④ a). $\because \quad V_{RS} = 200\,mV,$

$\therefore \quad I_{DS}\,R_S = 200\,mV$

$$I_{DS} = \frac{0.2}{100}$$

$$I_{DS} = 2\,mA.$$

For M_1 to stay in saturation,

$$V_{DS} \geqslant V_{GS} - V_{TH}.$$

$\because \quad V_{DS} = V_D - V_S$

$$= [1.8 - (2\times10^{-3})\times500] - 0.2$$

$$= 0.6,$$

$\therefore \quad V_{GS} - V_{TH} \leqslant 0.6,$

Since $\quad I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2,$

$\left(\frac{W}{L}\right)$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $\left(\frac{W}{L}\right)_1$ is when $(V_{GS} - V_{TH}) = 0.6V,$

$$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)_1 (0.6)^2$$

$\therefore$ min. $\left(\frac{W}{L}\right)_1 \approx 56$

b) With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

i.e. $1.8 \times \dfrac{R_2}{R_1 + R_2} = 1.2V$,

$$\frac{R_2}{R_1} = 2 \quad \text{—— (1)}$$

Input impedance $= R_2 // R_1$,

i.e. $R_2 // R_1 \geqslant 30k\Omega$ —— (2)

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$

will satisfy both (1) & (2).

⑤

$$V_S = V_{RS}$$
$$= I_{D1}\,(200) = 0.1\,V$$

$$I_{DS} = \frac{1}{2}\,\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$0.5mA = \frac{1}{2}\,(200 \times 10^{-6})\left(\frac{20}{0.18}\right)(V_{GS} - V_{TH})^2$$

$$\therefore\ V_{GS} = 0.612\ V$$

$$\therefore V_G = 0.612 + 0.1$$
$$= 0.712.$$

$$\because\ V_G = V_{DD} - I_{R1} \times R_1$$

$$R_1 = \underline{\underline{21.76\ k\Omega}}.$$

and $V_{GS} = I_{R2} \times R_2.$

$$\therefore R_2 = \frac{0.712}{0.05 \times 10^{-3}}$$

$$= \underline{\underline{14.24\,k\Omega}}$$

(8) Without defects,

$$V_{GS} = 1.8V$$

$$\text{i.e. } V_{DS} = (1.8 - 0.1)V$$

$$= 1.7V$$

$$I_{DS} = \frac{(1.8-1.7)V}{2000\,\Omega} = 0.05\,mA.$$

$$\therefore\ 0.05\,mA = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1.8-0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) = 0.255$$

b) With defects,

$$V_{GS} = V_{DS} + 50\,mV$$

$$\therefore\ V_{R_P} = 50\,mV$$

$$I_{RP} = \frac{50\,mV}{R_p}$$

$$V_{GS} = 1.8\,V - \frac{0.05V}{R_p} \times 30\,k\Omega \quad \text{— (1)}$$

$$\because\ V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{DS}$$

$$V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{GS} - 50\,mV \quad \text{— (2)}$$

$$\because \quad I_{DS} = \frac{1}{2}\left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})^2$$

$$= \frac{1}{2}(0.255)(200 \times 10^{-6})(V_{GS} - 0.4)^2$$

$$= 2.55 \times 10^{-5}(V_{GS} - 0.4)^2$$

$\therefore$ From ②,

$$1.8 - \left[2.55 \times 10^{-5}(V_{GS} - 0.4)^2 - \frac{0.2}{R_P}\right]2000$$

$$= V_{GS} - 0.05.$$

From ①, $\quad \frac{0.05}{R_P} = \frac{1.8 - V_{GS}}{30\ 000}$

$$\therefore\ 1.8 - \left[0.051(V_{GS} - 0.4)^2 - \frac{1.8 - V_{GS}}{15}\right] = V_{GS} - 0.3$$

$$1.85 - 0.051 V_{GS}^2 + 0.0408 V_{GS} - 0.00816 + \frac{1.8 - V_{GS}}{15} = V_{GS}$$

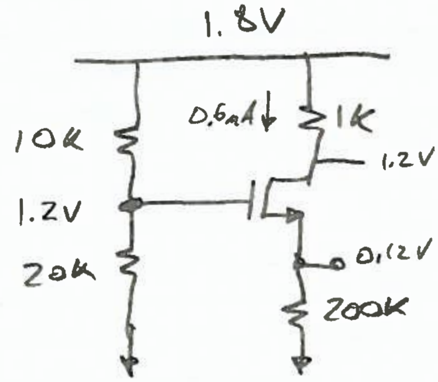
$$29.4276 - 15.388 V_{GS} - 0.765 V_{GS}^2 = 0$$

$$\therefore V_{GS} = 1.76\ V$$

$$R_P = \frac{0.05 \times 30\ 000}{1.8 - 1.76}$$

$$\approx 36.3\,k\Omega$$

7.9 w/o defect $V_{GS} = V_{DS} \Rightarrow V_G = V_D$



$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}(V_{GS}-V_T)^2$$

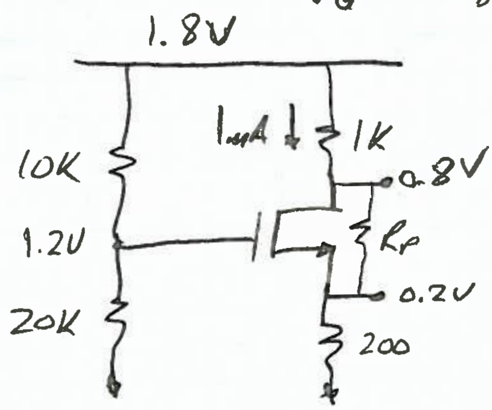
$$\frac{W}{L} = \frac{2I_D}{\mu C_{ox}(V_{GS}-V_T)^2} = \frac{2(0.6mA)}{(200\mu)(1.2-0.12-0.4)^2}$$

$$\boxed{\frac{W}{L} = 13.0}$$

W/defect $V_{GS} = V_{DS} + V_{TH}$

$\Rightarrow V_G = V_D + V_{TH} \Rightarrow V_D = V_G - V_{TH}$

$= 1.2V - 0.4V = 0.8V$

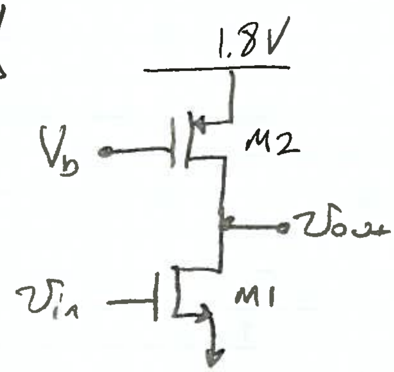


$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}(V_{GS}-V_T)^2 = \frac{200\mu}{2}(13)(1.2-0.2-0.4)^2 = 468\mu A$$

$$I_{RP} = 1mA - 468\mu A = 532\mu A$$

$$\boxed{R_P = \frac{V_{RP}}{I_{RP}} = \frac{0.6V}{532\mu A} = 1.13k\Omega}$$

7.20



$|A_V| = 10$ w/ $I_D = 0.5mA$

$\lambda_1 = 0.1V^{-1}$, $\lambda_2 = 0.15V^{-1}$

a. $\frac{W}{L}_1 = ?$

$$|A_V| = g_{m1}(r_{o1} \| r_{o2}) = \frac{g_{m1}}{g_{o1} + g_{o2}}$$

$$= \frac{\sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 2 I_D}}{\lambda_1 I_D + \lambda_2 I_D}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \frac{A_V^2 I_D (\lambda_1 + \lambda_2)^2}{2 \mu_n C_{ox}} = \frac{(10)^2 (0.5mA)(0.1 + 0.15)^2}{2(200\mu)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 7.81}$$

b. $\left(\frac{W}{L}\right)_2 = \frac{20}{0.18} \Rightarrow V_b = ?$

$$I_D = \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_2 (V_{SG} - |V_{THP}|)^2$$

$$= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{THP}|)^2$$

$$V_b = V_{DD} - |V_{THP}| - \sqrt{\frac{2 I_D}{\mu C_{ox} \left(\frac{W}{L}\right)_2}}$$

$$\boxed{V_b = 1.1V}$$

$$= 1.8V - |-0.4V| - \sqrt{\frac{2(0.5mA)}{100\mu \left(\frac{20}{0.18}\right)}} = 1.1V$$

(24). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "g_m" than (b),

$\because g_{m1} > g_{m2}$

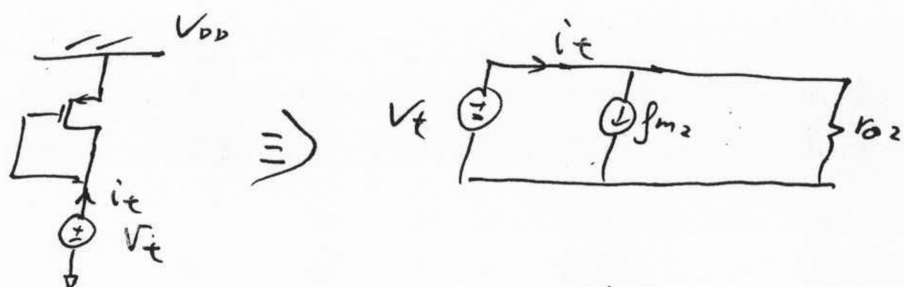
(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} // r_{o2})$ is the same for both cases.

(28) a). $A_v = -g_{m1}\, r_{o1} \,//\, Z_2$,

where Z_2 is the impedance presented by M_2.

To find Z_2, apply a test voltage (V_T) at the drain of M_2:



From the small-signal model,

$$i_t = g_{m2}\, V_t + \frac{V_t}{r_{o2}},$$

$$Z_2 = \frac{V_t}{i_t} = r_{o2} \,//\, \frac{1}{g_{m2}}.$$

$$\therefore\ A_v = -g_{m1}\left(r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right)$$

b) $A_v = -g_{m1}\left(r_{o1} \,//\, Z_2 \,//\, Z_3\right)$

where Z_2 and Z_3 are impedances presented by M_2 and M_3 respectively.

From (a) $Z_3 = r_{o3} \,//\, \frac{1}{g_{m3}}$

By inspection, $Z_2 = r_{o2}$

$$\therefore\ A_v = -g_{m1}\left(r_{o1} \,//\, r_{o2} \,//\, r_{o3} \,//\, \frac{1}{m_3}\right)$$

c) $A_v = -g_{m1}\, r_{o1} \,//\, Z_2 \,//\, Z_3.$

Similar to (b),

$$Z_2 = r_{o2},$$

and $Z_3 = r_{o3} \,//\, \frac{1}{g_{m3}}$

(the small signal model of M_3 in this case is equivalent to that of M_2 in (a))

$\therefore\ A_v = -g_{m1} \left(r_{o1} \,//\, r_{o2} \,//\, r_{o3} \,//\, \frac{1}{g_{m3}} \right)$

d). M_2 is in CS arrangement. (similar to (c)).

$$A_v = g_{m2}\, r_{o2} \,//\, Z_1 \,//\, Z_3.$$

$$Z_3 = \frac{1}{g_{m3}} \,//\, r_{o3}$$

$$Z_1 = r_{o1}$$

$\therefore\ A_v = g_{m2} \left(r_{o2} \,//\, r_{o1} \,//\, \frac{1}{g_{m3}} \,//\, r_{o3} \right)$

e). $A_v = g_{m2} \left(r_{o2} \,//\, Z_1 \,//\, Z_3 \right)$

$$Z_1 = r_{o1}$$

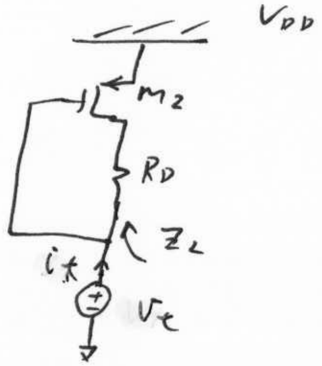
$$Z_3 = \frac{1}{g_{m3}} \,//\, r_{o3}$$

(recall: impedance looking into source $= \frac{1}{g_m}$)

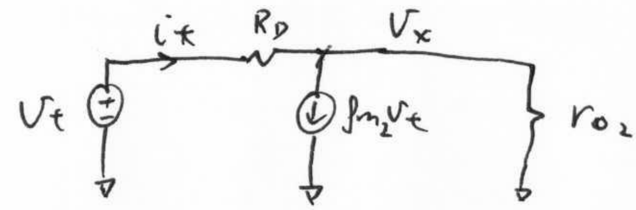
$\therefore\ A_v = g_{m2} \left(r_{o2} \,//\, r_{o1} \,//\, \frac{1}{g_{m3}} \,//\, r_{o3} \right)$

(28) f) $A_v = -g_{m1}(r_{o1} \parallel Z_L)$,

where Z_L is the impedance depicted as follows:



The equivalent small-signal model is:



$$i_t = g_{m2}V_t + \frac{V_x}{r_{o2}},$$

$$V_x = V_t - R_D i_t$$

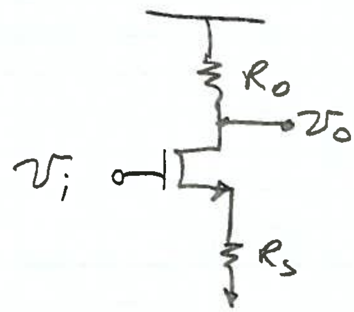
$$\therefore i_t = g_{m2}V_t + \frac{V_t}{r_{o2}} - \frac{R_D i_t}{r_{o2}}$$

$$i_t\left(1 + \frac{R_D}{r_{o2}}\right) = V_t\left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$\frac{V_t}{i_t} = \frac{r_{o2} + R_D}{g_{m2} r_{o2} + 1}$$

$$\therefore A_v = -g_{m1}\left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2} r_{o2}}\right)$$

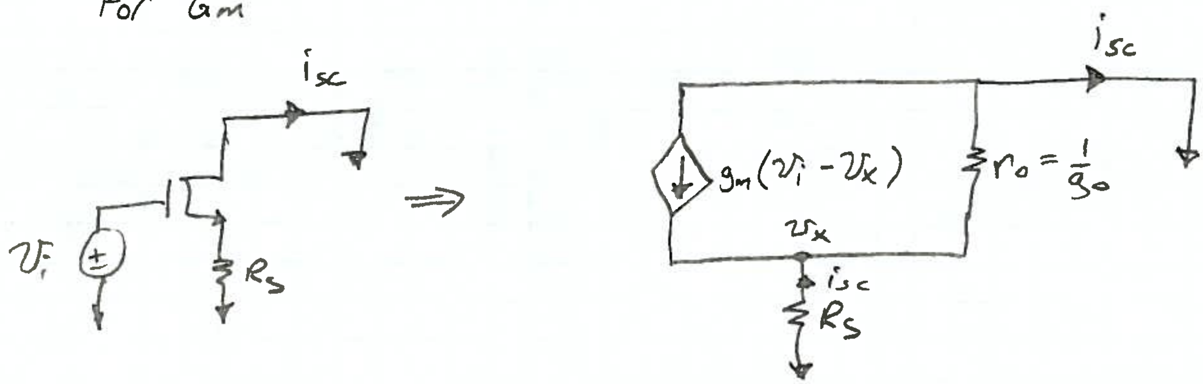
7.32



We can express the gain

$$A_v = \frac{v_o}{v_i} = G_m R_{out}$$

For G_m



$$v_x = -i_{sc} R_s$$

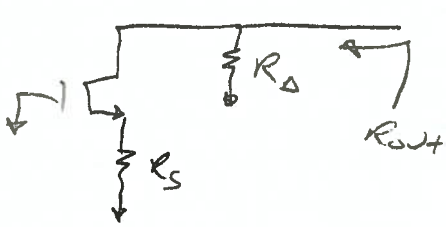
KCL @ output

$$i_{sc} + g_m(v_i + i_{sc} R_s) + i_{sc} R_s g_o = 0$$

$$i_{sc}\left[1 + (g_m + g_o) R_s\right] = -g_m v_i$$

$$G_m = \frac{i_{sc}}{v_i} = -\frac{g_m}{1 + (g_m + g_o) R_s}$$

For R_{out}



$$R_{out} = R_o \,\|\, \left[r_o + R_s + g_m r_o R_s\right]$$

$$A_V = G_m R_{out}$$

$$= -\left[\frac{g_m}{1+(g_m+g_o)R_S}\right]\left[R_D \,\|\, \left[r_o + R_S + g_m r_o R_S\right]\right]$$

w/ $g_m r_o = \frac{g_m}{g_o} \gg 1$

$$\approx -\left[\frac{g_m}{1+g_m R_S}\right]\left[R_D \,\|\, \left[r_o + g_m r_o R_S\right]\right]$$

OK to just have this term, as this often dominates

(37) a) $|\text{Voltage gain}| = g_m R_D$

$$= 5$$

$$\therefore g_m = \frac{5}{500}$$

$$= 10\,mS.$$

$$= \sqrt{2(200\times10^{-6})\left(\frac{W}{L}\right)\times10^{-3}}$$

$$\therefore \frac{W}{L} = 250$$

b) $V_D = 1.8 - 500\times10^{-3}$

$$= 1.3\,V$$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$$V_D \geq V_G - 0.2$$

$$\therefore V_G \leq 1.5$$

Also, $I_{R_1+R_2} = 0.1\times10^{-3}A.$

$$\therefore R_1 + R_2 = \frac{1.8}{0.1\times10^{-3}}$$

$$= 18\,k\Omega.$$

choose $R_2 = 15\,k\Omega$ & $R_1 = 3\,k\Omega$

c) With twice of (W/L), M_1 will go further away from triode. As (W/L) doubles, & I_{bias} is fixed by the current source, V_{GS} is forced to decrease (so M_1 will have same I_{DS}). Thus, $(V_{GS} - V_{TH})$ decreases, and V_{DS} can be allowed to drop more before M_1 goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(44) a) Voltage gain $(A_v) = \left[\dfrac{\frac{1}{g_{m1}}}{R_s + \frac{1}{g_{m1}}}\right] \dfrac{g_{m1}}{g_{m2}}$

$$= \frac{g_{m1}/g_{m2}}{1 + g_{m1}R_s}$$

b) Voltage gain $(A_v) = g_{m1} Z_L$

(similar to prob. 32(b))

$$= \frac{g_{m1}}{g_{m2}}$$

c) Voltage gain $= \left[\dfrac{\frac{1}{g_{m1}} // R_1}{R_s + \frac{1}{g_{m1}} // R_1}\right] \dfrac{g_{m1}}{g_{m2}}$

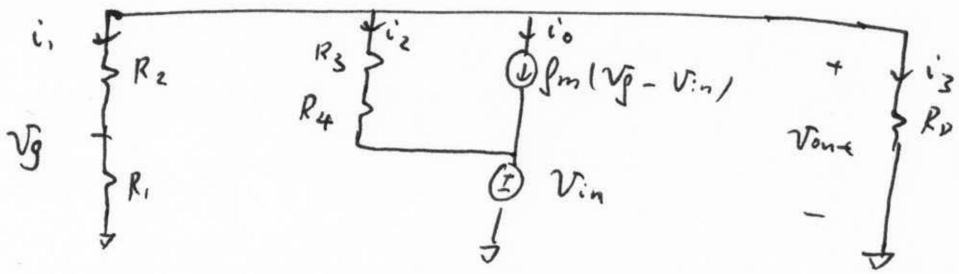
d) Voltage gain $= g_{m1}\left[R_D + r_{o3} // \frac{1}{g_{m2}}\right]$,

$\because r_{o3} = \infty$,

gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$.

e) Voltage gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$

(48) The small-signal model is:



$$\because \; -i_0 = i_1 + i_2 + i_3$$

$$-g_m(v_g - v_{in}) = \frac{v_{out}}{R_2 + R_1} + \frac{v_{out} - v_{in}}{R_3 + R_4} + \frac{v_{out}}{R_D}$$

$$g_m\left(v_{in} - \frac{R_1}{R_1+R_2} v_{out}\right) = v_{out}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right) - \frac{v_{in}}{R_3+R_4}$$

$$v_{in}\left(g_m + \frac{1}{R_3+R_4}\right) = v_{out}\left(\frac{g_m R_1 + 1}{R_1+R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right)$$

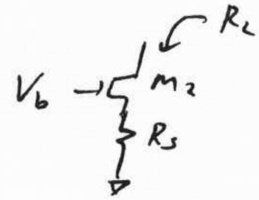
$$\frac{v_{out}}{v_{in}} = \frac{\left(g_m + \frac{1}{R_3+R_4}\right)}{\frac{1}{R_D} + \frac{1}{R_3+R_4} + \frac{g_m R_1 + 1}{R_1+R_2}}$$

(55)

a/ $$A_v = \frac{r_{o1} \,//\, (R_s + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \,//\, (R_s + r_{o2})}$$

b/ $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



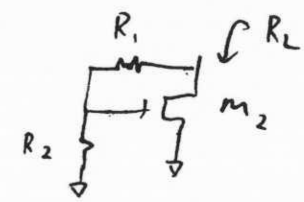
$$R_L = (1 + g_{m2} r_{o2}) R_s + r_{o2} \quad \text{Eq.(7.110)}$$

$$\therefore A_v = \frac{r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}{\frac{1}{g_{m1}} + r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}$$

c/. $$A_v = \frac{r_{o1} \,//\, \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + (r_{o1} \,//\, \frac{1}{g_{m2}})}$$

d/ $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



(c) Finding R_L with small-signal model:
(cont'd)

$$R_L = \frac{v_t}{i_t},$$

where
$$i_t = \frac{v_t}{r_{o2}} + g_{m2}v_1 + \frac{v_t}{R_1 + R_2}$$

$$= \frac{v_t}{r_{o2}} + \frac{g_{m2} R_2 v_t}{R_1 + R_2} + \frac{v_t}{R_1 + R_2}$$

$$\therefore R_L = \frac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}$$

$$\therefore A_v = \frac{r_{o1} \mathbin{/\!/} \dfrac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}{\dfrac{1}{g_{m1}} + r_{o1} \mathbin{/\!/} \dfrac{r_{o2}(R_2 + R_1)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}$$

e)
$$A_v = \frac{r_{o2} \mathbin{/\!/} \left(\frac{1}{g_{m1}} \mathbin{/\!/} r_{o3}\right)}{\frac{1}{g_{m2}} + r_{o2}\left(\frac{1}{g_{m1}} \mathbin{/\!/} r_{o3}\right)}$$

f)
$$A_v = \frac{r_{o1} \mathbin{/\!/} [(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}]}{\frac{1}{g_{m1}} + \{r_{o1} \mathbin{/\!/} [(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}]\}}$$