope of -45°/dec. until 1 dec. after the pole freq., for a total phase shift of -90°
   b. For negative zeros: 1 dec. before the zero freq., the phase will increase with a slope of +45°/dec. until 1 dec. after the zero freq., for a total phase shift of +90°
   c. Note, if you have positive poles or zeros, the phase change polarity is inverted
3. Note, the above algorithm is only valid for real poles and zeros.

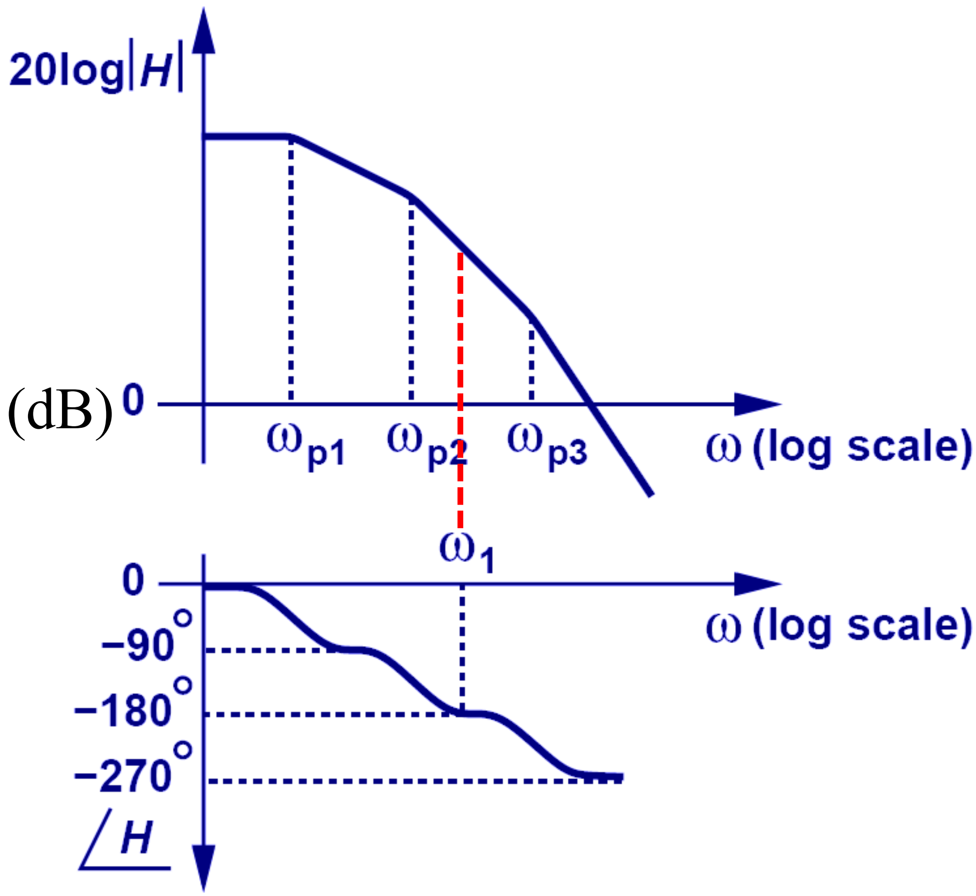
Example: Phase Response

Assuming $\omega_z \geq 100\omega_{p1}$ and $\omega_{p2} \geq 100\omega_z$



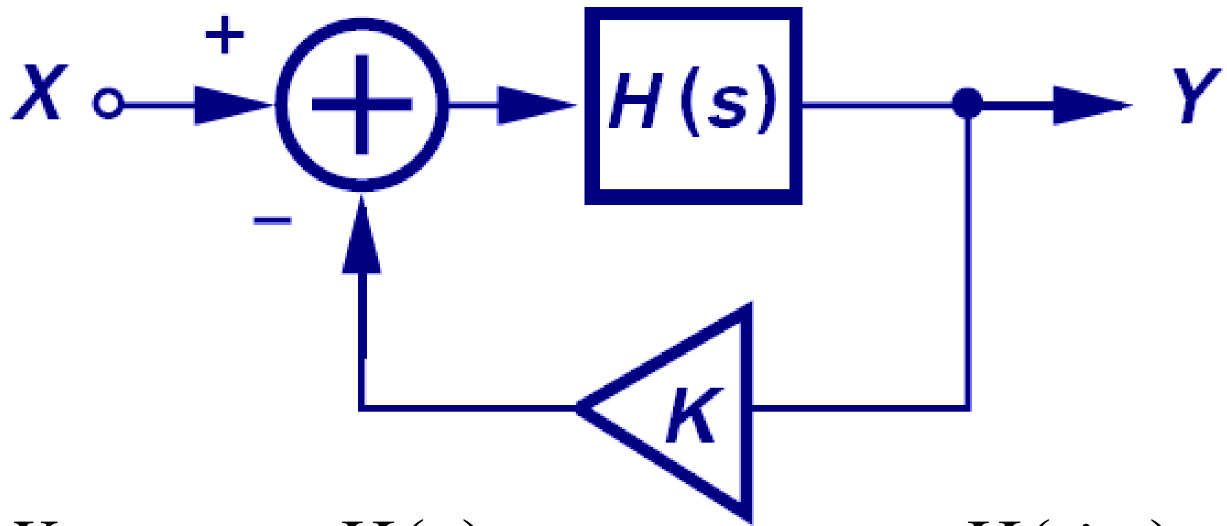
- **Assuming general negative (left-half plane) poles and zeros, the phase of H(jω) starts to drop at 1/10 of the pole, hits -45° at the pole, and approaches -90° at 10 times the pole.**

Example: Three-Pole System



- For a three-pole system, a finite frequency produces a phase of -180°, which means an input signal that operates at this frequency will have its output inverted.

Instability of a Negative Feedback Loop

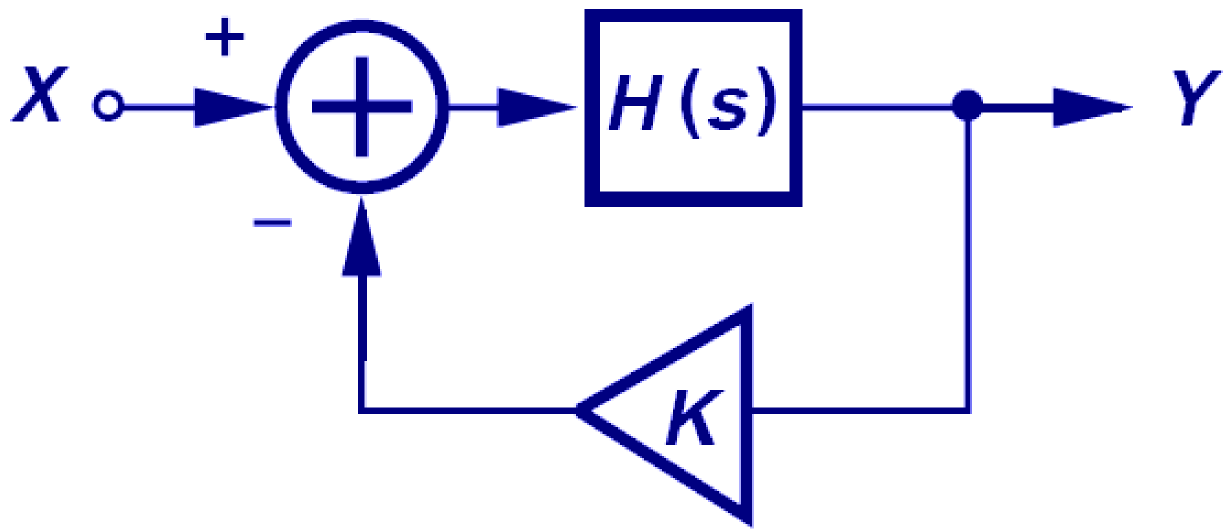


$$\frac{Y}{X}(s) = \frac{H(s)}{1+KH(s)} \xrightarrow{s=j\omega} \frac{H(j\omega)}{1+KH(j\omega)}$$

$$\text{If } KH(j\omega_1) = -1, \text{ then } \frac{Y}{X}(j\omega_1) = \frac{H(j\omega_1)}{0} = \infty$$

- **Substitute $j\omega$ for s. If for a certain ω_1, $KH(j\omega_1)$ reaches -1, the closed loop gain becomes infinite. This implies for a very small input signal (or inherent system noise) at ω_1, the output can be very large. Thus the system becomes unstable.**

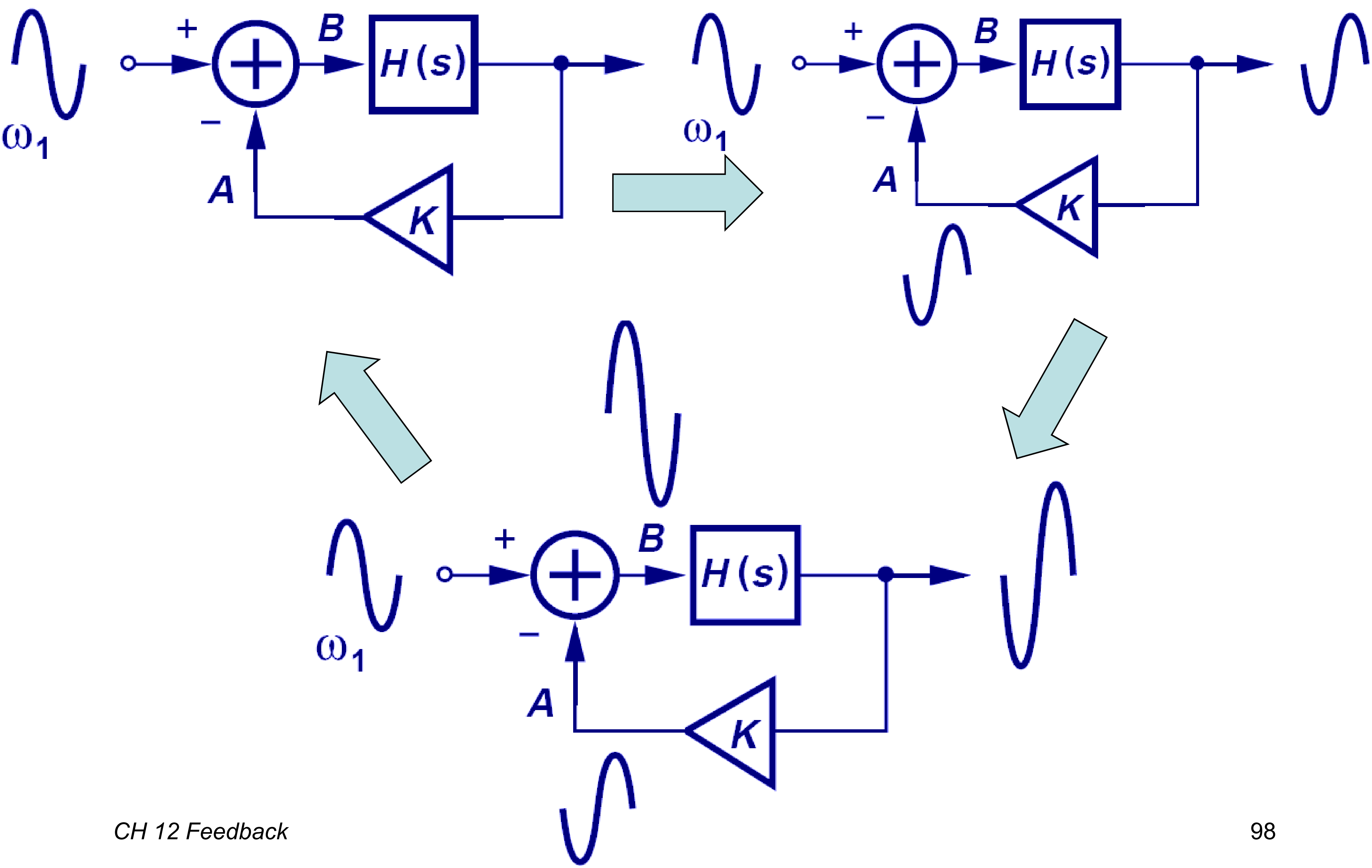
"Barkhausen's Criteria" for Oscillation



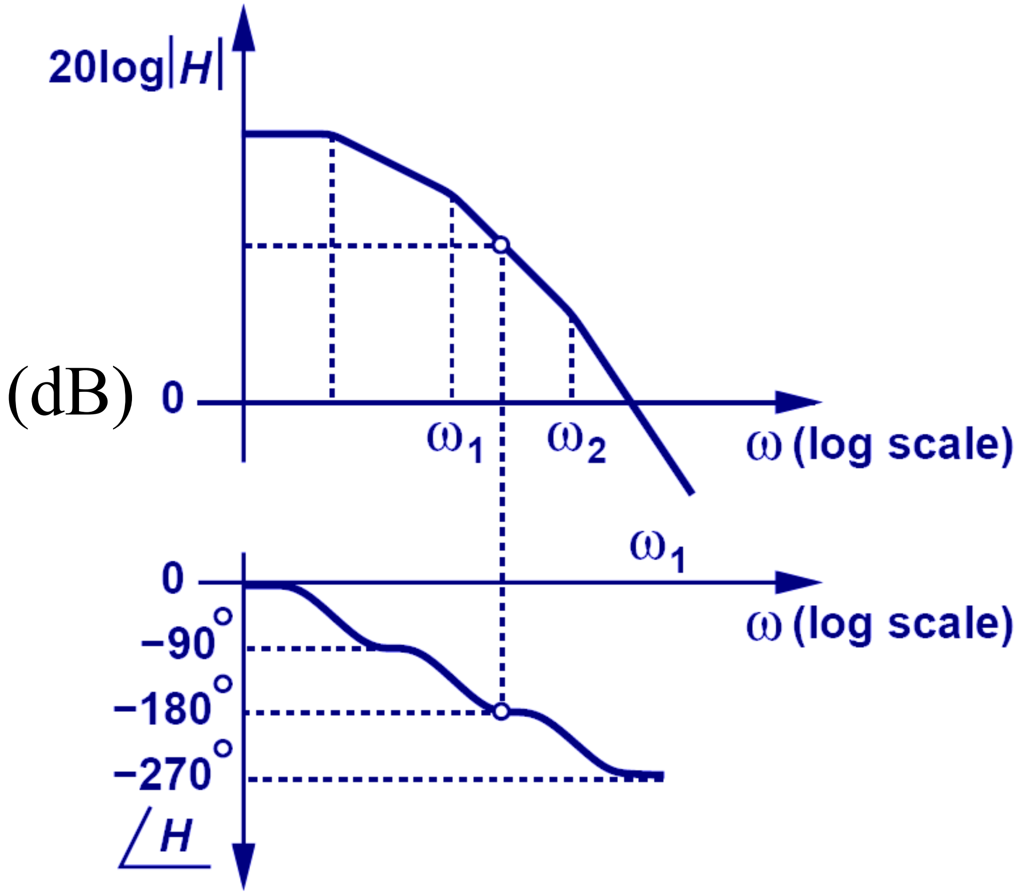
$$|KH(j\omega_1)| \geq 1$$
$$\angle KH(j\omega_1) = -180$$

- We want our linear amplifiers to be stable (not oscillate)
- Thus, we don't want this criteria to be satisfied

Time Evolution of Instability



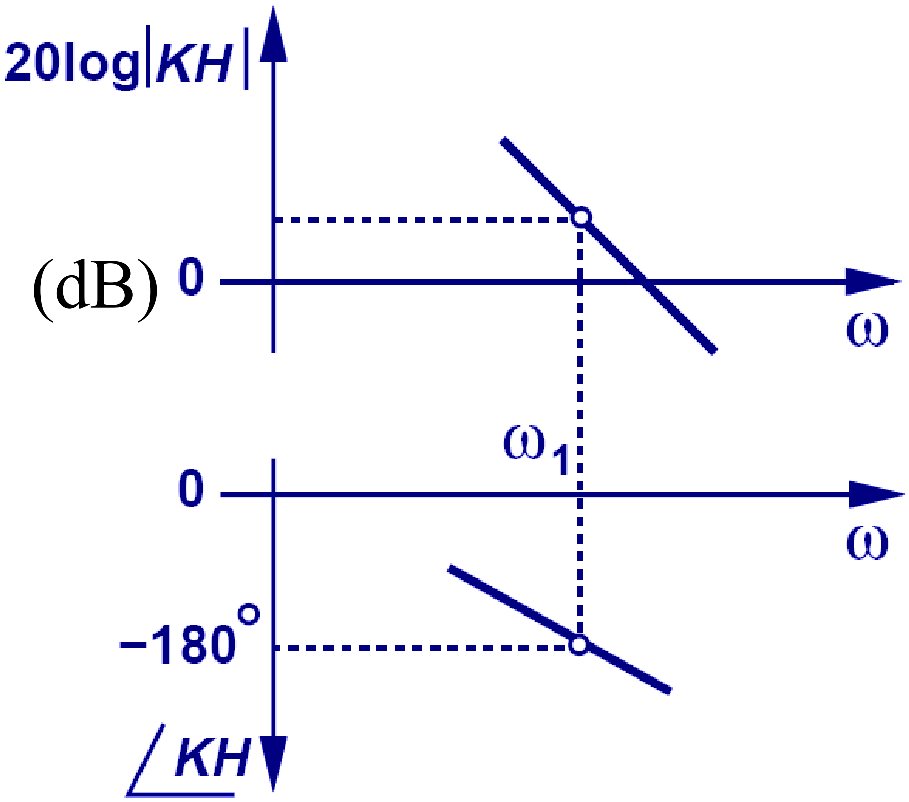
Oscillation Example



(c)

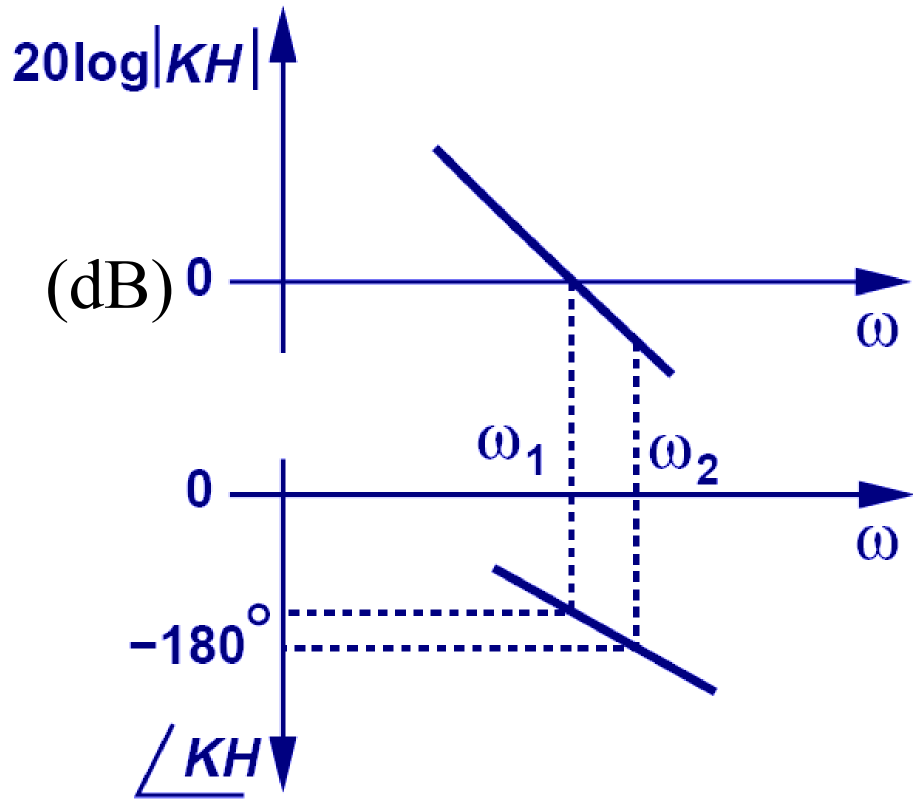
- This system oscillates, since there's a finite frequency at which the phase is -180° and the gain is greater than unity. In fact, this system exceeds the minimum oscillation requirement.

Condition for Oscillation



Un-Stable

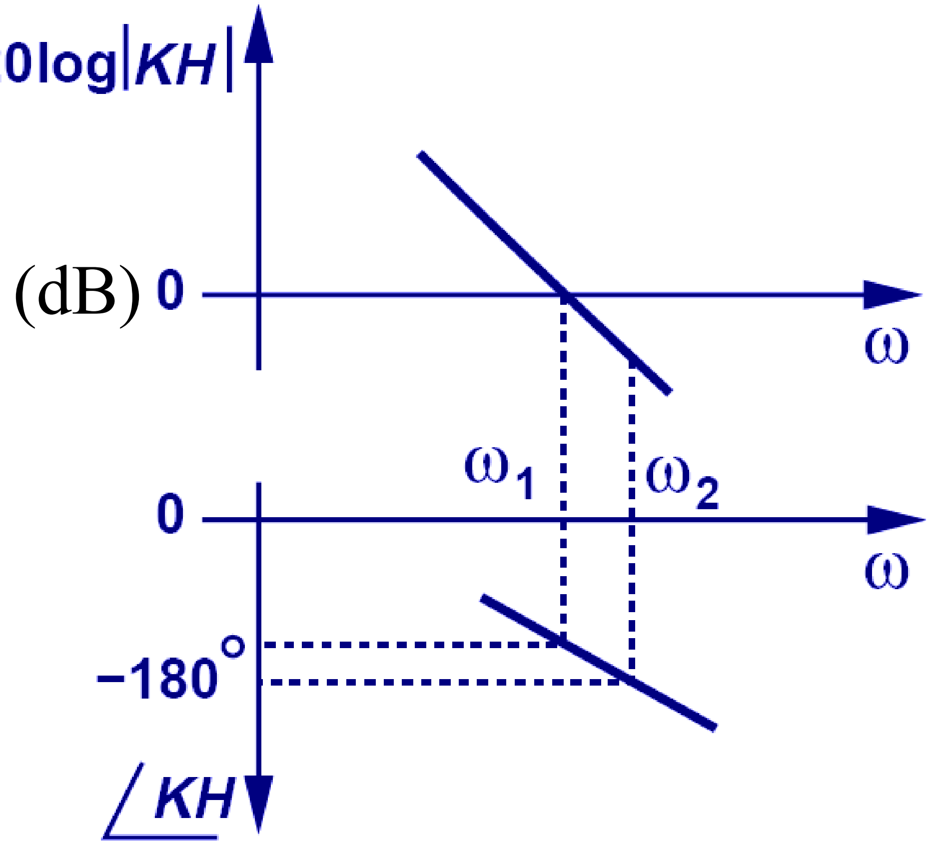
When $\angle KH = -180^\circ,\ |KH| > 1$



Stable

When $\angle KH = -180^\circ,\ |KH| < 1$

Condition for Stability



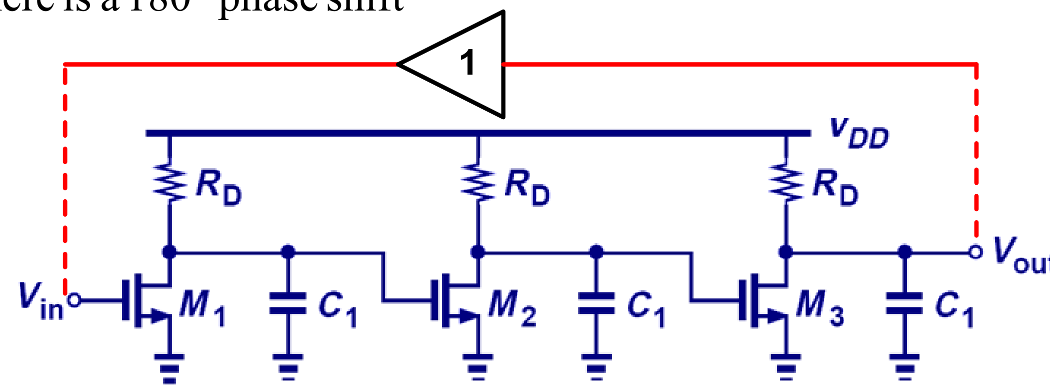
$$\text{For Stability}: \ \omega_{GX} < \omega_{PX}$$

- ω_{PX}, ("phase crossover"), is the frequency at which $\angle KH = -180^o$ (ω_2 above)
- ω_{GX}, ("gain crossover"), is the frequency where $|KH|=1$ (ω_1 above)

Stability Example I

For Stability with the worst-case feedback factor $(K=1)$, we need to find the open-loop magnitude response $|H_p|<1$ when there is a 180° phase shift

$$KH(s)=\frac{(-g_mR_D)^3}{\left(1+\frac{s}{\omega_p}\right)^3} \quad \text{where } \omega_p=\frac{1}{R_DC_1}$$



1. Find the phase crossover frequency, ω_{PX}

$$\angle H(j\omega_{PX})=-3\tan^{-1}\left(\frac{\omega_{PX}}{\omega_p}\right)=-180^\circ$$

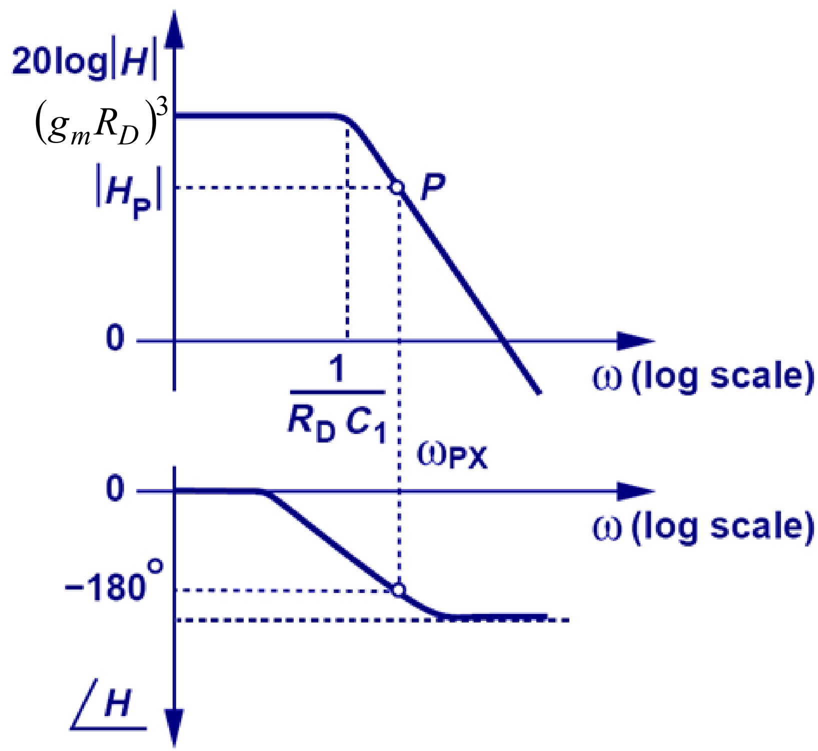
$$\omega_{PX}=\sqrt{3}\omega_p$$

2. The KH magnitude at ω_{PX} must be less than unity

$$\frac{(g_mR_D)^3}{\left[\sqrt{1+\left(\frac{\omega_{PX}}{\omega_p}\right)^2}\right]^3}=\frac{(g_mR_D)^3}{\left[\sqrt{1+\left(\sqrt{3}\right)^2}\right]^3}<1$$

3. This implies that the low-frequency gain

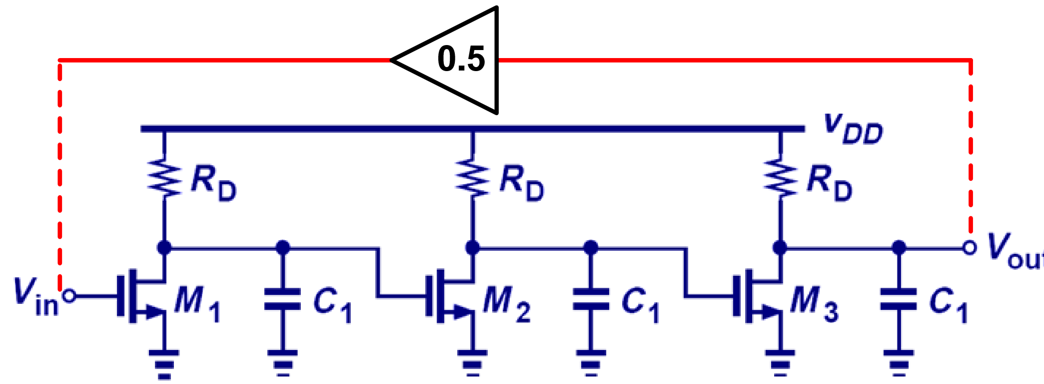
$$g_mR_D<2$$



Stability Example II

Now the feedback factor has been reduced to $K = 0.5$

$$KH(s) = \frac{(0.5)(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3} \qquad \text{where } \omega_p = \frac{1}{R_D C_1}$$



1. The phase crossover frequency is the same

$$\angle H(j\omega_{PX}) = -3\tan^{-1}\left(\frac{\omega_{PX}}{\omega_p}\right) = -180^\circ$$

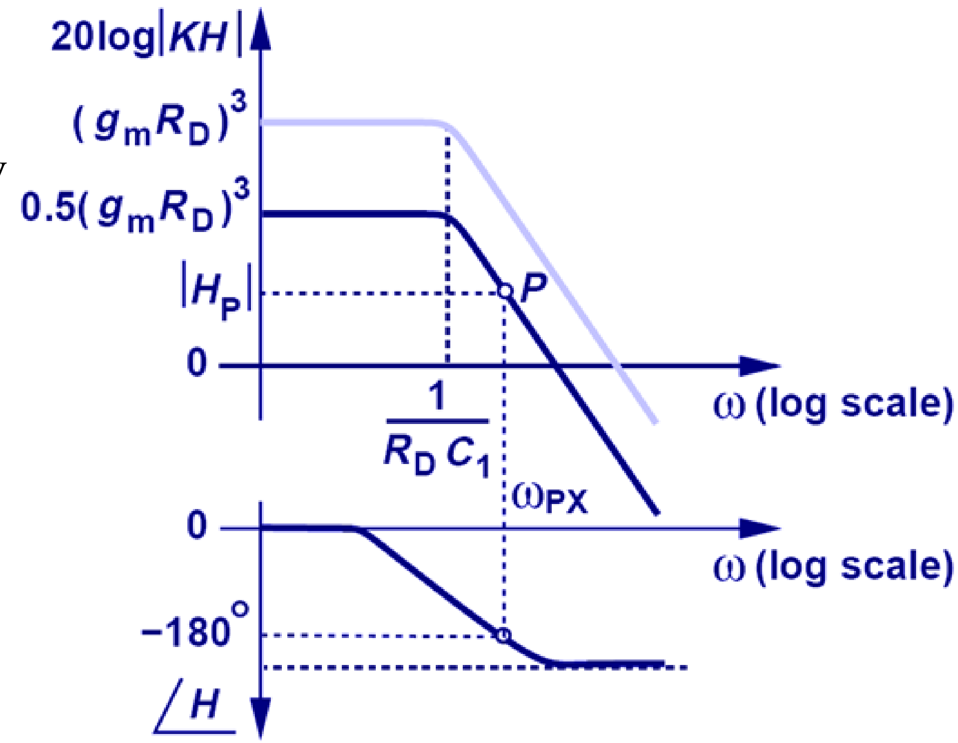
$$\omega_{PX} = \sqrt{3}\,\omega_p$$

2. The KH magnitude at ω_{PX} must be less than unity

$$\frac{(0.5)(g_m R_D)^3}{\left[\sqrt{1 + \left(\frac{\omega_{PX}}{\omega_p}\right)^2}\right]^3} = \frac{(0.5)(g_m R_D)^3}{\left[\sqrt{1 + \left(\sqrt{3}\right)^2}\right]^3} < 1$$

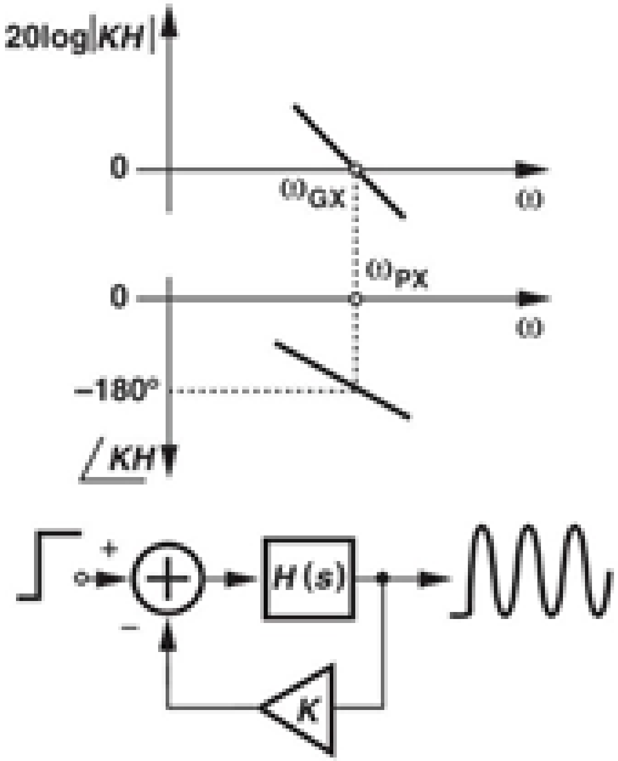
3. This implies that the low-frequency gain

$$g_m R_D < \sqrt[3]{\frac{2^3}{0.5}} = 2.52$$



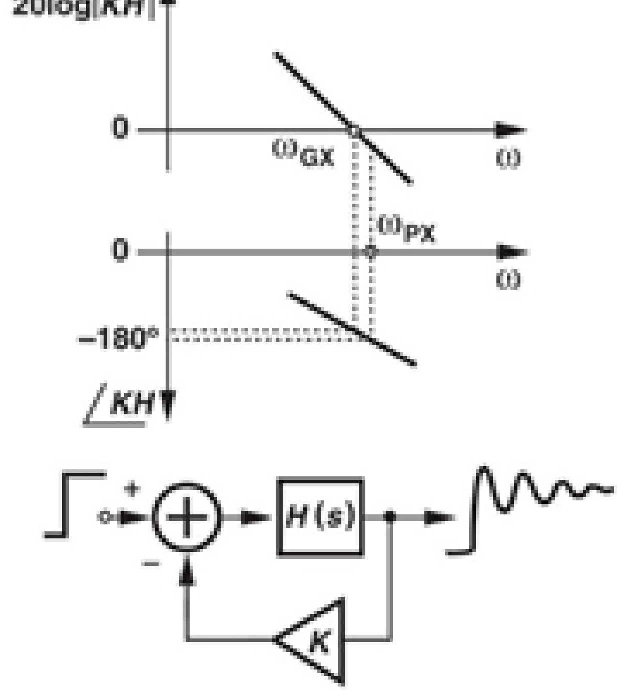
Un-Stable vs. Marginally Stable vs. Stable

$\omega_{GX} \geq \omega_{PX}$



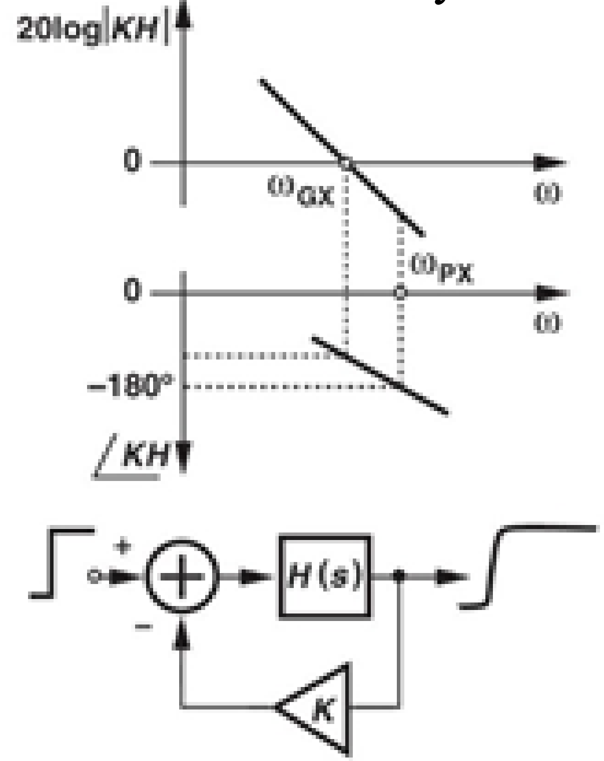
Un-Stable

$\omega_{GX} < \omega_{PX}$, but close



Marginally Stable

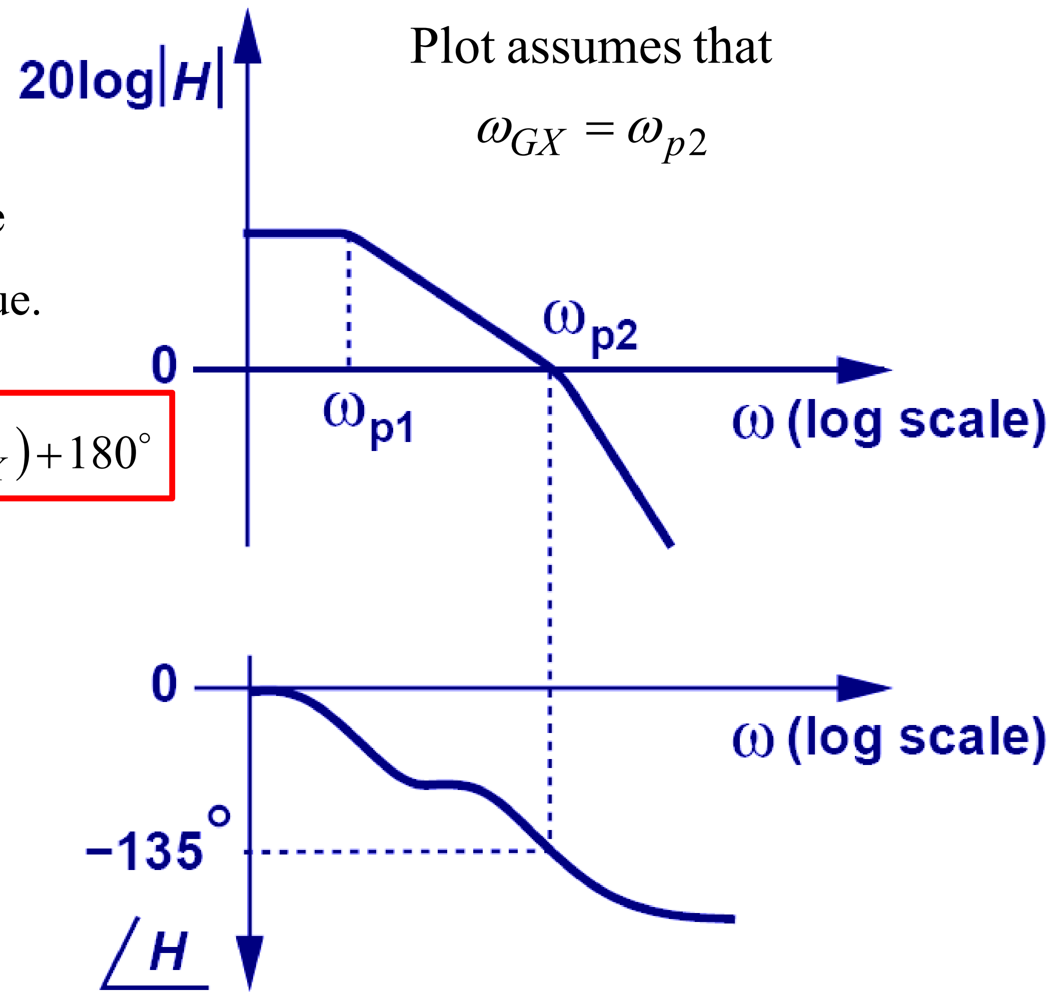
$\omega_{GX} < \omega_{PX}$ sufficiently



Stable

- While the middle system is "Marginally Stable", it has a poor transient step response, in that it displays large ringing which takes a long time to die out

Phase Margin



The "phase margin" quantifies the separation of the loop-gain phase shift from the unstable -180° value.

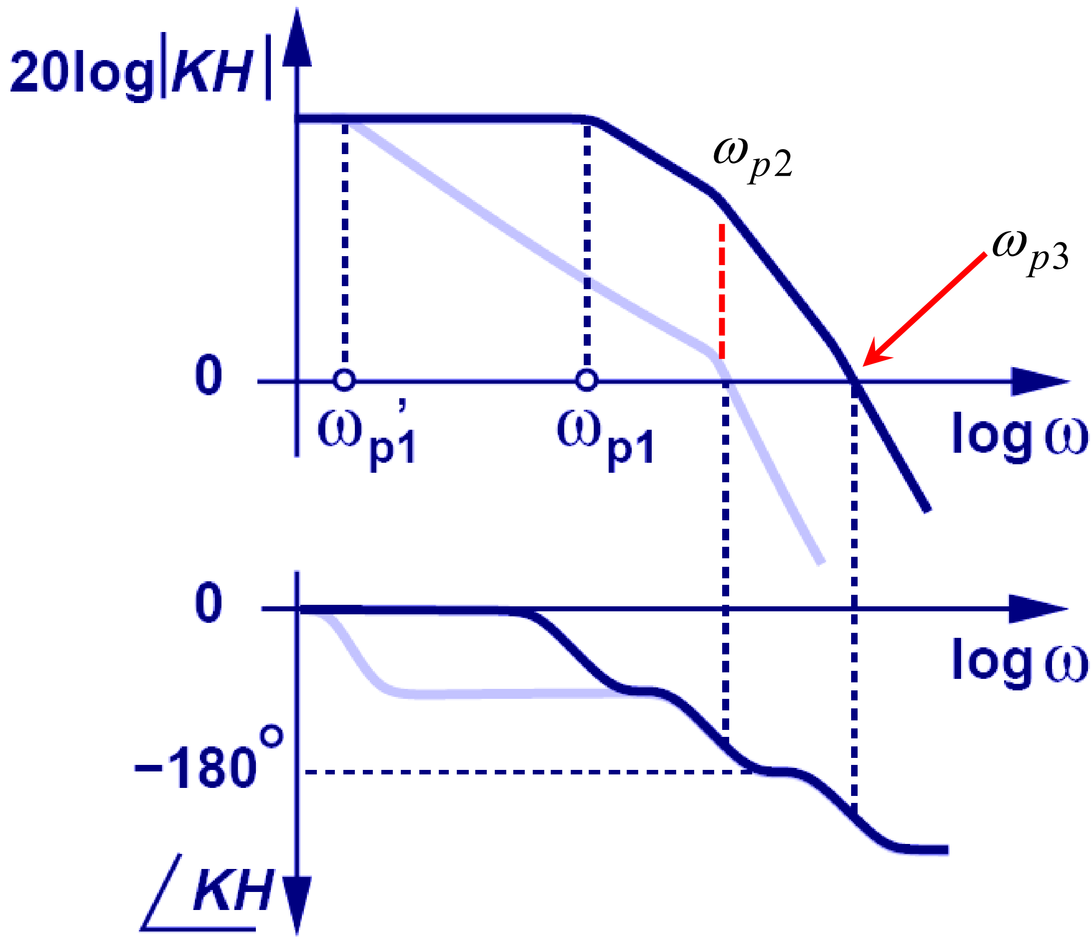
$$\text{Phase Margin} = \angle KH(\omega_{GX}) - (-180^\circ) = \angle KH(\omega_{GX}) + 180^\circ$$

The worst-case phase margin is when K = 1, and is often an amplifier design spec.

$$\text{Worst-Case Phase Margin} = \angle H(\omega_{GX}) + 180^\circ$$

$$\text{Worst-Case PM} = \angle H(\omega_{GX}) + 180^\circ = -135^\circ + 180^\circ = 45^\circ$$

Frequency Compensation



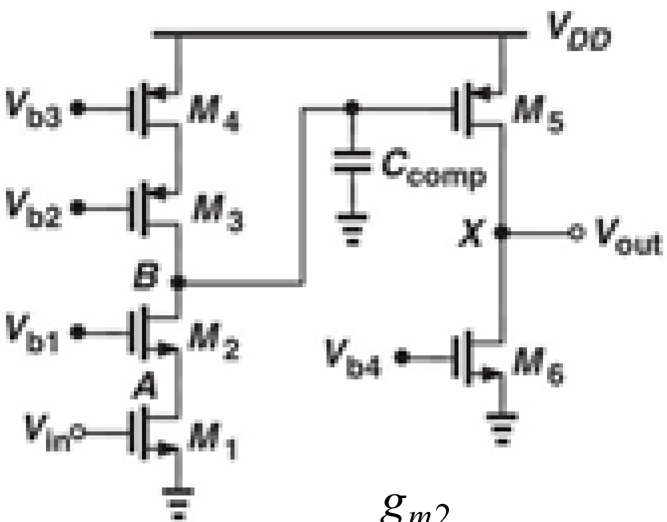
Original system is not stable

$\omega_{GX} > \omega_{PX}$

Moving ω_{p1}' to a lower frequency causes a lower ω_{GX} and allows stability

- **Phase margin can be improved by moving ω_{GX} closer to origin while maintaining ω_{PX} unchanged.**

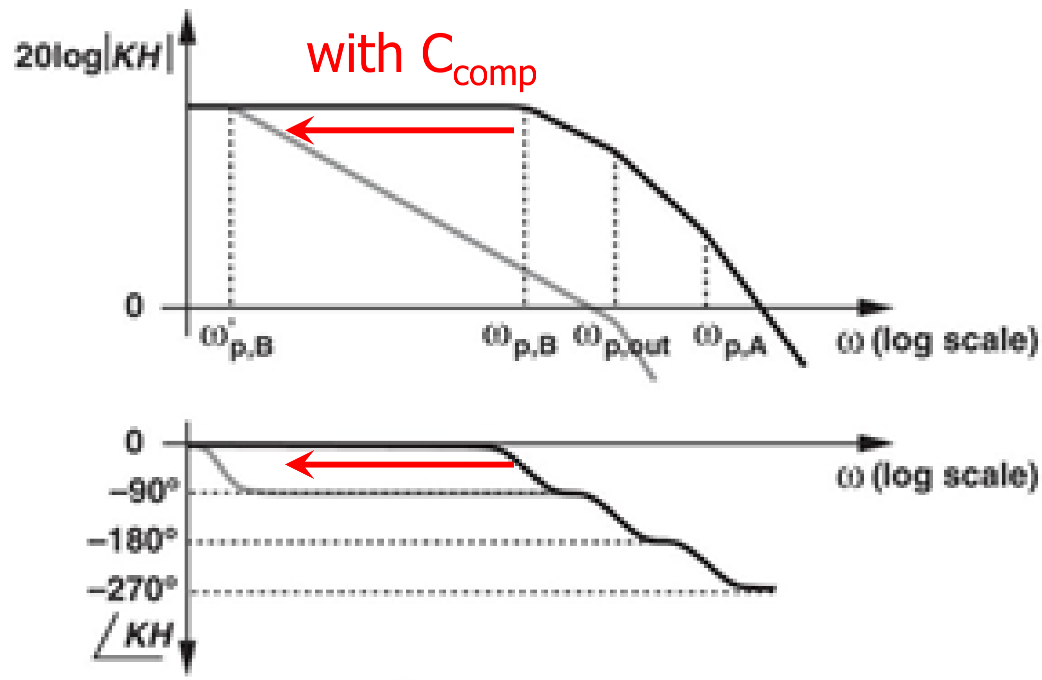
Frequency Compensation Example



$$\omega_{p,A} \approx \frac{g_{m2}}{C_A}$$

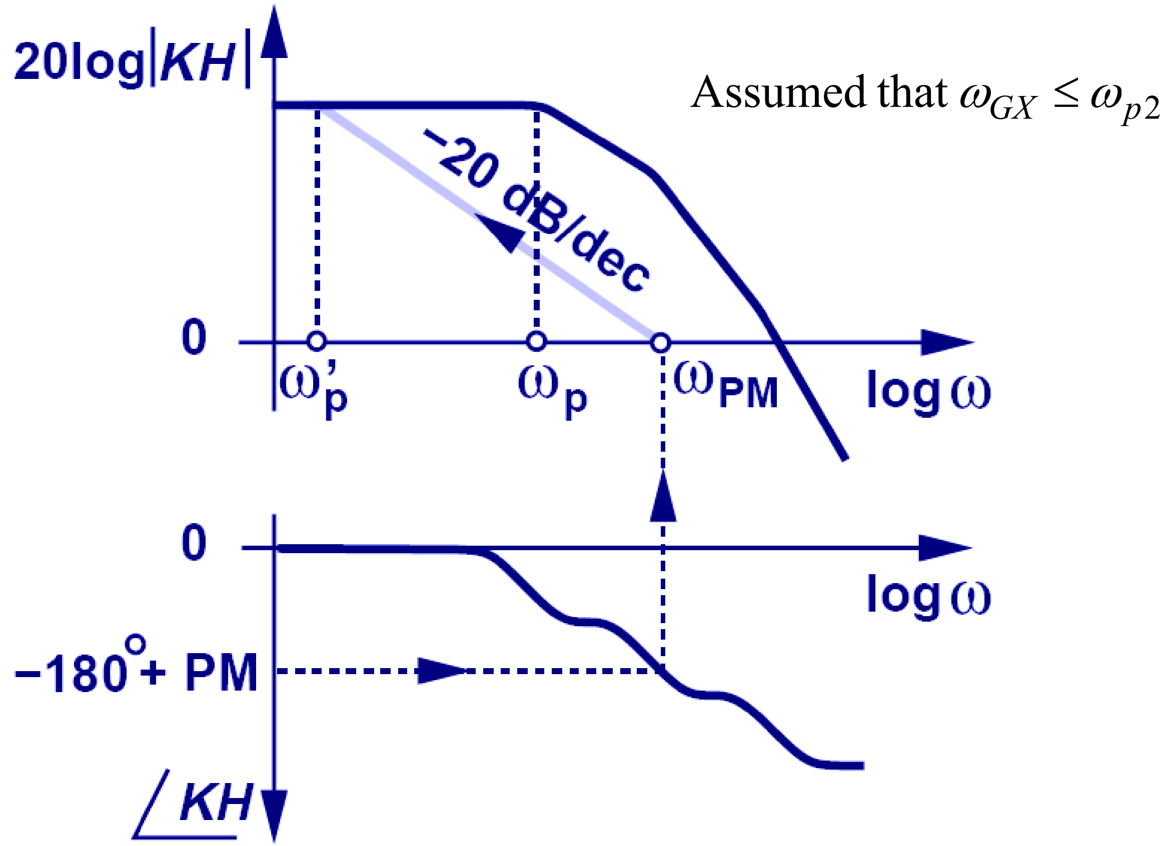
$$\omega_{p,B} \approx \frac{1}{[(g_{m2}r_{O2}r_{O1} \| g_{m3}r_{O3}r_{O4})]C_B}$$

$$\omega_{p,out} = \frac{1}{(r_{O5} \| r_{O6})C_{out}}$$



- C_{comp} is added to lower the dominant pole so that ω_{GX} occurs at a lower frequency than before, which means phase margin increases.

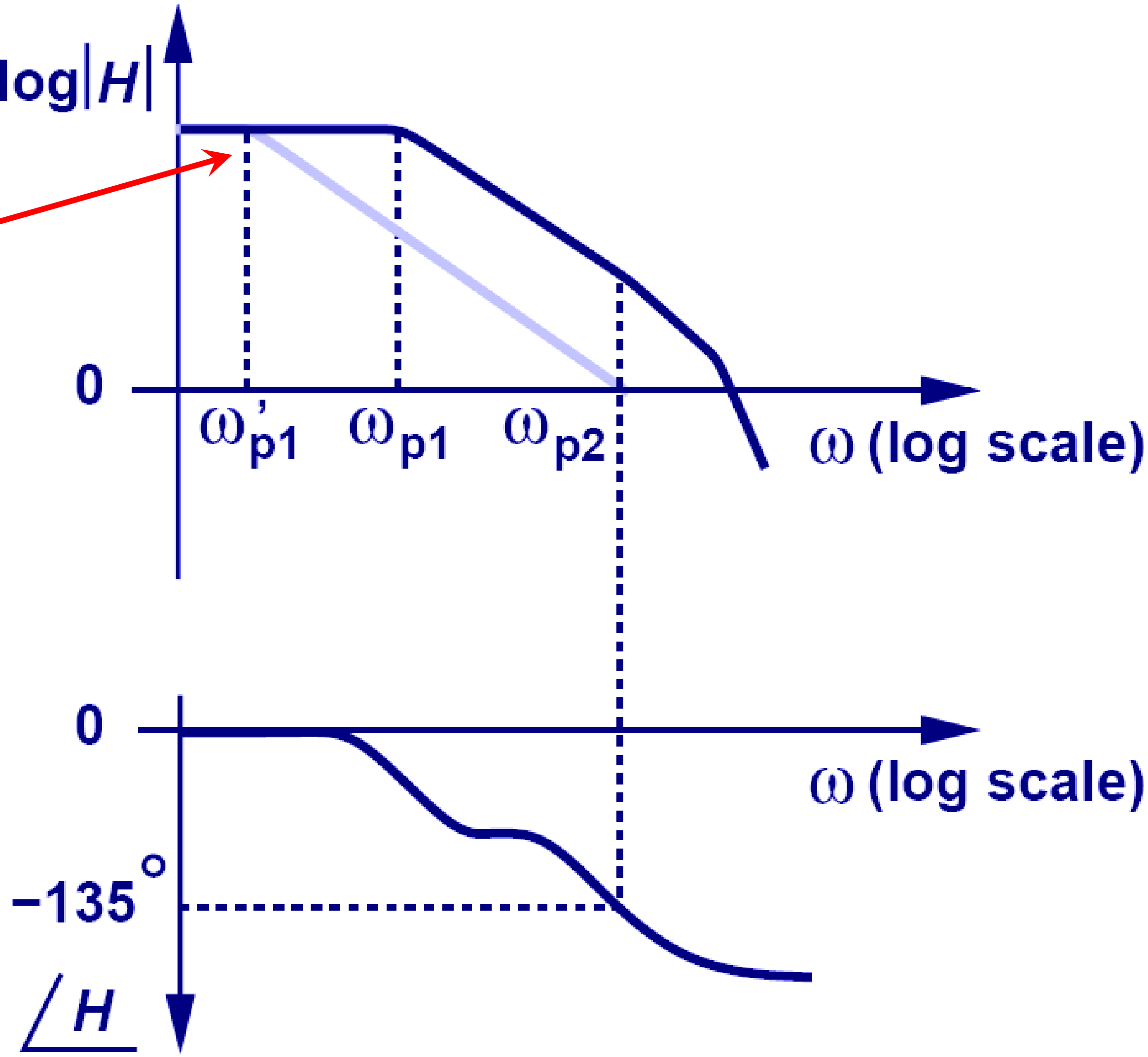
Frequency Compensation Procedure



- 1) We identify a PM, then -180°+PM gives us the new ω_{GX}, or ω_{PM}.
- 2) On the magnitude plot at ω_{PM}, we extrapolate up with a slope of +20dB/dec until we hit the low frequency gain then we look "down" and the frequency we see is our new dominant pole, ω_P'.
  - A slope of 20dB/dec is used, as we assume that we want a PM ≥45°

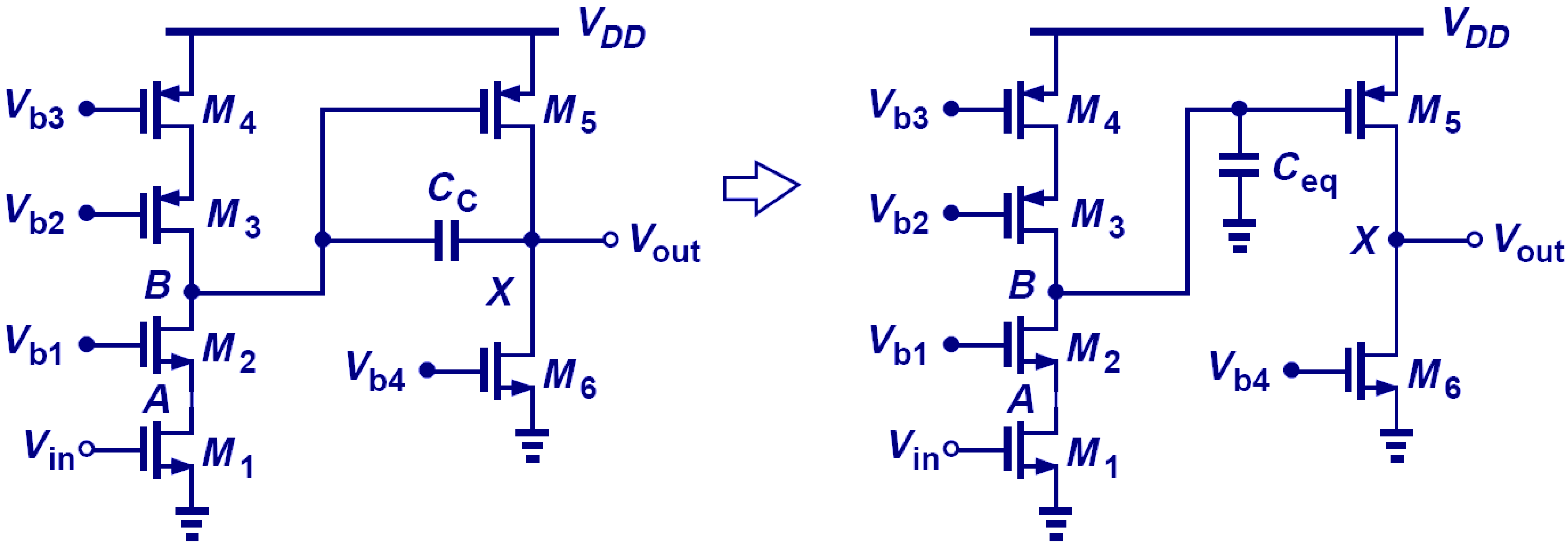
Example: 45° Phase Margin Compensation

$$\omega'_{p1} = \frac{\omega_{p2}}{H_0}$$



$$\omega_{PM} = \omega_{p2}$$

Miller Compensation



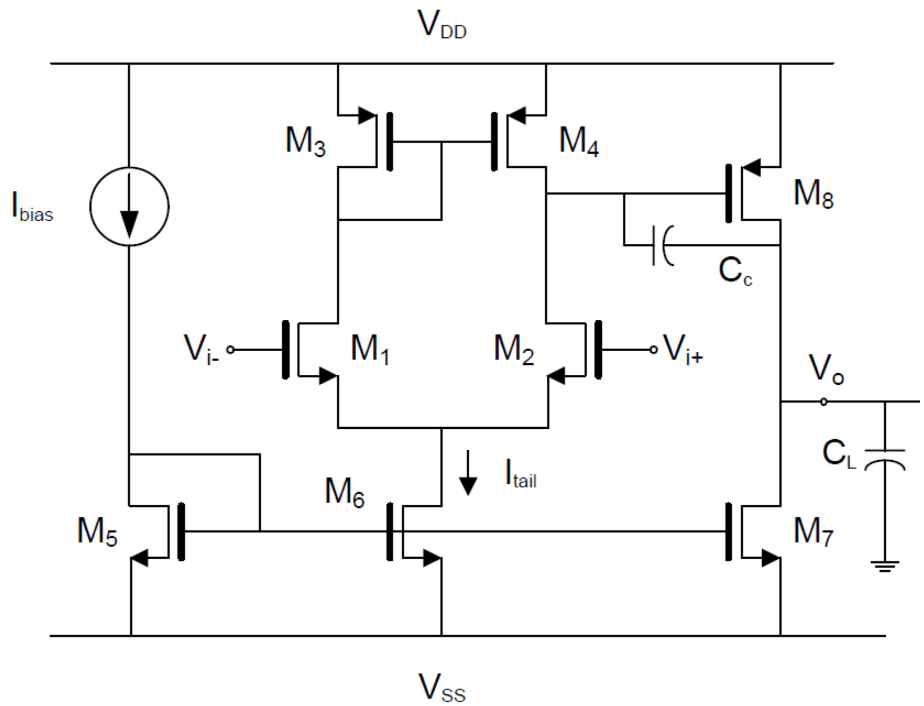
$$C_{eq} = [1 + g_{m5}(r_{O5} \| r_{O6})]C_c$$

- To save chip area, Miller multiplication of a smaller capacitance creates an equivalent effect.

Agenda

- Feedback Overview
- Feedback Properties
- Amplifier Types
- Sense and Return Techniques
- Feedback Polarity
- Feedback Topologies
- Effect of Nonideal I/O Impedances
- Stability
- Two-Stage Miller OTA

Two-Stage Miller OTA – DC Gain



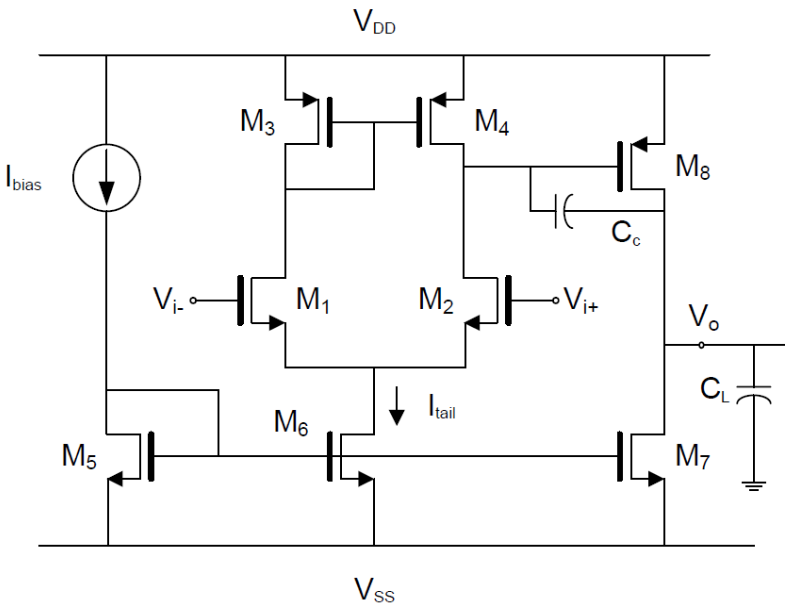
DC Gain $$A_{VDC} = A_{v1}A_{v2} = \left(-\frac{g_{m2}}{g_{o2}+g_{o4}}\right)\left(-\frac{g_{m8}}{g_{o8}+g_{o7}}\right) = \frac{g_{m2}g_{m8}}{(g_{o2}+g_{o4})(g_{o8}+g_{o7})}$$

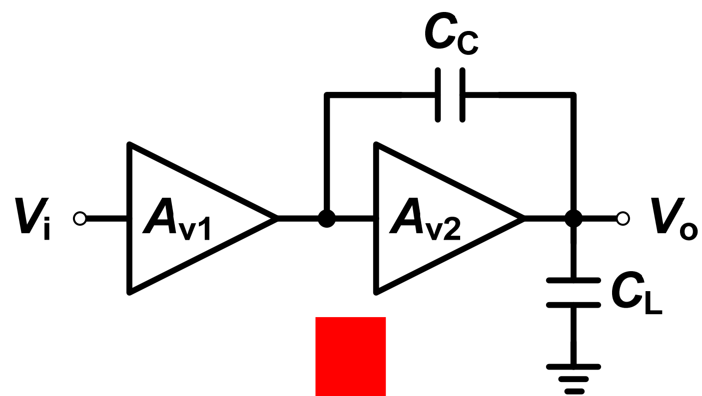
$$A_{VDC} = -G_m R_{out}$$

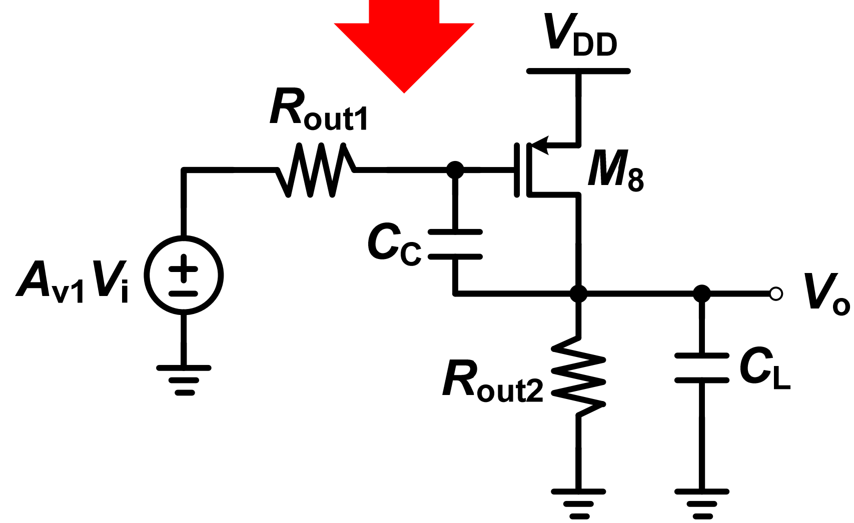
$$R_{out} = \frac{1}{g_{o8}+g_{o7}}$$

$$G_m = g_{m8}A_{v1} = -\frac{g_{m8}g_{m2}}{g_{o2}+g_{o4}}$$

Two-Stage Miller OTA – Frequency Response

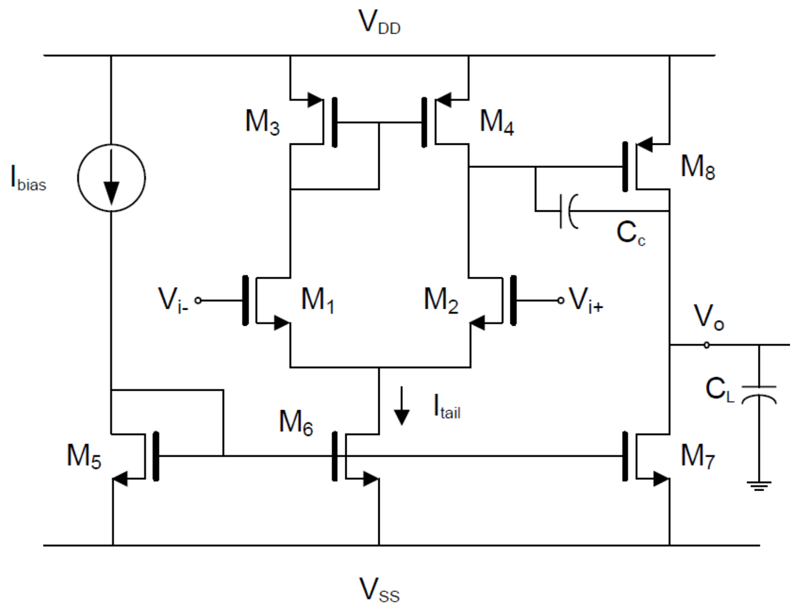


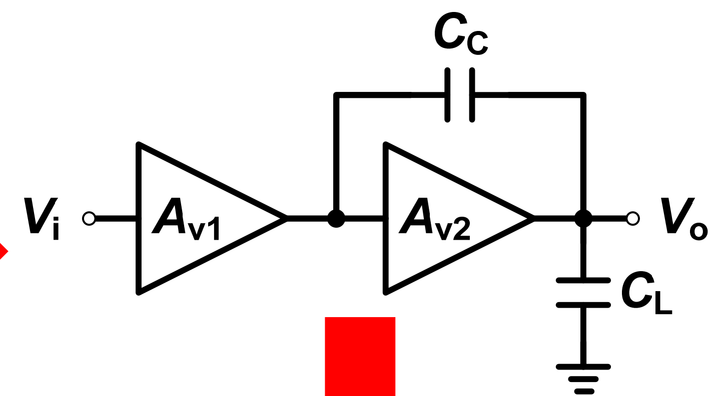


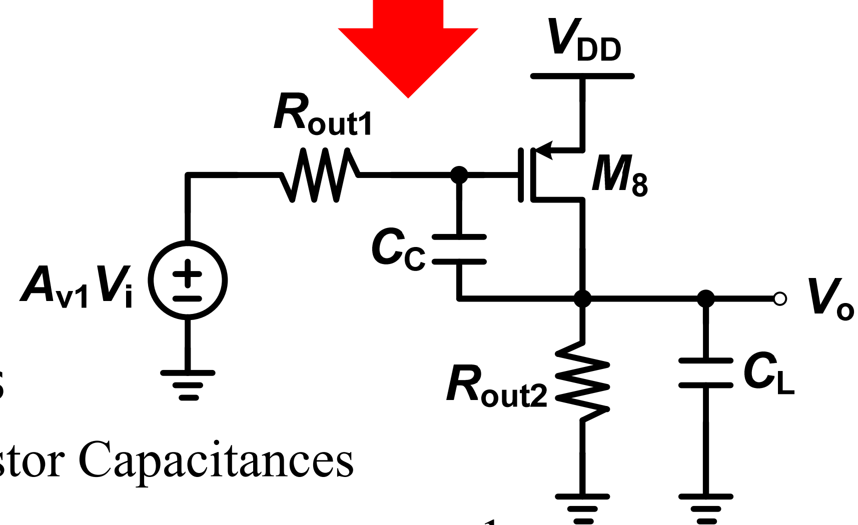


- Stage 1 is a differential amplifier with an active load
- Stage 2 is a common-source amplifier with a large miller capacitor
- Using a Thevenin equivalent for Stage 1, we can use the common-source equations from Lecture 6

Two-Stage Miller OTA – Frequency Response







- The amplifier should be designed to yield one dominant pole, so we use the dominant pole approximation equations
  - Lecture 6, Slide 44

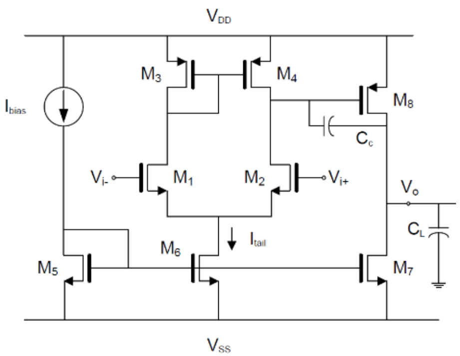
Neglecting Transistor Capacitances

$$\left|\omega_{p1}\right| = \frac{1}{R_{out1}\left(1+g_{m8}R_{out2}\right)C_C + R_{out2}\left(C_C + C_L\right)} \approx \frac{1}{R_{out1}g_{m8}R_{out2}C_C}$$

$$\left|\omega_{p2}\right| = \frac{R_{out1}\left(1+g_{m8}R_{out2}\right)C_C + R_{out2}\left(C_C + C_L\right)}{R_{out1}R_{out2}C_C C_L} \approx \frac{g_{m8}}{C_L}$$

where $R_{out1} = r_{O2} \| r_{O4}$ and $R_{out2} = r_{O7} \| r_{O8}$

Two-Stage Miller OTA – Unity Gain Frequency



The Two - Stage Miller OTA has the following transfer function

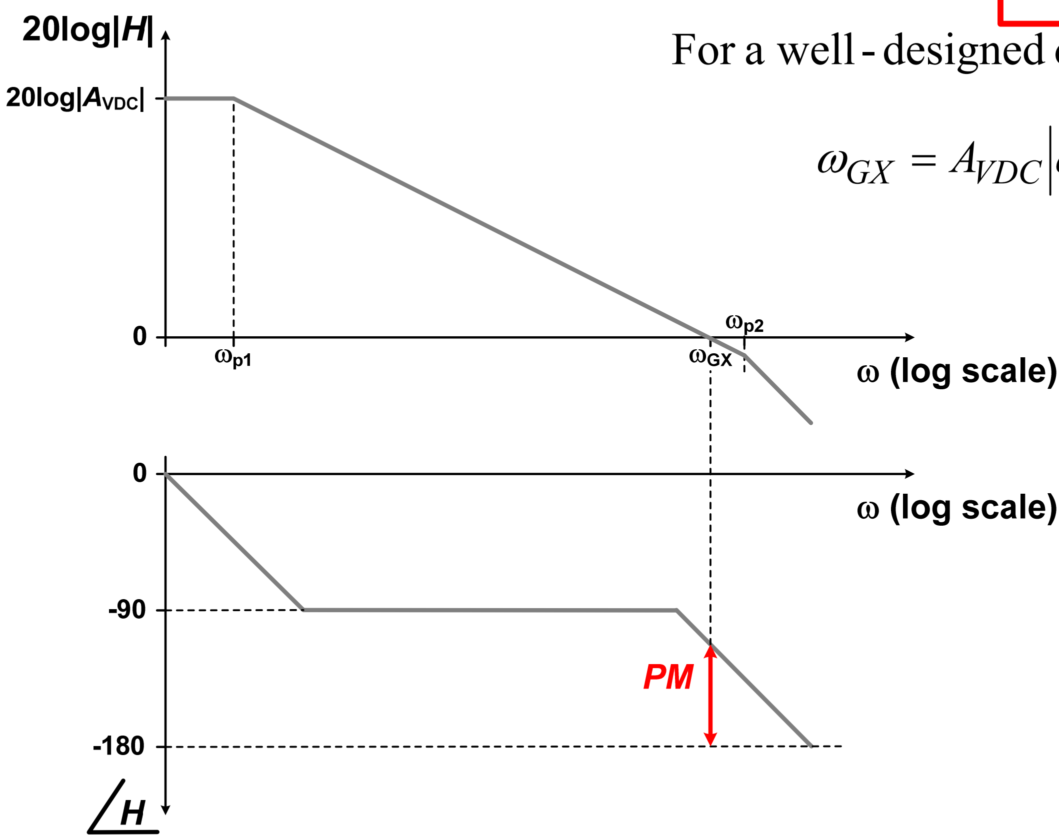
$$\frac{V_o}{V_i}(s) = H(s) = \frac{A_{VDC}}{\left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right)}$$

where $\omega_{p1} \approx -\frac{1}{R_{out1} g_{m8} R_{out2} C_C}$ and $\omega_{p2} \approx -\frac{g_{m8}}{C_L}$

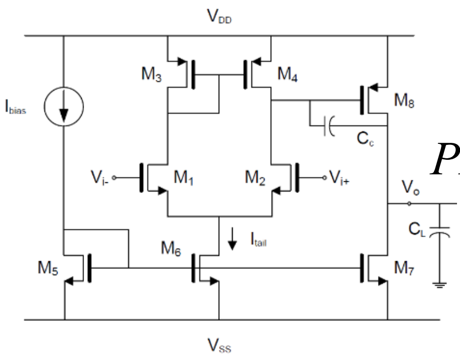
For a well - designed dominant - pole amplifer, the unity - gain frequency is

$$\omega_{GX} = A_{VDC}\left|\omega_{p1}\right| = g_{m2} R_{out1} g_{m8} R_{out2}\left(\frac{1}{R_{out1} g_{m8} R_{out2} C_C}\right)$$

$$\omega_{GX} = \frac{g_{m2}}{C_C}$$



Two-Stage Miller OTA – Phase Margin



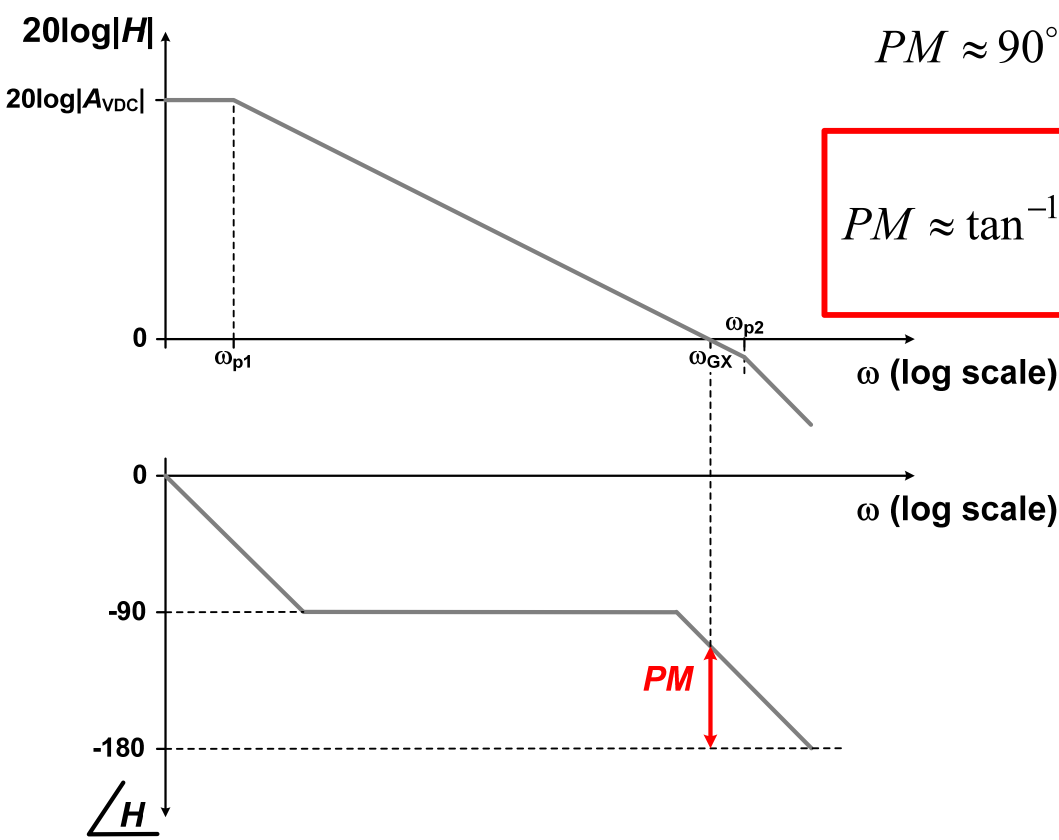
The worst-case phase margin $(f = 1)$

$$PM = \angle H(\omega_{GX}) + 180^\circ = 180^\circ - \tan^{-1}\left(\frac{\omega_{GX}}{|\omega_{p1}|}\right) - \tan^{-1}\left(\frac{\omega_{GX}}{|\omega_{p2}|}\right) \approx 90^\circ - \tan^{-1}\left(\frac{\omega_{GX}}{|\omega_{p2}|}\right)$$

Using $\tan^{-1}(x) = 90^\circ - \tan^{-1}\left(\frac{1}{x}\right)$

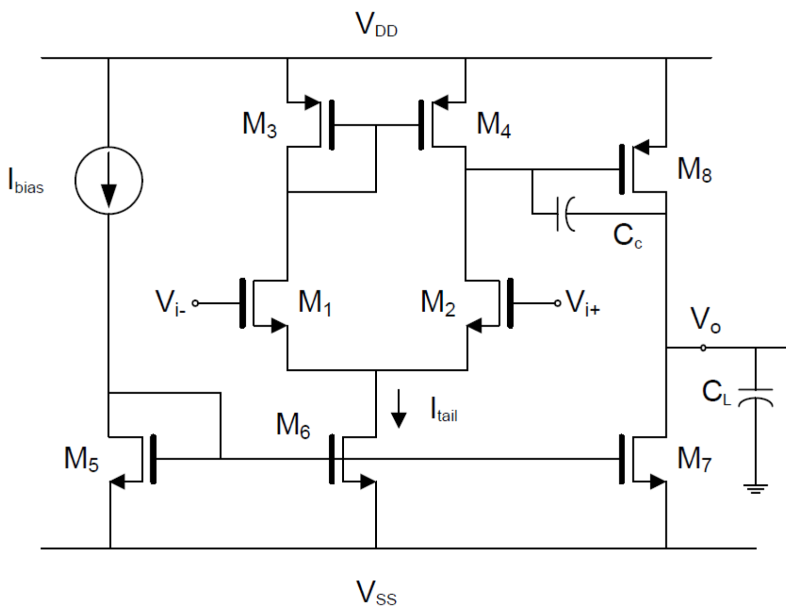
$$PM \approx 90^\circ - \left(90^\circ - \tan^{-1}\left(\frac{|\omega_{p2}|}{\omega_{GX}}\right)\right)$$

$$PM \approx \tan^{-1}\left(\frac{|\omega_{p2}|}{\omega_{GX}}\right) = \tan^{-1}\left(\frac{g_{m8}C_C}{g_{m2}C_L}\right)$$



- Note: We are neglecting a right-half plane (positive) zero that is introduced with C_C, which can potentially degrade the amplifier stability
- We will talk more about this in 474

Two-Stage Miller OTA – Example



- Again, we are neglecting a right-half plane (positive) zero, which can potentially degrade the amplifier stability

$$\omega_z = \frac{g_{m8}}{C_C}$$

- This is OK, as long as $C_C << C_L$
- More about this in 474

Assume that for the first stage

$$g_{m2} = 0.5\frac{mA}{V} \text{ and } r_{O2} = r_{O4} = 20k\Omega$$

and for the second stage

$$g_{m8} = 10\frac{mA}{V},\ r_{O7} = r_{O8} = 10k\Omega,\ \text{and}\ C_L = 1pF$$

1. What should C_C be for $f_{GX} = 500\text{MHz}$?

$$\omega_{GX} = \frac{g_{m2}}{C_C} = \frac{0.5\frac{mA}{V}}{C_C} = 2\pi(500\text{MHz}) \Rightarrow C_C = 159\,fF$$

2. What is f_{p1}, f_{p2}, and the phase margin?

$$f_{p1} = \frac{1}{2\pi R_{out1} g_{m8} R_{out2} C_C} = \frac{1}{2\pi(10k\Omega)\left(10\frac{mA}{V}\right)(5k\Omega)(159\,fF)} = 2\text{MHz}$$

$$f_{p2} = \frac{g_{m8}}{2\pi C_L} = \frac{10\frac{mA}{V}}{2\pi(1pF)} = 1.59\text{GHz}$$

$$PM = \tan^{-1}\left(\frac{f_{p2}}{f_{GX}}\right) = \tan^{-1}\left(\frac{1.59\text{GHz}}{500\text{MHz}}\right) = 72.5^\circ$$

Next Time

- Output Stages and Power Amplifiers
  - Razavi Chapter 14