Analytic Rasterization of Curves with Polynomial Filters

Josiah Manson and Scott Schaefer
Texas A&M University

Motivation


2PEASBILLBOARD
2peasMisterGiggles
ACTIONPACKED
AdLib
Allembert
AlternateGothic
Americana
Annie
Arial
Arial Black
BahiaScript
BALLOON
Batang
BELLCENT
Bernhard BdCn
Blackadder
Black Jack
Bodoni
BoyzrGross
BradleyHand
BrownCow
Brush
Calligraph
Cheddar Salad
ChillyMoe
CK Fantasy
COPPERPLGOTHL
Comic Sans
CrazyGirlBlond
Curlz
Dannette
Dear Joe
DellaRobbia
DeVinne
Donnys Hand
Dotum
DreamerOne
EastMarket
EdwardianScript
Elise
English
ENGRAVERS
EngrverOldEng
Eras Demi
EscpeTyWriterC
Euclid
FestivalFlourish
FG Adam
FG ALBIN
FG amelia
FG Isak
FG Josefina
FG Maria
FG ScriptElegant
FontDinDot
FontOnAStick
Formal
Freehand
French Script
French
Georgia
GOLIATH
GoodDogPlain
Greer
Grilled Cheese
Gumsey
Hank
HardCompound
HastyPudding

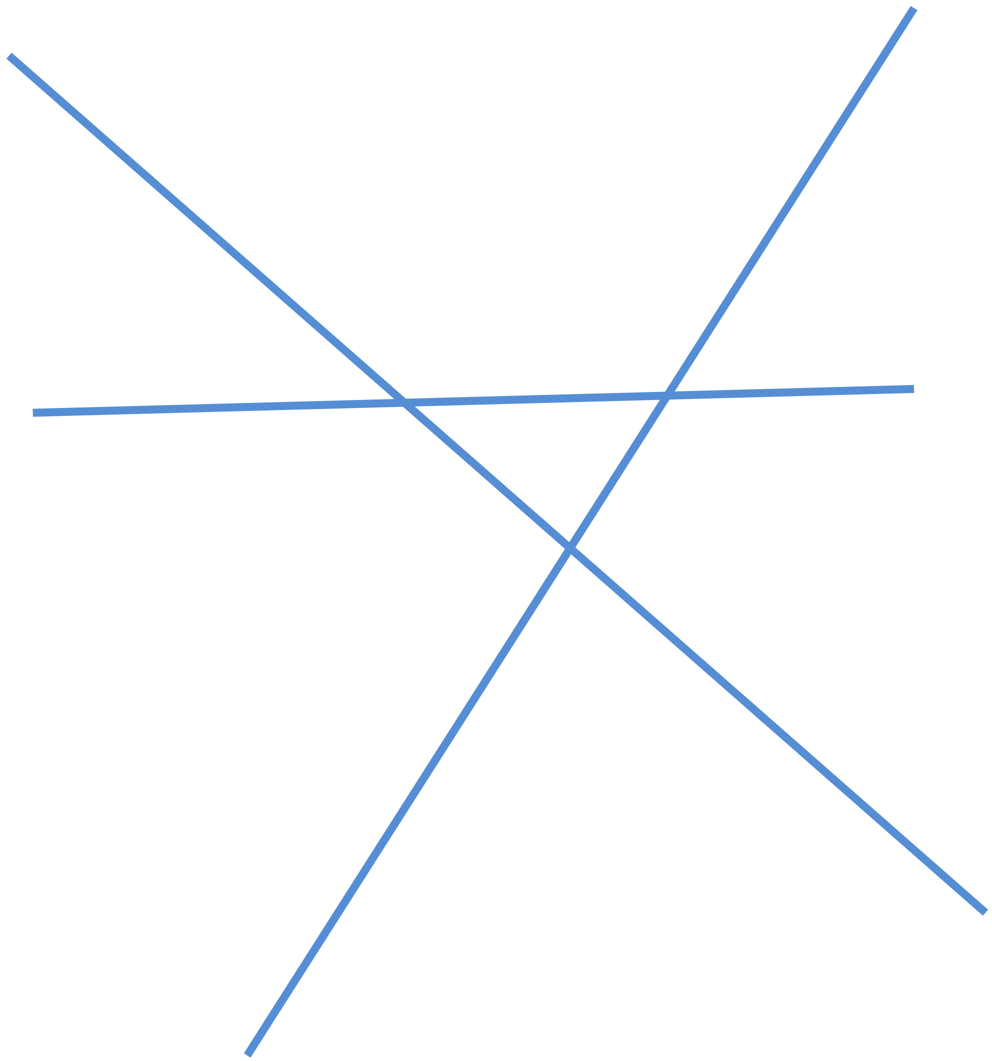
Motivation


Castrol
Chevron
chili's
Cirrus
MSNBC
Colgate
Comfort
Coors Light
Corona Extra
DQ
Daytona
Dell
Denny's
DHL
DirecTV
Domino's
ESPN
Esso
FedEx
Finlandia Vodka
Firestone
Flowmaster
Fruit of the Loom
Gatorade
Hanes
Goodyear
Heineken
Holiday Inn Express
Hooters
Hot Wheels
IKEA
Jack in the Box
JVC
Kmart
KFC
KitKat
Kool-Aid
Land Rover
Lay's
Lego
Lipton
Mitsubishi
Mobil
MasterCard
VISA
NASA
NBC
Nestle
Nintendo 64
Nivea body
NOS
Ocean Spray
Old Navy
Oral-B
Oreo
Pepsi
Pennzoil
Pizza Hut
PowerBar
Red Lobster
Safeway Food & Drug
STP
Subway
swissair
Taco Bell
Texaco
tic tac
Toro
Tropicana
United States Postal Service
United Way
Valvoline
Virgin
Wal*Mart
Xerox
Zellers
zippo
Zantac

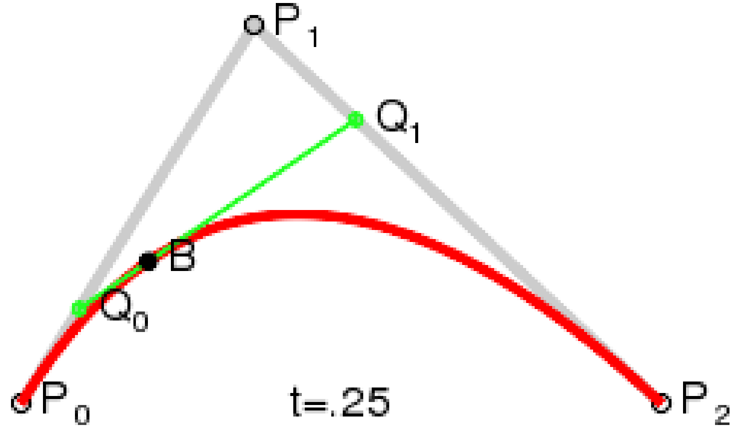
Motivation

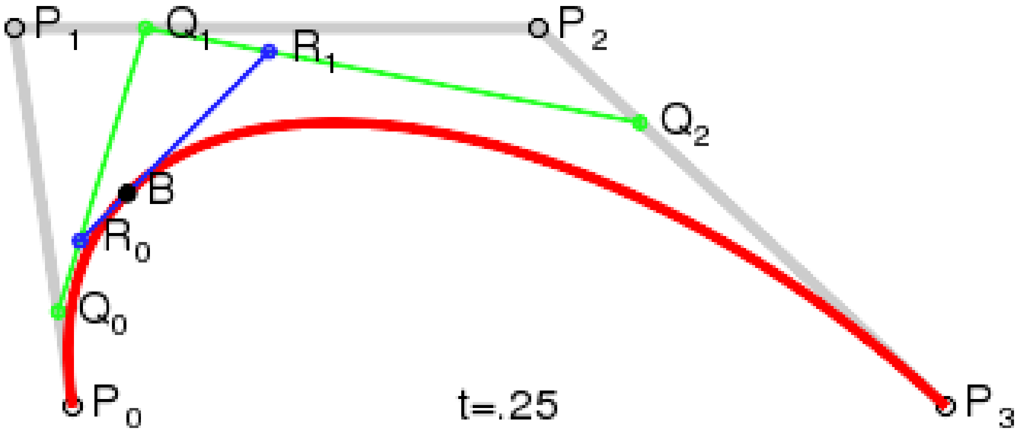
Portugal
Spain
Balearic Sea
Alboran Sea
Andorra
Gibraltar
Ortigueira
Viveiro
Cedeira
A Coruña
Arteixo
Betanzos
Carballo
Lugo
Santiago
Muros
Lalín
Ribeira
Carballiño
Pontevedra
Ourense
Vigo
Gijón
Oviedo
Infiesto
Llanes
Langreo
Santander
Laredo
Torrelavega
Getxo
Bilbao
Vitoria
Donostia-San Sebastián
Irún
Bayonne
Dax
Orthez
Pau
Tarbes
Lourdes
Auch
Toulouse
Castres
Muret
Mazamet
Béziers
Carcassonne
Narbonne
Perpignan
Iparragirre
Pamplona
Logroño
Tudela
Tarazona
Huesca
Barbastro
Monzón
Burgos
Ponferrada
León
Astorga
Las Médulas
Benavente
Palencia
Aranda de Duero
Soria
Zaragoza
Sariñena
Lleida
Manresa
Girona
El Moianès
Sabadell
Mataró
Barcelona
Vilanova i la Geltrú
Reus
Tarragona
Caspe
Alcañiz
Valladolid
Zamora
Viana do Castelo
Braga
Guimarães
Póvoa de Varzim
Maia
Vila Real
Oporto
Valongo
S. João da Madeira
Aveiro
Viseu
Coimbra
Figueira da Foz
Pombal
Salamanca
Segovia
Sigüenza
Calatayud
Alcobendas
Guadalajara
Madrid
Fuenlabrada
Aranjuez
Toledo
Cuenca
Teruel
Ciudad Rodrigo
Guijuelo
Béjar
Plasencia
Coria
Talavera de la Reina
Navalmoral de la Mata
Cáceres
Trujillo
Castellón de la Plana
Borriana
Peñíscola
Sagunt
Valencia
Torrent
Palma de Mallorca
Llevant
Inca
Alcúdia
Artà
Felanitx
Sant Antoni de Portmany
Ibiza
Caldas da Rainha
Santarém
Torres Vedras
Loures
Queluz
Lisbon
Setúbal
Évora
Badajoz
Montijo
Mérida
Don Benito
Almendralejo
Zafra
Ciudad Real
Manzanares
La Solana
Puertollano
Valdepeñas
Albacete
Gandia
Alcoi
Benidorm
Elda
Alicante
Elx-Elche
Santa Pola
Peñarroya-Pueblonuevo
La Carolina
Linares
Úbeda
Córdoba
Andújar
Jaén
Murcia
Torrevella
Lorca
Cartagena
Baza
Águilas
Seville
Huelva
Dos Hermanas
Granada
Portimão
Loulé
Tavira
Faro
Sanlúcar de Barrameda
Jerez
Cádiz
Marbella
Fuengirola
Málaga
Vélez-Málaga
Adra
Roquetas de Mar
Almería
Algiers
Baraki
Blida
Chlef
Médéa

Motivation

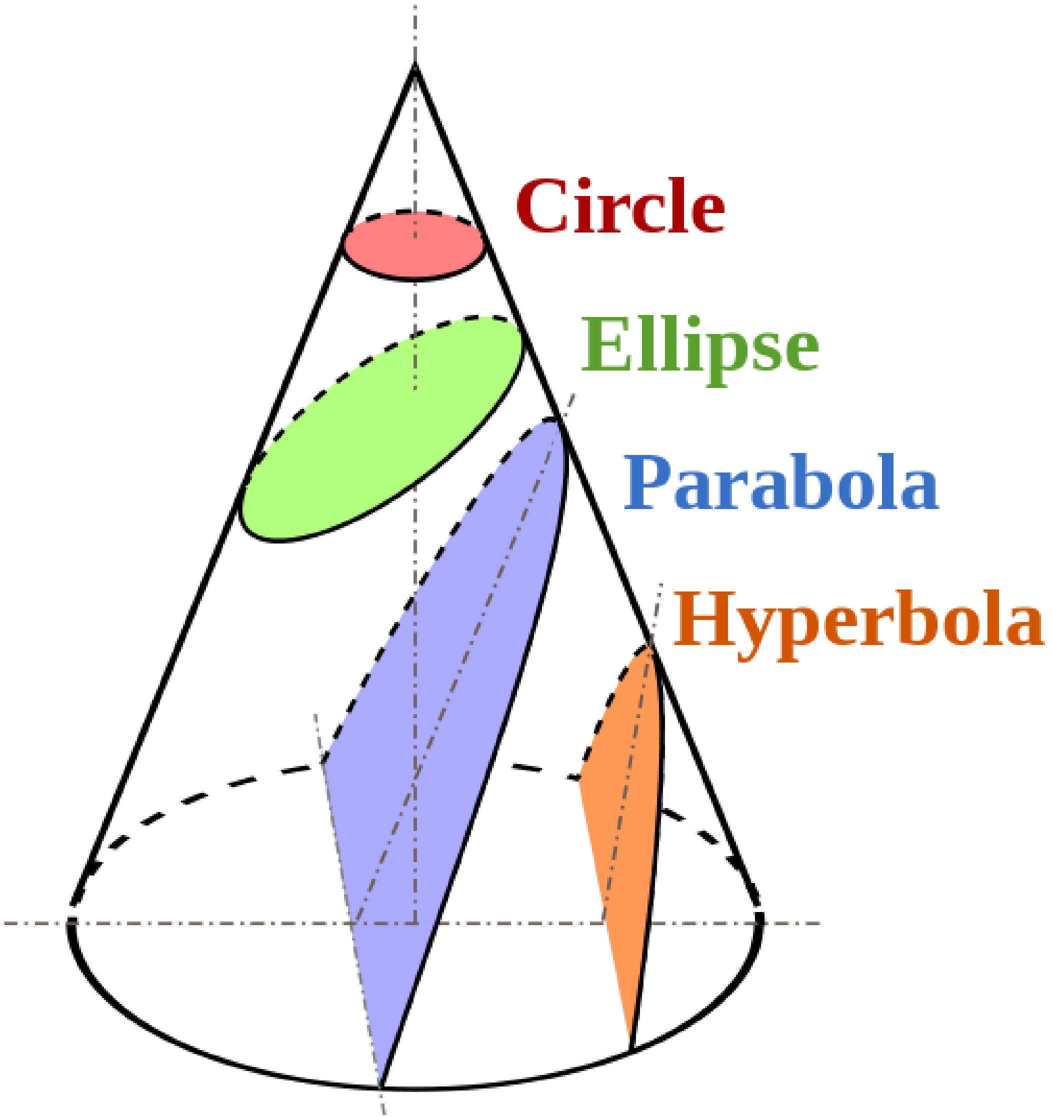


Motivation

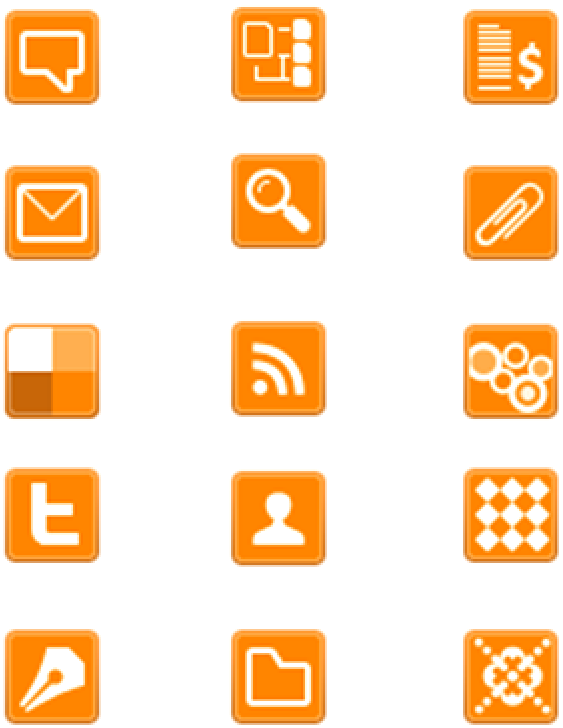




Motivation



Motivation

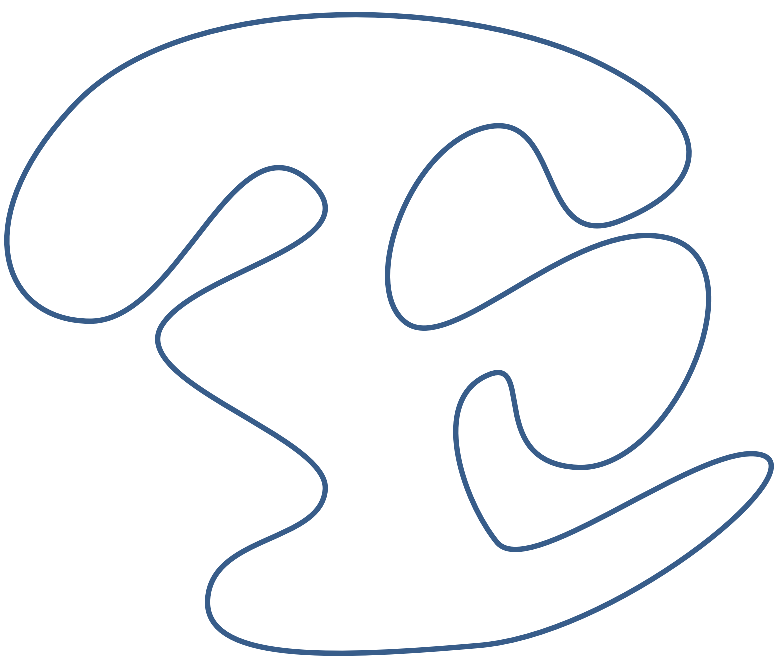


Constant colors

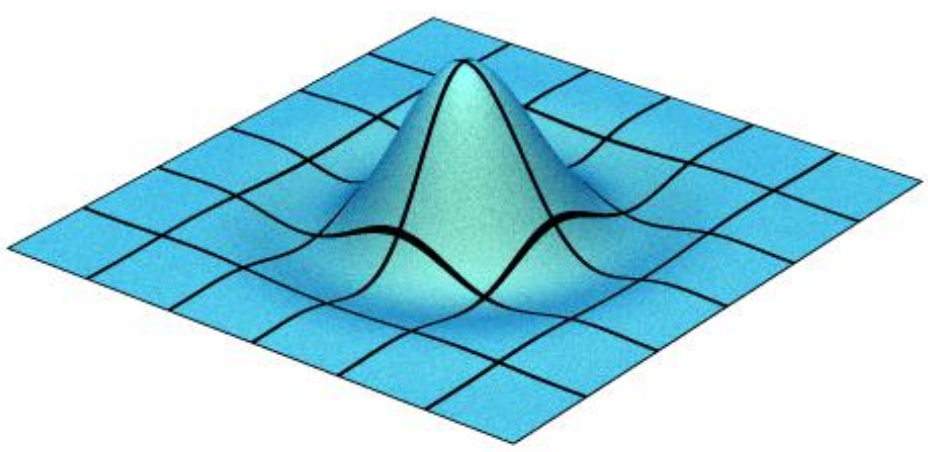
Color gradients

Motivation

Input



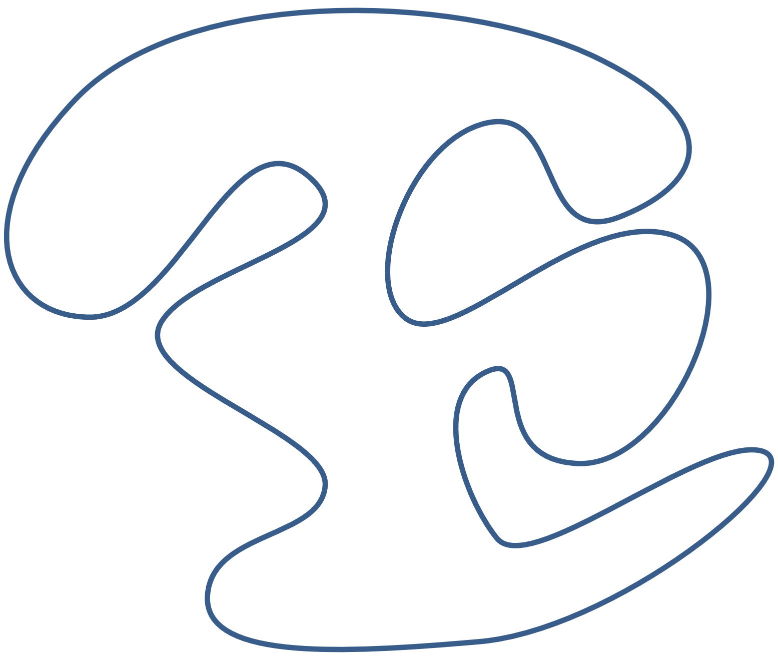
Curve Boundary



Piecewise Filter

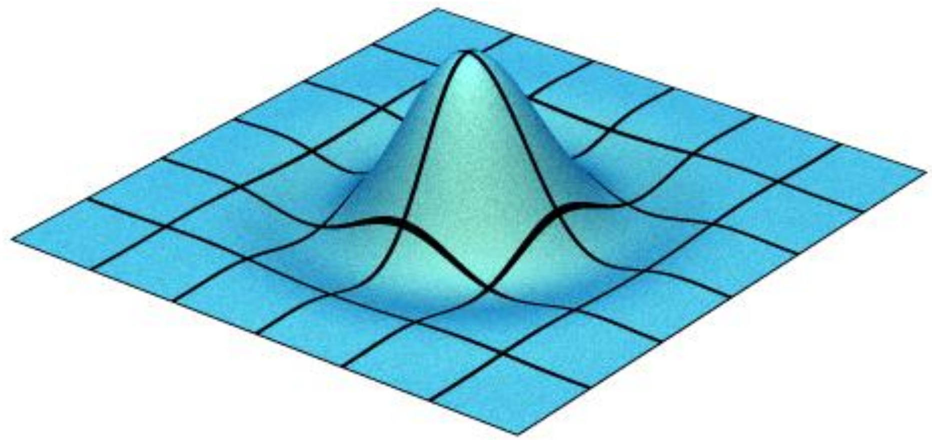
Input

[Manson and Schaefer, 2011]
"Wavelet Rasterization"



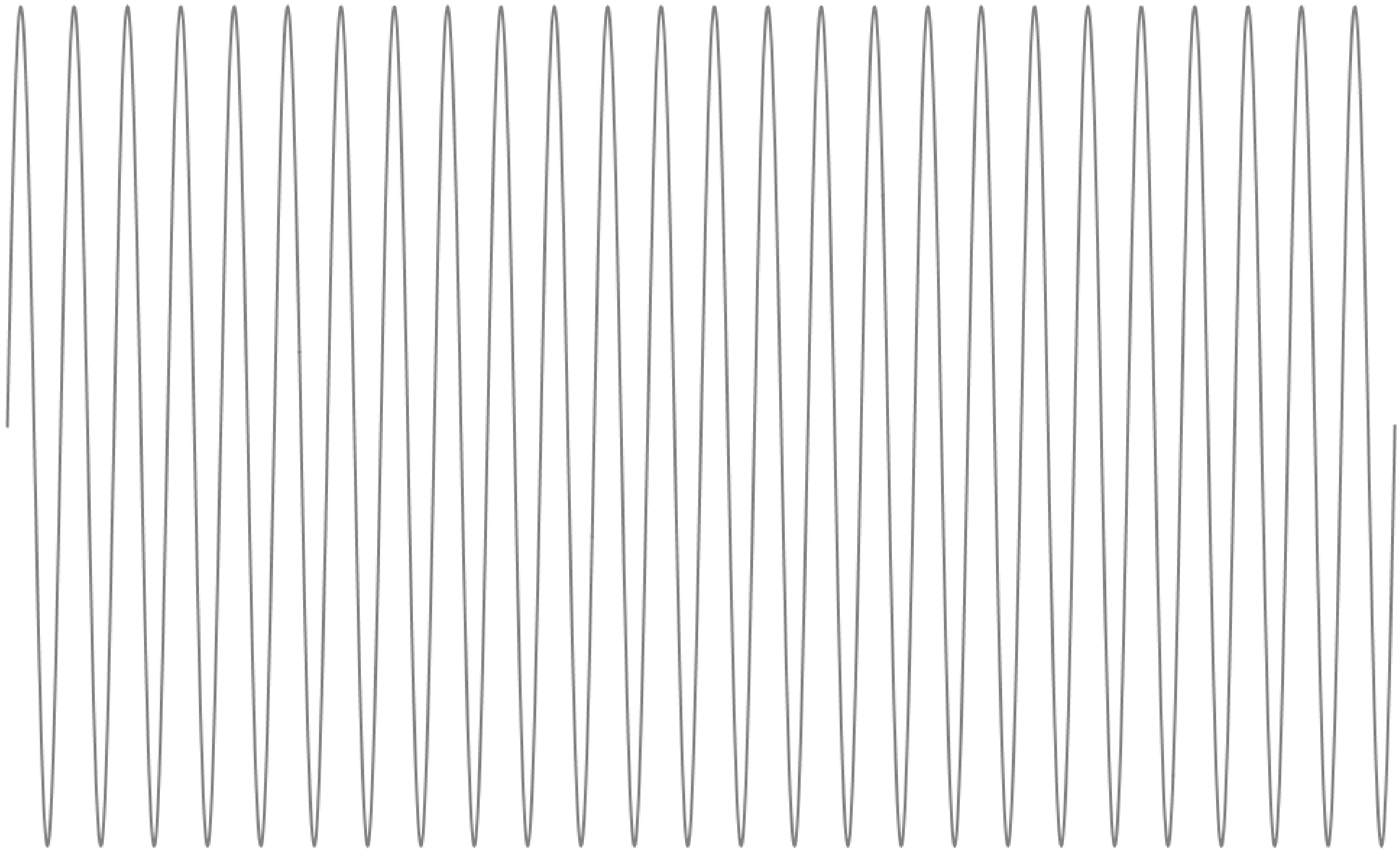
Curve Boundary

[Duff, 1989]
"Polygon scan conversion by exact convolution"

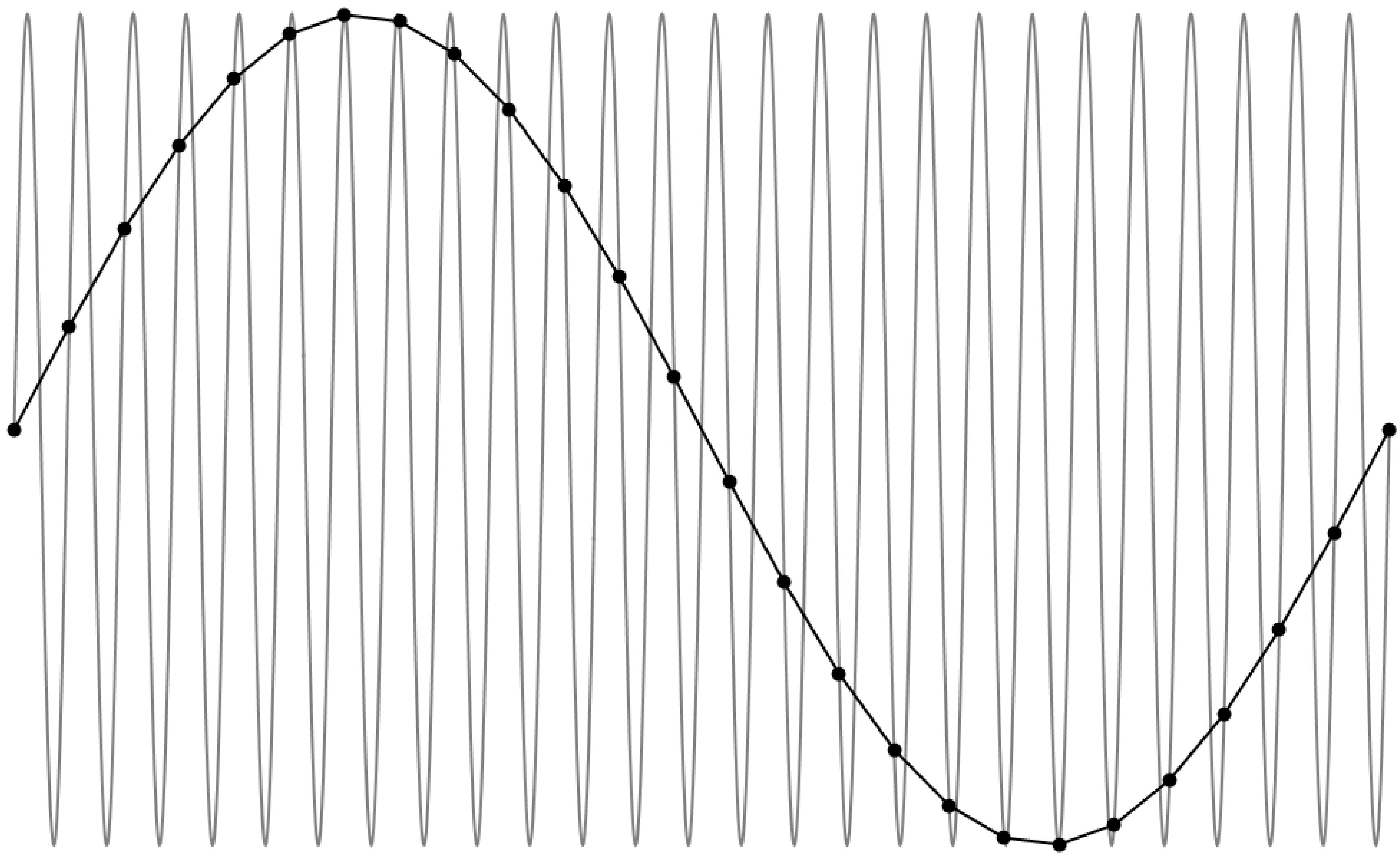


Piecewise Filter

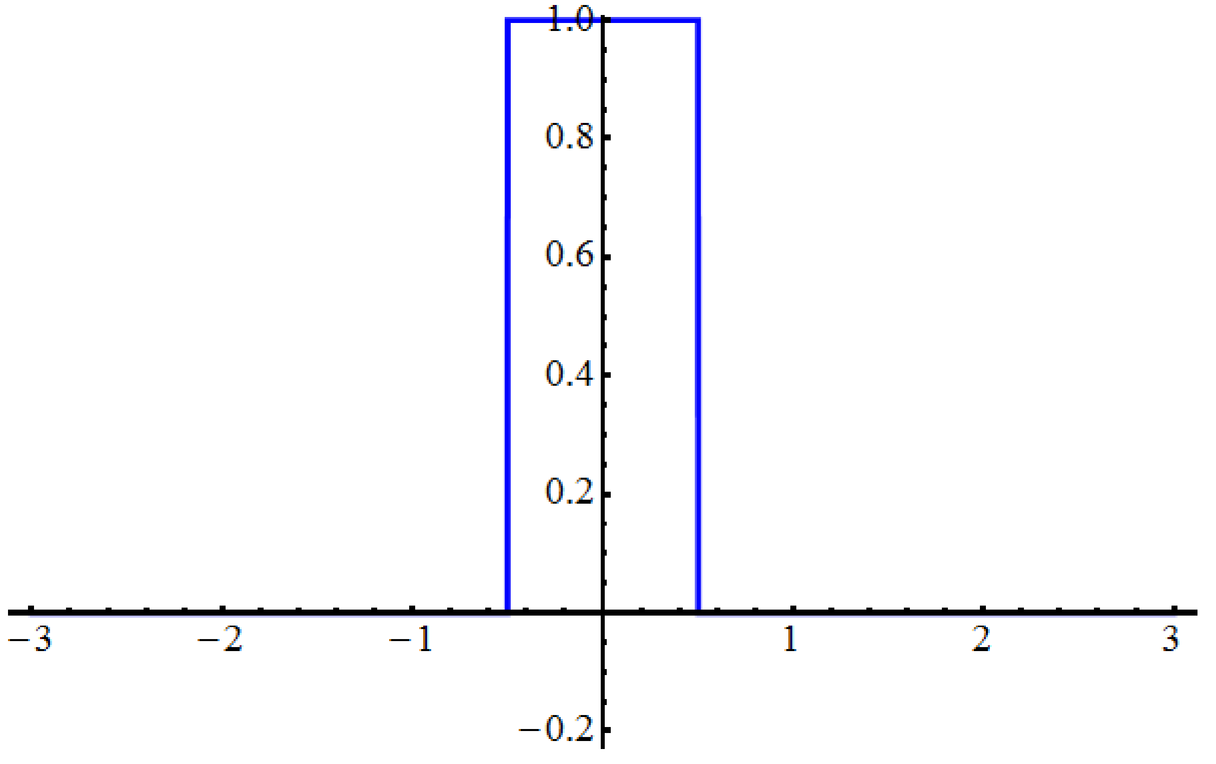
Signal Sampling



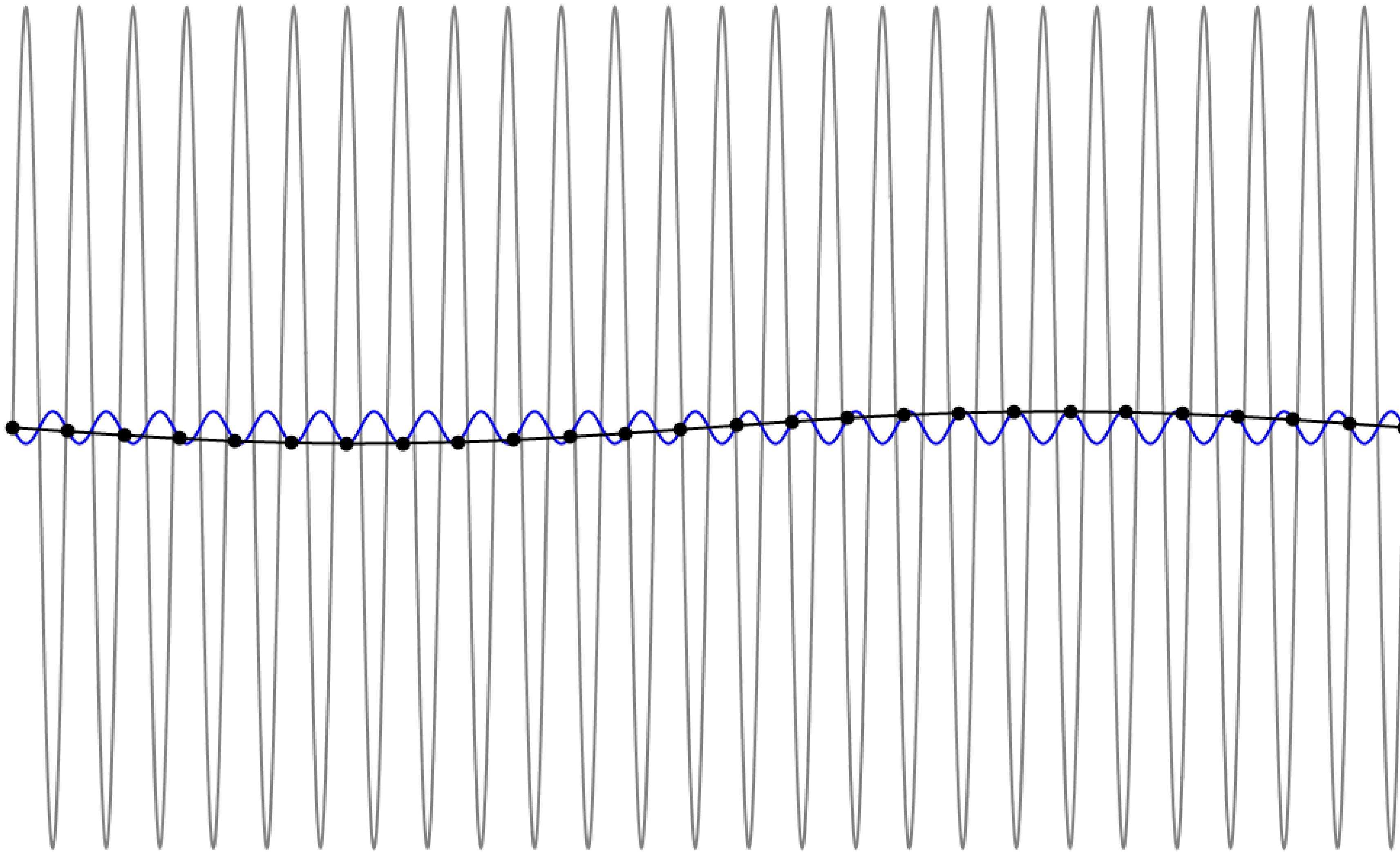
Signal Sampling



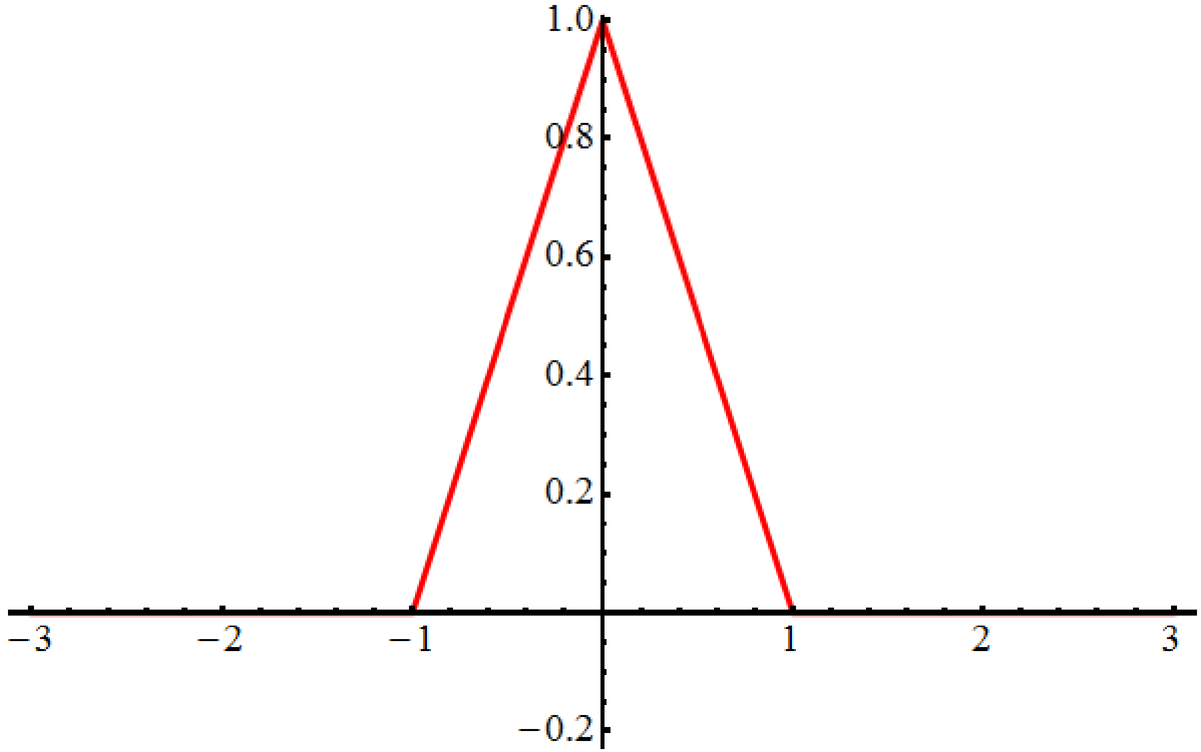
Signal Sampling



Signal Sampling



Signal Sampling



Signal Sampling

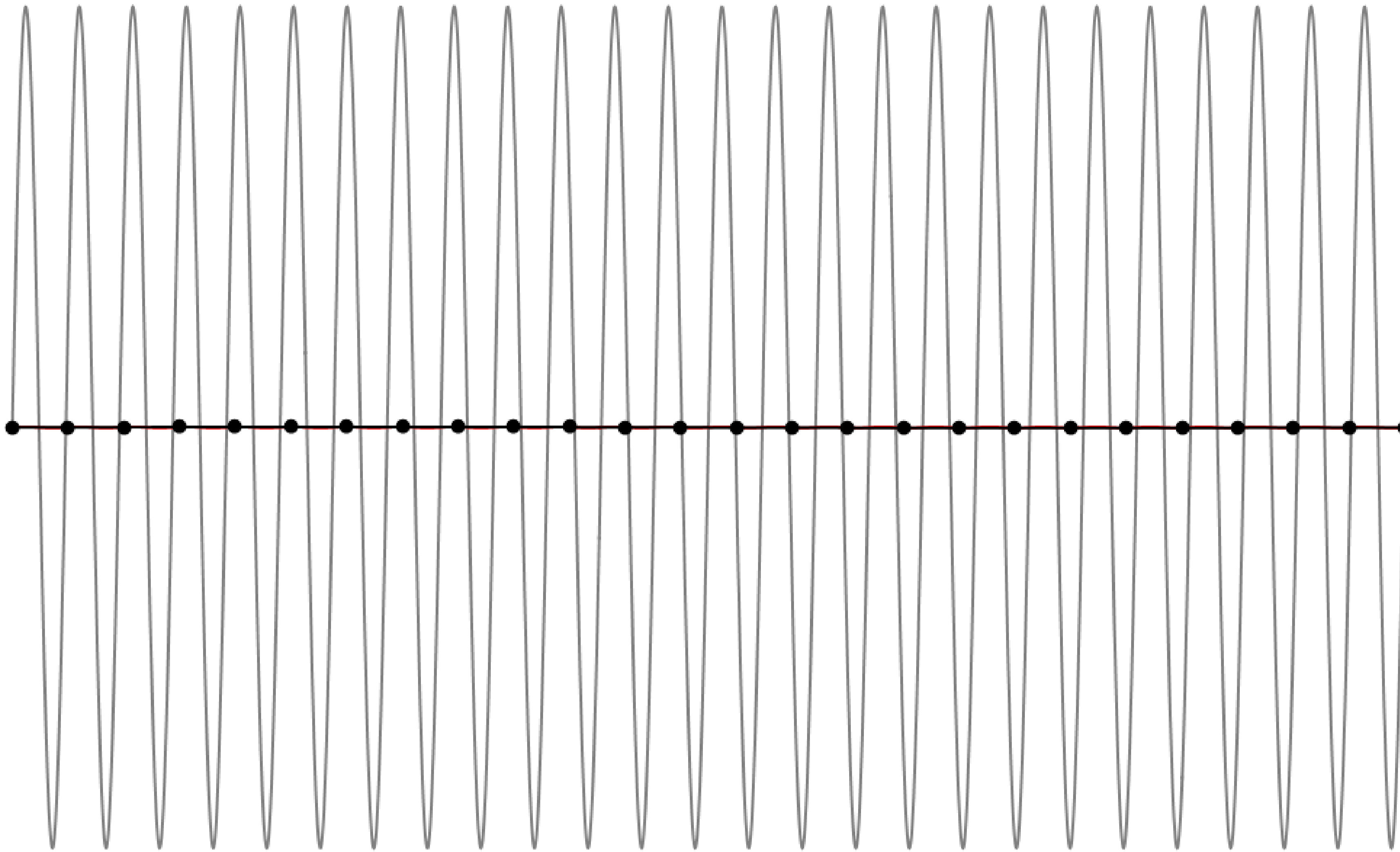


Image Sampling

Image

Image Sampling

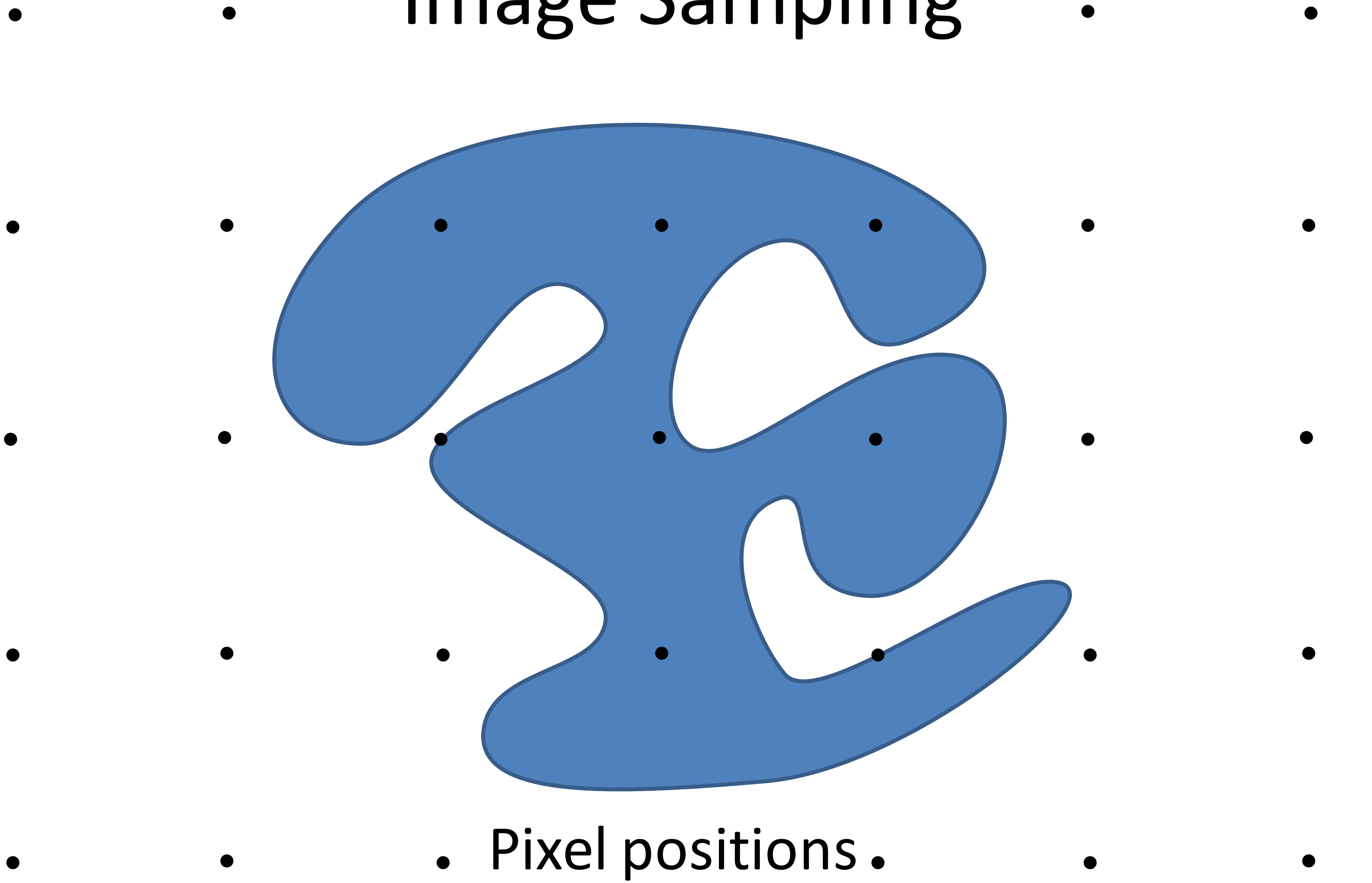
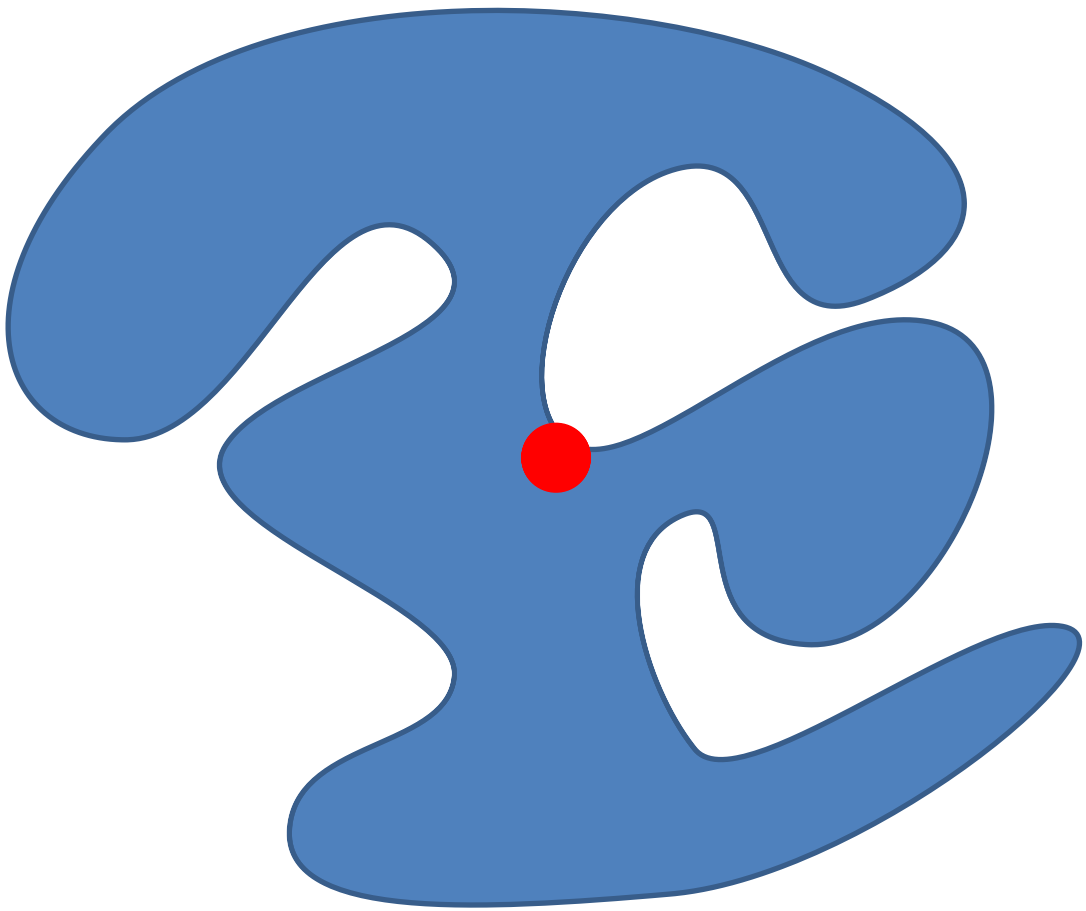
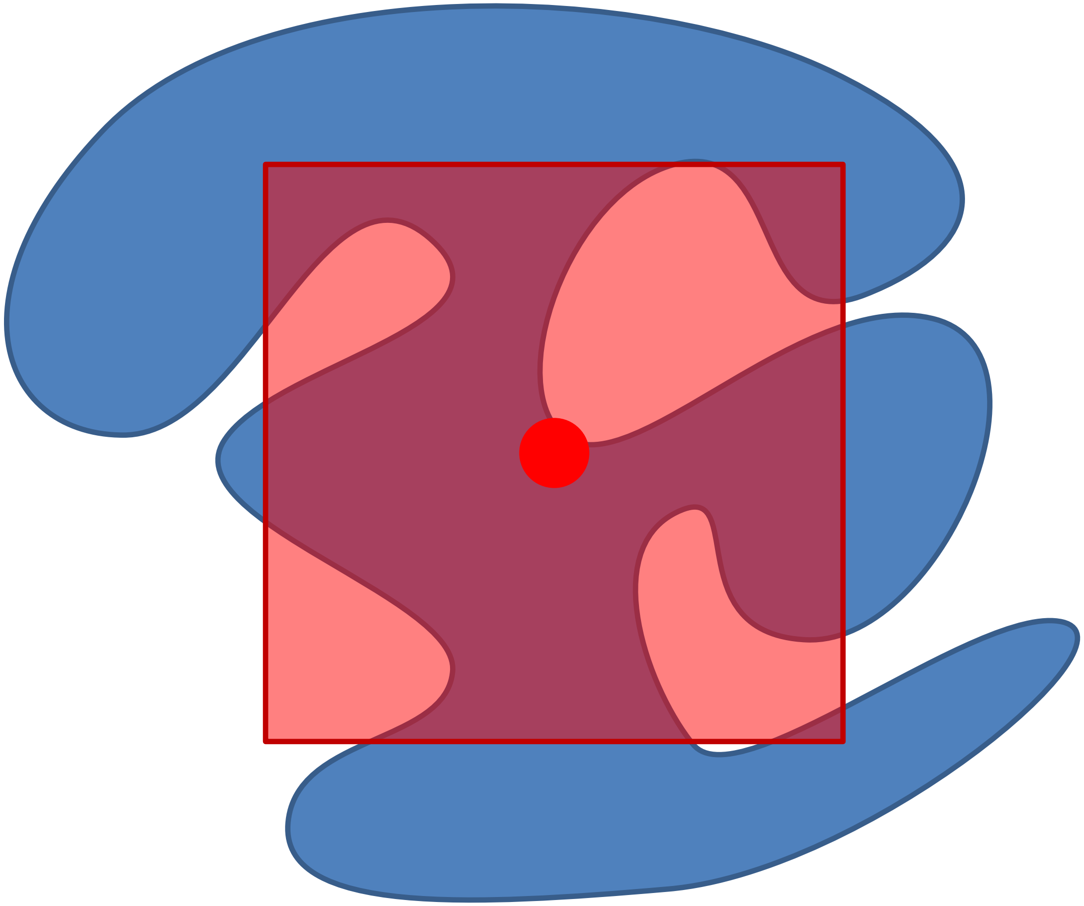


Image Sampling



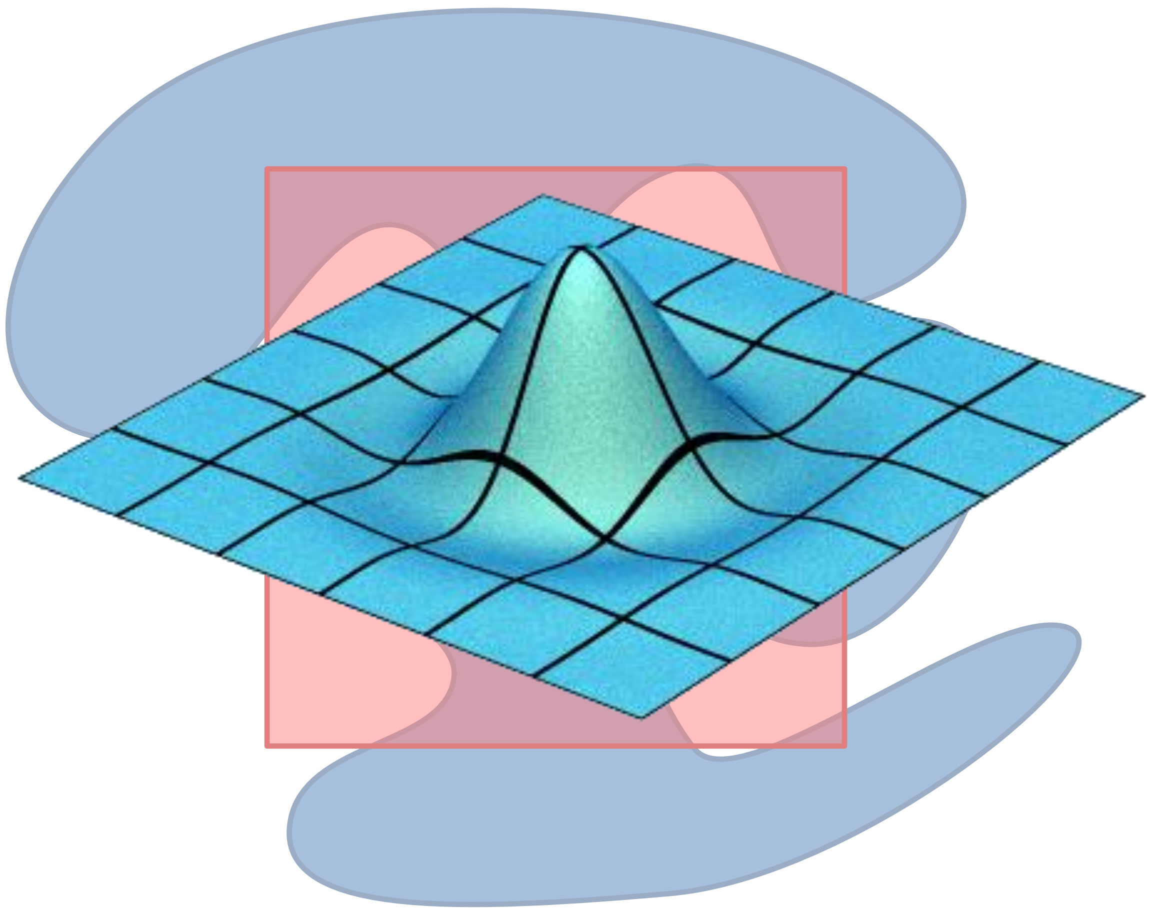
Evaluate at point

Image Sampling



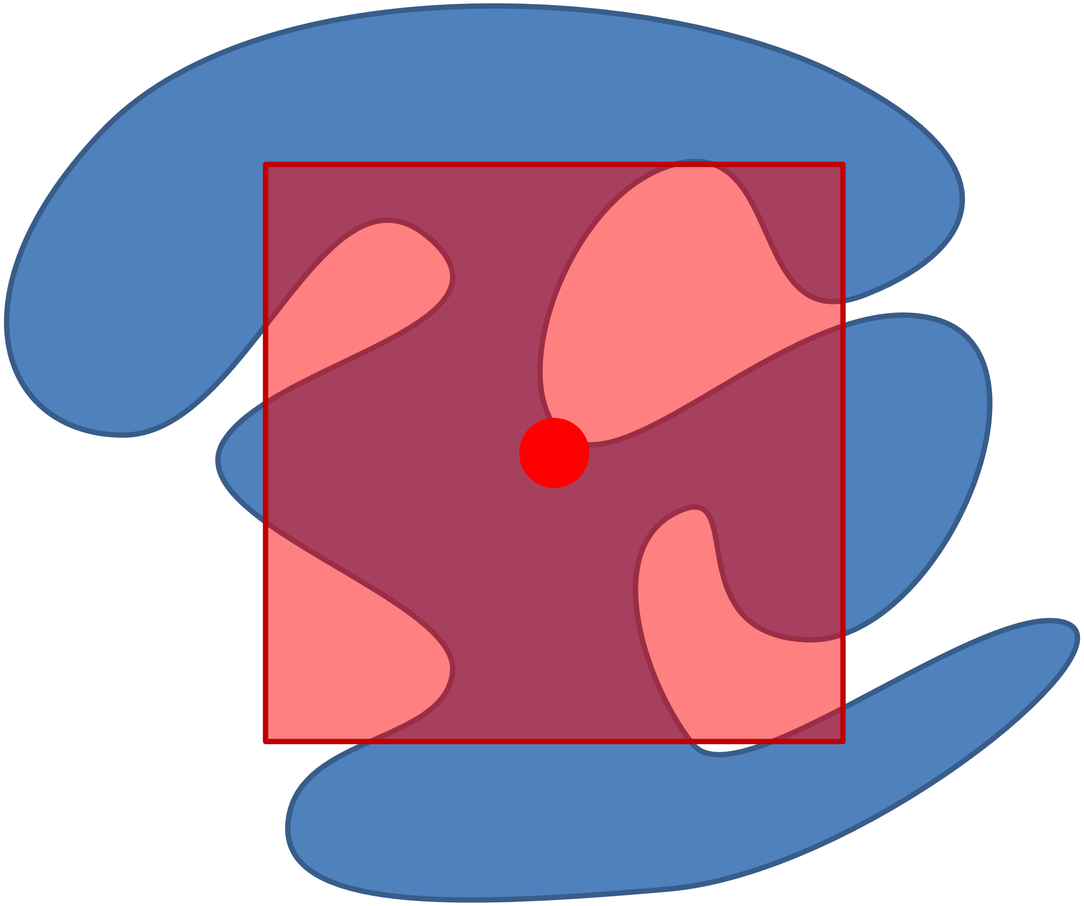
Center filter at point

Image Sampling



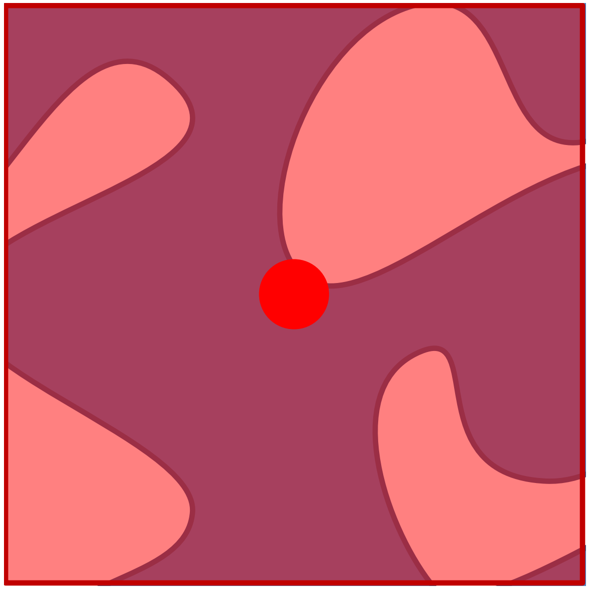
Center filter at point

Image Sampling



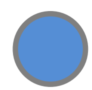
Center filter at point

Image Sampling



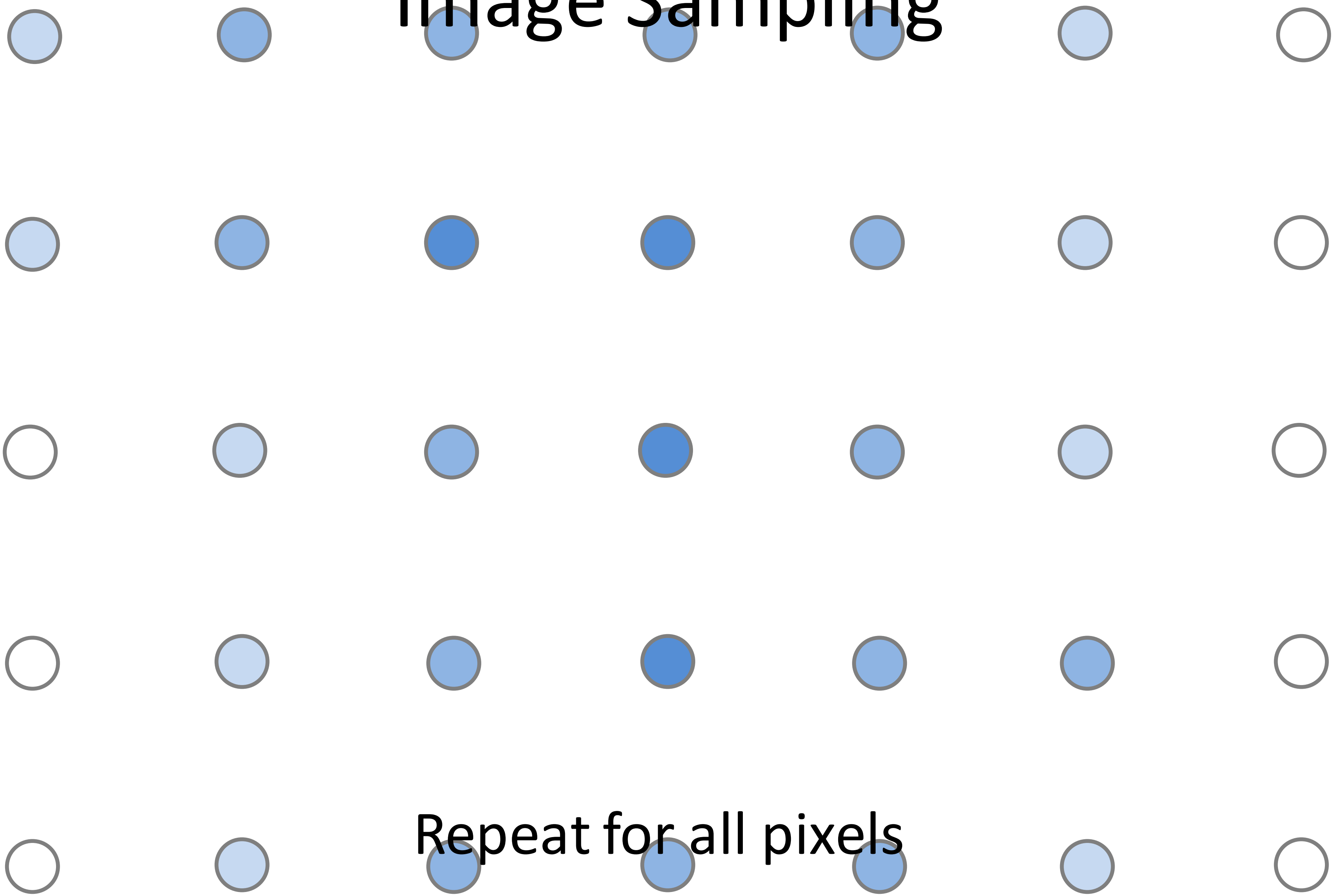
Multiply

Image Sampling



Integrate

Image Sampling



Repeat for all pixels

Derivation

$$\iint_{\mathbb{R}^2} I(x,y)h(x-\ell_x, y-\ell_y)\; dx\, dy$$

Derivation

$$\iint_{x,y \in M} c(x,y) h(x - \ell_x, y - \ell_y)\, dx\, dy$$

Derivation

$$\iint_{x,y\in M} c(x,y)h(x-\ell_x, y-\ell_y)\;dx\,dy$$

$$\oint_{p(s)\in\partial M} F(p(s))\cdot n(s)\;ds = \iint_{x,y\in M} \nabla\cdot F(x,y)\;dx\,dy$$

Derivation

$$\iint_{x,y\in M} \boxed{c(x,y)h(x-\ell_x, y-\ell_y)}\, dx\, dy$$

$$\oint_{p(s)\in\partial M} F(p(s))\cdot n(s)\, ds = \iint_{x,y\in M} \boxed{\nabla\cdot F(x,y)}\, dx\, dy$$

$$\nabla\cdot F(x,y) = c(x,y)h(x-\ell_x, y-\ell_y)$$

Derivation

$$\iint_{x,y\in M} \boxed{c(x,y)h(x-\ell_x, y-\ell_y)}\, dx\, dy$$

$$\oint_{p(s)\in\partial M} F(p(s))\cdot n(s)\, ds = \iint_{x,y\in M} \boxed{\nabla\cdot F(x,y)}\, dx\, dy$$

$$\nabla\cdot F(x,y) = \underbrace{c(x,y)h(x-\ell_x, y-\ell_y)}_{f(x,y)}$$

Derivation

$$\iint_{x,y\in M} \boxed{c(x,y)h(x-\ell_x, y-\ell_y)}\, dx\, dy$$

$$\oint_{p(s)\in\partial M} F(p(s))\cdot n(s)\, ds = \iint_{x,y\in M} \boxed{\nabla\cdot F(x,y)}\, dx\, dy$$

$$\nabla\cdot F(x,y) = \underbrace{c(x,y)h(x-\ell_x, y-\ell_y)}_{f(x,y)}$$

$$F(x,y) = \begin{pmatrix} (1-\alpha)\int_{-\infty}^{x} f(u,y)\, du \\ \alpha\int_{-\infty}^{y} f(x,u)\, du \end{pmatrix}$$

Derivation

$$\iint_{x,y\in M} \boxed{c(x,y)h(x-\ell_x, y-\ell_y)}\, dx\, dy$$

$$\oint_{p(s)\in\partial M} F(p(s))\cdot n(s)\, ds = \iint_{x,y\in M} \boxed{\nabla\cdot F(x,y)}\, dx\, dy$$

$$\nabla\cdot F(x,y) = \underbrace{c(x,y)h(x-\ell_x, y-\ell_y)}_{f(x,y)}$$

$$F(x,y) = \begin{pmatrix} \int_{-\infty}^{x} f(u,y)\, du \\ 0 \end{pmatrix}$$

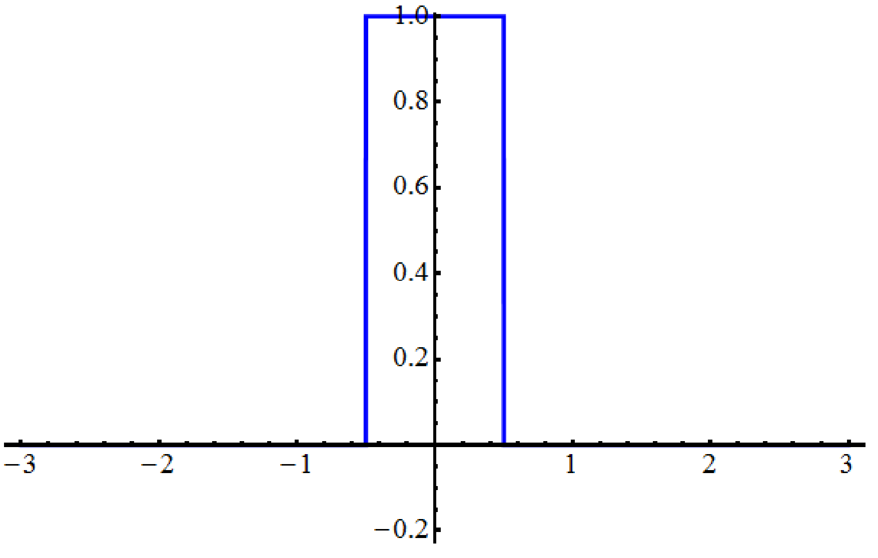
Derivation

$$\iint_{x,y \in M} c(x,y) h(x - \ell_x, y - \ell_y)\, dx\, dy$$

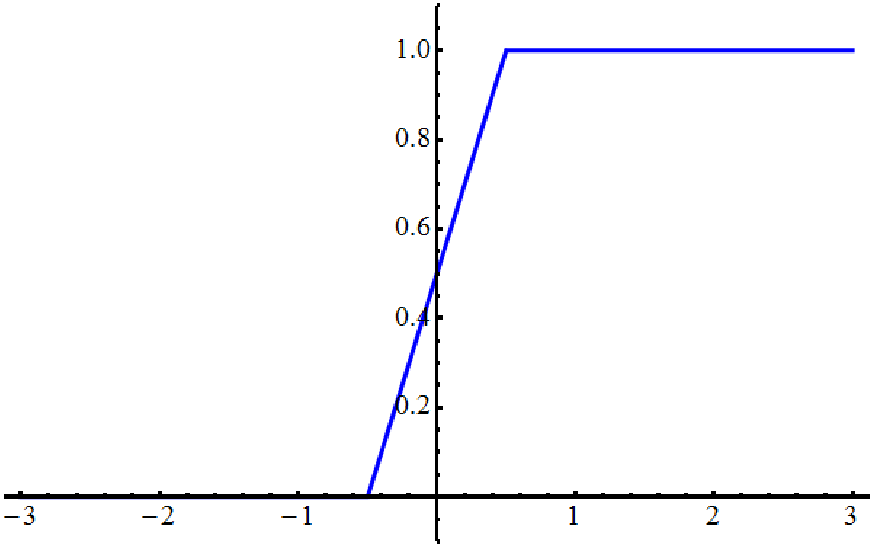
$$\boxed{\oint_{p(s) \in \partial M} F(p(s)) \cdot n(s)\, ds} = \iint_{x,y \in M} \nabla \cdot F(x,y)\, dx\, dy$$

$$\oint F_x(p(t))\, p_y'(t)\, dt$$

Filter Integrals

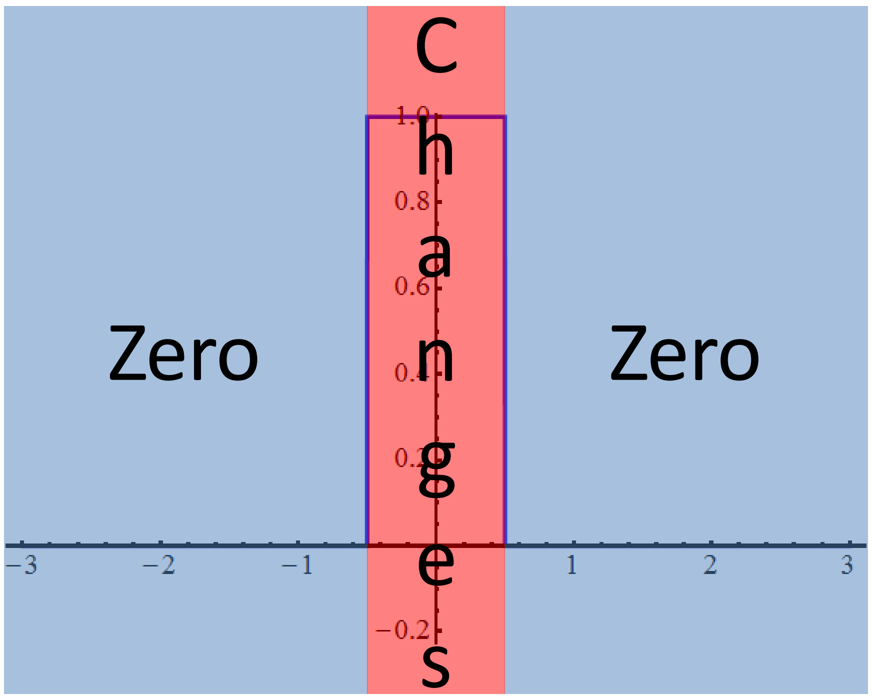


$f(x, y)$

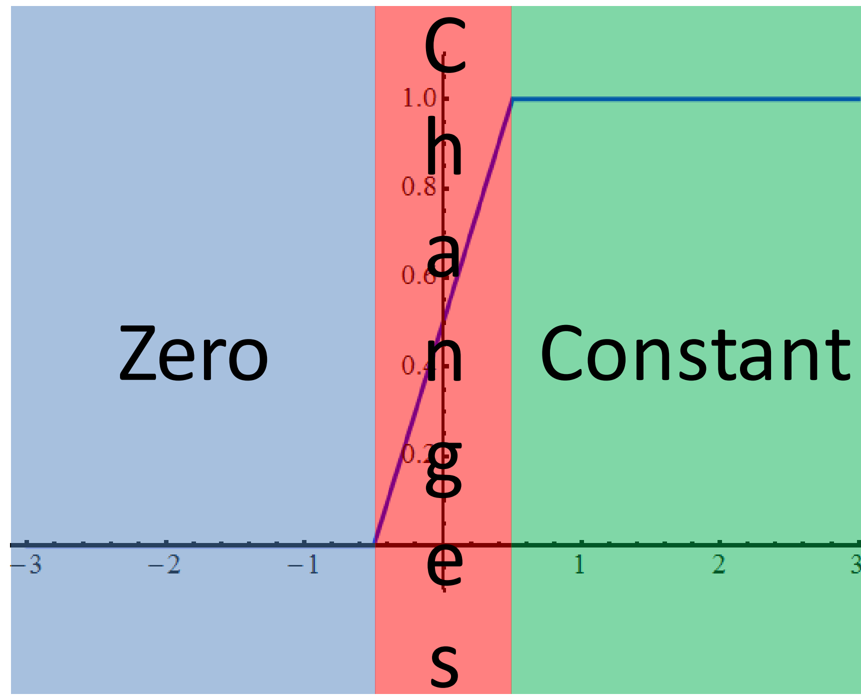


$F(x, y)$

Filter Integrals

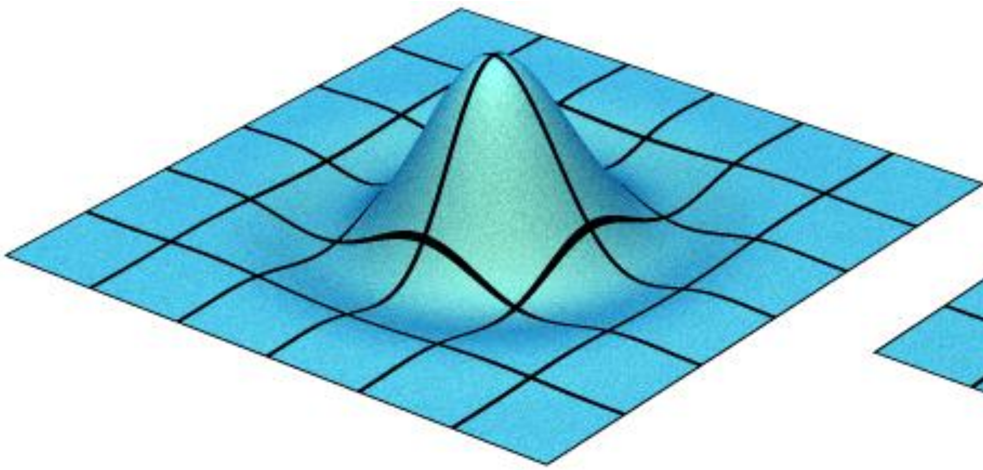


$f(x,y)$

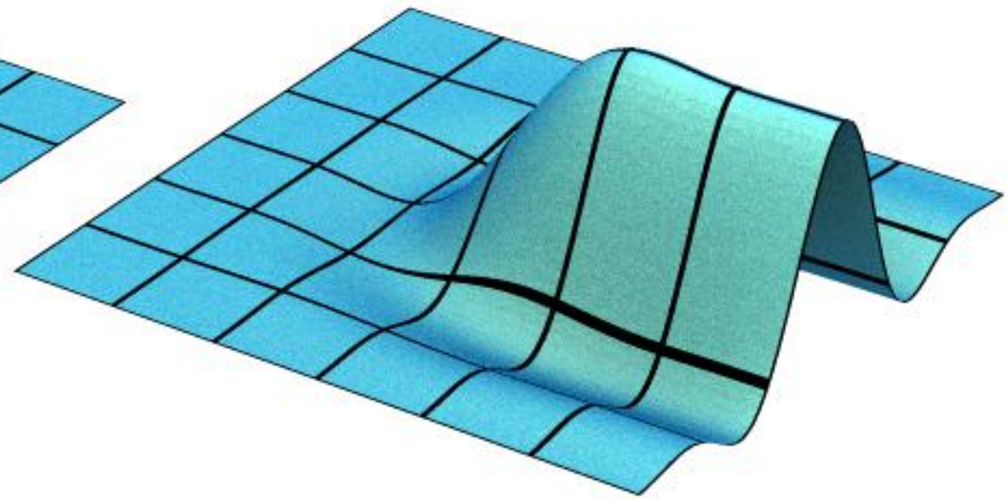


$F(x,y)$

Filter Pieces

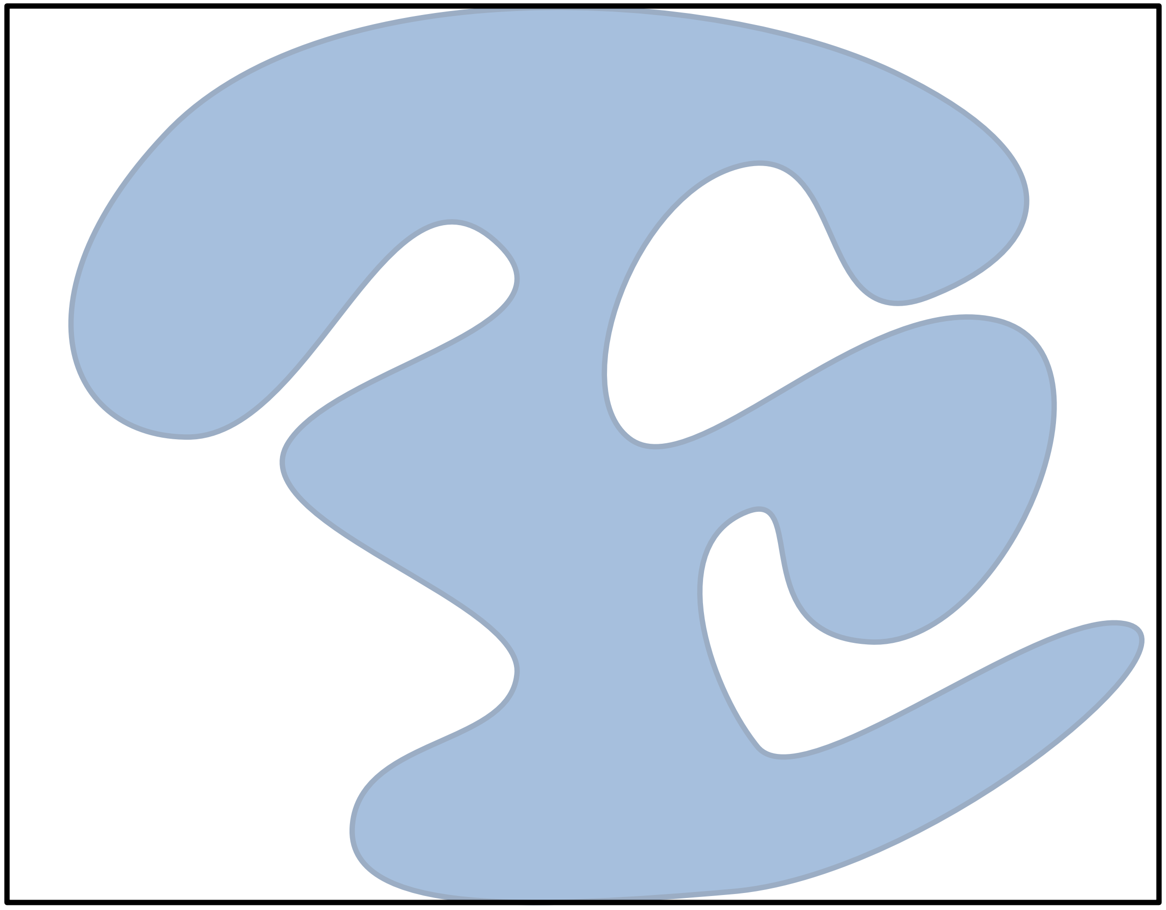


$f(x, y)$

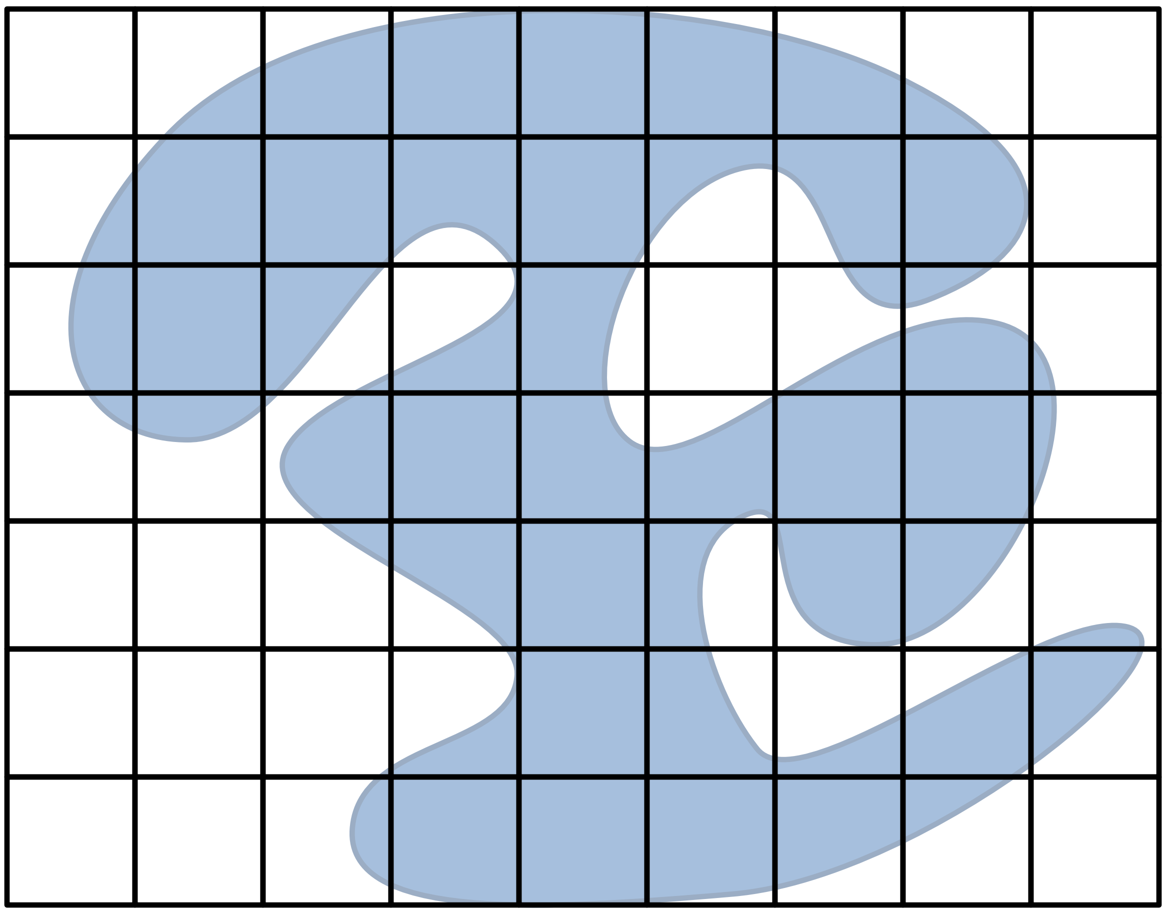


$F(x, y)$

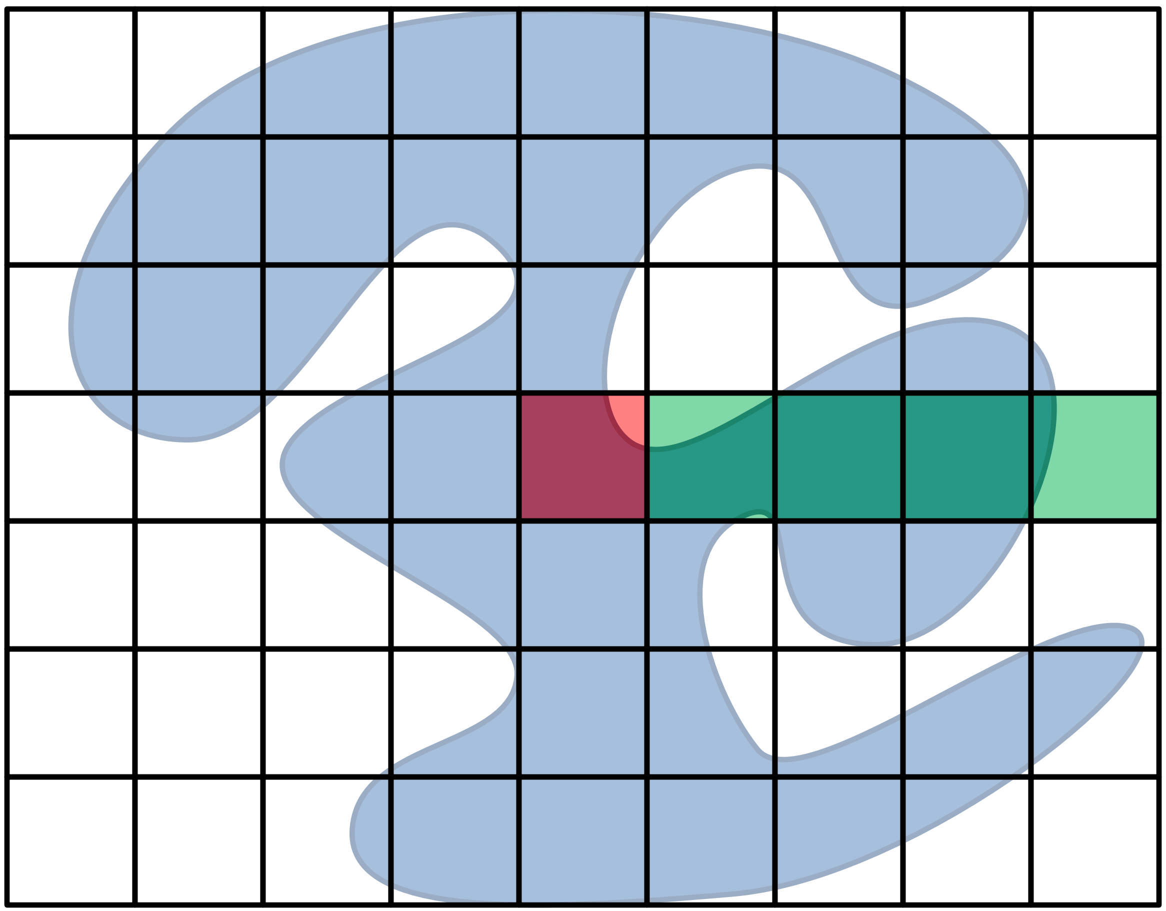
Scanline Rasterization



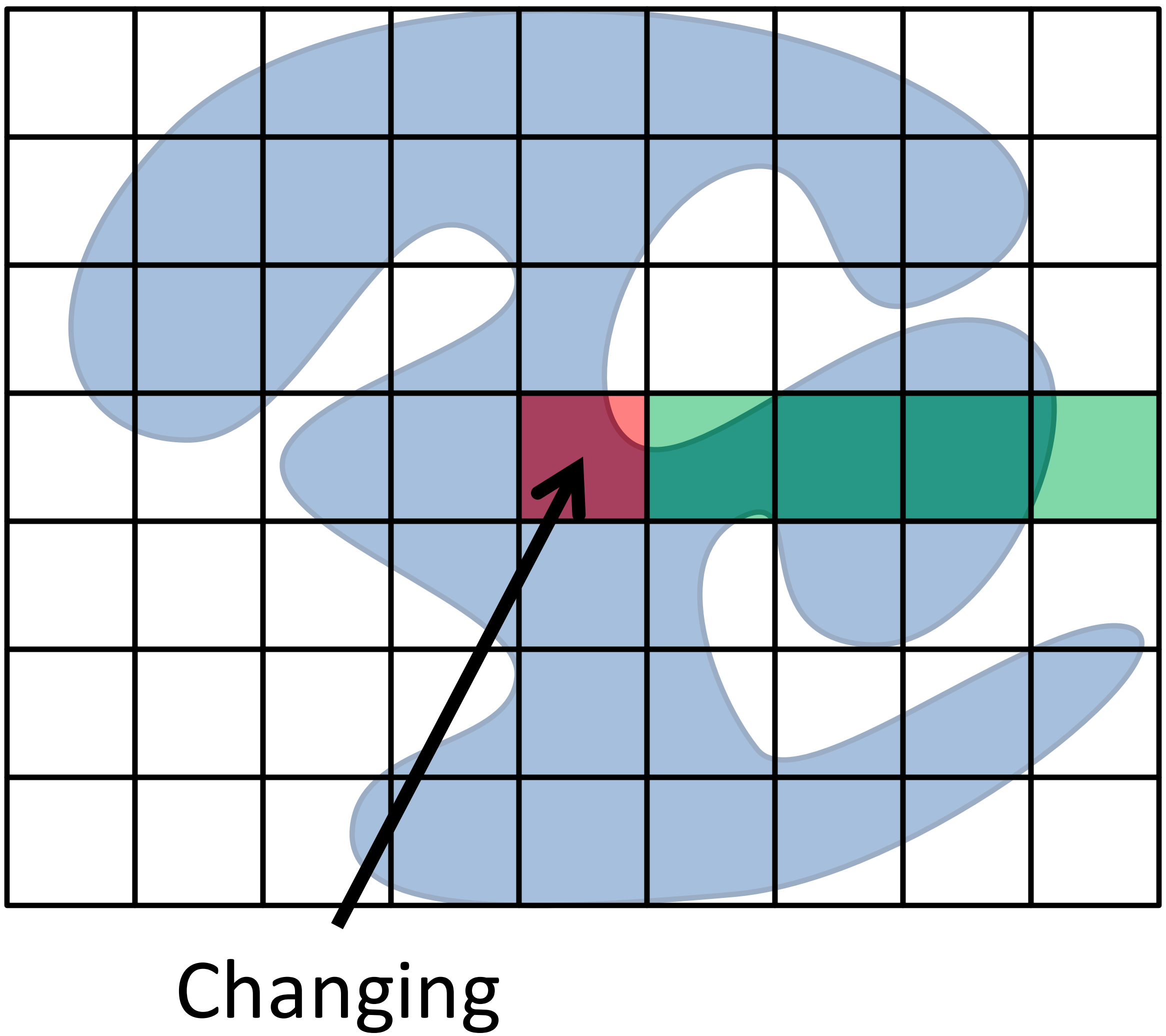
Scanline Rasterization



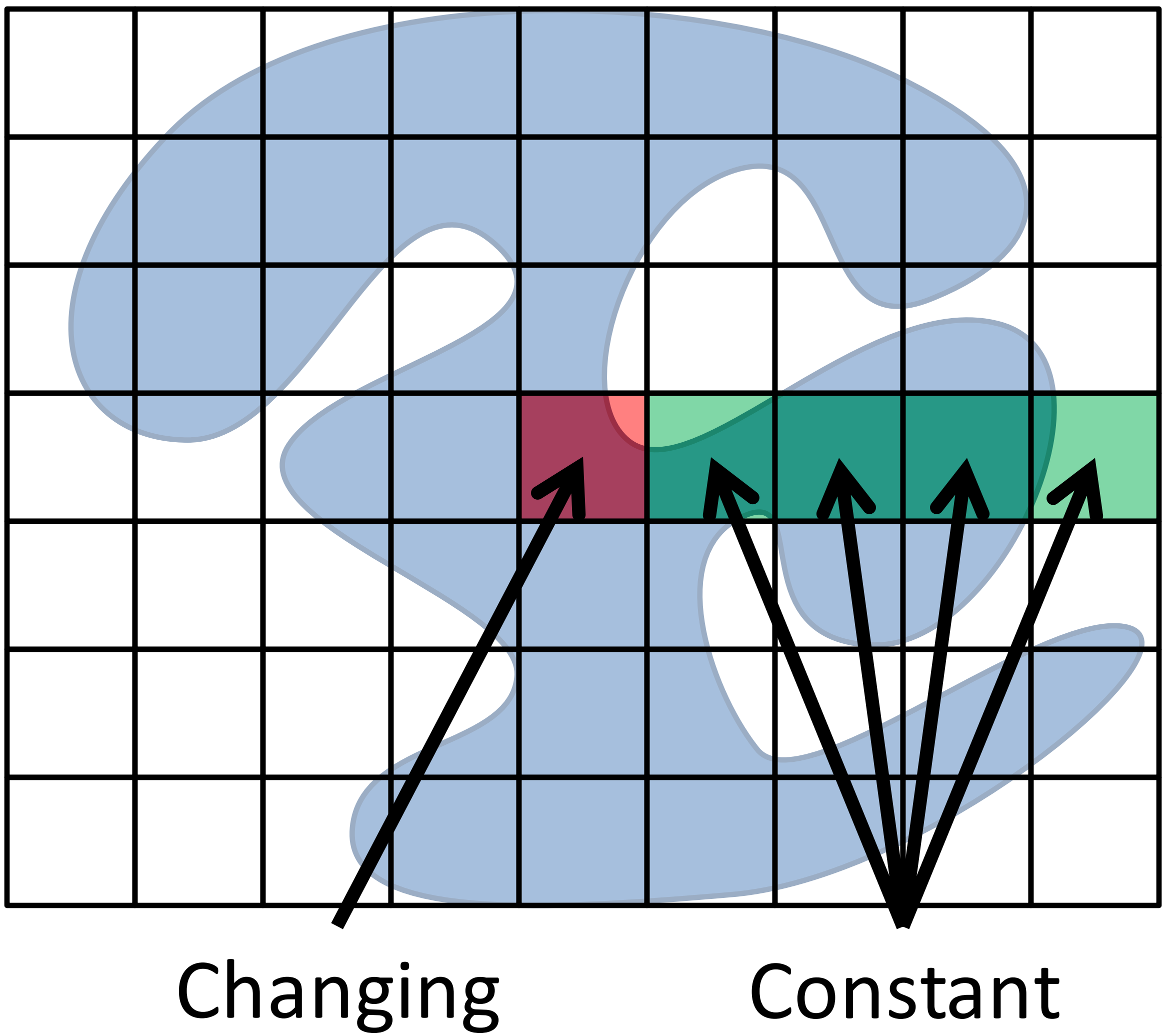
Scanline Rasterization



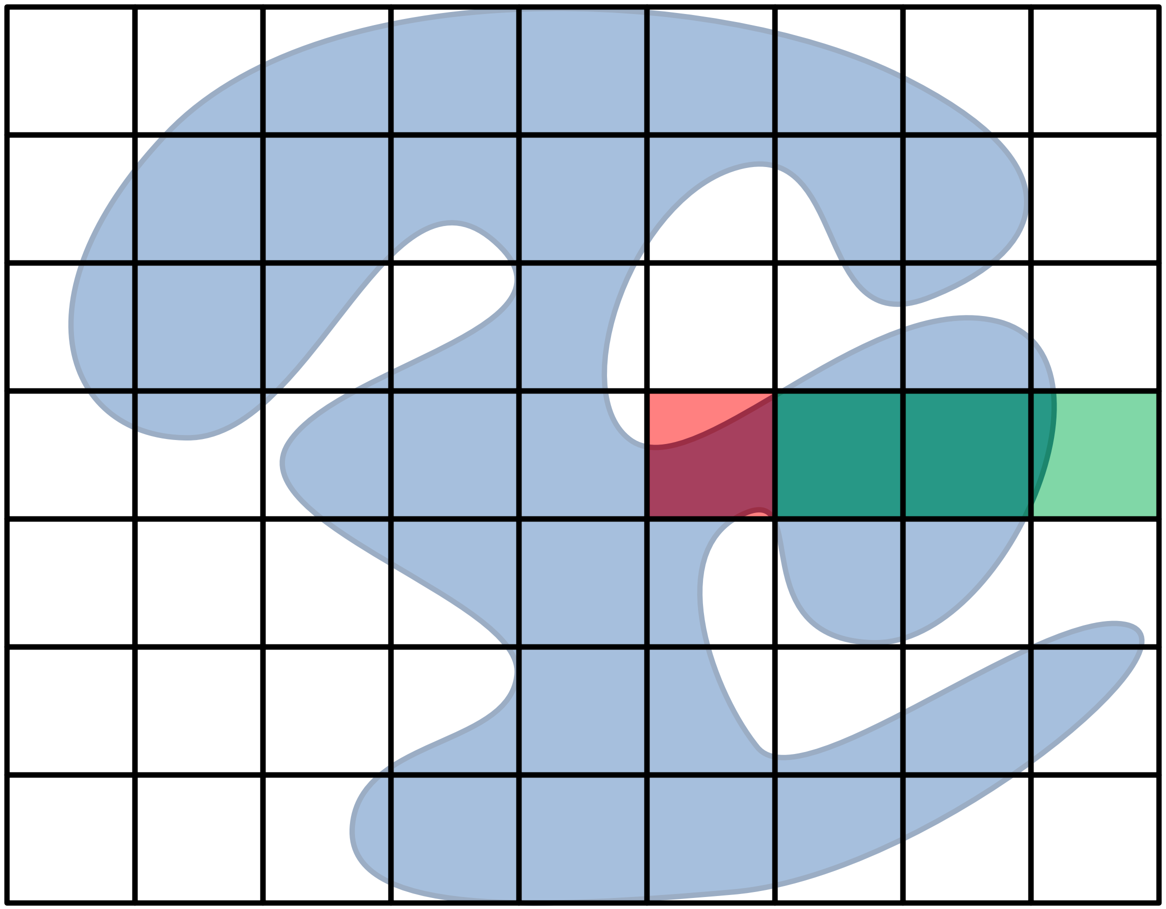
Scanline Rasterization



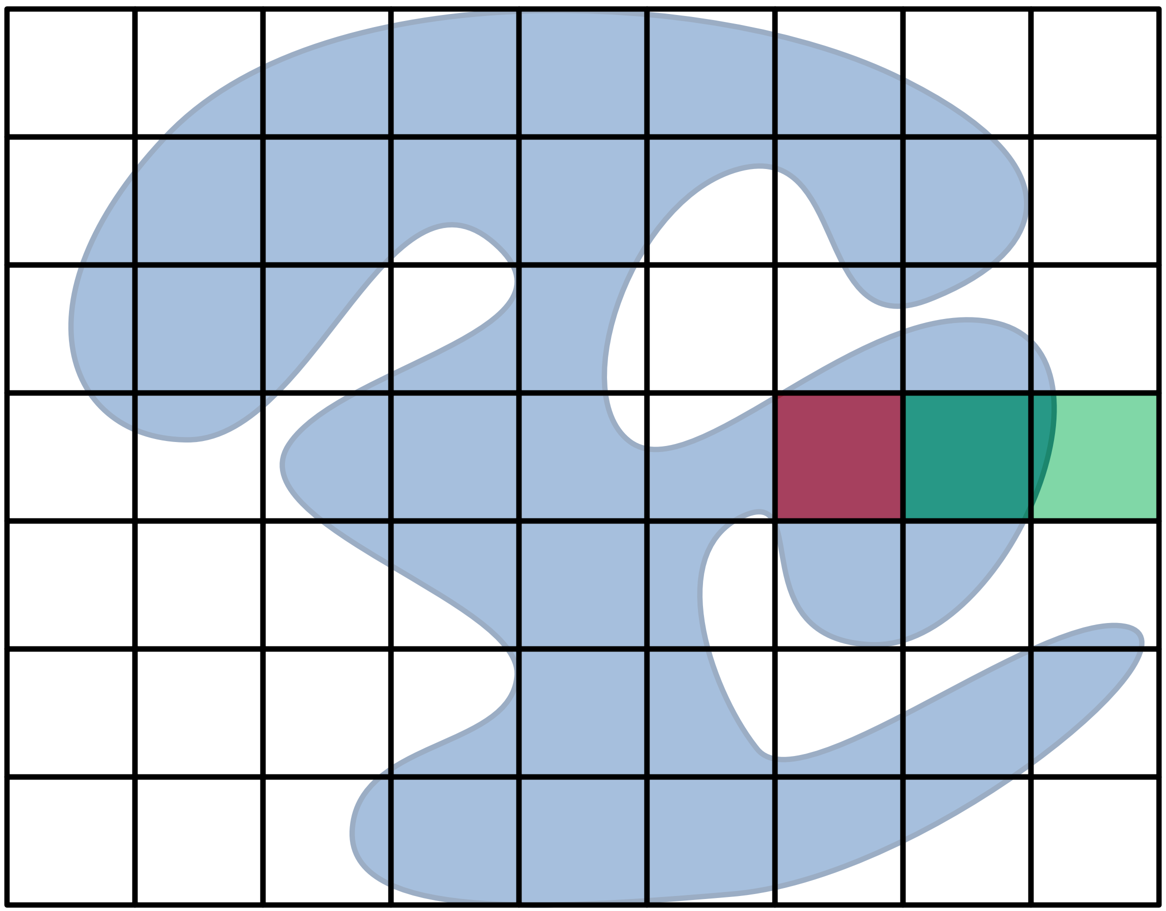
Scanline Rasterization



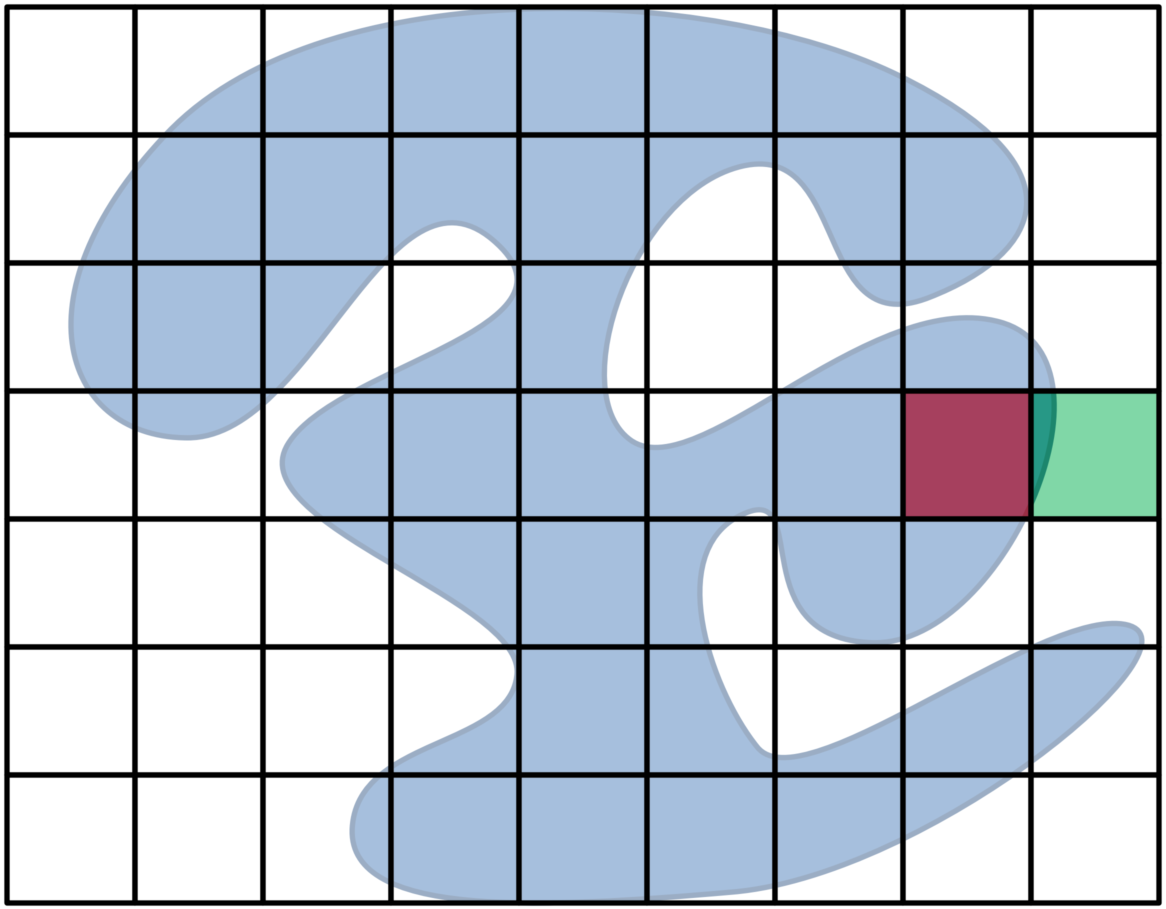
Scanline Rasterization



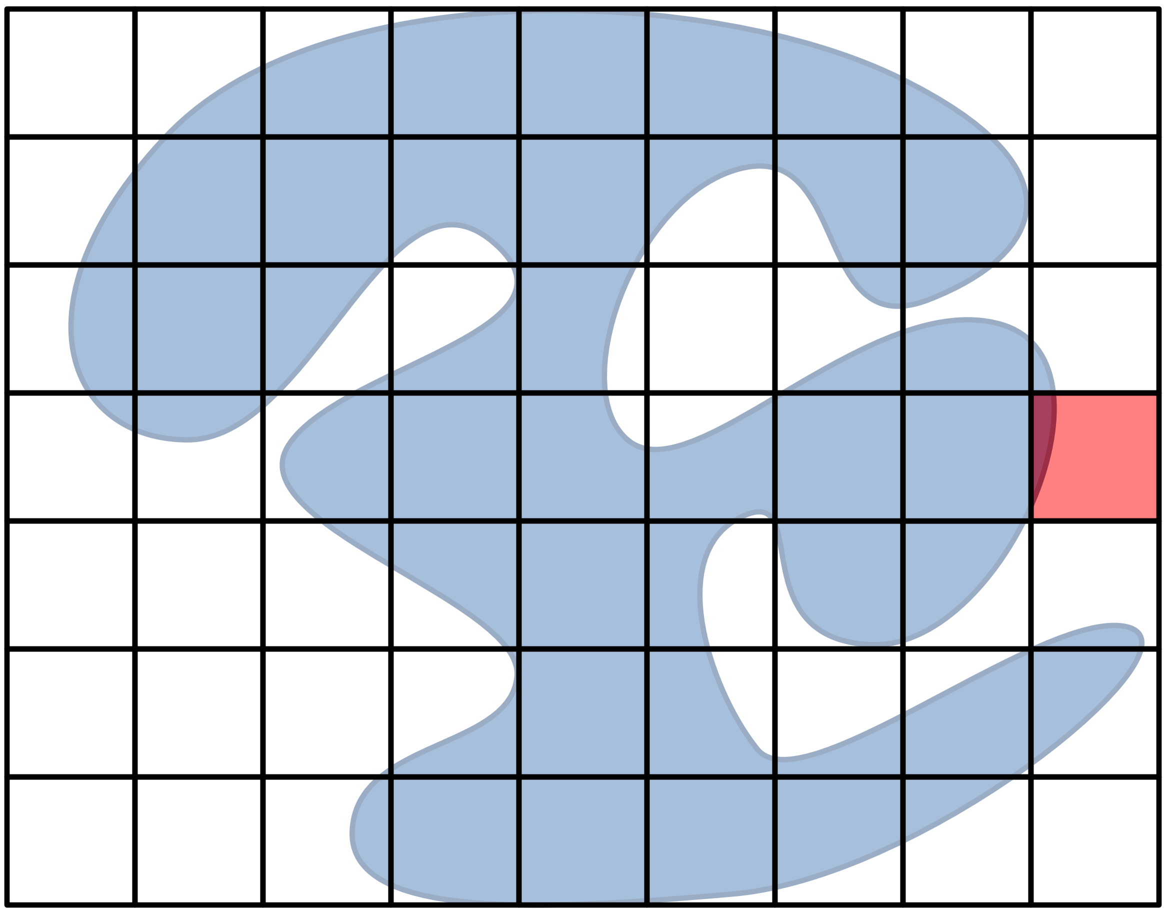
Scanline Rasterization



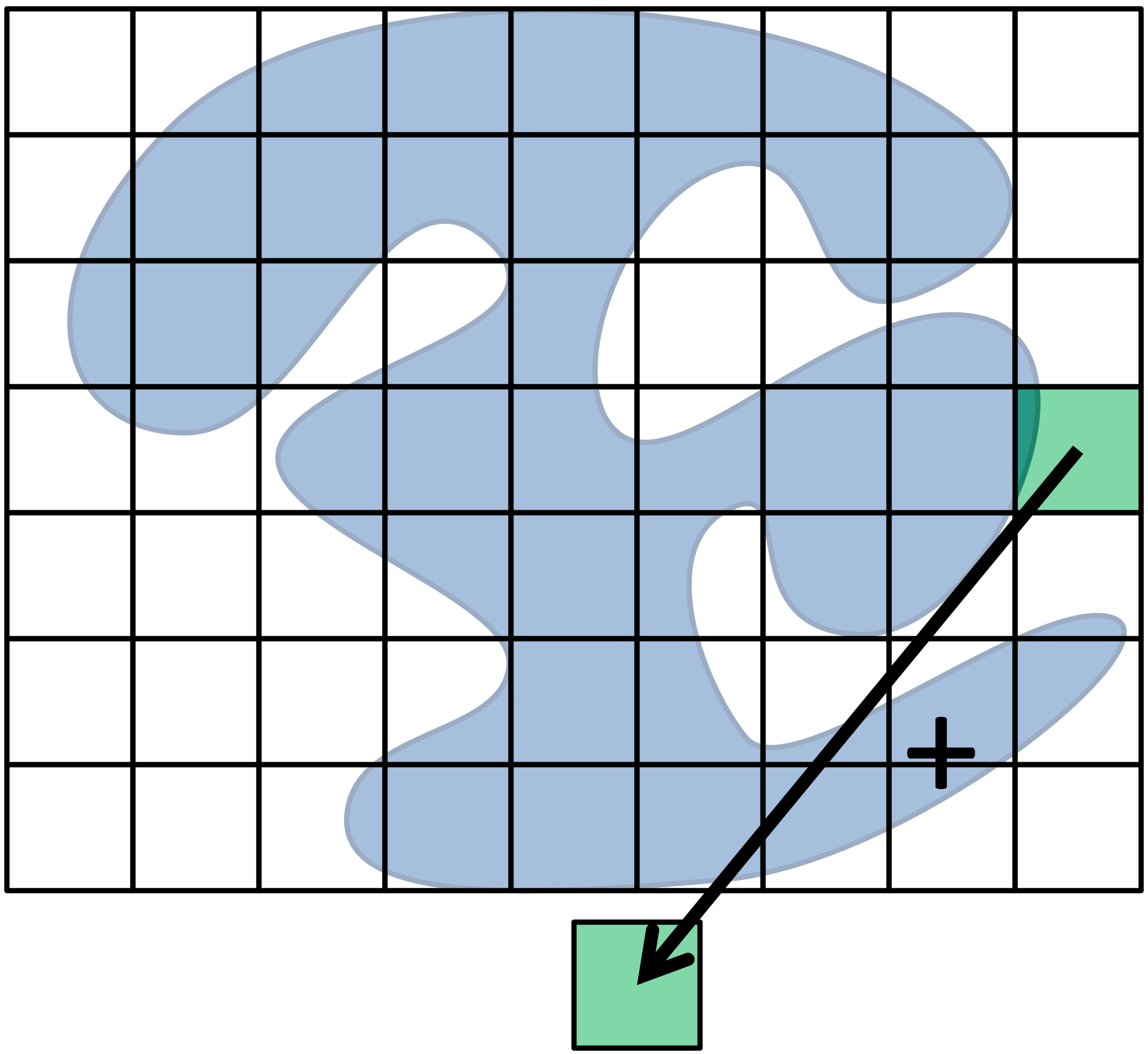
Scanline Rasterization



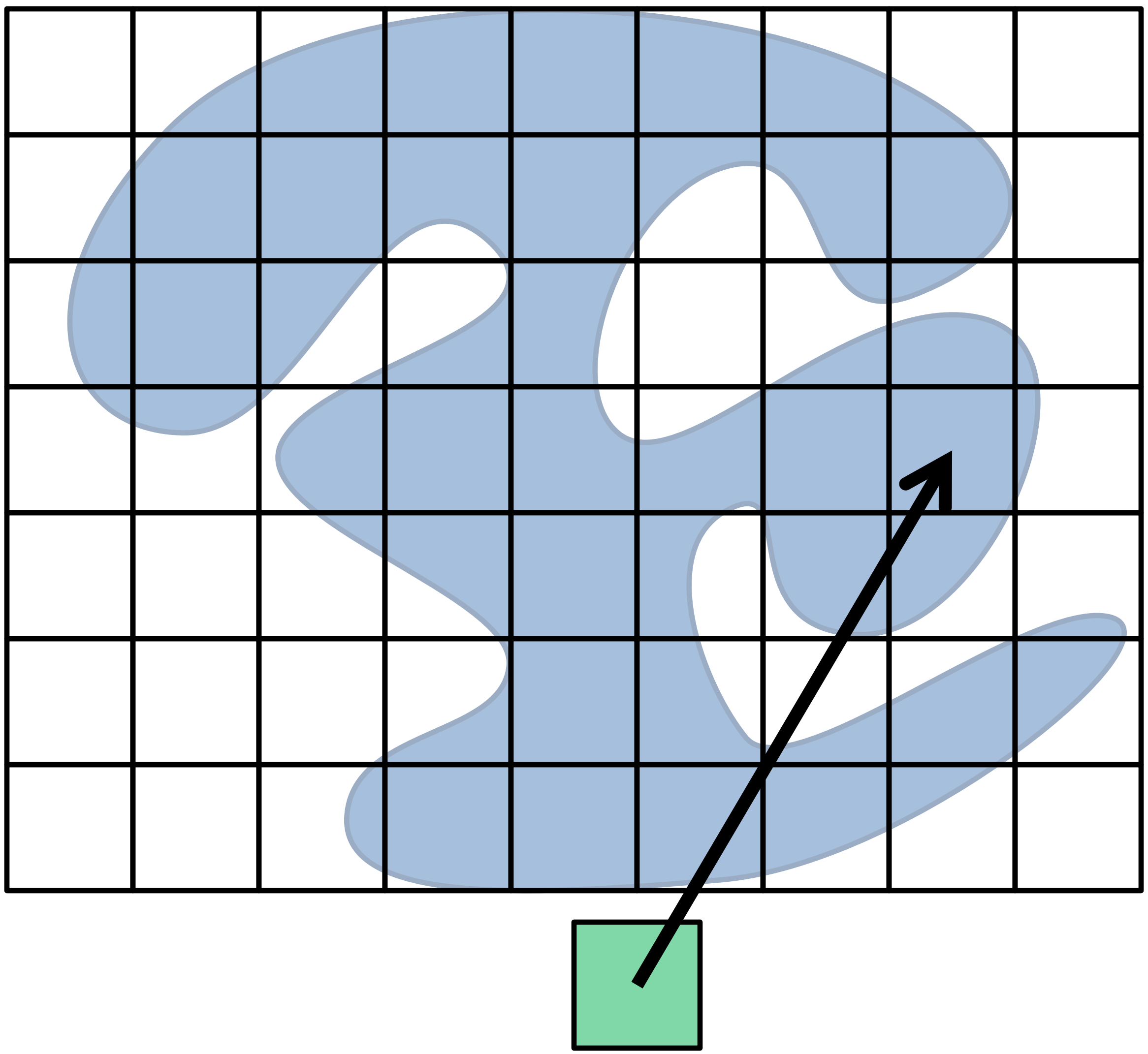
Scanline Rasterization



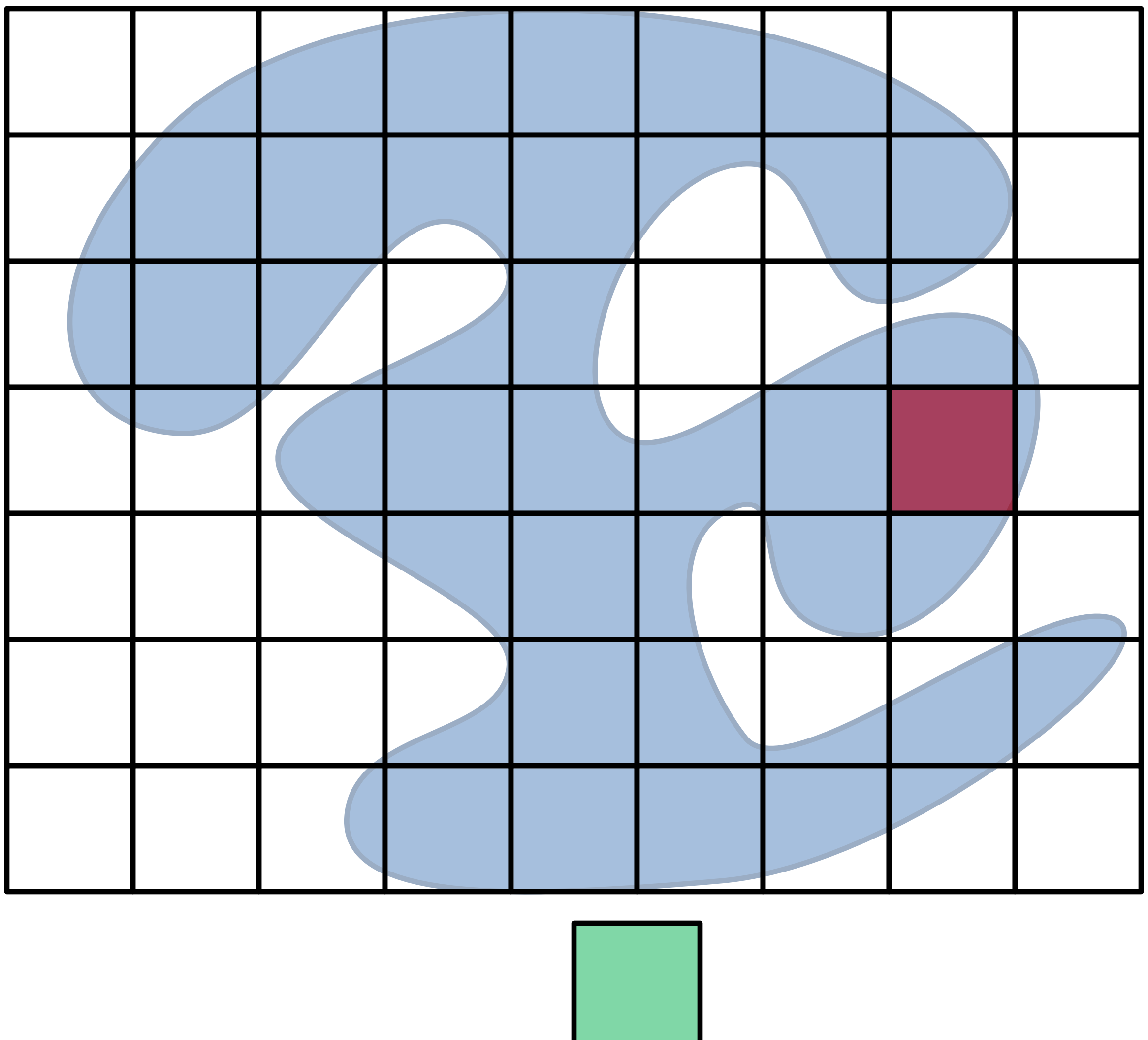
Scanline Rasterization



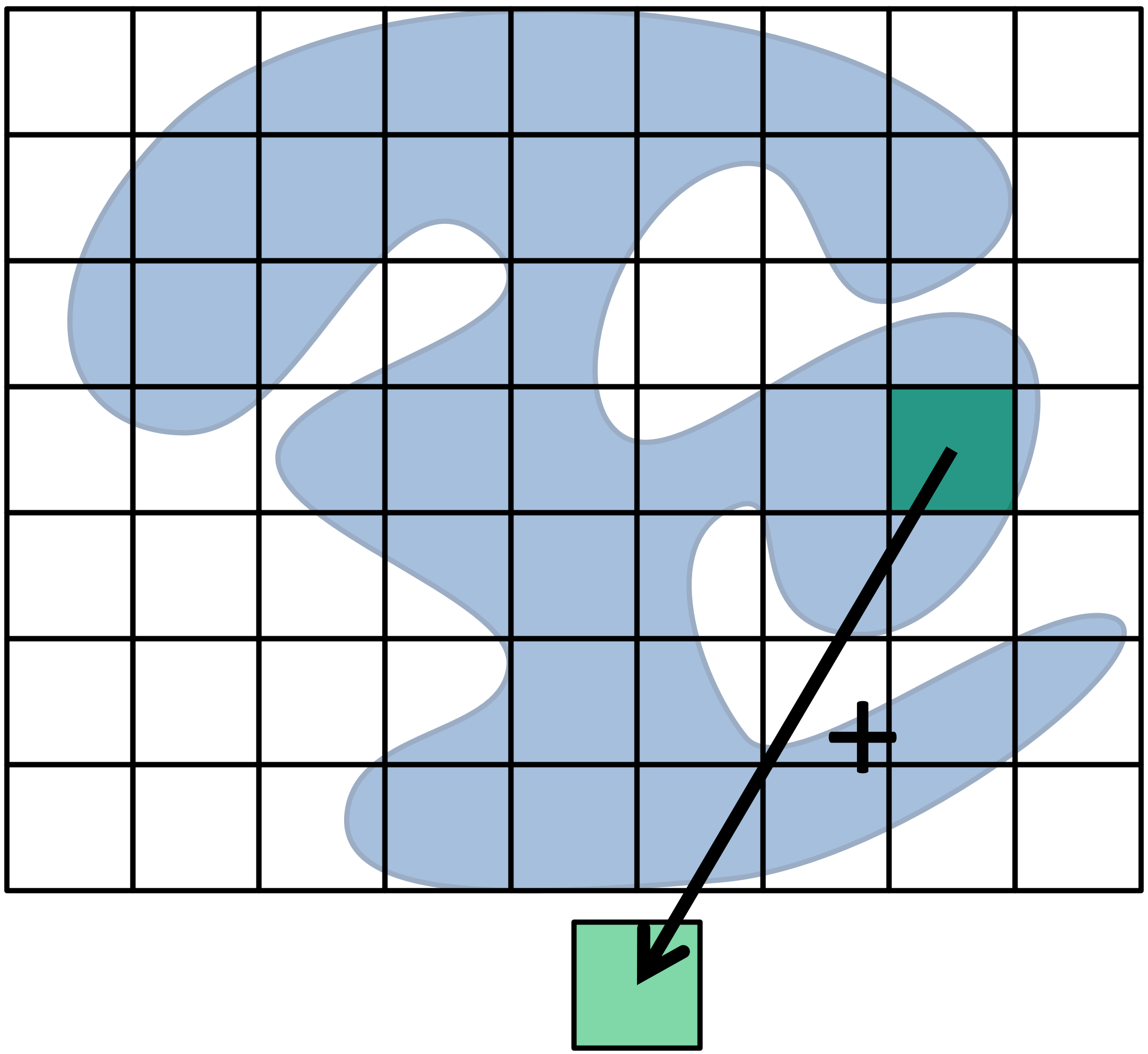
Scanline Rasterization



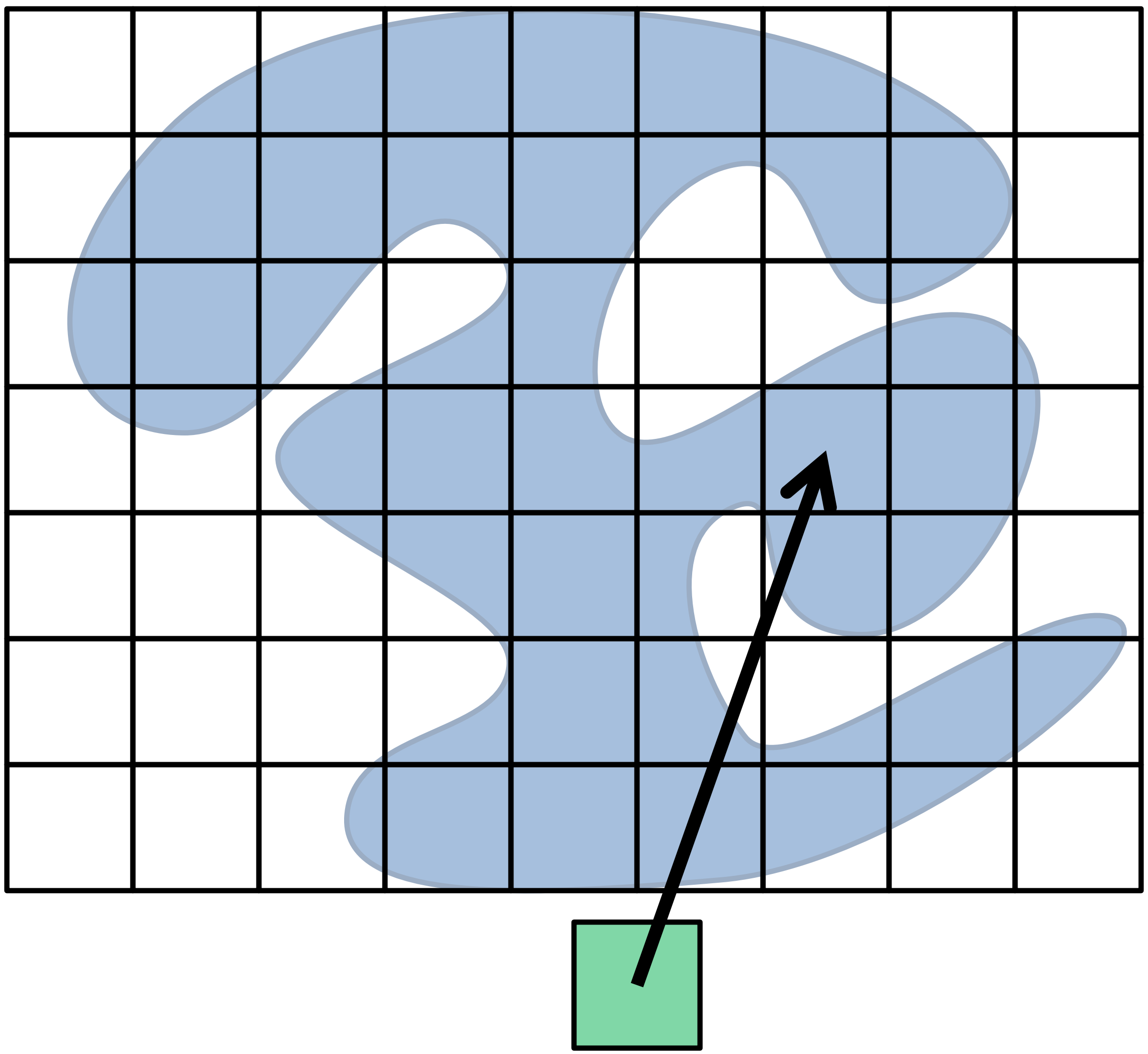
Scanline Rasterization



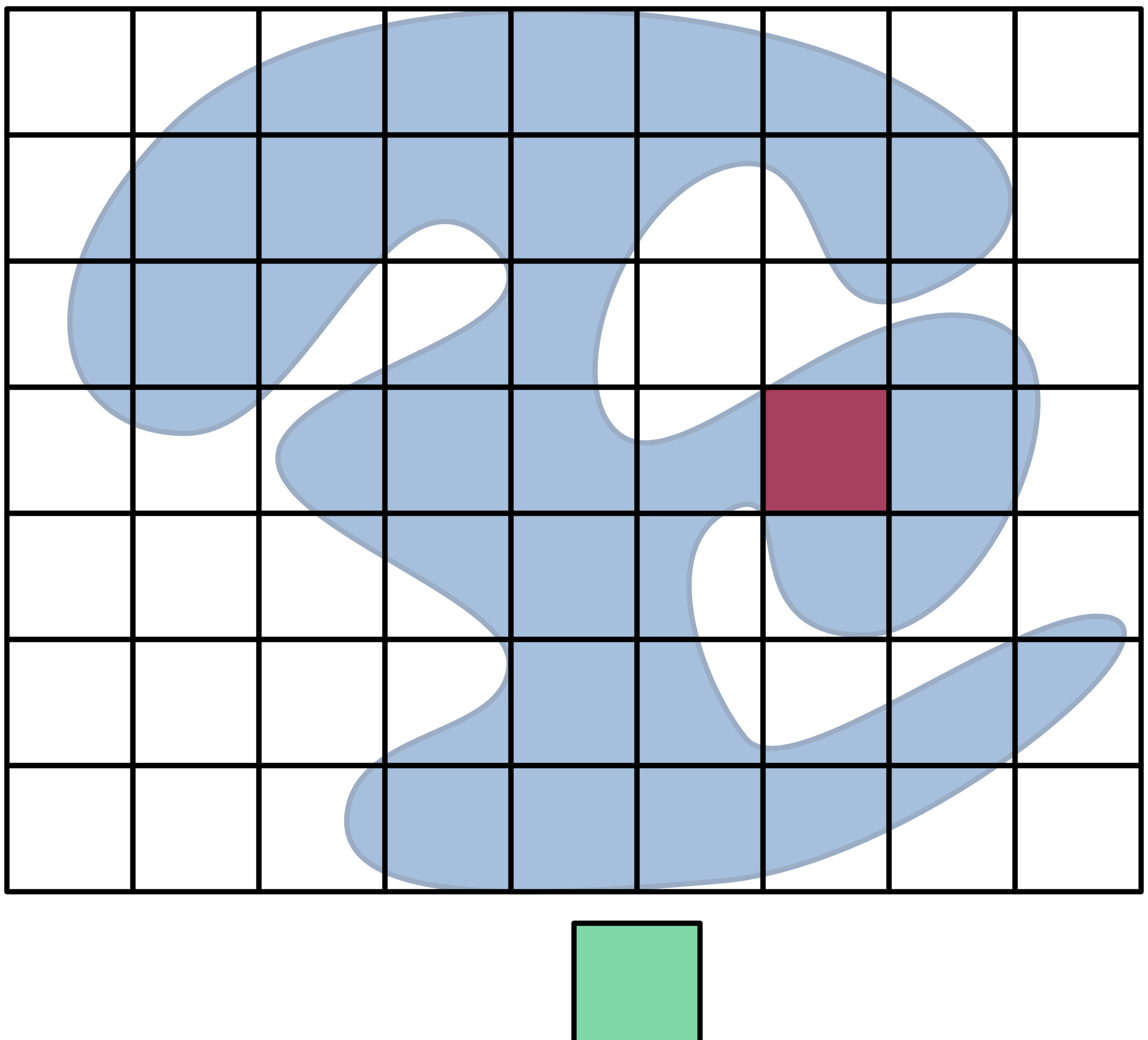
Scanline Rasterization



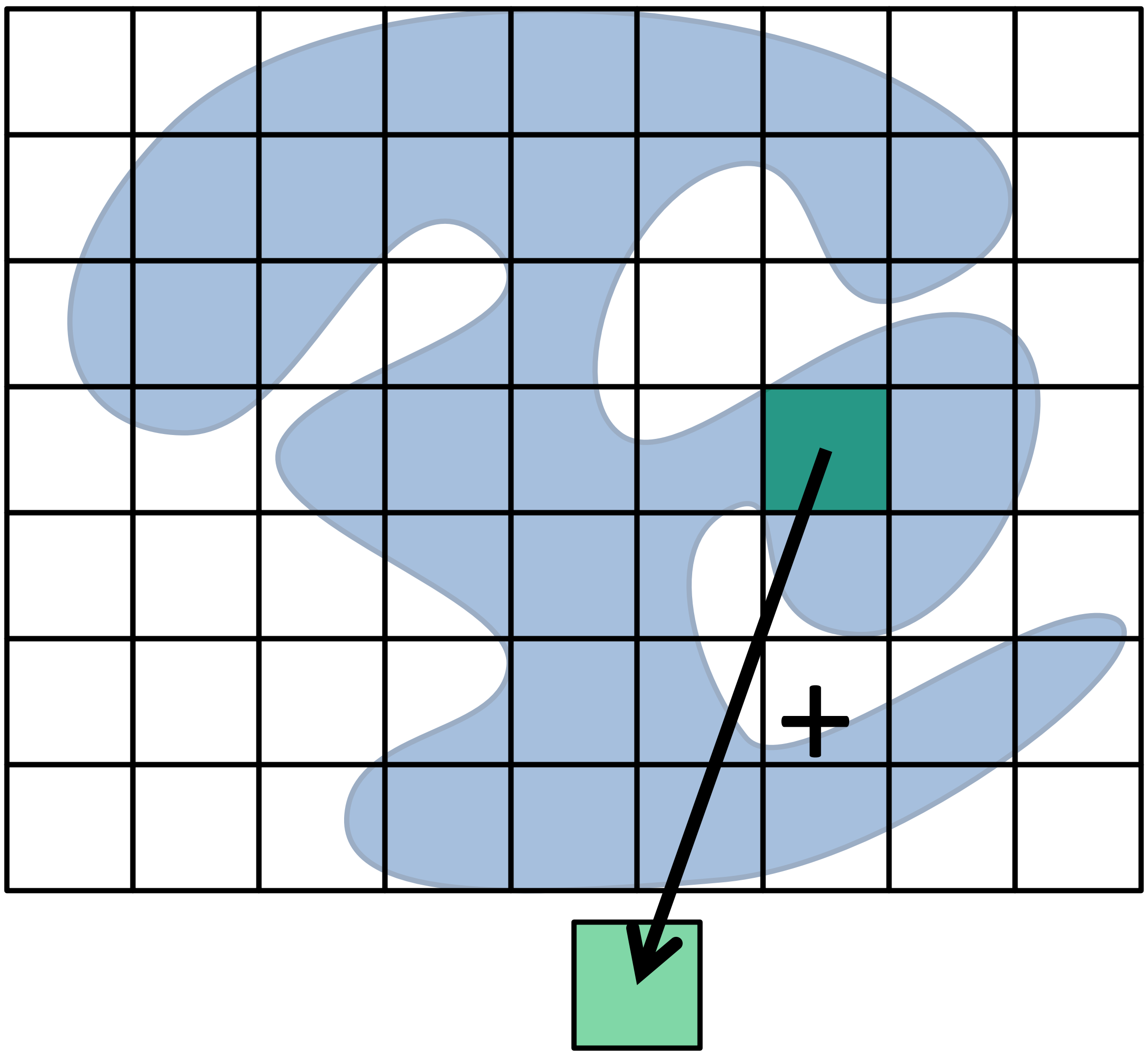
Scanline Rasterization



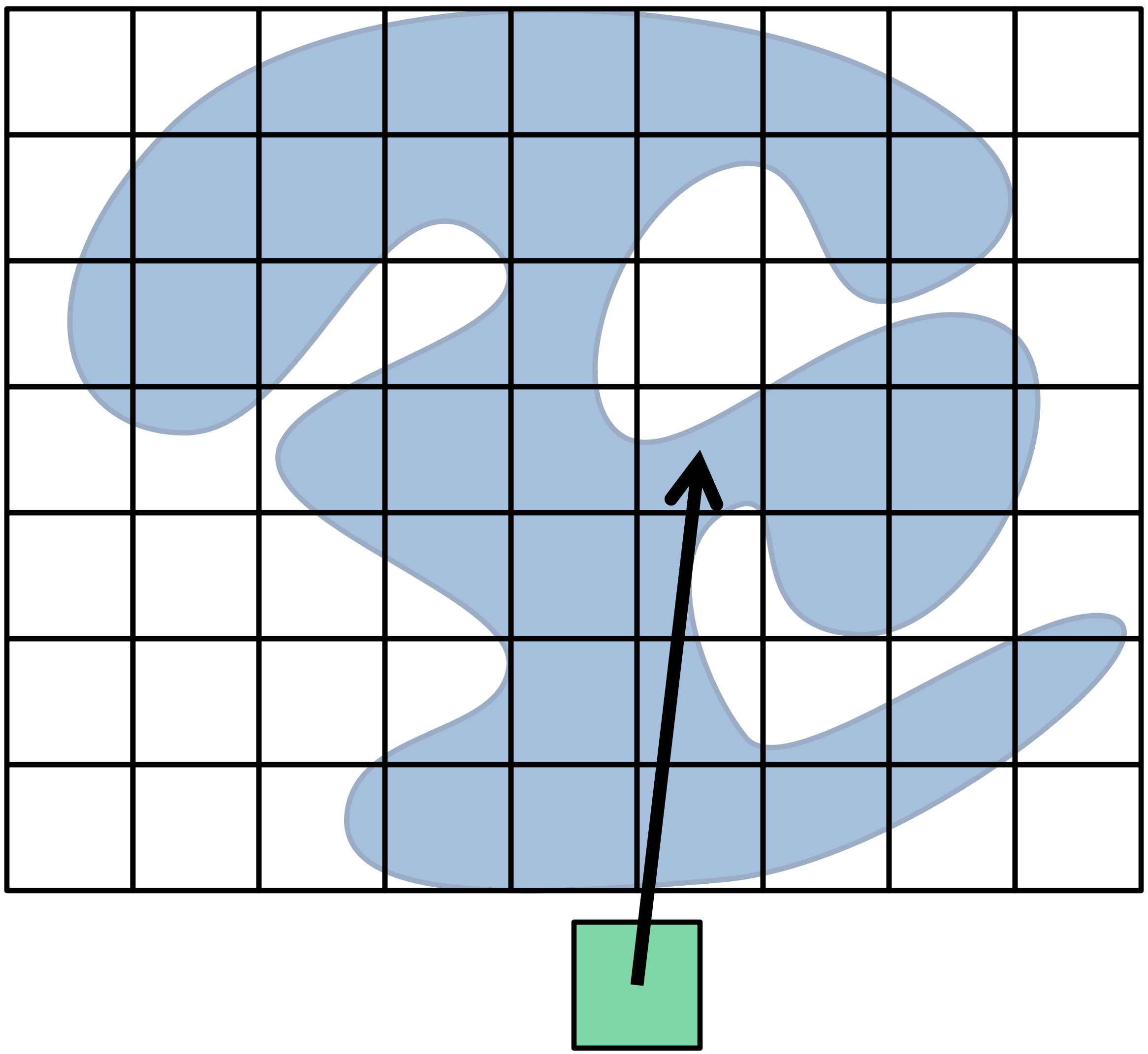
Scanline Rasterization



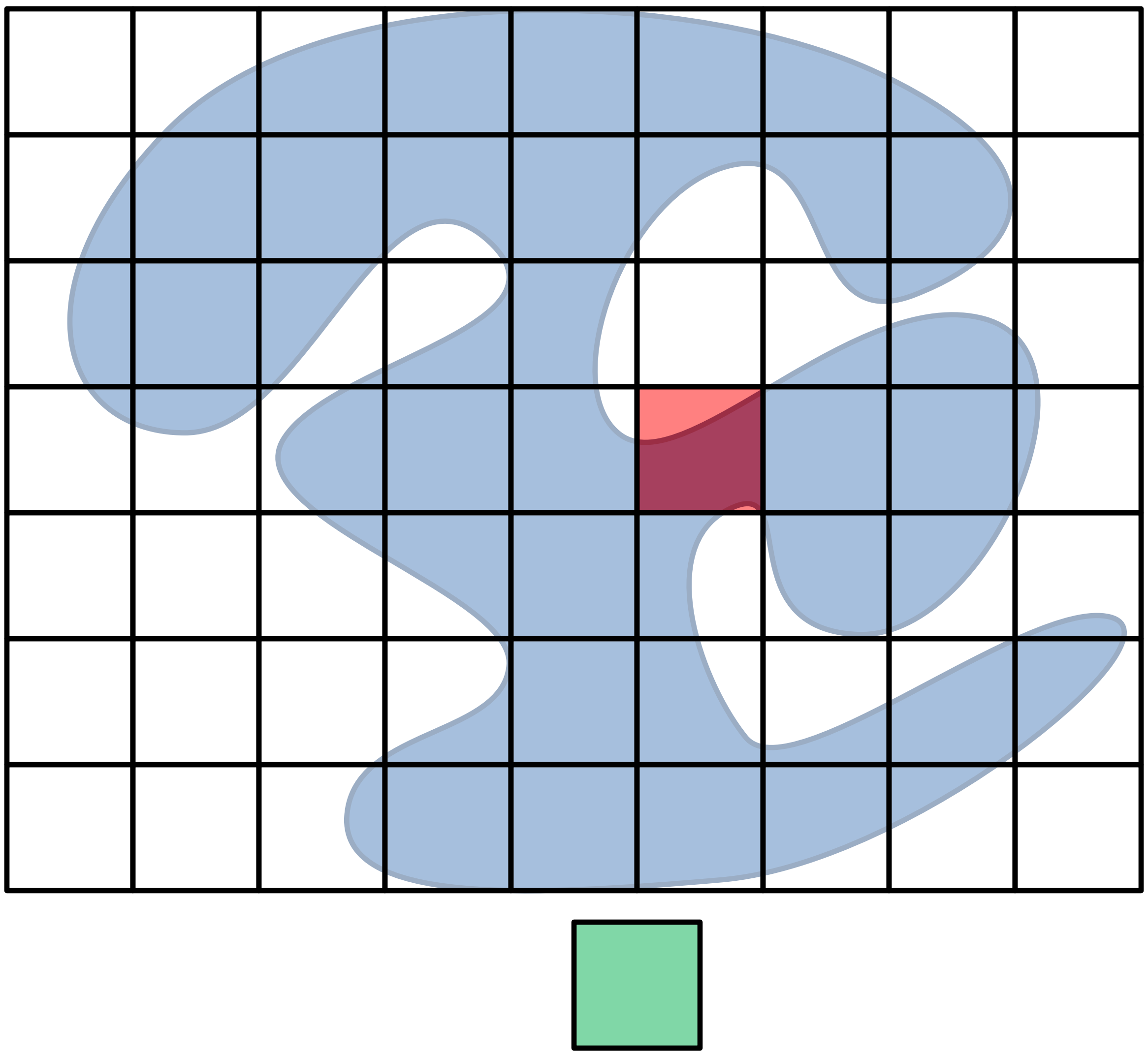
Scanline Rasterization



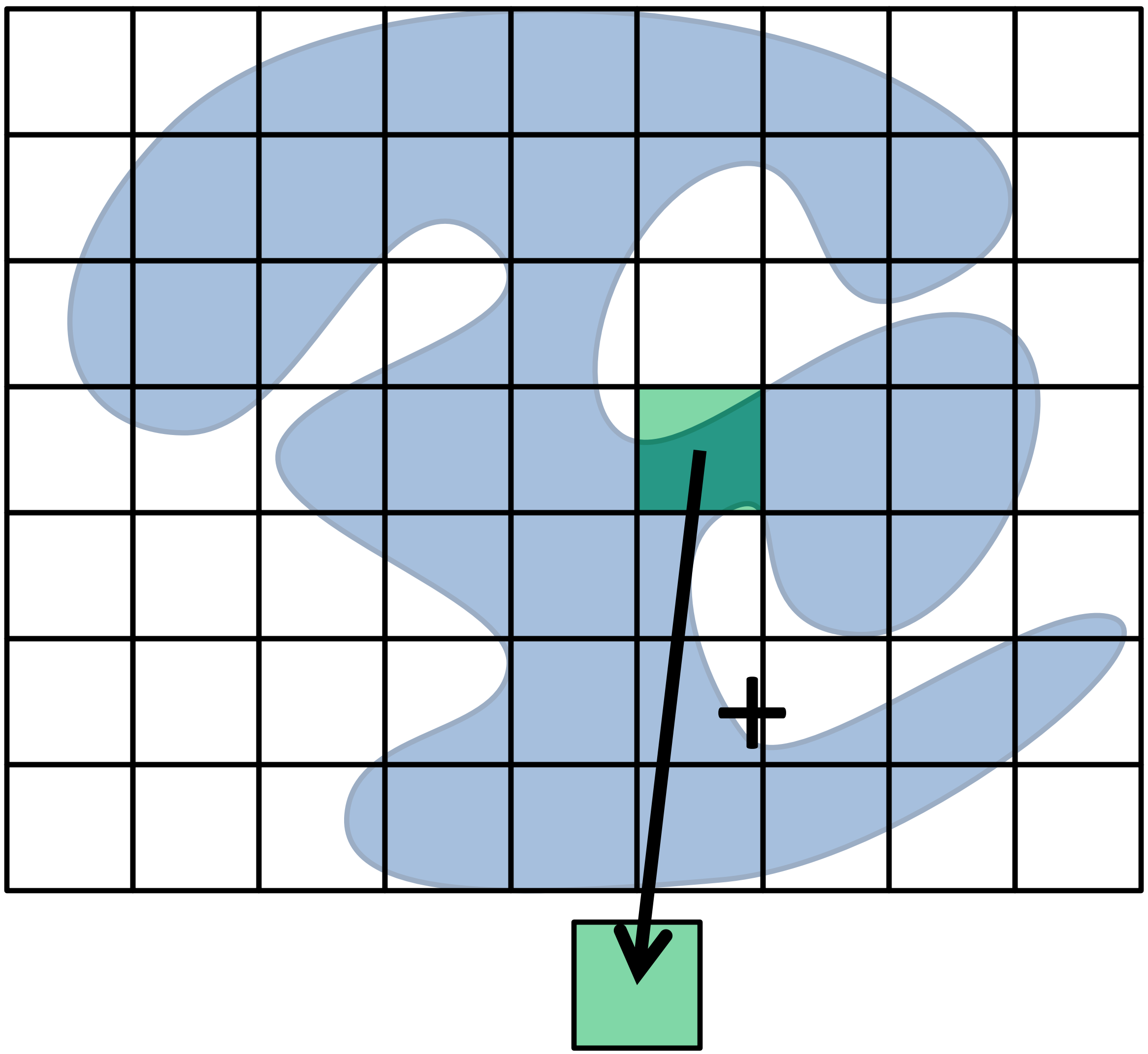
Scanline Rasterization



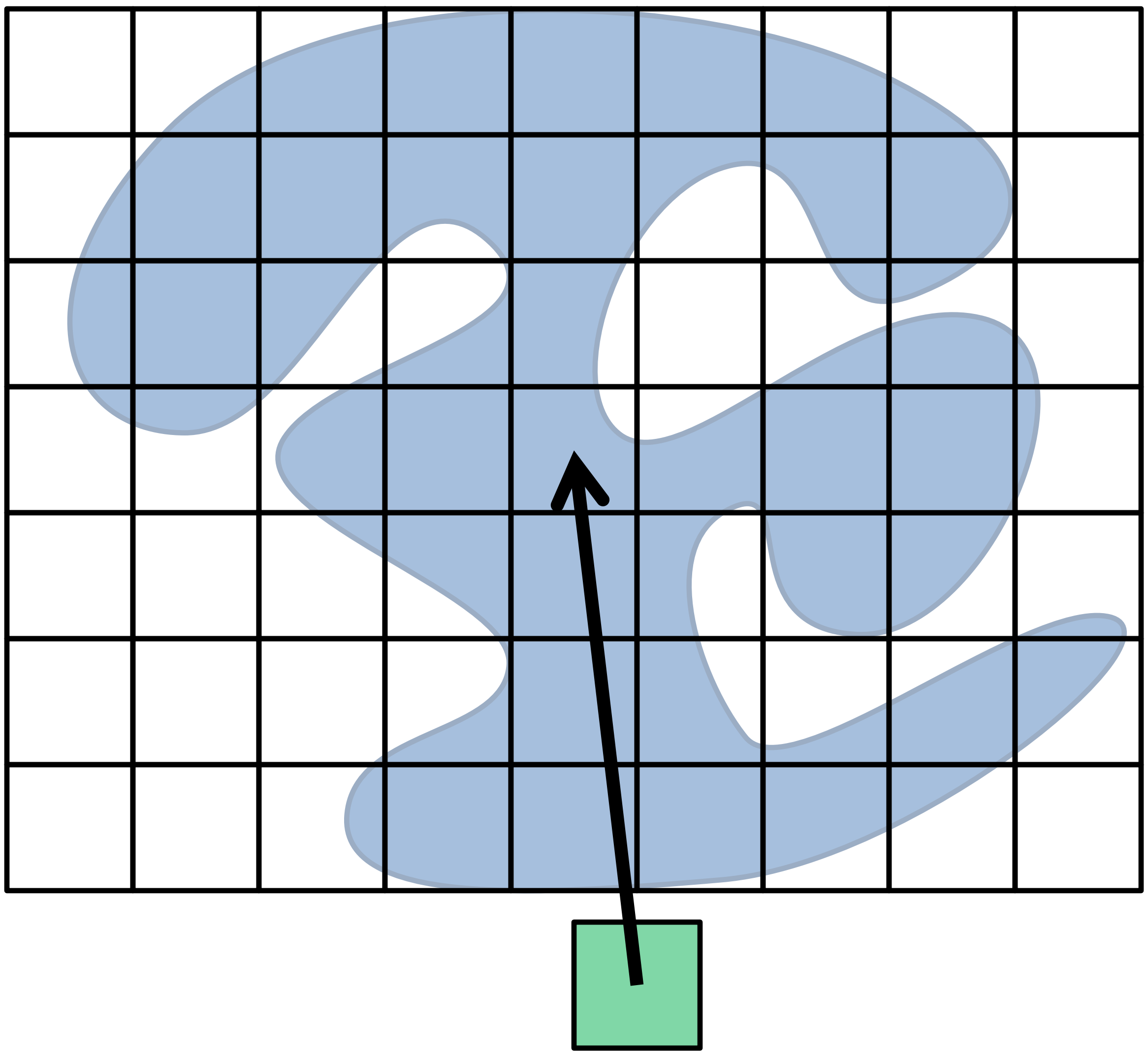
Scanline Rasterization



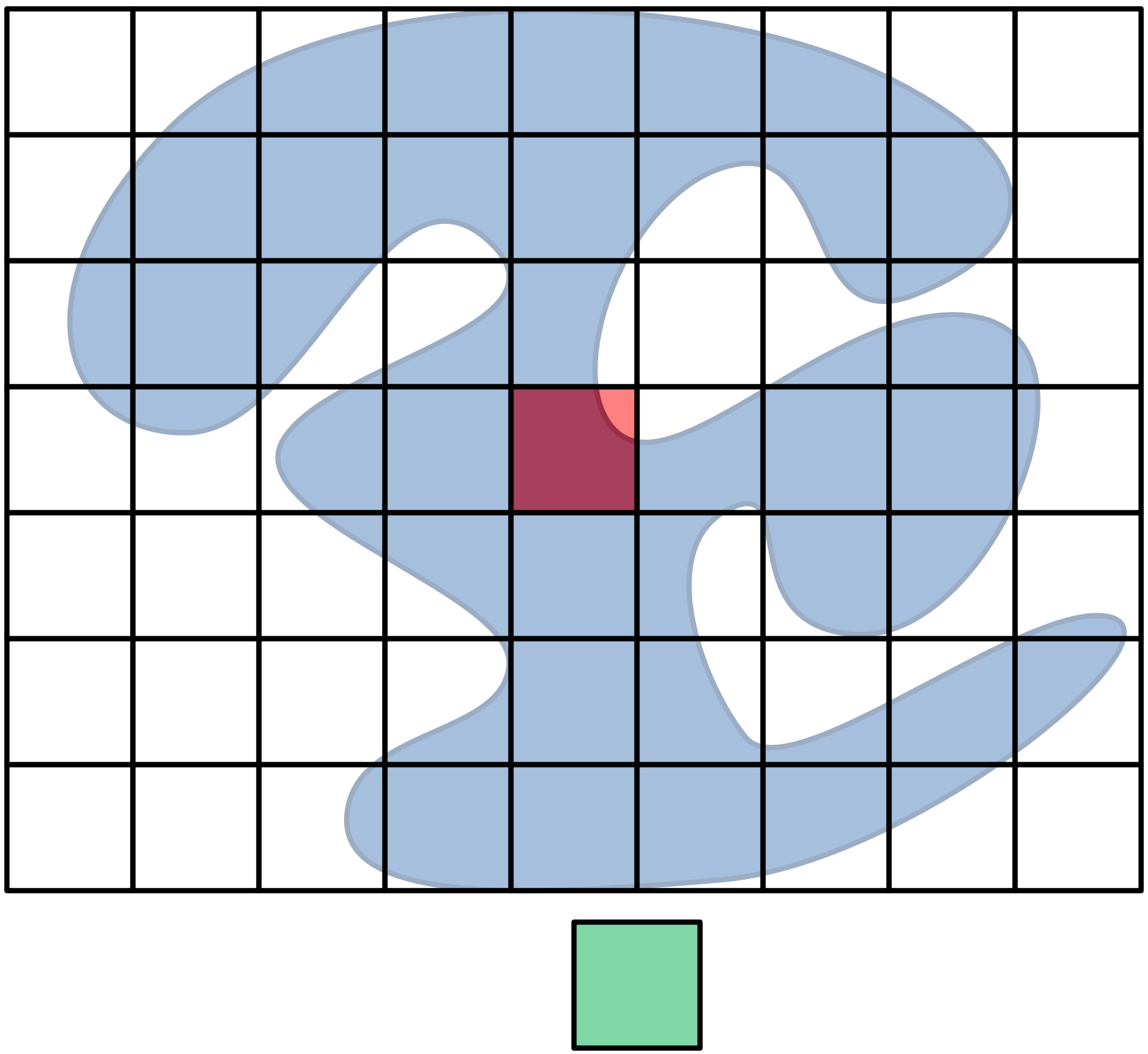
Scanline Rasterization



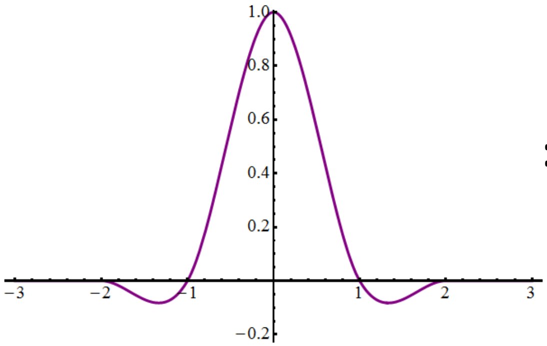
Scanline Rasterization



Scanline Rasterization

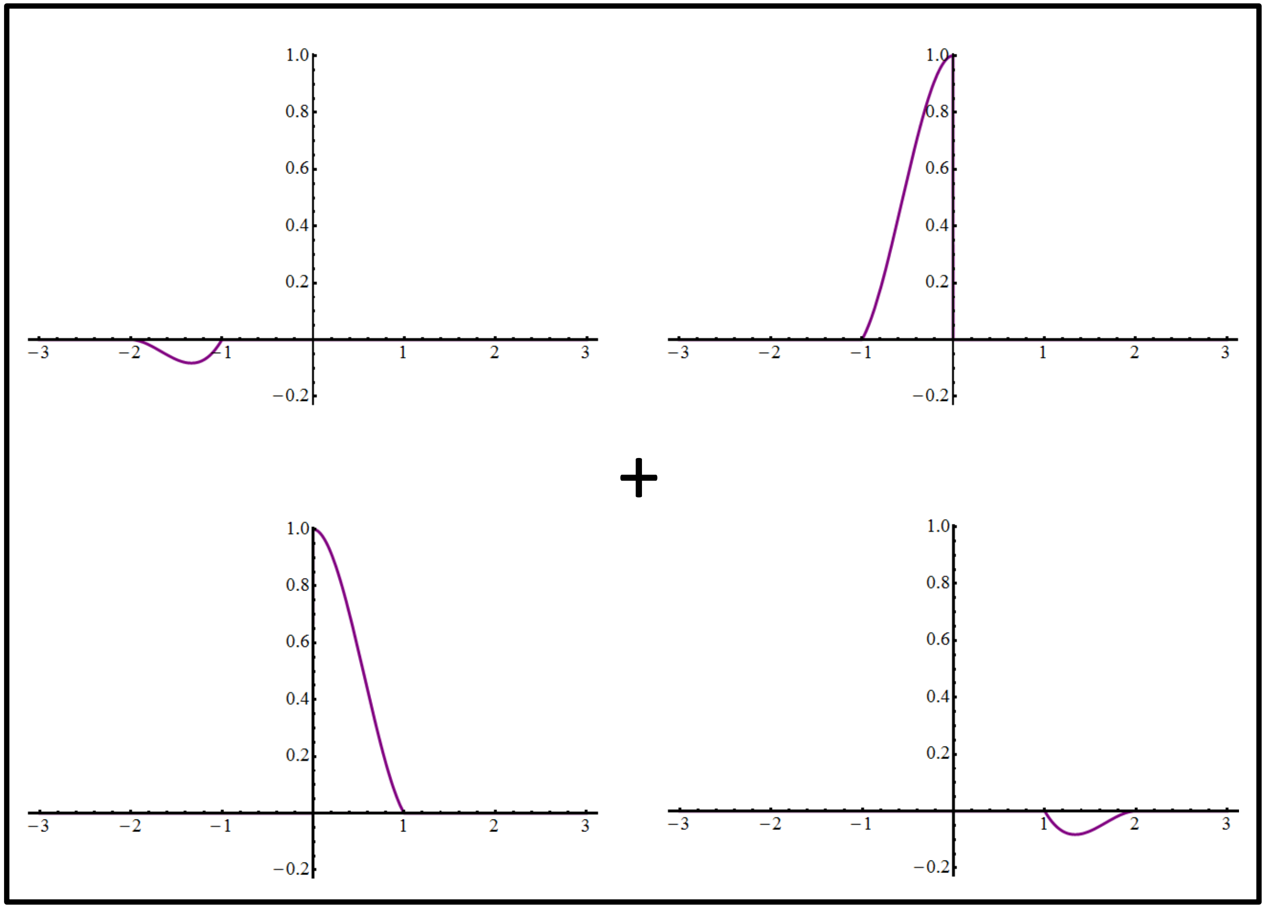


Filter Pieces

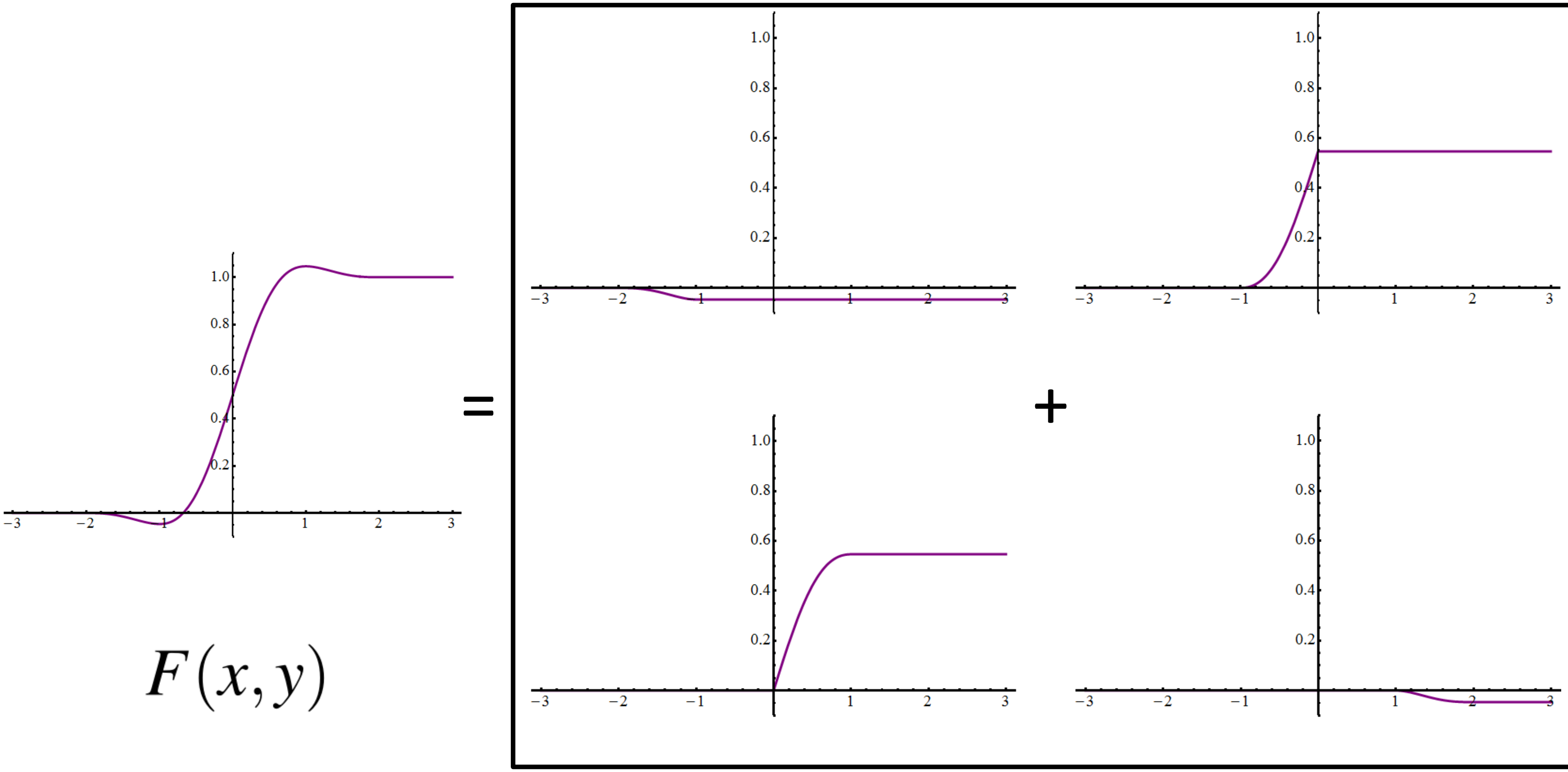


$$f(x,y)$$

=



Filter Pieces



$$F(x,y)$$

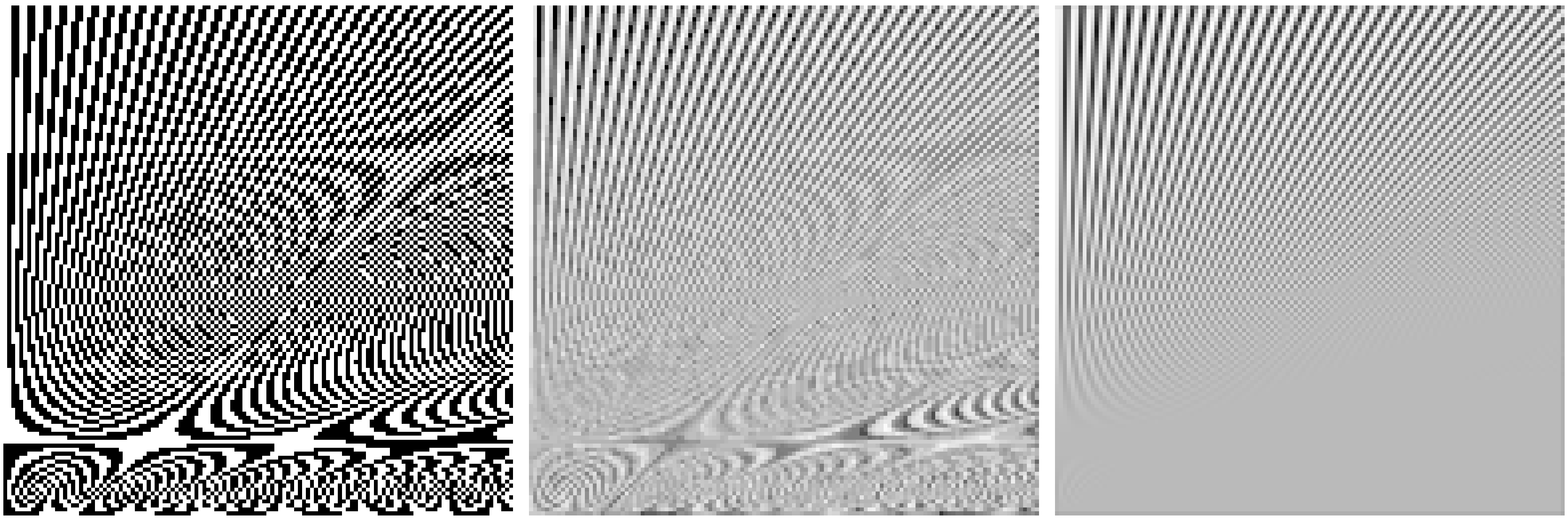
Rational Curves

Cubic Curves

Input image

Mitchell-Netravali

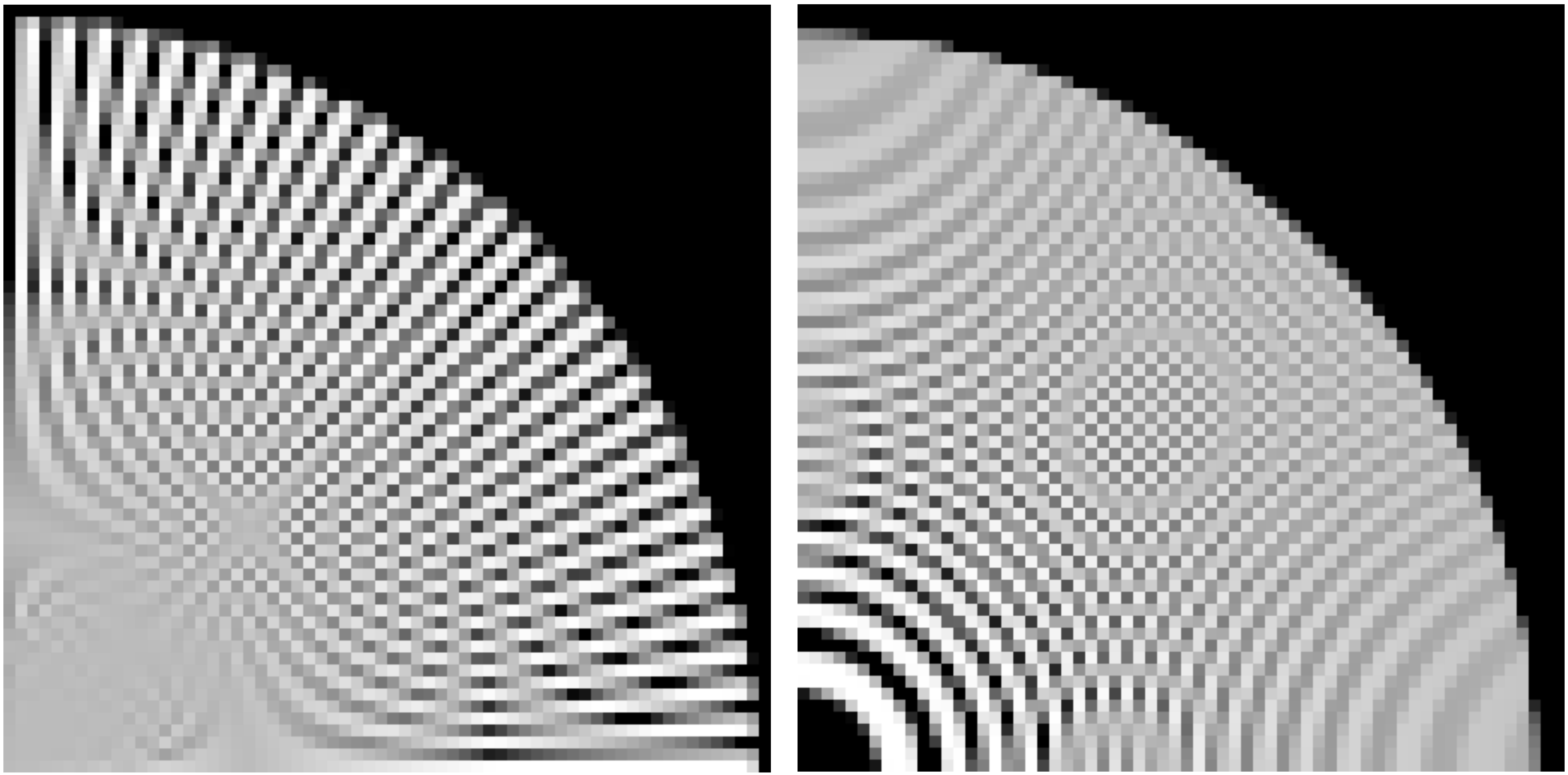
Anti-aliasing



Point sampling 16x MSAA tent Analytic tent

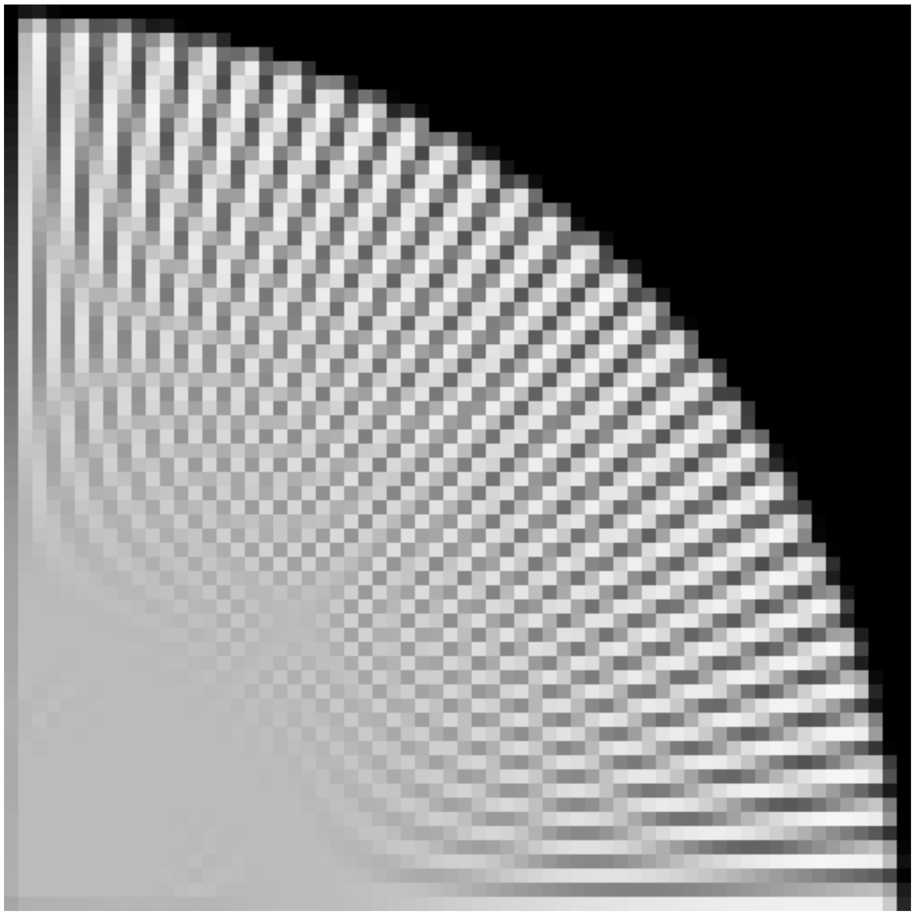
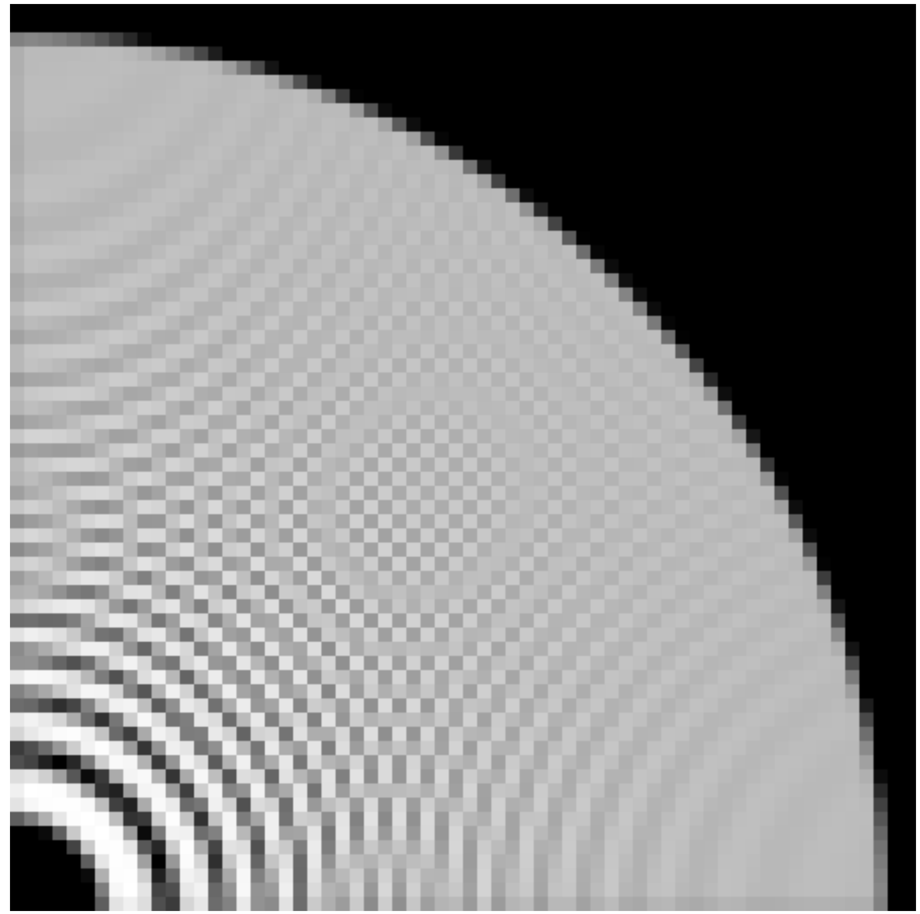
ATI Radeon HD 5700

Anti-aliasing



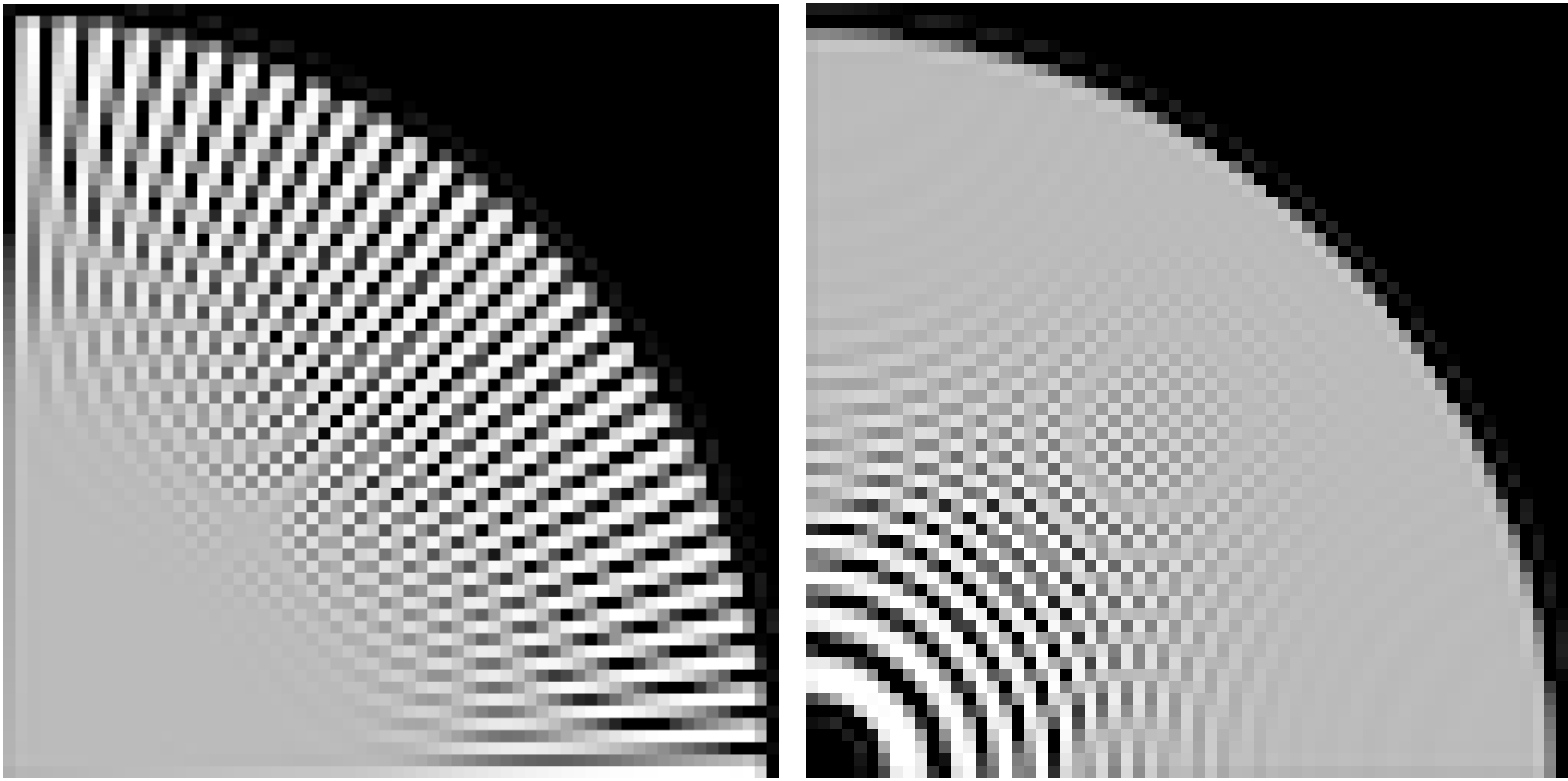
Box

Anti-aliasing

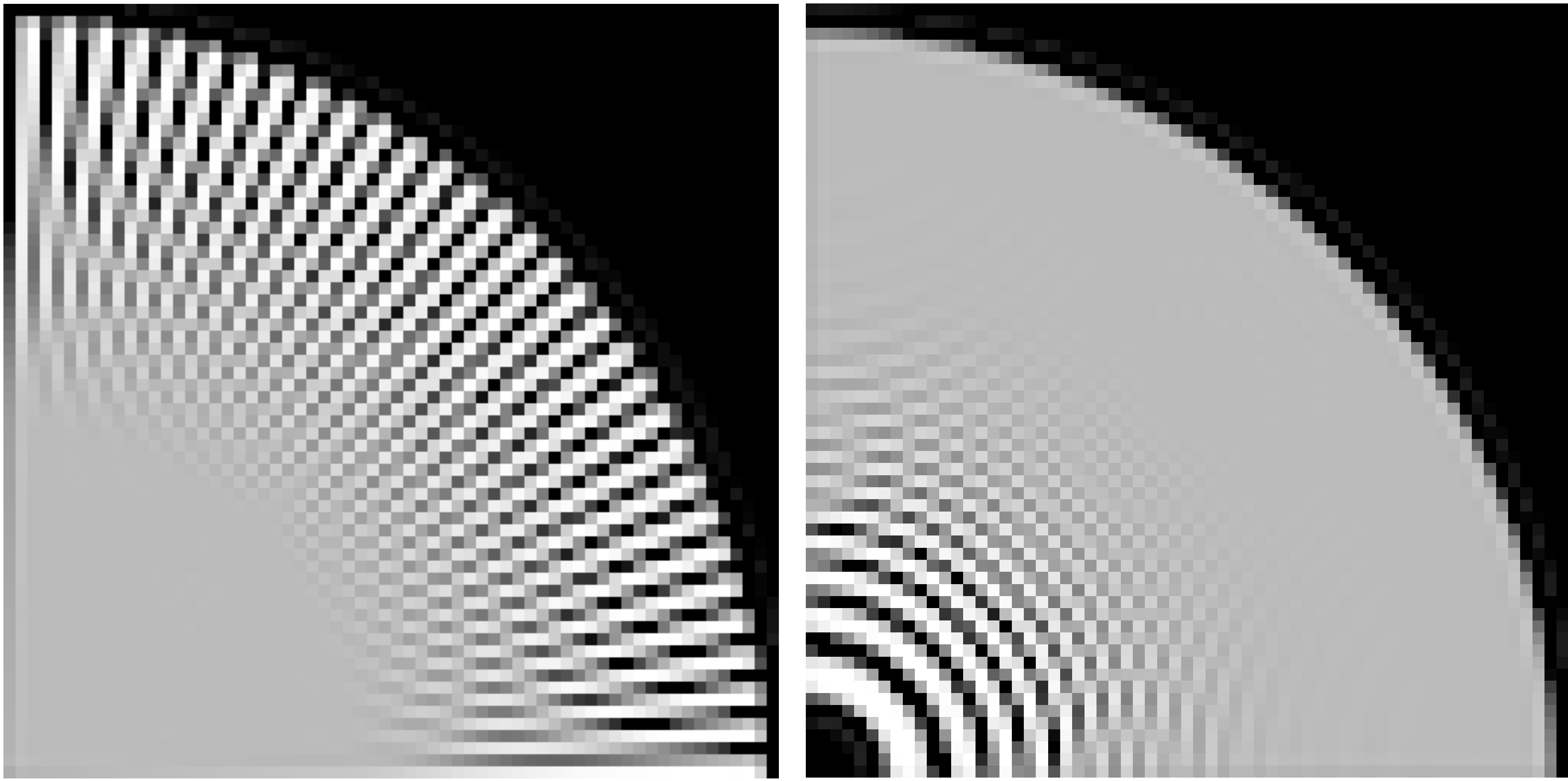
Tent

Anti-aliasing



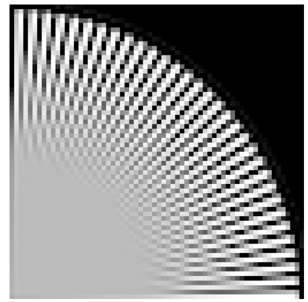
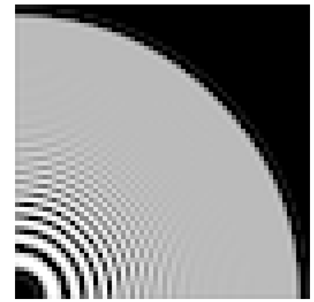
Lanczos 3

Anti-aliasing

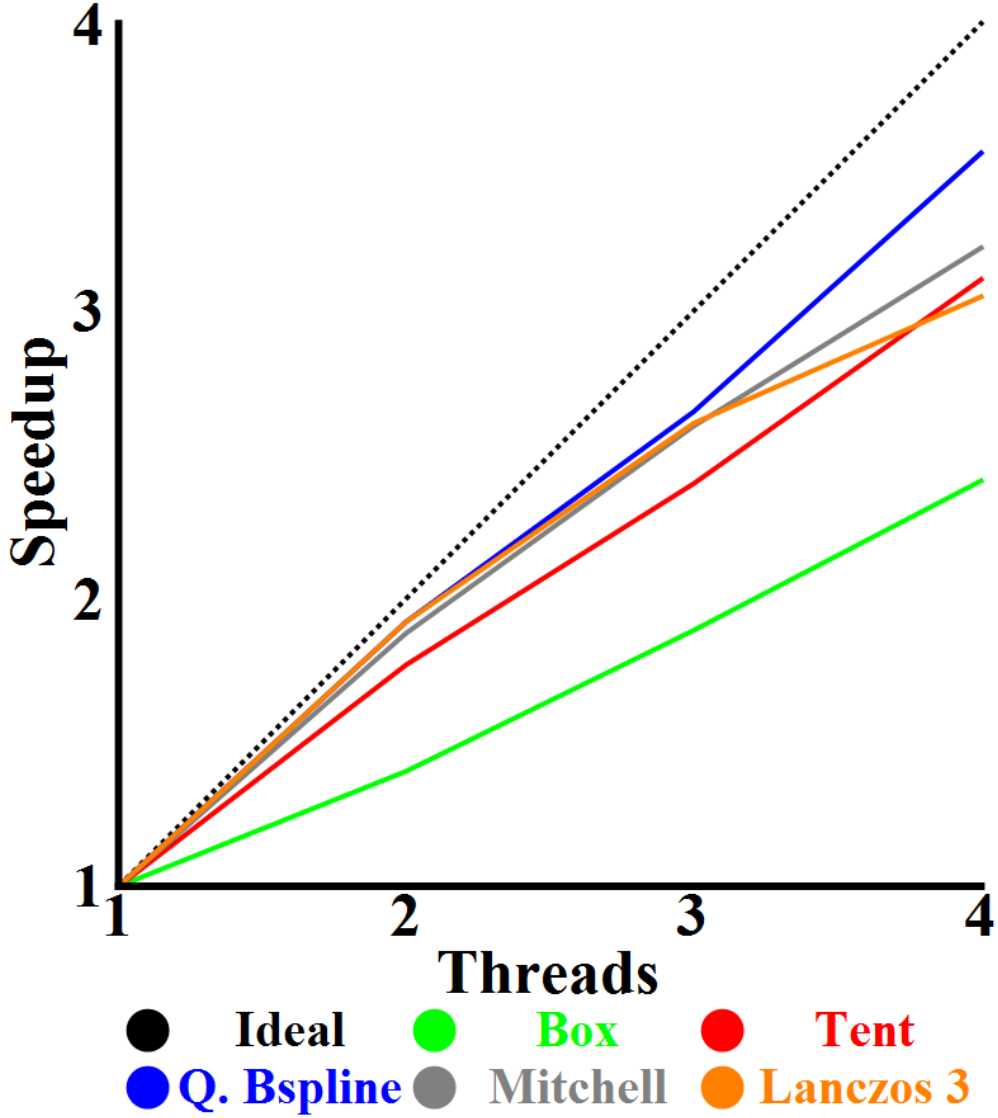


Radial 3

Timings (ms)

		
AGG	0.25	**0.35**
Cairo	0.57	**2.34**
Wavelet	0.42	3.14
Box	0.14 *(0.07)*	0.37 *(0.17)*
Tent	0.27 *(0.12)*	0.75 *(0.26)*
Q. B-spline	1.02 *(0.31)*	4.45 *(1.26)*
Mitchell	2.26 *(0.79)*	37.0 *(11.4)*
Lánczos 2	2.71 *(0.96)*	27.6 *(7.54)*

Parallelism



Conclusions

- Analytic prefiltering
  - Smooth curves
  - High-quality filters
  - Color gradients
- Fast algorithm
  - Independent curves and filter pieces
  - High parallelism