Resonant Coupling Analysis for a Two-Coil Wireless Power Transfer System

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***Abstract*—Inductive or non-radiative wireless power transfer (WPT) is a popular short range power delivery mechanism for transcutaneous biomedical implants. In this work, the relative performance of a two-coil WPT system is analyzed with each of the coils in series or parallel resonance. This analysis helps in choosing the optimum resonance configuration for a given pair of coils that can maximize the efficiency of the WPT system. The analysis described in this work shows that for a given pair of coils at a fixed distance apart and with the transmitter coil driven by a source with significant impedance, choosing parallel resonance configuration at the transmitter and receiver coils can offer up to 20dB and 25dB higher efficiencies respectively when compared to the series configurations. Thus, there is scope for improving the WPT efficiency with a simple rearrangement of the circuit components.**

I. Introduction

The idea of not being tethered to a power source by wires opens up exciting prospects. Wirelessly transferring power is critical to some applications like transcutaneous biomedical implants where it helps increase operational lifespan of such implants. In recent years, wireless power is garnering attention in consumer electronics as well- in the form of remote cordless charging of batteries.

Inductive or non-radiative wireless power transfer is a popular technique for wireless power transfer (WPT) over short distances. Though WPT systems has been extensively studied in literature, the choice of resonance configuration has not been discussed in detail in existing works. [1] analyses the transmit and receive circuits in a two-coil WPT system using reflected load theory. An iterative design procedure using HFSS to optimize coil design for a WPT system utilizing a pair of PCB coils is described in [2]. [3] summarizes the analysis of multi-coil WPT systems using reflected load theory and coupled mode theory described in other works and concludes that the two approaches lead to similar results as long as the resonant coupling is near-field and non-radiative. The circuits shown in [1] and [3] uses a series resonance at the transmitter and parallel resonance at the receiver, whereas [4] depicts series resonance at the receiver as well. The purpose of the analysis presented in this work is to develop some intuition on resonance configurations for a given pair of coils with the discussion limited to a two-coil WPT system.

II. Wireless Power Transfer Using Coils

Any change in the magnetic flux through a coil of wire induces a voltage across it. The simplest WPT scheme consists of two coils as shown in Fig. 1(a). The source V_G sets up an alternating current through the transmitter coil (inductance) L_T which in turn creates a time varying magnetic field around it. A portion of this magnetic flux is coupled into the receiver coil L_R which is in the vicinity of the transmitter coil. This coupled flux induces a voltage across the receiver coil and delivers power to the load R_L. In principle, such a scheme closely resembles a transformer with a key difference that the coupling efficiency between coils is quite low in a WPT system owing to the larger distance separating the two coils.

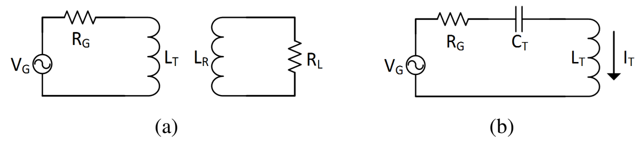

Fig. 1. (a) A simple two-coil WPT system (R_G models the source impedance) and (b) Transmitter coil in resonance.

Using Biot-Savart law, it can be shown that the magnetic field at a distance z along the centre line of a coil of radius R carrying a current I is described by

$$B_z = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}} \tag{1}$$

From this expression, it is evident that in order to maximize magnetic field generated by a given coil at a given distance, the current through the coil needs to be maximized.

Now consider the WPT system shown in Fig. 1(a). Assuming weak coupling between the two coils, the current through the transmitter coil can be given by

$$I_T = \frac{V_G}{R_G + Z_T}, \tag{2}$$

where Z_T is the impedance offered by transmitter coil. Z_T can be modelled as a coil inductance $j\omega L_T$ in series with an equivalent series resistance (ESR) of the coil, R_T. Therefore, the current through the transmitter coil is

$$I_T = \frac{V_G}{R_G + R_T + j\omega L_T}. \tag{3}$$

In Fig. 1(a) the transmitter coil current, and hence the magnetic flux generated by the transmitter coil, is dependant on the impedance (and ESR) of the coil at the frequency of operation of the WPT system. This dependence of the magnetic flux generated by the transmitter coil (and so, of the WPT efficiency) on the coil impedance can be easily removed by introducing resonance. Consider a capacitor in series with

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the transmitter coil as shown in Fig. 1(b). Then, the current through the coil becomes

$$I_T = \frac{V_G}{R_G + R_T + j\omega L_T - \dfrac{j}{\omega C_T}}. \quad (4)$$

Now, if the source frequency is chosen to be $\omega_0 = \frac{1}{\sqrt{L_T C_T}}$, then the inductive and capacitive reactances cancel out, maximizing the magnitude of current I_T as

$$I_{T,max} = \frac{V_G}{R_G + R_T};\ if\ \omega_0 = \frac{1}{\sqrt{L_T C_T}}. \quad (5)$$

This, in turn, maximizes the magnetic field generated by the coil and hence the efficiency of coupling as well.

Having established the motivation to have the transmitter coil in resonance, the next logical step is to enquire if resonance in the receiver coil can benefit the WPT system. Moreover, the possibility of employing a parallel resonance (capacitor in shunt with the coil) in place of the series resonance described above can be investigated. The aim of the following analysis will be to determine whether one resonance configuration is better than the other for a given pair of coils.

III. Analysis of Coil Coupling in a WPT System

In the following subsections, a comparison of the WPT performance with series and parallel resonances at transmitter and receiver coils is made. A coil is said to be in series resonance when the capacitor resonating with the coil is in series with it. Similarly, for the coil to be in parallel resonance, the capacitor is in shunt with it. In a real WPT system, the transmitter coil will see an additional load because of the load R_L connected to the receiver coil. Thus the load the the receiver is "reflected" onto the transmitter coil. This reflected load can be shown in series with the transmitter coil and so can be bracketed with R_T shown in Fig. 2. In this analysis, a source V_G driving the transmitter coil is considered to have a source impedance R_G. Assuming Q_T to be the quality factor of the transmitter coil, the ESR of the transmitter coils is $R_T = Q_T \omega L_T$.

A. Series Versus Parallel Resonance at Transmitter

The transmitter coil can be configured with series or shunt capacitors as shown in Fig. 2(a) and Fig. 2(b) respectively. From the previous section it was concluded that efficiency of WPT can be maximized for a given transmitter coil and a given distance of seperation between the coils by maximizing the magnitude of alternating current through the coil.

In the series resonance configuration shown in Fig. 2(a), the current through the coil is given by (5) as derived in the previous section. Now, if the transmitter coil was considered to be high-Q, then the ESR of the coil will be very small and the current through the coil can be approximated as

$$I_{Tseries} \approx \frac{V_G}{R_G};\ if\ R_G \gg R_T. \quad (6)$$

To analyse the parallel resonance configuration shown in Fig. 2(b), a narrowband impedance transformation can be

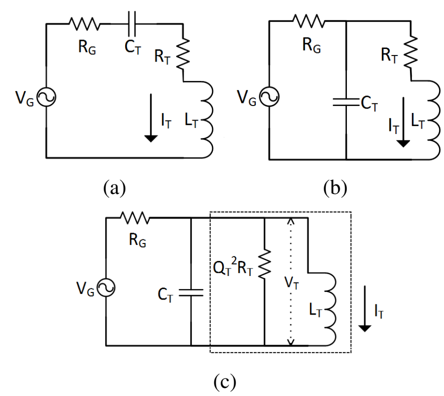


Fig. 2. Transmitter coil in (a) series resonance and (b) parallel resonance. (c) depicts the circuit in (b) after narrowband impedance transformation.

applied as shown in Fig. 2(c). This transformation of the circuit is justified since the source V_G is driving the circuit at frequency ω_0. For this circuit, at resonance, the admittances of the capacitance C_T and the inductance L_T together add up to zero. That means that the effective impedance seen by the driver consists of just the resistance $Q_T^2 R_T$. So at resonance, the voltage across this resistance $Q_T^2 R_T$ can be derived as

$$V_T = \frac{Q_T^2 R_T}{R_G + Q_T^2 R_T} V_G \approx V_G;\ if\ R_G \ll Q_T^2 R_T. \quad (7)$$

If the transmitter coil was considered to have very high-Q, then $Q_T^2 R_T$ will be much larger than the driver impedance R_G. Then the current through the coil can be written as

$$I_{Tparallel} = \frac{V_T}{j\omega_0 L} \approx \frac{V_G}{j\omega_0 L};\ if\ R_G \ll Q_T^2 R_T \quad (8)$$

From equations (6) and (8), it can be observed that the expression for current through the coil at resonance is different for series and parallel resonance configurations. For a given high-Q coil, if the source resistance R_G is greater than the coil impedance $\omega_0 L$, then the magnitude of $I_{Tseries}$ will be smaller than the magnitude of $I_{Tparallel}$ and so the parallel configuration is preferred. On the other hand, if the source resistance R_G is lower than the coil impedance $\omega_0 L$, then $I_{Tseries}$ will have a larger magnitude than $I_{Tparallel}$ and using series configuration is more efficient for WPT.

Note that the assumption of a high-Q transmitter coil was invoked in the above derivation. As typical biomedical applications employ a PCB transmitter coils which can provide very high Q, this assumption is justified. Generally, reflected load seen at the transmitter coil due to the load R_L at receiver can be approximated as $k^2 \frac{L_T}{L_R} R_L$ [1]. Since the coupling coefficient $k \ll 1$, and assuming moderate to low load R_L at the receiver, this reflected load at the transmitter can be neglected.

B. Series Versus Parallel Resonance at Receiver

The two resonance configurations at the receiver coil is compared in this section. The voltage that couples into the receiver coil can be modelled as a voltage source V_I in series with the receiver coil. The following analysis compares the power delivered to the load R_L connected at the receiver coil for a given induced voltage V_I at the receiver coil, for the two configurations shown in Fig. 3(a) and Fig. 3(b). The equivalent series resistance of the receiver coil is represented by the resistance R_R in Fig. 3.

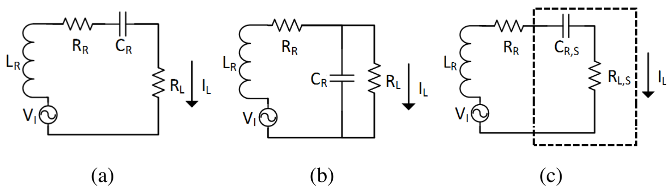


Fig. 3. Receiver coil in (a) series resonance and (b) parallel resonance. (c) depicts the circuit in (b) after narrowband impedance transformation.

Consider the series resonance configuration as shown in Fig. 3(a). The current through the load R_L is given by

$$I_{Lseries} = \frac{V_I}{R_L + R_R + j\omega L_R - \dfrac{j}{\omega C_R}} \tag{9a}$$

$$\implies I_{Lseries} = \frac{V_I}{R_L + R_R};\ if\ \omega_0 = \frac{1}{\sqrt{L_R C_R}}. \tag{9b}$$

Now, consider the parallel resonance configuration as shown in Fig. 3(b). At resonance frequency ω_0, a quality factor Q_R is defined as

$$Q_R = \omega_0 C_R R_L. \tag{10}$$

Q_R helps transform the parallel impedances of R_L and C_R shown in Fig. 3(b) to a series arrangement of $R_{L,S}$ and $C_{R,S}$ as shown in Fig. 3(c) through a narrowband impedance transformation. The equivalent series capacitance $C_{R,S}$ and the equivalent load resistance $R_{L,S}$ in Fig. 3(c) is related to C_R and R_L in Fig. 3(b) by

$$C_{R,S} = \frac{Q_R^2 + 1}{Q_R^2} C_R,\ \&\ R_{L,S} = \frac{R_L}{Q_R^2 + 1}. \tag{11}$$

Note that the effective load seen by the coil has now decreased by a factor of $Q_R^2 + 1$ when compared to the series resonance configuration. For this transformed circuit, the current through the equivalent load $R_{L,S}$ is

$$I_{Lparallel} = \frac{V_I}{R_{L,S} + R_R + j\omega L_R - \dfrac{j}{\omega C_{R,S}}}, \tag{12a}$$

$$\implies I_{Lparallel} = \frac{V_I}{R_{L,S} + R_R};\ if\ \omega_0 = \frac{1}{\sqrt{L_R C_{R,S}}}. \tag{12b}$$

Notice that the resonance frequency ω_0 is not exactly the same as the condition derived in (9b) for the series resonance configuration. The resonance frequencies in these two cases are equal only if $Q_R \gg 1$. But unlike the assumption made on the quality factor Q_T of the transmitter coil, Q_R may not always be greater than 1.

The power delivered to the load at resonance in the two configurations are

$$P_{series} = I_{Lseries}^2 R_L,\ \&\ P_{parallel} = I_{Lparallel}^2 R_{L,S}. \tag{13}$$

Assuming the receiver coil to be high-Q (not to be confused with Q_R of considered in the derivation of $I_{Lparallel}$) then, the ESR of the receiver coil is much smaller than the load. That is, $R_R \ll R_L$ and $R_R \ll R_{L,S}$. Under this condition, substituting for currents in expressions (13) from (9b) and (12b), the power delivered to the load in the two configurations are

$$P_{series} = \frac{V_I^2}{R_L},\ \&\ P_{Parallel} = \frac{V_I^2}{R_{L,S}}. \tag{14}$$

Since $R_{L,S} < R_L$, the power delivered will be more in the parallel resonance configuration.

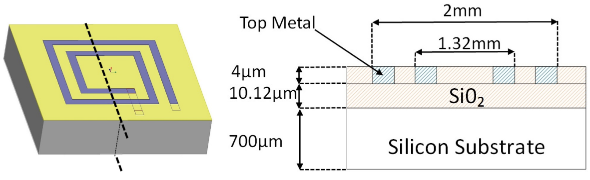
Revisiting the expressions for load current in series and parallel resonances ((9a) and (12a)), it can be observed that if the load resistances $R_L \gg \omega L$, then there is no benefit of having a capacitance cancel out the $j\omega L$ term. This is because the magnitude of the load current is going to be dominated by R_L. Typically, in any application, the load R_L is usually a rectifier circuit which can be modelled as a resistance and a capacitance in series. Hence even in the absence of an explicit capacitance C_R in Fig. 3(a), it is still possible to have cancellation of $j\omega L$ term. It may be possible that the $\frac{-j}{\omega C}$ term introduced by the rectifier load can dominate over $j\omega L$ of the coil. In such a case, the WPT system is best served by redesigning the coil such that $j\omega L$ term matches $\frac{-j}{\omega C}$ at $\omega = \omega_0$.

In a typical application in biomedical implants, the receiver coil can be an on-chip integrated inductor. The Q of such on-chip inductor is significantly lower than the Q of a PCB inductor. In this case, the optimum resonance configuration is determined by the relative values of $R_{L,S}$, R_L and R_R.

IV. Results

To explore the analysis made in the previous section, a WPT system involving a PCB coil transmitting power to an on-chip integrated coil was considered. Coil dimensions from [5], summarized in Table I, were utilized with a 5mm separation in air. The coil coupling was simulated using Ansys HFSS Electromagnetic simulator and a two-port model obtained, which was then plugged into a circuit simulator (Cadence Virtuoso) to simulate the various resonance configurations discussed in the previous section. Choosing the resonant frequency to be 100MHz and based on the inductance of the coils extracted from HFSS simulations, the resonance capacitances C_T and C_R were chosen to be 204pF and 265pF respectively. Fig. 4 shows the on-chip coil structure used in HFSS simulation.

The transmitting PCB coil was simulated to have a quality factor of close to 167 and gives the R_T of around 50mΩ. Now, setting up the on-chip receiver coil with parallel resonance and a load R_L = 1kΩ, and driving the transmitter through R_G= 100mΩ, it can be seen from the plots in Fig. 5(a) that series or parallel resonance gives about the same efficiency of around



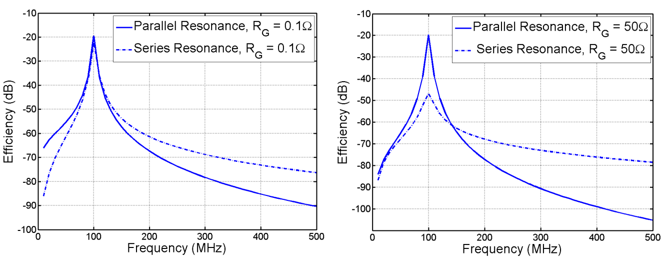
(a) (b)

Fig. 4. (a) Integrated coil model used in HFSS. (b) shows the cross section of (a) along the dotted line.

TABLE I
COIL DIMENSIONS

	PCB Coil	On-Chip Coil
Turns	1	2
Outer Diameter (mm)	14	2
Trace Width (μm)	3500	140
Trace Spacing (μm)	500	200
Substrate	1-oz FR-4	0.18μm CMOS
Inductance(nH)	12	9.4

-20dB at 100MHz. On the other hand, when transmitter coil is driven through an R_G= 50Ω, it can be seen from the plots in Fig. 5(b) that parallel resonance offers close to 25dB higher efficiency than series resonance at 100MHz, as predicted by the analysis presented in the previous section.

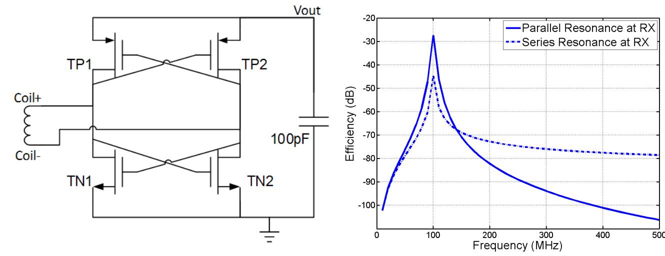


(a) (b)

Fig. 5. Coupling efficiency with parallel resonance at receiver coil with R_L= 1kΩ when transmitter coil is driven through (a) $R_G = 100m\Omega$ and (b) $R_G = 50\Omega$.

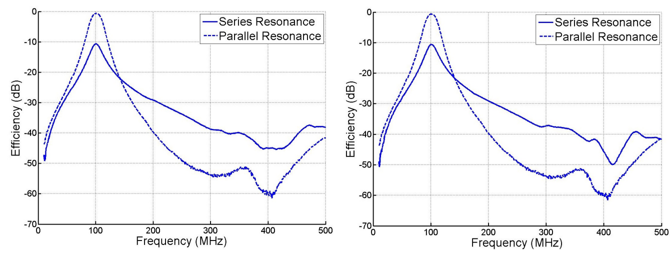
Since in any WPT system, the receiver coil generally drives a rectifier, a simple gate-cross coupled rectifier was designed in 0.18 μm CMOS as shown in Fig. 6(a) and the resonance configurations at the receiver coil was contrasted with this circuit driving 1kΩ as load. With the transmitter in parallel resonance configuration with R_G = 50Ω, it can be seen from the plots in Fig. 6(b) that parallel resonance configuration at the receiver offers close to 20dB higher efficiency at 100MHz when compared to series resonance configuration. This observation is also in concordance with the predictions made in the analysis presented in the previous section.

To validate the predictions in a real-world setting, a pair of PCB coils described in Table I was manufactured and WPT efficiencies were measured using a network analyser (R_G = R_L = 50Ω) for cases involving series and parallel resonances at transmitter and receiver coils. As shown in Fig. 7, in both TX and RX coils, parallel resonance seems to offer close to 10dB higher performance.



(a) (b)

Fig. 6. (a) Rectifier Circuit (b) Coupling efficiency with the receiver coil driving the rectifier.



(a) (b)

Fig. 7. Measurement results for (a) Series versus parallel resonance at transmitter with receiver coil in parallel resonance, and (b) Series versus parallel resonance at receiver with transmitter coil in parallel resonance.

V. CONCLUSION

Various combinations of resonance configurations can be employed in a inductive WPT system with two coils. The analysis has shown that in a system with a high-Q transmitter coil driven by a source with significant source impedance, a parallel resonance configuration is preferred at the transmitter coil. Moreover, for a typical system, parallel resonance seems to be more efficient at the receiver coil as well. With the right configuration, the efficiency of WPT can improve by as much as 20dB. Though the analysis neglected the loading seen by the transmitter coil due to the receiver, the trend predicted by the analysis has been validated with laboratory measurements on PCB based WPT system.

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