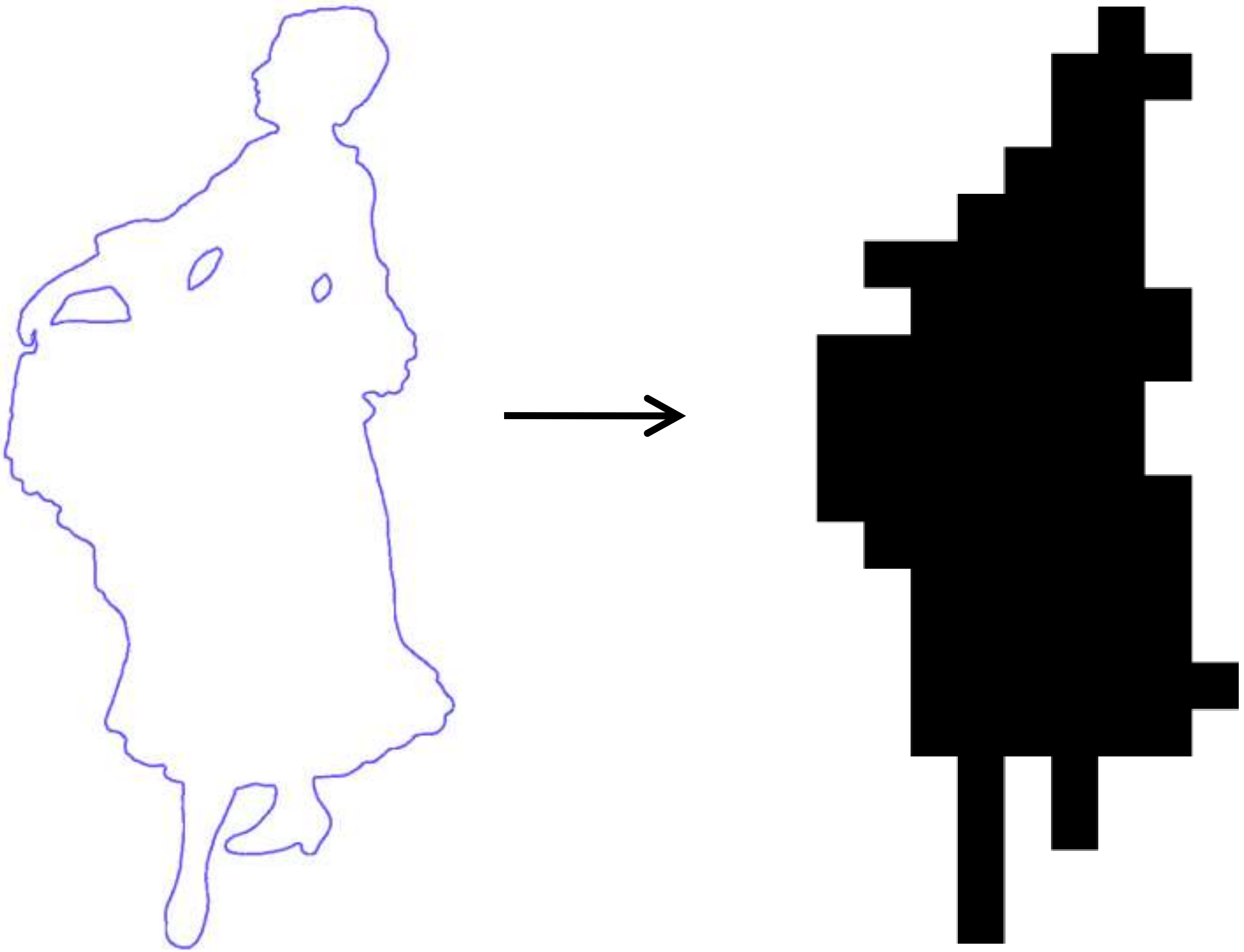
Wavelet Rasterization

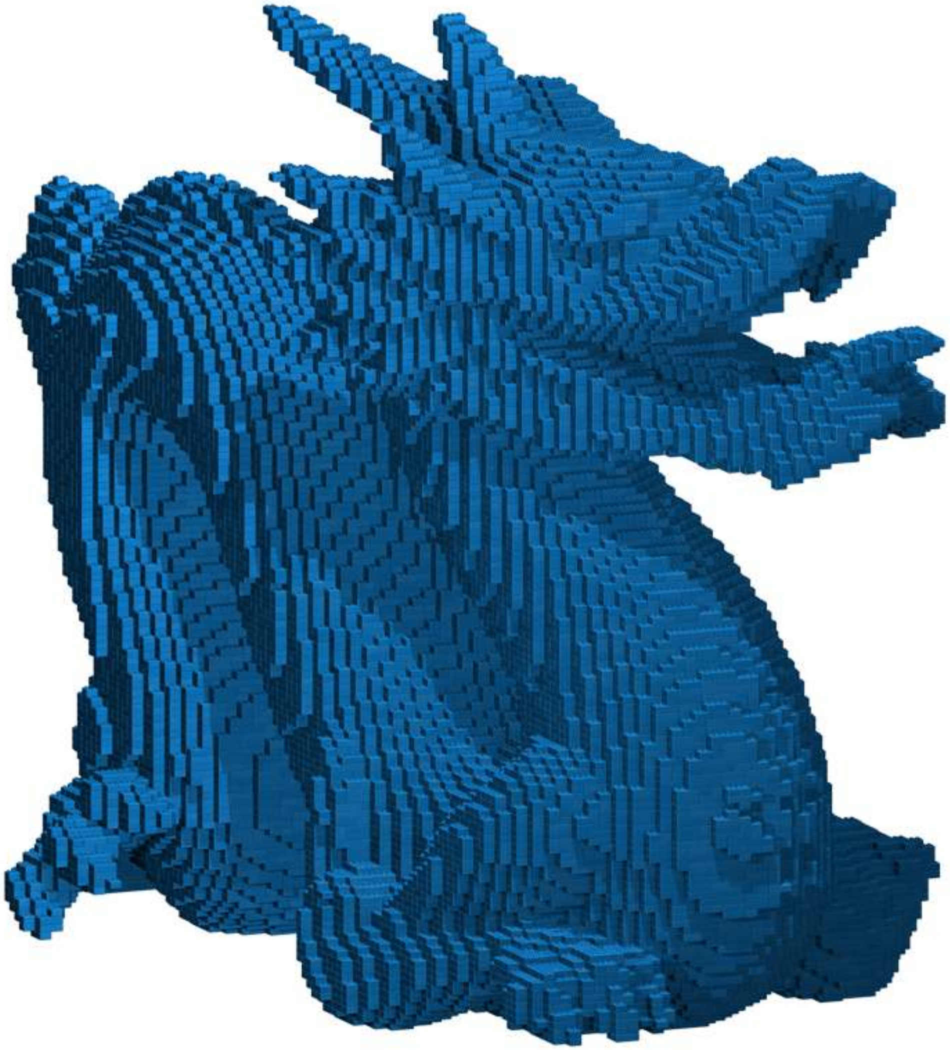
Josiah Manson and Scott Schaefer

Rasterization in 2D

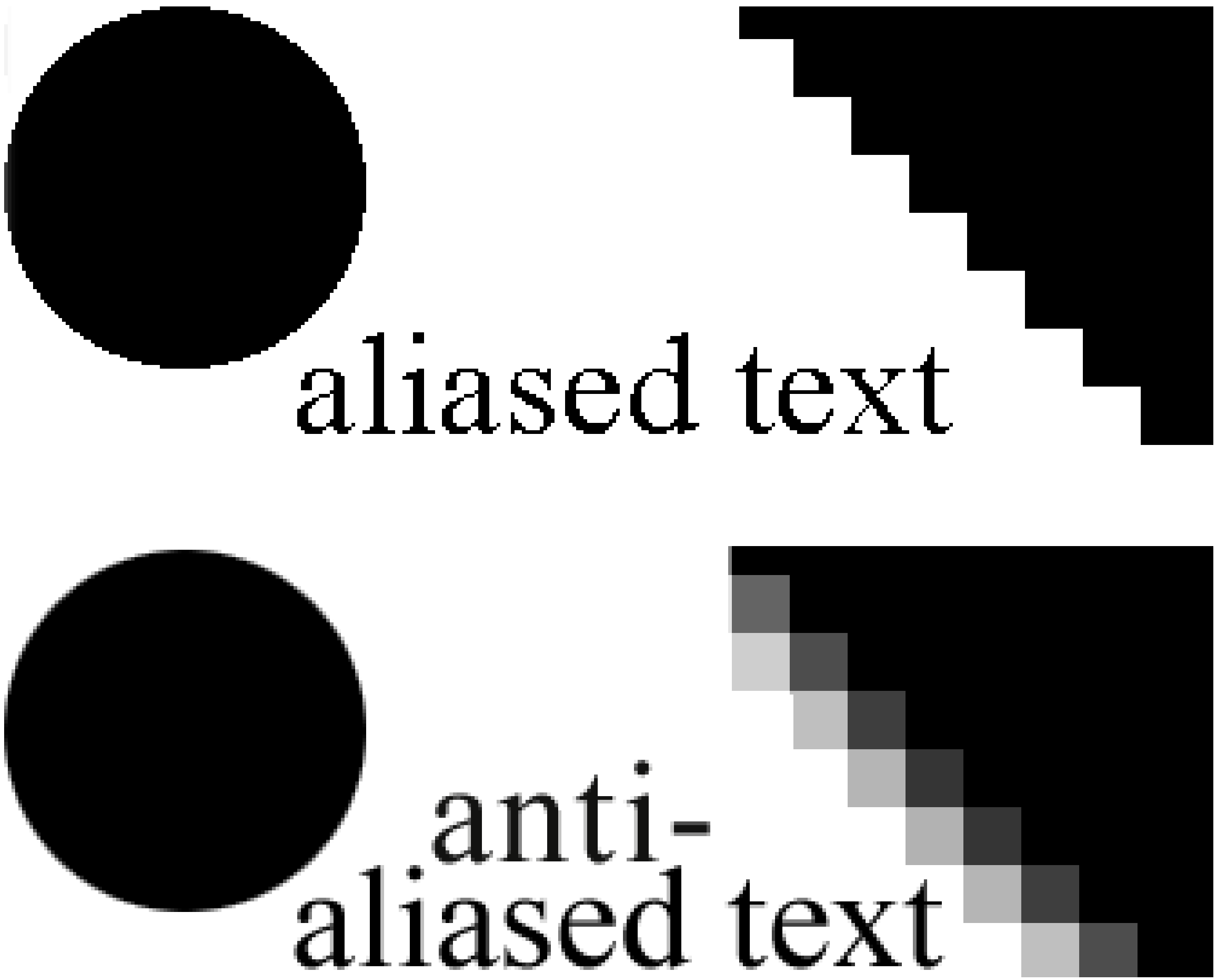


KarensWhimsy.com

Rasterization in 3D

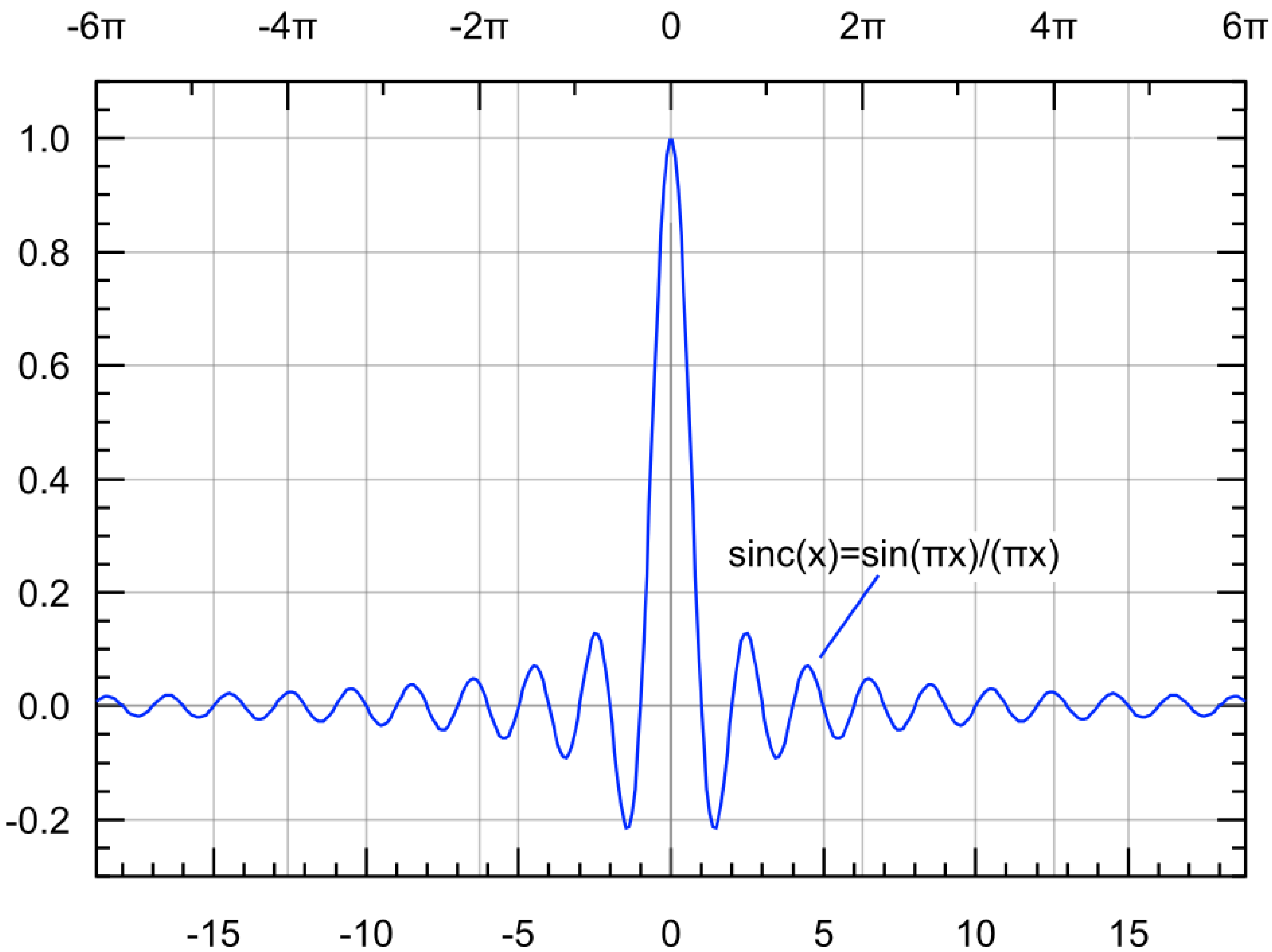


The aliasing problem

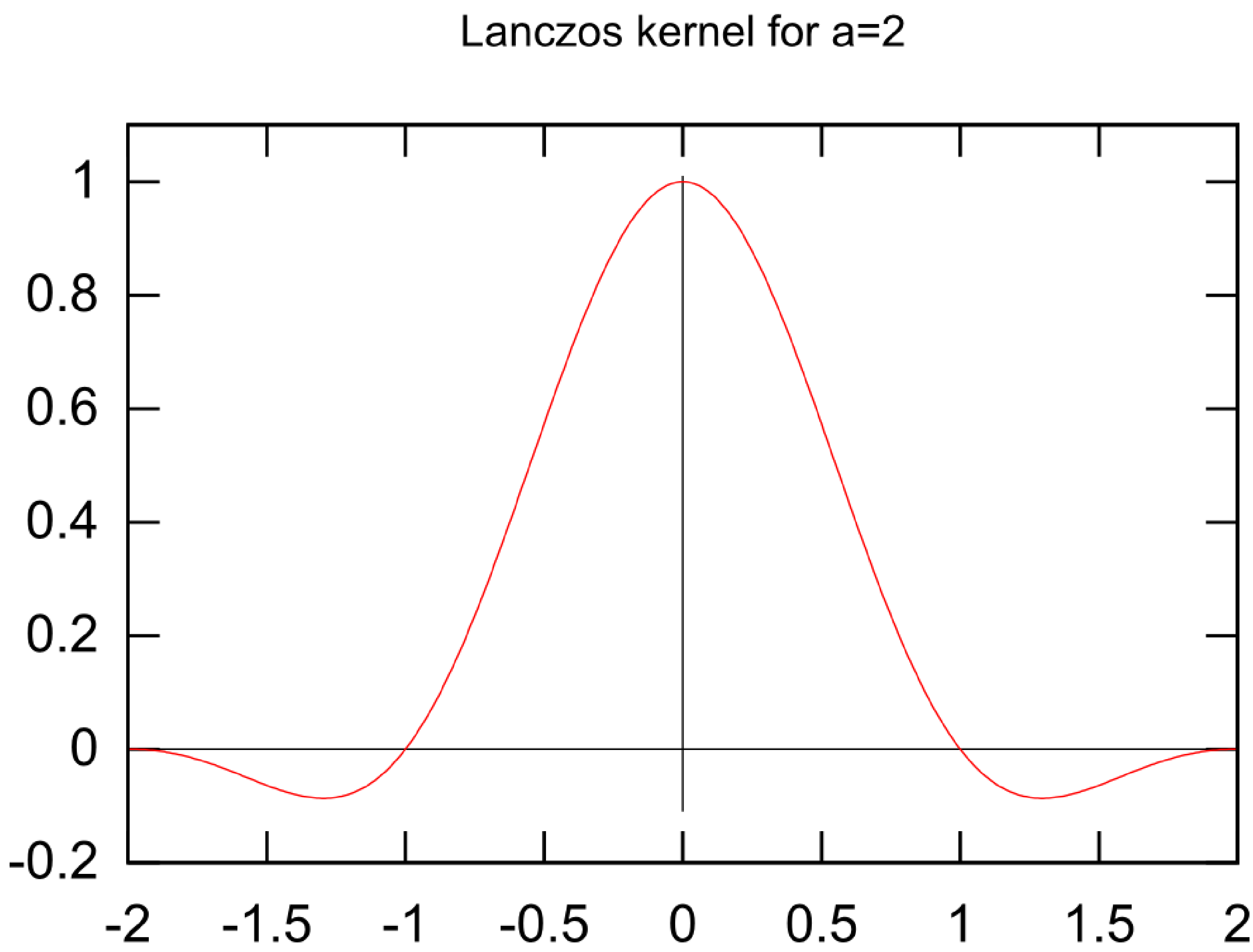


http://www.webpages.uidaho.edu/cti/photoshop/intro/anti-aliasing.htm

Anti-aliasing

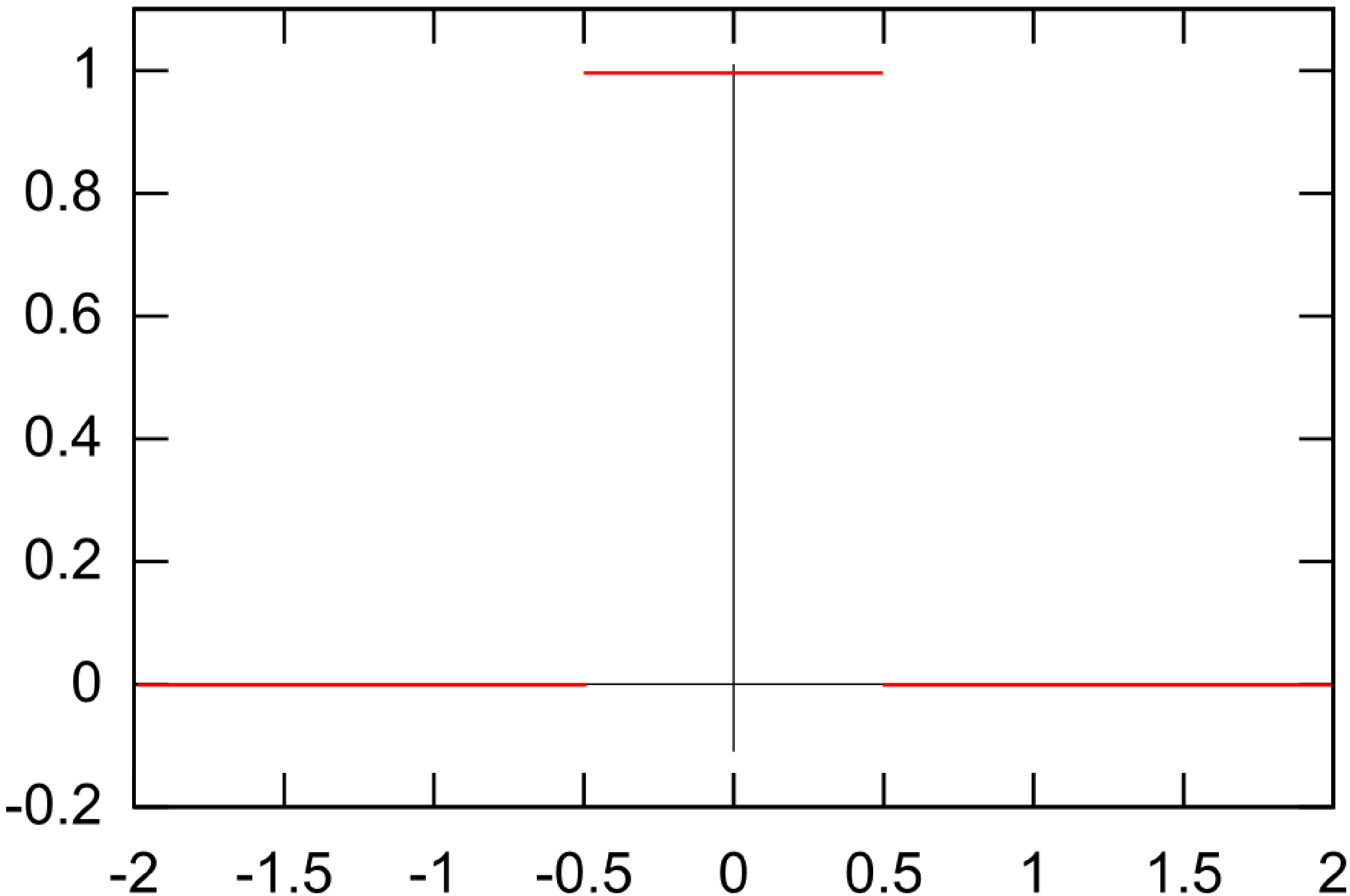


Anti-aliasing

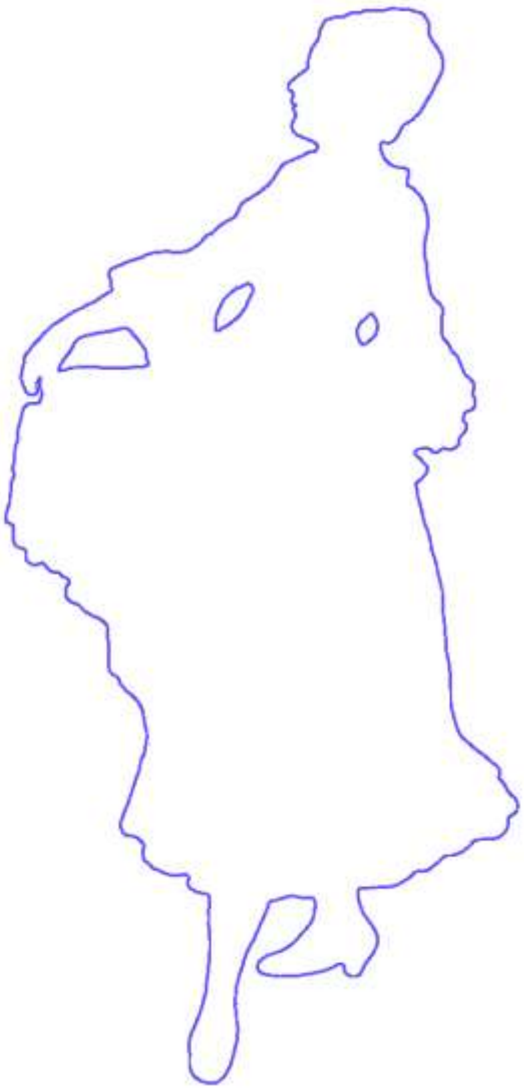


Anti-aliasing

Box filter

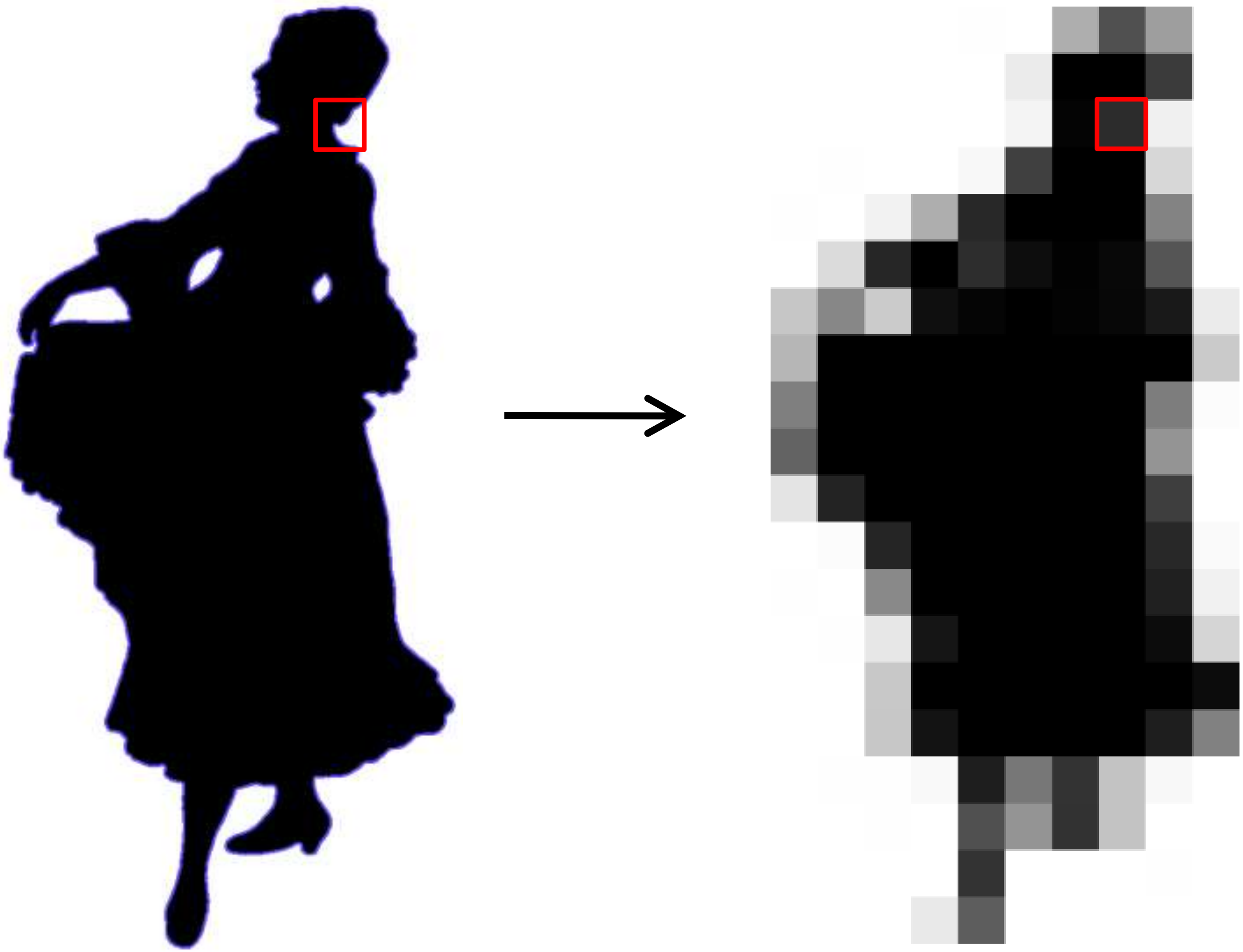


Pixel Raster Equation

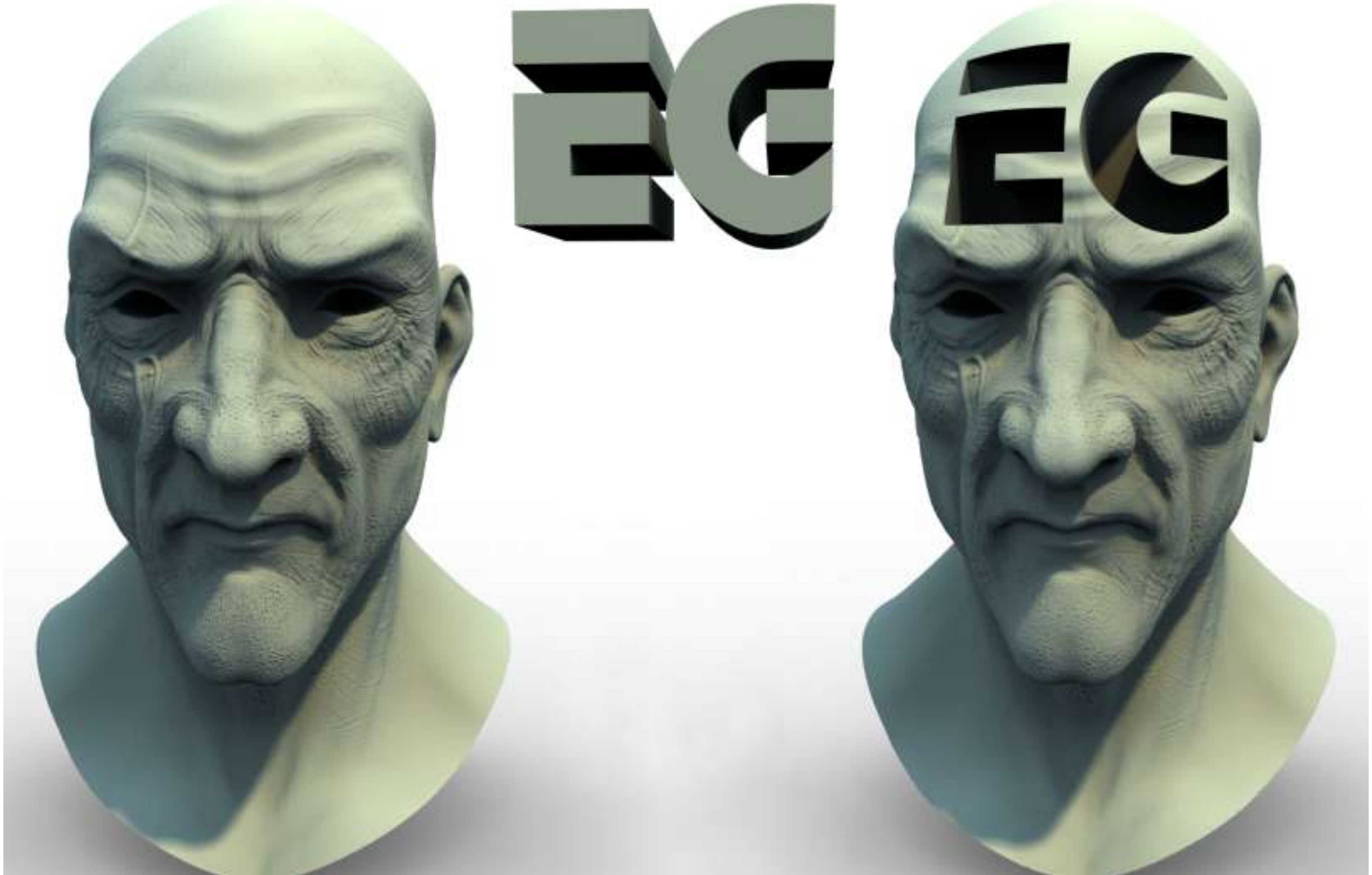


Pixel Raster Equation

Pixel Raster Equation



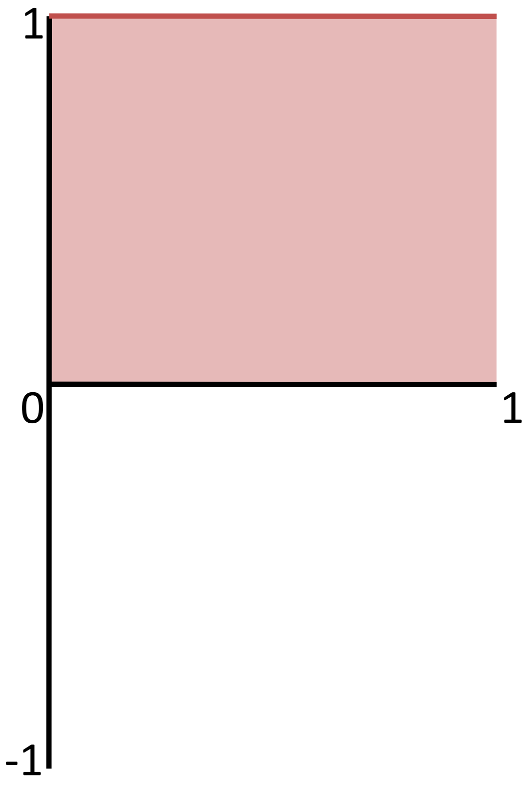
Applications



Applications

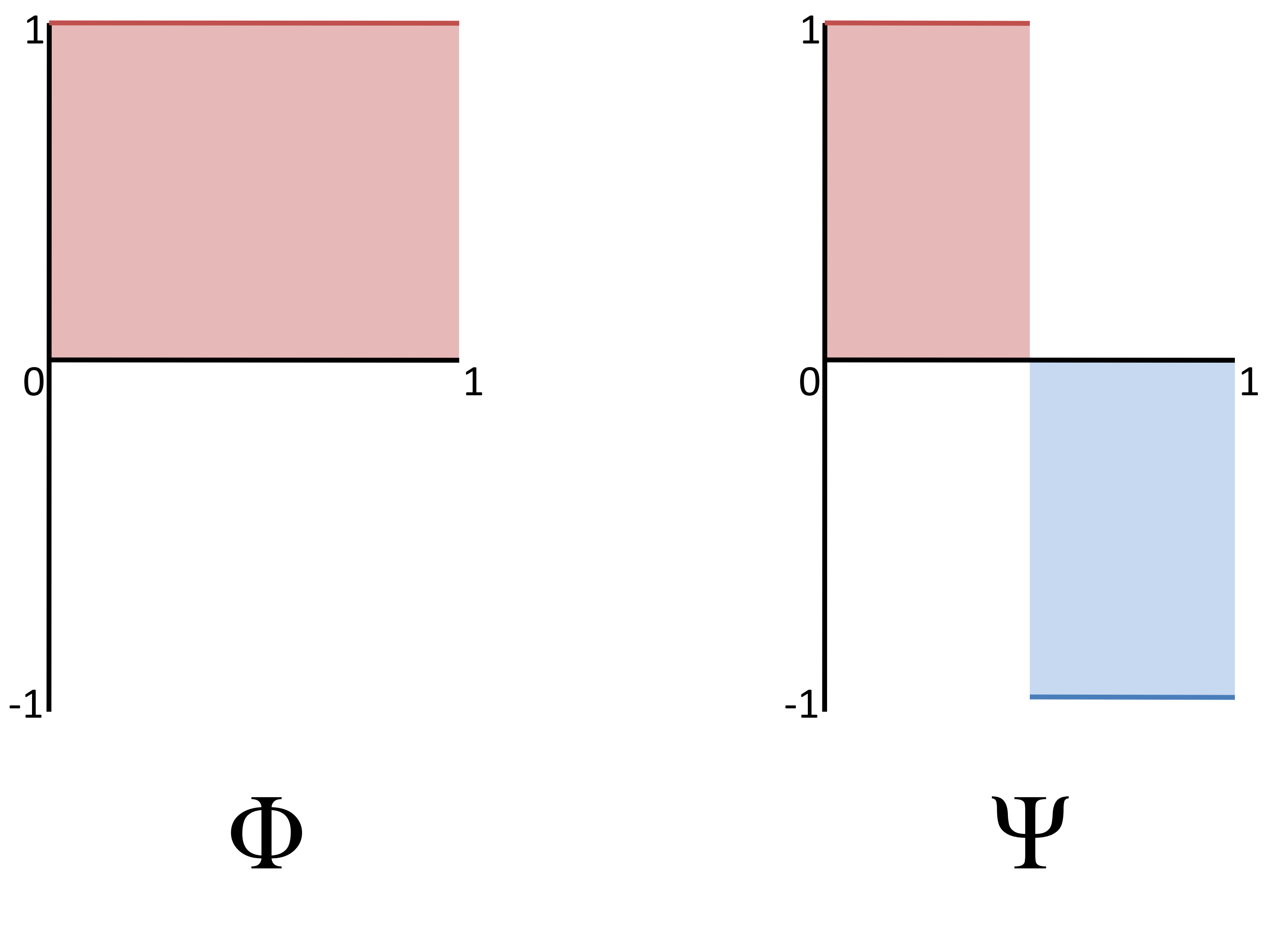
"A Multiscale Approach to Mesh-based Surface Tension Flows" Nils Thuerey, Chris Wojtan, Markus Gross, and Greg Turk

Haar Wavelets

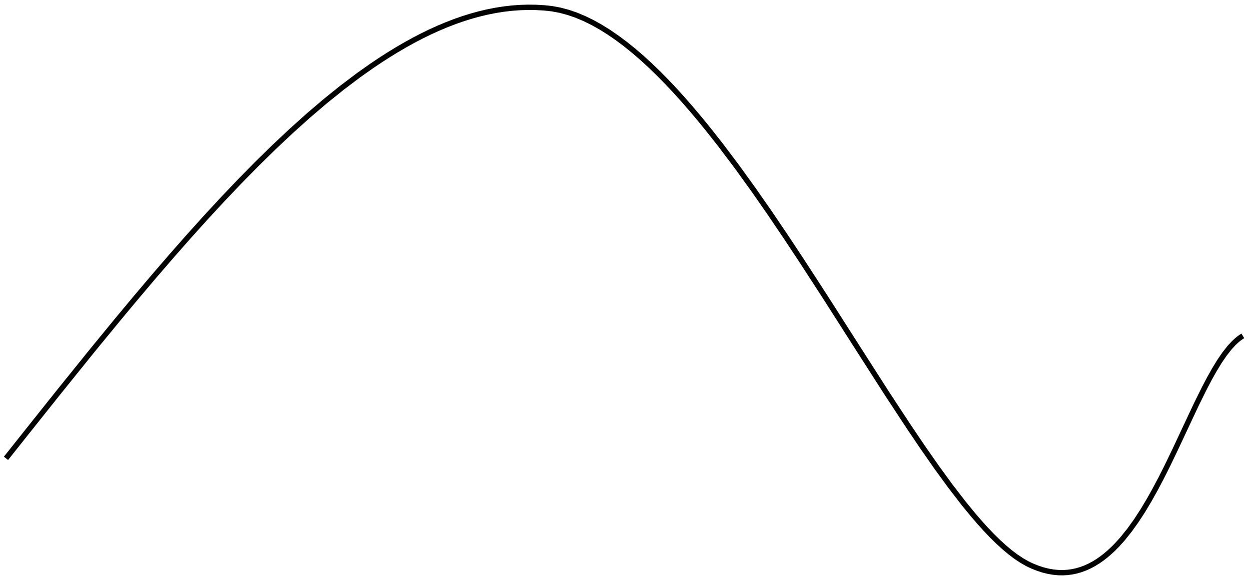


Φ

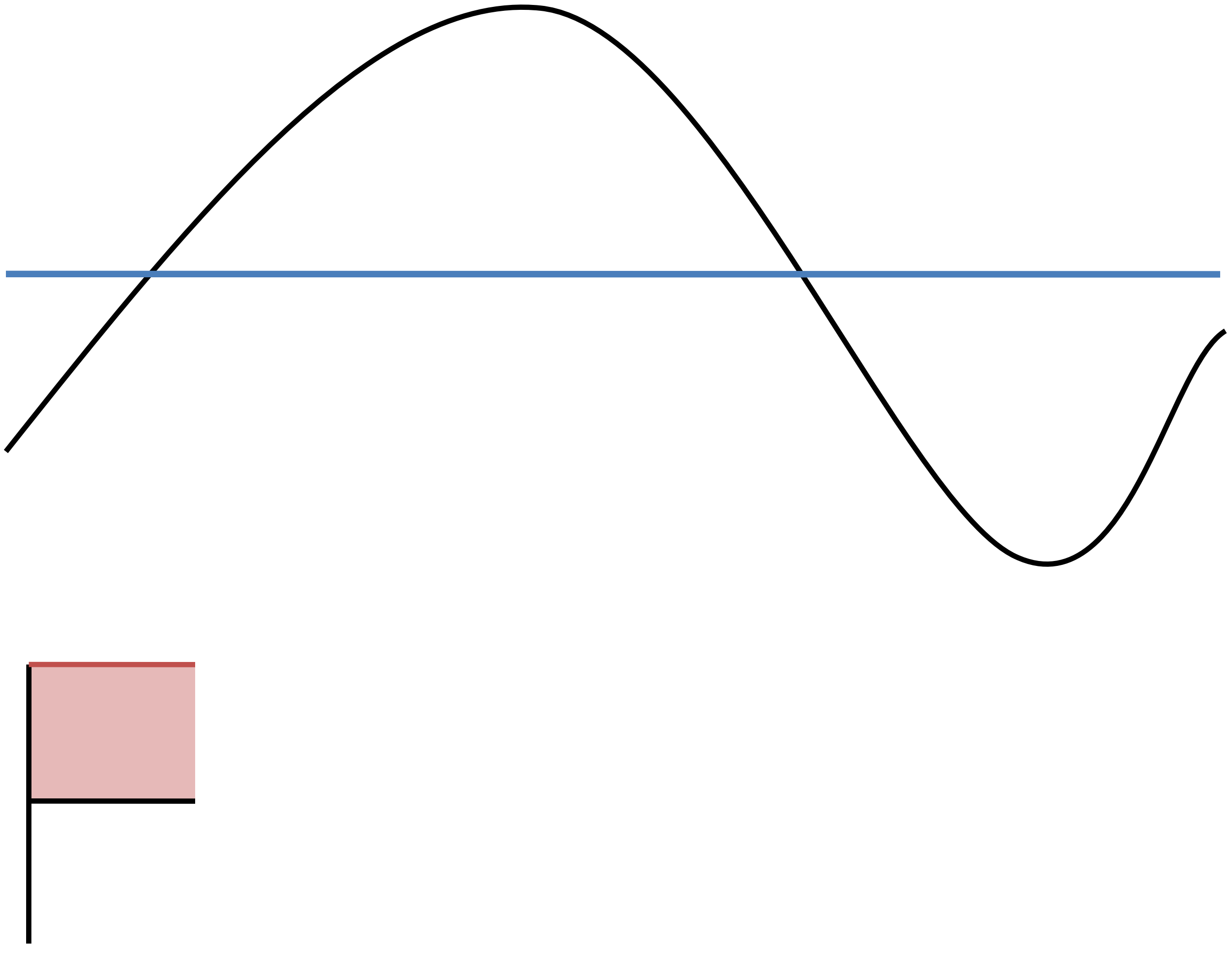
Haar Wavelets



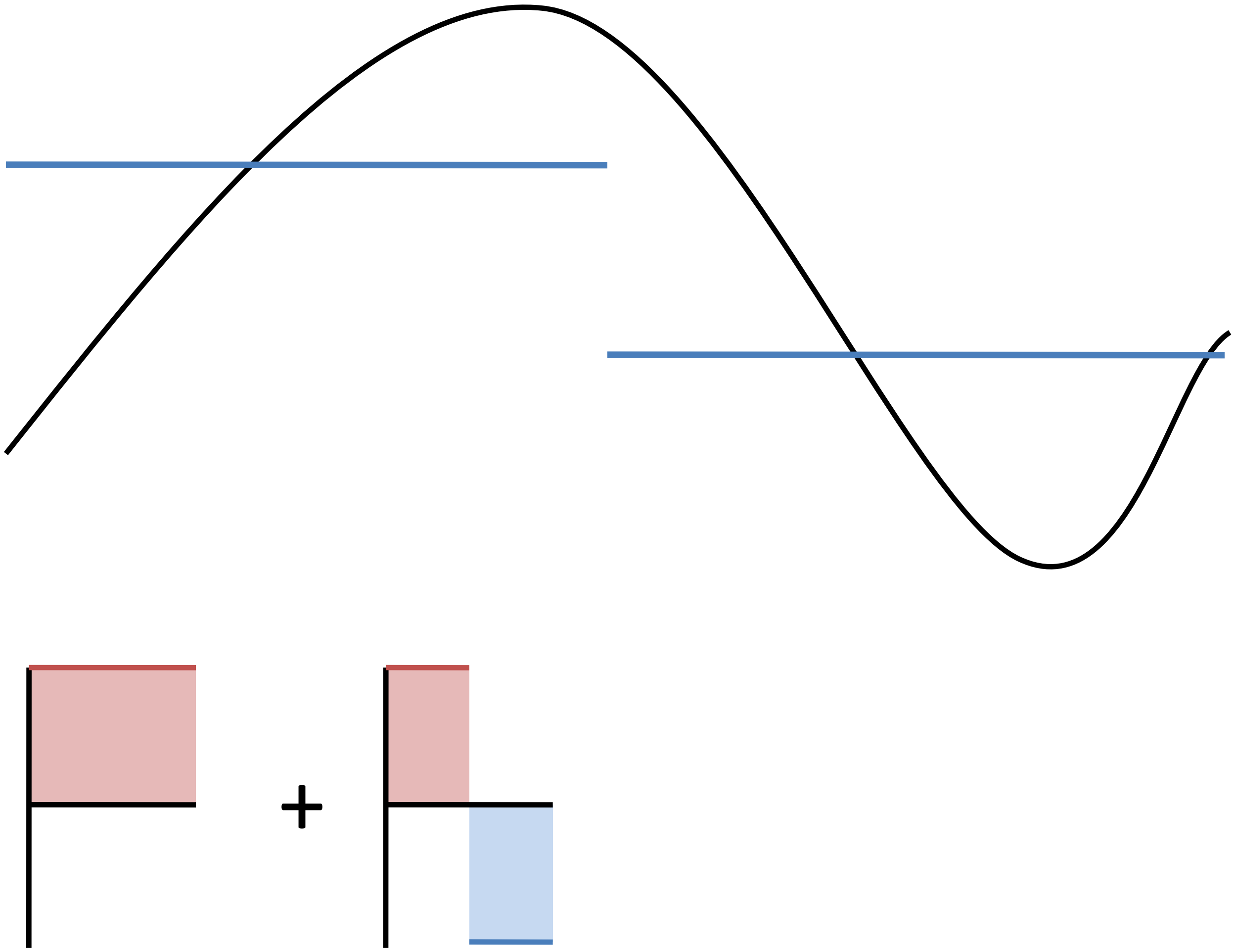
Haar Wavelets



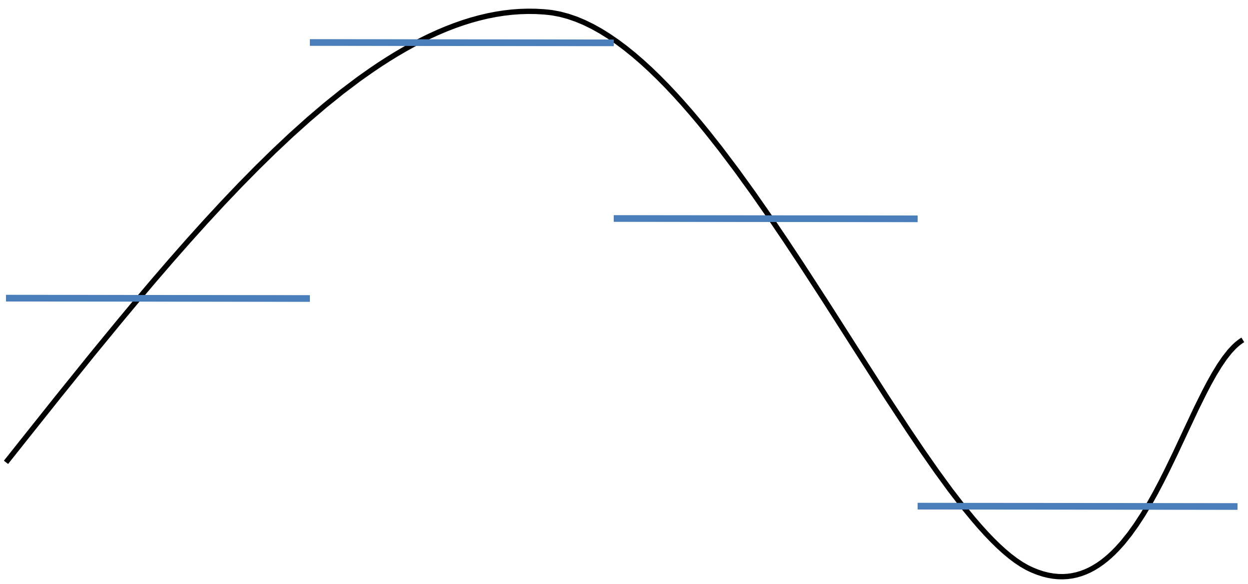
Haar Wavelets

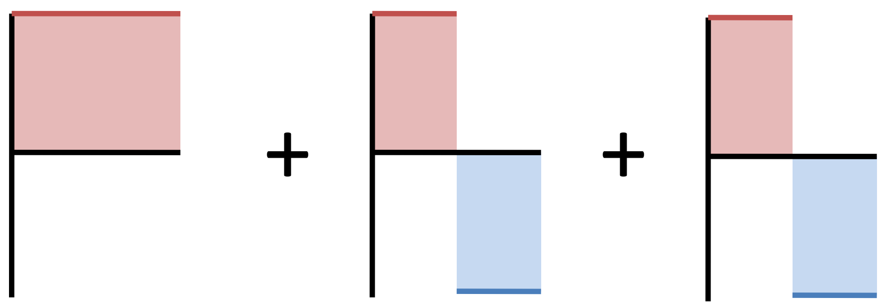


Haar Wavelets

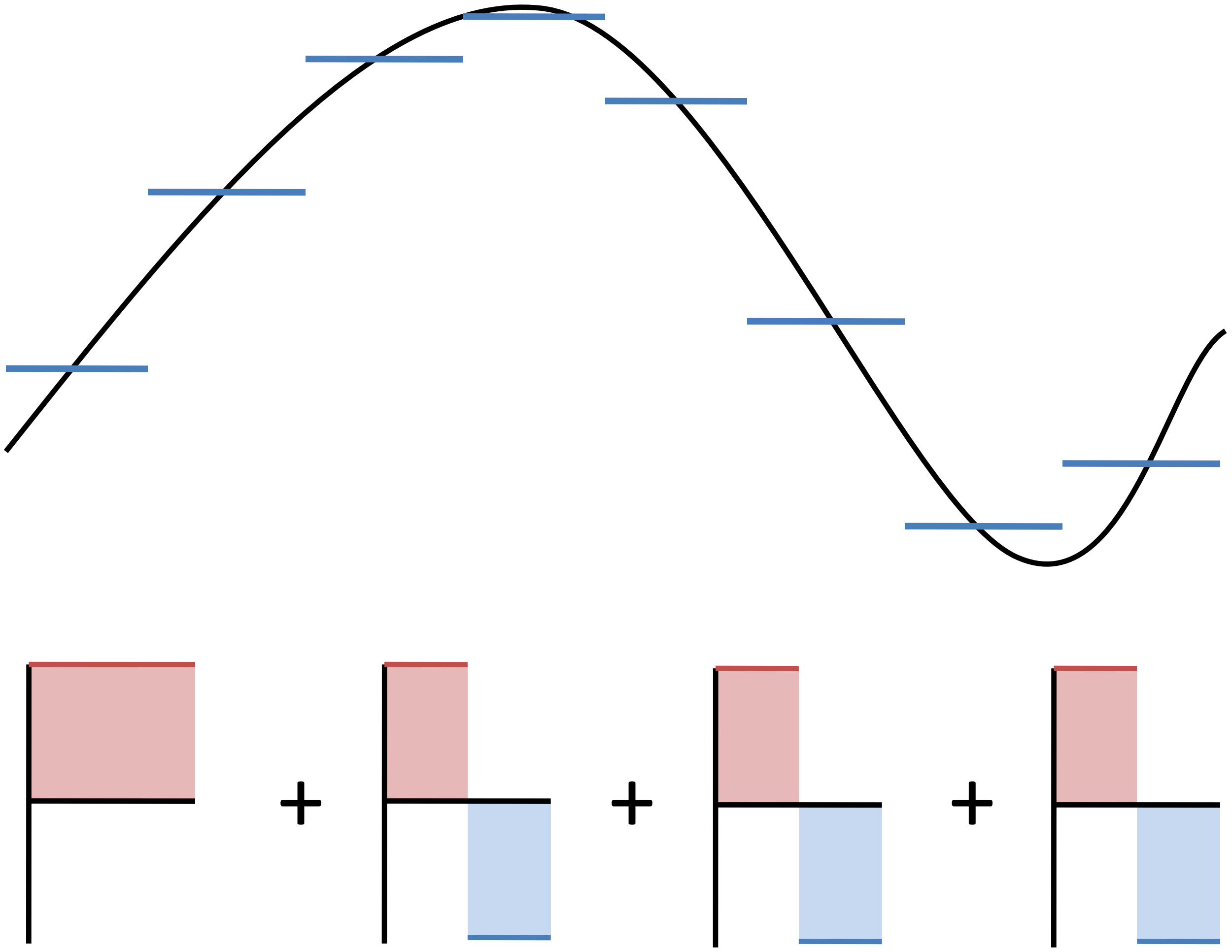


Haar Wavelets





Haar Wavelets

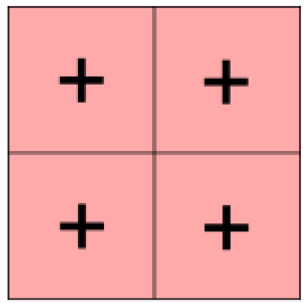


2D Haar Wavelets

$\chi_M(p)$

2D Haar Wavelets

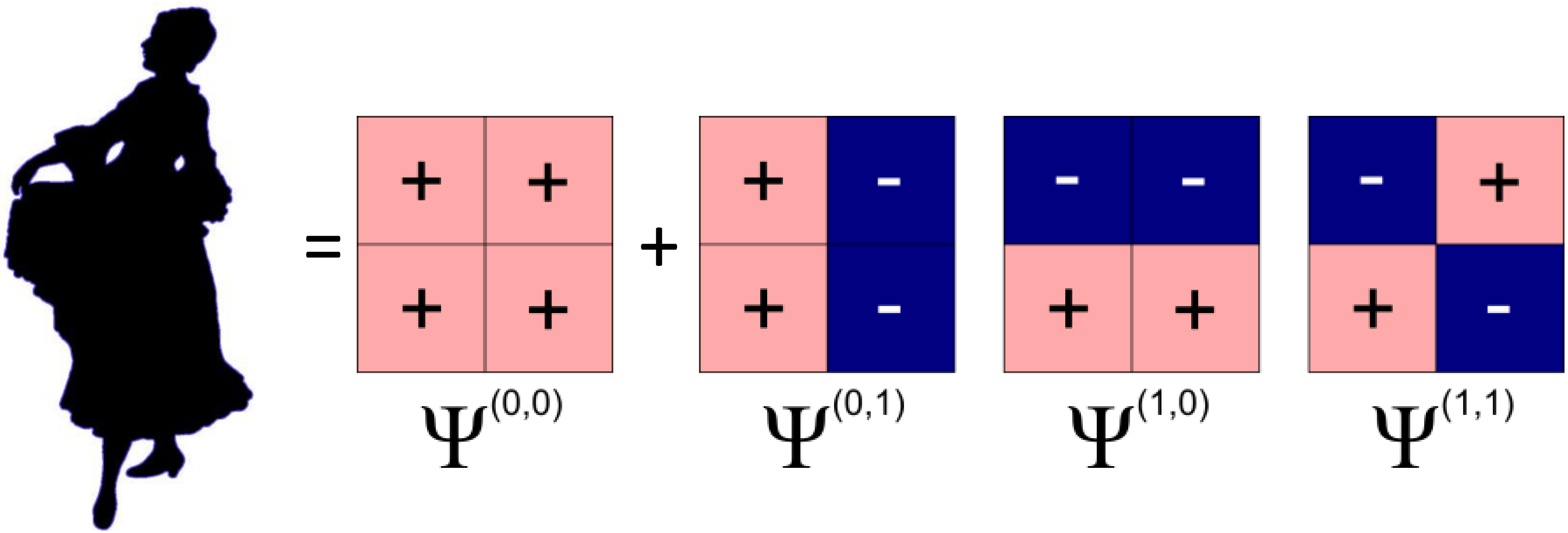
=



$\Psi^{(0,0)}$

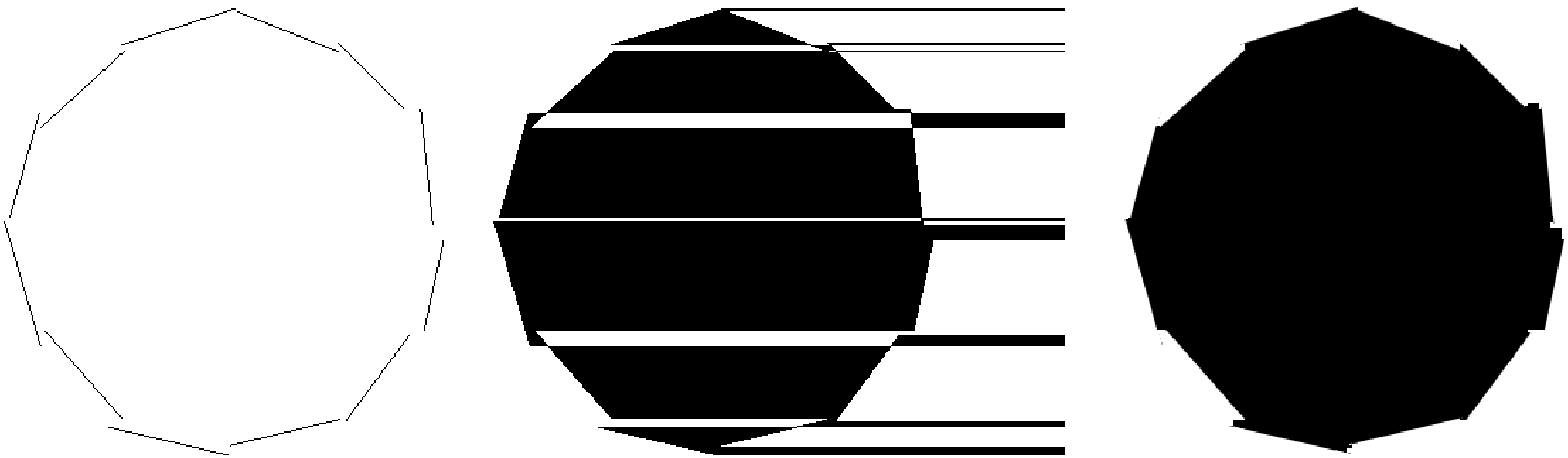
$$\chi_M(p) = \sum_{k \in \mathbb{Z}^2} c_{0,k}^{(0,0)} \psi_{0,k}^{(0,0)}(p)$$

2D Haar Wavelets

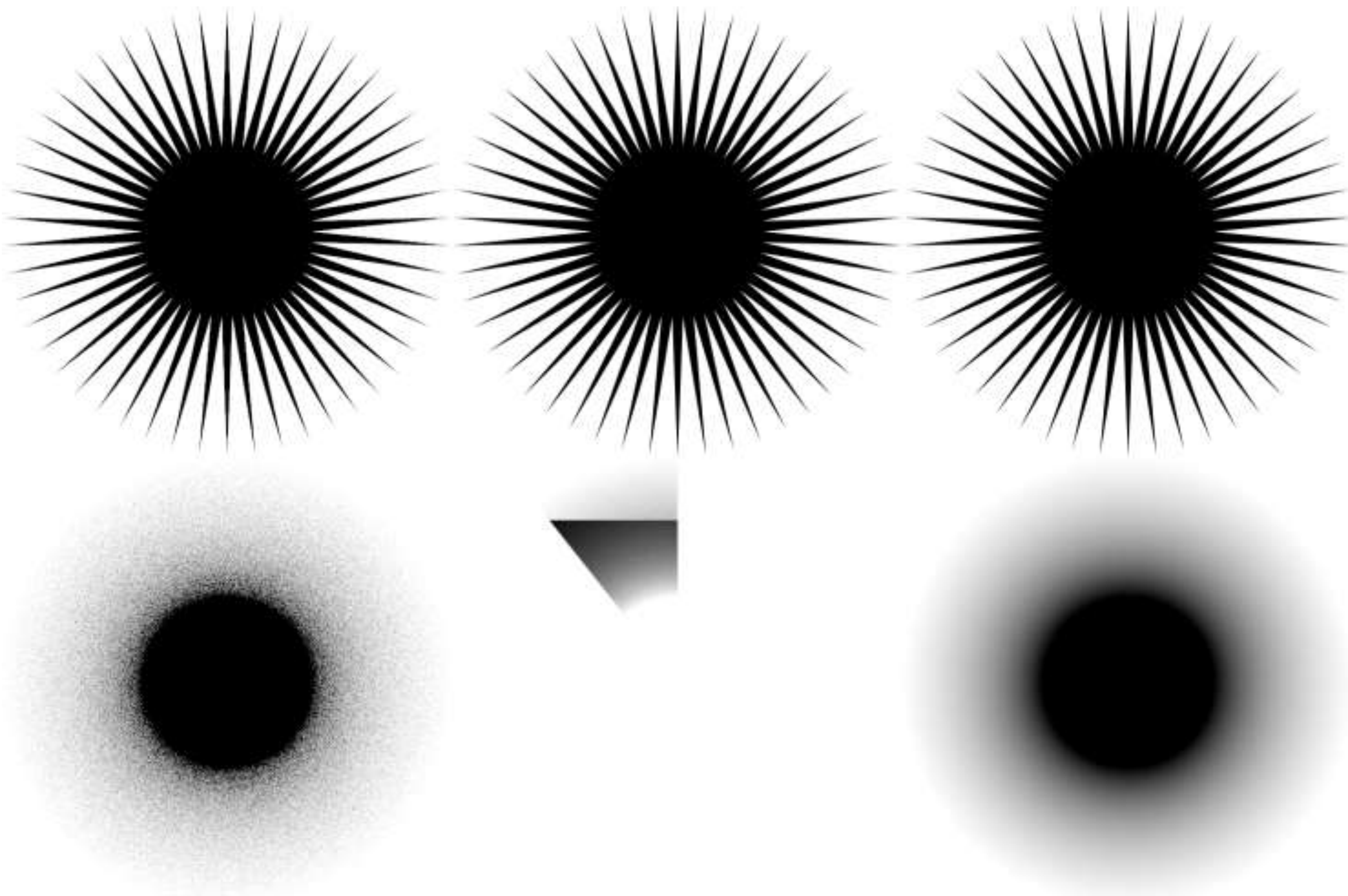


$$\chi_M(p) = \sum_{k \in \mathbb{Z}^2} c_{0,k}^{(0,0)} \psi_{0,k}^{(0,0)}(p) + \sum_{j \in \mathbb{N}} \sum_{k \in \mathbb{Z}^2} \sum_{e \in E} c_{j,k}^{e} \psi_{j,k}^{e}(p)$$

Why Wavelets?



Why Wavelets?



GPU AGG Wavelet

Calculating Wavelet Coefficients

$$c^e_{j,k} = \iint_{\mathbb{R}^2} \chi_M(p)\psi^e_{j,k}(p)\, dp$$

Calculating Wavelet Coefficients

$$
\begin{aligned}
c^e_{j,k} &= \iint\limits_{\mathbb{R}^2} \chi_M(p)\psi^e_{j,k}(p)\,dp \\
&= \iint\limits_{M} \psi^e_{j,k}(p)\,dp
\end{aligned}
$$

Calculating Wavelet Coefficients

$$c_{j,k}^{e} = \iint\limits_{\mathbb{R}^2} \chi_M(p)\psi_{j,k}^{e}(p)\,dp$$

$$= \iint\limits_{M} \psi_{j,k}^{e}(p)\,dp$$

$$\iint\limits_{M} \nabla \cdot F_{j,k}^{e}(p)\,dp = \oint_{p\in\partial M} F_{j,k}^{e}(p)\cdot n(p)\,d\sigma$$

Calculating Wavelet Coefficients

$$c^e_{j,k} = \iint\limits_{\mathbb{R}^2} \chi_M(p)\psi^e_{j,k}(p)\,dp$$

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Calculating Wavelet Coefficients

$$
\begin{aligned}
c^e_{j,k} &= \iint\limits_{\mathbb{R}^2} \chi_M(p)\psi^e_{j,k}(p)\, dp \\
&= \iint\limits_{M} \psi^e_{j,k}(p)\, dp \\
&= \sum_i \int_0^1 F^e_{j,k}(P_i(t)) \cdot n(P_i(t)) \|P_i'(t)\| dt
\end{aligned}
$$

Choosing *F*

$$\nabla \cdot F^e(p) = \psi^e(p)$$

Choosing *F*

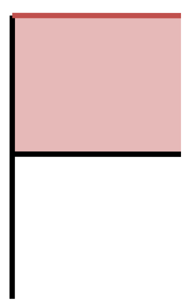
$$\nabla \cdot F^e(p) = \psi^e(p)$$

$$\bar{\Phi}(t) = \int_0^t \phi(s)\, ds \qquad \bar{\Psi}(t) = \int_0^t \psi(s)\, ds$$

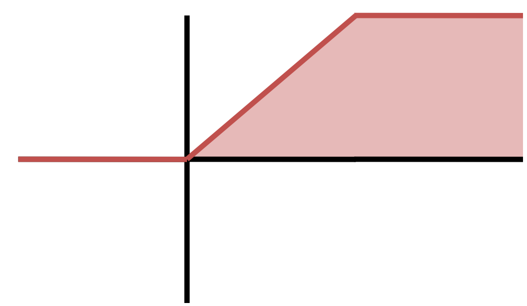
Choosing F

$$\nabla \cdot F^e(p) = \psi^e(p)$$

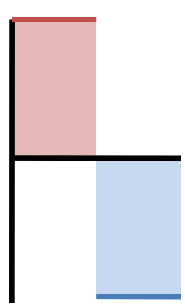
$\phi(s) =$



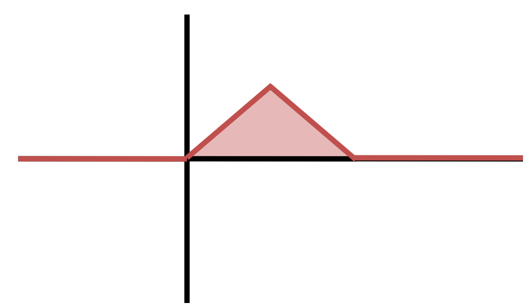
$\bar{\Phi}(t) =$



$\psi(s) =$



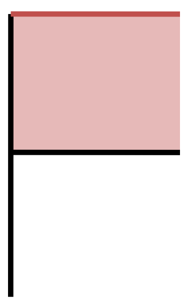
$\bar{\Psi}(t) =$

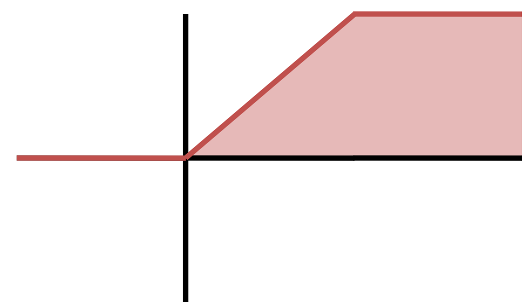


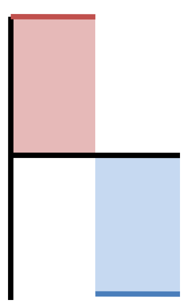
Choosing F

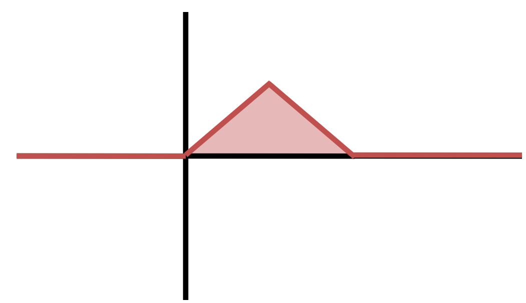
$$\nabla \cdot F^e(p) = \psi^e(p)$$

$$F^e(p) = \left(\alpha \bar{\Psi}^{e_x}(p_x)\psi^{e_y}(p_y), \beta \psi^{e_x}(p_x)\bar{\Psi}^{e_y}(p_y)\right)$$

$\phi(s) =$ 

$\bar{\Phi}(t) =$ 

$\psi(s) =$ 

$\bar{\Psi}(t) =$ 

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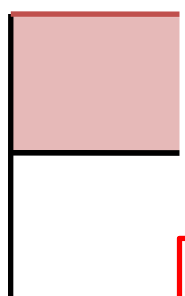
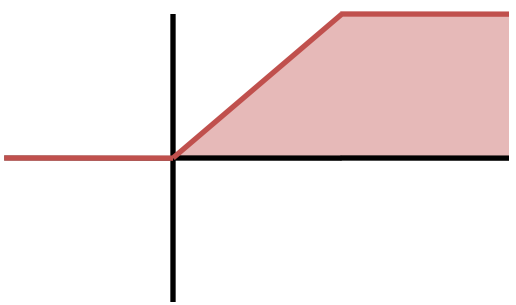
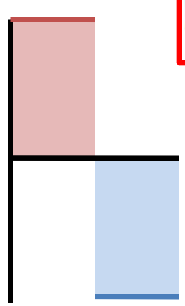
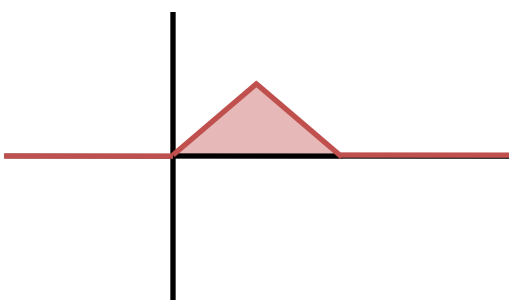
$\phi(s) =$ 

$\bar{\Phi}(t) =$ 

$$\alpha + \beta = 1$$

$\psi(s) =$ 

$\bar{\Psi}(t) =$ 

Choosing *F*

$$F^{(0,0)}(p) \quad = \quad \frac{1}{2}(\bar{\Phi}(p_x)\phi(p_y), \phi(p_x)\bar{\Phi}(p_y))$$

$$\alpha = \beta = \frac{1}{2}$$

Choosing *F*

$$
\begin{aligned}
F^{(0,0)}(p) &= \tfrac{1}{2}(\bar{\Phi}(p_x)\phi(p_y), \phi(p_x)\bar{\Phi}(p_y)) \\
F^{(1,0)}(p) &= (\bar{\Psi}(p_x), 0)
\end{aligned}
$$

$$\alpha = 1 \qquad \beta = 0$$

Choosing F

$$\begin{aligned} F^{(0,0)}(p) &= \tfrac{1}{2}(\bar{\Phi}(p_x)\phi(p_y), \phi(p_x)\bar{\Phi}(p_y)) \\ F^{(1,0)}(p) &= (\bar{\Psi}(p_x), 0) \\ F^{(0,1)}(p) &= (0, \bar{\Psi}(p_y)) \end{aligned}$$

$$\alpha = 0 \qquad \beta = 1$$

Choosing F

$$\begin{aligned}
F^{(0,0)}(p) &= \tfrac{1}{2}(\bar{\Phi}(p_x)\phi(p_y), \phi(p_x)\bar{\Phi}(p_y)) \\
F^{(1,0)}(p) &= (\bar{\Psi}(p_x), 0) \\
F^{(0,1)}(p) &= (0, \bar{\Psi}(p_y)) \\
F^{(1,1)}(p) &= (\bar{\Psi}(p_x)\psi(p_y), 0)
\end{aligned}$$

$$\alpha = 1 \qquad \beta = 0$$

Line Segments

$$F^{(0,0)}(p) \quad = \quad \tfrac{1}{2}(\bar{\Phi}(p_x)\phi(p_y), \phi(p_x)\bar{\Phi}(p_y))$$

$$\int_0^1 F^{(0,0)}(P(t)) \cdot n(P(t)) \|P'(t)\| dt = \frac{1}{2} det\,(v_0, v_1)$$

Line Segments

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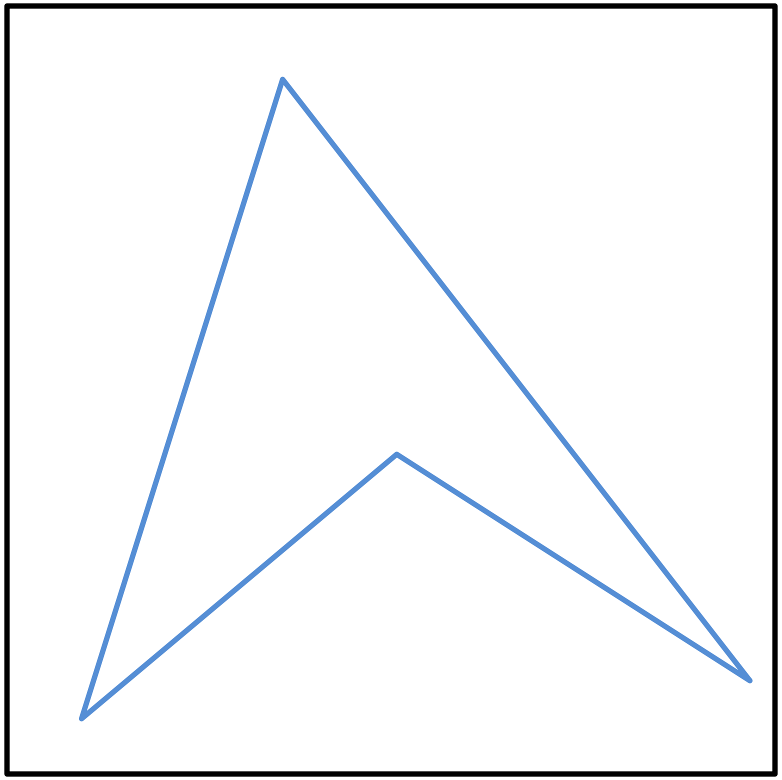
Details in paper

The Algorithm, Step-by-step



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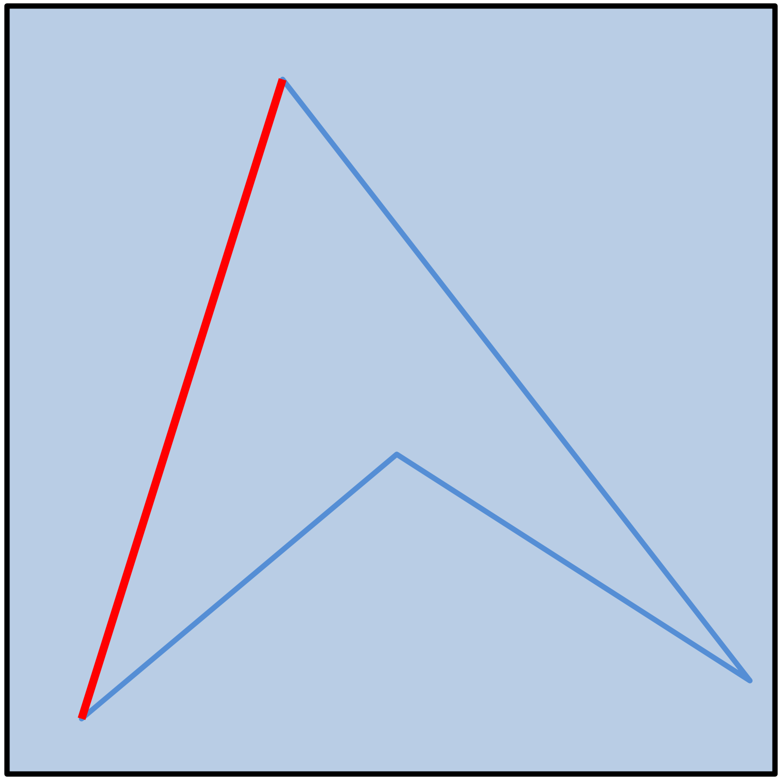

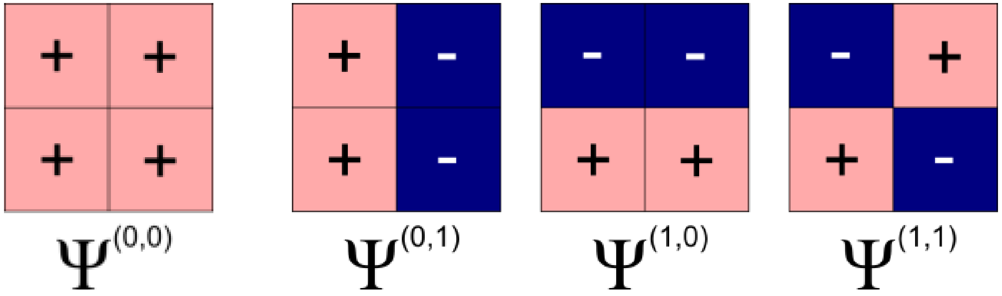

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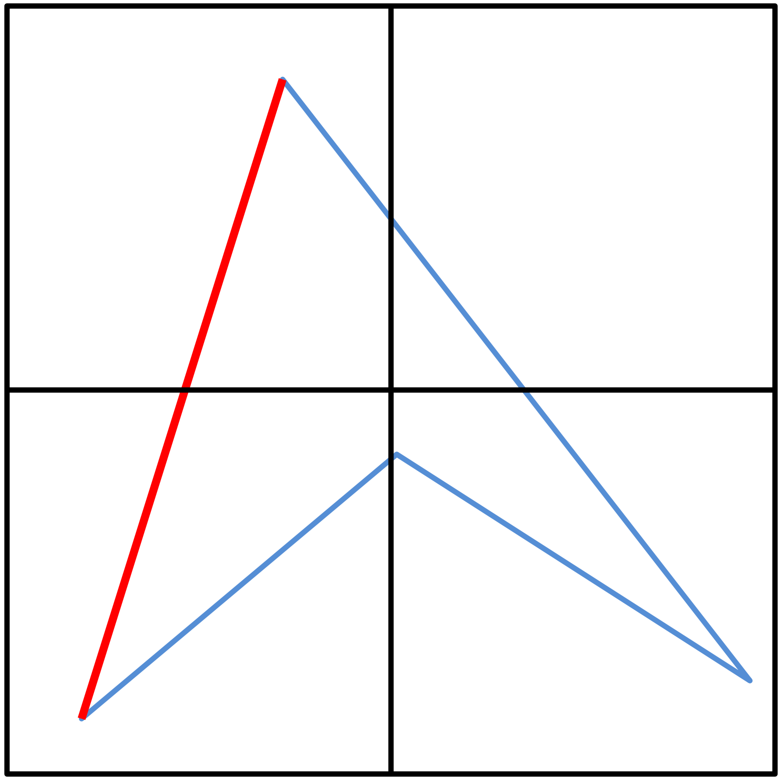

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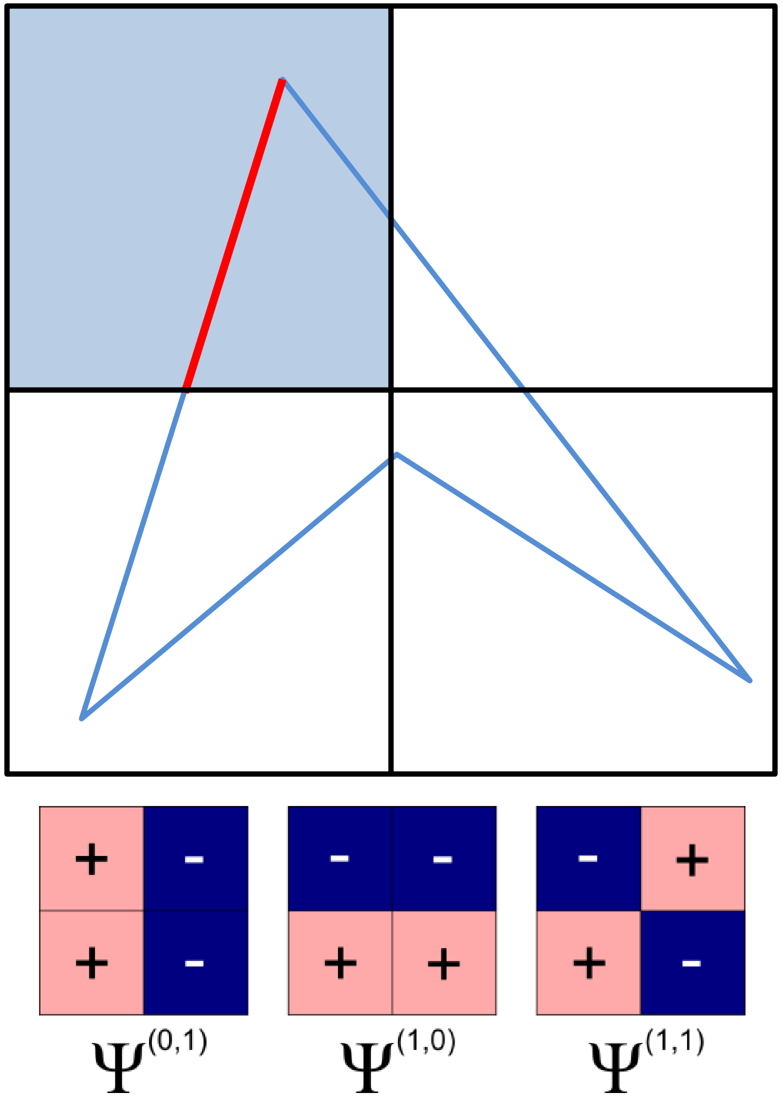

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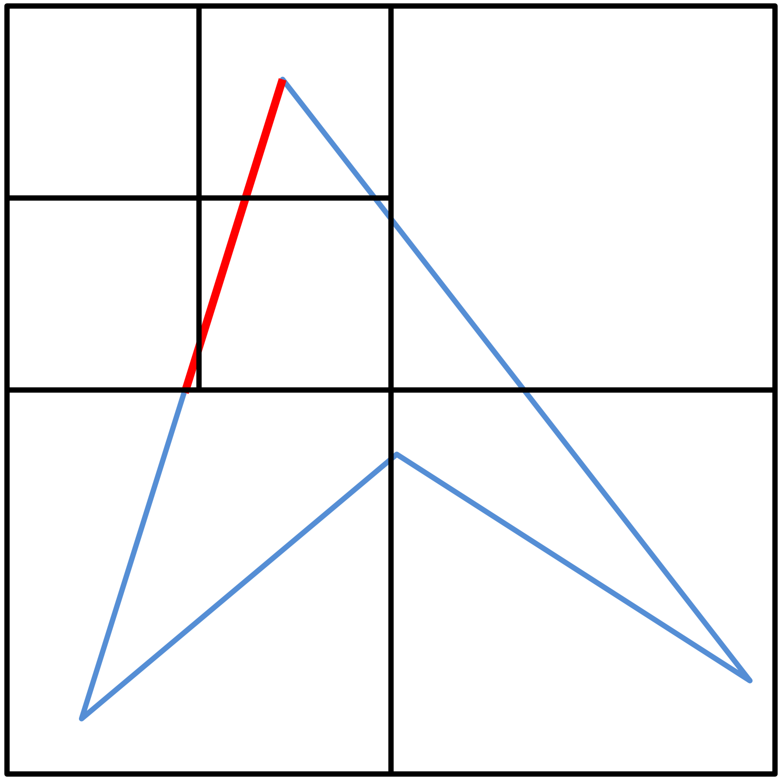

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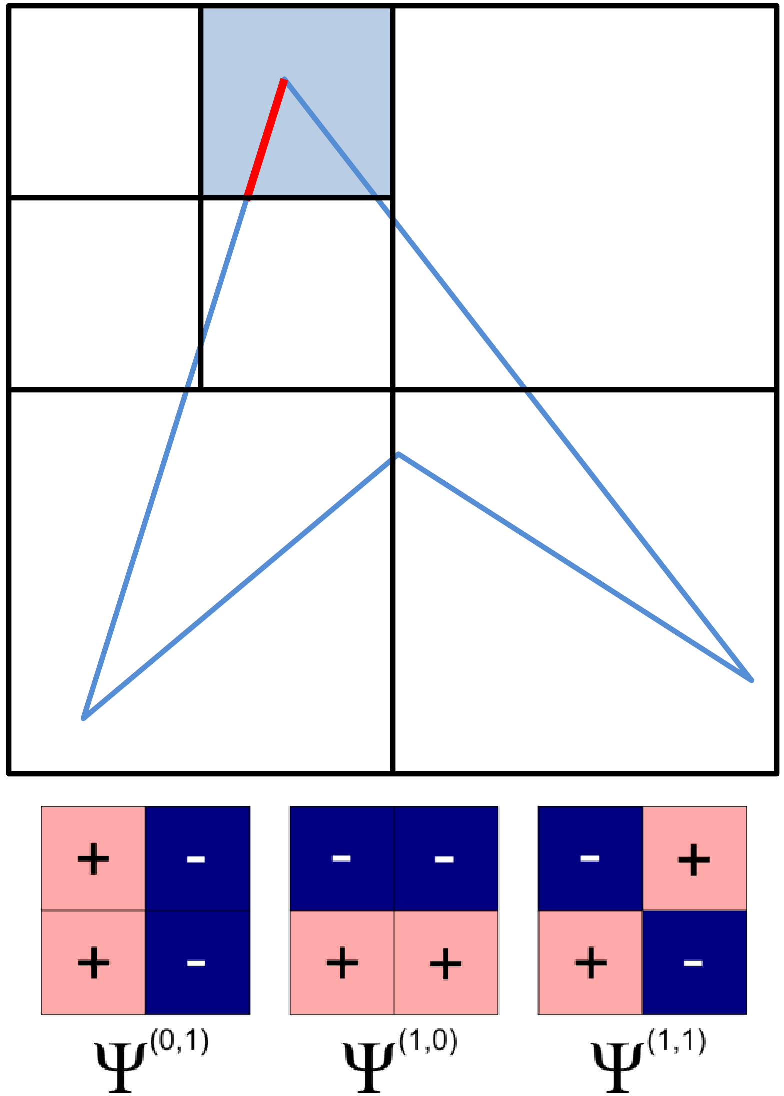

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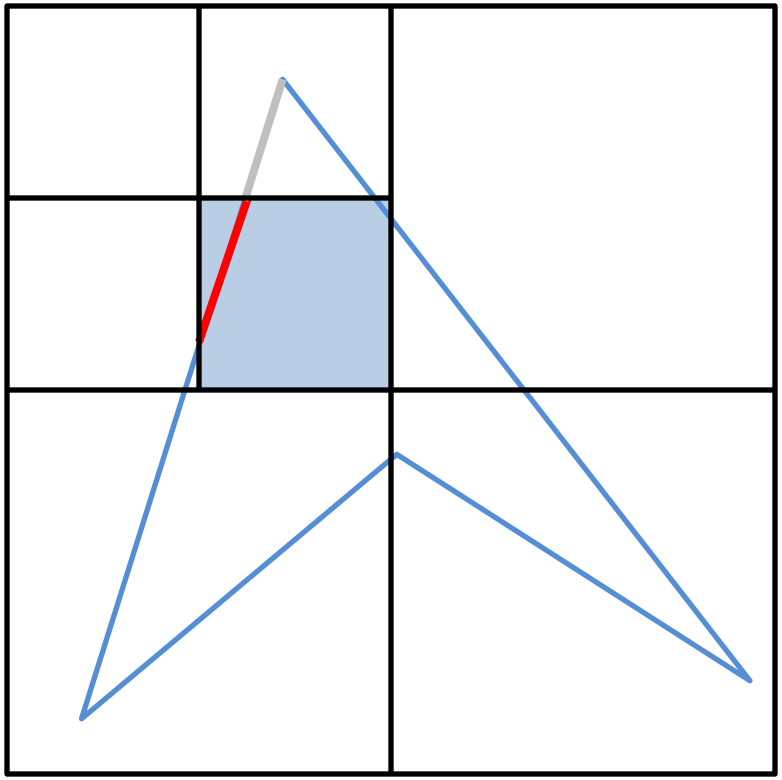
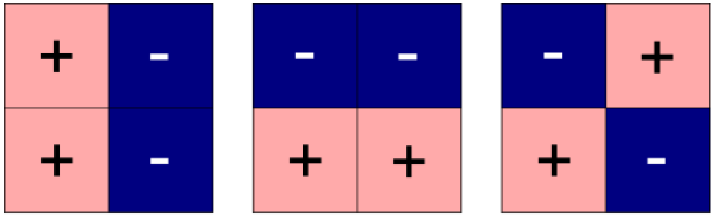
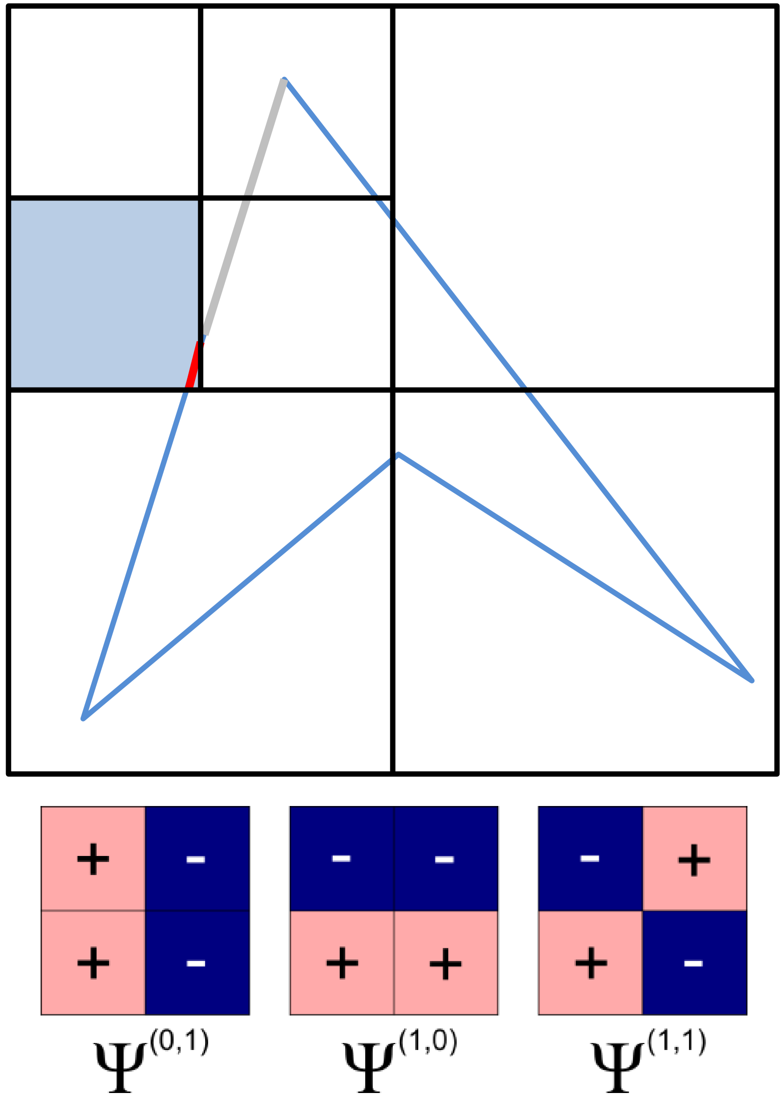




$\Psi^{(0,1)}$ $\Psi^{(1,0)}$ $\Psi^{(1,1)}$

The Algorithm, Step-by-step



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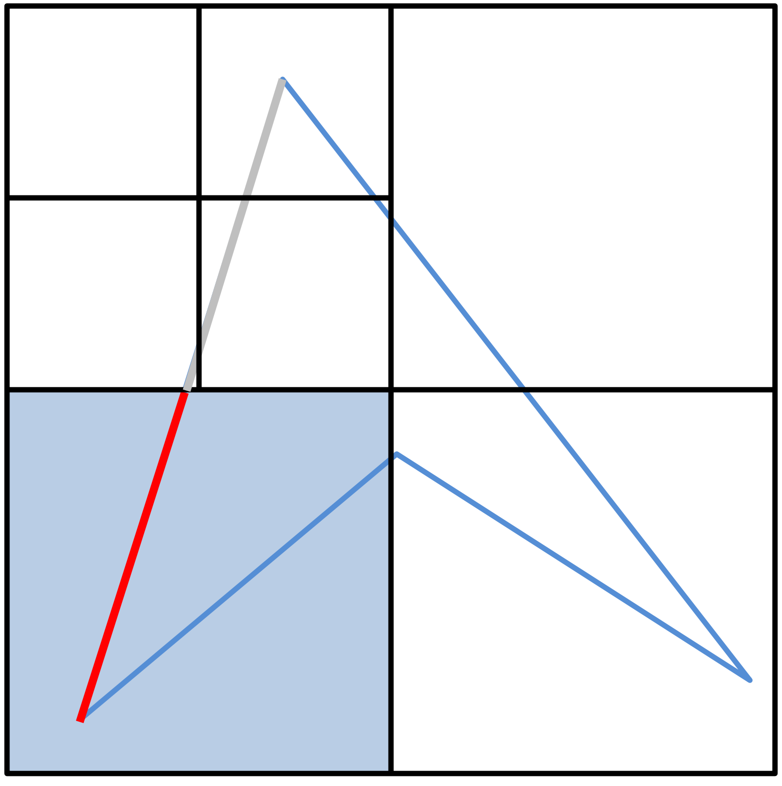
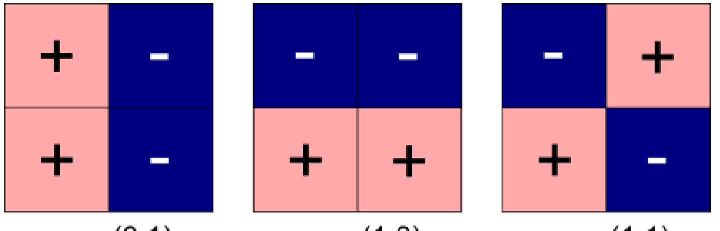
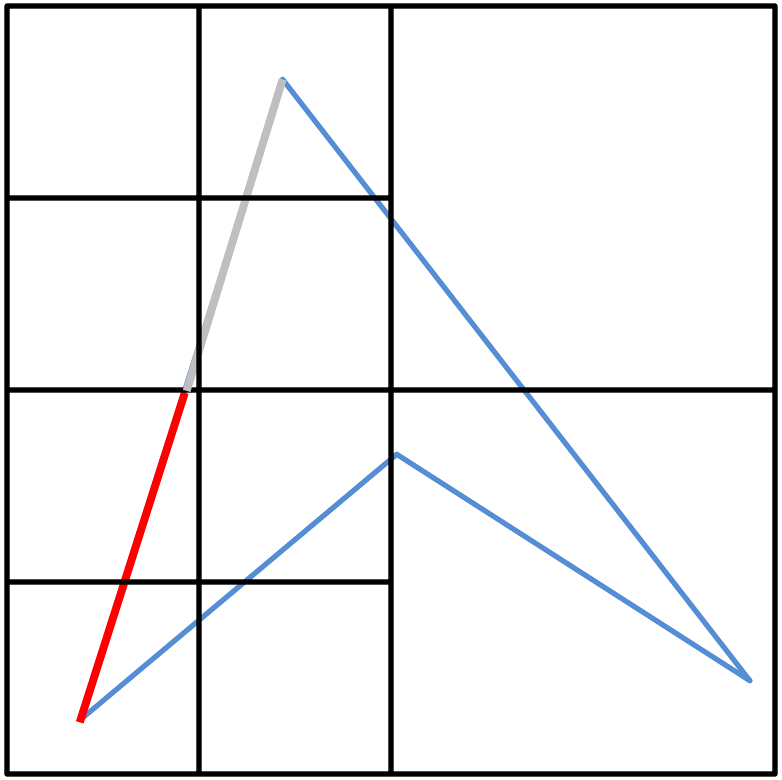




$\Psi^{(0,1)}$ $\Psi^{(1,0)}$ $\Psi^{(1,1)}$

The Algorithm, Step-by-step



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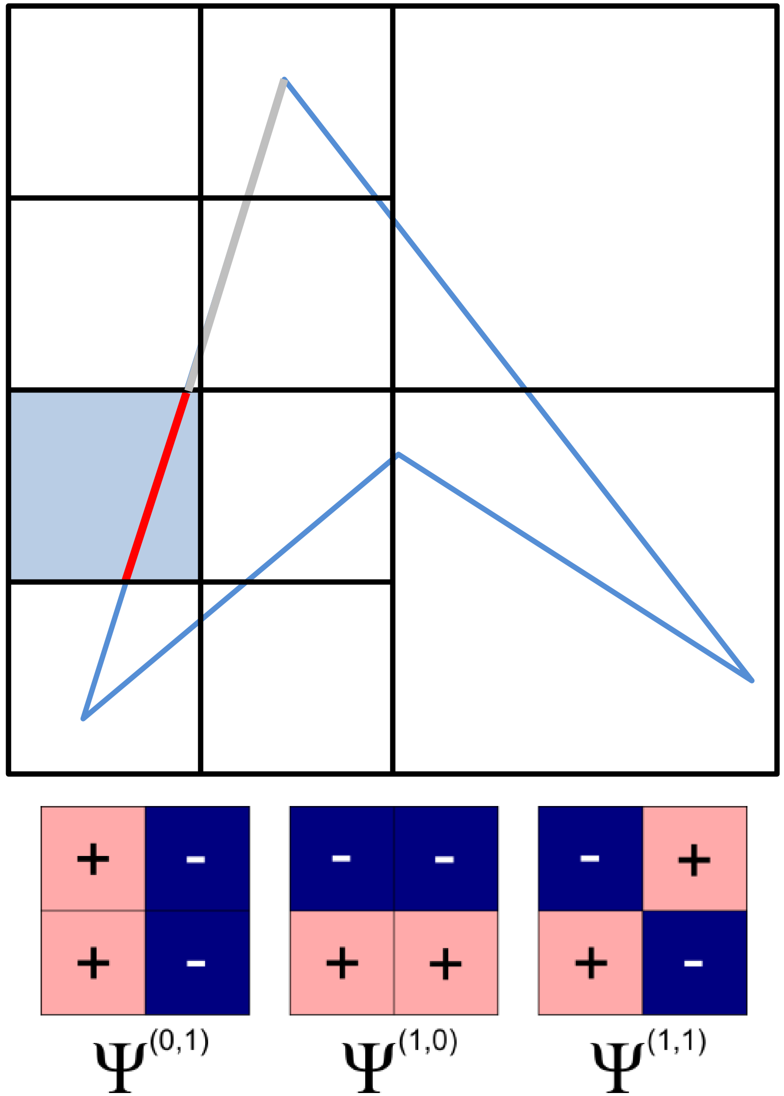

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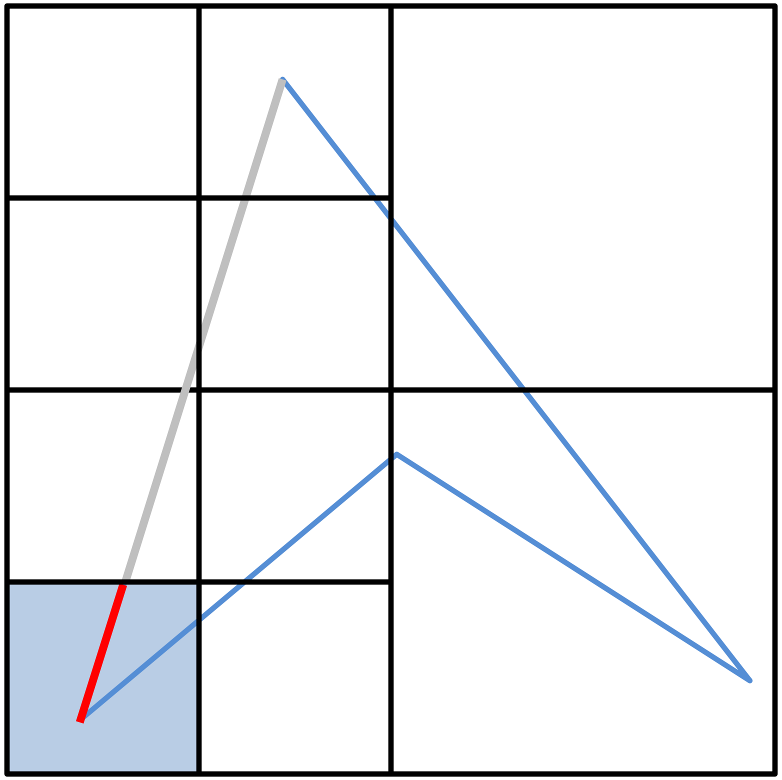
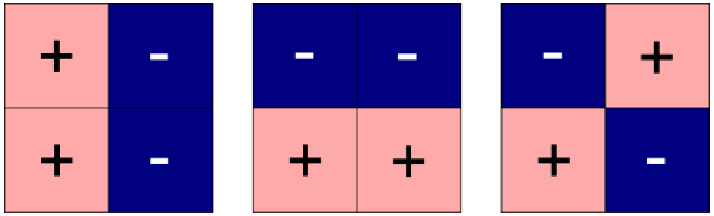
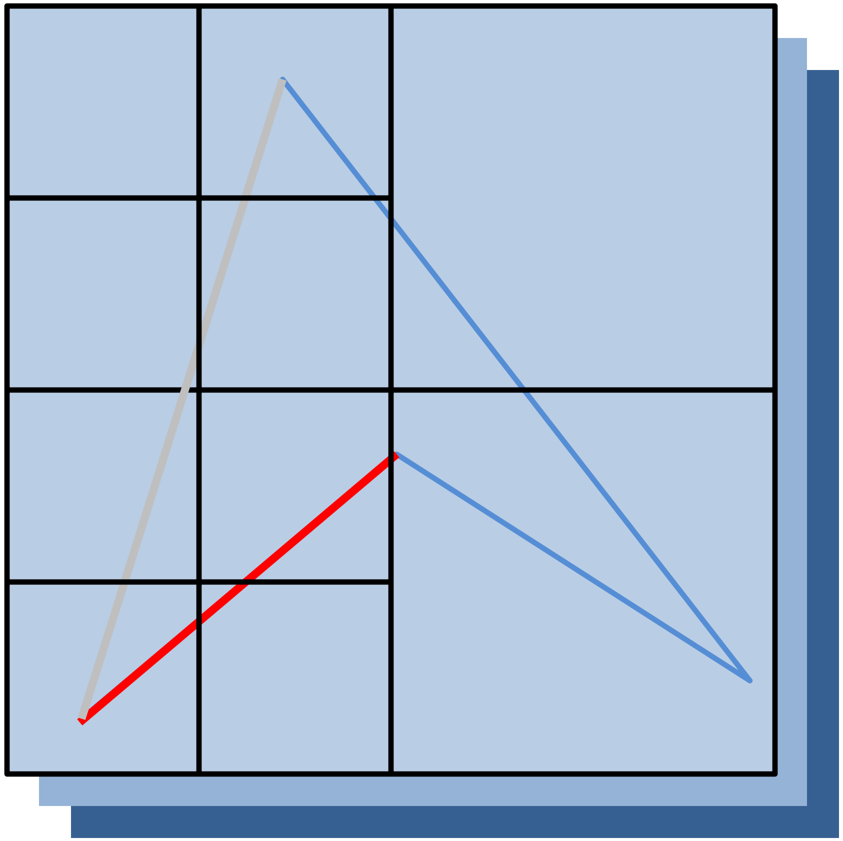




$\Psi^{(0,1)}$ $\Psi^{(1,0)}$ $\Psi^{(1,1)}$

The Algorithm, Step-by-step



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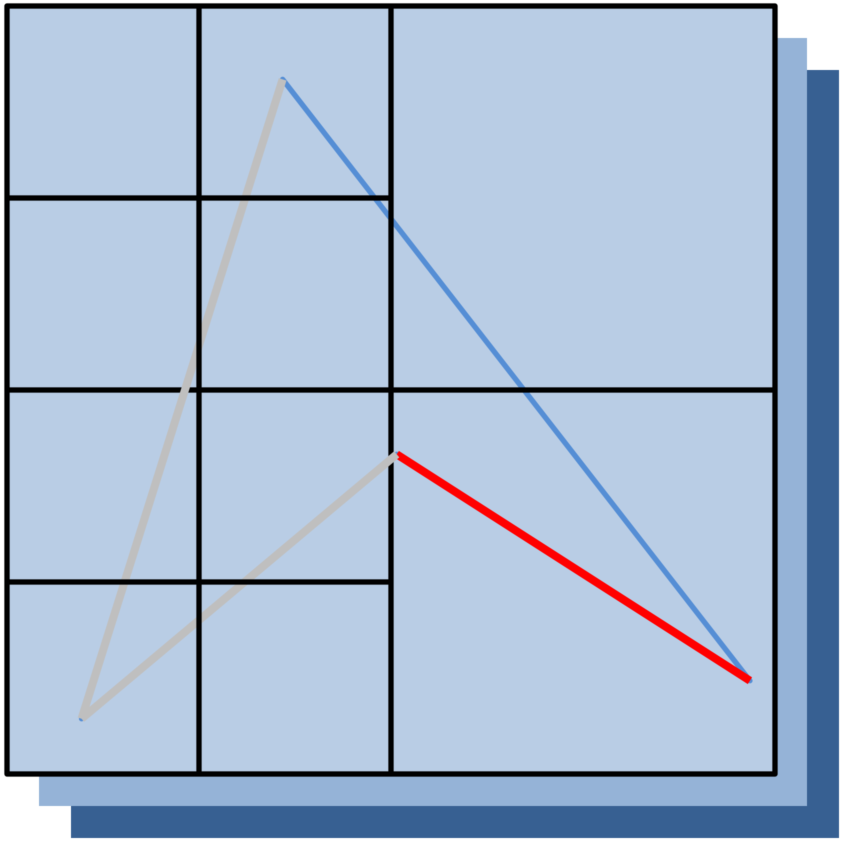

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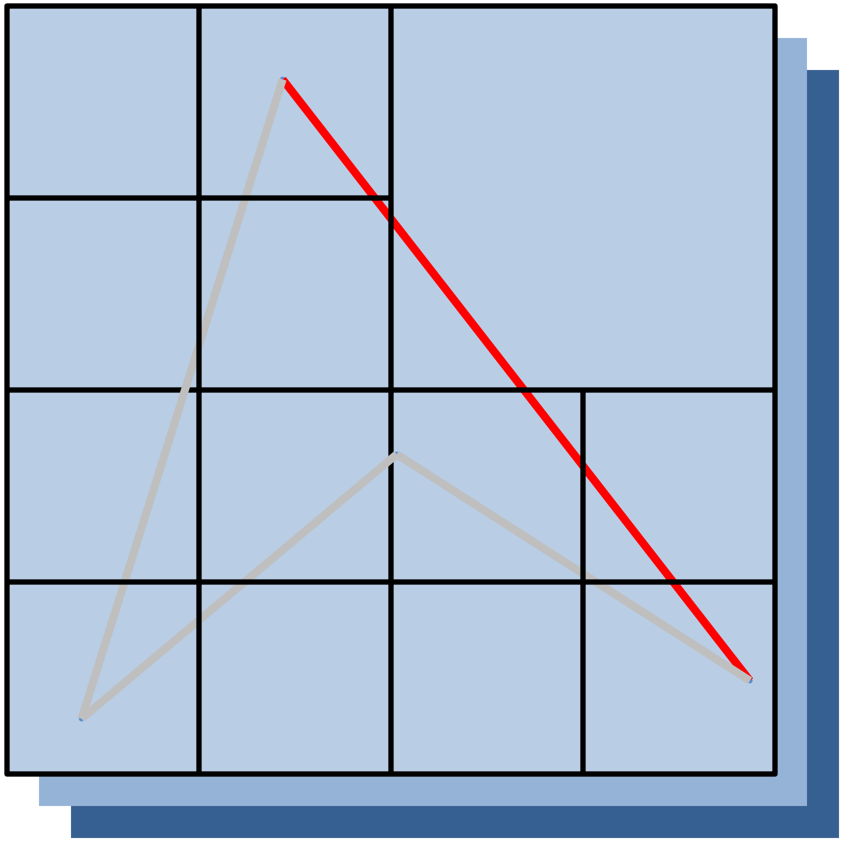

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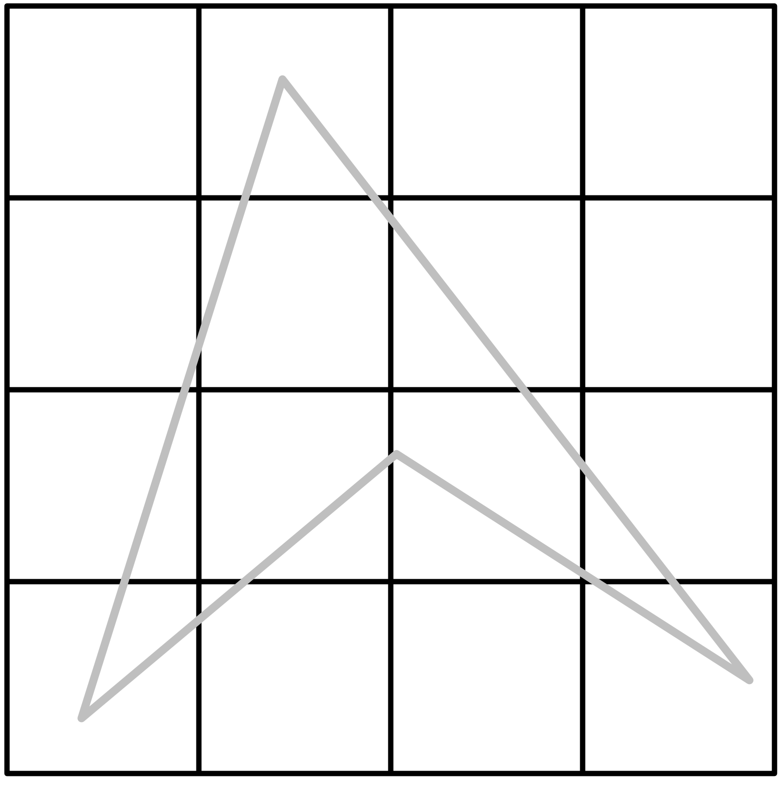

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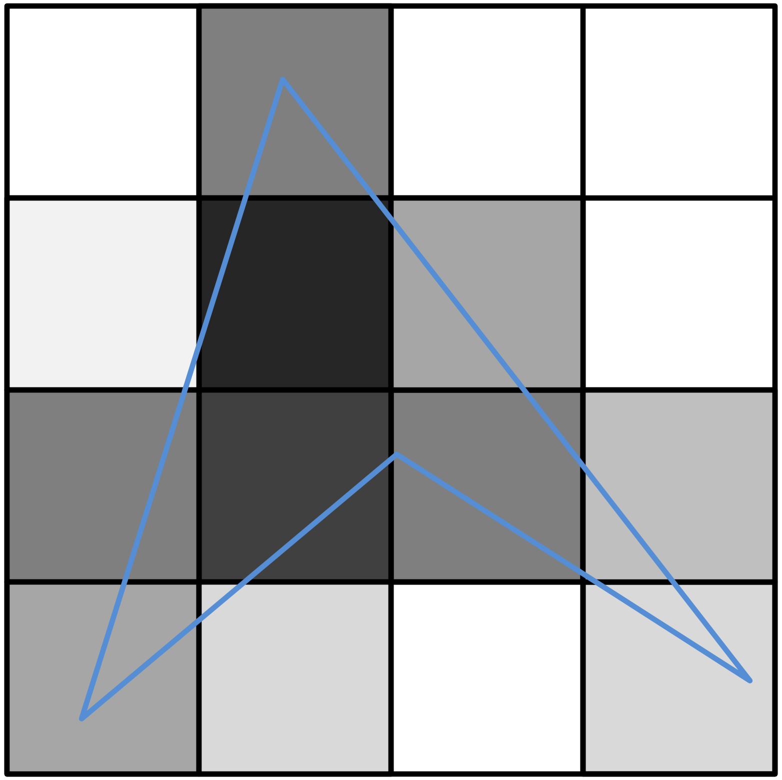
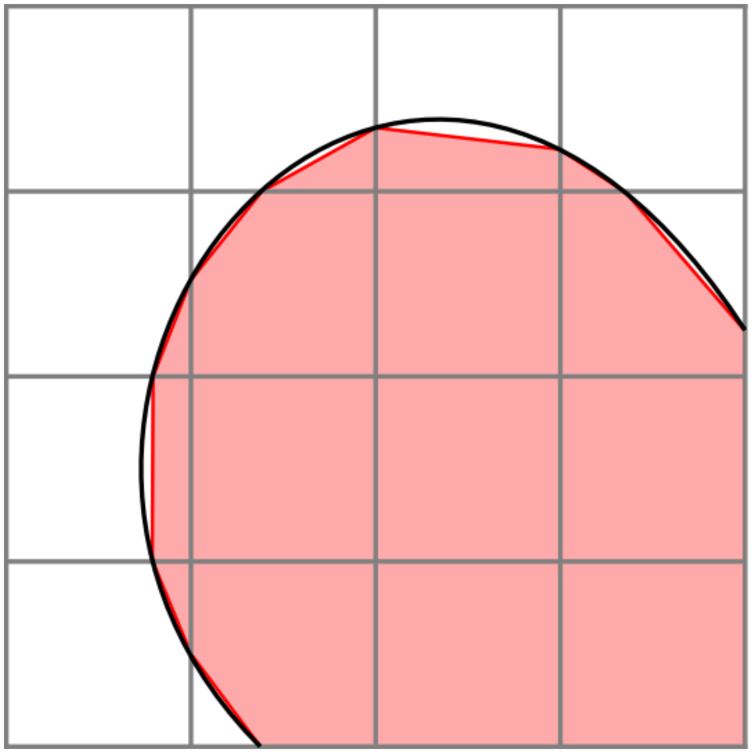
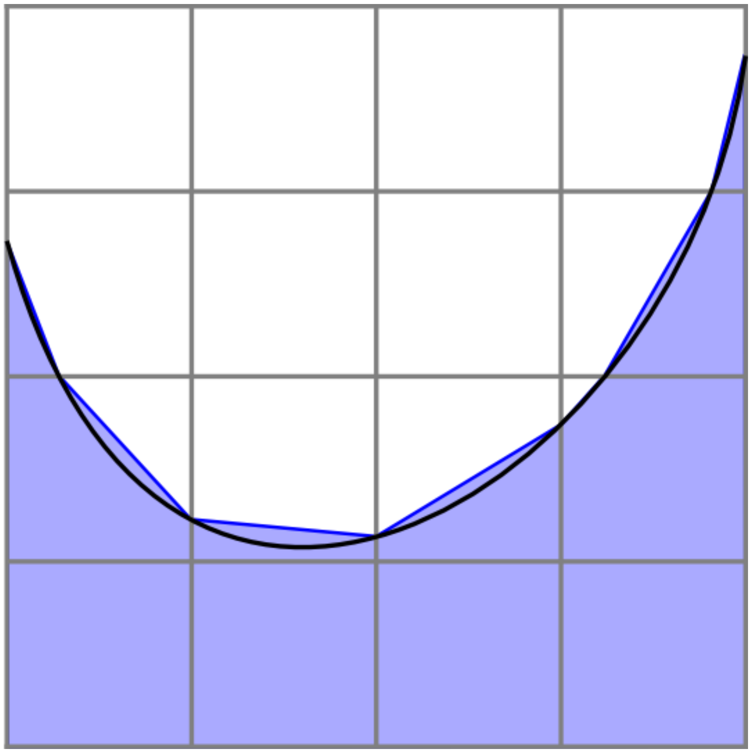


Bézier Curves





Bézier Curves

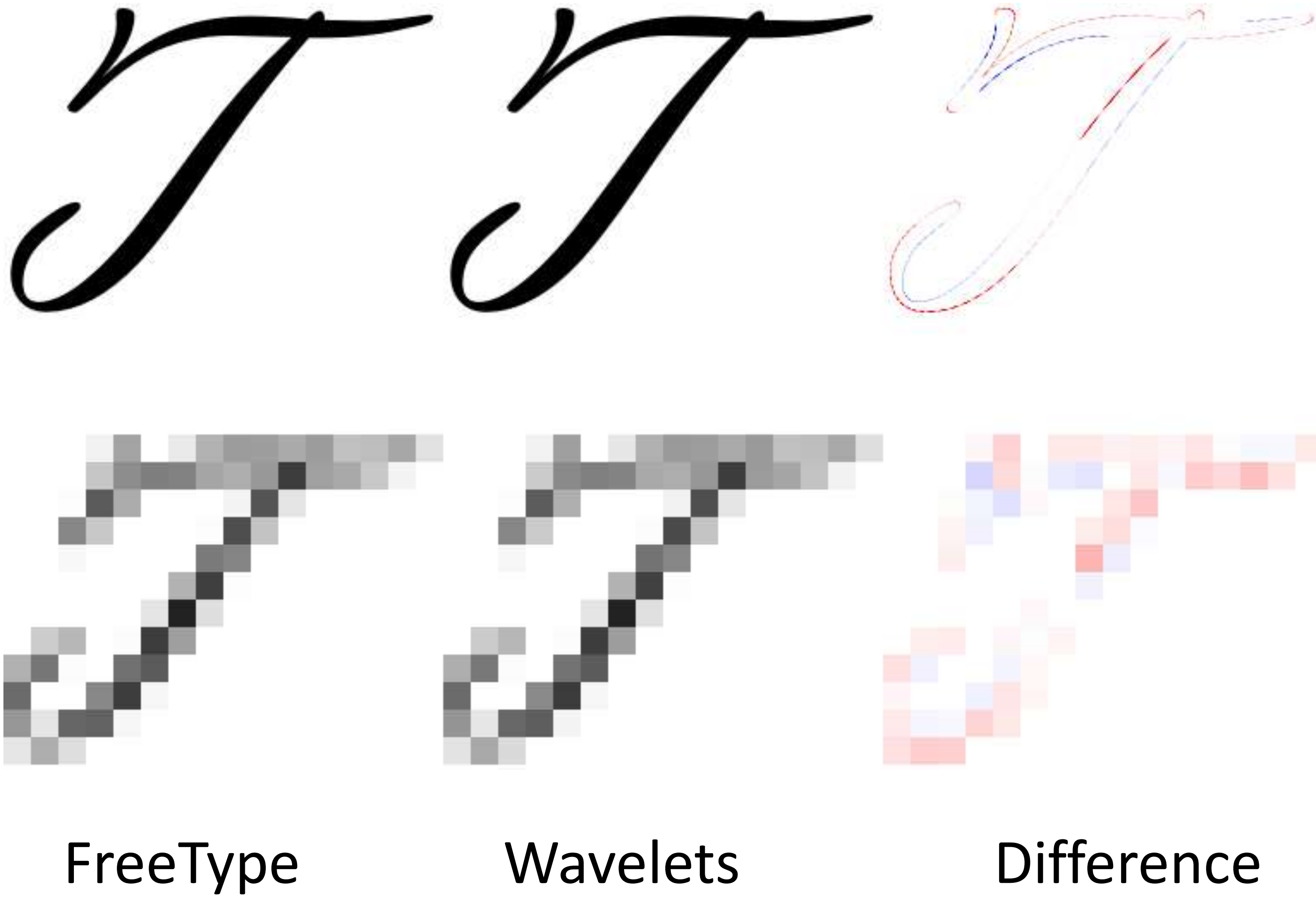
$$c^e = \sum_i \int_0^1 F^e(P_i(t)) \cdot n(P_i(t)) d\sigma$$

Bézier Curves

$$c^e = \sum_i \int_0^1 F^e(P_i(t)) \cdot n(P_i(t))\, d\sigma$$

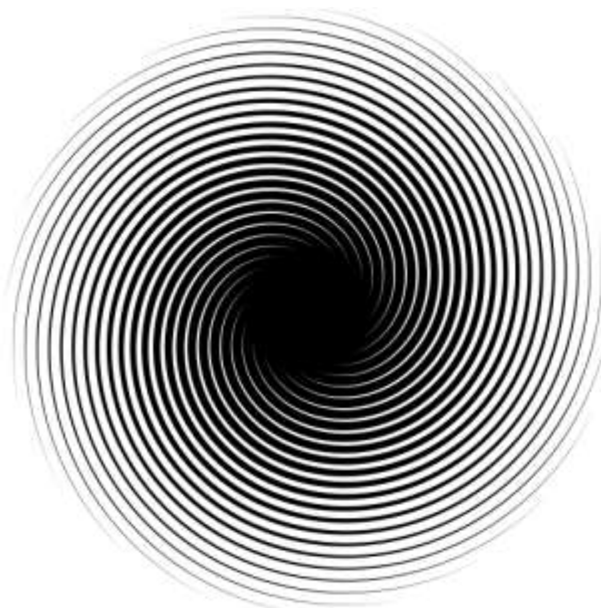
$$d\sigma = ||P'(t)||dt$$
$$n(P(t)) = P^{\perp}(t)/||P'(t)||$$

Results

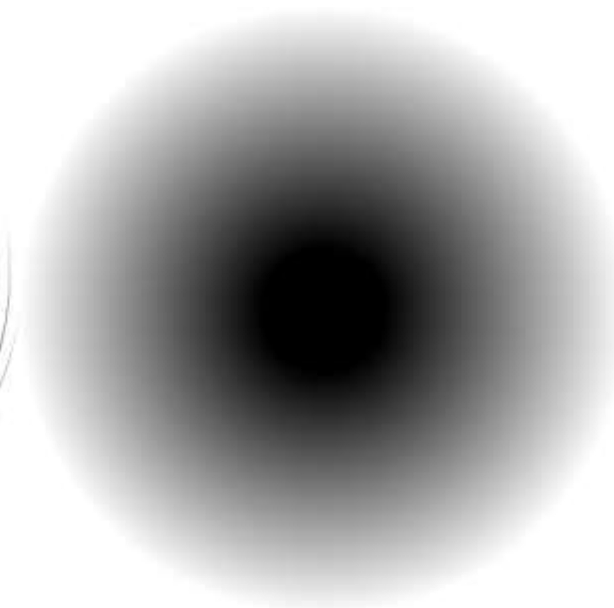


Results

2

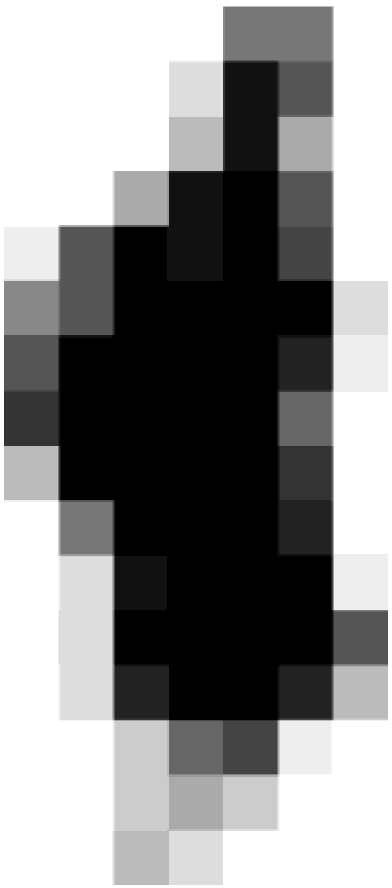


20



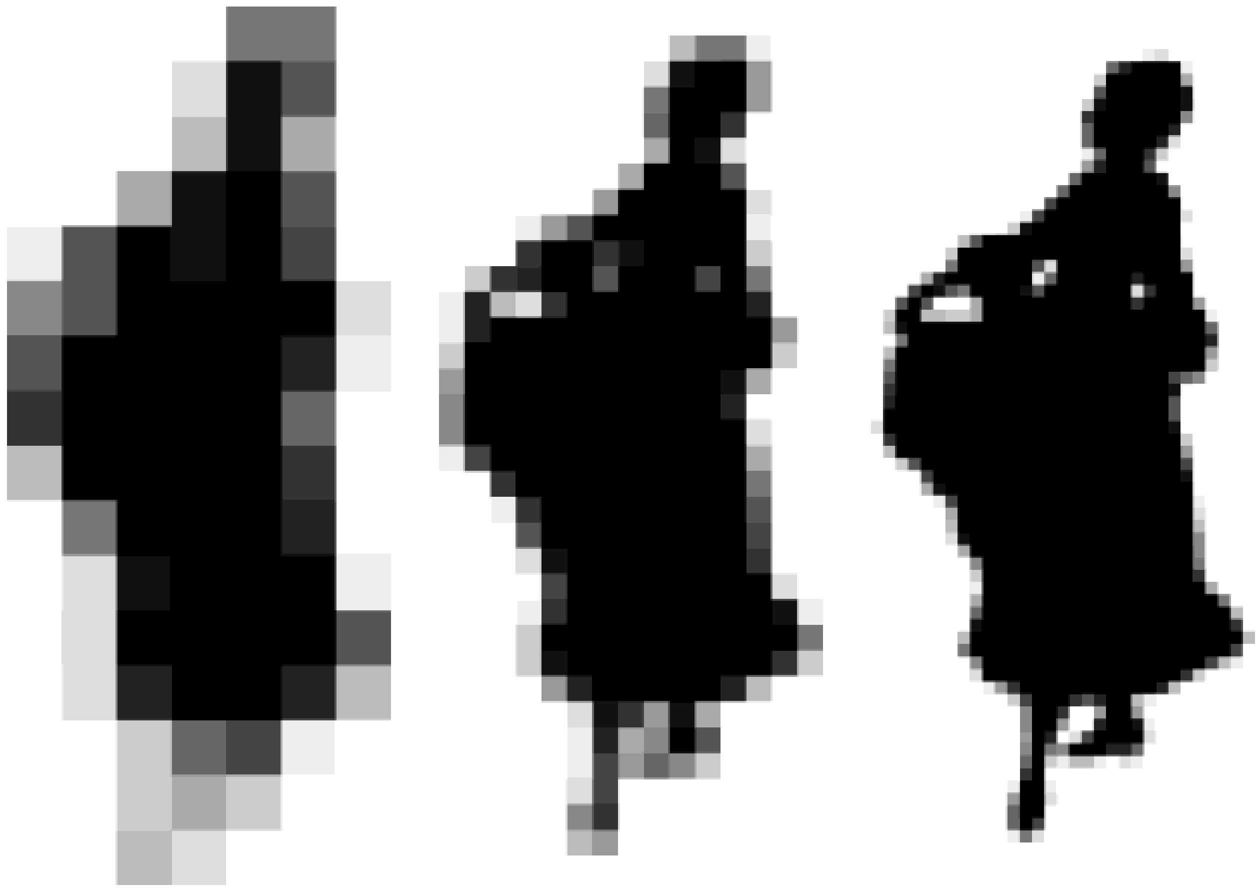
10000

Progressive Rasterization



Progressive Rasterization

Progressive Rasterization



Progressive Rasterization

Results

Speed

	polys	256^3		4096^3	
		coeff	synth	coeff	synth
Armadilloman	30.0k	.113	.022	7.31	3.99
Head	477k	.393	.023	12.0	4.74
Buddha	1.09M	.557	.021	10.7	3.34
David 2mm	7.23M	2.25	.019	14.8	1.79

The Future

- Use higher order wavelets
- Implement on GPU
- Progressive rasterization

Bézier Curves

$$c^e = \sum_i \int_0^1 F^e(P_i(t)) \cdot n(P_i(t)) d\sigma$$

$$= \sum_i \int_0^1 F^e(P_i(t)) \cdot P_i^{\perp}(t)\, dt$$

Bézier Curves

$$c^e = \sum_i \int_0^1 F^e(P_i(t)) \cdot n(P_i(t)) d\sigma$$

$$= \sum_i \int_0^1 F^e(P_i(t)) \cdot P_i^{\perp}(t)\, dt$$

$$c^{(0,0)} = \frac{1}{3} det(v_0, v_1) + \frac{1}{3} det(v_1, v_2) + \frac{1}{6} det(v_0, v_2)$$

Bézier Curves

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Details in paper

3D Formulation

$$
\begin{aligned}
F^{(0,0,0)}(p) &= \tfrac{1}{3}(\bar{\Phi}(p_x), \bar{\Phi}(p_y), \bar{\Phi}(p_z)) \\
F^{(1,0,0)}(p) &= (\bar{\Psi}(p_x), 0, 0) \\
F^{(0,1,0)}(p) &= (0, \bar{\Psi}(p_y), 0) \\
F^{(0,0,1)}(p) &= (0, 0, \bar{\Psi}(p_z)) \\
F^{(1,1,0)}(p) &= (\bar{\Psi}(p_x)\psi(p_y), 0, 0) \\
F^{(1,0,1)}(p) &= (\psi(p_x)\bar{\Psi}(p_z), 0, 0) \\
F^{(0,1,1)}(p) &= (0, \bar{\Psi}(p_y)\psi(p_z), 0) \\
F^{(1,1,1)}(p) &= (\bar{\Psi}(p_x)\psi(p_y)\psi(p_z), 0, 0)
\end{aligned}
$$

3D Formulation

$$F^{(0,0,0)}(p) \quad = \quad \tfrac{1}{3}(\bar{\Phi}(p_x), \bar{\Phi}(p_y), \bar{\Phi}(p_z))$$

$$c^{(0,0,0)} = \int_{p \in T} F^{(0,0,0)}(p) \cdot n d\boldsymbol{\sigma} = \frac{1}{6} det\left(v_0, v_1, v_2\right)$$