



Parameterizing Subdivision Surfaces

Lei He

Texas A&M University

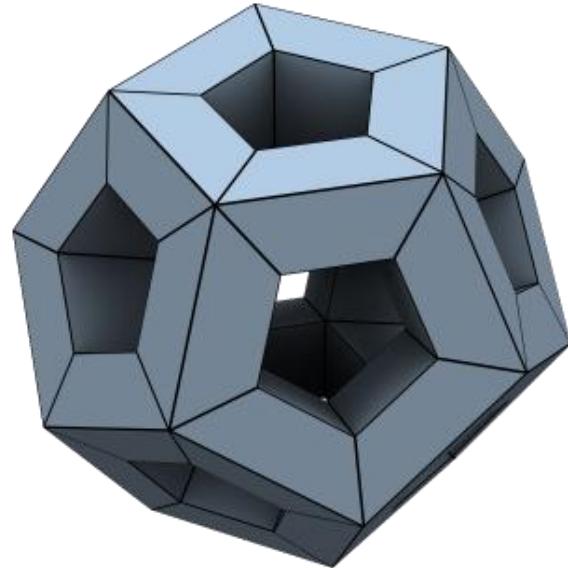
Scott Schaefer

Texas A&M University

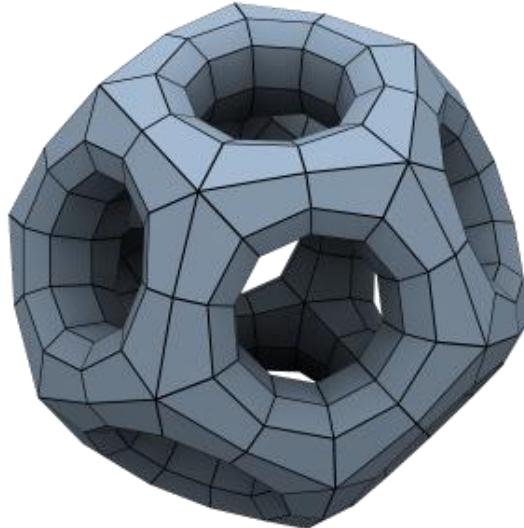
Kai Hormann

University of Lugano

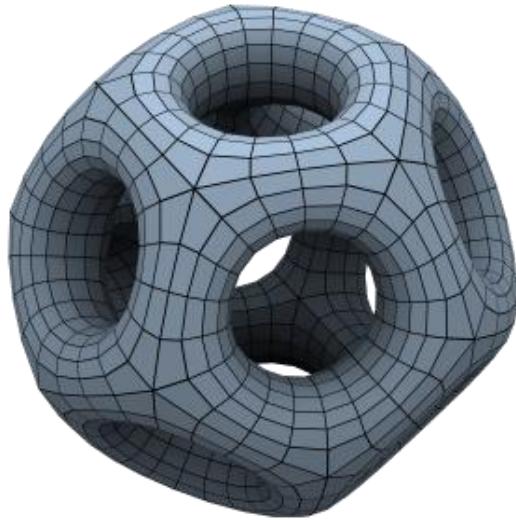
Subdivision Surface



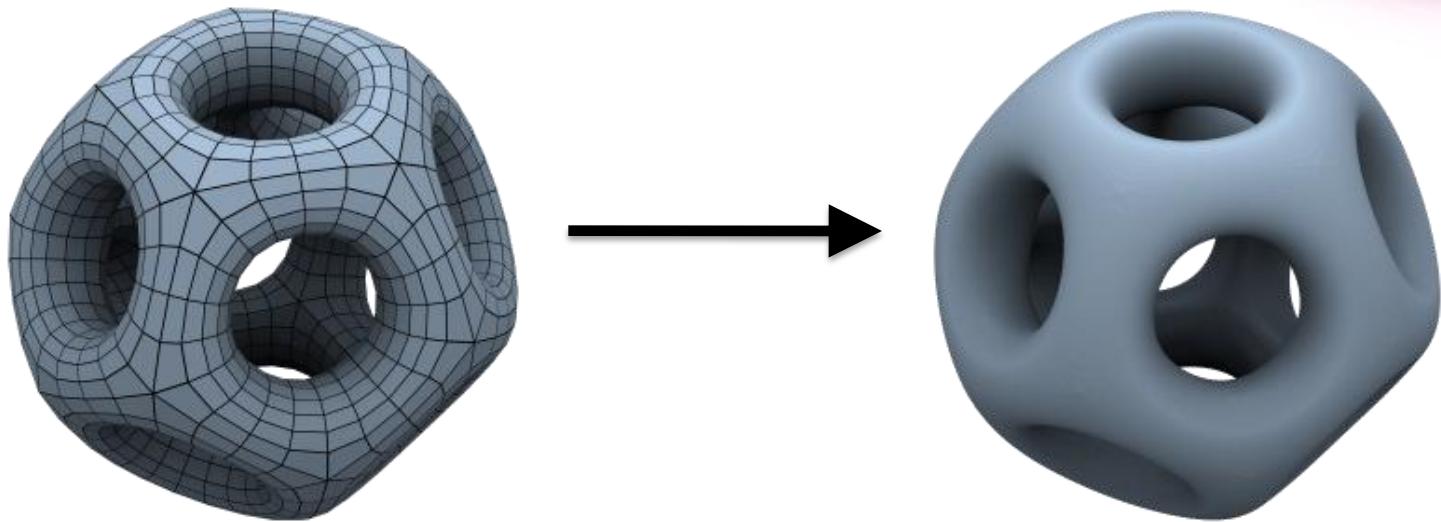
Subdivision Surface



Subdivision Surface



Subdivision Surface



Motivation



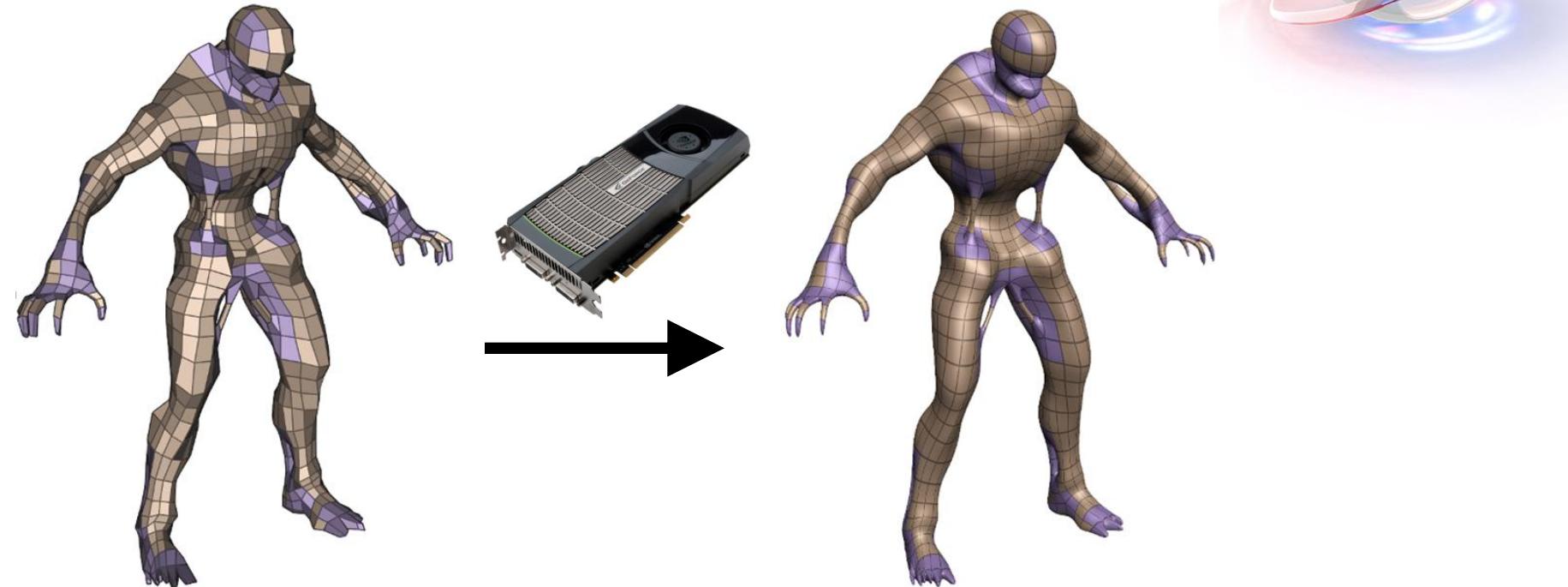
Toy Story © Disney / Pixar



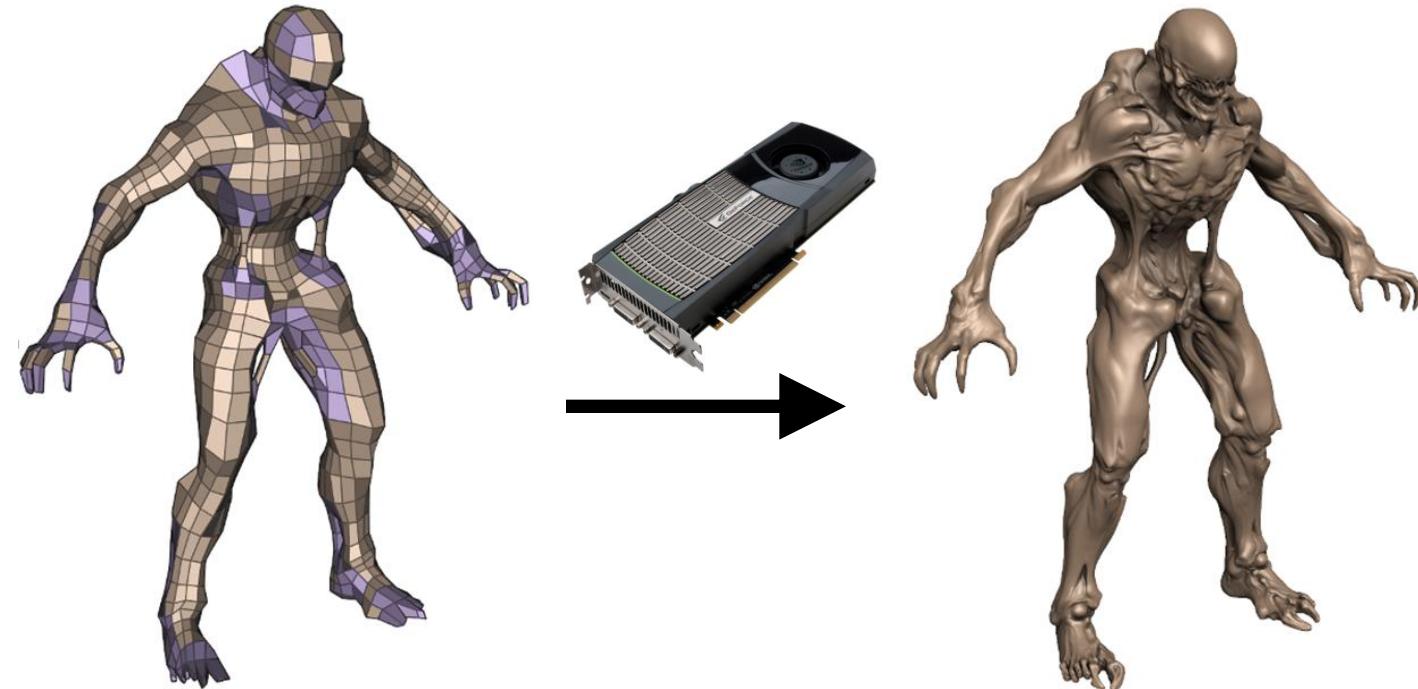
Geri's Game © Pixar Animation Studios



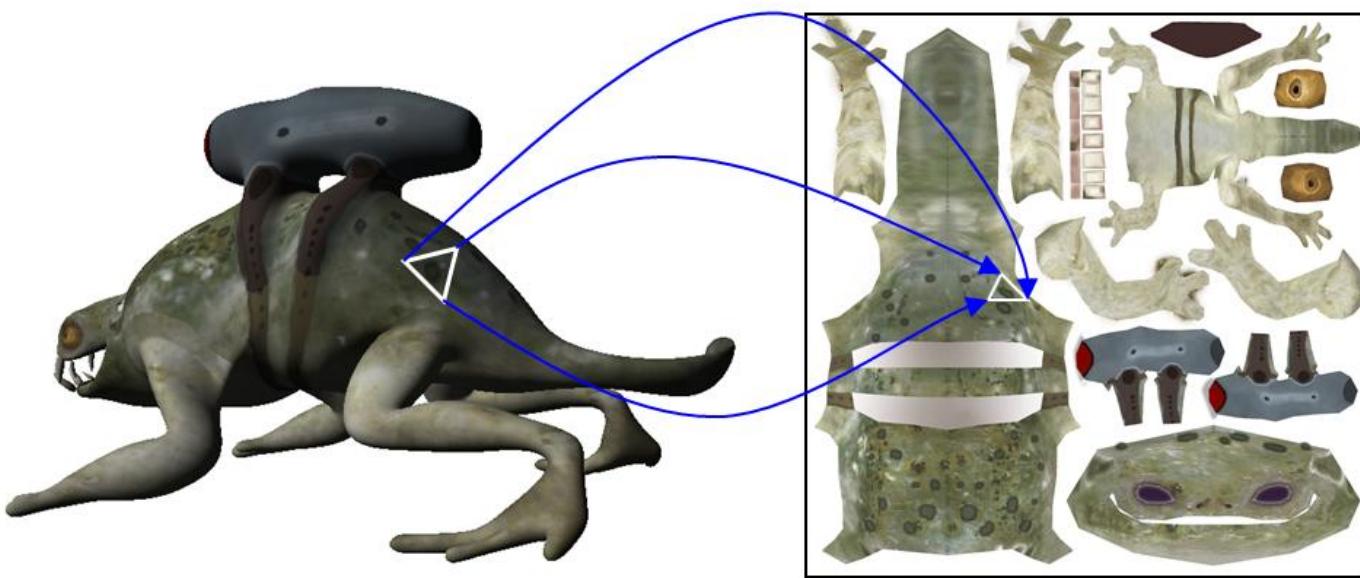
Motivation



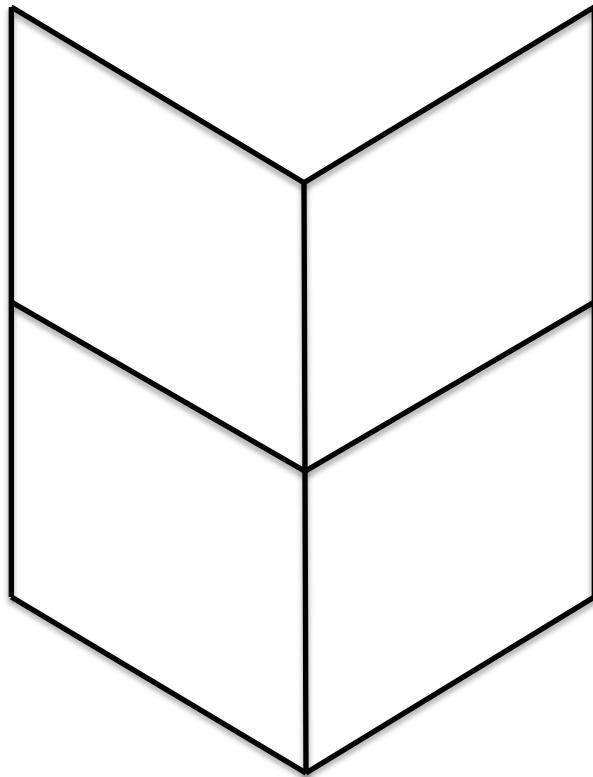
Motivation



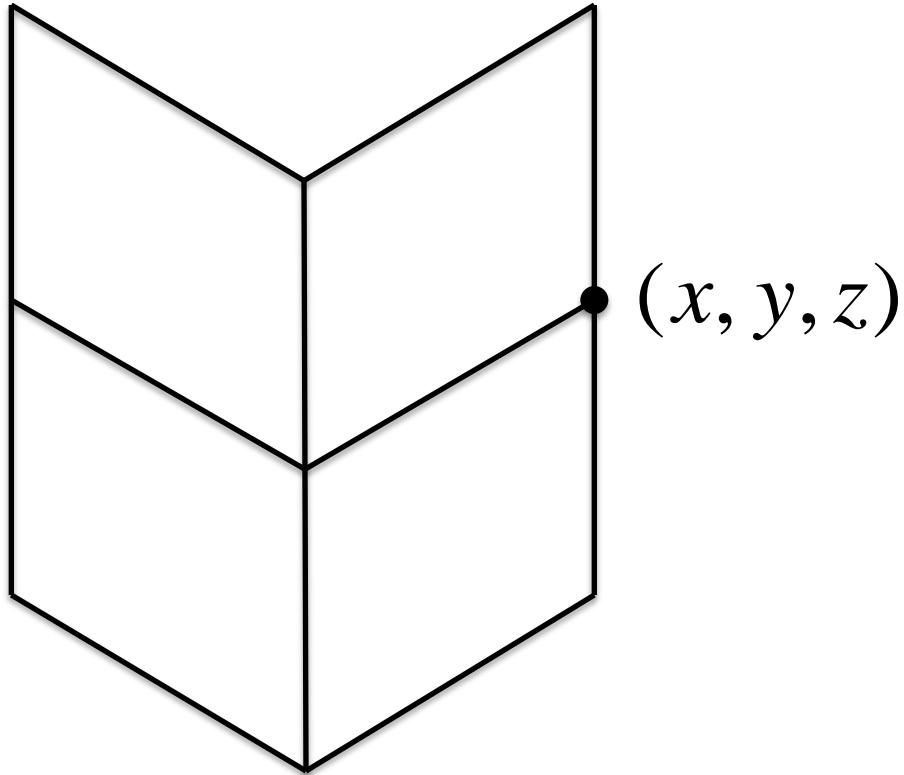
Parameterization



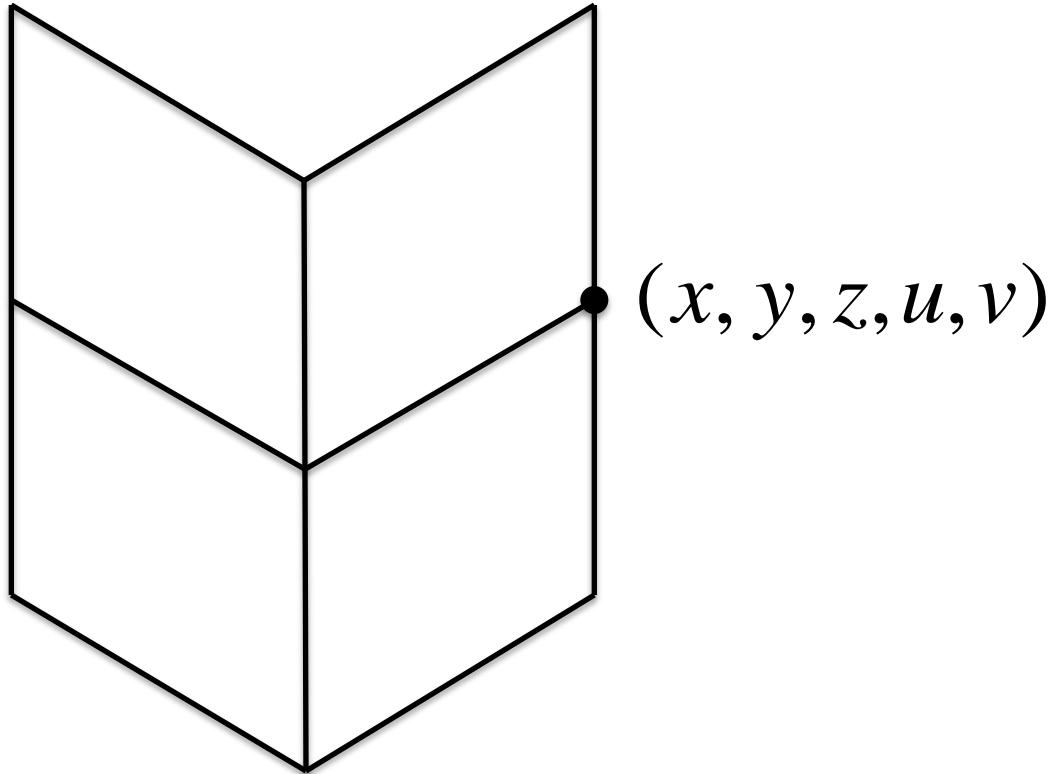
Texturing Subdivision Surfaces



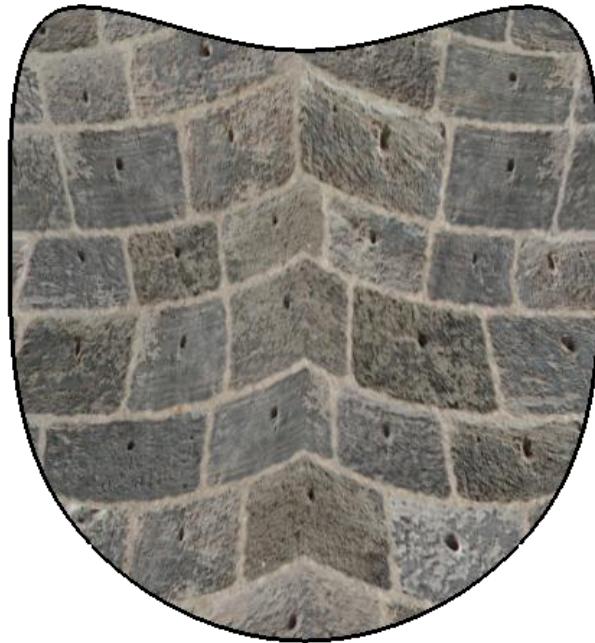
Texturing Subdivision Surfaces



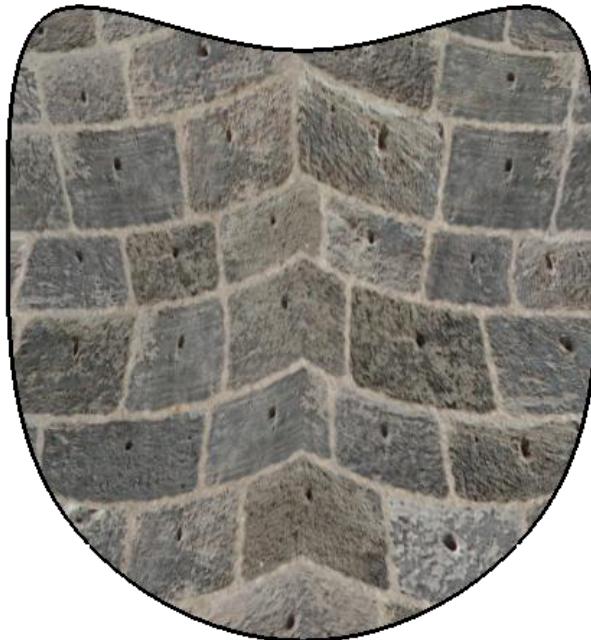
Texturing Subdivision Surfaces



Texturing Subdivision Surfaces



Texturing Subdivision Surfaces



[Derose et al. 98]

Previous Work

- Angles
 - [Sheffer et al. 2005] [Hormann et al. 2000]
[Lévy et al. 2002] [Desbrun et al. 2002]
[Kharevich et al. 2006] [Kälberer et al. 2007]
[Ben-Chen et al. 2008] [Springborn et al. 2008]
- Stretch
 - [Sander et al. 2001] [Sorkine et al. 2002]
- Balance
 - [Degener et al. 2003] [Tarini et al. 2004] [Liu et al. 2008]
[Dominitz et al. 2010] [Pietroni et al. 2010]

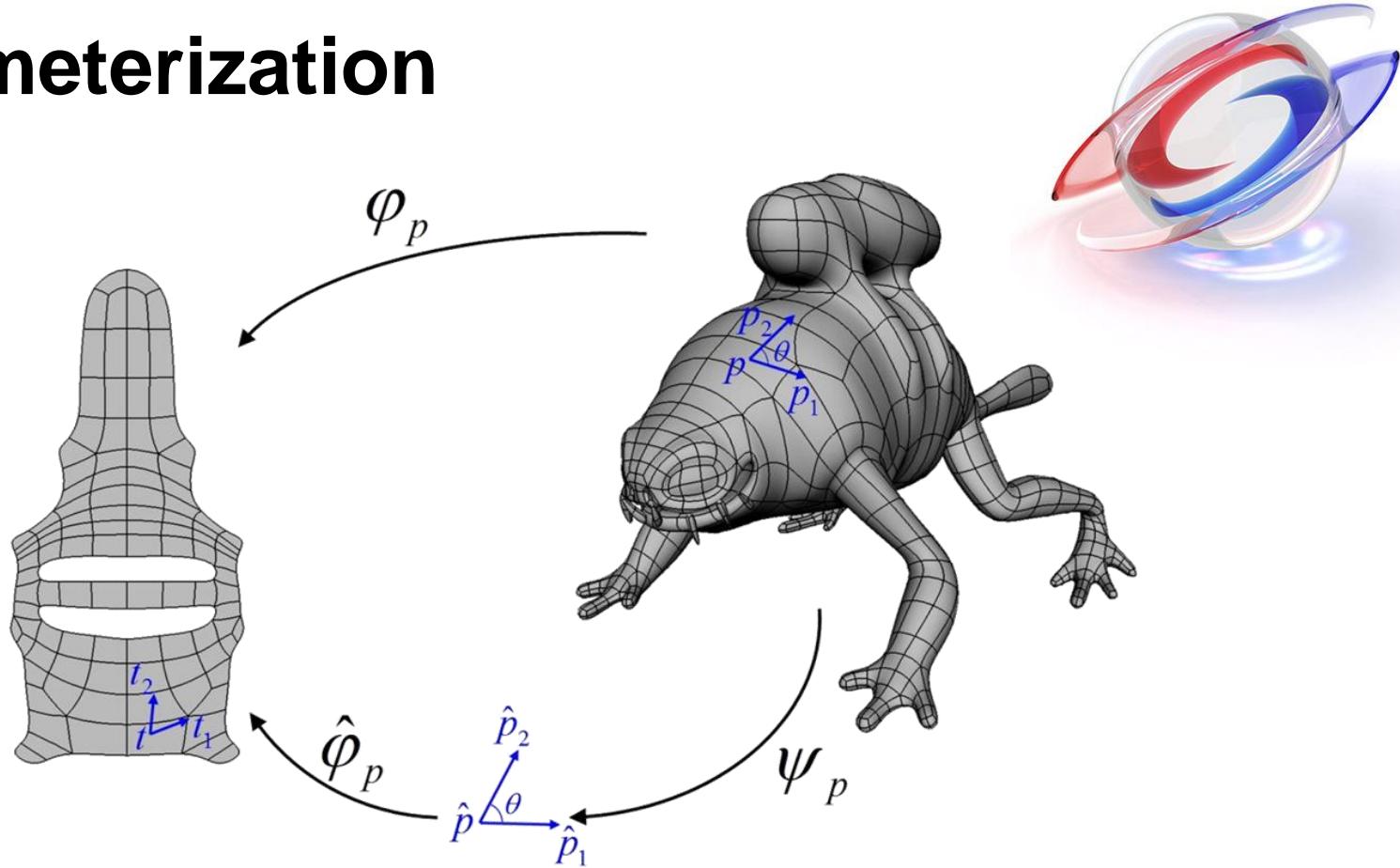


Previous Work

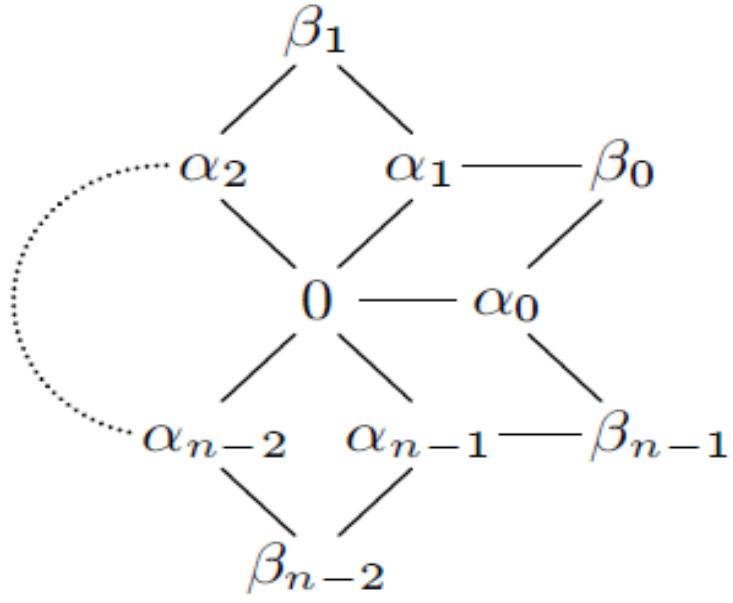
- Angles
 - [Sheffer et al. 2005] [Hormann et al. 2000]
[Lévy et al. 2002] [Desbrun et al. 2002]
[Kharevich et al. 2006] [Kälberer et al. 2007]
[Ben-Chen et al. 2008] [Springborn et al. 2008]
- Stretch
 - [Sander et al. 2001] [Sorkine et al. 2002]
- Balance
 - [Degener et al. 2003] [Tarini et al. 2004] **[Liu et al. 2008]**
[Dominitz et al. 2010] [Pietroni et al. 2010]



Parameterization

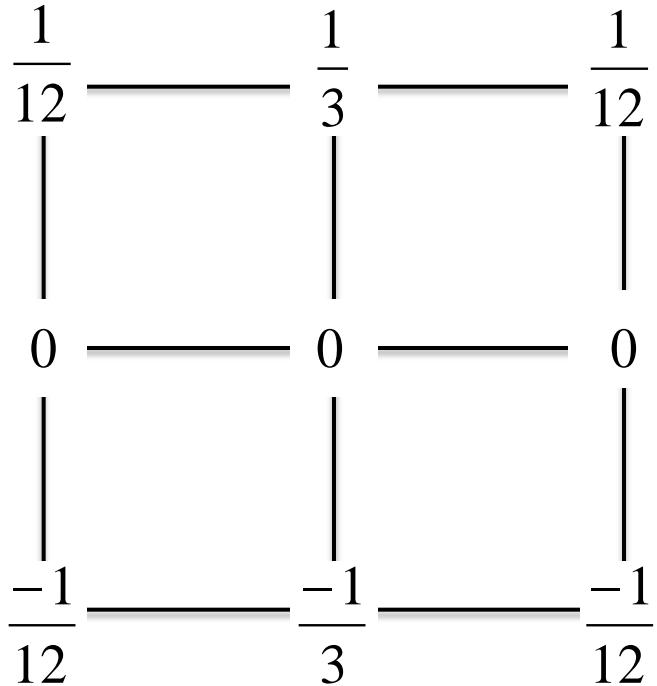


Tangent Vectors



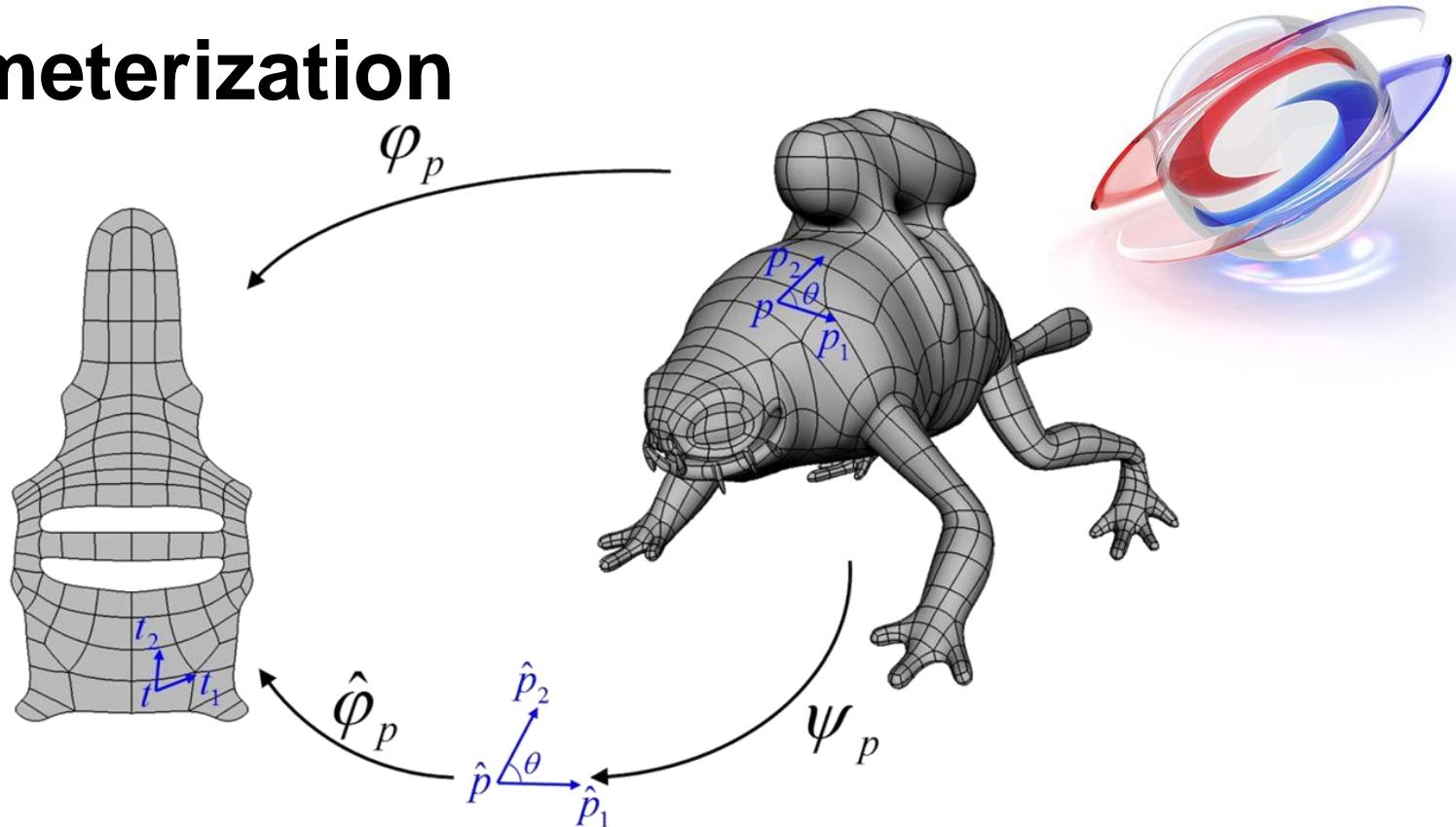
$$\alpha_i = \left(\frac{1}{n} + \frac{\cos\left(\frac{\pi}{n}\right)}{n\sqrt{4 + \cos^2\left(\frac{\pi}{n}\right)}} \right) \cos\left(\frac{2\pi i}{n}\right)$$
$$\beta_i = \left(\frac{1}{n\sqrt{4 + \cos^2\left(\frac{\pi}{n}\right)}} \right) \cos\left(\frac{2\pi i + \pi}{n}\right)$$

Tangent Vectors



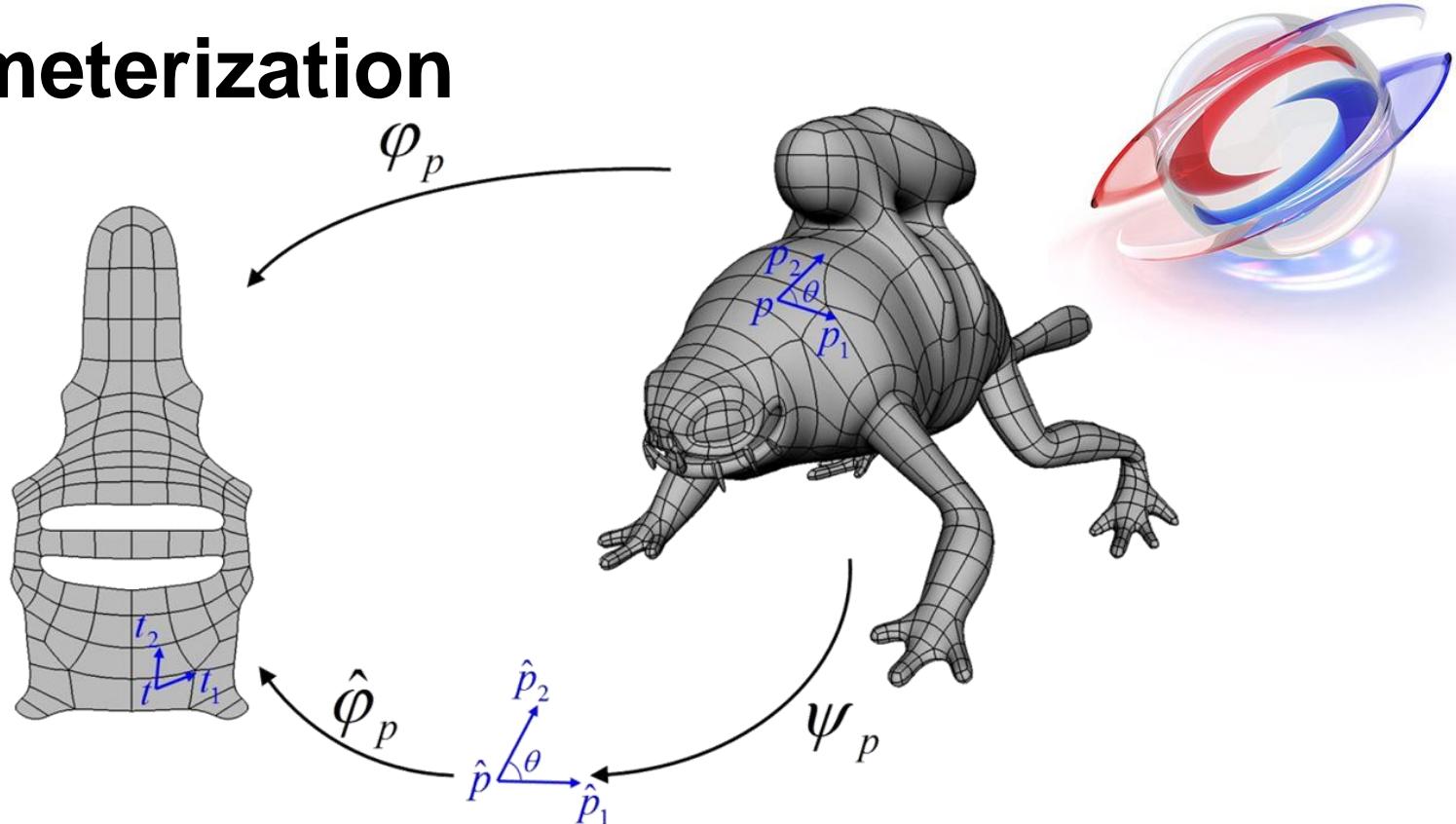
$$\alpha_i = \left(\frac{1}{n} + \frac{\cos\left(\frac{\pi}{n}\right)}{n\sqrt{4 + \cos^2\left(\frac{\pi}{n}\right)}} \right) \cos\left(\frac{2\pi i}{n}\right)$$
$$\beta_i = \left(\frac{1}{n\sqrt{4 + \cos^2\left(\frac{\pi}{n}\right)}} \right) \cos\left(\frac{2\pi i + \pi}{n}\right)$$

Parameterization



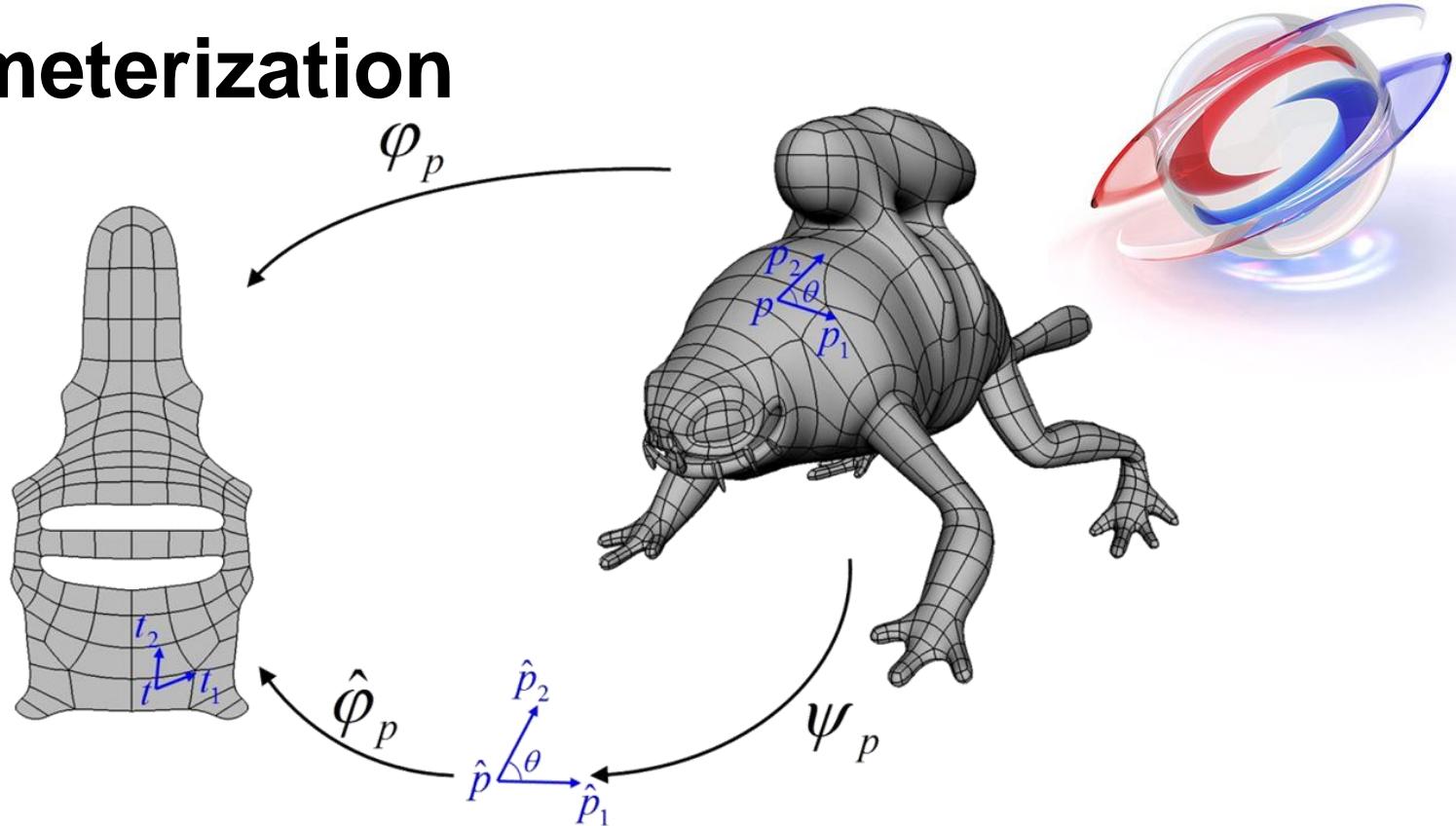
$$\min \int_{P^\infty} \left\| \nabla \hat{\phi}_p - R_p \right\|_F^2 dp$$

Parameterization



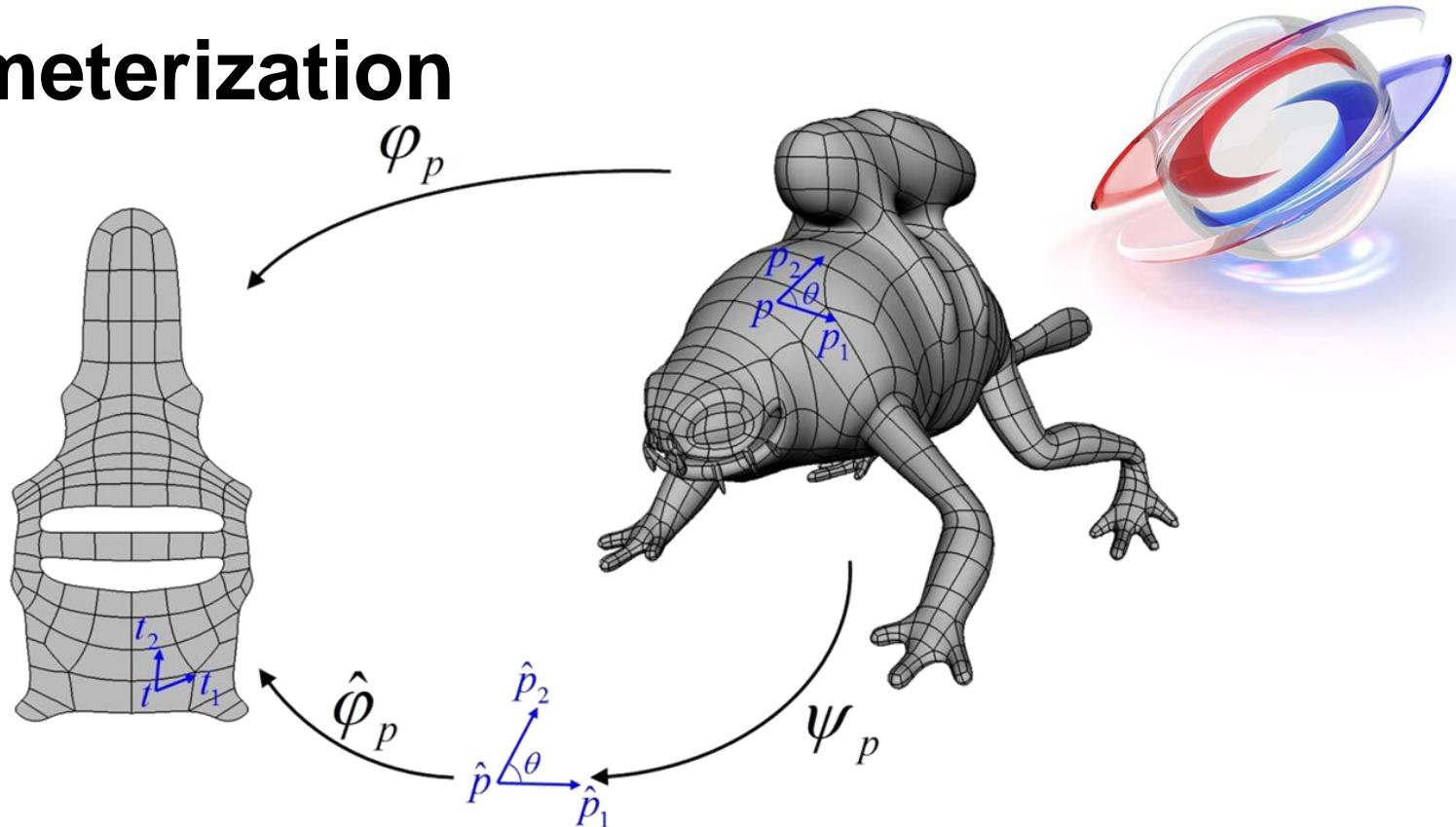
$$\min_{T^0} \sum_{p_j^k \in P^k} \left\| (t_{j,1}^k, t_{j,2}^k) (\hat{p}_{j,1}^k, \hat{p}_{j,2}^k)^{-1} - R_j^k \right\|_F^2 \|p_{j,1}^k \times p_{j,2}^k\|$$

Parameterization



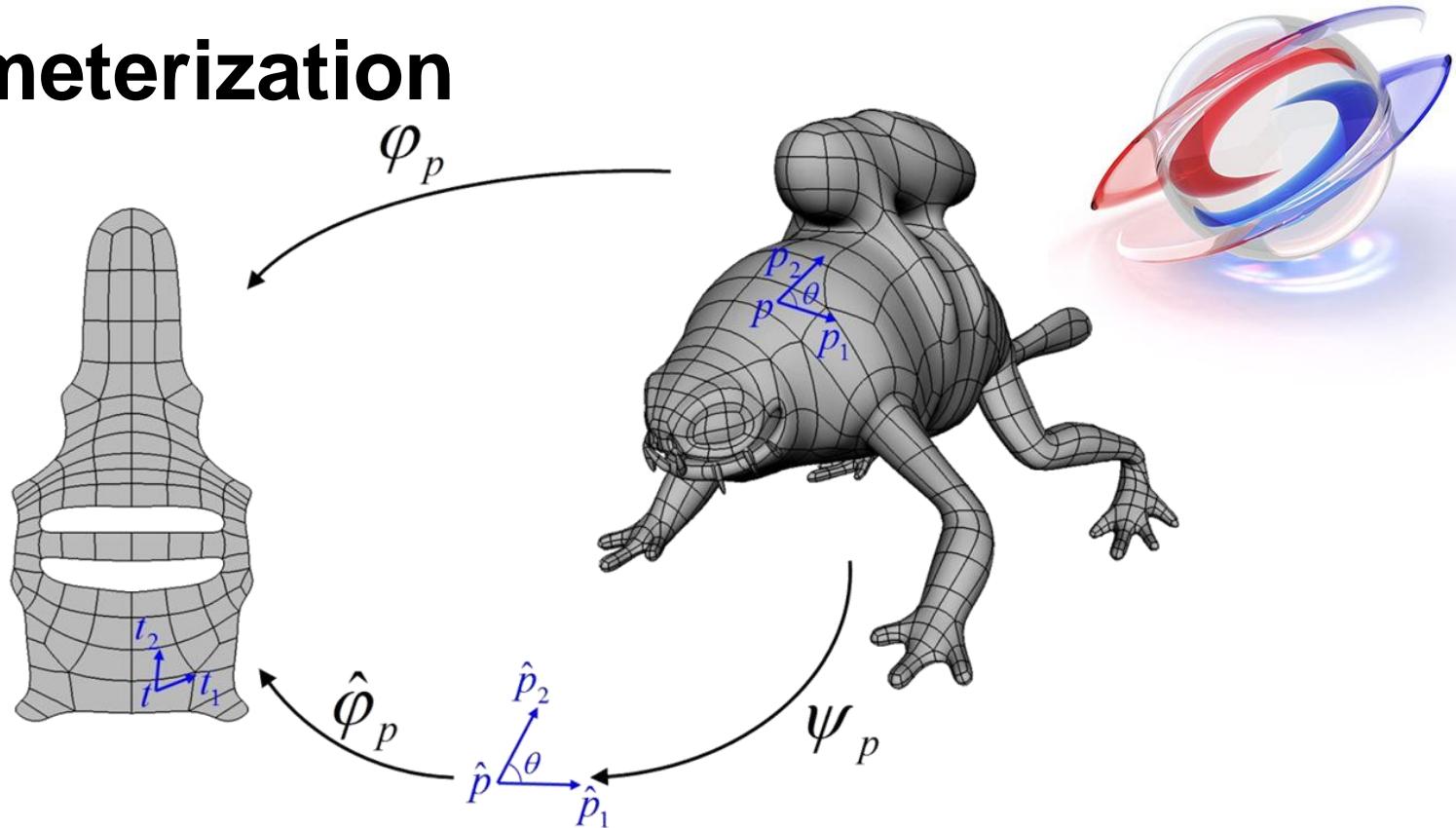
$$\min_{T^0} \sum_{p_j^k \in P^k} \left\| (t_{j,1}^k, t_{j,2}^k) (\hat{p}_{j,1}^k, \hat{p}_{j,2}^k)^{-1} - R_j^k \right\|_F^2 \left\| p_{j,1}^k \times p_{j,2}^k \right\|$$

Parameterization



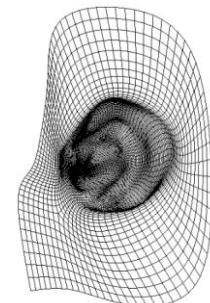
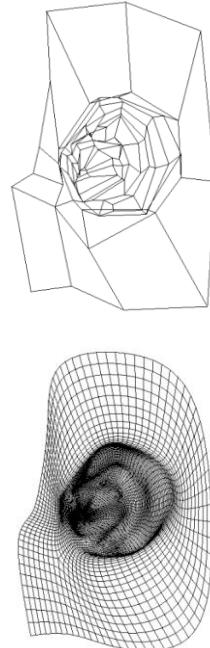
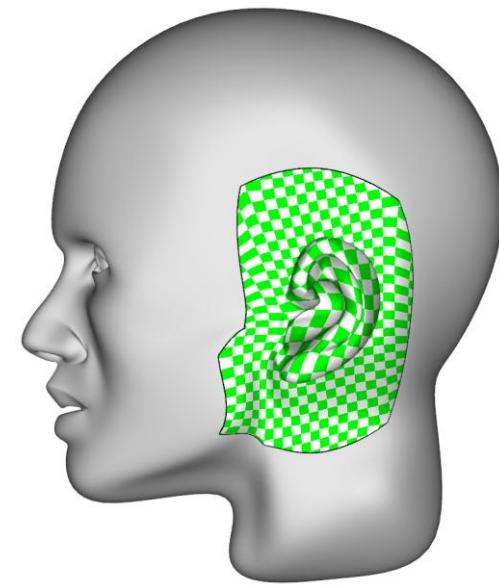
$$\min_{T^0} \sum_{P_j^k \in P^k} \left\| (t_{j,1}^k, t_{j,2}^k) (\hat{p}_{j,1}^k, \hat{p}_{j,2}^k)^{-1} - \boxed{R_j^k} \right\|_F^2 \|p_{j,1}^k \times p_{j,2}^k\|$$

Parameterization



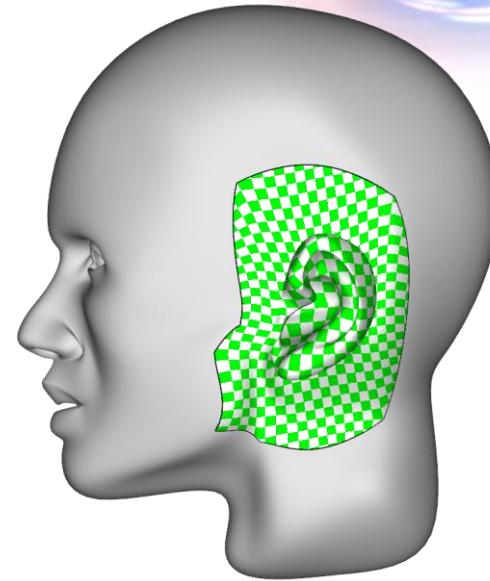
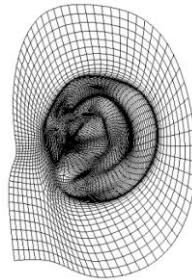
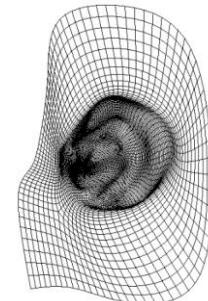
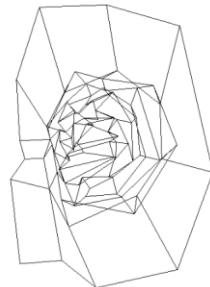
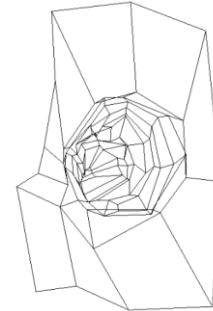
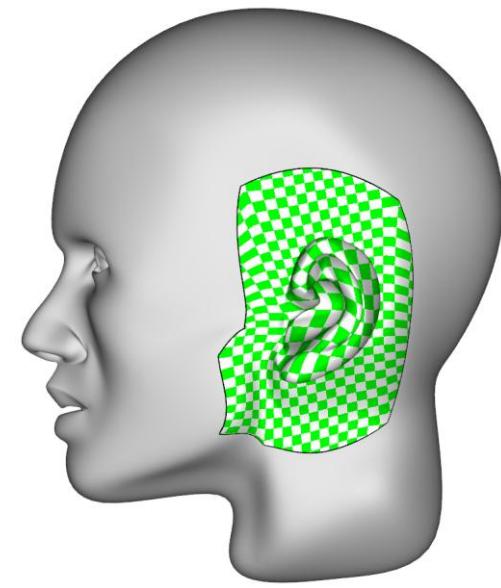
$$\min_{T^0} \sum_{p_j^k \in P^k} \left\| (t_{j,1}^k, t_{j,2}^k) (\hat{p}_{j,1}^k, \hat{p}_{j,2}^k)^{-1} - R_j^k \right\|_F^2 \| p_{j,1}^k \times p_{j,2}^k \|$$

Comparison



$$E_{angle} = 2.625 \quad E_{area} = 2.616 \quad E_{stretch} = 5.120$$

Comparison

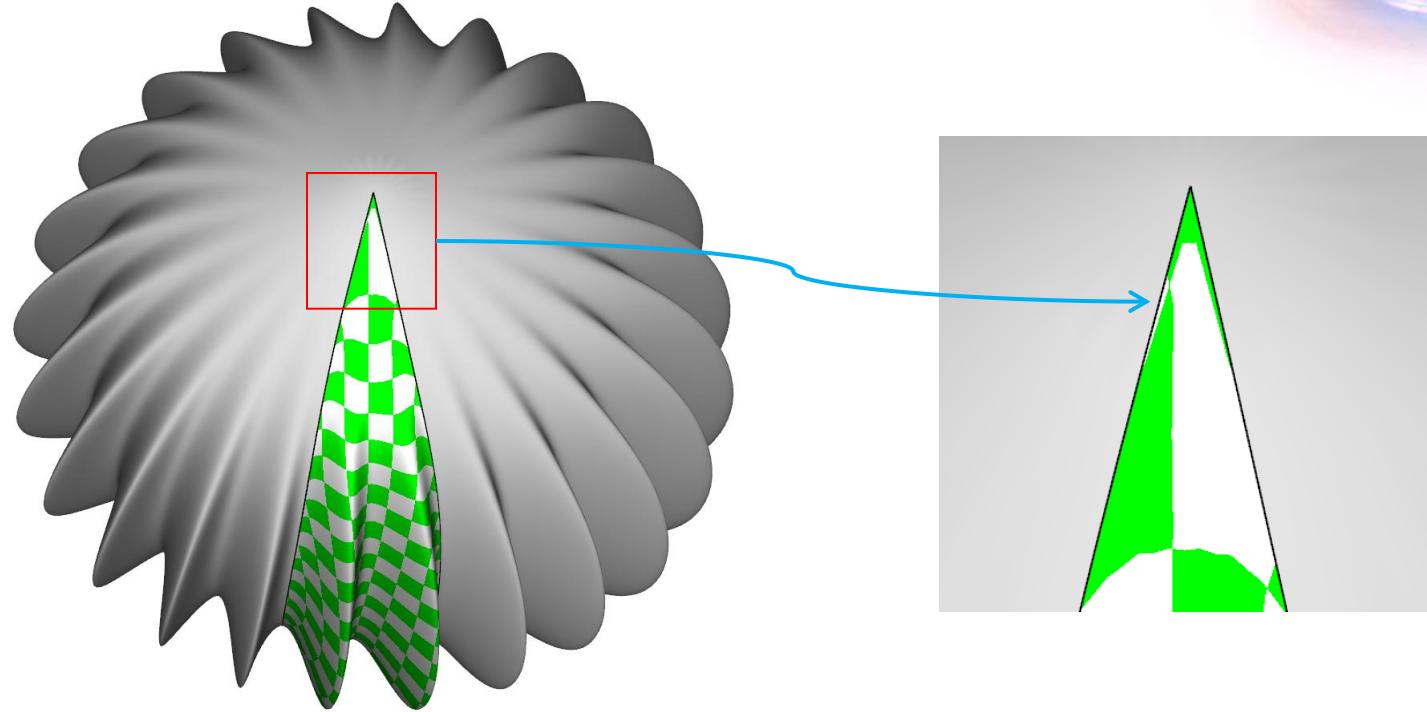


$$E_{angle} = 2.625 \quad E_{area} = 2.616 \quad E_{stretch} = 5.120$$

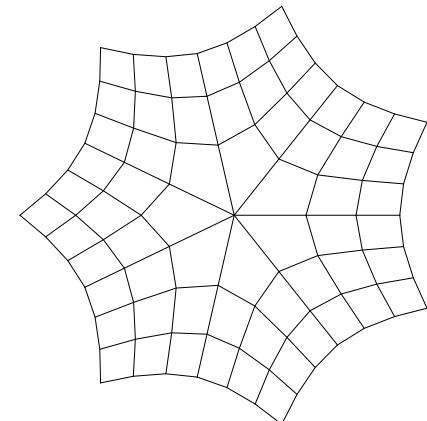
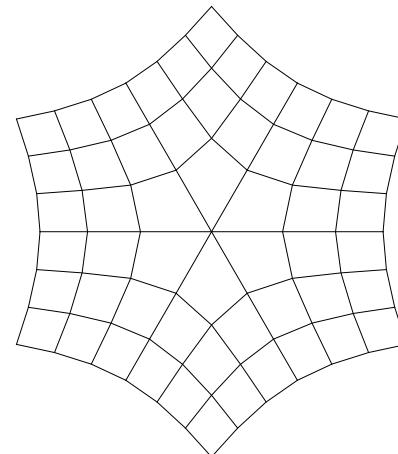
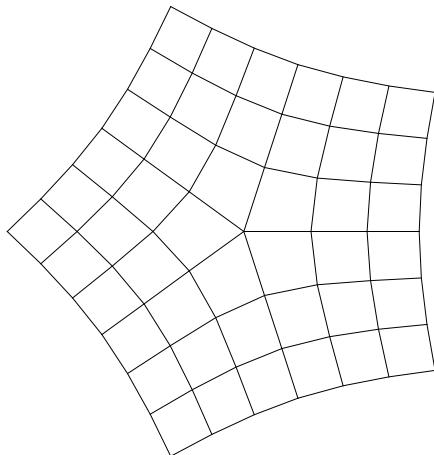
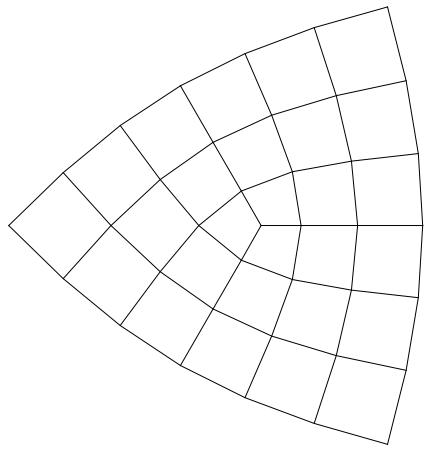
$$E_{angle} = 2.292 \quad E_{area} = 2.261 \quad E_{stretch} = 2.151$$

Problems

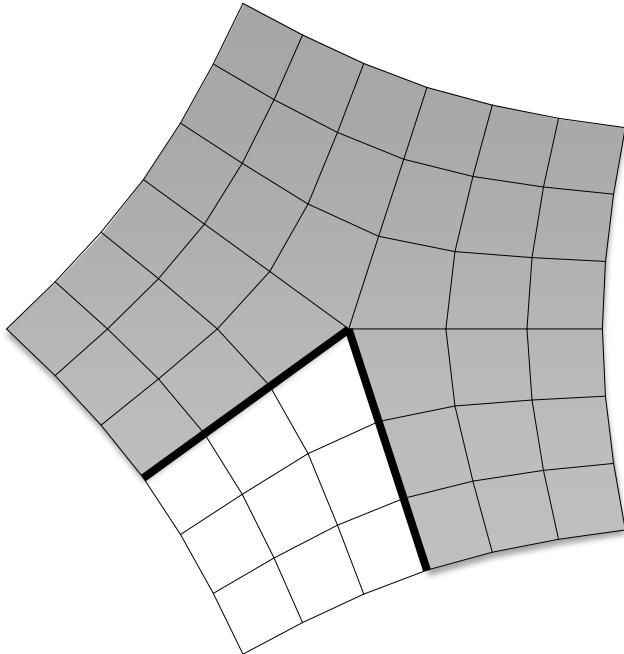
- Extra distortion along boundaries



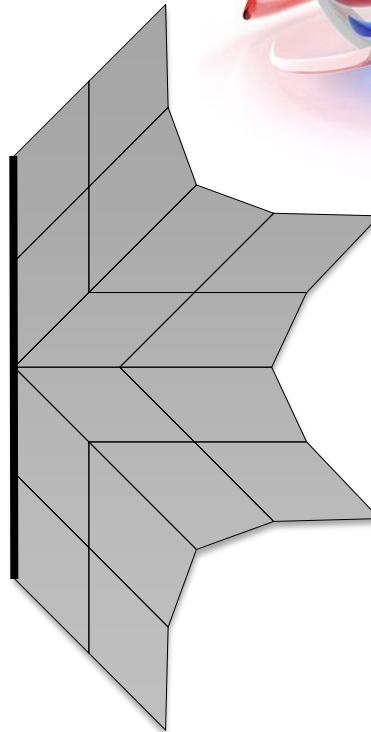
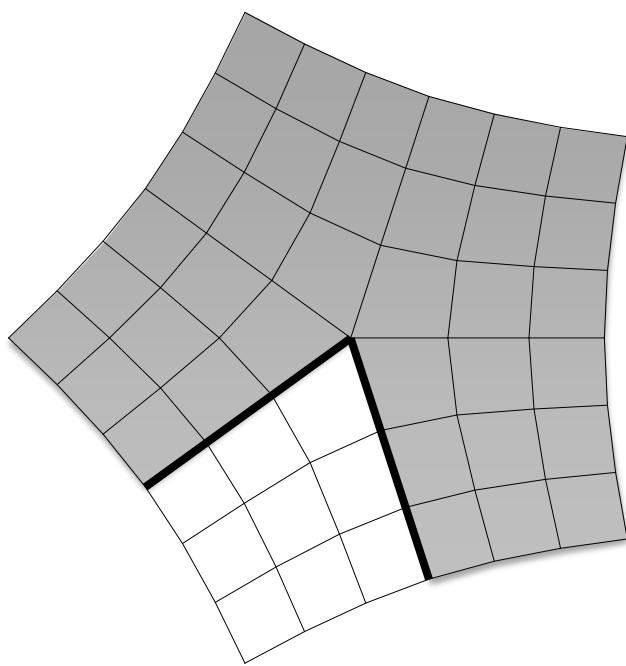
Problems



Problems

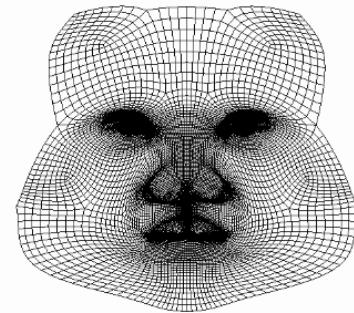
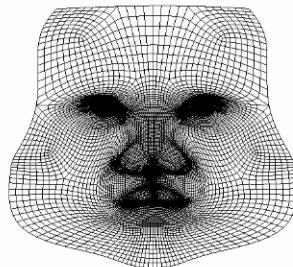
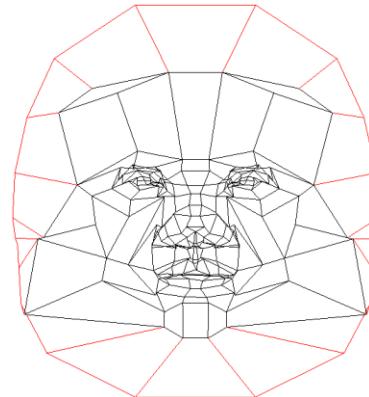
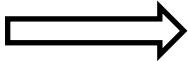
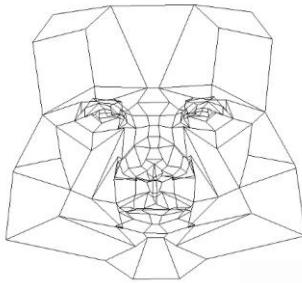


Problems

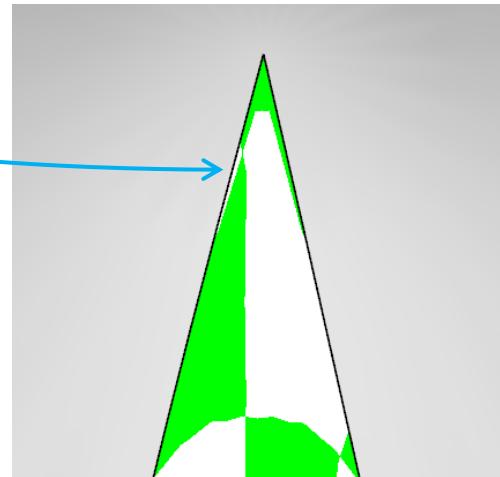
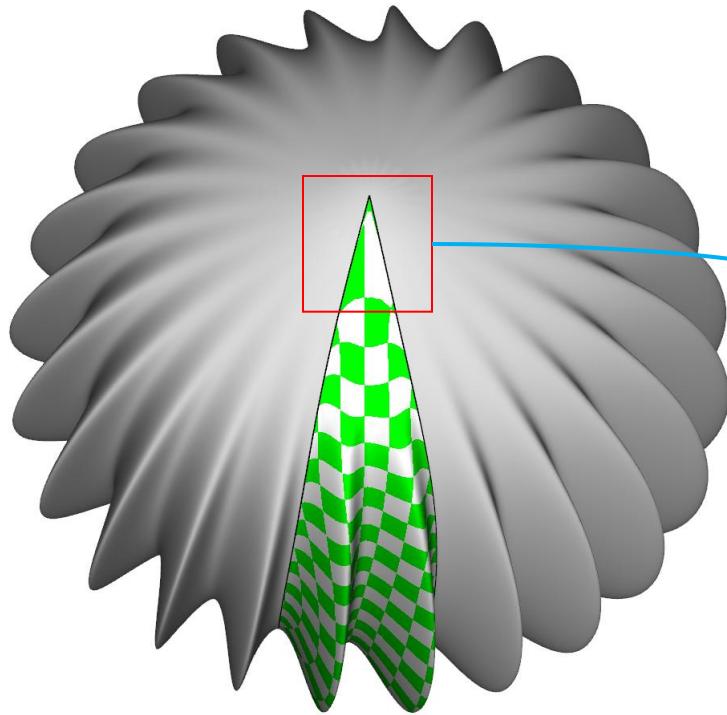


Extended Charts

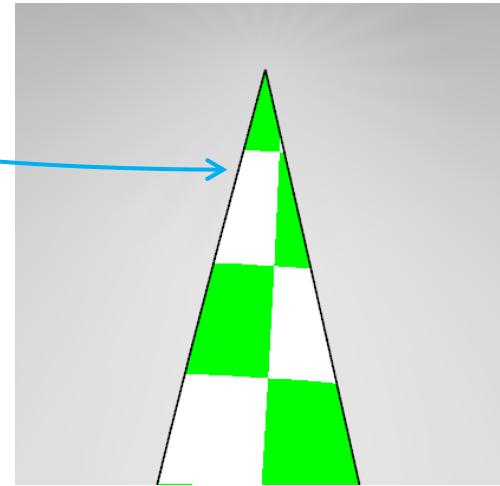
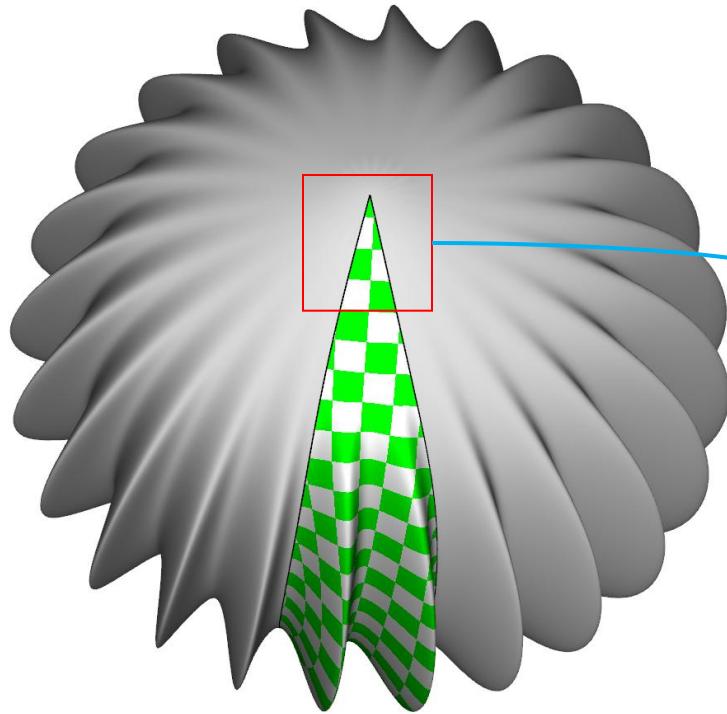
- Same topology as 3D surface
- Additional degrees of freedom



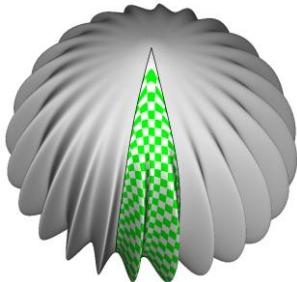
Extended Charts



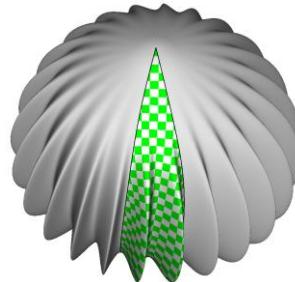
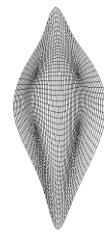
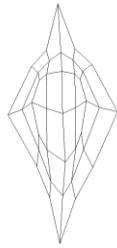
Extended Charts



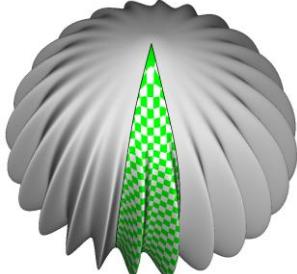
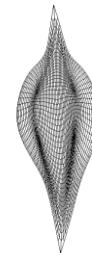
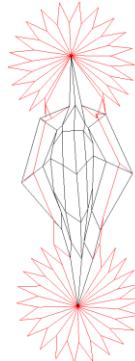
Comparison



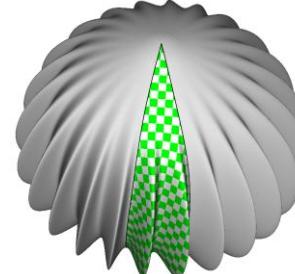
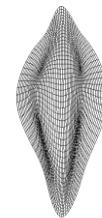
Poly



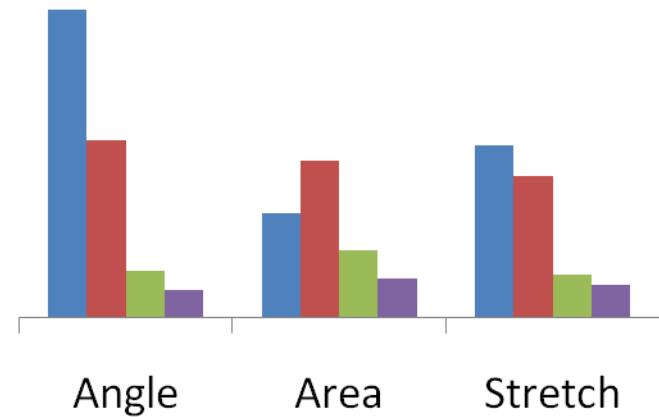
Extended



Subd



Subd Poly

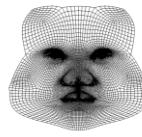
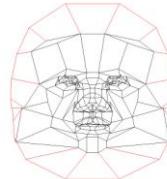
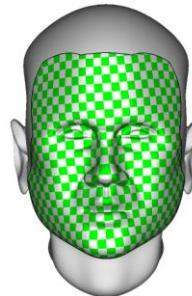
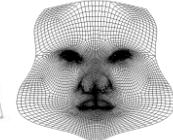
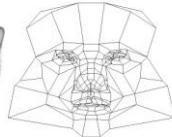
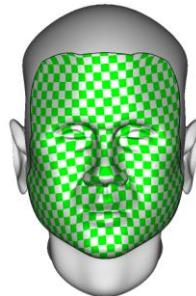


Angle

Area

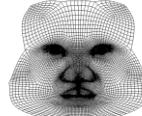
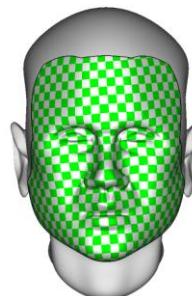
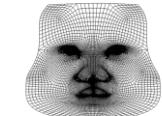
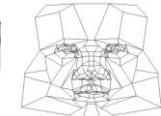
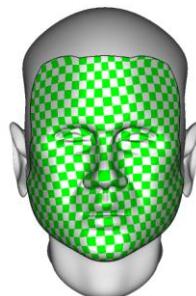
Stretch

Comparison



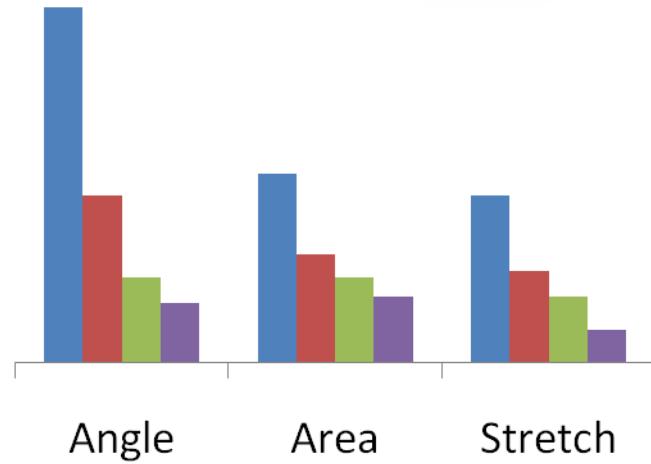
Poly

Extended

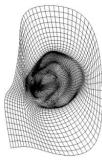
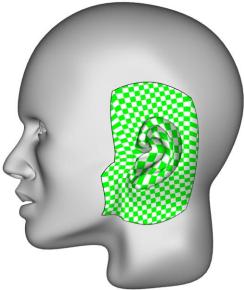


Subd

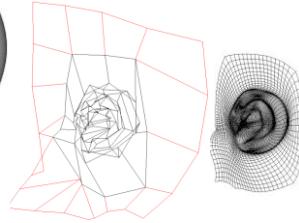
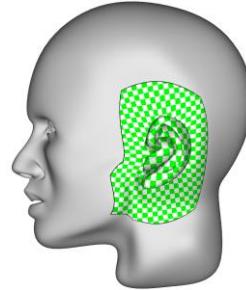
Subd Poly



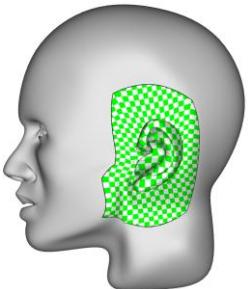
Comparison



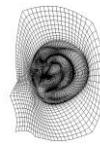
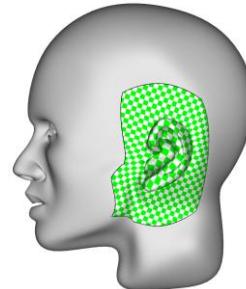
Poly



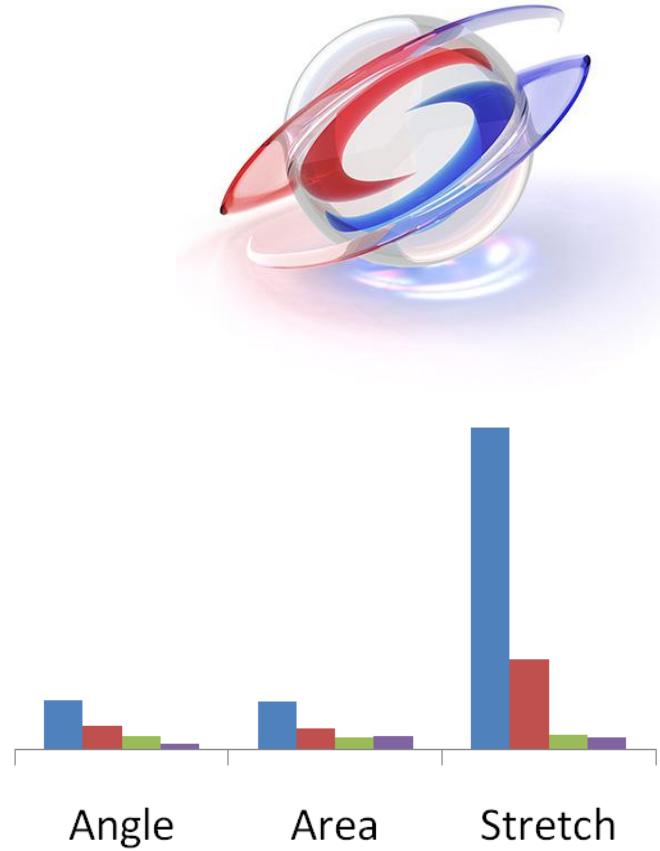
Extended



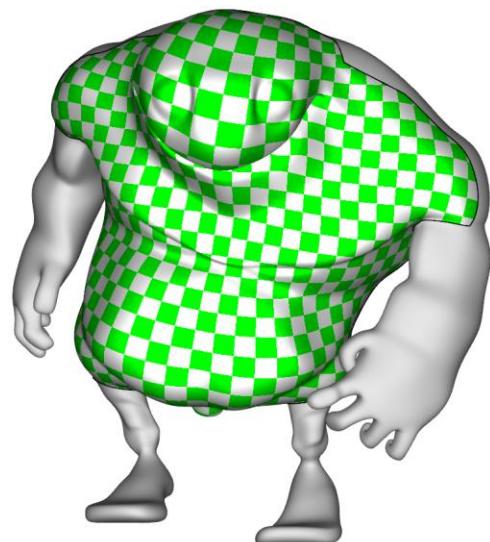
Subd



Subd Poly

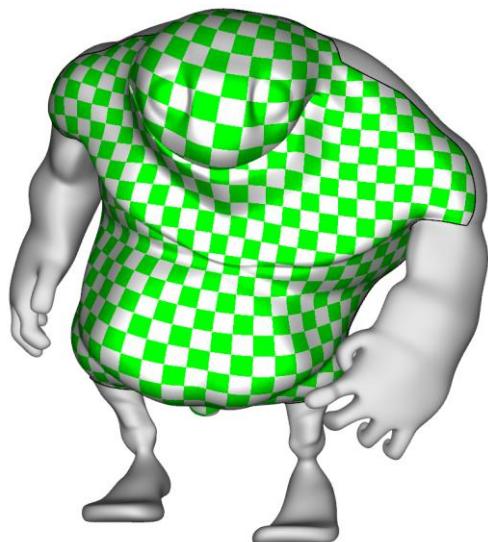


Multi-resolution optimization



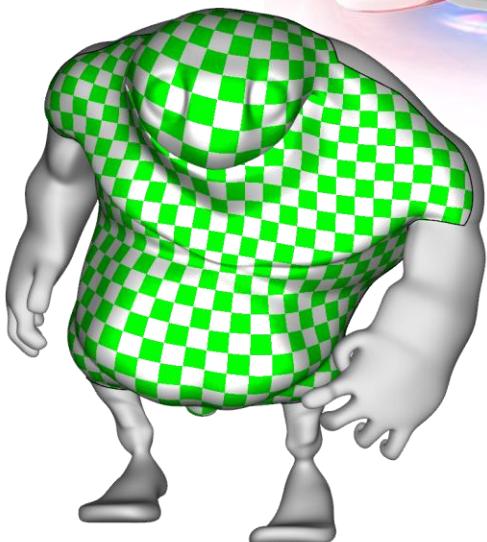
2.472, 2.366, 1.527

Level 1, 0.11s



2.293, 2.250, 1.447

Level 2, 0.41s

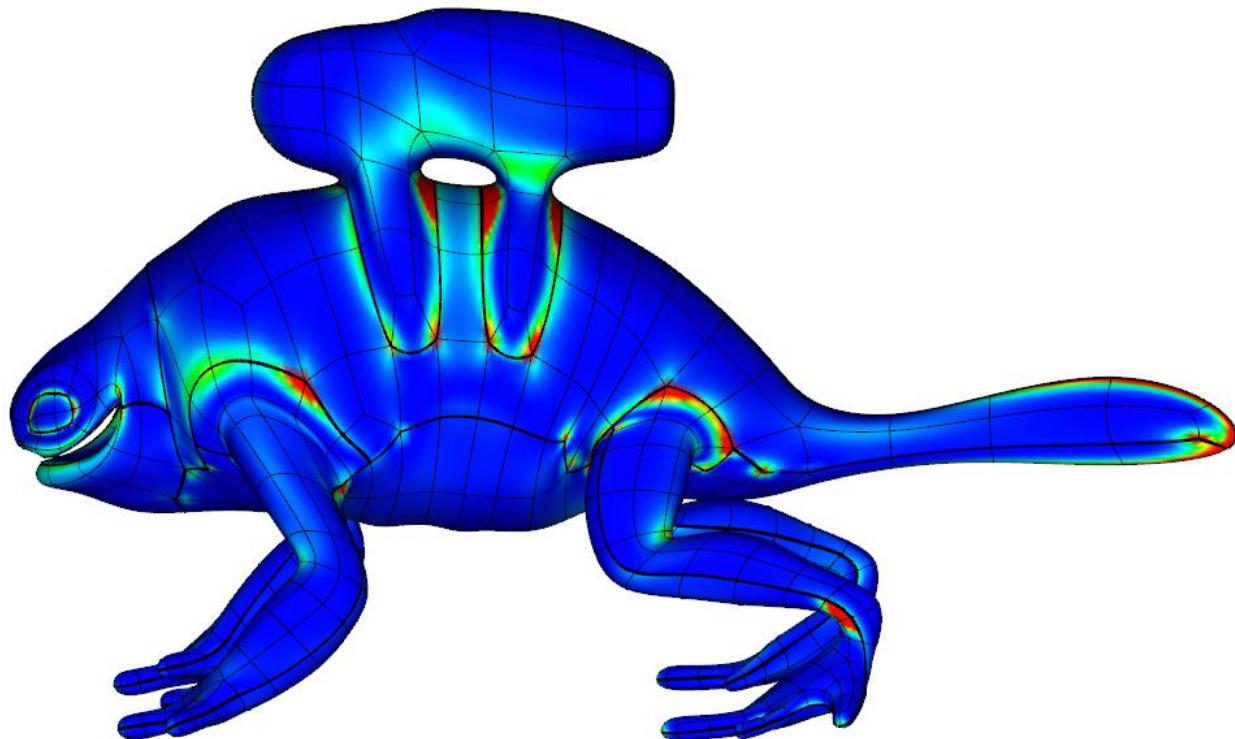


2.202, 2.187, 1.232

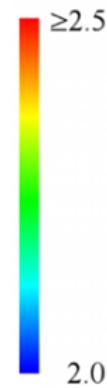
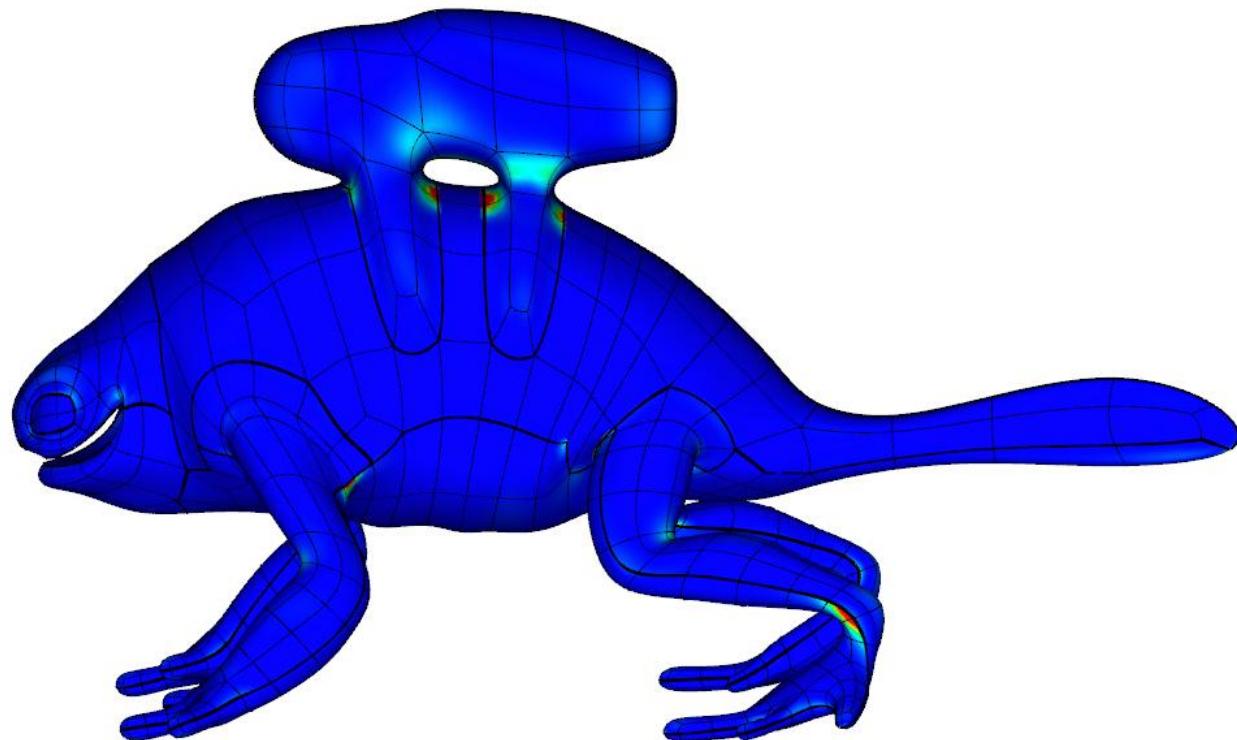
Level 3, 1.77s



Comparison



Comparison



Conclusions

- Select your favorite methods
 - subdivision surfaces (e.g. Catmull–Clark)
 - parameterization (e.g. As-rigid-as-possible)



Conclusions



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 - subdivision surfaces (e.g. Catmull–Clark)
 - parameterization (e.g. As-rigid-as-possible)
- Parameterize the subdivided mesh, not the base mesh
 - same number of DOFs, basically same cost
 - fast convergence by exploiting subdivision hierarchy
 - reduces distortion of each patch

Conclusions

- Select your favorite methods
 - subdivision surfaces (e.g. Catmull–Clark)
 - parameterization (e.g. As-rigid-as-possible)
- Parameterize the subdivided mesh, not the base mesh
 - same number of DOFs, basically same cost
 - fast convergence by exploiting subdivision hierarchy
 - reduces distortion of each patch
- Use extended charts
 - reduces distortion at patch boundaries

