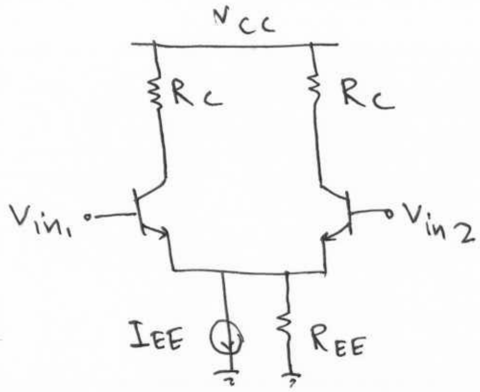
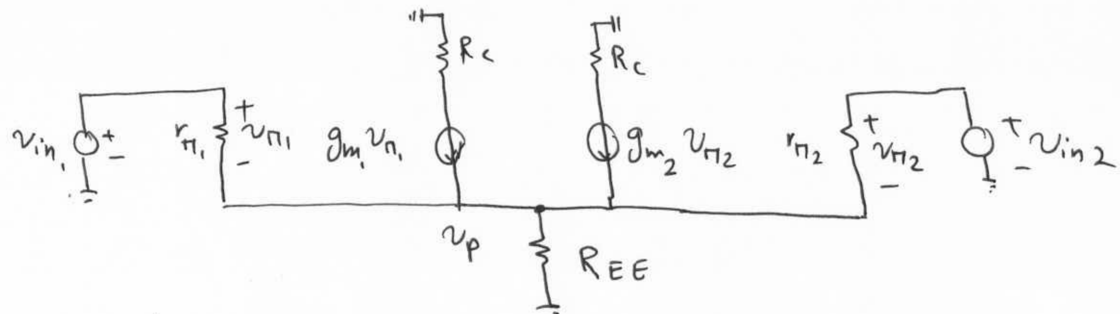
(27)



The small signal model is:

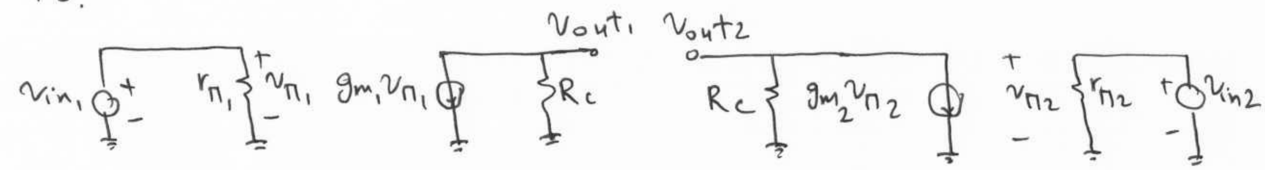


writing the node equation at P we have:

$$\frac{v_p}{R_{EE}} + \frac{v_p - v_{in_1}}{r_{\pi_1}} + g_{m_1}(v_p - v_{in_1}) + \frac{v_p - v_{in2}}{r_{\pi_2}} + g_{m_2}(v_p - v_{in_2}) = 0$$

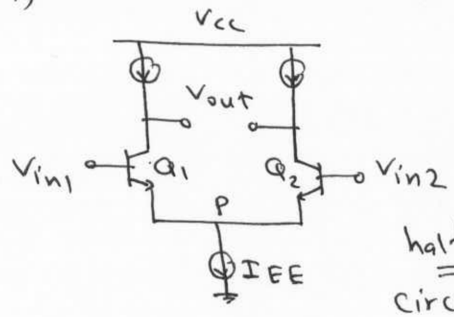
Since $v_{in_1} = -v_{in2}$ and $\begin{cases} r_{\pi_1} = r_{\pi_2} \\ g_{m_1} = g_{m_2} \end{cases}$, the above equation Simplifies to:

$$\frac{v_p}{R_{EE}} + \frac{2v_p}{r_{\pi_1}} + 2g_{m_1} v_p = 0 \Rightarrow v_p = 0 \Rightarrow$$ the small signal model is:



$$A_v = \frac{v_{out_1} - v_{out2}}{v_{in_1} - v_{in2}} = \frac{-g_{m_1} v_{in_1} R_C + g_{m_2} v_{in_2} R_C}{v_{in_1} - v_{in2}} = -g_{m_1} R_C$$

(29)



$I_{EE} = 1mA$, $V_A = 5V$

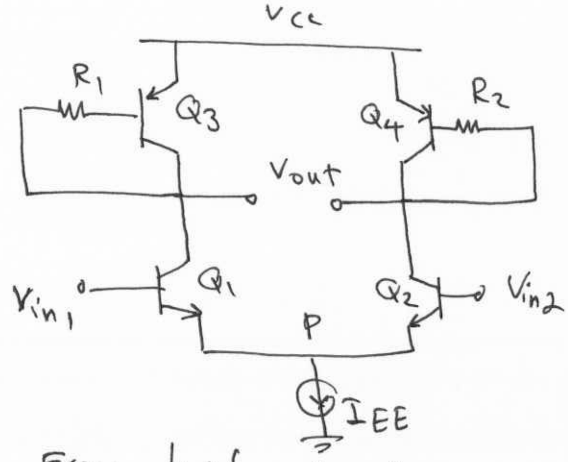
half $\Longrightarrow$ circuit

$$A_v = -g_{m_1} \cdot r_{o_1} = -\frac{I_{EE}}{2V_T} \frac{V_A}{\frac{I_{EE}}{2}} = -\frac{V_A}{V_T} = \frac{-5}{0.026}$$

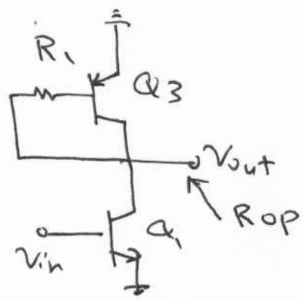
$\rightarrow A_v = -192.31$

32

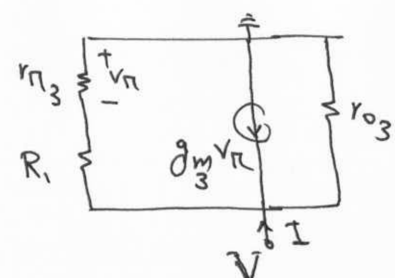
(a)



From half circuit concept we have: $A_v = -g_{m_1}(r_{o_1} \| R_{op})$



To calculate Rop, from small signal model we have:



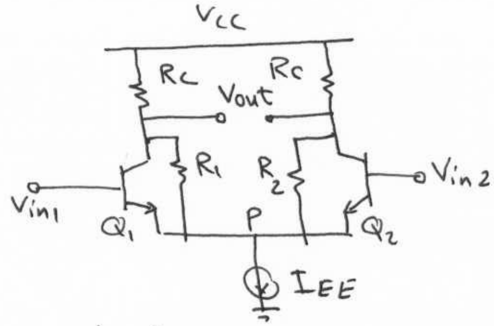
$$I = \frac{V}{r_{o_3}} - g_{m_3}V_\pi + \frac{V}{R_1 + r_{\pi_3}} = V\left[\frac{1}{r_{o_3}} + \frac{1}{R_1 + r_{\pi_3}}\right] + g_{m_3}\frac{r_{\pi_3}}{R_1 + r_{\pi_3}}V$$

$$\rightarrow R_{op} = \frac{V}{I} = r_{o_3} \,\|\, (R_1 + r_{\pi_3}) \,\|\, \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)$$

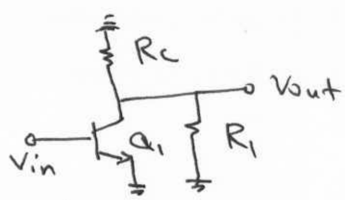
$$\rightarrow A_v = -g_{m_1}\left[r_{o_1} \,\|\, r_{o_3} \,\|\, (R_1 + r_{\pi_3}) \,\|\, \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)\right]$$

(32)

(b)

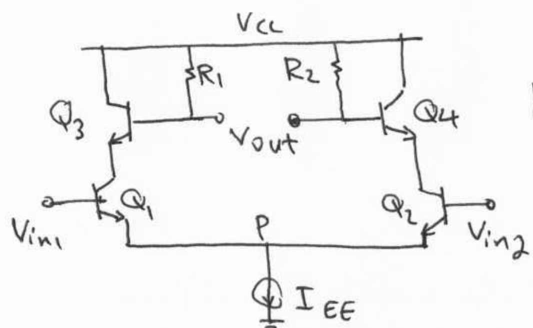


From half circuit concept:

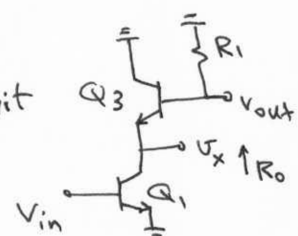


$\rightarrow A_v = -g_{m_1} (R_1 \| R_C \| r_{o_1})$

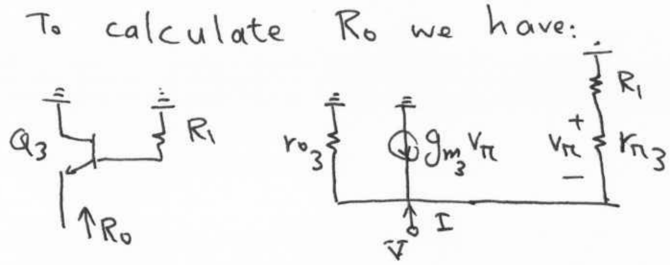
(C)



half circuit $\Longrightarrow$



To calculate R_o we have:



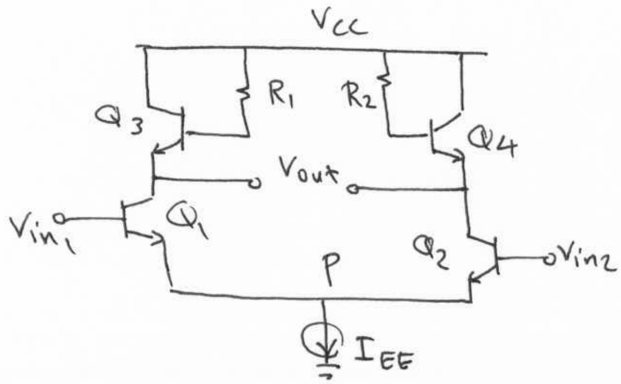
$$R_o = \frac{V}{I}$$

$$I = \frac{V}{r_{o_3}} + \frac{V}{R_1 + r_{\pi_3}} + g_{m_3} \frac{r_{\pi 3}}{r_{\pi 3} + R_1} V$$

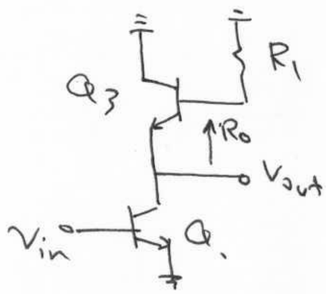
$$\Longrightarrow R_o = r_{o3} \| (R_1 + r_{\pi 3}) \| \left(1 + \frac{R_1}{r_{\pi 3}}\right) \frac{1}{g_{m_3}}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{v_x}{v_{in}} \, \frac{v_{out}}{v_x} = -g_{m_1} (r_{o_1} \| R_o) \frac{R_1}{R_1 + r_{\pi 3}}$$

(32) (d)



From half circuit concept:

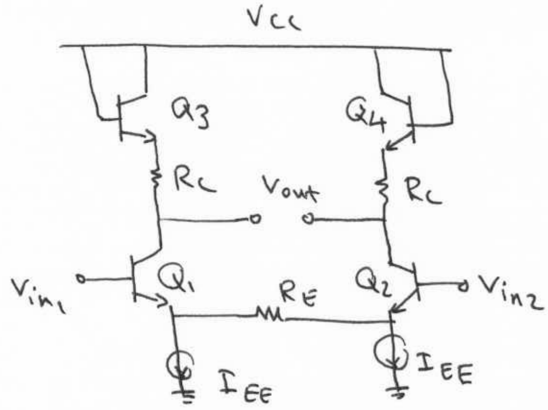


we already proved in part (c) that

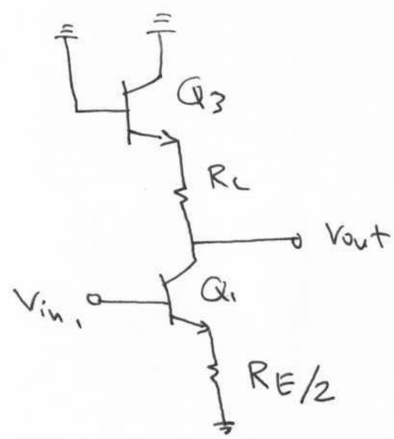
$$R_o = r_{o3} \parallel (R_1 + r_{\pi 3}) \parallel \left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m3}}$$

$$\rightarrow A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} \parallel R_o)$$

(3.3)



The half circuit is shown as:



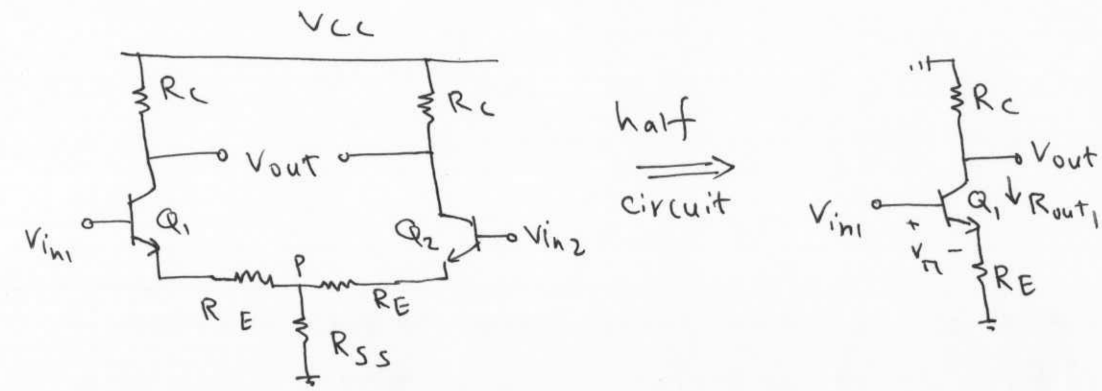
a) $$A_v = \frac{v_{out}}{v_{in_1}} = -\frac{R_C + 1/g_{m3}}{R_E/2 + 1/g_{m1}}$$

b) if $\frac{R_C}{R_E/2} = A$, then if $\frac{1/g_{m3}}{1/g_{m1}} = A$

we conclude $A_v = -A$. So the circuit is very linear.

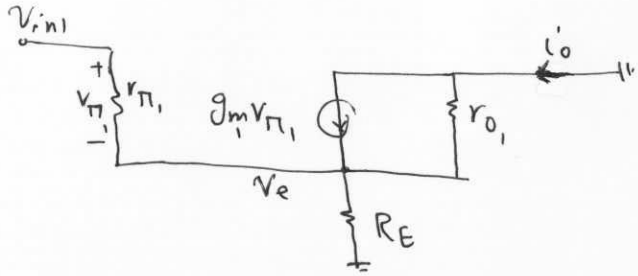
(34)

(a)



$$R_{out} = R_C \| R_{out_1} = R_C \| \left(g_{m_1} r_{o_1} (R_E \| r_{\pi_1}) + r_{o_1} + (R_E \| r_{\pi_1}) \right)$$

To calculate G_m, the small signal model is:



writing node equation at node v_e:

$$\frac{v_e}{R_E \| r_{o_1}} = \left(g_{m_1} + \frac{1}{r_{\pi_1}}\right) v_{\pi_1} = \underbrace{\left(g_{m_1} + \frac{1}{r_{\pi_1}}\right)}_{\approx g_{m_1}} (v_{in_1} - v_e)$$

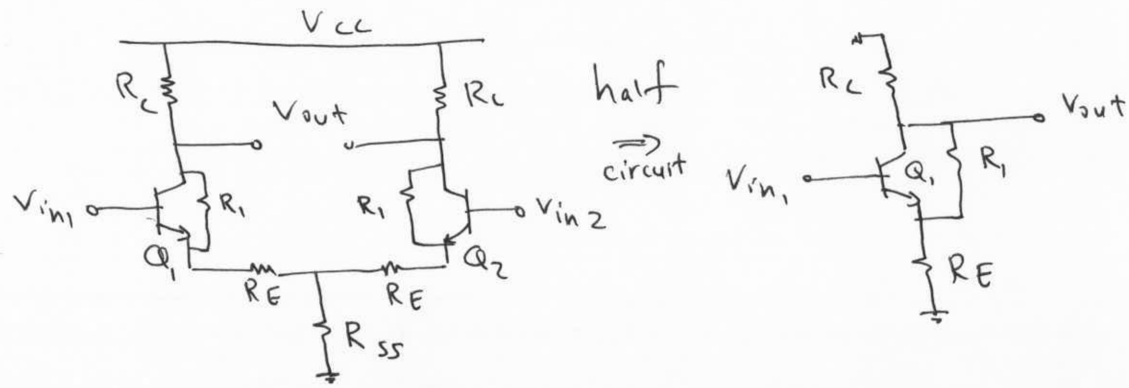
$$\Rightarrow v_e = \frac{g_{m_1}}{g_{m_1} + \frac{1}{R_E \| r_{o_1}}} v_{in_1} \Rightarrow i_o = \frac{v_e}{r_{o_1}} - g_{m_1} v_{\pi_1}$$

$$= \frac{v_e}{r_{o_1}} + g_{m_1}(v_e - v_{in_1}) \Rightarrow G_m = \frac{i_o}{v_{in_1}} = + \frac{g_{m_1} r_{o_1}}{g_{m_1} r_{o_1} R_E + r_{o_1} + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

(34)

(b)



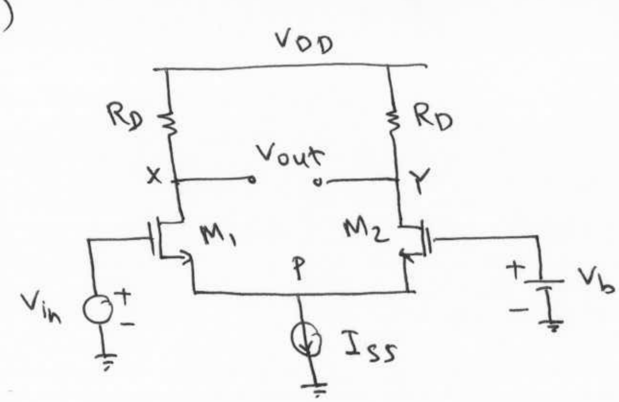
$$R_{out} = R_C \parallel \left(g_{m_1}(r_{o_1} \parallel R_1)(R_E \parallel r_{\pi_1}) + (r_{o_1} \parallel R_1) + (R_E \parallel r_{\pi_1}) \right)$$

Similar to the approach in part (a)

$$G_m = + \frac{g_{m_1}(r_{o_1} \parallel R_1)}{g_{m_1}(r_{o_1} \parallel R_1) R_E + (r_{o_1} \parallel R_1) + R_E}$$

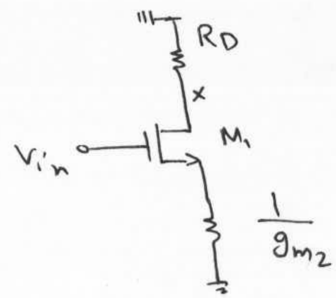
$$\Rightarrow A_v = -G_m R_{out}$$

(51)



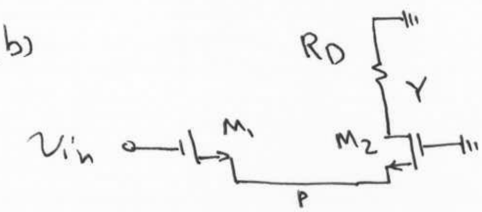
$g_{m1} = g_{m2} = g_m$

(a)



$$v_x = -g_{m1} v_{gs1} R_D = -g_{m1} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in} R_D = -\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in} = -\frac{g_m}{2} R_D v_{in}$$

(b)



$$v_P = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in}$$

$$\Rightarrow v_P = \frac{g_{m1}}{g_{m1} + g_{m2}} v_{in} \Rightarrow$$

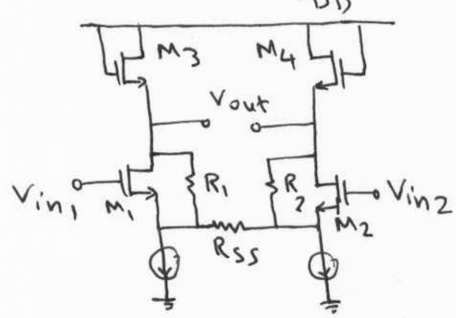
$$v_y = -g_{m2} v_{gs2} R_D = g_{m2} v_P R_D = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in}$$

$$\rightarrow v_y = \frac{g_m}{2} R_D v_{in}$$

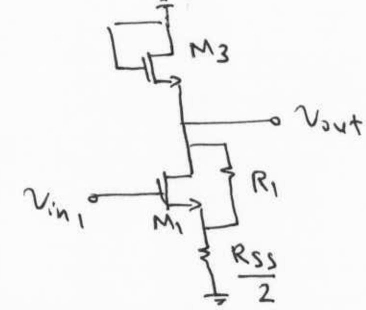
(c) $\frac{v_x - v_y}{v_{in}} = -g_m R_D$ This value is equal to the gain of the differential amplifier.

(53)

(a)

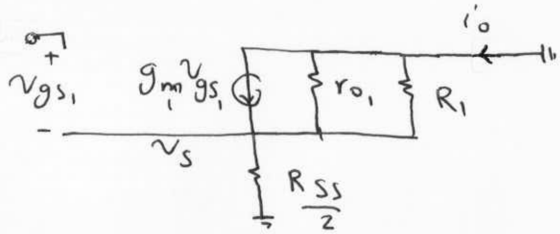


half $\Longrightarrow$ circuit



$$R_{out} = \left(r_{o3} \,\|\, \frac{1}{g_{m3}}\right) \Big\| \left(g_{m1}\,(R_1 \| r_{o1})\,\frac{R_{SS}}{2} + \frac{R_{SS}}{2} + R_1 \| r_{o1}\right)$$

To calculate G_m:



$v_{gs1} = v_{in} - v_s$

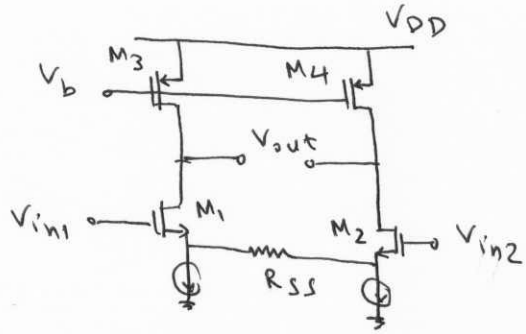
$$\frac{v_s}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}} + g_{m1} v_s = g_{m1} v_{in} \Longrightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

$$i_o = + \frac{v_s}{\frac{R_{SS}}{2}} = + \frac{1}{\frac{R_{SS}}{2}} \, \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}} \Longrightarrow$$

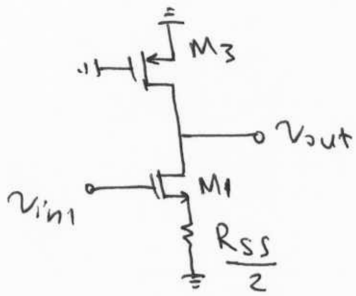
$$G_m = \frac{i_o}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \, \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

$A_v = -G_m R_{out}$

53 (b)

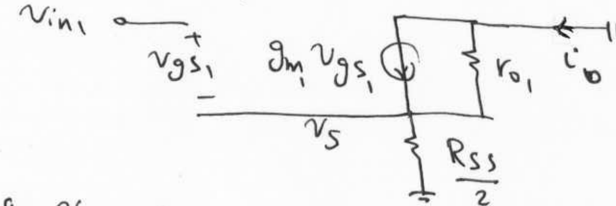


half $\Longrightarrow$ circuit



$$R_{out} = r_{o3} \,\|\, \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

To calculate G_m:

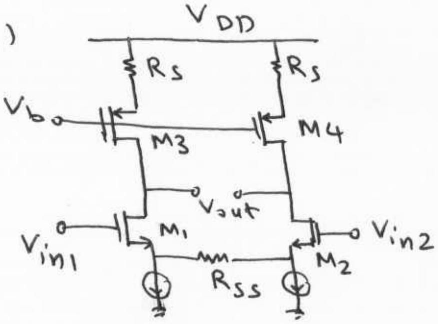


$$\frac{v_s}{r_{o_1} \| \frac{R_{SS}}{2}} + g_{m_1} v_s = g_{m_1} v_{in} \Longrightarrow v_s = \frac{g_{m_1} v_{in}}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \| r_{o_1}}}$$

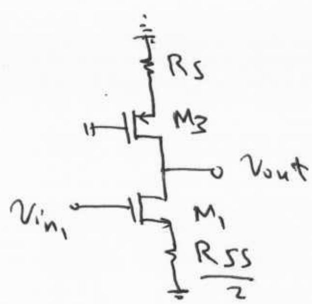
$$G_m = \frac{i_o}{v_{in}} = + \frac{v_s}{\frac{R_{SS}}{2}} \frac{1}{v_{in}} = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \| r_{o_1}}}$$

$\rightarrow A_v = -G_m R_{out}$

(c)



half $\Longrightarrow$ Circuit

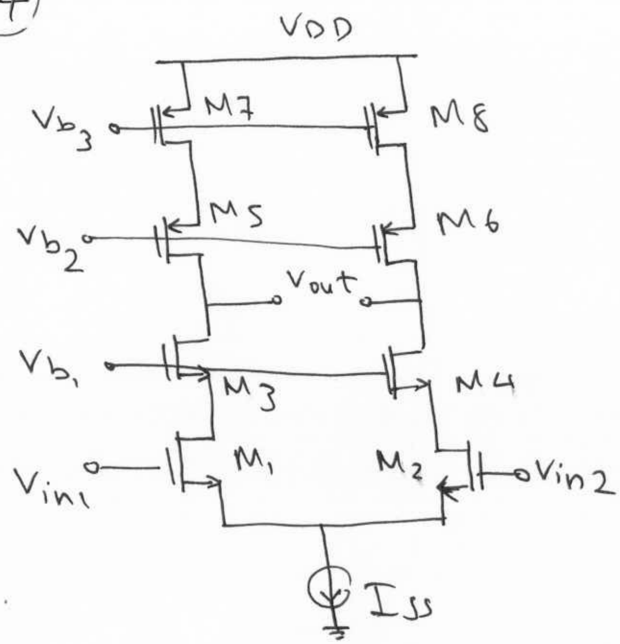


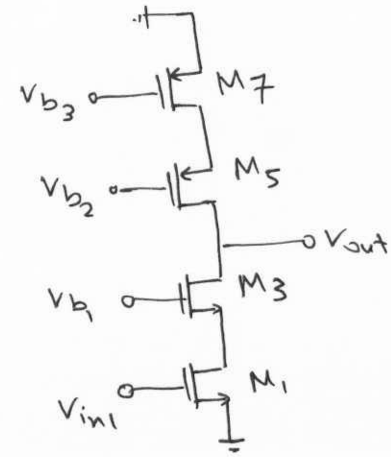
$$R_{out} = (g_{m_3} r_{o_3} R_s + r_{o_3} + R_s) \,\|\, \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

G_m for this circuit is equal to the one for part (b) so:

$$G_m = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \| r_{o_1}}} \Longrightarrow A_v = -G_m R_{out}$$

(64)





$A_v = 200$, $I_{SS} = 1mA$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$\mu_p C_{ox} = 50\,\mu A/V^2$, $\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$(\frac{W}{L})_1 = \cdots = (W/L)_8 = ?$

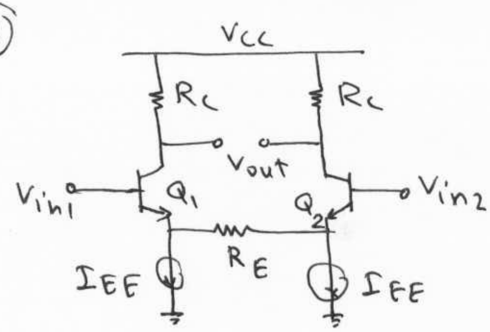
$$A_v \approx -g_{m_1}\left[(g_{m_3} r_{o3} r_{o1}) \| (g_{m_5} r_{o5} r_{o7})\right] \Rightarrow$$

$$200 = \sqrt{\mu_n C_{ox} (\tfrac{W}{L})_1 I_{SS}}\left[\left(\sqrt{\mu_n C_{ox} (\tfrac{W}{L})_3 I_{SS}}\,(\frac{2}{\lambda_n I_{SS}})^2\right) \| \left(\sqrt{\mu_p C_{ox} (\tfrac{W}{L})_5 I_{SS}}\,(\frac{2}{\lambda_p I_{SS}})^2\right)\right]$$

$$\Rightarrow 200 = \sqrt{10^{-4} (\tfrac{W}{L})_1 10^{-3}}\left[\left(\sqrt{10^{-4} (\tfrac{W}{L}) 10^{-3}}\,(\frac{20}{10^{-3}})^2\right) \| \left(\sqrt{0.5\times 10^{-4} (\tfrac{W}{L}) 10^{-3}}\,(\frac{10}{10^{-3}})^2\right)\right]$$

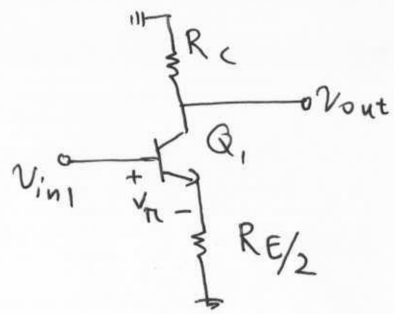
$$\Rightarrow \frac{W}{L} = 33.28$$

(81)



$A_v = 5$
$P = 4\,mW$
$V_{CC} = 2.5\,V$
$V_A = \infty$

The half circuit is:



$$A_v = \frac{v_{out}}{v_{in1}} \simeq \frac{-g_{m1} v_\pi R_C}{v_{in1}} = -\frac{g_{m1} R_C}{v_{in1}} \, \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_E}{2}} v_{in1}$$

$$= -\frac{R_C}{\frac{R_E}{2} + \frac{1}{g_{m1}}}$$

$P = 4\,mW = 2 I_{EE} V_{CC} = 5 I_{EE} \Rightarrow I_{EE} = 0.8\,mA$

$g_m = \frac{I_{EE}}{V_T} = 0.03077$

$$A_v = 5 \Rightarrow \frac{R_C}{\frac{R_E}{2} + 32.5} = 5 \quad (1)$$

if I_{EE} increases by 10%, the gain will be:

$$A_v = \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} \Rightarrow 5 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} < 5 \times 1.02 \quad (2)$$

if I_{EE} decreases by 10% then:

$$5 \times 0.98 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} < 5 \quad (3)$$

The worse case is:

$$\begin{cases} \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 5 \times 1.02 & (4) \\[2ex] \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 5 \times 0.98 & (5) \end{cases}$$

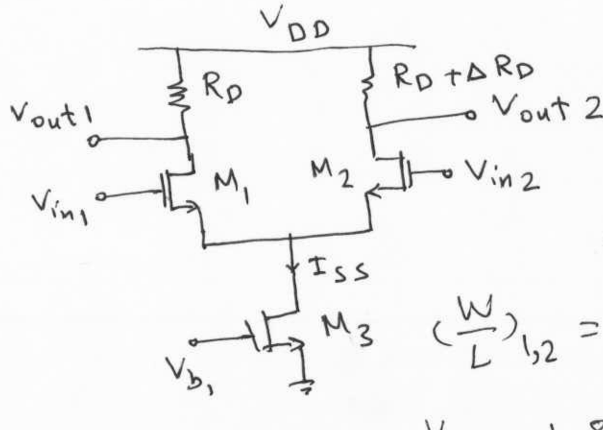
dividing (4) and (5) to (1) leads to:

$$\begin{cases} \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 1.02 \Rightarrow R_E = 236.36\,\Omega \\[3ex] \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 0.98 \Rightarrow R_E = 288.89\,\Omega \end{cases}$$

To ensure less than 2% gain variation for 10% current variation $R_E = 288.89\,\Omega$

From (1) $\quad R_C = 5\left(\frac{R_E}{2} + 32.5\right) = 884.72\,\Omega$

(91)



CMMR = 60dB

P = 2mW

$A_{DM} = 5$

$\lambda(M_3) = ?$

$\left(\frac{W}{L}\right)_{1,2} = \frac{10}{0.18}$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$V_{DD} = 1.8$, $\frac{\Delta R_D}{R_D} = 2\%$

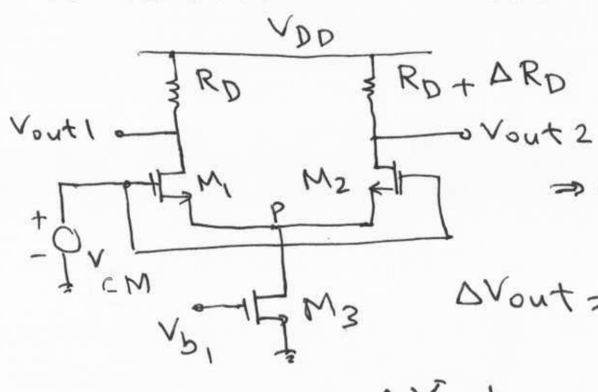
$$P = 2mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\ mA$$

$$A_{DM} = -g_{m1} R_D$$

$$g_{m1} = \sqrt{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{SS}} = \sqrt{10^{-4}\times\frac{10}{0.18}\times1.11\times10^{-3}} = 2.4845\ mS$$

$$\Rightarrow R_D = \frac{|A_{DM}|}{g_{m1}} = \frac{5}{2.4845\times10^{-3}} = 2.012\ K\Omega$$

To calculate $A_{CM,DM}$ we have:



$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3}$$

$$\Rightarrow \Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m1}} + r_{o3}\right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D I_D$$

$$\Rightarrow A_{CM,DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = -\frac{\Delta R_D/2}{\frac{1}{2g_{m1}} + r_{o3}}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM,DM}} = (1+2g_{m1} r_{o3})\frac{R_D}{\Delta R_D}, \quad r_{o3} = \frac{1}{\lambda_3 I_{SS}}$$

$$\Rightarrow \quad CMMR = 60\,dB = 10^{3} = \left(1 + 2 \times 2.4845 \times 10^{-3} \frac{1}{\lambda_3 \times 1.11 \times 10^{-3}}\right) 50$$

$$\Rightarrow \quad \lambda_3 = 0.2354$$