Large Mesh Deformation Using the Volumetric Graph Laplacian(VGL)

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Review

- Laplacian of graph

$$\delta_i = \mathcal{L}_G(p_i) = p_i - \sum_{j \in \mathcal{N}(i)} w_{ij} p_j$$

$$w_{ij} = 1/|\mathcal{N}(i)|$$

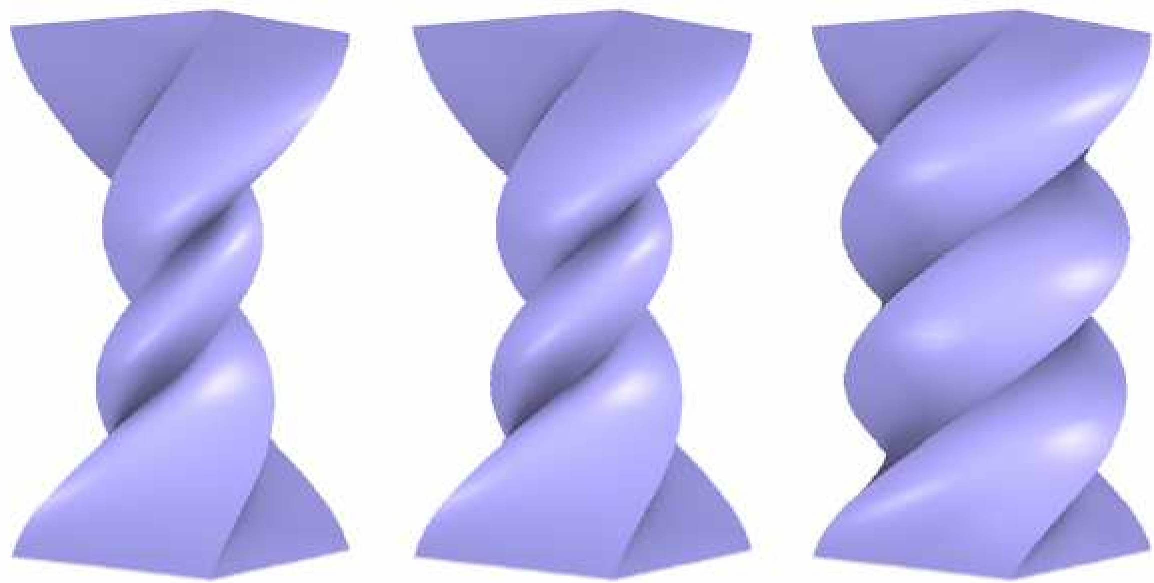
- Compute the new position of the vertices

$$\min_{p'_i} \left(\sum_{i=1}^{N} \|\mathcal{L}_G(p'_i) - \delta'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 \right)$$

Review

- Disadvantages:

  The result not so good in some large deformations



(a) Laplacian surface (b) Poisson mesh (c) VGL

Review

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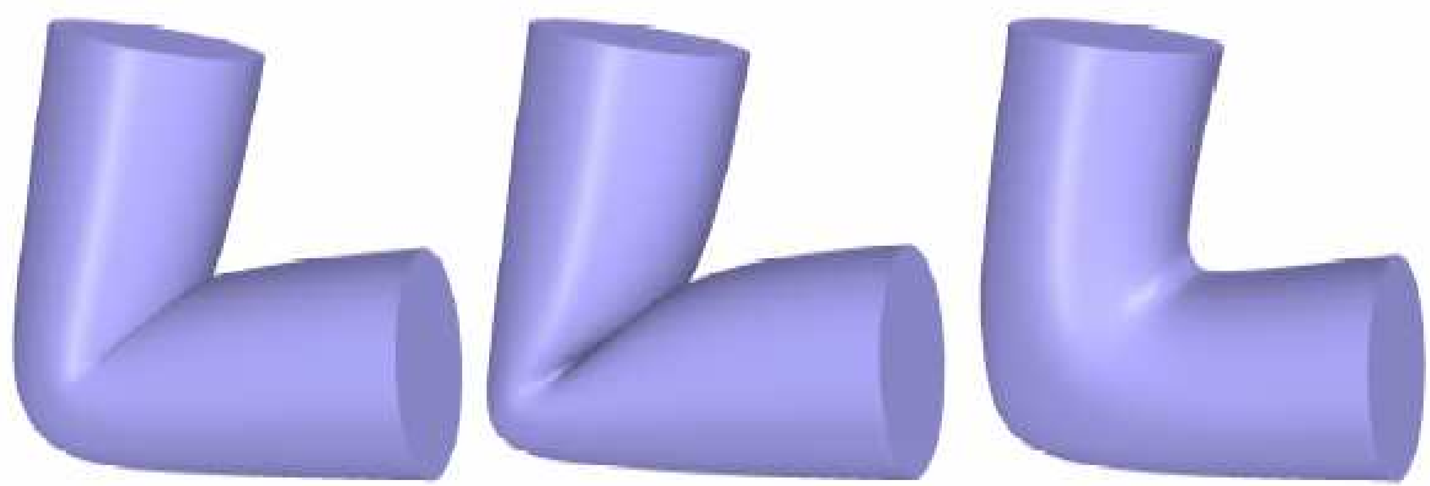


(a) Laplacian surface (b) Poisson mesh (c) VGL

Advantage of VGL

An inside graph --------- prevent large volume change

An outside graph -------- prevent local self-intersaction

A better defined weight ----- improve the result

Process of mesh deformation using VGL

- Constructing inner graph
- Constructing outer graph
- Calculate Laplacian for each of the vertices
- Perform a deformation (curve-based)
- Calculate new positions for vertices

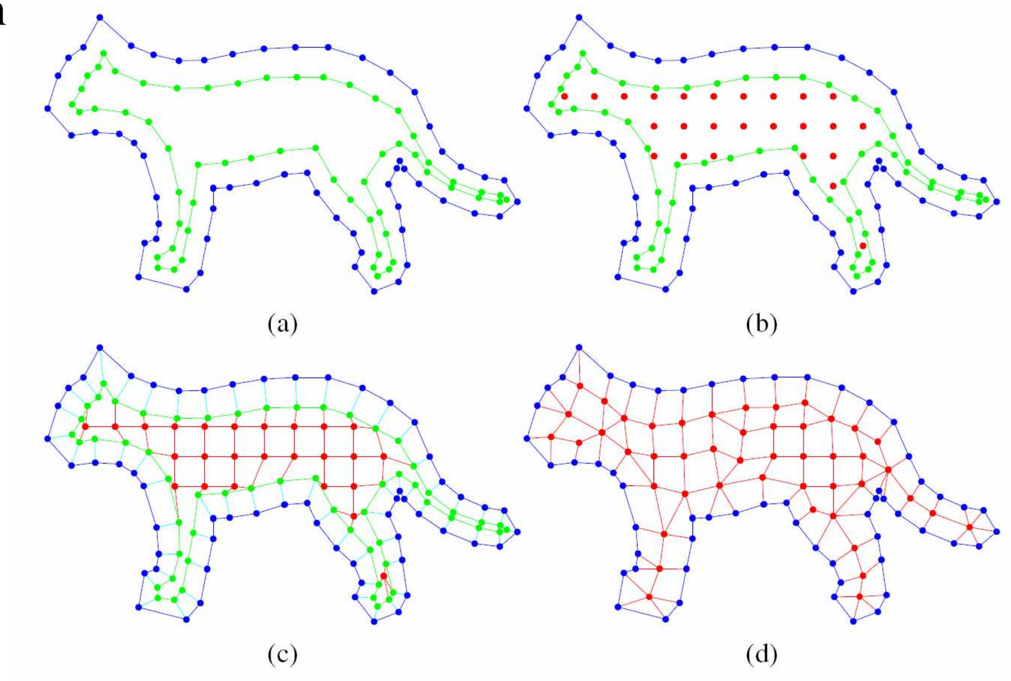
Constructing inner graph

• Construct an inner shell *Min* for the mesh*M* by offsetting each vertex a distance in the direction opposite its normal .

• Embed *Min* and *M* in a body-centered cubic (BCC) lattice. Remove lattice nodes outside *Min* .

• Build edge connections among *M*, *Min*, and lattice nodes.

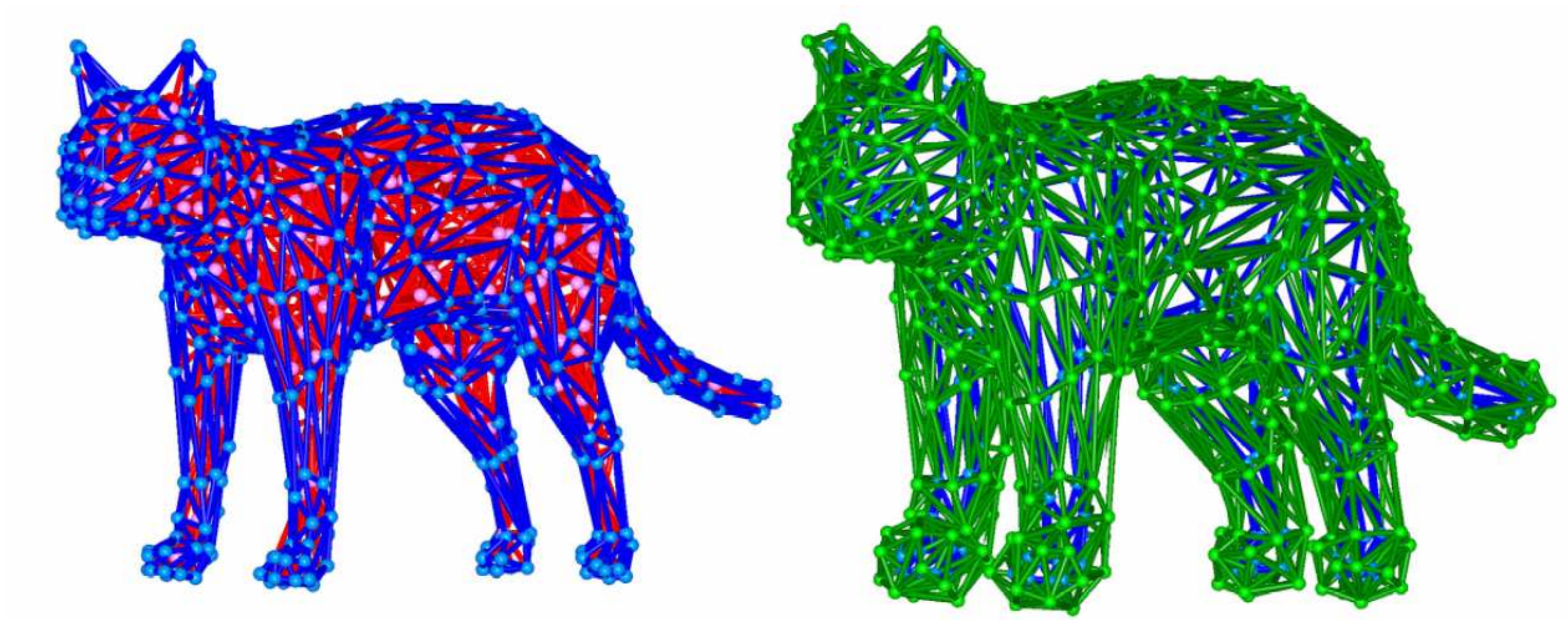
• Simplify the graph using edge collapse and smooth the graph.



Constructing outer graph

• *use the iterative normal-offset method to construct Mout just as creating Min*

• *Build connections between M and Mout.*



Laplacian of the Volumetric Graph

$$\boxed{\sum_{i=1}^{N} \|\mathscr{L}_G(p'_i) - \delta'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2}$$

$$\Downarrow$$

$$\sum_{i=1}^{n} \|\mathscr{L}_M(p'_i) - \varepsilon'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 + \beta \sum_{i=1}^{N} \|\mathscr{L}_{G'}(p'_i) - \delta'_i\|^2$$

$\mathscr{L}_M$ is laplacian for original mesh

$\mathscr{L}_{G'}$ is laplacian for inner and outside graph

β balances between surface and volumetric details

Laplacian of the Volumetric Graph Weighting Scheme

- For the mesh Laplacian $\mathcal{L}_M$

$$w_{ij} \propto (\cot \alpha_{ij} + \cot \beta_{ij})$$

where $\alpha_{ij} = \angle(p_i, p_{j-1}, p_j)$ and $\beta_{ij} = \angle(p_i, p_{j+1}, p_j)$

Laplacian of the Volumetric Graph Weighting Scheme

- For the graph Laplacian $\mathscr{L}_{G'}$

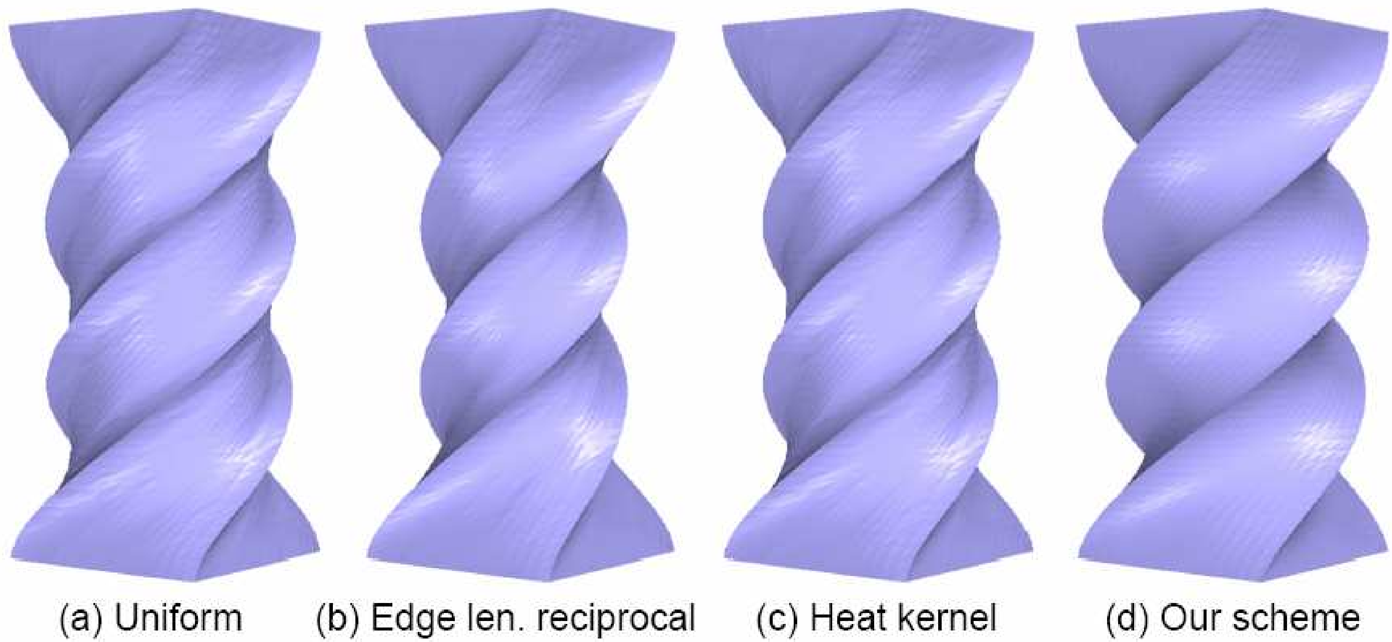
$$\min_{w_j} \left(\|p_i - \sum_{j \in \mathcal{N}(i)} w_j p_j\|^2 + \lambda \left(\sum_{j \in \mathcal{N}(i)} w_j \|p_i - p_j\| \right)^2 \right)$$

$$\text{subject to} \quad \sum_{j \in \mathcal{N}(i)} w_j = 1 \text{ and } w_j > \xi .$$

generate Laplacian coordinates of smallest magnitude

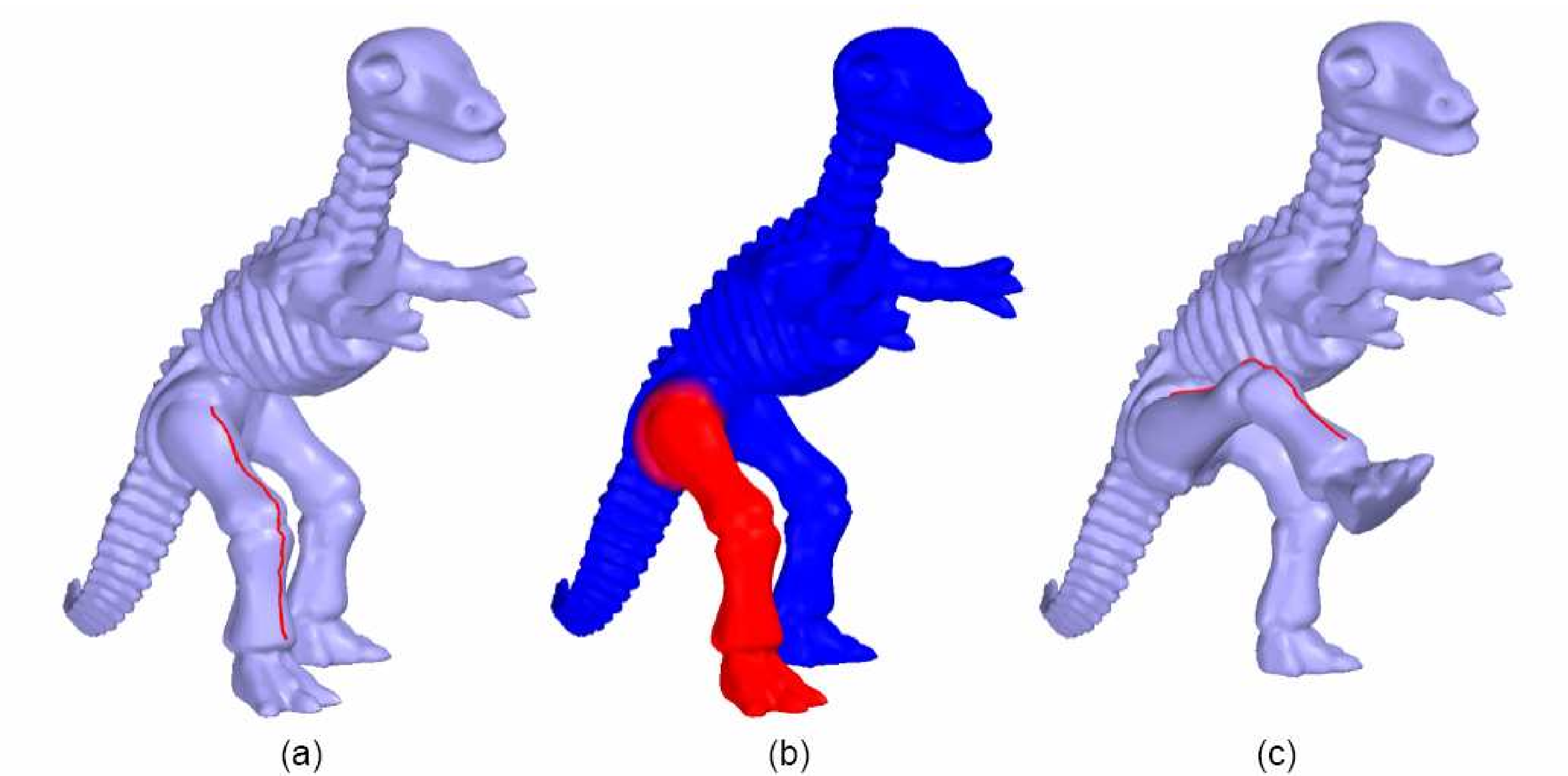
based on a *scale-dependent umbrella operator* which prefers weights in inverse proportion to the edge lengths.

Laplacian of the Volumetric Graph
Weighting Scheme



decaying exponential
function of squared distance

Deformation of the Volumetric Graph
Curve-based deformation



Deformation of the Volumetric Graph
Curve-based deformation

- Select control curve (control points)

- Calculate deformed positions for control points (WIRE)

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right)$$

- Propagation the deformation to the rest points of the graph

Strength field----based on the shortest edge path
(discrete geodesic distance)
from p to the curve.
----constant, linear, and gaussian

Deformation of the Volumetric Graph
Curve-based deformation

- Weighting all the vertices on the control curve----smoother

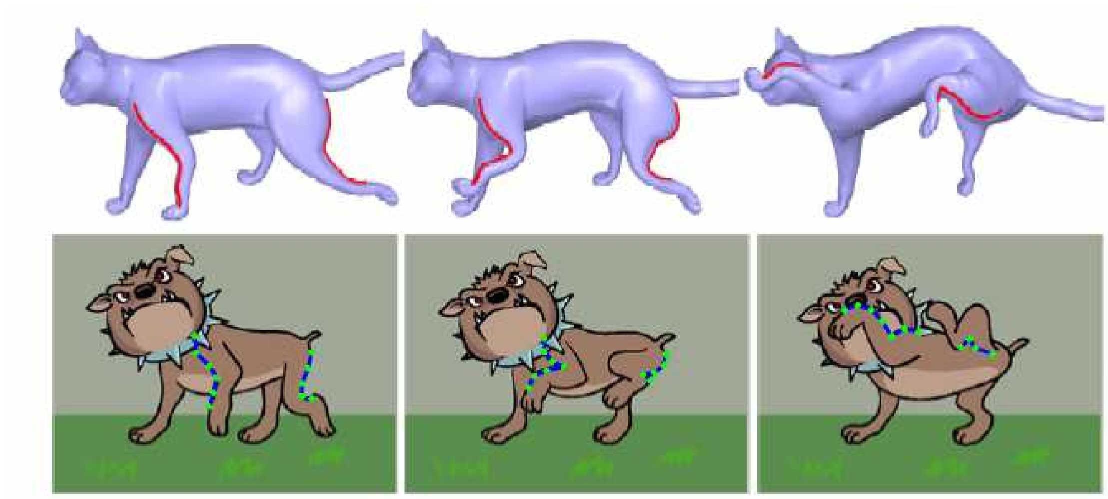
reciprocal of distance $1/\|p-q_i\|_g$

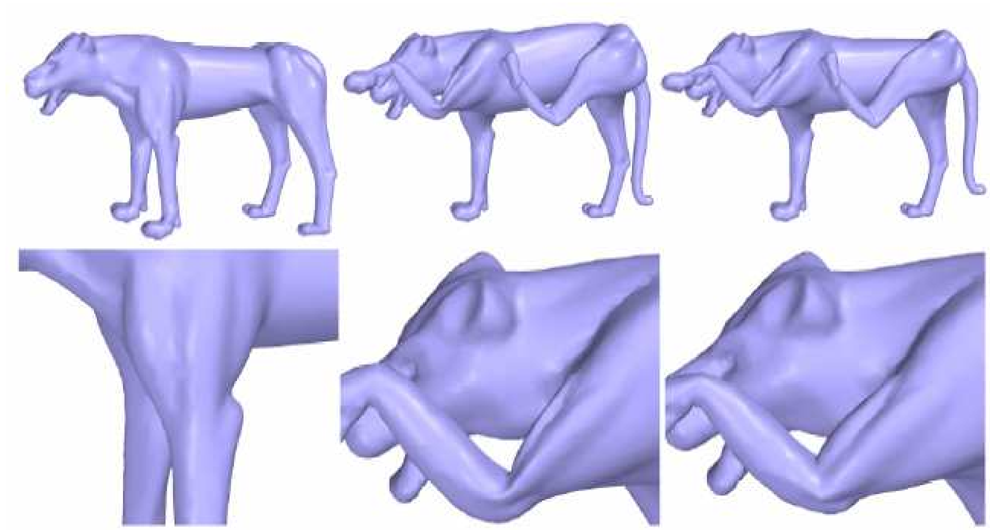
Gaussian function $\exp\left(-\frac{(\|p-q_i\|_g-\|p-q_p\|_g)^2}{2\sigma^2}\right)$

$\|p-q\|_g$ discrete geodesic distance from p to q

σ the width of the Gaussian

Result of VGL





Result of VGL

	arma	dino	cat	lioness	dog
# mesh vertices	15,002	10,002	7,207	5,000	10,002
# graph points	28,142	15,895	14,170	8,409	17,190
graph generation	2.679s	1.456s	1.175s	1.367s	1.348s
LU decomposition	0.524s	0.286s	0.348s	0.197s	0.118s
back substitution	0.064s	0.028s	0.030s	0.019s	0.011s
# control curves	6	5	4	5	
# key frames	10	9	8	8	
session time (min)	$\sim$120	$\sim$90	$\sim$30	$\sim$90	