Simplification of Articulated Meshes

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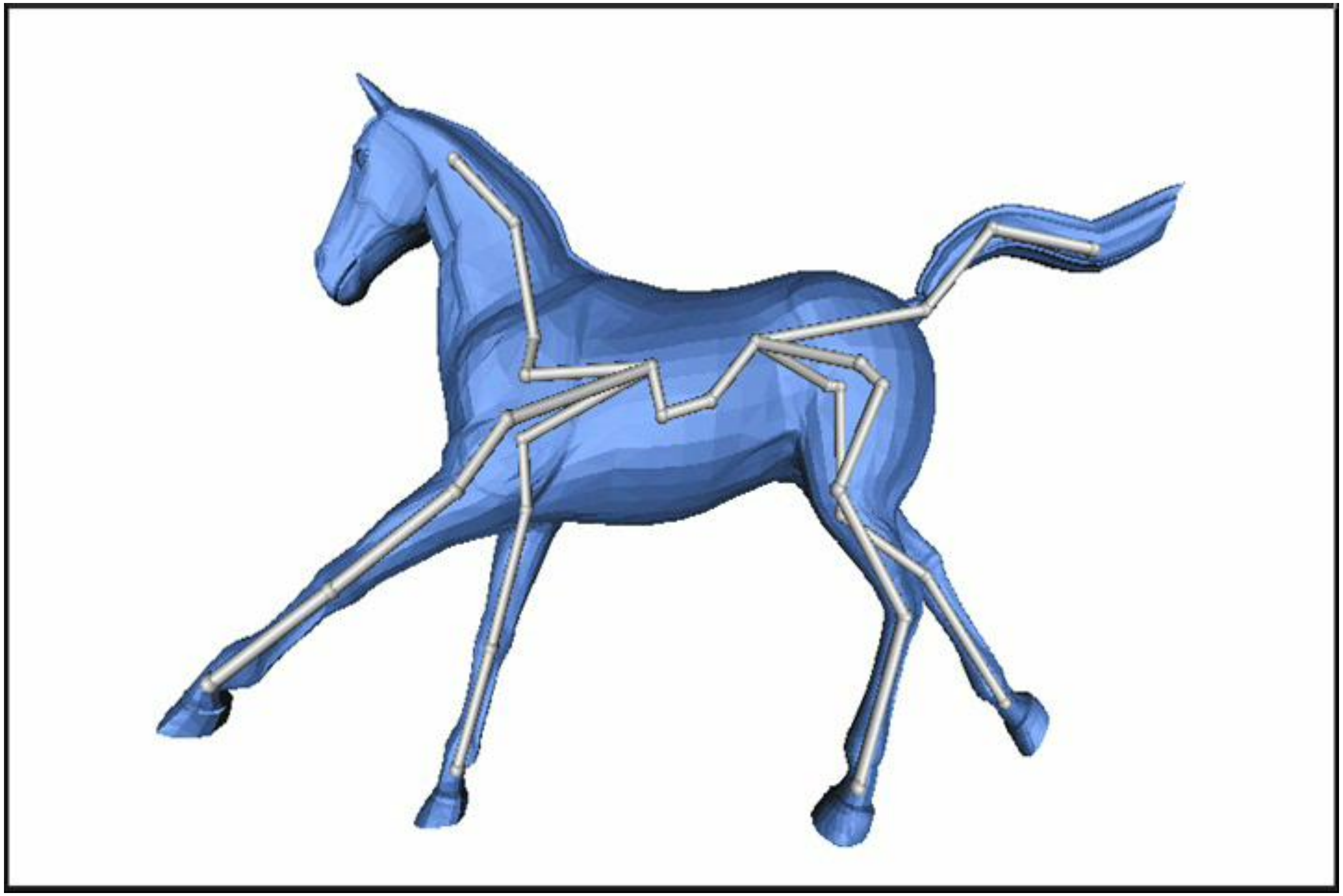
Introduction

Simplification

1,087,716 faces → 10,000 faces

Introduction

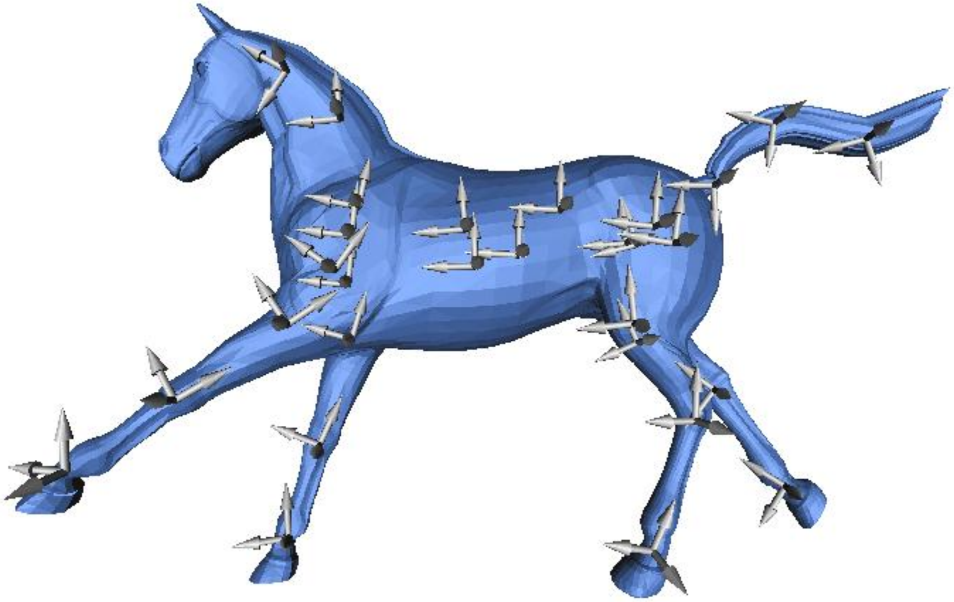
Articulated meshes



Introduction

Articulated meshes

$$\hat{v} = \sum_k \alpha_k (M_k v)$$

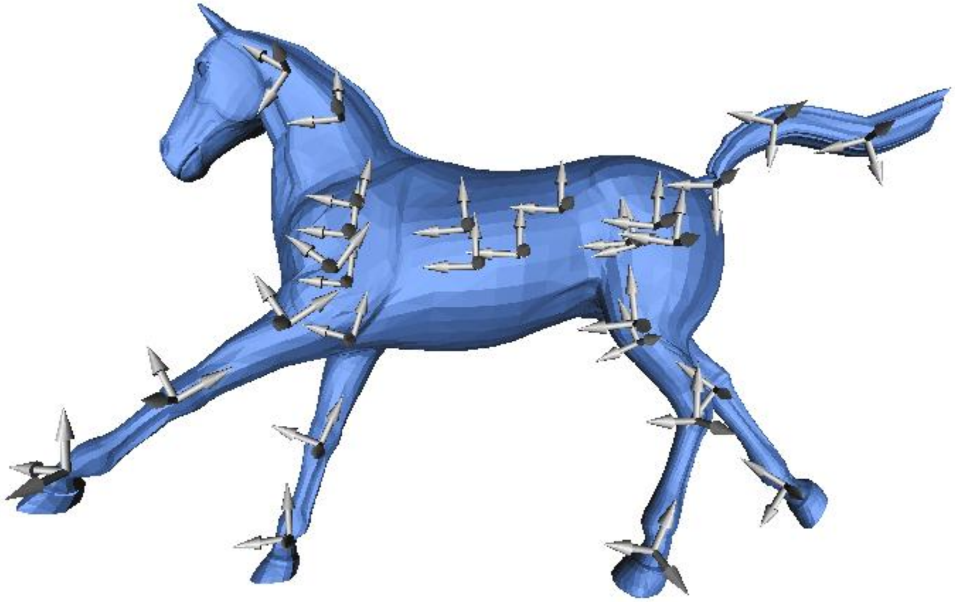


Introduction

Articulated meshes

$$\hat{v} = \sum_k \alpha_k (M_k v)$$

M_k : Bone Transformation Matrix



Introduction

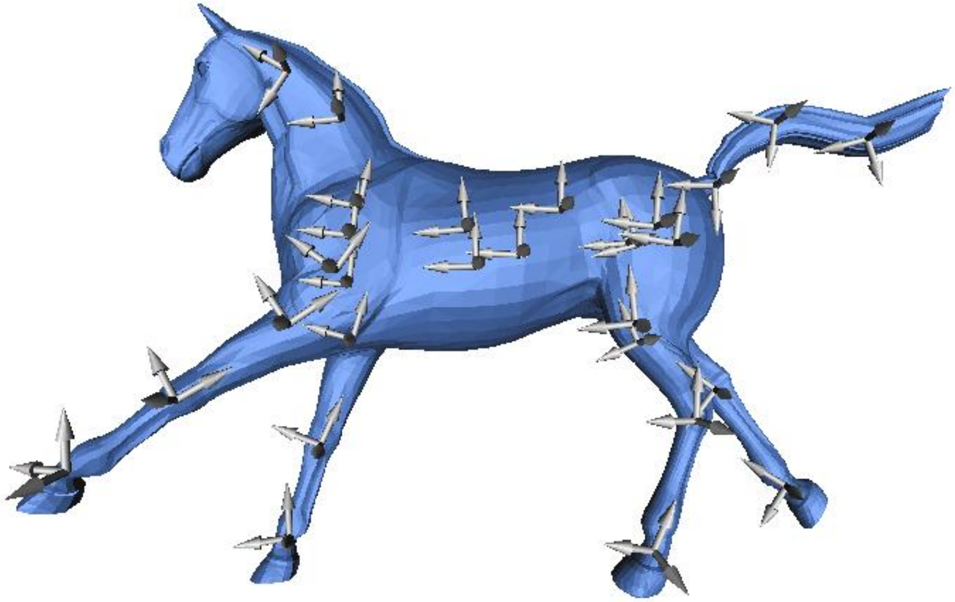
Articulated meshes

$$\hat{v} = \sum_k \alpha_k (M_k v)$$

M_k : Bone Transformation Matrix

α_k : Skin Weights

$$\sum_k \alpha_k = 1,\ \alpha_k \geq 0$$



Introduction

Unsimplified

Introduction

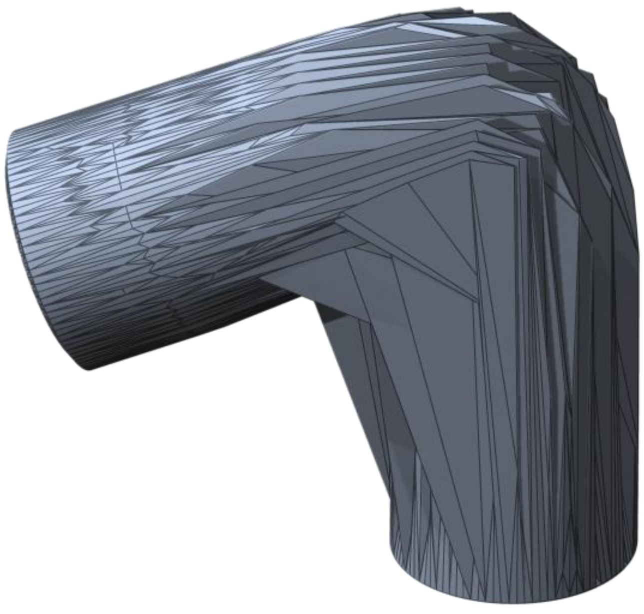
Unsimplified

Introduction

Introduction

Static simplification

Introduction



Static simplification

insufficient for deformable models

Quadric Error Functions

Basic QEF equation:

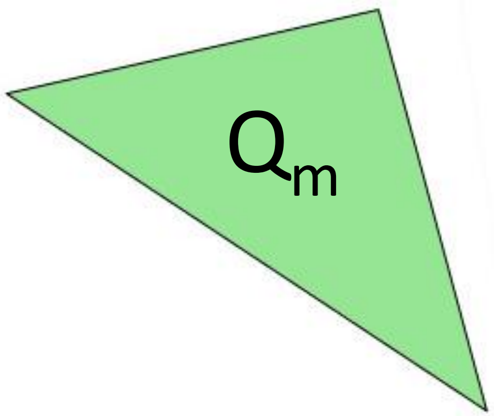
$$E_i(v) = \sum_m \left(n_m \cdot (v - p_i)\right)^2 = v^T Q_i v$$

p_i : *i*th vertex *p* in mesh

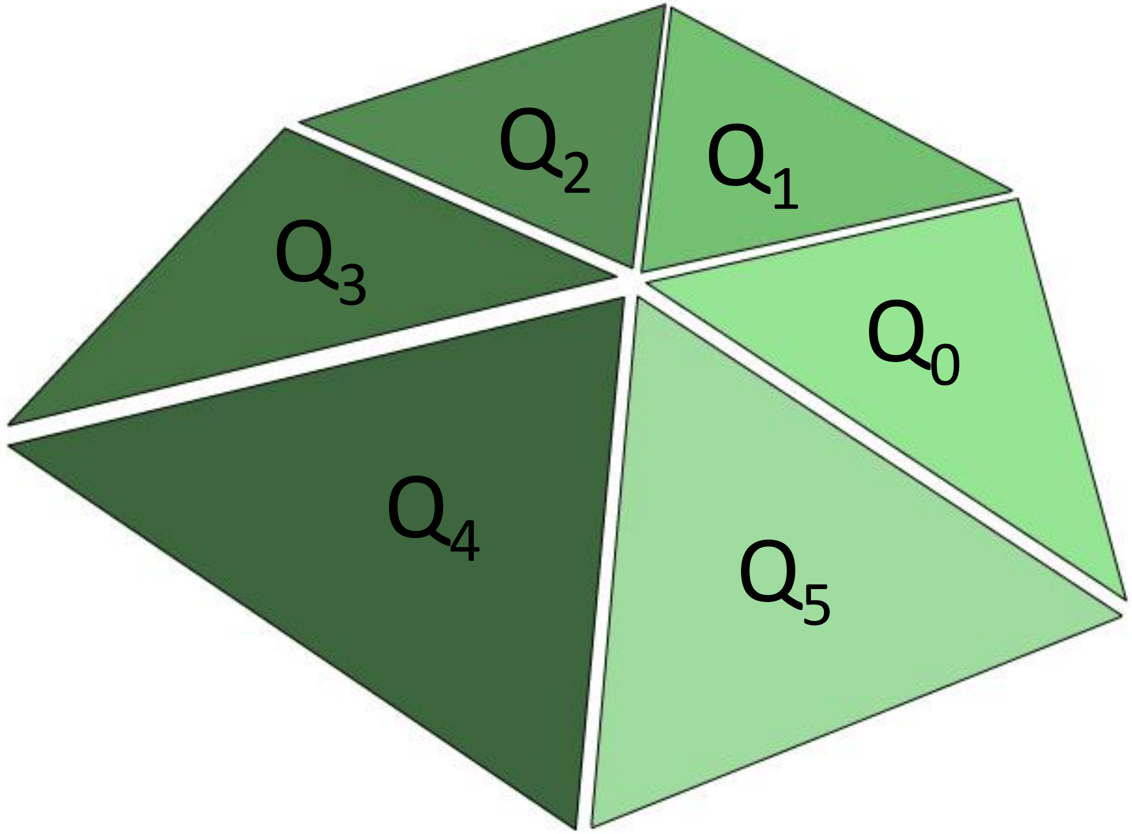
n_m : normal of *m*th adjacent face

QEF Edge Collapses

Q_m = Quadric Error Function
(distance to plane on face m)

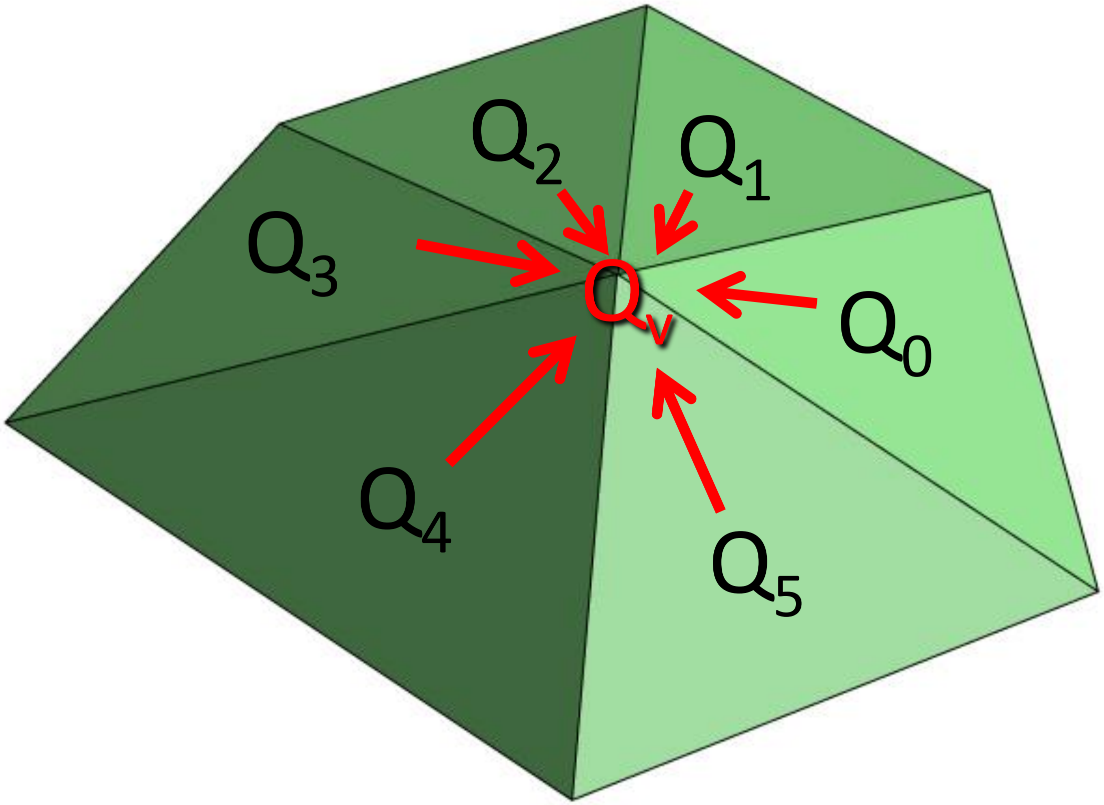


QEF Edge Collapses



QEF Edge Collapses

$$Q_v = Q_0 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$



QEF Edge Collapses

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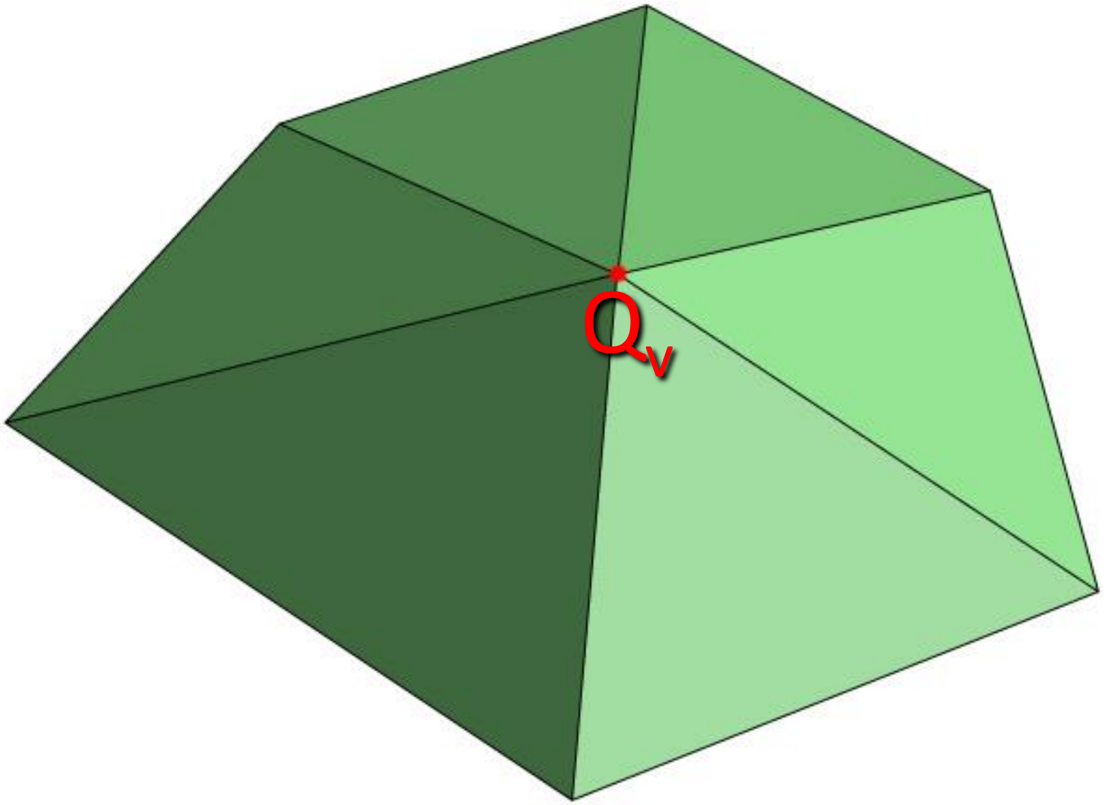
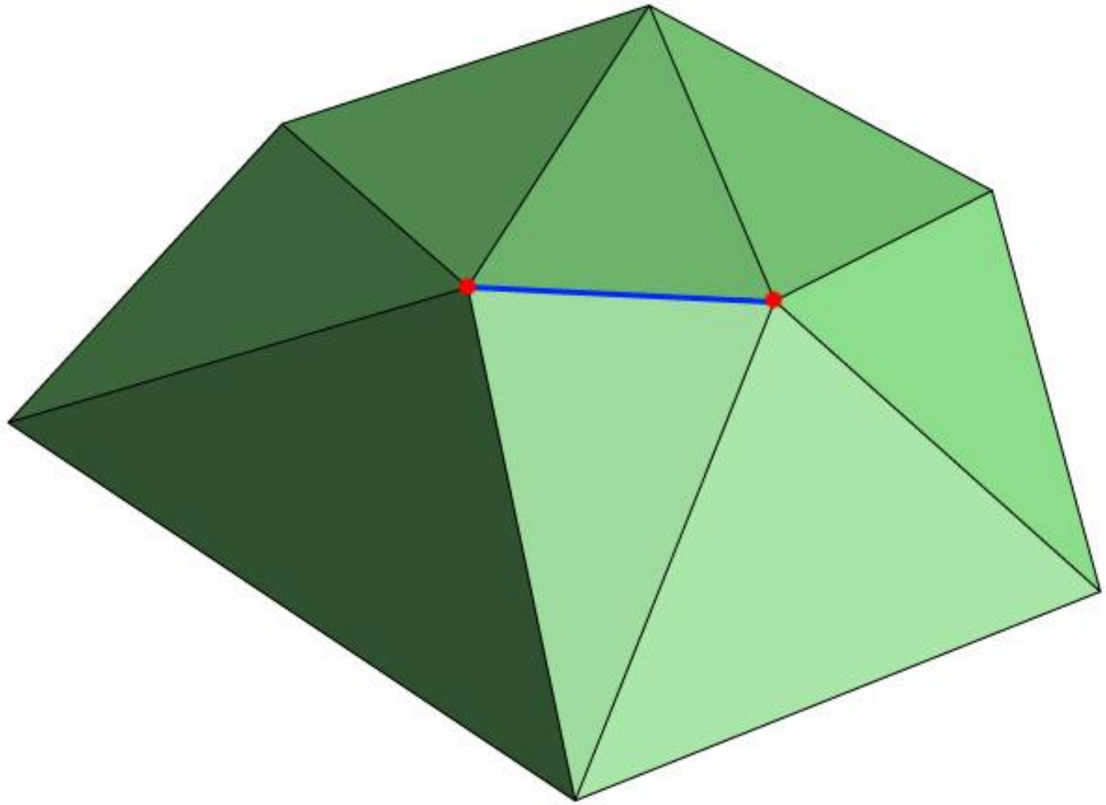
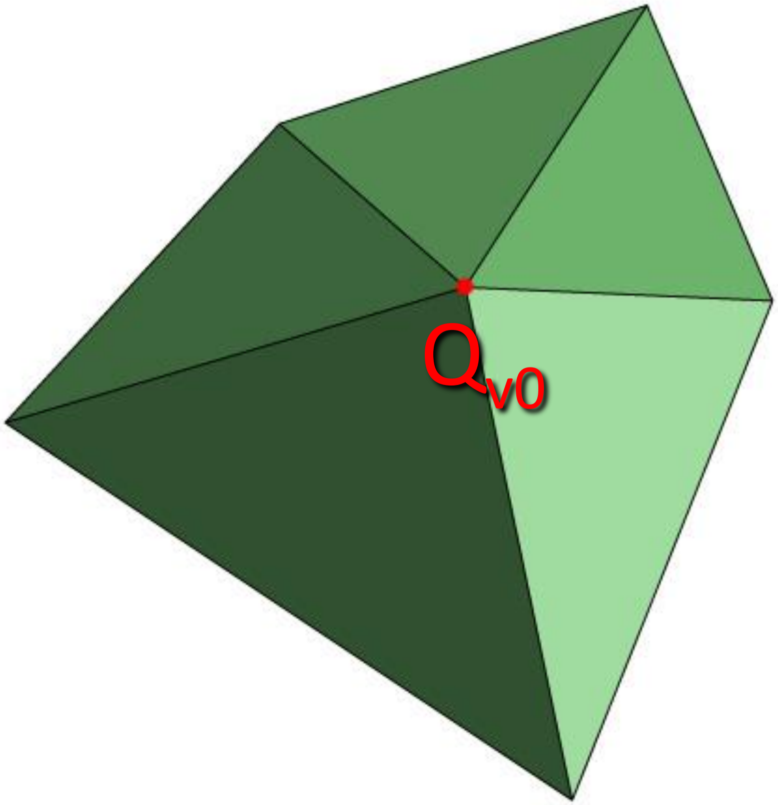


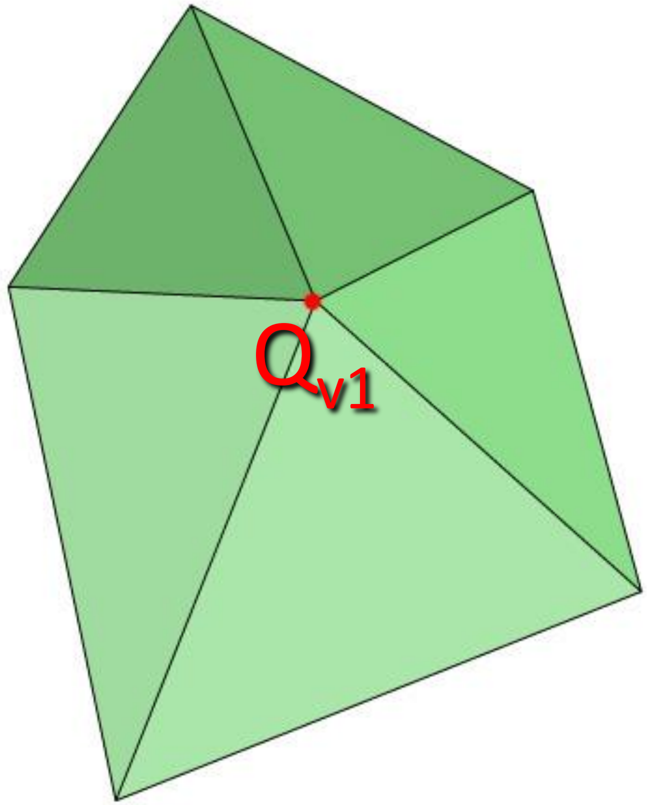
QEF Edge Collapses



QEF Edge Collapses

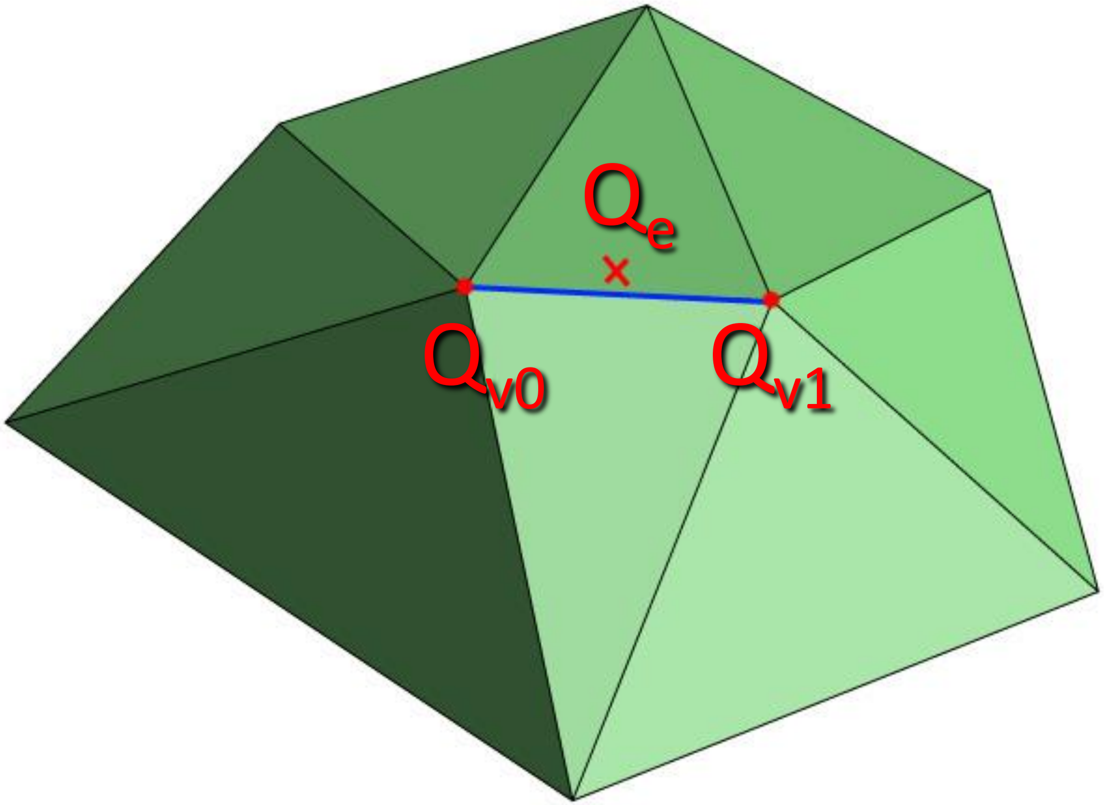


QEF Edge Collapses

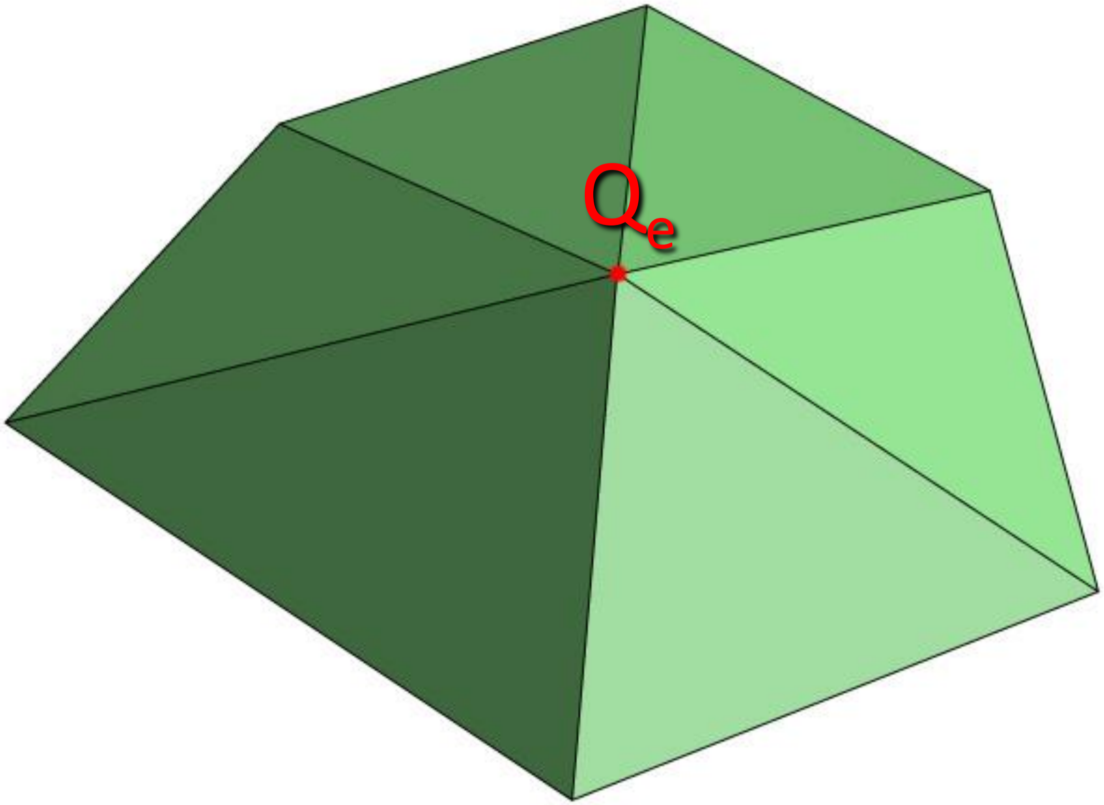


QEF Edge Collapses

$$Q_e = Q_{v0} + Q_{v1}$$

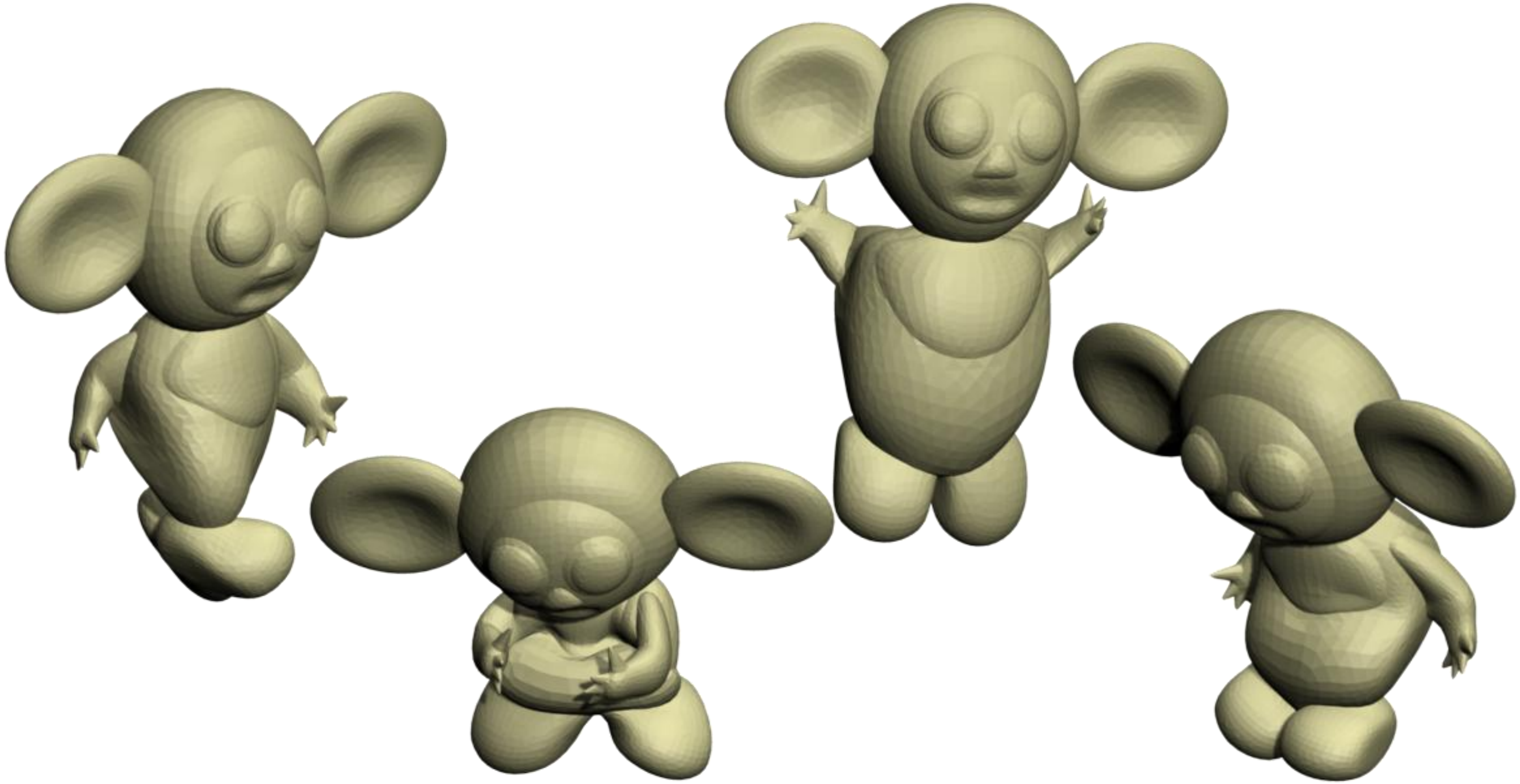


QEF Edge Collapses



Our Method

Example Poses



Our Method

Modify QEF Equation:

$$\sum_j \sum_m \left(n_m^j \cdot (\hat{v}^j - p_i^j) \right)^2 = (\hat{v}^j)^T Q_i^j \hat{v}^j$$

Our Method

Modify QEF Equation:

$$\sum_j \sum_m \left(n_m^j \cdot (\hat{v}^j - p_i^j) \right)^2 = (\hat{v}^j)^T Q_i^j \hat{v}^j$$

$$\hat{v}^j = \sum_k \alpha_k M_k^j v$$

Our Method

Modify QEF Equation:

$$E_i(v,\alpha_k) = \sum_j \left(\sum_k \alpha_k M_k^j v \right)^T Q_i^j \left(\sum_k \alpha_k M_k^j v \right)$$

$$\hat{v}^j = \sum_k \alpha_k M_k^j v$$

Our Method

Modify QEF Equation:

$$E_i(v,\alpha_k) = \sum_j \left(\sum_k \alpha_k M_k^j v \right)^T Q_i^j \left(\sum_k \alpha_k M_k^j v \right)$$

Problem: equation is quartic

Solution: split into alternating quadratic equations

Our Method

Quadratic #1 – Solve for position

$$\min_{v} E_i(v) = v^T \left(\sum_j \left(\sum_k \alpha_k M_k^j \right)^T Q_i^j \left(\sum_k \alpha_k M_k^j \right) \right) v$$

Hold weights constant and solve for position v

Our Method

Quadratic #2 – Solve for weights

$$\min_{\alpha} E_i(\alpha_k) = \alpha^T \left(\sum_j V_j^T Q_i^j V_j \right) \alpha$$

Hold V constant and solve for weights

$$V_j = \begin{pmatrix} M_0^j v & M_1^j v & \cdots & M_k^j v \end{pmatrix}$$

Our Method

Quadratic #2 – Solve for weights

$$\min_{\alpha} E_i(\alpha_k) = \alpha^T \left(\sum_j V_j^T Q_i^j V_j \right) \alpha$$

subject to $\sum_k \alpha_k = 1$

Hold V constant and solve for weights

$$V_j = \begin{pmatrix} M_0^j v & M_1^j v & \cdots & M_k^j v \end{pmatrix}$$

Our Method

Quadratic #2 – Solve for weights

$$\min_{\alpha} E_i(\alpha_k) = \alpha^T \left(\sum_j V_j^T Q_i^j V_j \right) \alpha$$

subject to $\sum_k \alpha_k = 1, \alpha_k \geq 0$

Hold V constant and solve for weights

$$V_j = \begin{pmatrix} M_0^j v & M_1^j v & \cdots & M_k^j v \end{pmatrix}$$

Our Method

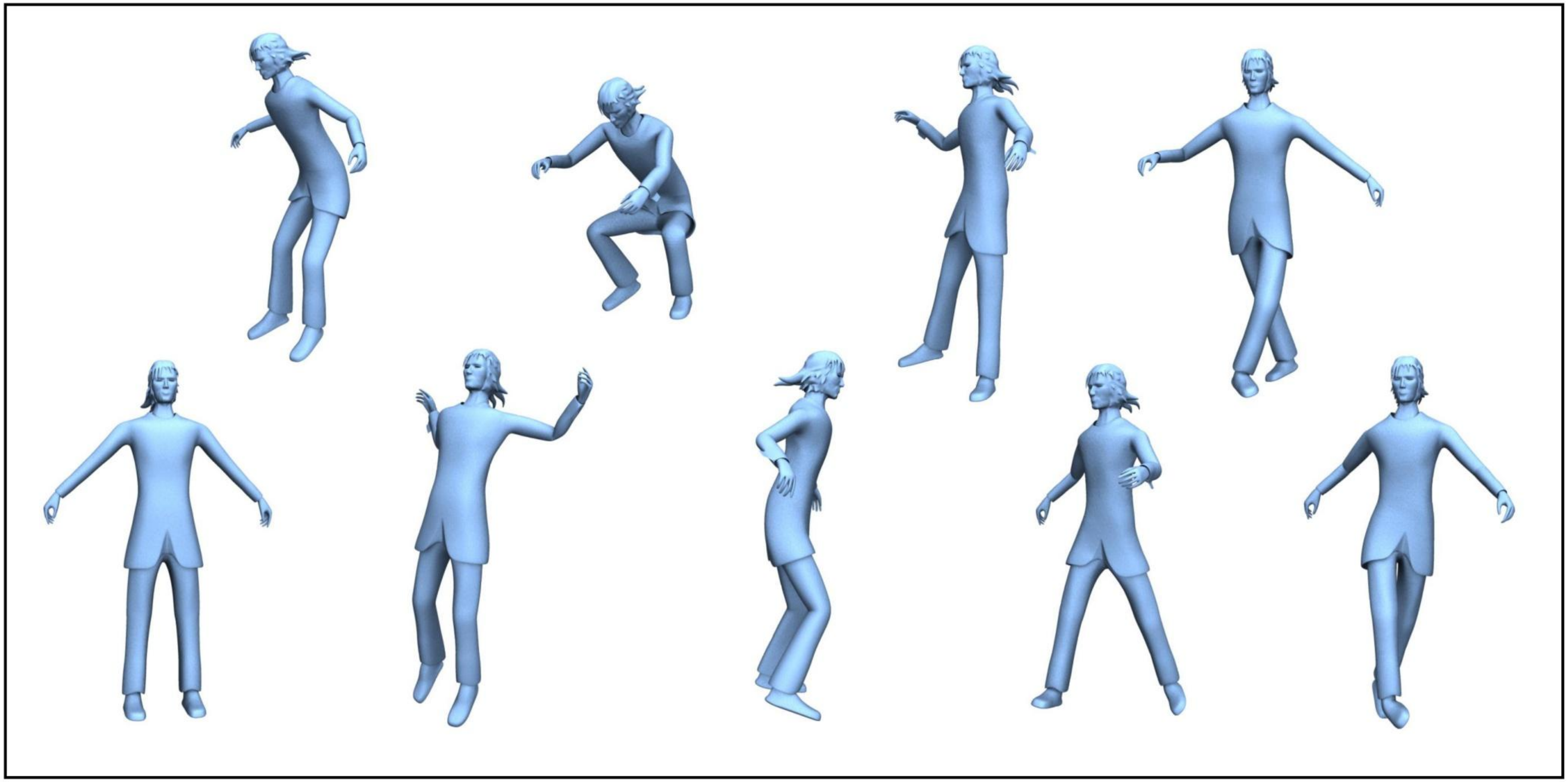
Alternating minimization

$$E_i(v) \quad = \quad v^T \left(\sum_j \left(\sum_k \alpha_k M_k^j \right)^T Q_i^j \left(\sum_k \alpha_k M_k^j \right) \right) v$$

$$E_i(\alpha_k) = \alpha^T \left(\sum_j V_j^T Q_i^j V_j \right) \alpha$$

Results

Input Poses



240,448 poly

Results

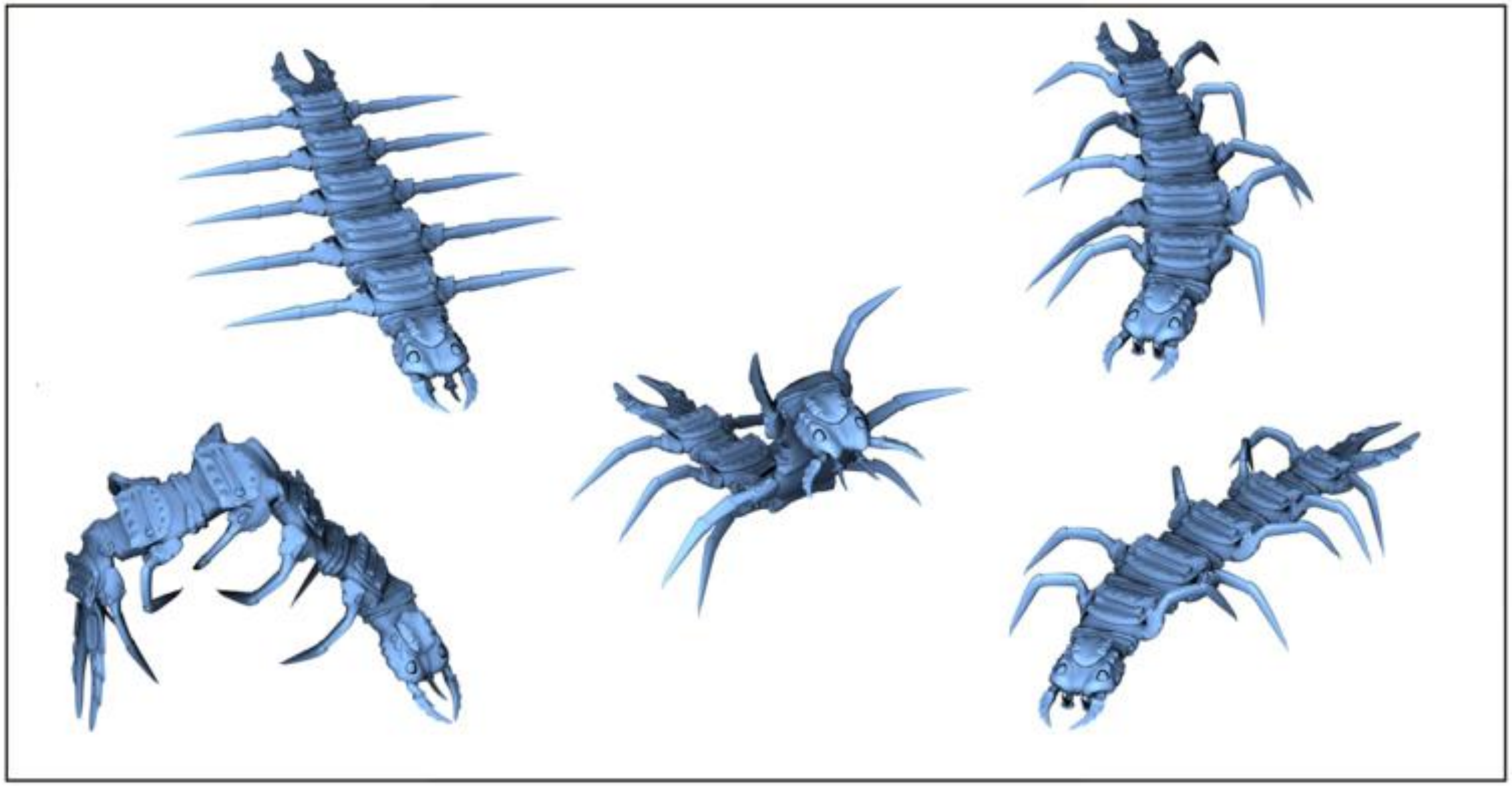
10,000 poly

5,000 poly

2,000 poly

Results

Input Poses

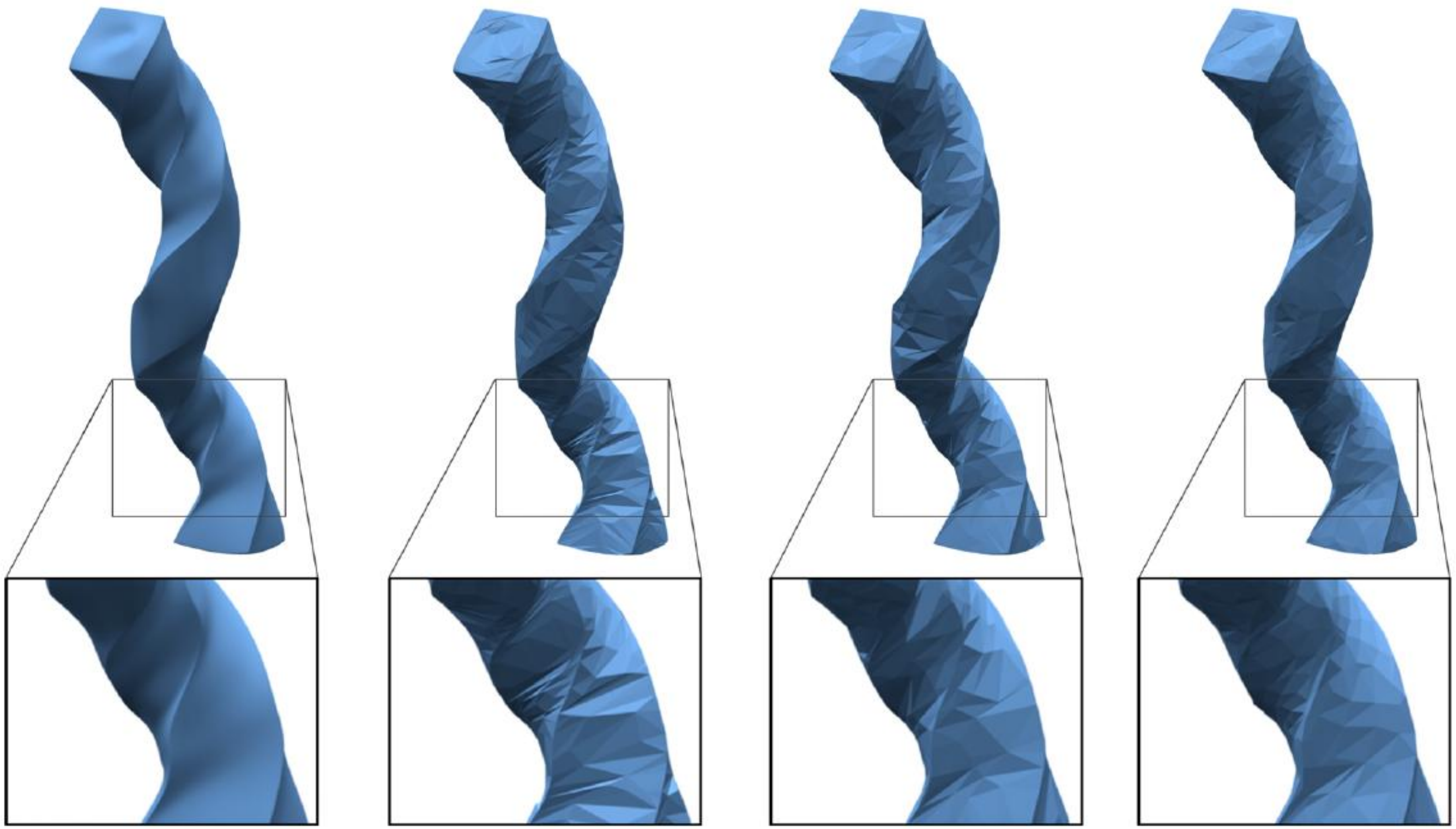


206,672 poly

Results

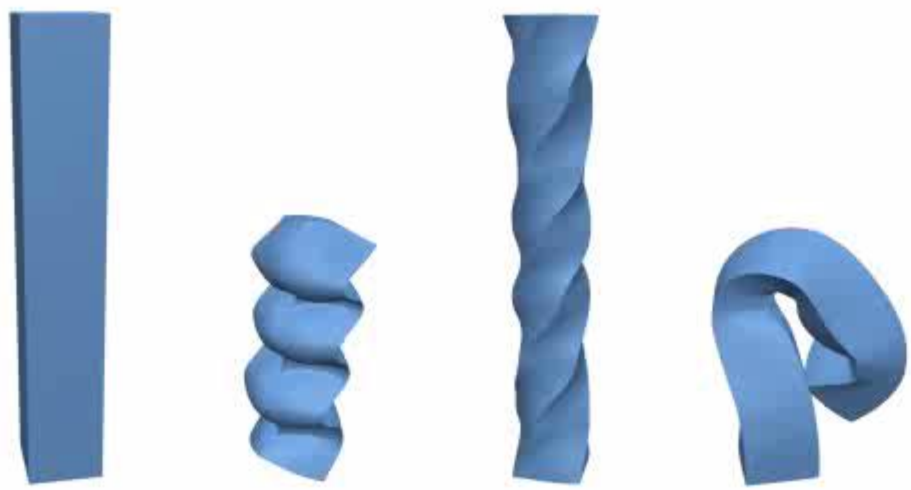
Results

Comparison with previous techniques



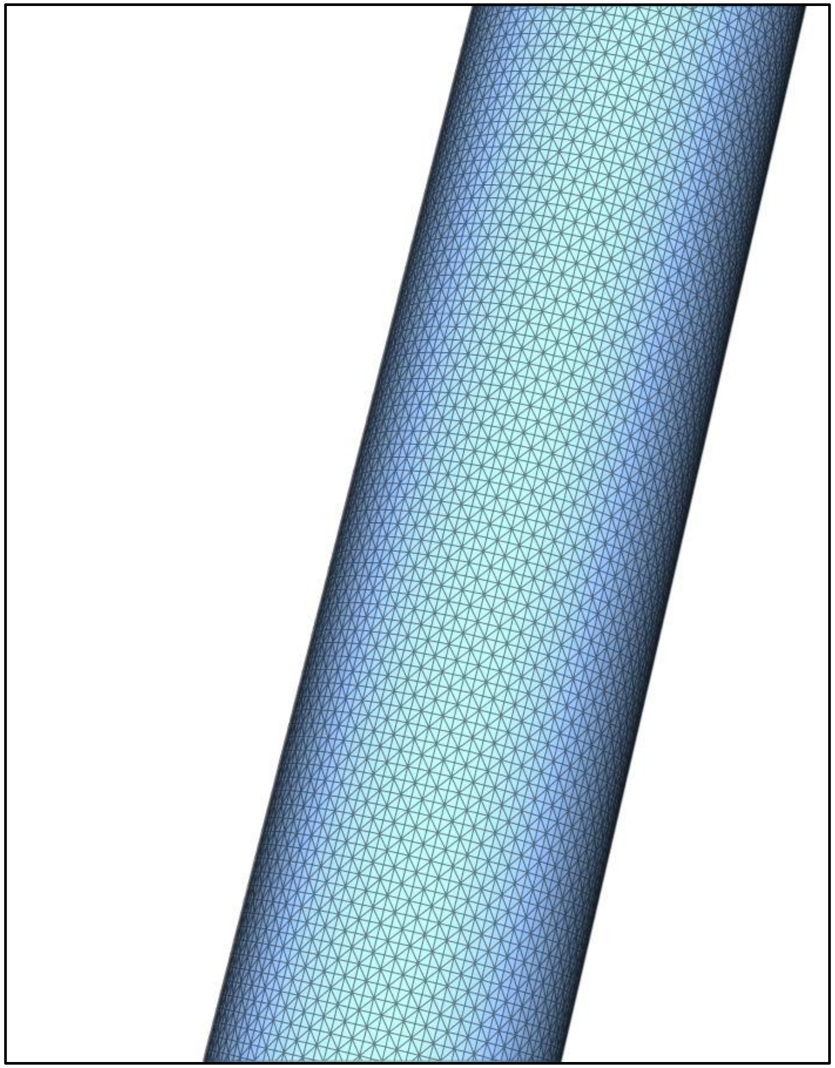
Original deCoro et al. Mohr et al. Ours

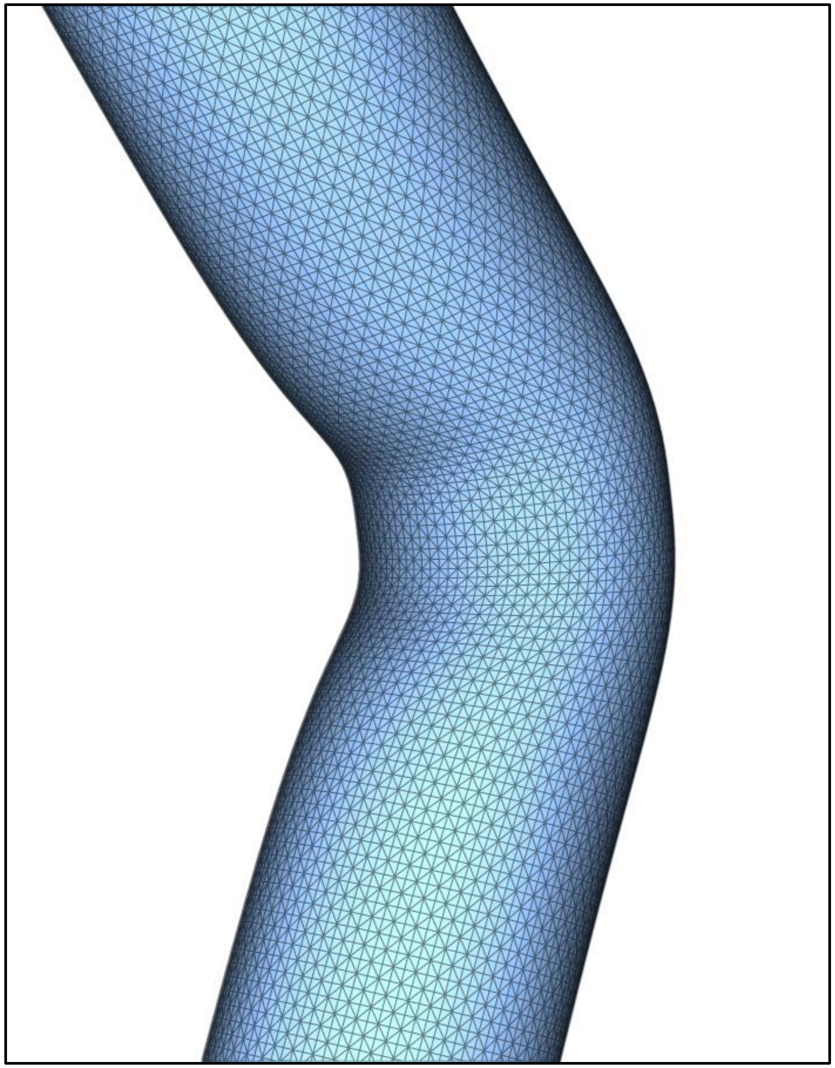
Results



4 Input Poses

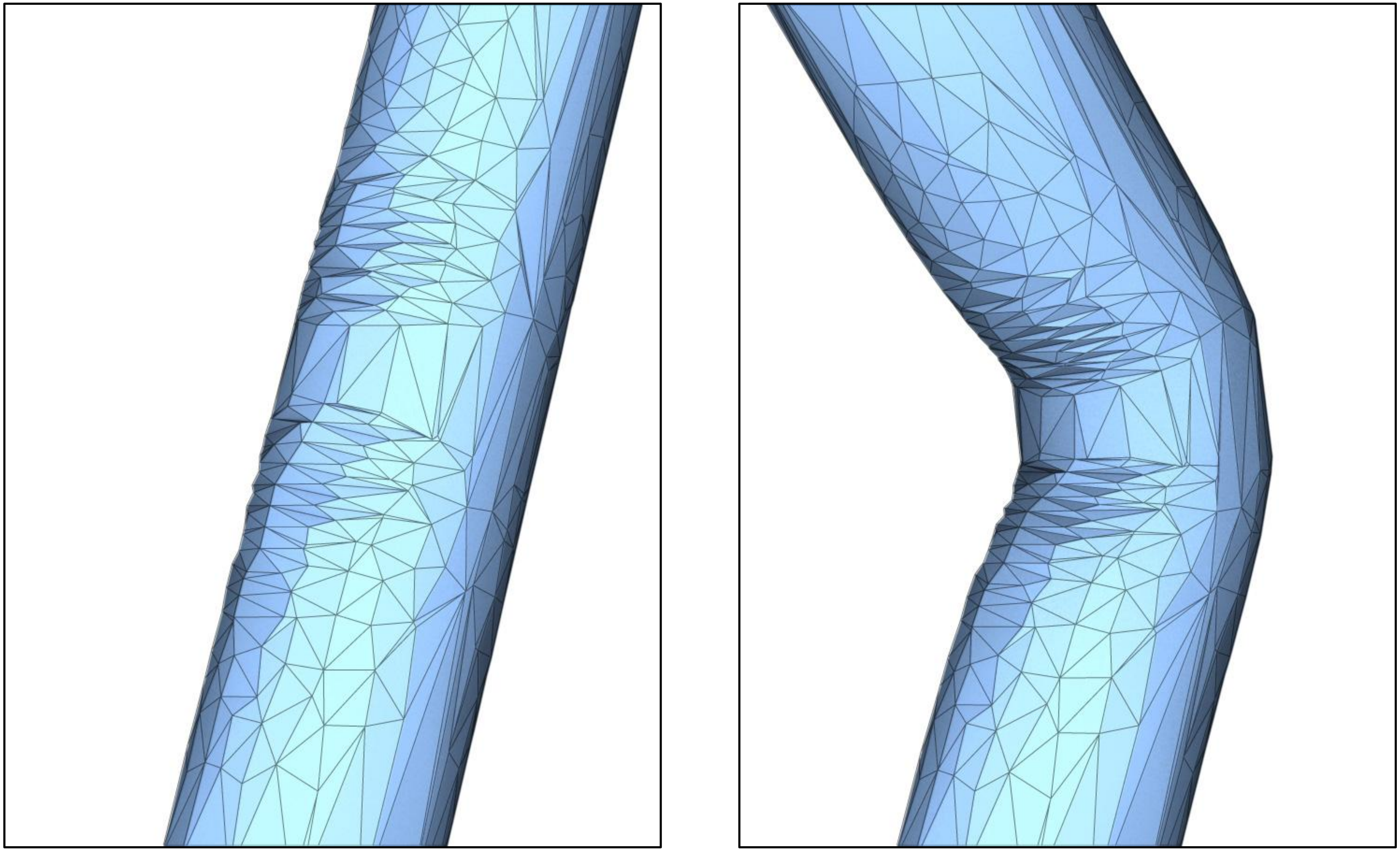
Results





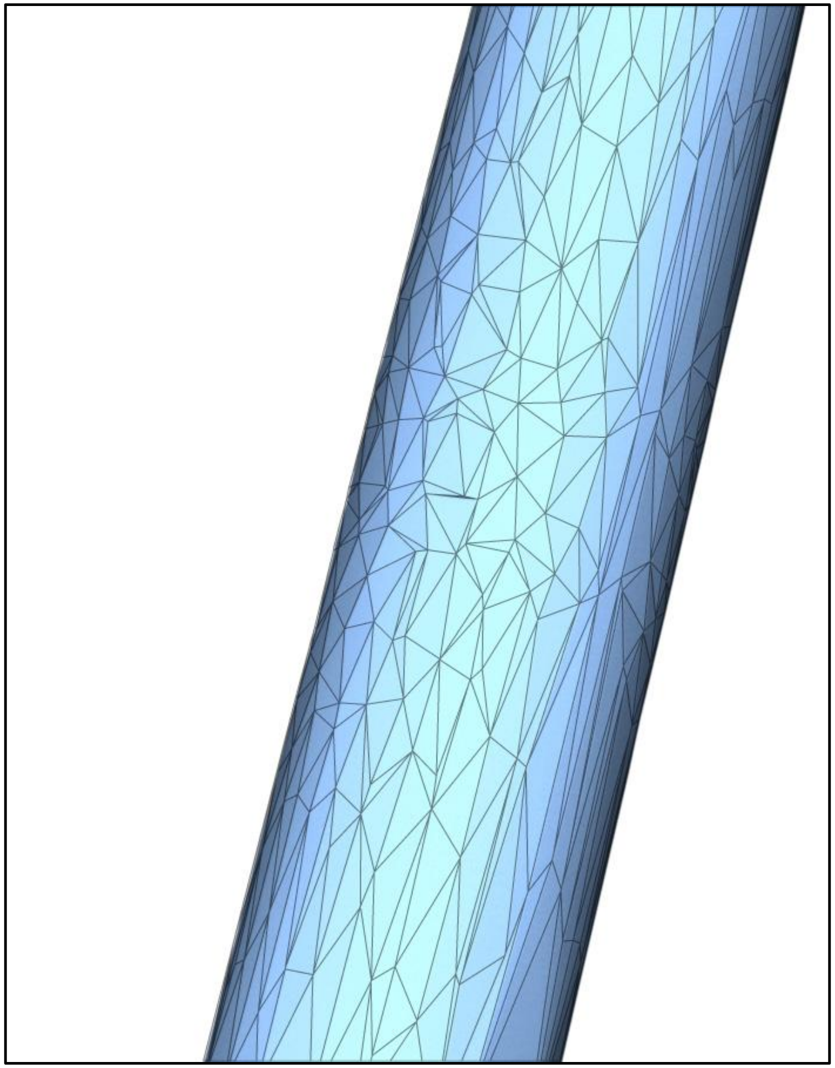
Results

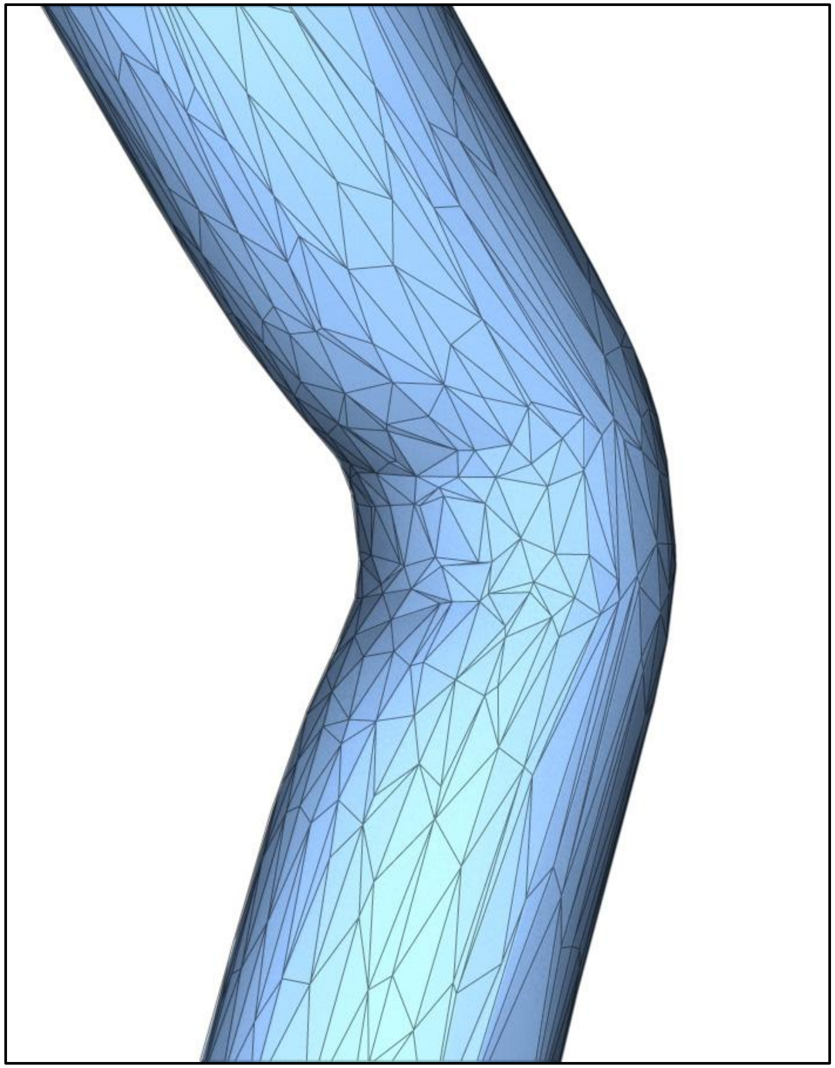
DeCoro et al.



Results

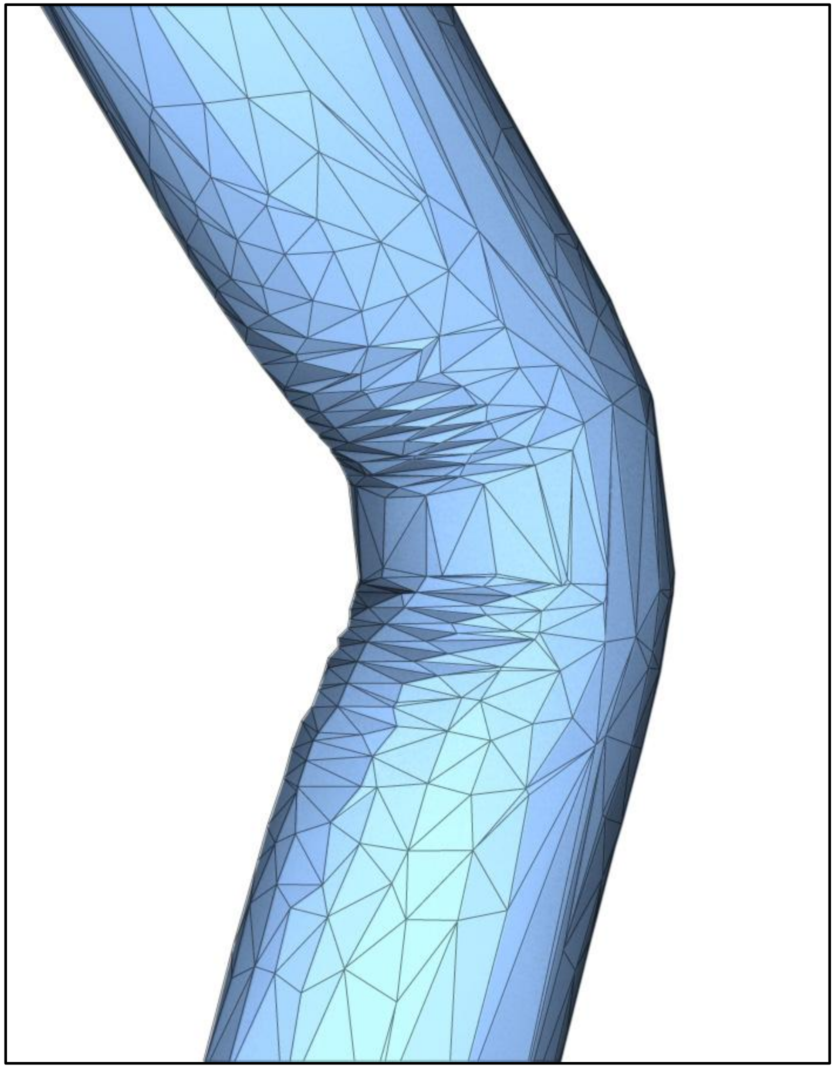
Ours



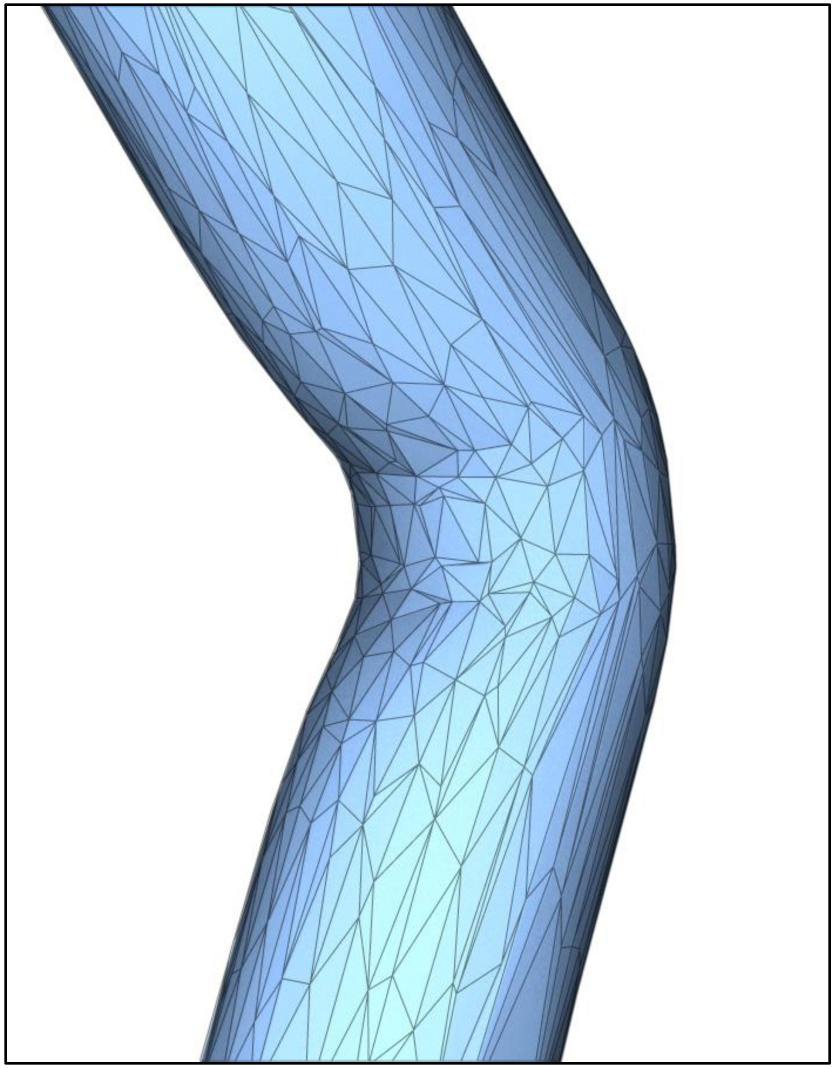


Results

DeCoro et al.

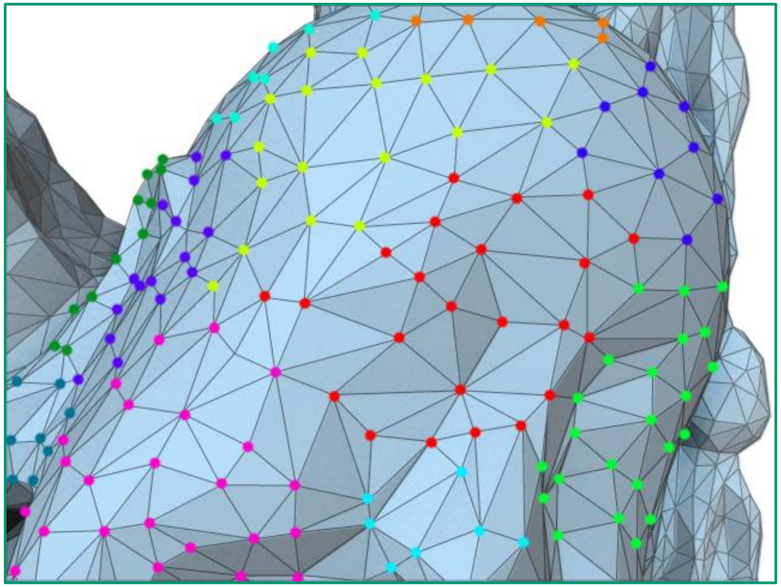


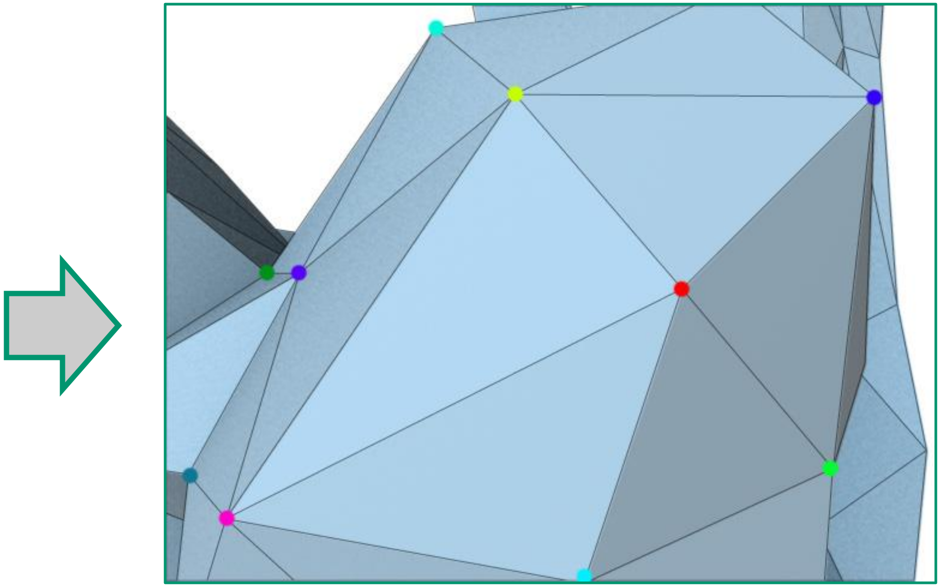
Ours



Results

Weight Influences





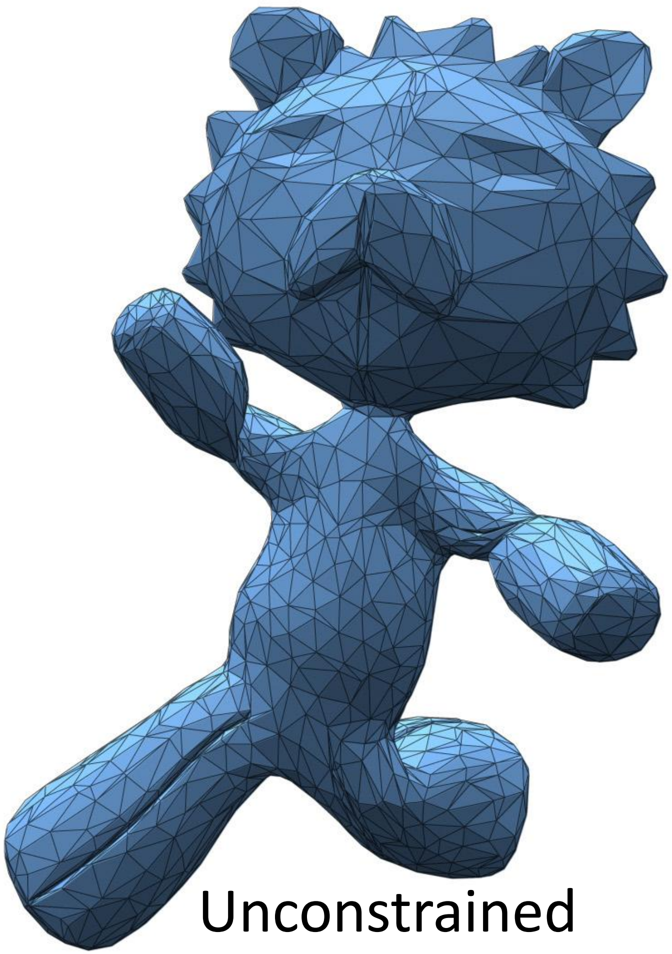
Weight reduction

Restriction to **n** weight influences:

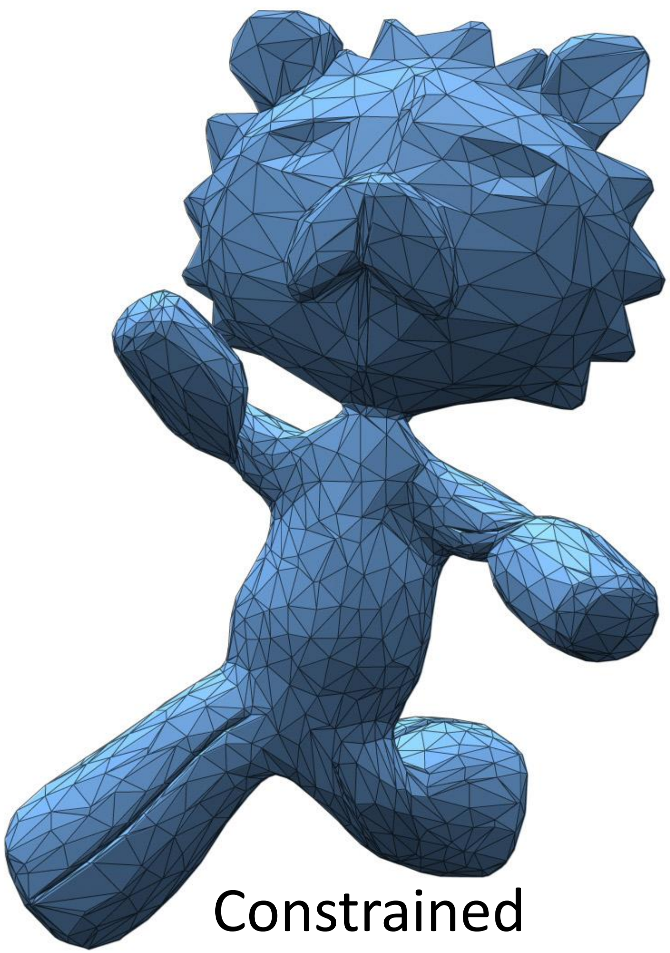
- Minimize $E_i(\alpha_k)$
- Prune down to ***n*** largest weights
- Minimize $E_i(\alpha_k)$ again

Results

Weight Reduction



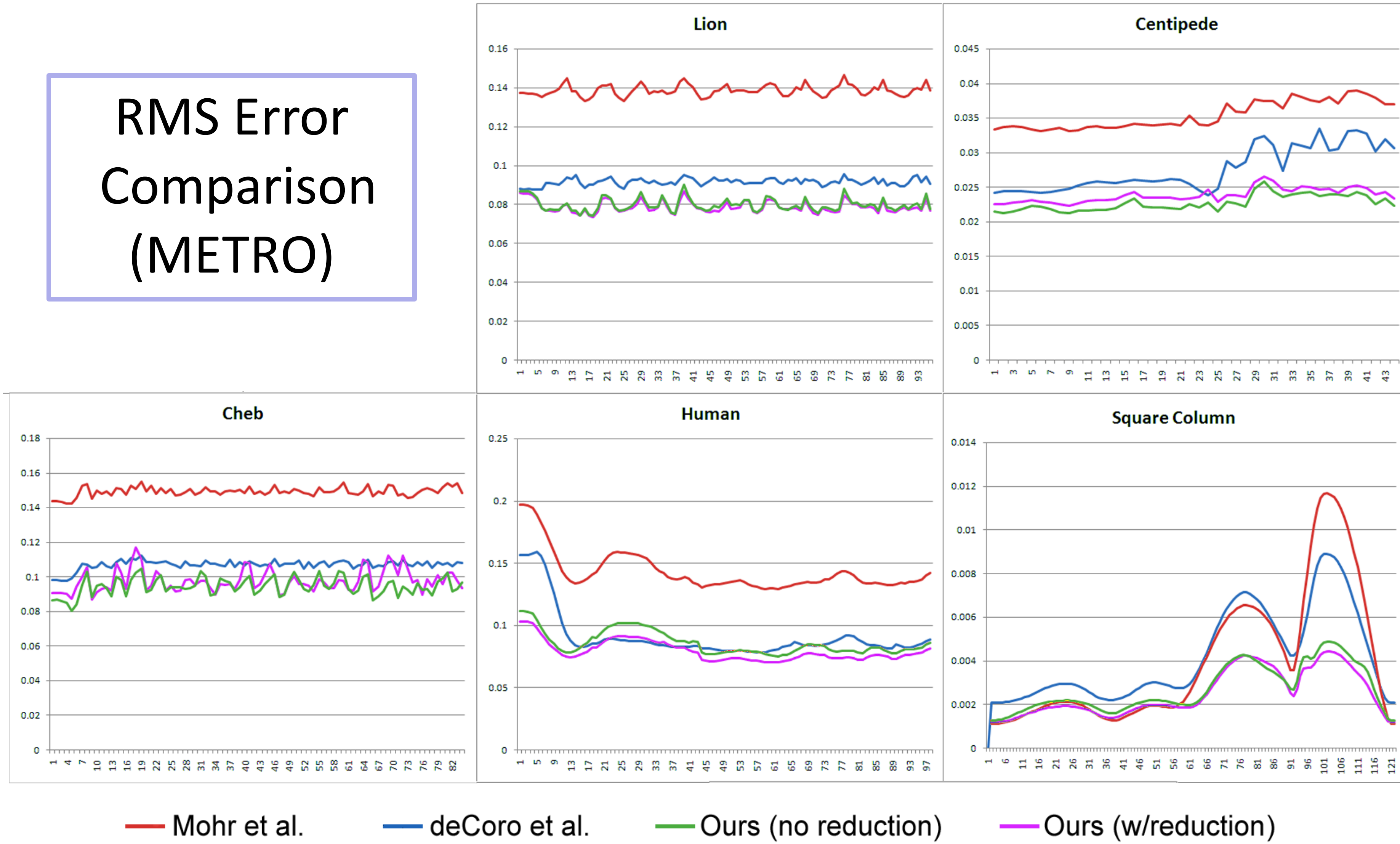
Unconstrained
up to 11 weights/vertex



Constrained
5 weights/vertex

Results

RMS Error Comparison (METRO)



Results

Model	Polys	Poses	Mohr	DeCoro	Our Method
Centipede	206672	5	6.769	5.180	22.727
Cheb	13334	27	2.025	.536	5.806
Lion	35152	33	6.733	1.704	13.720
Square Column	114688	4	1.927	2.221	19.580
Human	240448	9	12.066	6.123	24.452

Conclusions

- Minimizes both skin weights and vertex positions
- Easy to implement (quadratic minimization)
- Requires few example poses
- Reduces to a specified number of weights everywhere in the hierarchy

Questions?