ECEN325 HW#1 Solutions

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Complex Number Review

a. $a + jb$

$$\text{magnitude} = \sqrt{a^2 + b^2}$$

$$\text{phase} = \tan^{-1}\left(\frac{b}{a}\right)$$

b. $\dfrac{a+jb}{c+jd}$

$$\text{magnitude} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$\text{phase} = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)$$

c. $(a+jb)(a-jb)$

$$\text{magnitude} = \sqrt{a^2+b^2}\,\sqrt{a^2+b^2} = a^2+b^2$$

$$\text{phase} = \tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{-b}{a}\right)$$

$$= \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{b}{a}\right) = \emptyset$$

d. $100 + j10$

$$\text{magnitude} = \sqrt{100^2 + 10^2} = 100.5$$

$$\text{phase} = \tan^{-1}\left(\frac{10}{100}\right) = 5.71^\circ$$

e. $$\frac{100+j10}{(1+j10)(10+j10)}$$

$$\text{magnitude} = \frac{\sqrt{100^2 + 10^2}}{\sqrt{1^2+10^2}\,\sqrt{10^2+10^2}} = 0.707$$

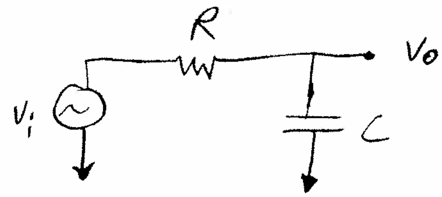
$$\text{phase} = \tan^{-1}\left(\frac{10}{100}\right) - \tan^{-1}\left(\frac{10}{1}\right) - \tan^{-1}\left(\frac{10}{10}\right) = -123.6^\circ$$

f. $$\frac{(1+j10)(10+j10)}{(100+j10)}$$

$$\text{magnitude} = \frac{\sqrt{1^2+10^2}\,\sqrt{10^2+10^2}}{\sqrt{100^2+10^2}} = 1.414$$

$$\text{phase} = \tan^{-1}\left(\frac{10}{1}\right) + \tan^{-1}\left(\frac{10}{10}\right) - \tan^{-1}\left(\frac{10}{100}\right) = 123.6^\circ$$

1,



$$V_o(s) = \frac{Z_C}{Z_R + Z_C} V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + 1/sC} = \frac{1/RC}{s + 1/RC}$$

W/ $R = 1k\Omega$, $C = 10nF$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{10^5}{s + 10^5}$$

1 pole at $\omega = -10^5$ rad/s or $f = -15{,}9$ kHz

$$\lim_{s \to 0} F(s) = \frac{10^5}{10^5} = 1$$

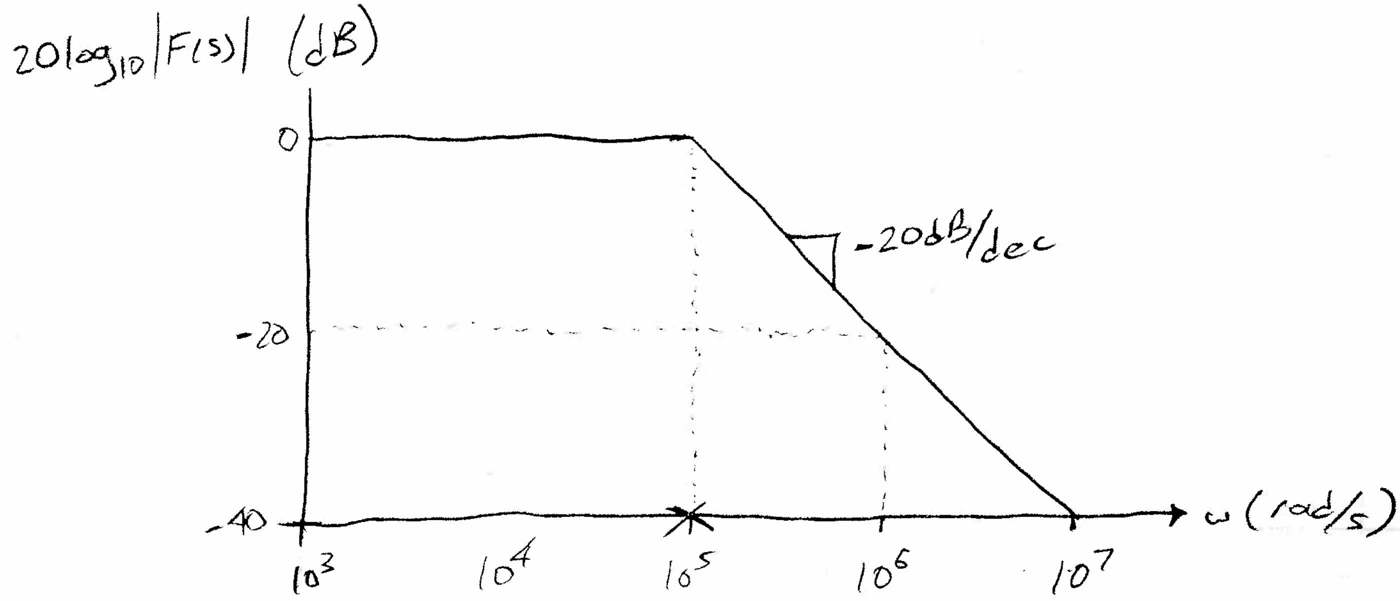
$$\lim_{s \to \infty} F(s) = \frac{10^5}{\infty + 10^5} = 0$$

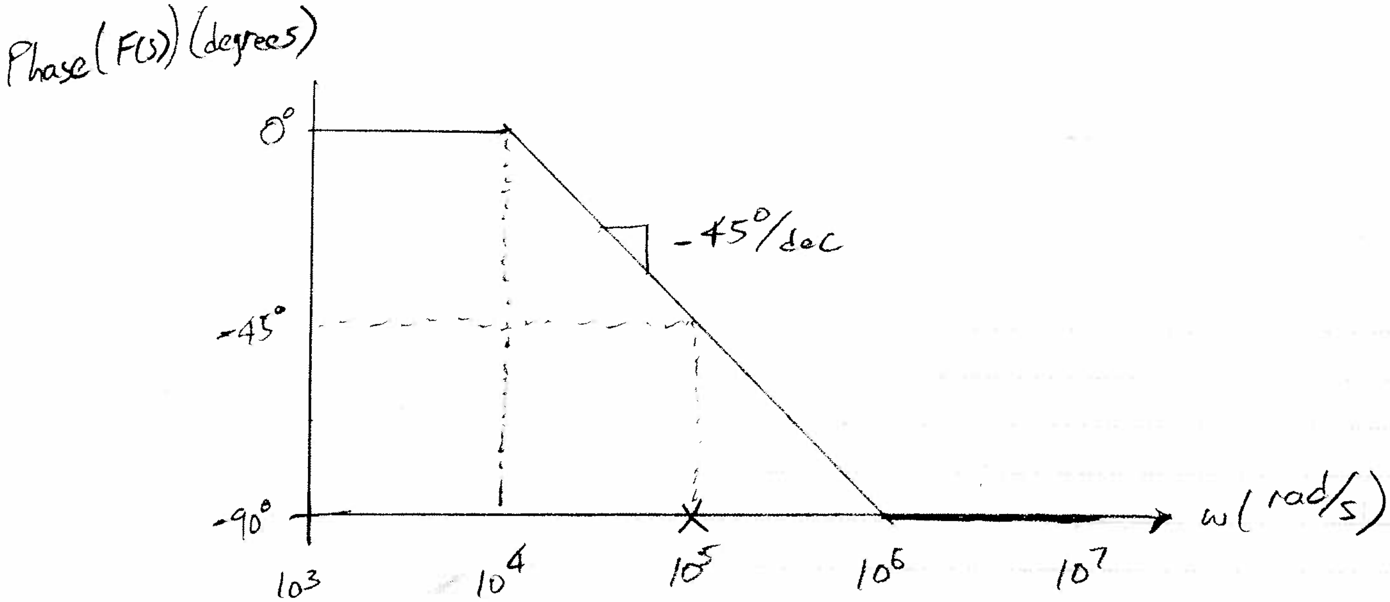
DC gain = 1

High freq. gain = 0

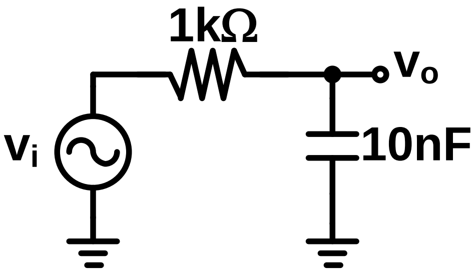
Low-Pass Filter

Bode Approximate Plot

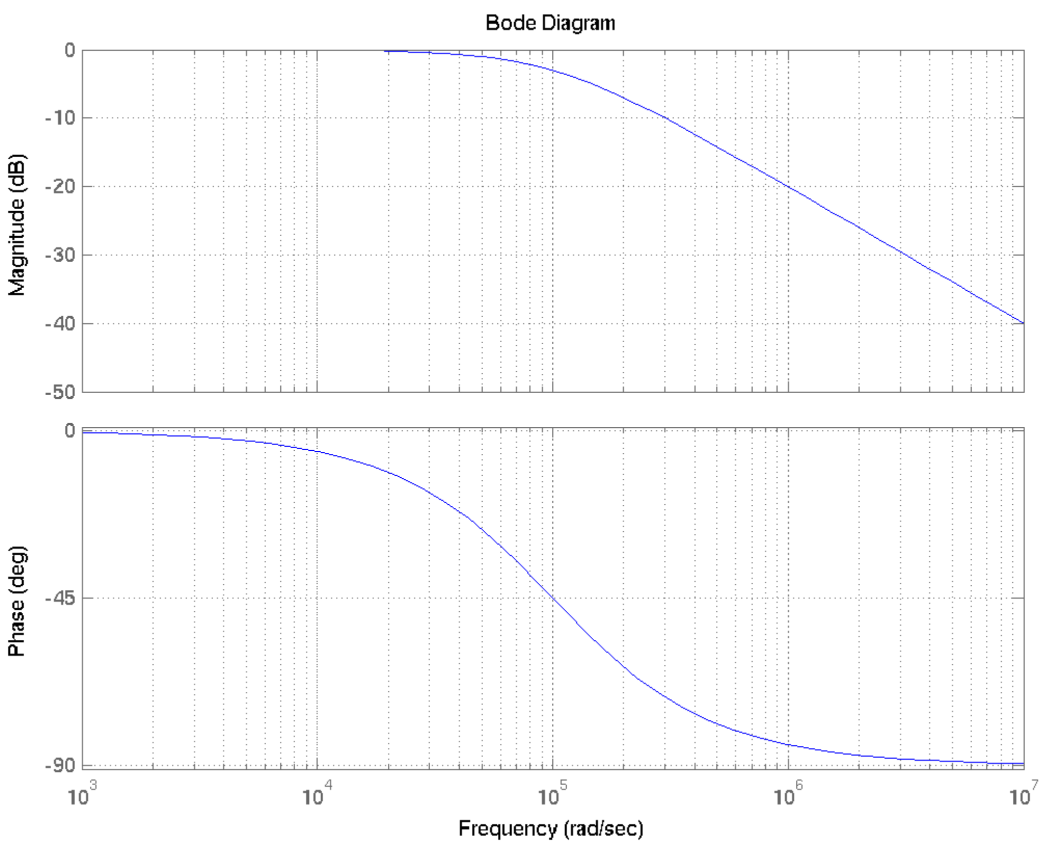




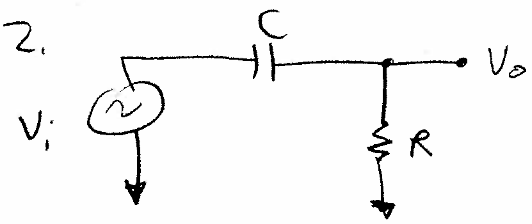
1.


1kΩ
vo
vi
10nF

Bode Plot


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
Phase (deg)
0
-45
-90
10^3
10^4
10^5
10^6
10^7
Frequency (rad/sec)

2.



$$V_o(s) = \frac{Z_R}{Z_R + Z_C} V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{s}{s + \frac{1}{RC}}$$

w/ $R = 1K\Omega$, $C = 10nF$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s + 10^5}$$

$$\lim_{s \to 0} F(s) = \frac{0}{0 + 10^5} = 0$$

$$\lim_{s \to \infty} F(s) = \frac{\infty}{\infty + 10^5} \Rightarrow 1$$

DC gain = 0

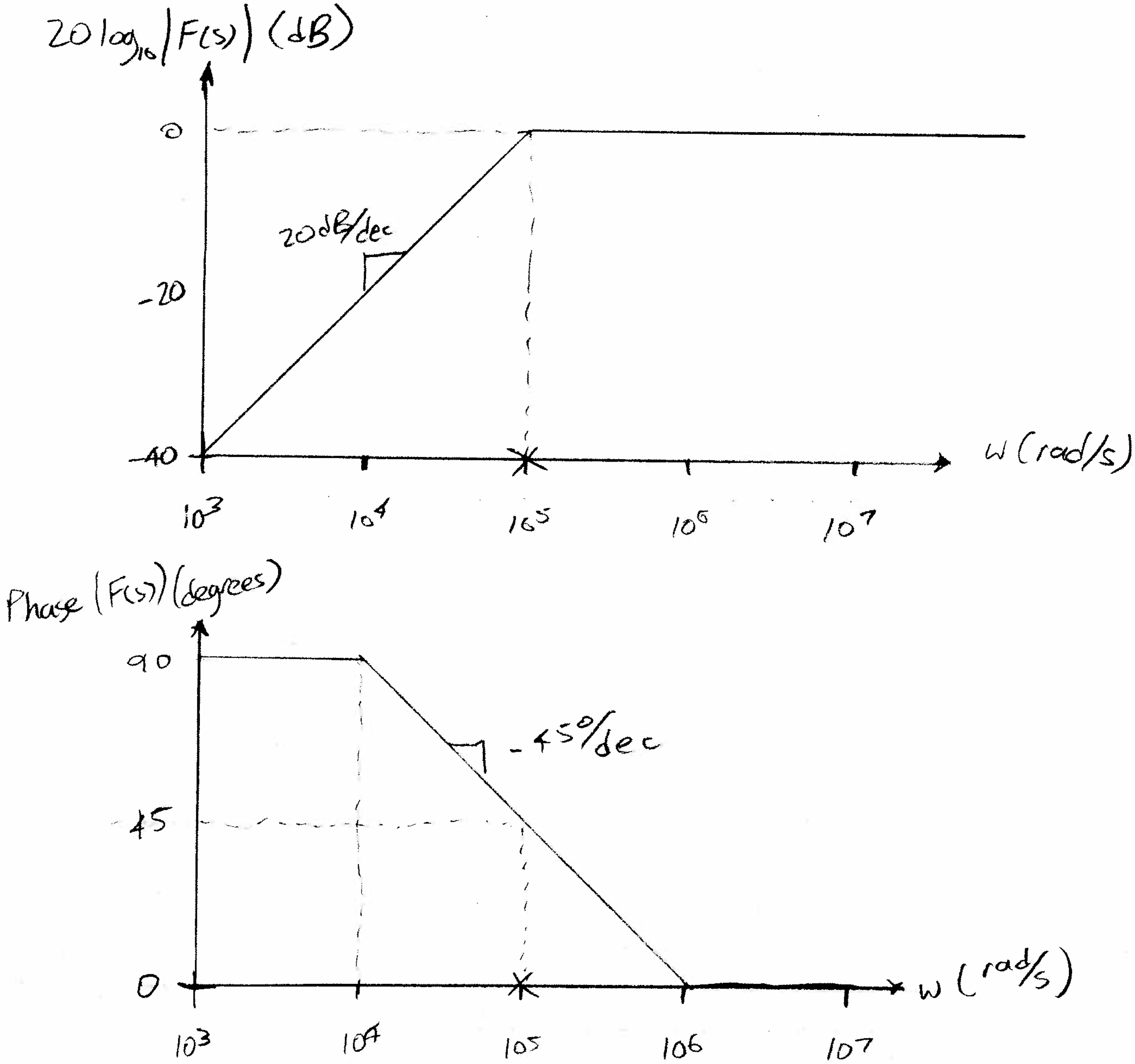
High Freq gain = 1

High-Pass Filter

1 zero at 0

1 pole at -10^5 rad/s

Approximate Bode Plot



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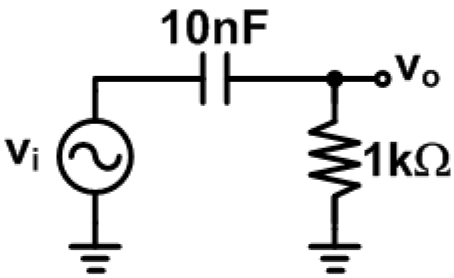
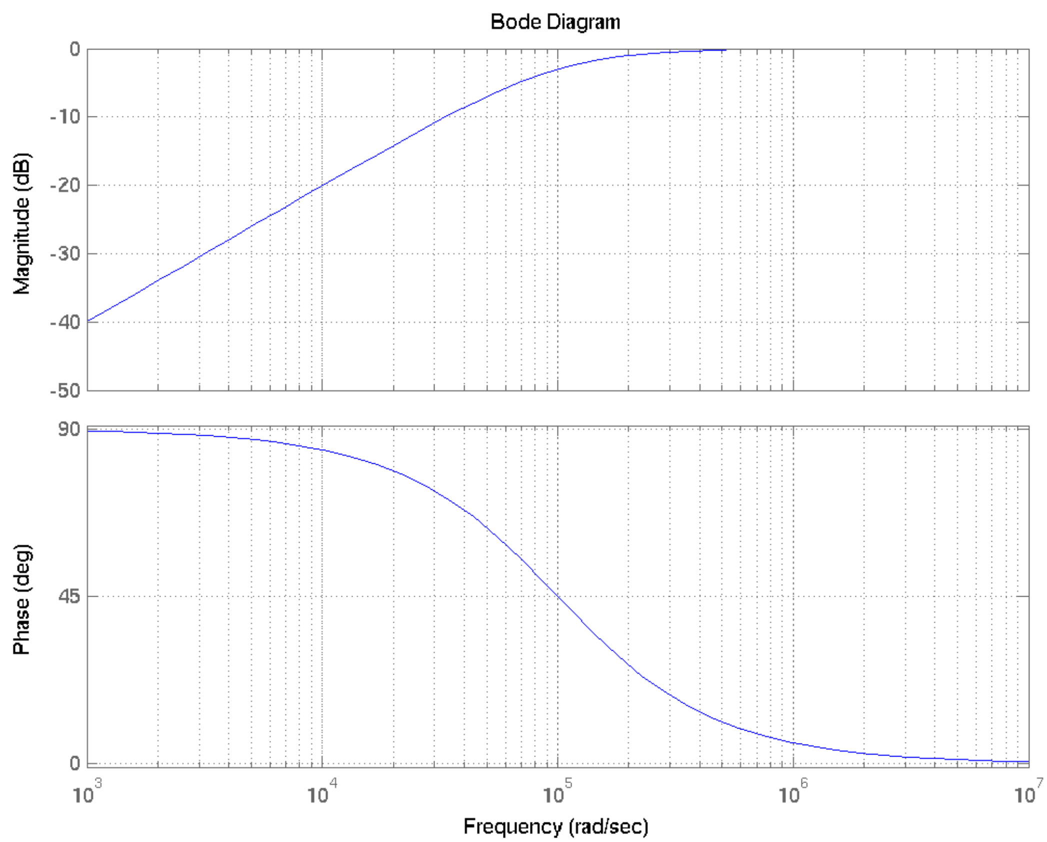
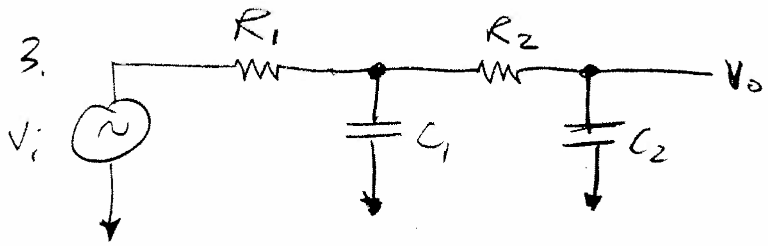

10nF
vo
vi
1kΩ

Bode Plot


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
Phase (deg)
90
45
0
10^3
10^4
10^5
10^6
10^7
Frequency (rad/sec)

5

3.



$$V_o(s) = \left[\frac{Z_{C_1} \parallel (Z_{R_2} + Z_{C2})}{Z_{R_1} + Z_{C_1} \parallel (Z_{R_2} + Z_{C2})}\right]\left[\frac{Z_{C2}}{Z_{R_2} + Z_{C2}}\right] V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \left[\frac{\dfrac{1}{sC_1 + \dfrac{1}{R_2 + \frac{1}{sC_2}}}}{R_1 + \dfrac{1}{sC_1 + \dfrac{1}{R_2 + \frac{1}{sC_2}}}}\right]\left[\frac{\frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}}\right]$$

$$= \left[\frac{1}{sR_1C_1 + \dfrac{R_1}{R_2 + \frac{1}{sC_2}} + 1}\right]\left[\frac{1/sC_2}{R_2 + \frac{1}{sC_2}}\right]$$

$$= \frac{\frac{1}{sC_2}}{sR_1R_2C_1 + R_1\frac{C_1}{C_2} + R_1 + R_2 + \frac{1}{sC_2}}$$

$$= \frac{1}{s^2R_1R_2C_1C_2 + sR_1C_1 + sR_1C_2 + sR_2C_2 + 1}$$

$$= \frac{1/R_1R_2C_1C_2}{s^2 + s\left[\dfrac{R_1C_1 + R_1C_2 + R_2C_2}{R_1R_2C_1C_2}\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

w/ $R_1 = R_2 = 1k\Omega \quad C_1 = C_2 = 10nF$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{10^{10}}{s^2 + 3\times10^5 s + 10^{10}}$$

$$\lim_{s\to0} F(s) = \frac{10^{10}}{0 + 0 + 10^{10}} = 1$$

$$\lim_{s\to\infty} F(s) = \frac{10^{10}}{\infty} = 0$$

DC gain = 1

High Freq gain = 0

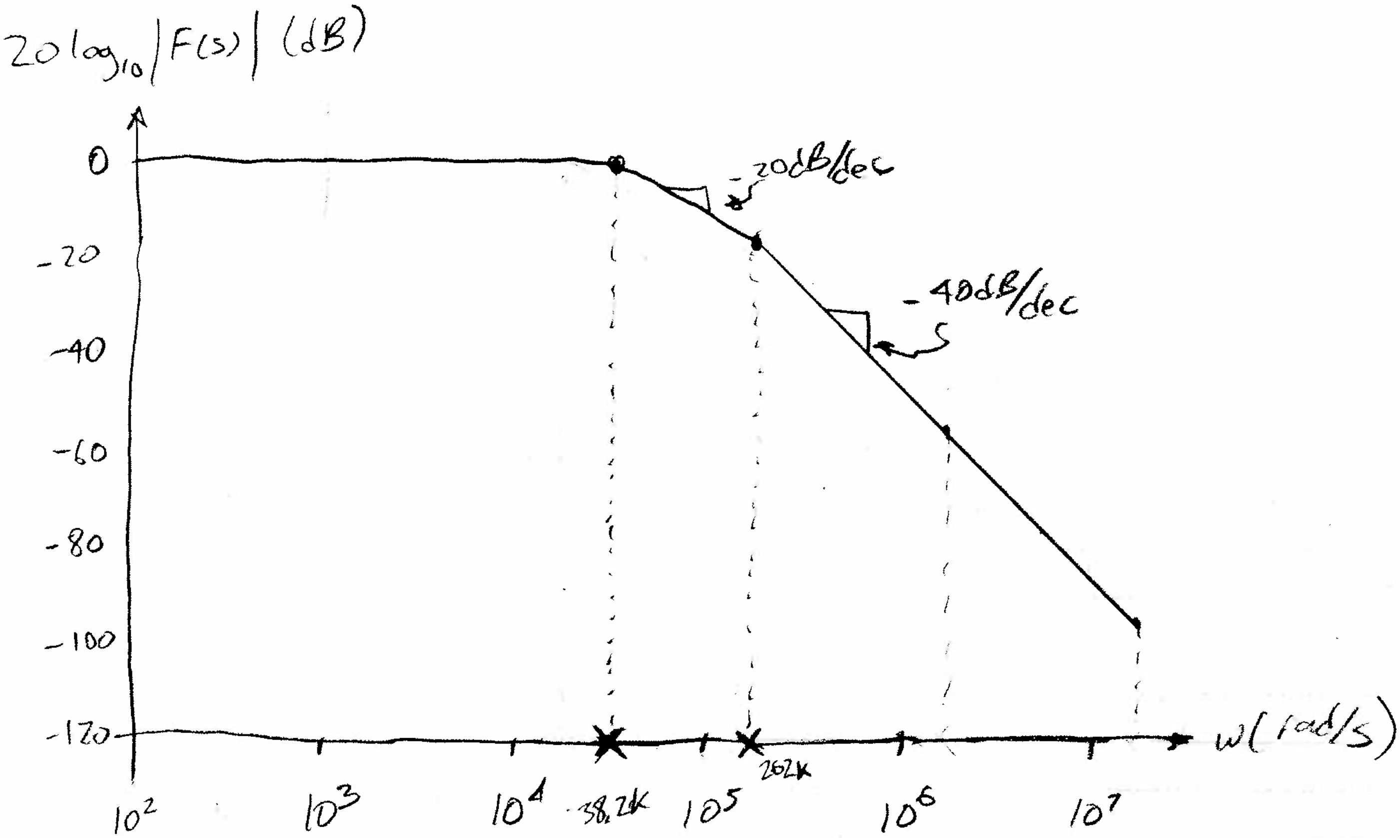
Low Pass Filter

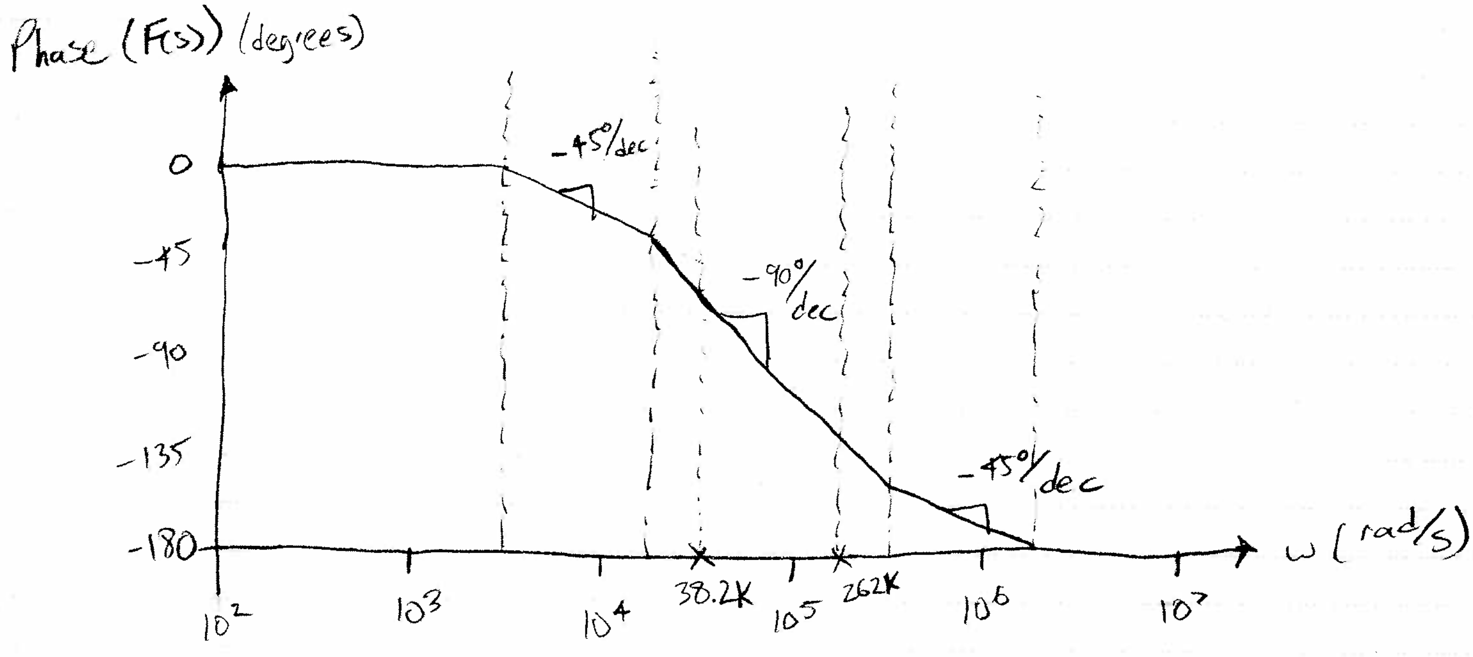
2 poles at $\frac{-3\times10^5 \pm \sqrt{9\times10^{10} - 4\times10^{10}}}{2}$

Poles at -38.2 krad/s

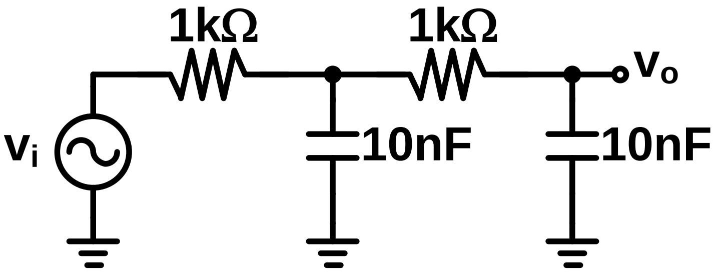
-262 krad/s

Approximate Bode Plot

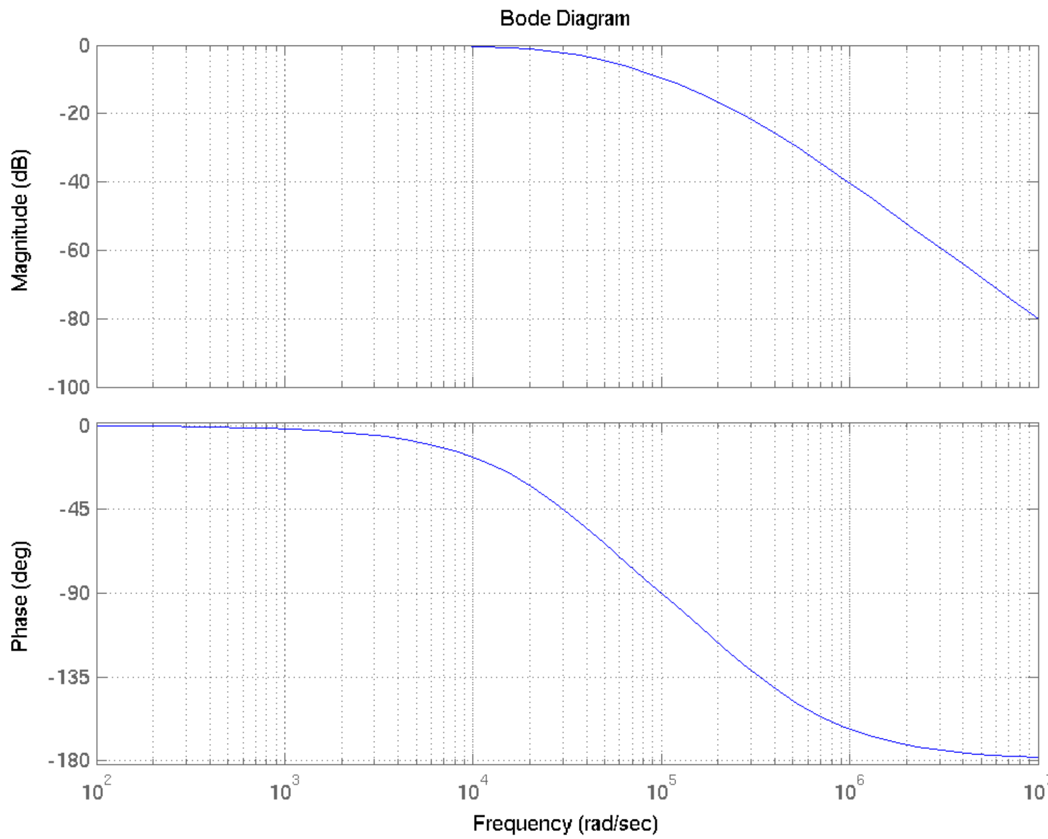




3.

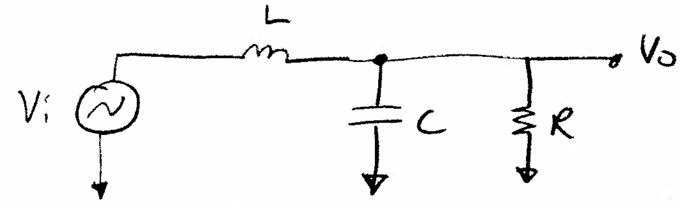

1kΩ
1kΩ
vo
vi
10nF
10nF

Bode Plot


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
0
-20
-40
-60
-80
-100
0
-45
-90
-135
-180
10^2
10^3
10^4
10^5
10^6
10^7

8

4.



$$V_o(s) = \frac{Z_C \parallel Z_R}{Z_L + Z_C \parallel Z_R} V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC + \frac{1}{R}}}{sL + \frac{1}{sC + \frac{1}{R}}} = \frac{1}{s^2 LC + s\frac{L}{R} + 1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

w/ $L = 1\,nH$, $C = 1\,pF$, $R = 25\,\Omega$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{10^{21}}{s^2 + 4\times 10^{10} s + 10^{21}}$$

$$\lim_{s\to 0} F(s) = \frac{10^{21}}{10^{21}} = 1$$

$$\lim_{s\to \infty} F(s) = \frac{10^{21}}{\infty} = 0$$

DC gain = 1

High Freq Gain = 0

Low-Pass Filter

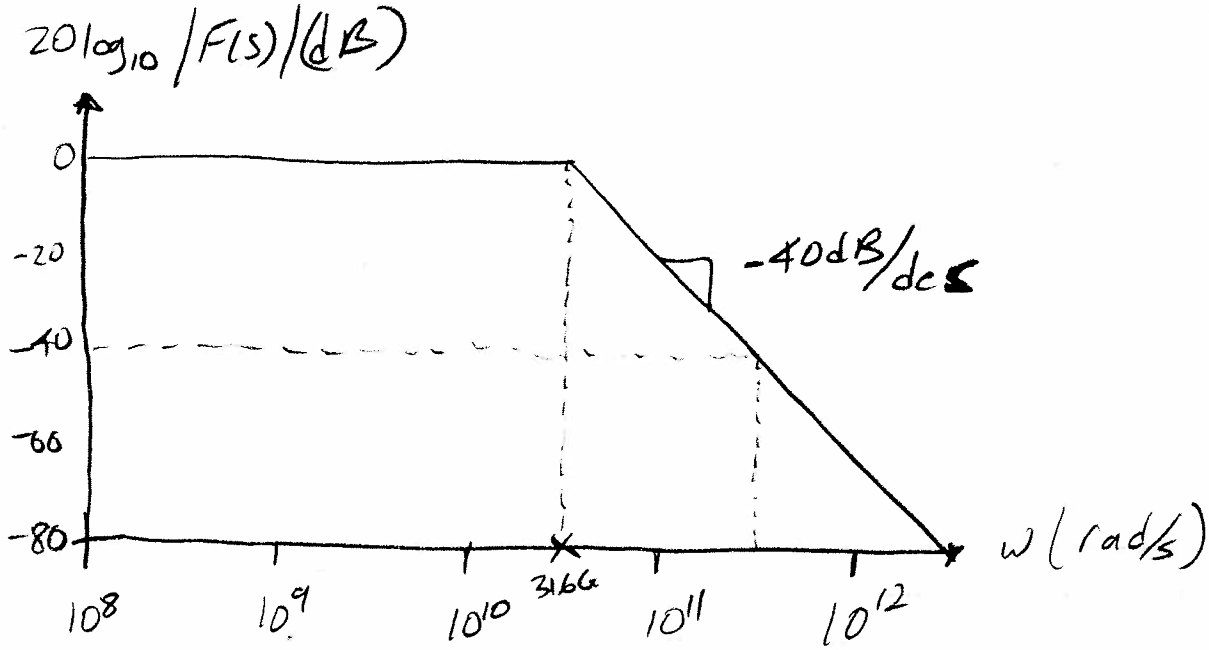
2 poles at $-2\times 10^{10} \pm j\,2.4\times 10^{10}$ rad/s

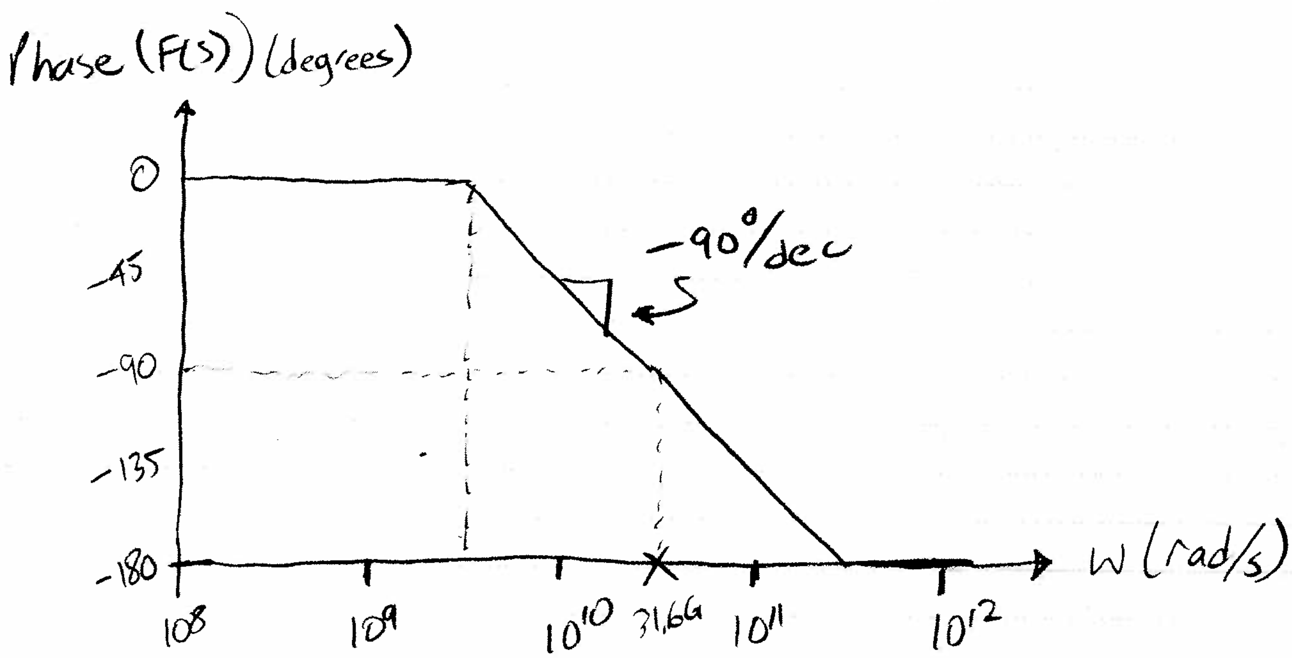
From F(s) denominator...

$$s^2 + 2\zeta\omega_n + \omega_n^2 = s^2 + 4\times10^{10}s + 10^{21}$$

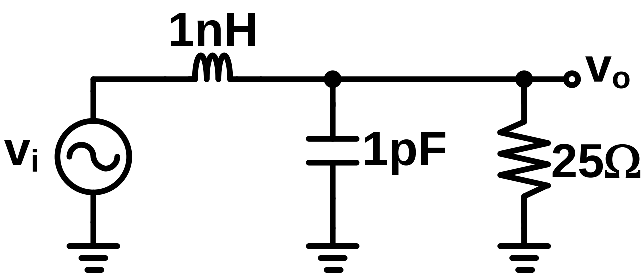
$$\omega_n = 3.16\times10^{10}\ \text{rad/s}, \quad \zeta = 0.63$$

For Approximate Bode Plot, both poles at $\approx \omega_n$

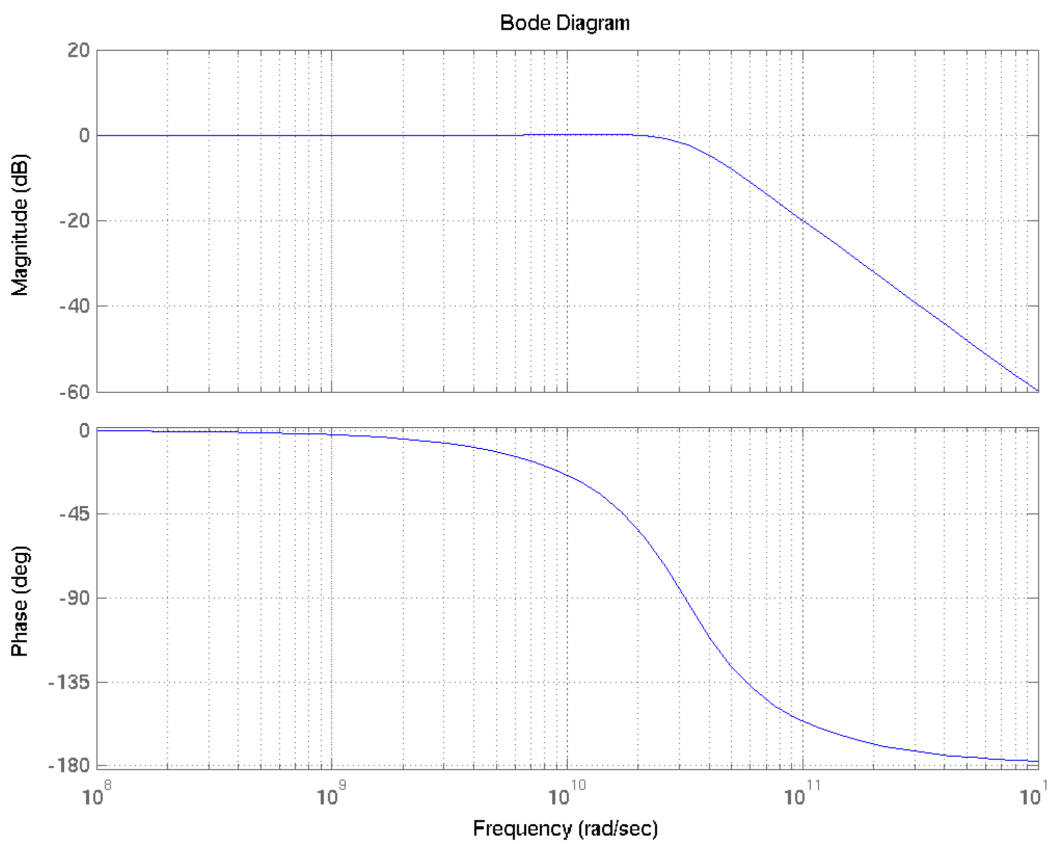




4.


1nH
vo
vi
1pF
25Ω

Bode Plot


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)