Free-Form Deformation of Solid Geometric Models

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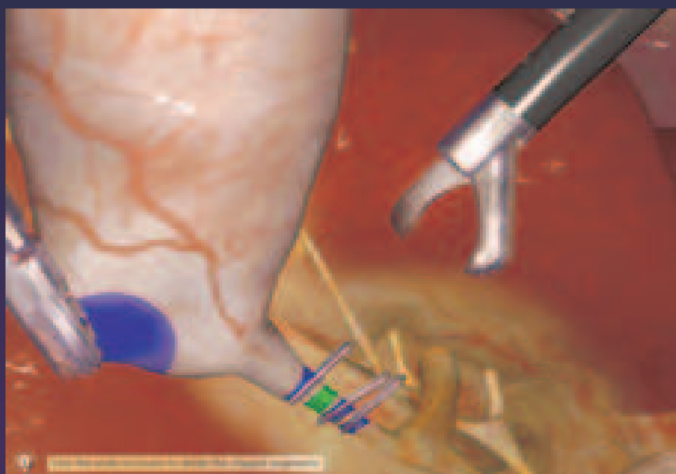
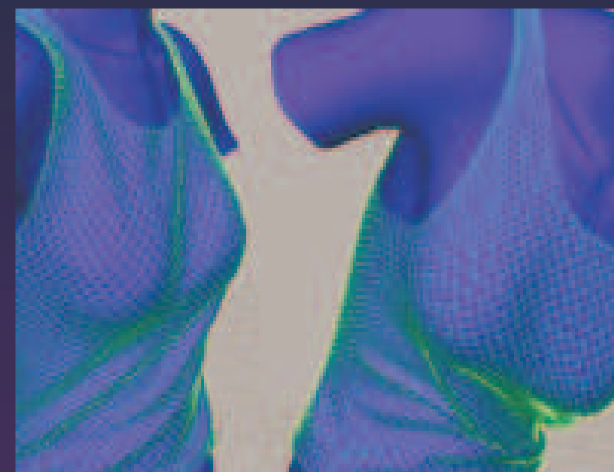
TAMU
Computer
Science

Deformation

Deformation

Deformation Applications

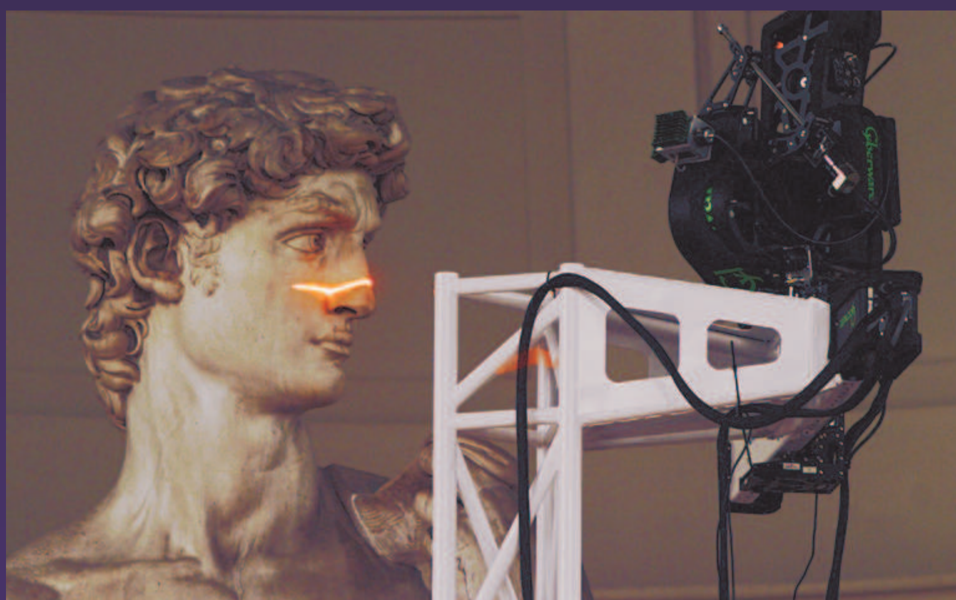


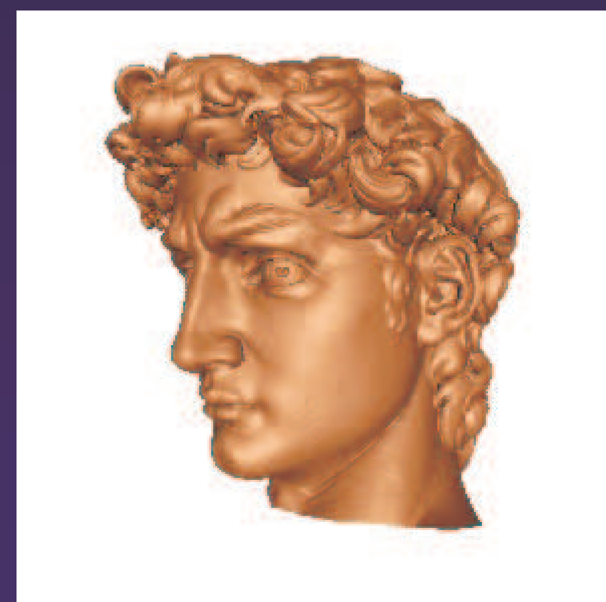


Toy Story © Disney / Pixar

Challenges in Deformation

- Large meshes – millions of polygons
- Need efficient techniques for computing and specifying the deformation





Digital Michelangelo Project

FFD Contributions

- Smooth deformations of arbitrary shapes
- Local control of deformation
- Performing deformation is fast

- Widely used
  - Game/Movie industry
  - Part of nearly every 3D modeler

Bernstein Polynomials

- Different polynomial basis

$$1 \qquad t \qquad t^2 \qquad t^3$$

$$(1-t)^3 \quad 3(1-t)^2 t \quad 3(1-t)t^2 \quad t^3$$

- Arbitrary degree polynomials

$$(1-t) \qquad t$$

$$(1-t)^2 \quad 2(1-t)t \quad t^2$$

$$(1-t)^3 \quad 3(1-t)^2 t \quad 3(1-t)t^2 \quad t^3$$

$$\vdots$$

Properties of Bernstein Polynomials

- All polynomials can be written as Bernstein polynomials

$$\sum_{i=0}^{n} a_i t^i = \sum_{i=0}^{n} b_i \frac{n!}{(n-i)!i!}(1-t)^{n-i} t^i$$

- Polynomials sum to one

$$\sum_{i=0}^{n} \frac{n!}{(n-i)!i!}(1-t)^{n-i} t^i = ((1-t)+t)^n$$

Geometric Properties of Bernstein Polynomials

$$F(t) = \sum_{i=0}^{n} b_i \frac{n!}{(n-i)!\,i!} (1-t)^{n-i} t^i$$

- Interpolates its end-points

$$F(0) = b_0 \qquad\qquad F(1) = b_n$$

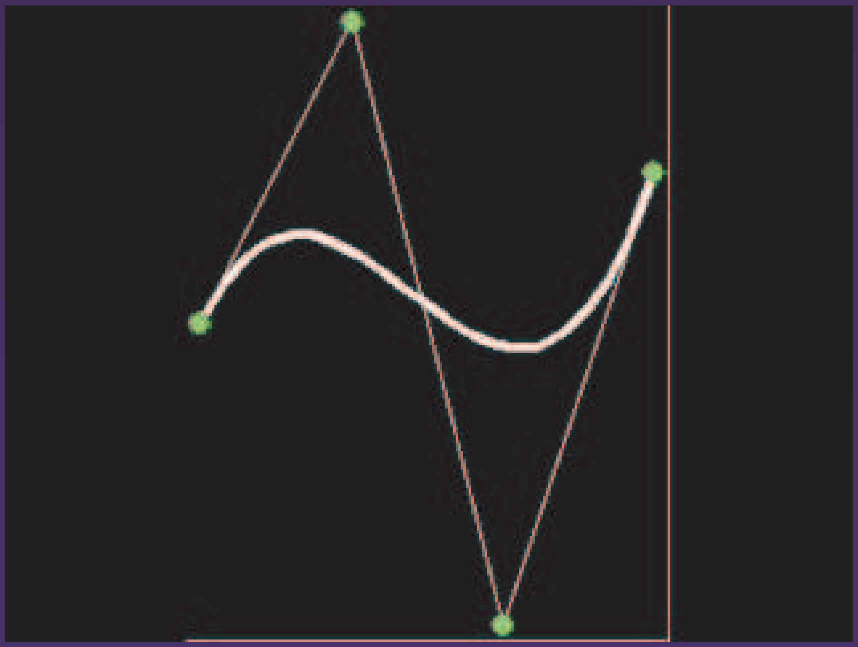
- End-point derivatives given by differences

$$F'(0) = n(b_1 - b_0) \qquad\qquad F'(1) = n(b_n - b_{n-1})$$

Bezier Curves

- Parametric curves defined by Bernstein polynomials

$$(x(t), y(t)) = \sum_{i=0}^{n} (x_i, y_i) \frac{n!}{(n-i)!i!} (1-t)^{n-i} t^i$$

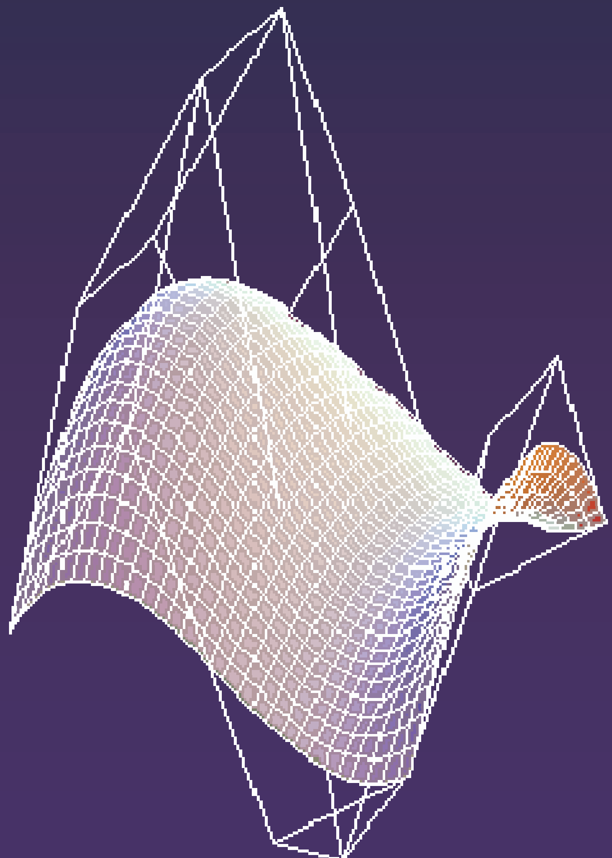


The Tensor Product Operation

$$x_j(u)=\sum_{i=0}^{n} x_{i,j} B_i(u) \qquad y_i(v)=\sum_{j=0}^{n} y_{i,j} B_j(v)$$

$$p(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{n}(x_{i,j},y_{i,j})B_i(u)B_j(v)$$

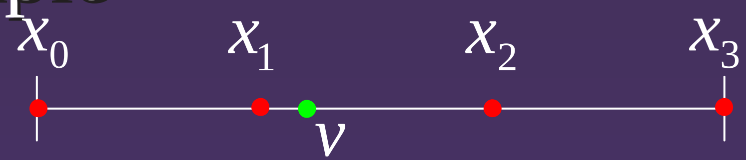
v	$(1-u)v$	uv
$(1-v)$	$(1-u)(1-v)$	$u(1-v)$
	$(1-u)$	u



Free-Form Deformations

- Embed object in uniform grid
- Represent every point in space as a weighted combination of the control points

- 1D Example



$$v = x(t) = \sum_{i=0}^{3} x_i \frac{3!}{(3-i)!i!} (1-t)^{3-i} t^i$$

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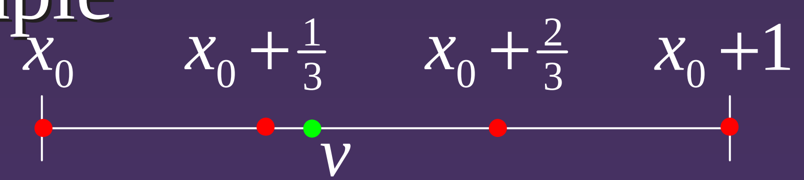

$$v = x(t) = \sum_{i=0}^{3} \left(x_0 + \tfrac{i}{3}\right) \frac{3!}{(3-i)!\,i!} (1-t)^{3-i} t^{i}$$

Free-Form Deformations

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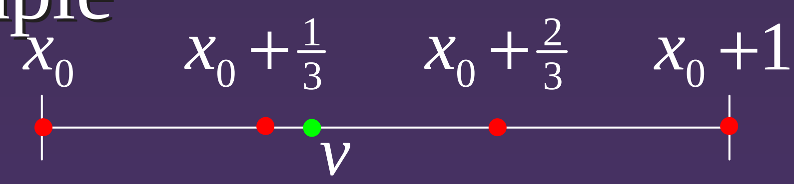
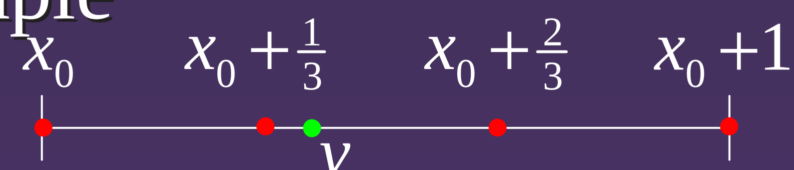
- 1D Example



$$v = x(t) = \sum_{i=0}^{3} \left(x_0 + \tfrac{i}{3}\right) \tfrac{3!}{(3-i)!i!} (1-t)^{3-i} t^i = x_0 + t$$

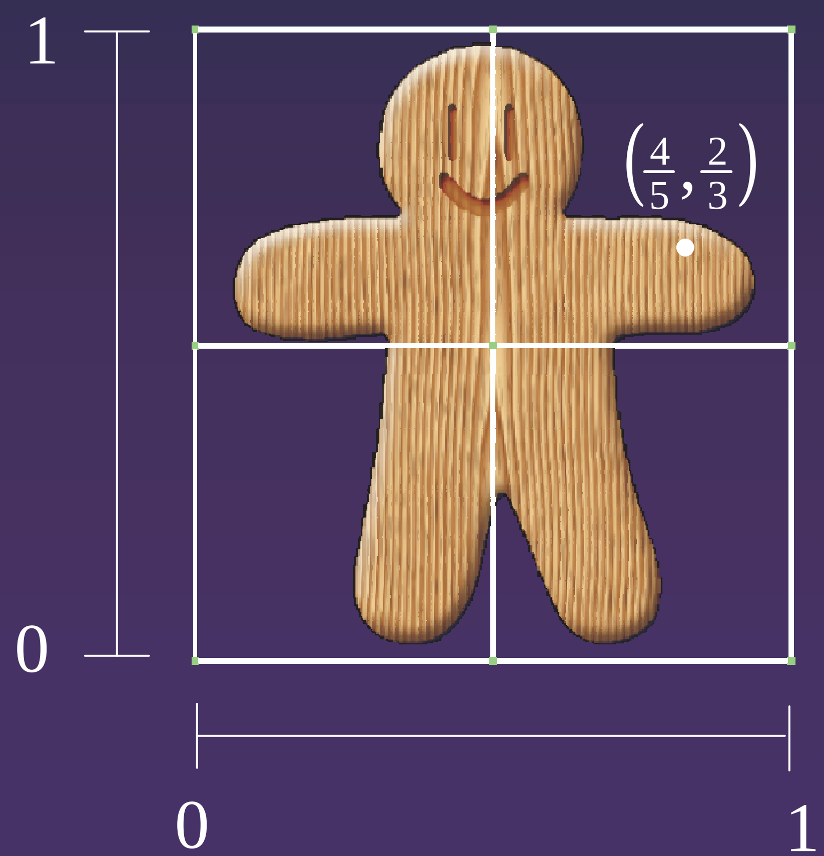
Free-Form Deformations

- Embed object in uniform grid
- Represent every point in space as a weighted combination of the control points
- 1D Example



$$v = \sum_{i=0}^{3} x_i \frac{3!}{(3-i)!\,i!} (1-(v-x_0))^{3-i} (v-x_0)^i = \sum_{i=0}^{3} x_i \alpha_i$$

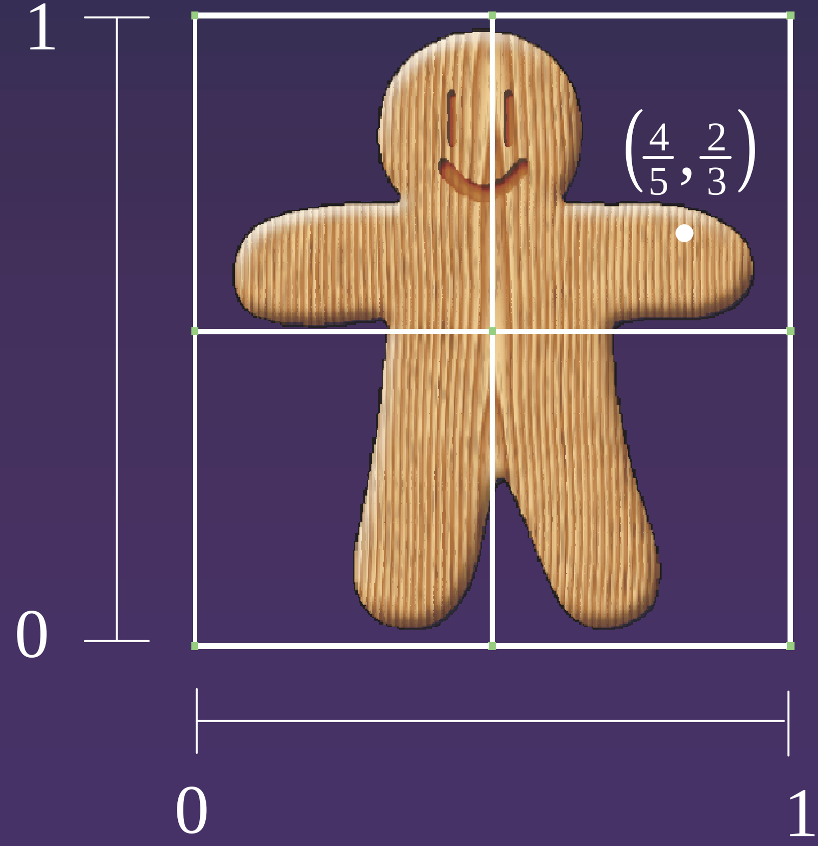
2D Example



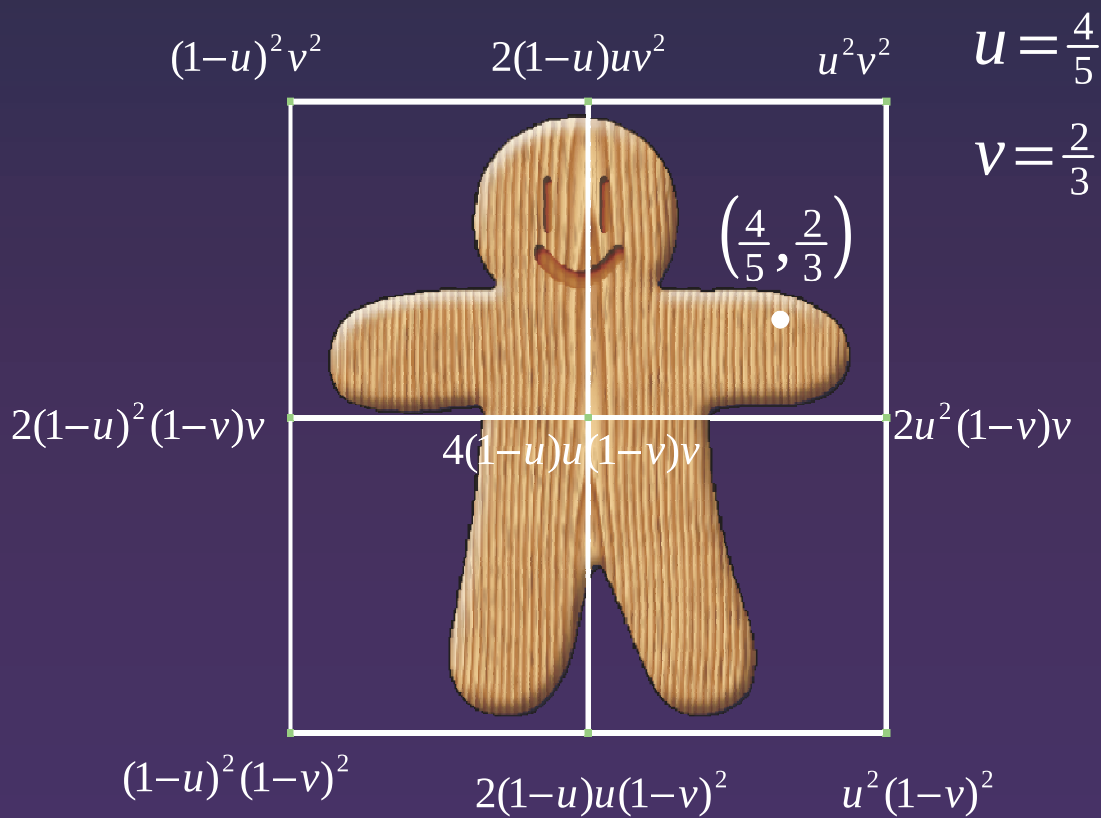
2D Example

$u = \frac{4}{5}$

$v = \frac{2}{3}$

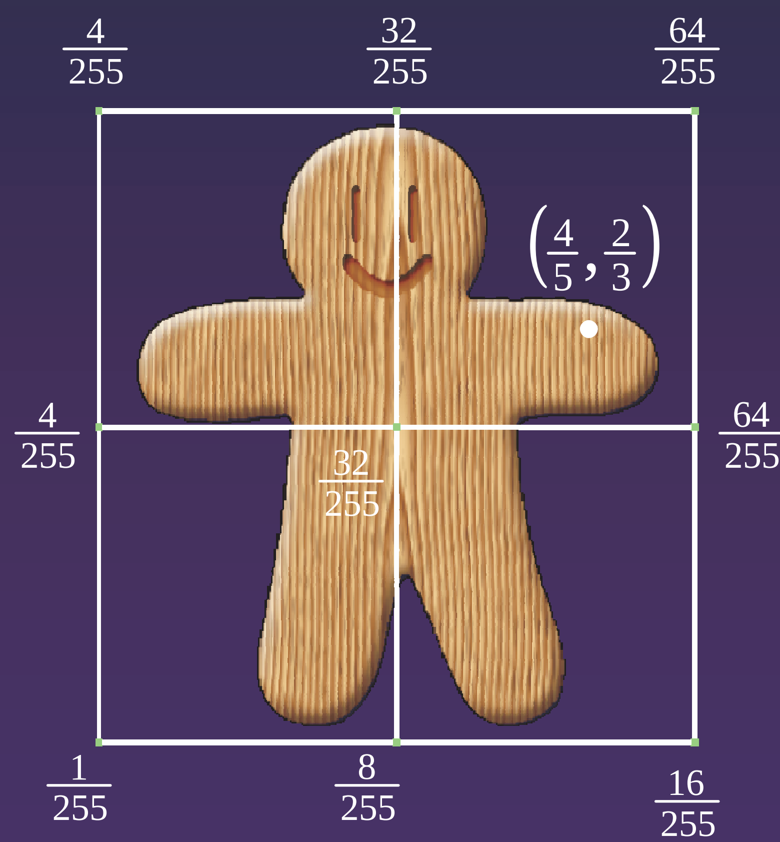


2D Example



$(1-u)^2 v^2$
$2(1-u)uv^2$
$u^2 v^2$
$u = \frac{4}{5}$
$v = \frac{2}{3}$
$\left(\frac{4}{5}, \frac{2}{3}\right)$
$2(1-u)^2(1-v)v$
$4(1-u)u(1-v)v$
$2u^2(1-v)v$
$(1-u)^2(1-v)^2$
$2(1-u)u(1-v)^2$
$u^2(1-v)^2$

2D Example

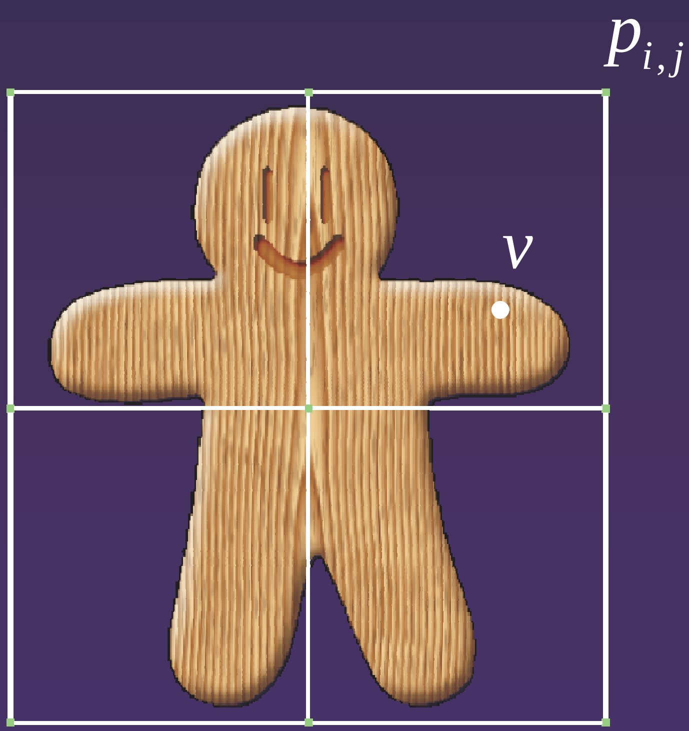


$u = \frac{4}{5}$

$v = \frac{2}{3}$

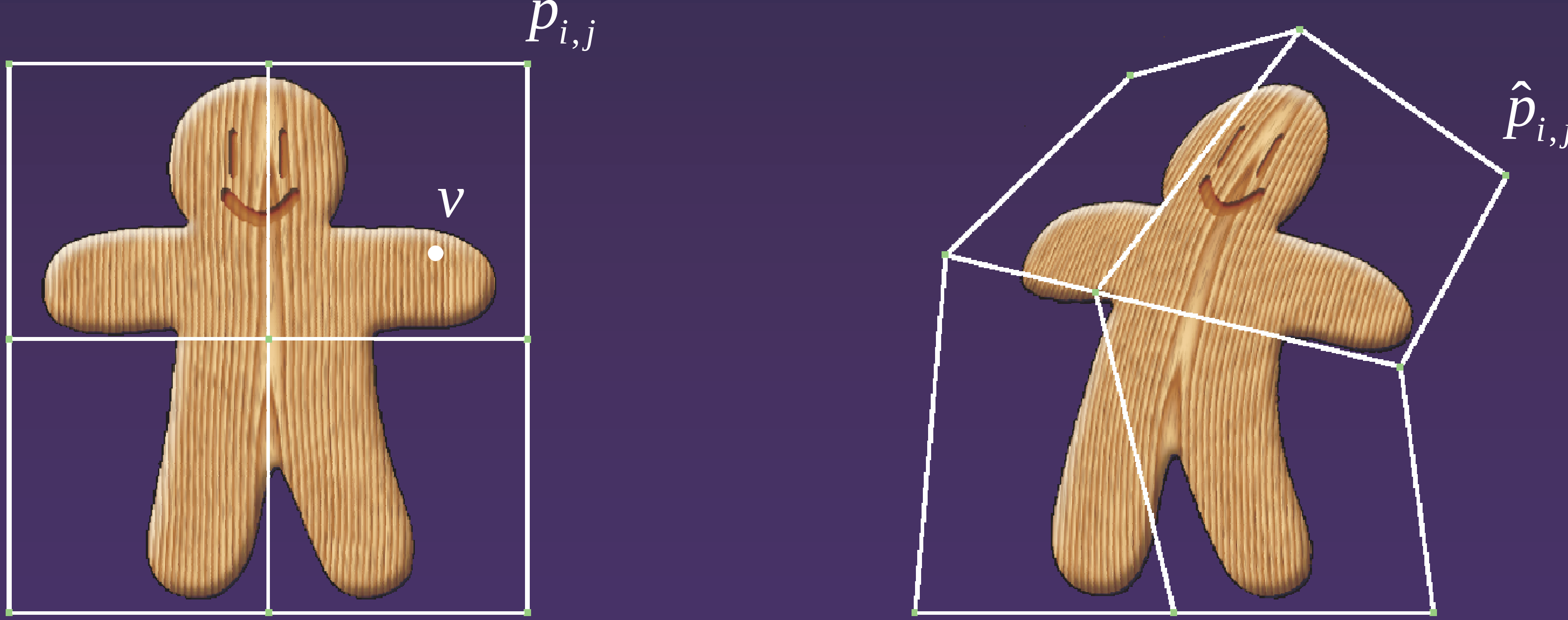
Applying the Deformation

$$v = \sum_i \sum_j \alpha_{i,j} p_{i,j}$$



Applying the Deformation

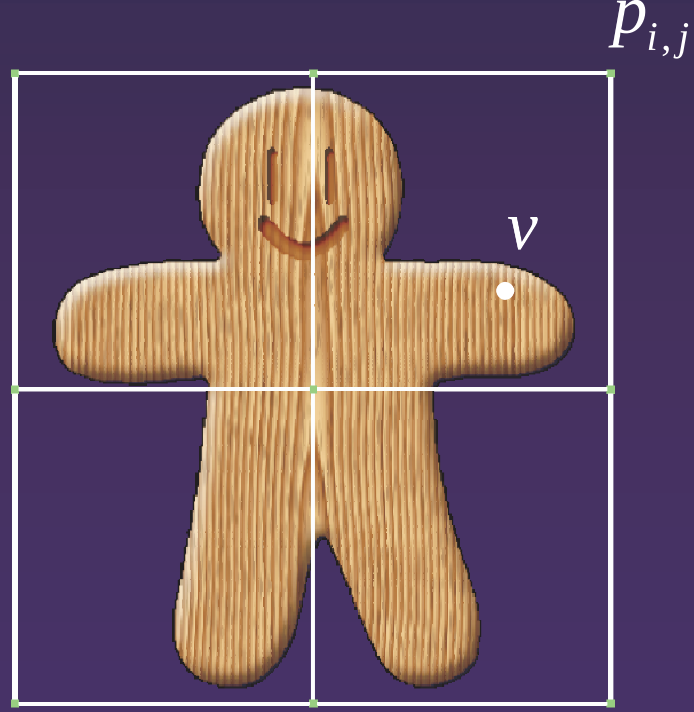
$$v = \sum_i \sum_j \alpha_{i,j} \, p_{i,j}$$

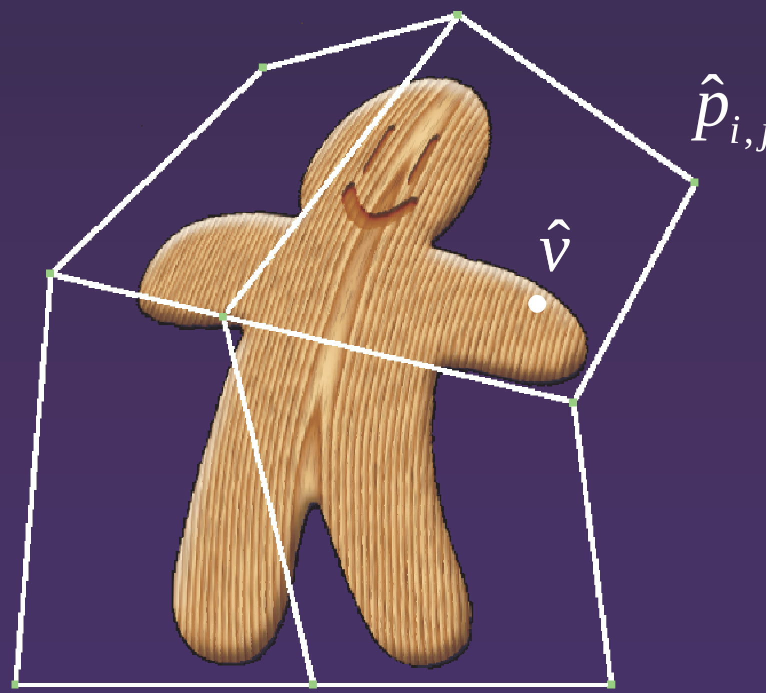


Applying the Deformation

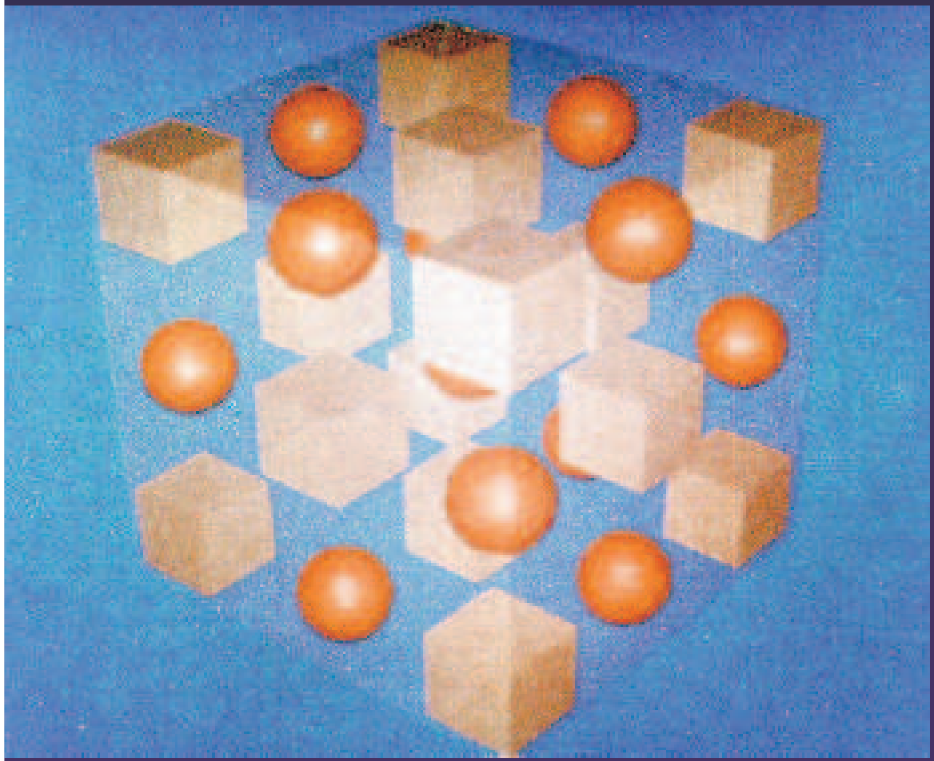
$$v = \sum_i \sum_j \alpha_{i,j} p_{i,j}$$

$$\hat{v} = \sum_i \sum_j \alpha_{i,j} \hat{p}_{i,j}$$

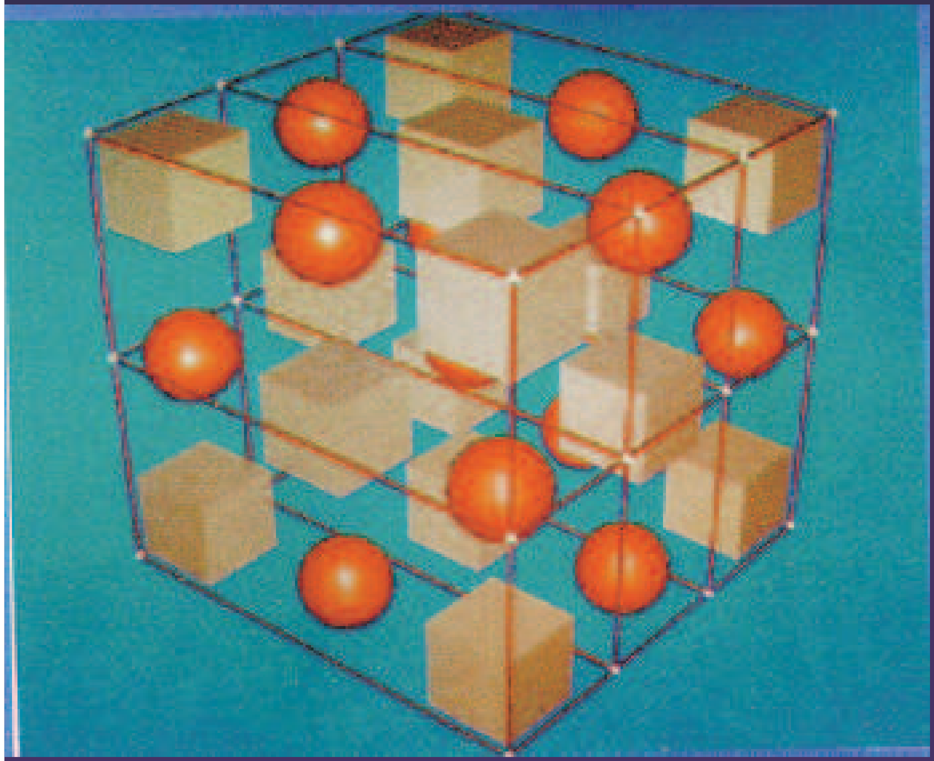




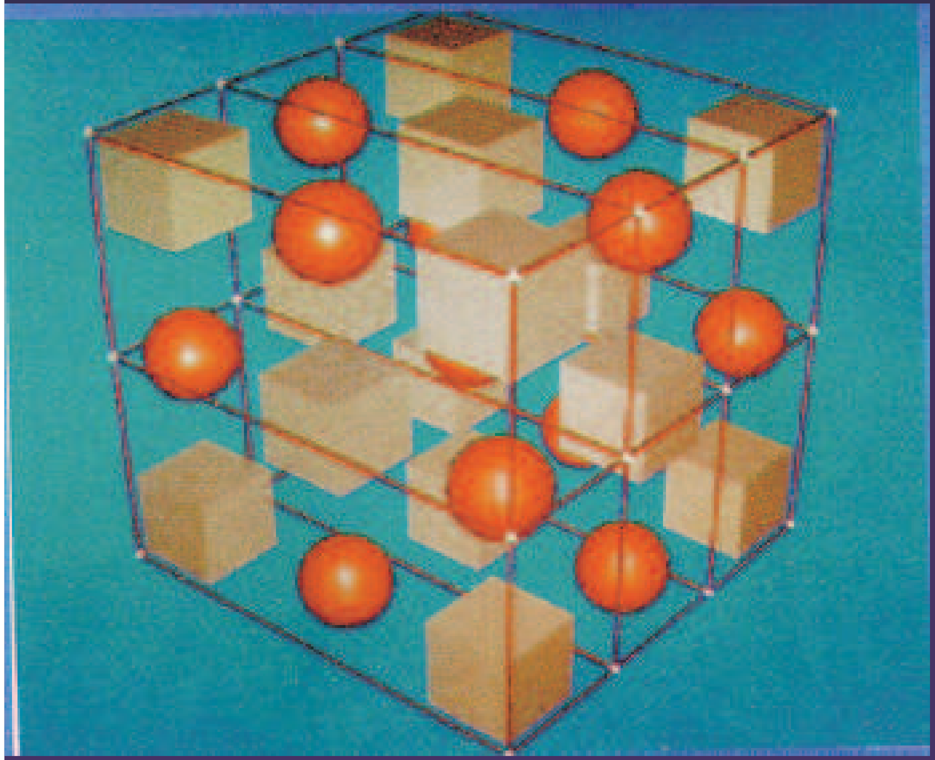
Examples



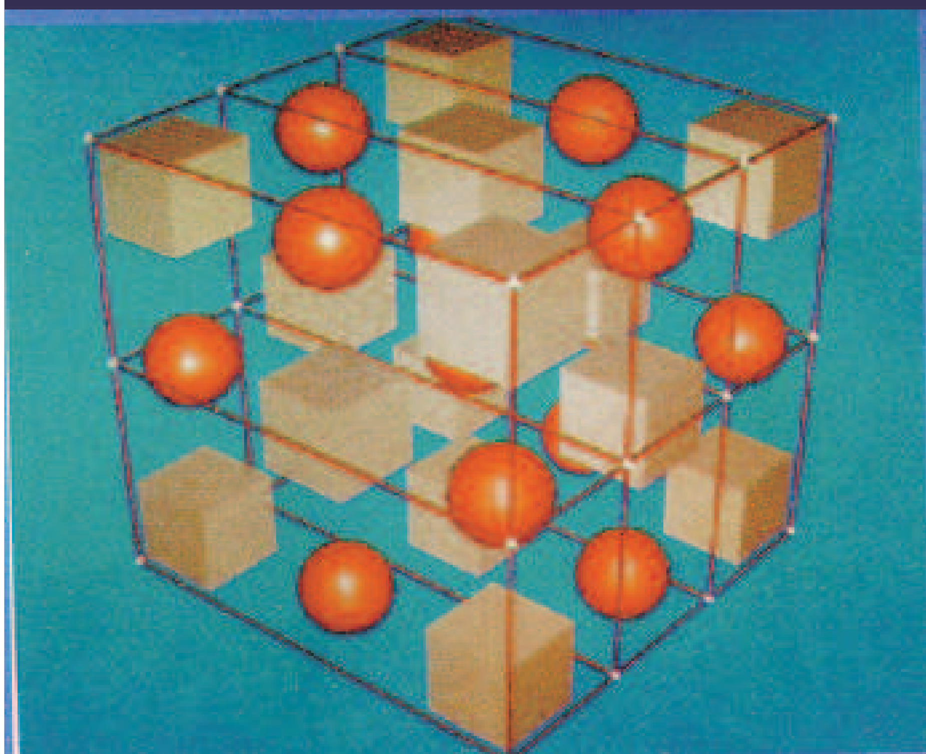
Examples



Examples



Examples



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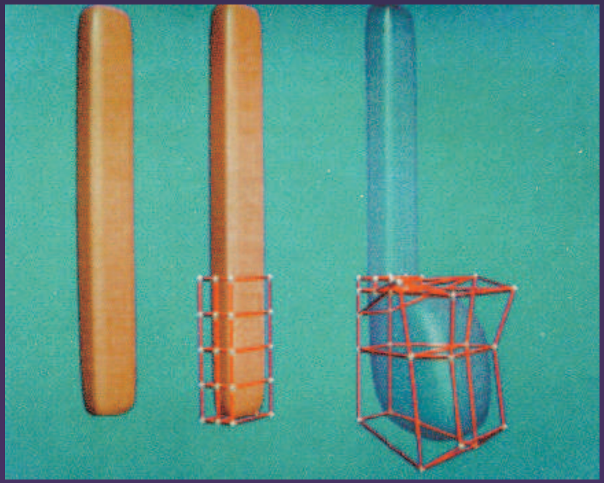

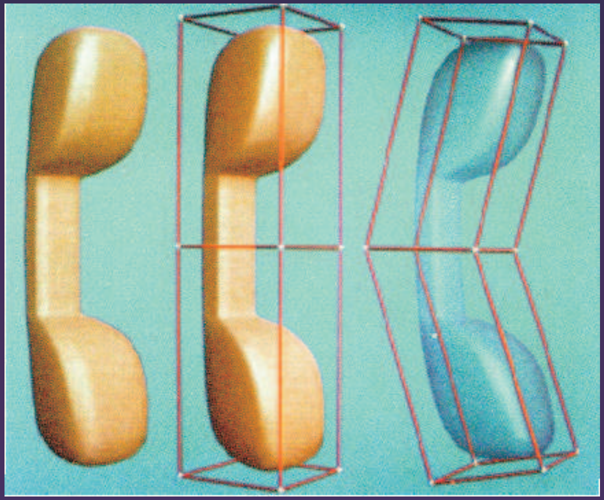

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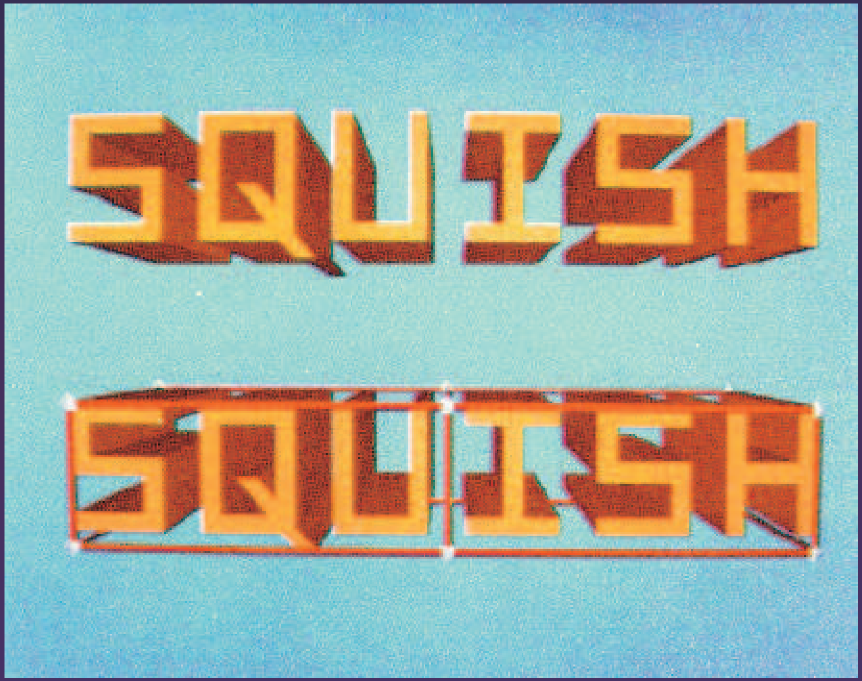
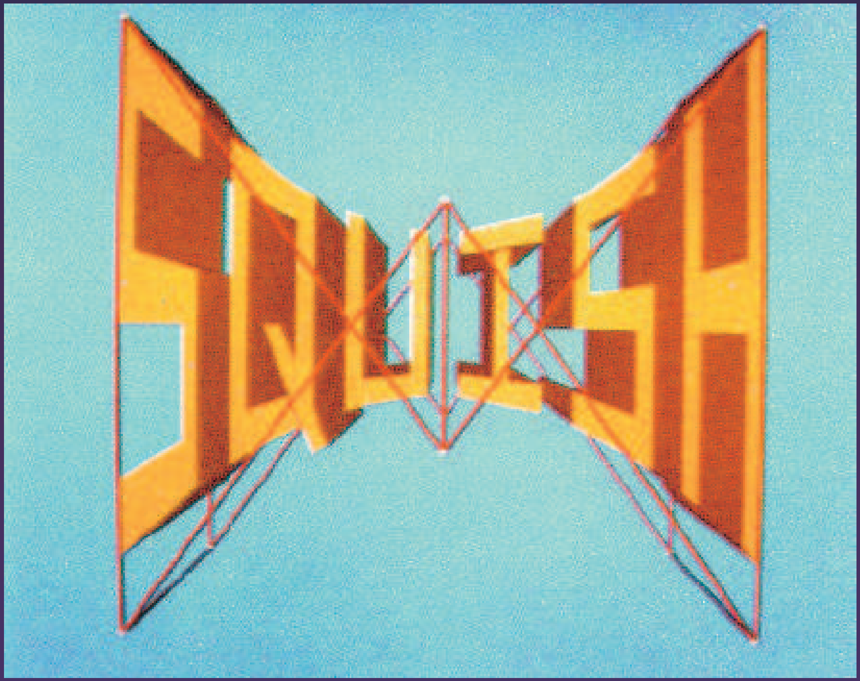
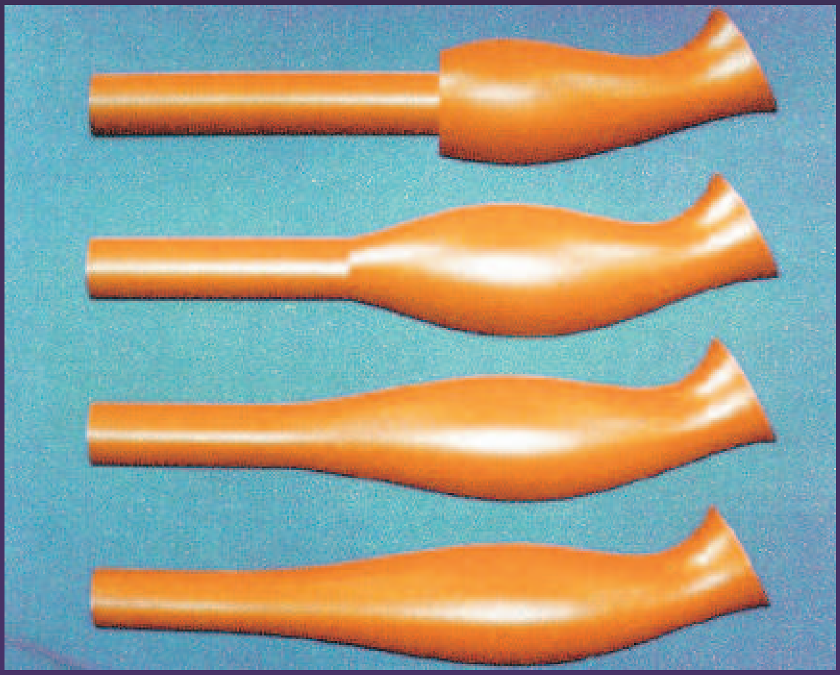




Examples

- Smoothness of deformation: C^{-1}, C^{0}, C^{1}, C^{2}
- Creates conditions on Bezier control points

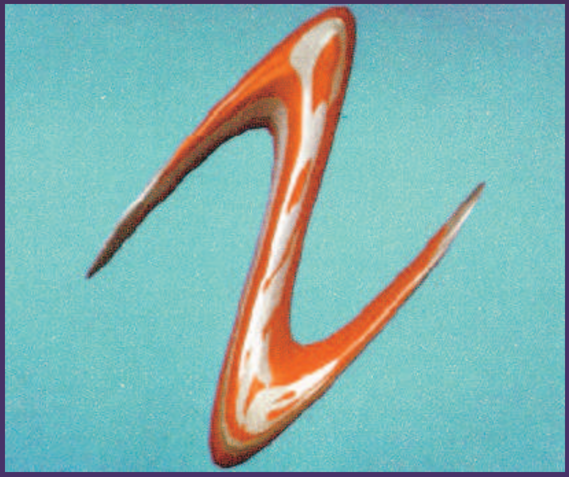


Volume Preservation

- Ensure that the Jacobian of the FFD is 1

$$(\hat{x},\hat{y},\hat{z})=(F(x,y,z),G(x,y,z),H(x,y,z))$$

$$\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z} \end{vmatrix}=1$$




Advantages

- Smooth deformations of arbitrary shapes
- Local control of deformation
- Computing the deformations is easy
- Deformations are very fast

Disadvantages

- Must use cubical cells for deformation
- Restricted to uniform grid
- Space warping
  - Deformations do not take into account structure of surface
- May need many FFD’s to achieve a simple deformation

Summary

- Widely used deformation technique
- Fast, easy to compute
- Some control over volume preservation
- Controllable degrees of smoothness

- Uniform grids are restrictive