

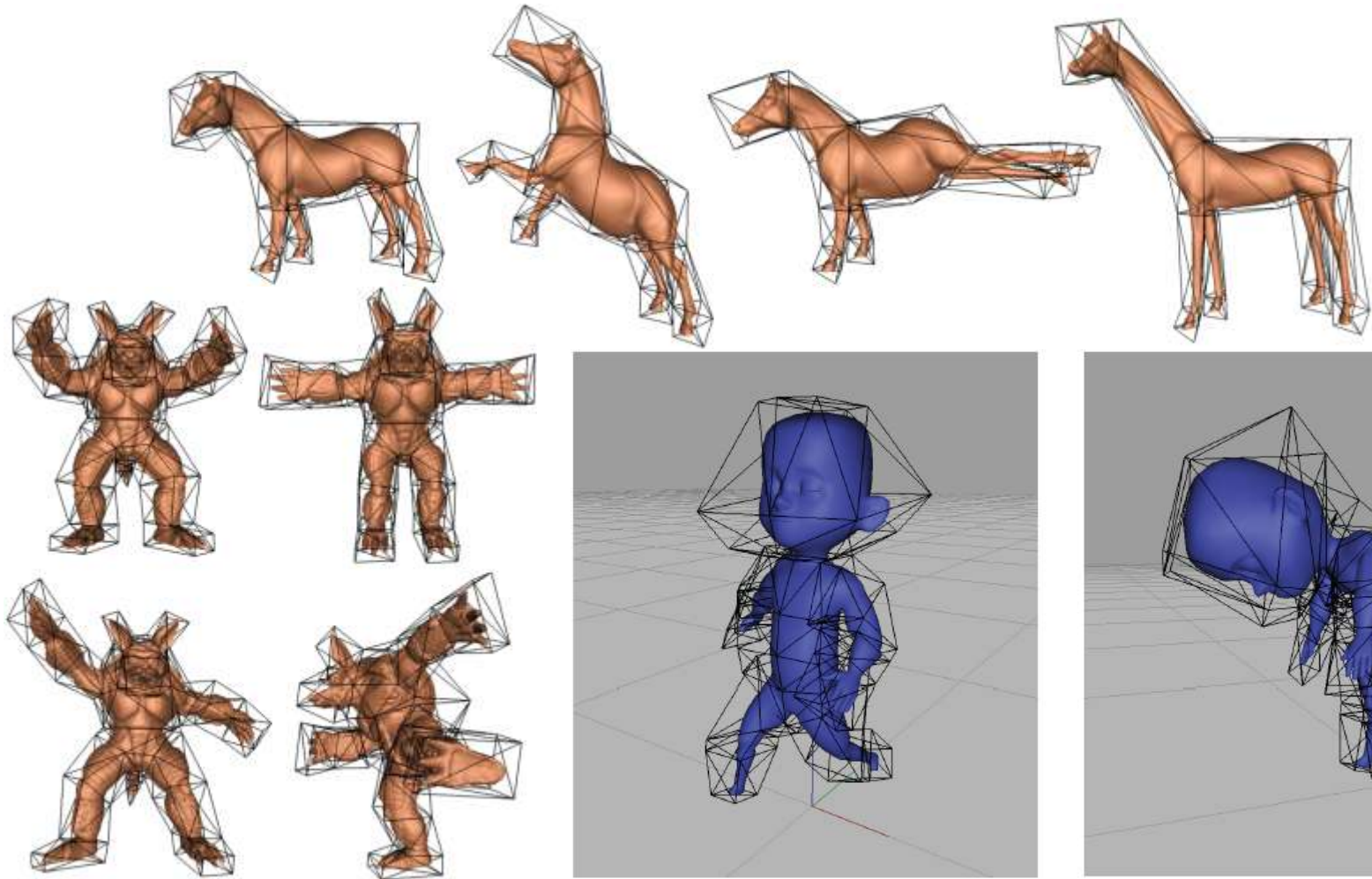
Positive Gordon–Wixom Coordinates

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¹ Texas A&M University

² Intel

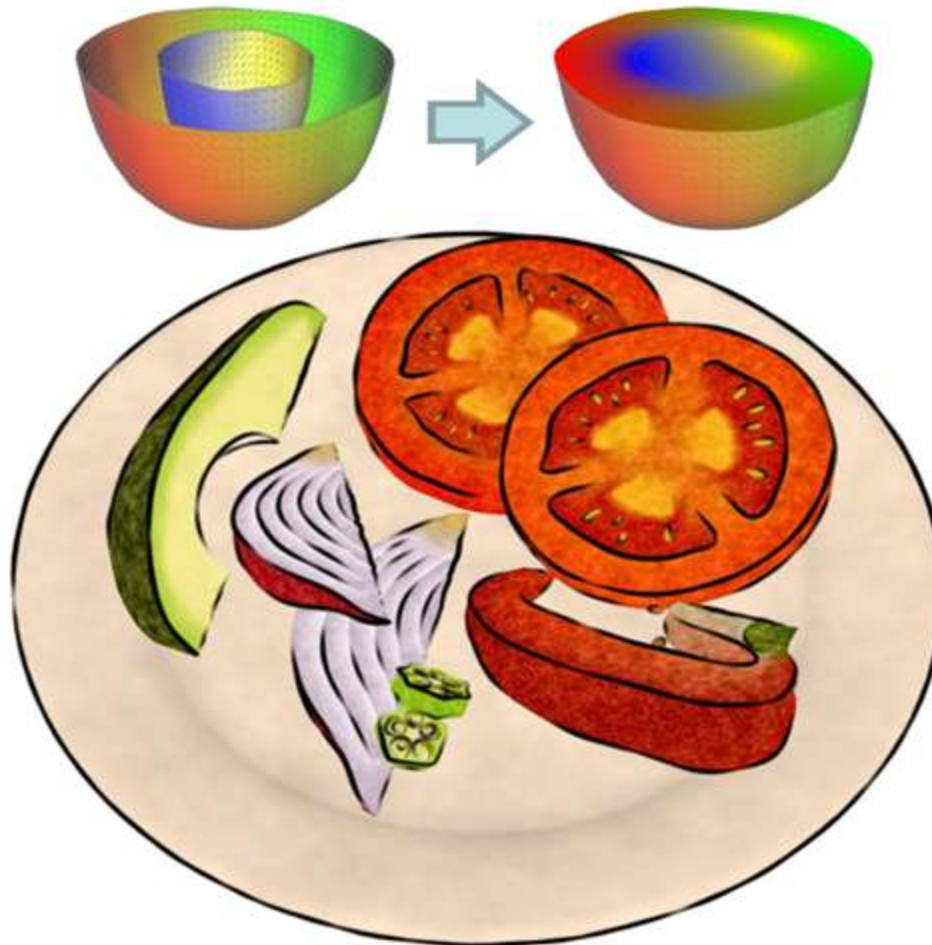
Barycentric Coordinates: Mesh deformation



[Ju et al. 2005]

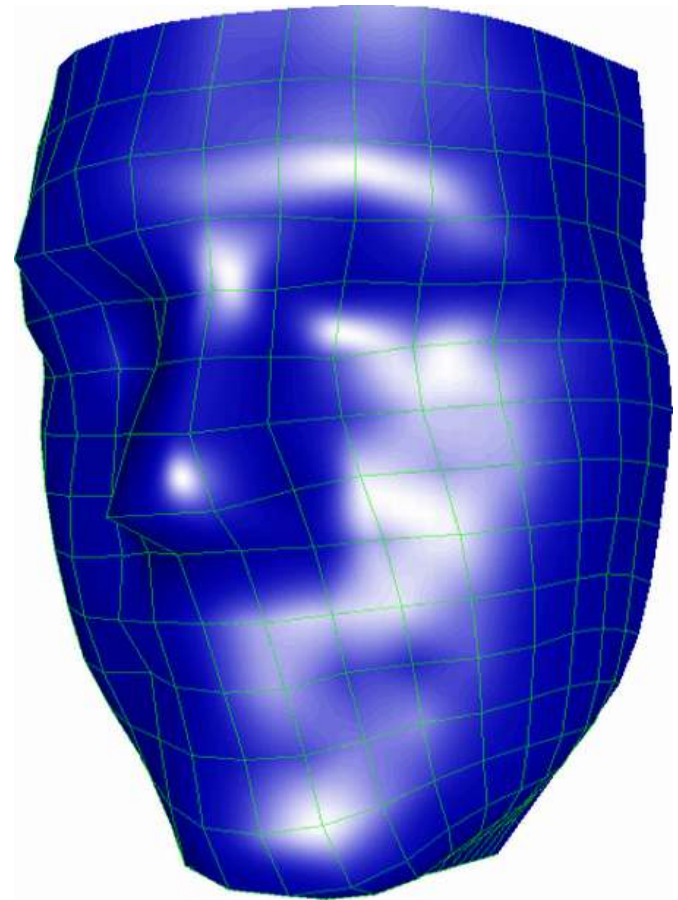
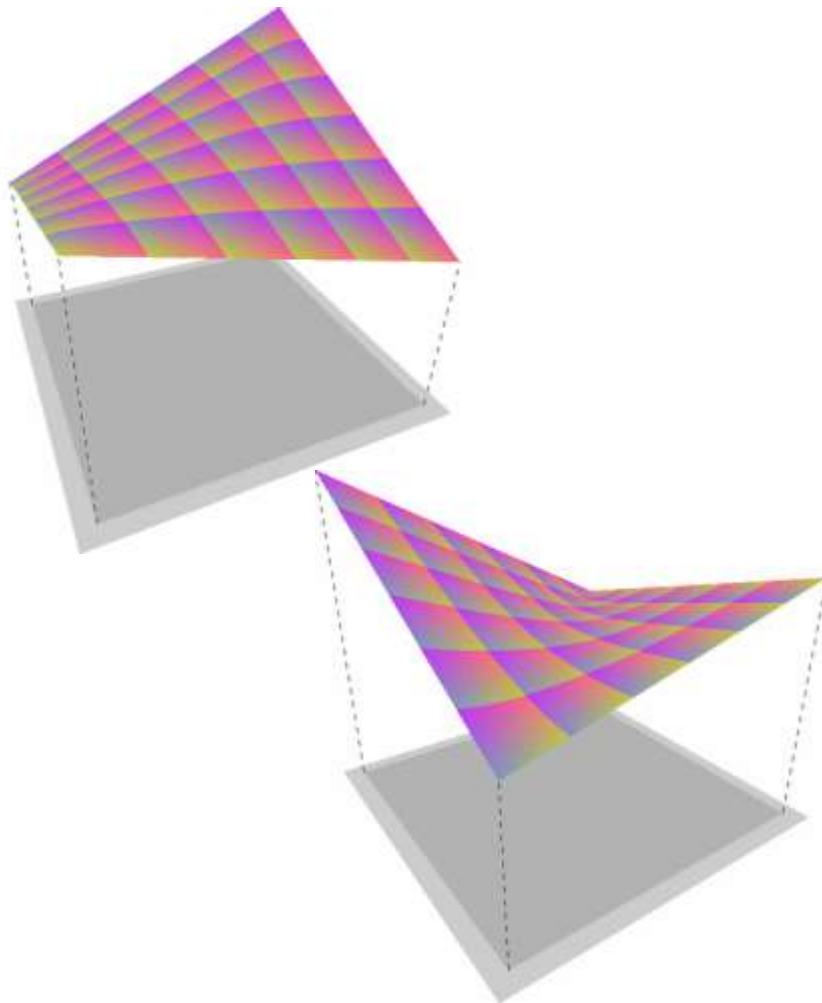
[Joshi et al. 2007]

Barycentric Coordinates: Volumetric textures



[Takayama et al. 2010]

Barycentric Coordinates: Rasterize polygons



[Hormann and Tarini 2004]

Barycentric Coordinates: Surface representation

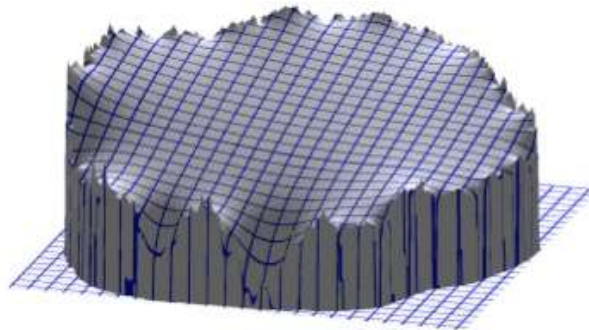


[Loop et al. 1989]

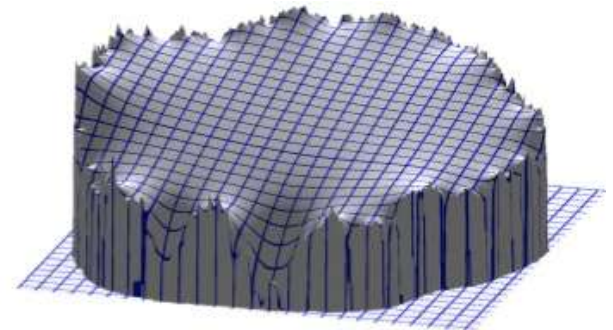
Barycentric Coordinates: Image editing



(a) Source patch



(b) Laplace membrane



(c) Mean-value membrane



(d) Target image



(e) Poisson cloning

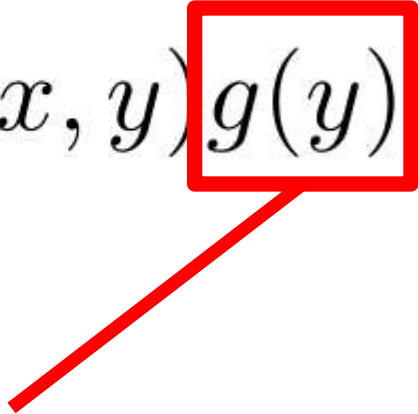


(f) Mean-value cloning

Barycentric Interpolant

$$f(x) = \int_{\partial\Omega} b(x, y) g(y) dy$$

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boundary values

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basis functions
coordinates

Basis Function Constraints

Boundary
interpolation

$$x \in \partial\Omega \Rightarrow b(x, y) = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{if } x \neq y \end{cases}$$

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$$\forall g \in \mathbb{P}^1, f(x) = g(x)$$

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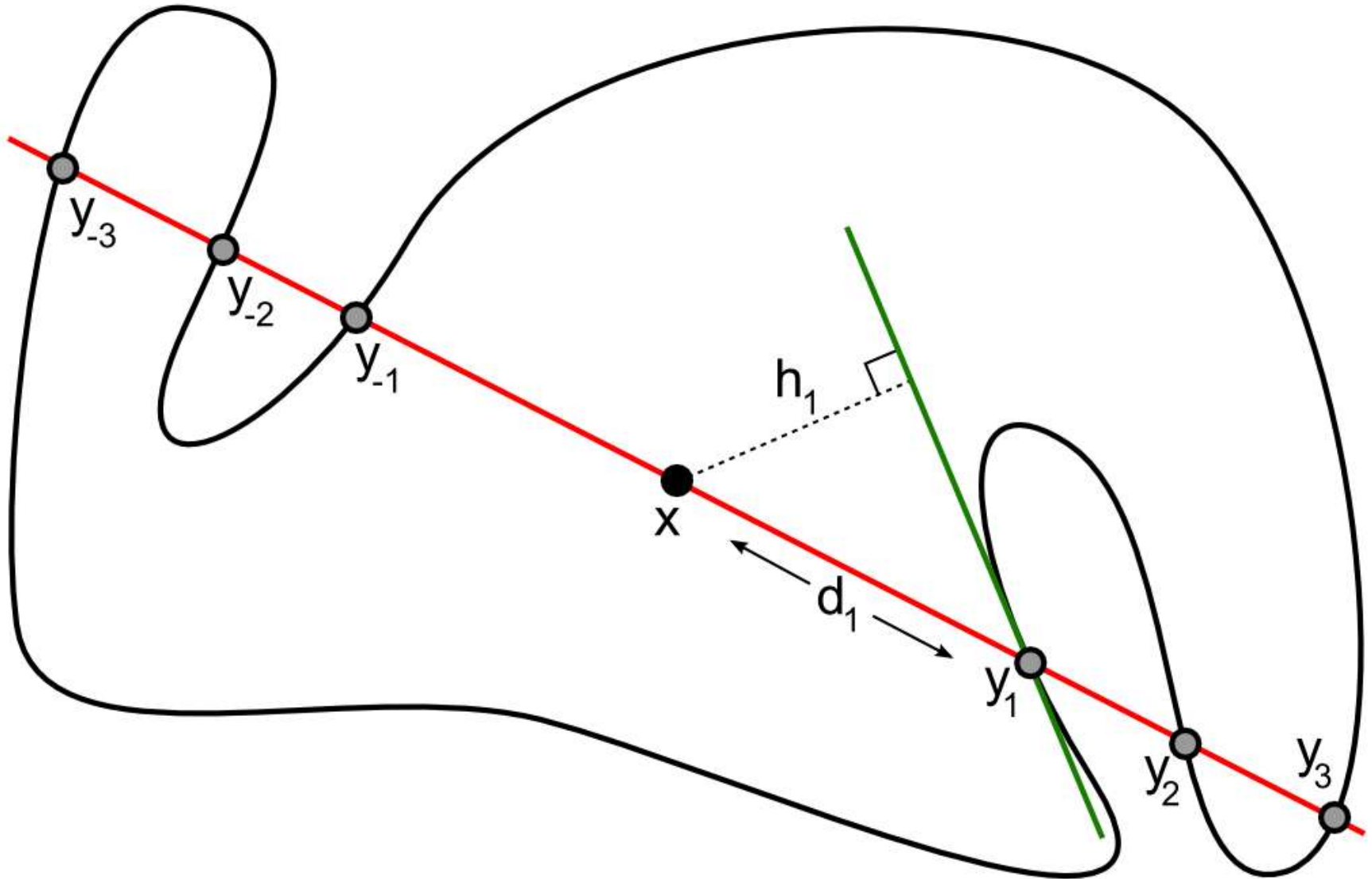
Positivity

$$b(x, y) \geq 0$$

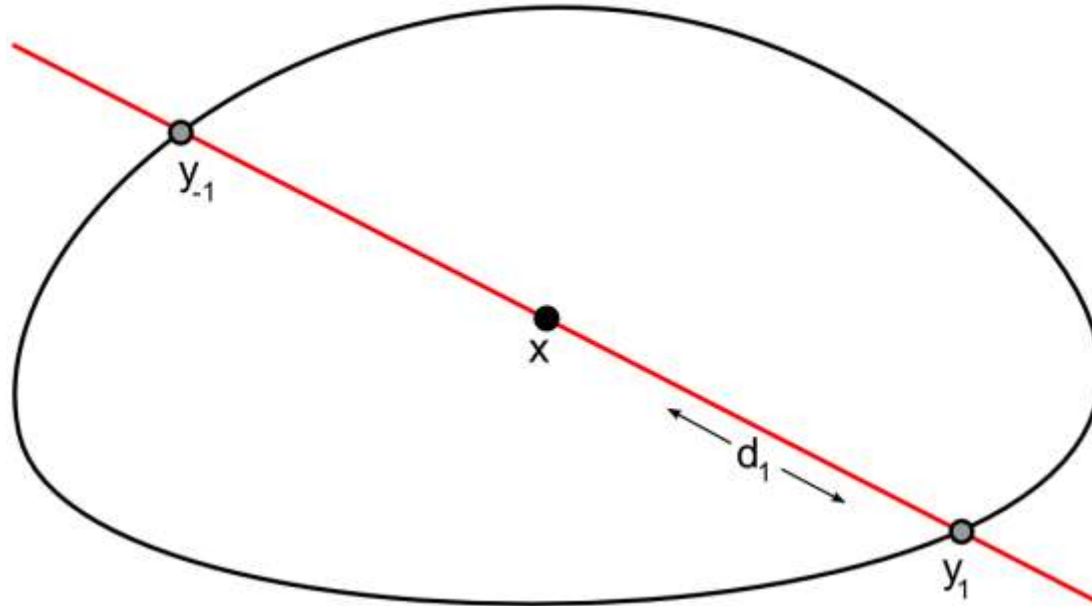
Types of Coordinates

- Wachspress [[Wachspress 1975](#)]
 - Only convex domains, obtuse angles bad
- Gordon-Wixom [[Gordon and Wixom 1974](#)]
 - Only convex domains
- Mean Value [[Floater 2003](#)]
 - Negative, but fast
- Moving Least Squares [[Manson and Schaefer 2010](#)]
 - Negative, but less so, slower
- Harmonic [[Joshi et al. 2007](#)]
 - Positive, ideal, very slow
- Maximum Entropy [[Hormann and Sukumar 2008](#)]
 - Positive, non-linear optimization, probably smooth
- Positive Gordon Wixom
 - Positive, evaluate integral, smooth for smooth boundaries

Notation

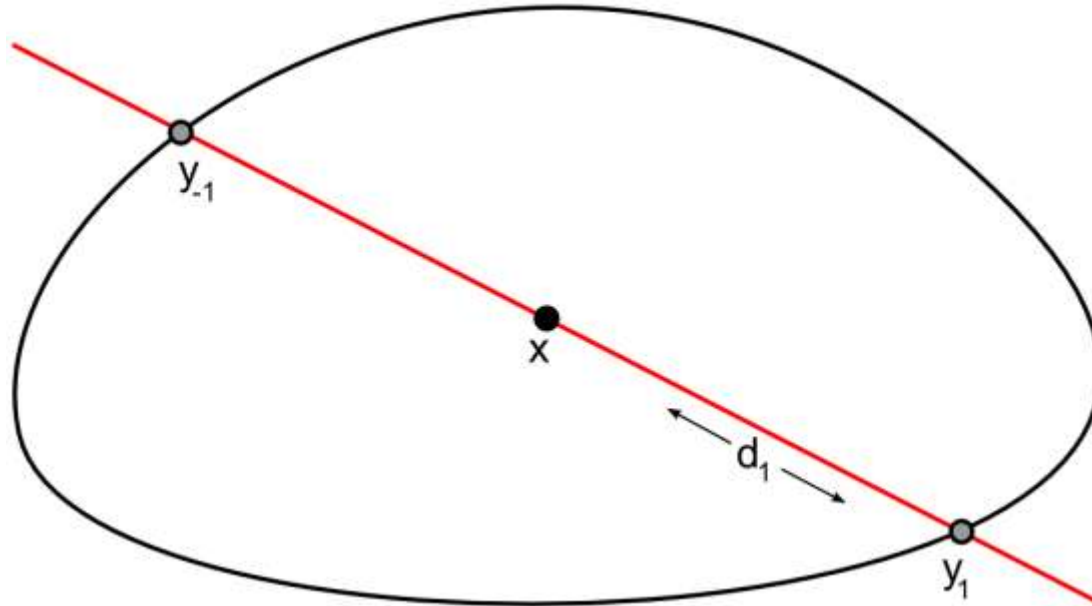


Linear Interpolant



$$L_{i,j}(x, \theta) = \frac{d_j}{d_i + d_j} g(y_i) + \frac{d_i}{d_i + d_j} g(y_j)$$

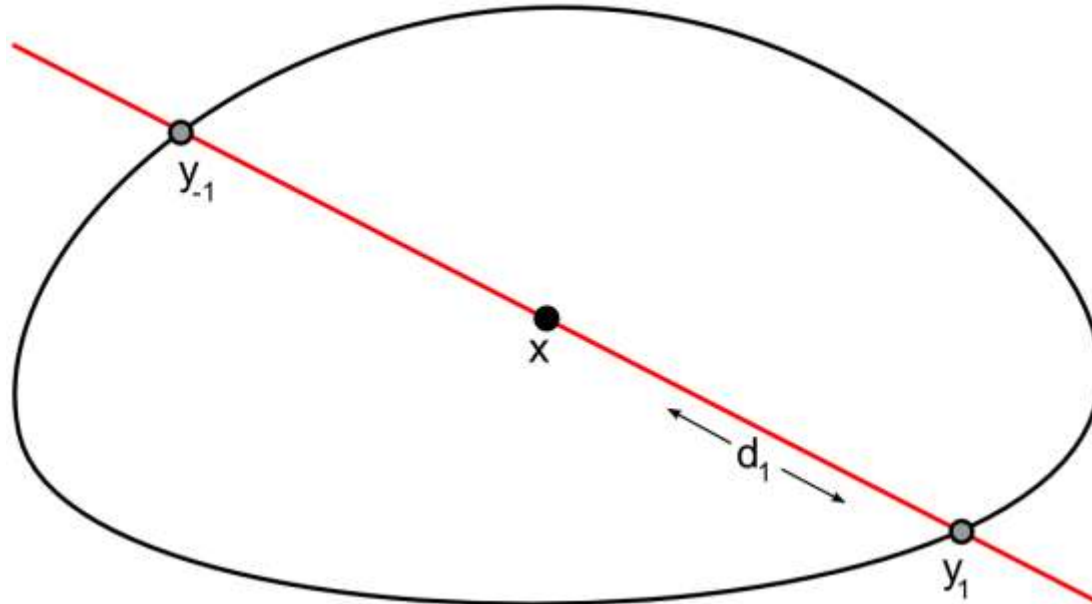
Gordon-Wixom



$$f(x) = \frac{1}{2\pi} \int_0^{2\pi} L_{-1,1}(x, \theta) d\theta$$

[Gordon and Wixom 1974]

Weighted Gordon-Wixom



$$f(x) = \frac{\int_0^{2\pi} L_{-1,1}(x, \theta) W_{-1,1}(x, \theta) d\theta}{\int_0^{2\pi} W_{-1,1}(x, \theta) d\theta}$$

Mean Value Coordinates (MVC)

$$W_{-1,1}(x, \theta) = \frac{d_1 + d_{-1}}{d_1 d_{-1}}$$

$$L_{i,j}(x, \theta) = \frac{d_j}{d_i + d_j} g(y_i) + \frac{d_i}{d_i + d_j} g(y_j)$$

$$f(x) = \frac{\int_0^{2\pi} \left(\frac{g(y_1)}{d_1} + \frac{g(y_{-1})}{d_{-1}} \right) d\theta}{\int_0^{2\pi} \left(\frac{1}{d_1} + \frac{1}{d_{-1}} \right) d\theta}$$

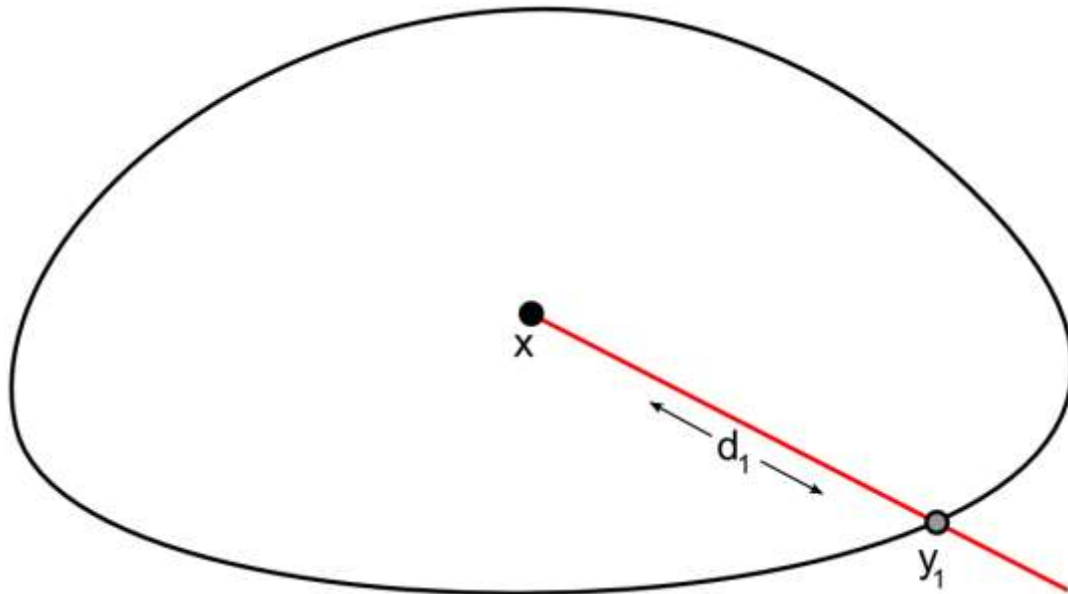
Mean Value Coordinates (MVC)

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$$L_{i,j}(x, \theta) = \frac{d_j}{d_i + d_j} g(y_i) + \frac{d_i}{d_i + d_j} g(y_j)$$

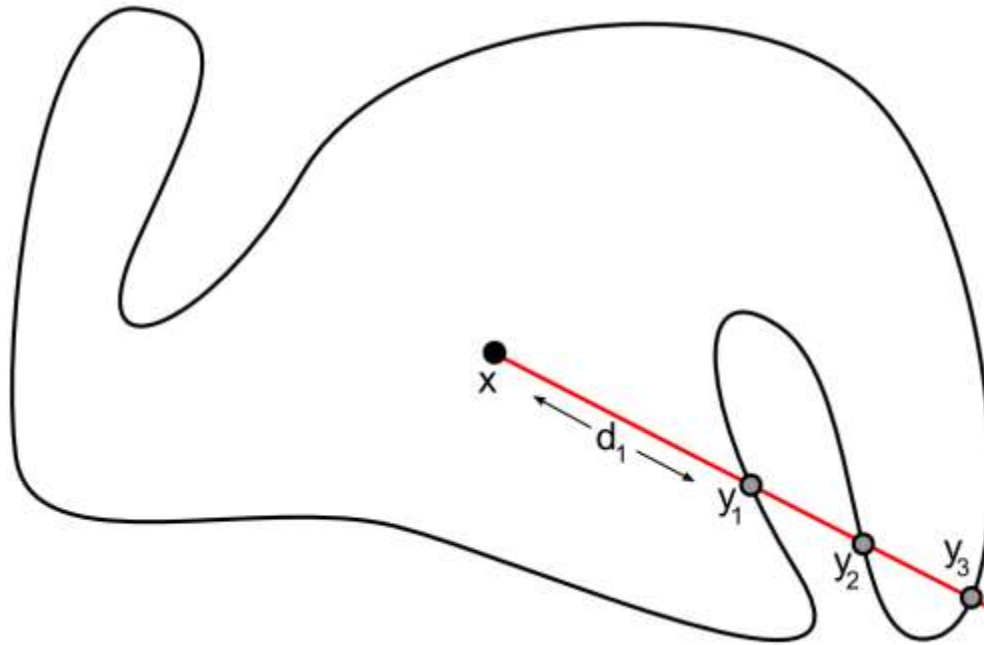
$$f(x) = \frac{\int_0^{2\pi} \left(\frac{g(y_1)}{d_1} + \frac{g(y_{-1})}{d_{-1}} \right) d\theta}{\int_0^{2\pi} \left(\frac{1}{d_1} + \frac{1}{d_{-1}} \right) d\theta} = \frac{\int_0^{2\pi} \frac{g(y_1)}{d_1} d\theta}{\int_0^{2\pi} \frac{1}{d_1} d\theta}$$

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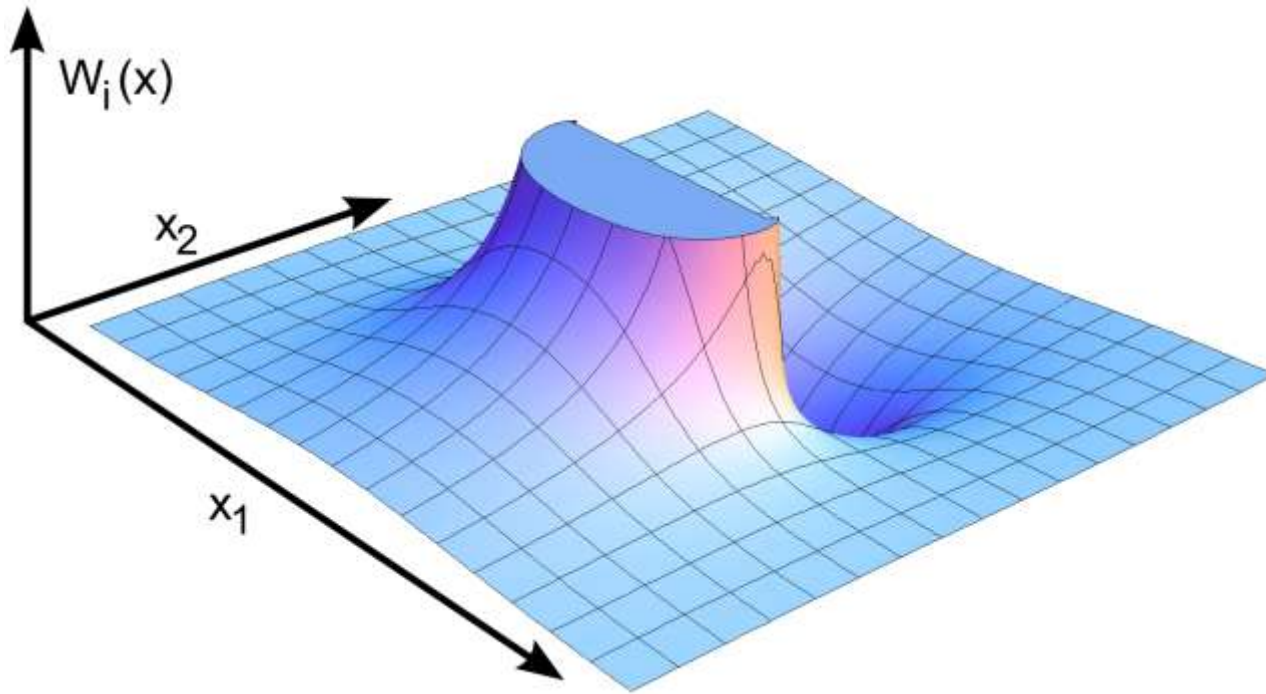
Concave MVC



$$f(x) = \frac{\int_0^{2\pi} \sum_{i=1}^m g(y_i) \frac{(-1)^{i+1}}{d_i} d\theta}{\int_0^{2\pi} \sum_{i=1}^m \frac{(-1)^{i+1}}{d_i} d\theta}$$

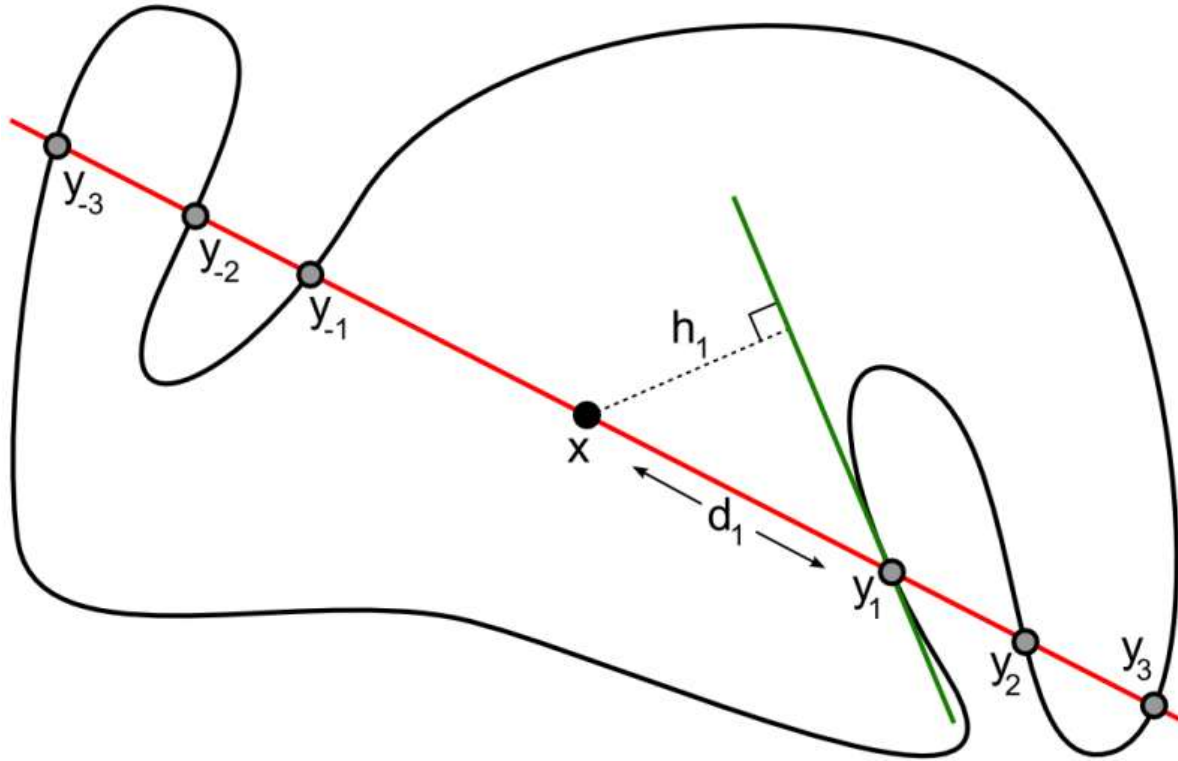
[Hormann and Floater 2006]

Concave MVC



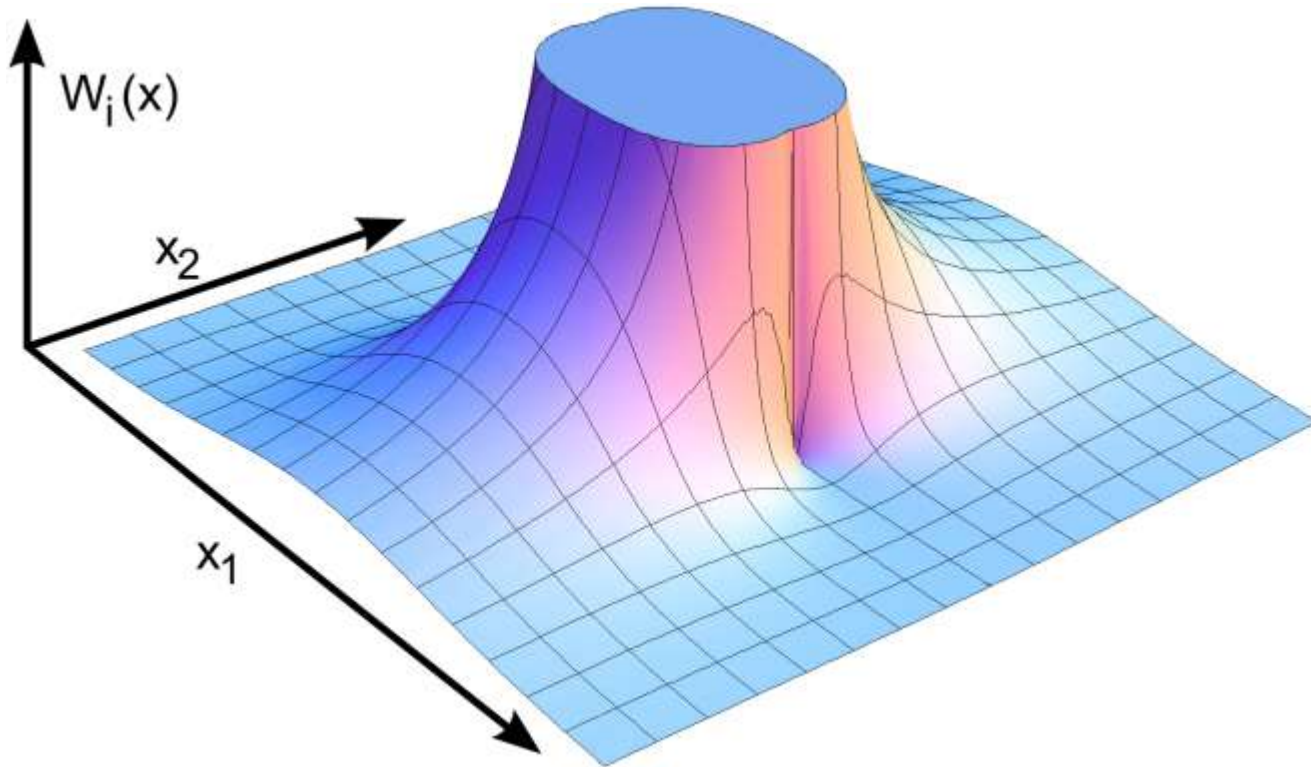
$$W_i(x, \theta) = \frac{1}{d_i}$$

Our Coordinates



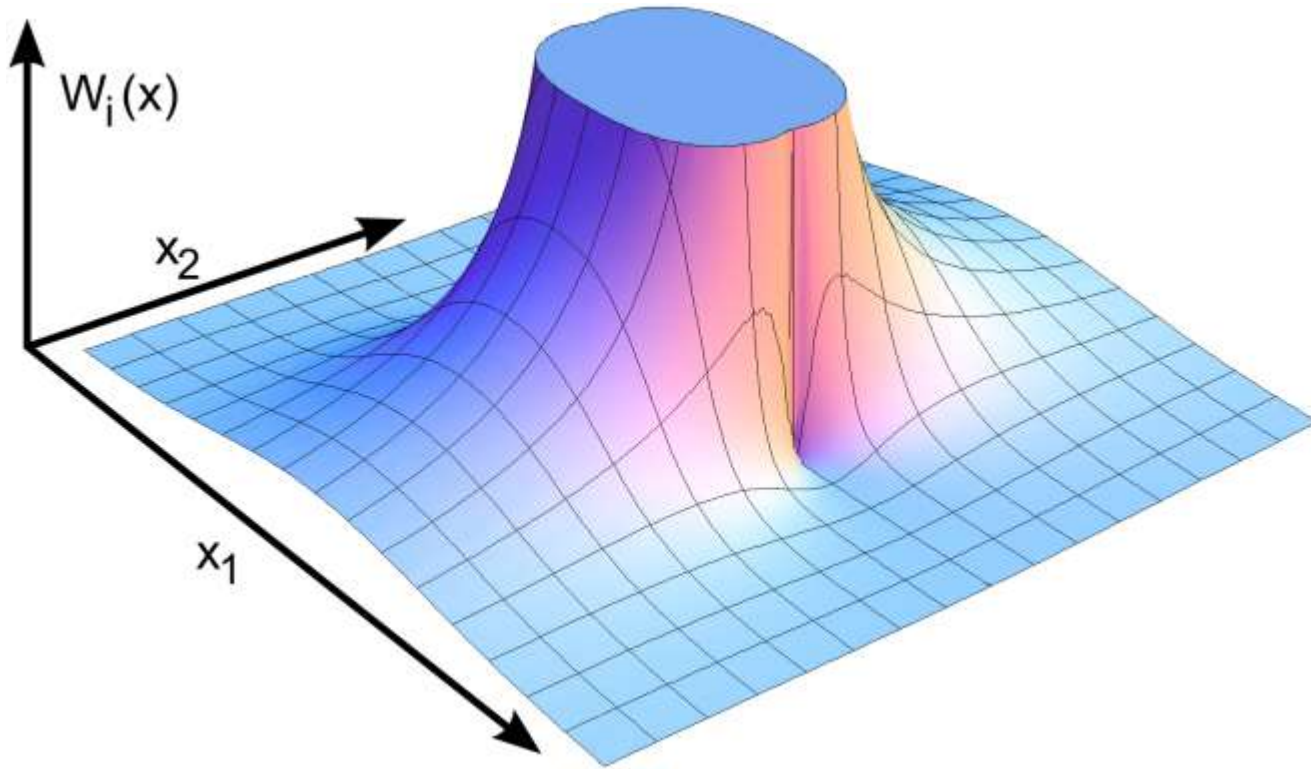
$$f(x) = \frac{\int_0^{2\pi} \sum_{i=1}^m \sum_{j=-n}^{-1} L_{i,j}(x, \theta) W_{i,j}(x, \theta) d\theta}{\int_0^{2\pi} \sum_{i=1}^m \sum_{j=-n}^{-1} W_{i,j}(x, \theta) d\theta}$$

Our Weight Function



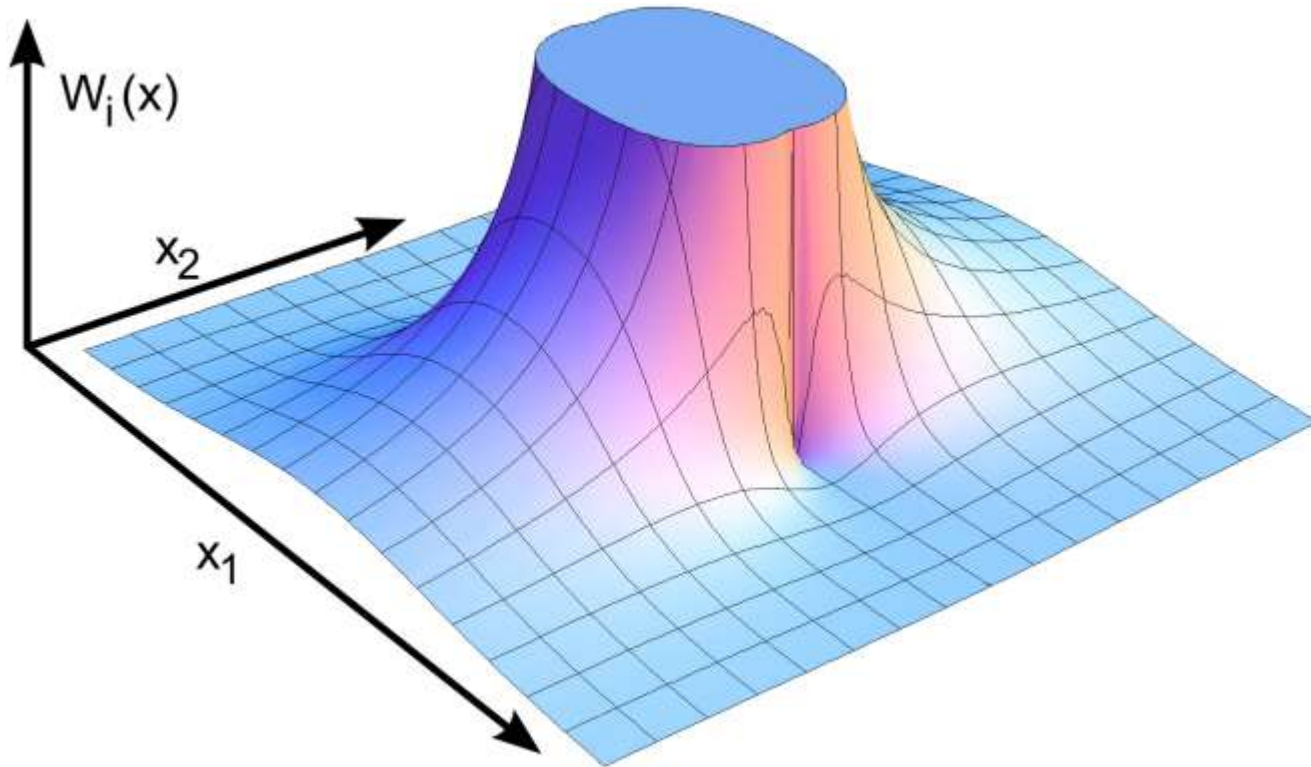
$$W_i(x, \theta) = \frac{h_i}{d_i^2}$$

Our Weight Function



$$W_{i,j}(x, \theta) = (d_i + d_j)W_i(x, \theta)W_j(x, \theta)$$

Our Weight Function



$$W_{i,j}(x, \theta) = \frac{(d_i + d_j)h_i h_j}{d_i^2 d_j^2}$$

Basis Functions

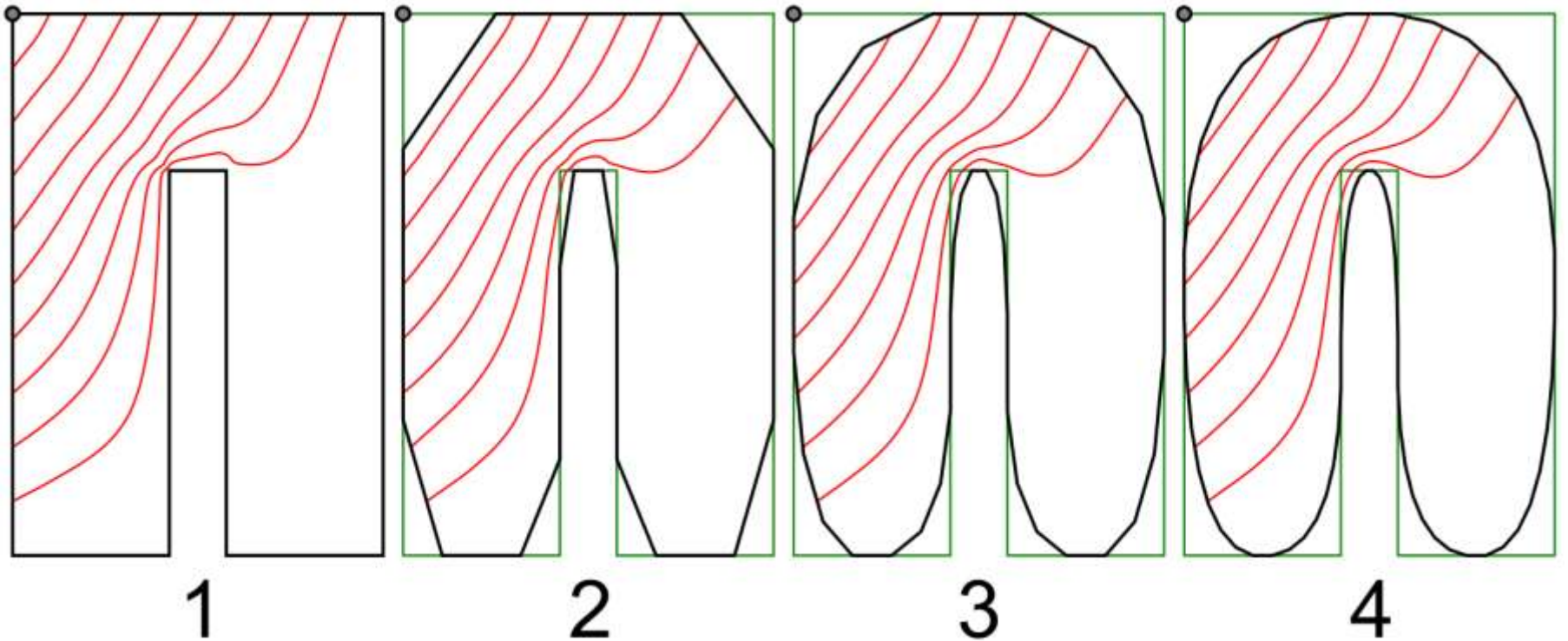
$$f(x) = \frac{\int_0^{2\pi} \sum_{i=1}^m \sum_{j=-n}^{-1} L_{i,j}(x, \theta) W_{i,j}(x, \theta) d\theta}{\int_0^{2\pi} \sum_{i=1}^m \sum_{j=-n}^{-1} W_{i,j}(x, \theta) d\theta}$$

$$W_{i,j}(x, \theta) = \frac{(d_i + d_j)h_i h_j}{d_i^2 d_j^2}$$

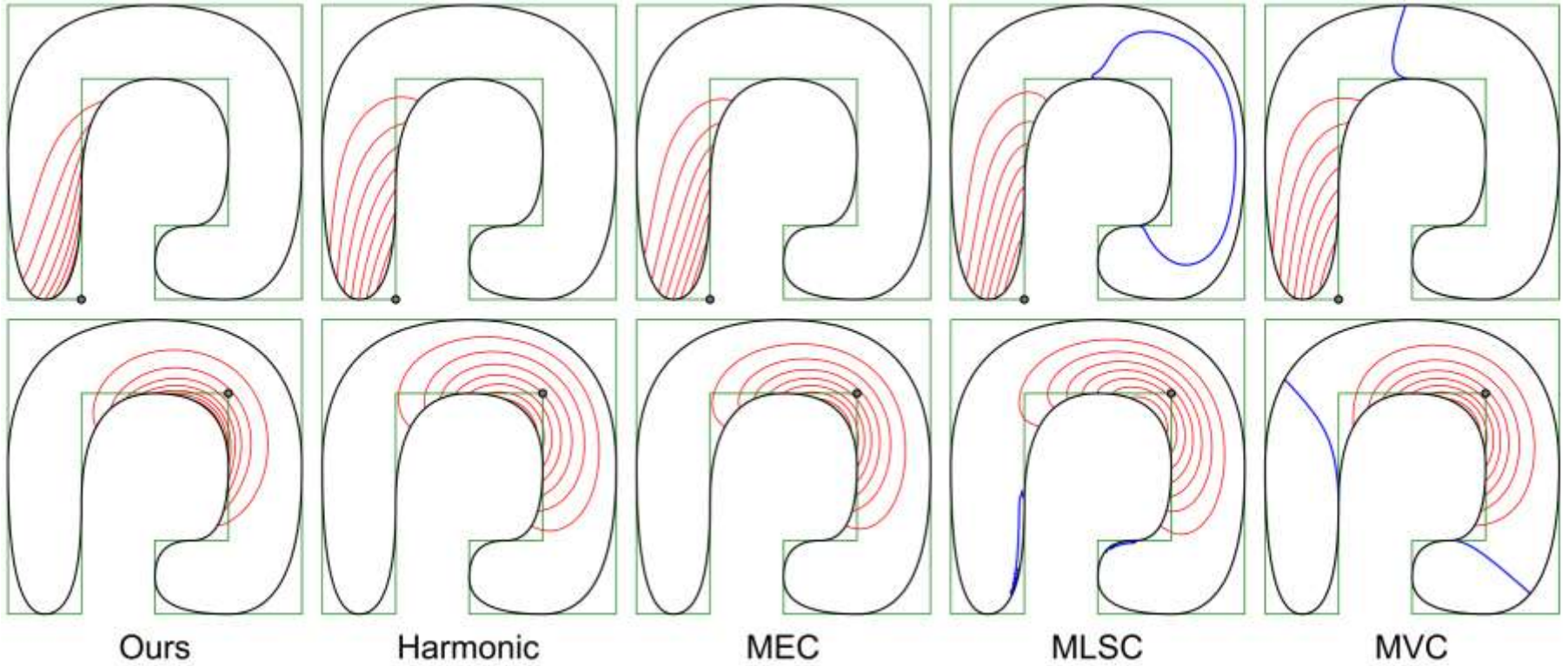


$$b(x, y_i) = \frac{\sum_{j=-n}^{-1} \frac{h_i h_j}{d_i^2 d_j^2}}{\int_0^{2\pi} \sum_{i=1}^m \sum_{j=-n}^{-1} W_{i,j}(x, \theta) d\theta}$$

Approximating Smooth Boundaries



Comparison



Conclusion

- Our coordinates are:
 - Positive
 - Smooth for smooth boundary
 - Evaluated through integral
 - Closed-form for polygons
- Need visibility through sample point
 - Logarithmic lookup
 - Slows computation
- Evidence that closed-form for polygons exists