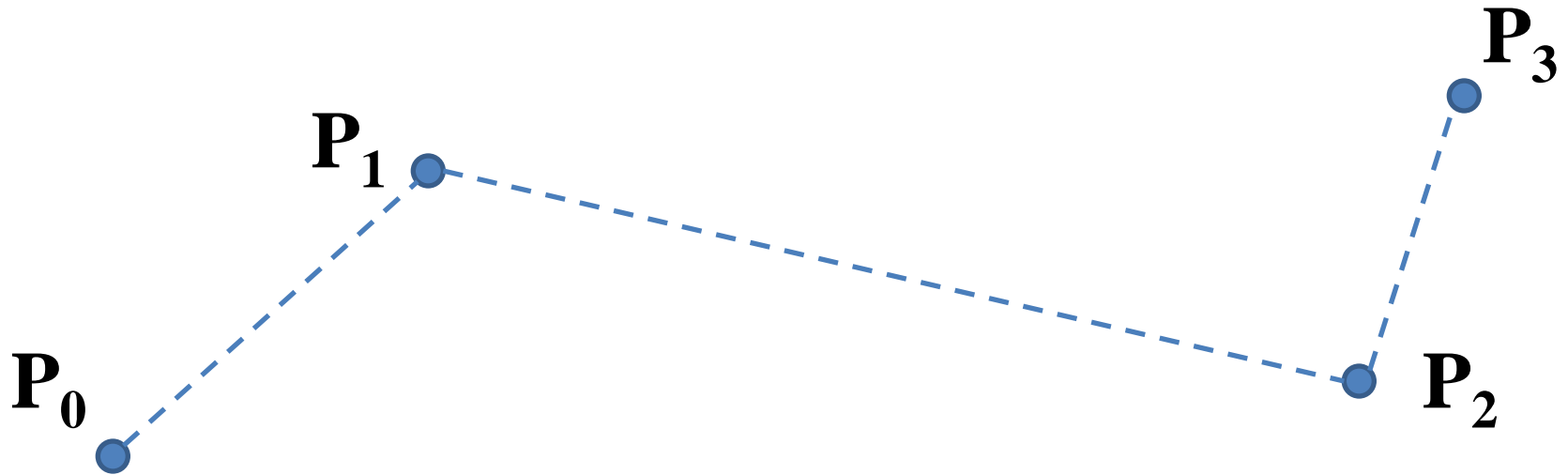


On the Parameterization of Catmull-Rom Curves

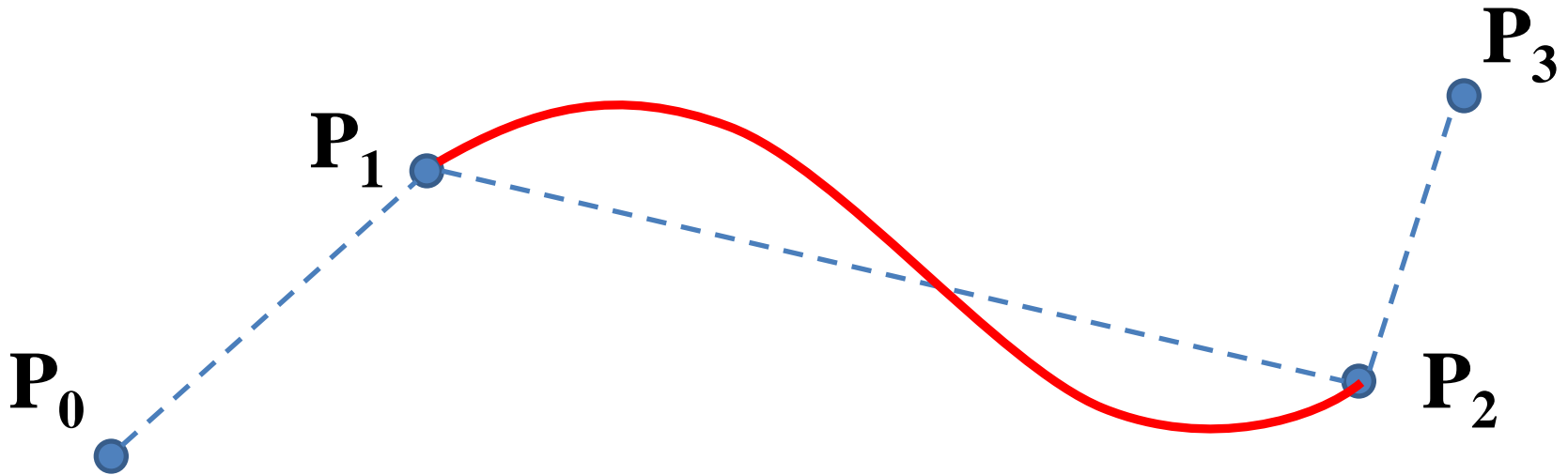
Cem Yuksel Scott Schaefer John Keyser

Texas A&M University

Catmull-Rom Curves

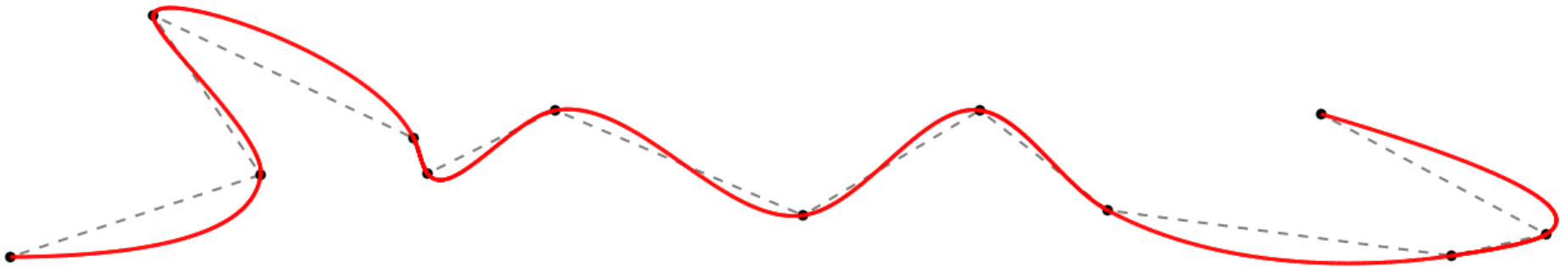


Catmull-Rom Curves



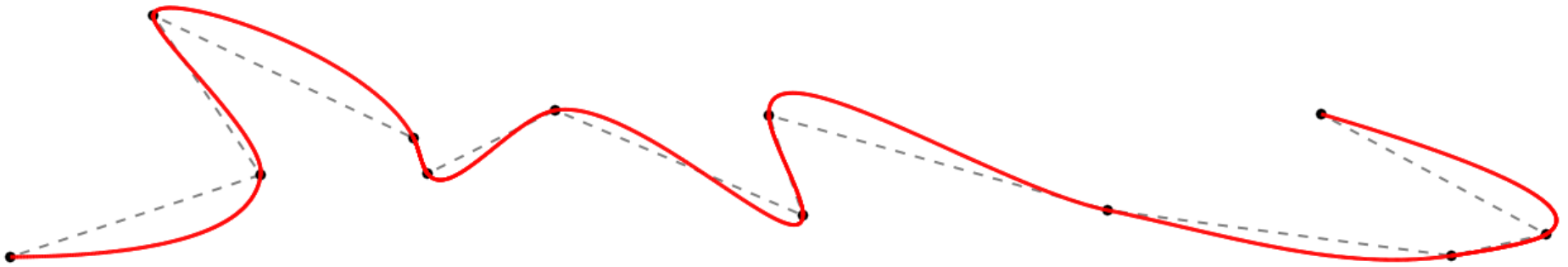
Catmull-Rom Curves

- Important Properties
 - Interpolate control points
 - Local support
 - Piecewise polynomial representation

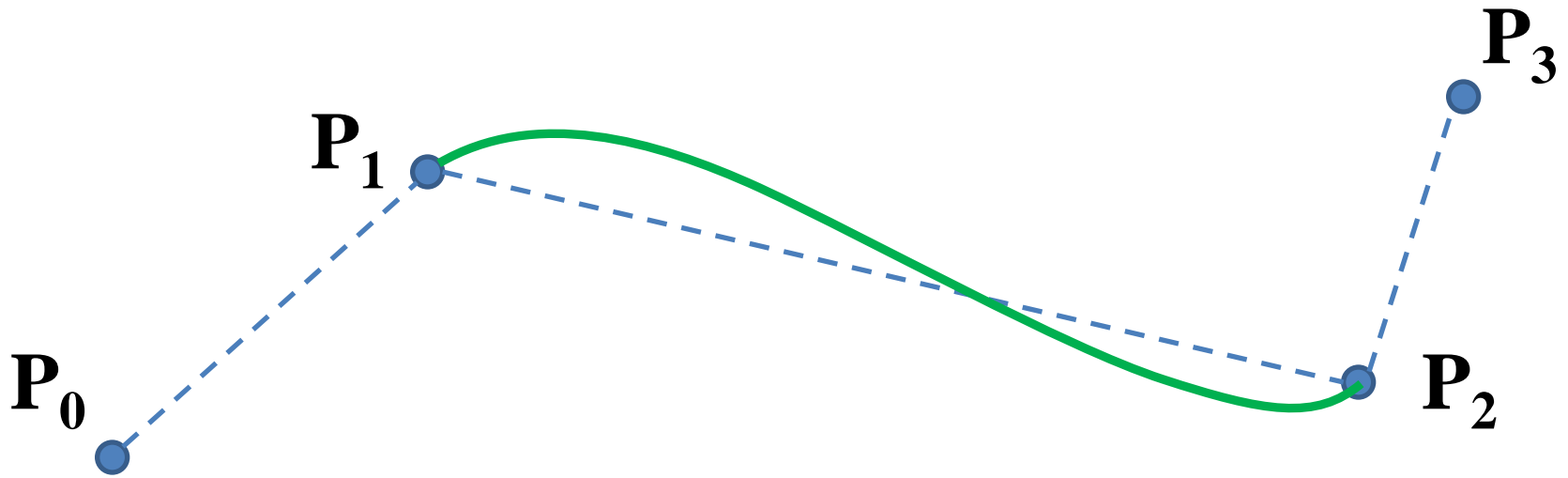


Catmull-Rom Curves

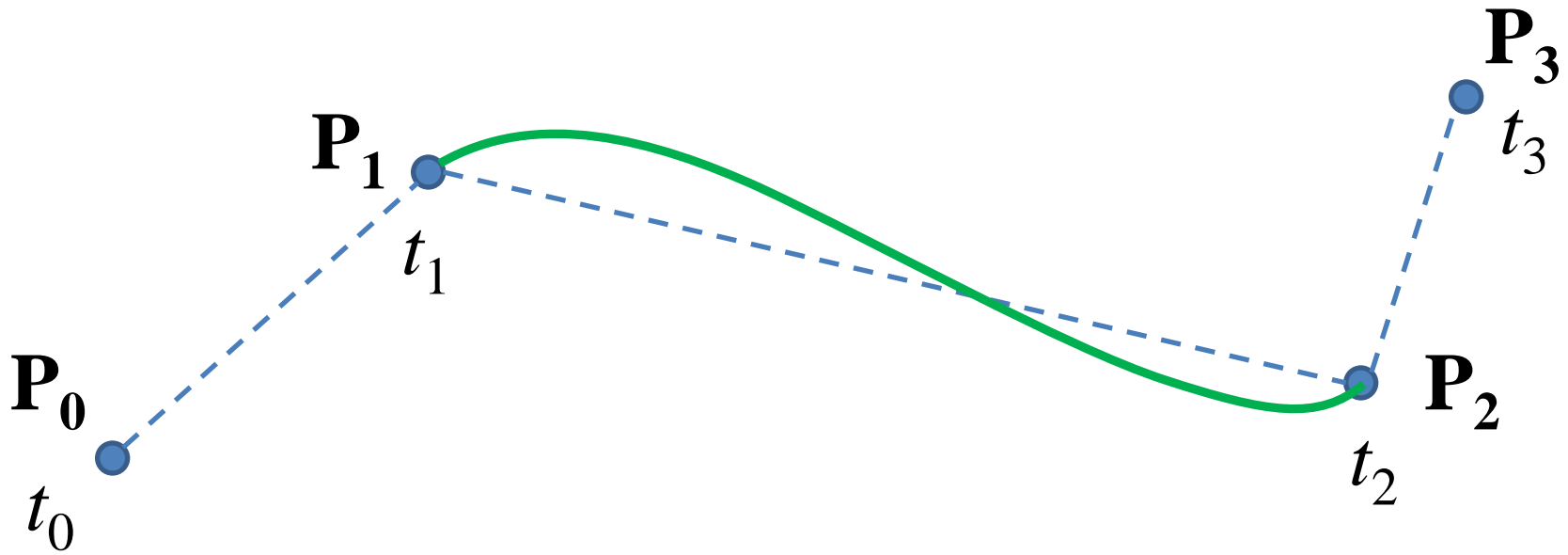
- Important Properties
 - Interpolate control points
 - Local support
 - Piecewise polynomial representation



Catmull-Rom Curves

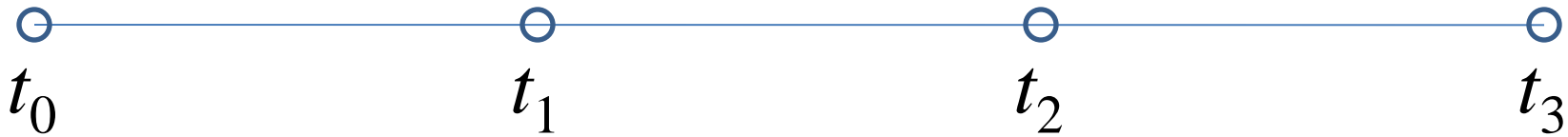
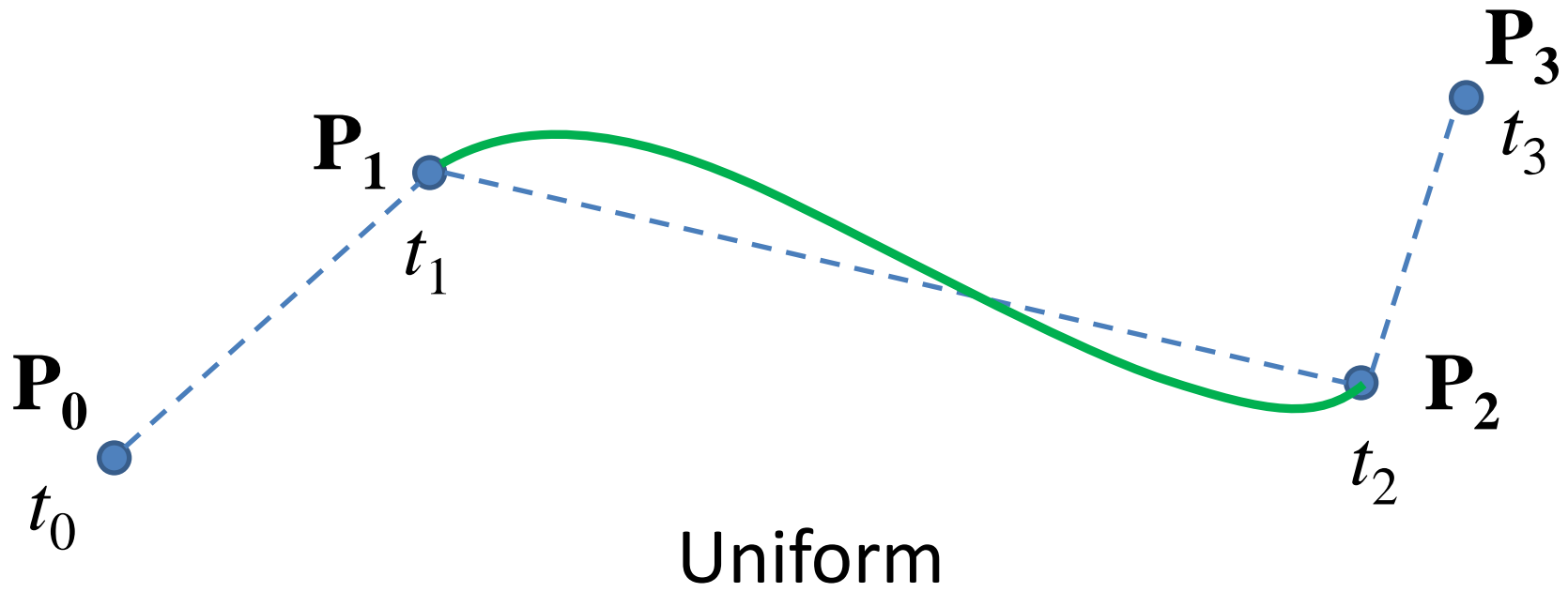


Catmull-Rom Curves



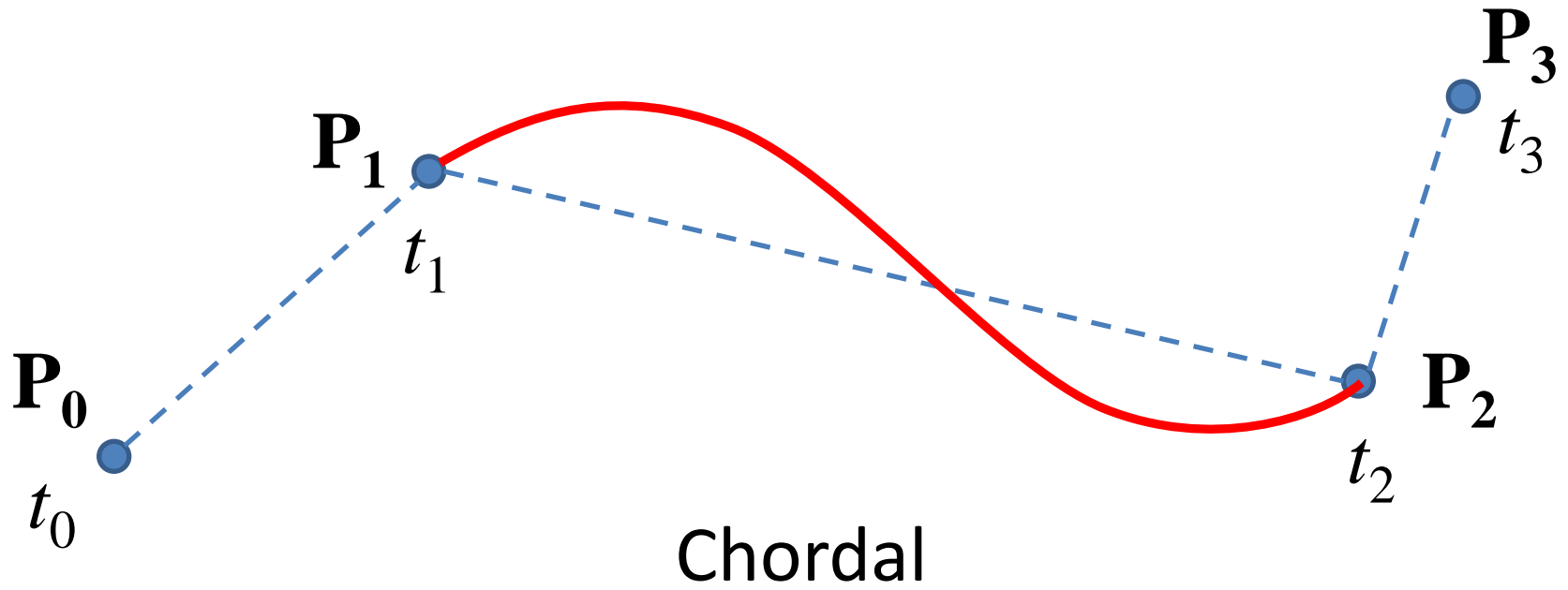
$$C_{12} (P_{0,1,2,3}, t_{0,1,2,3})$$

Catmull-Rom Curves



$$t_{i+1} = t_i + 1$$

Catmull-Rom Curves



$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |$$

Catmull-Rom Curves

- Parameterization

- Uniform:

$$t_{i+1} = t_i + 1$$

- Chordal:

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |$$

Catmull-Rom Curves

- Parameterization

- Uniform:

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^0$$

- Chordal:

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^1$$

Catmull-Rom Curves

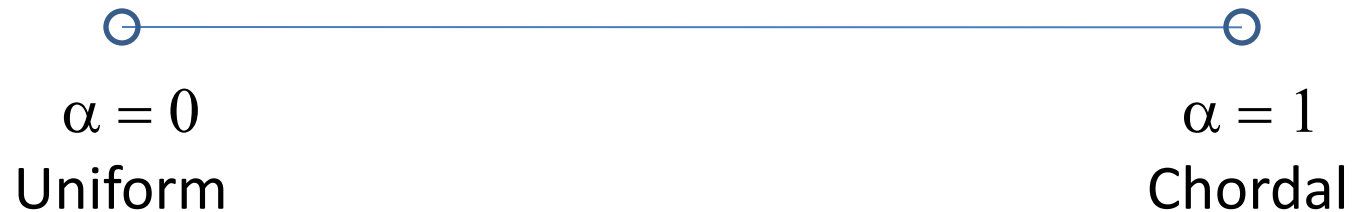
- Parameterization

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$

Catmull-Rom Curves

- Parameterization

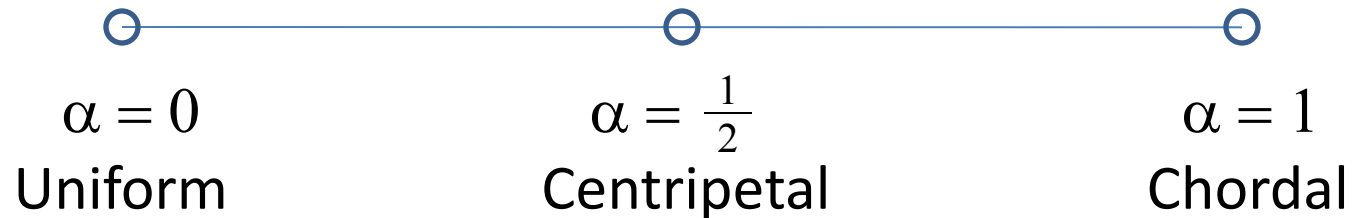
$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$



Catmull-Rom Curves

- Parameterization

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$



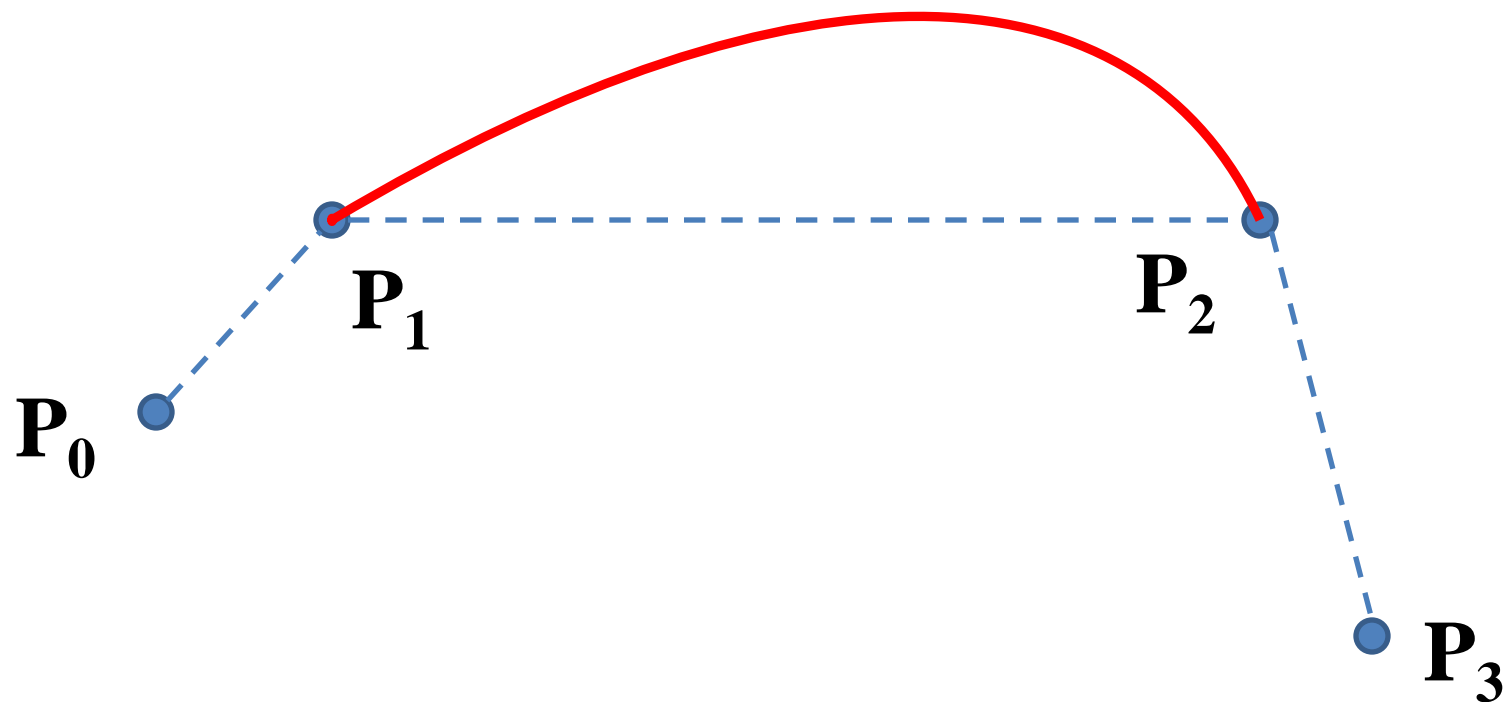
On the Parameterization of Catmull-Rom Curves

DEMO

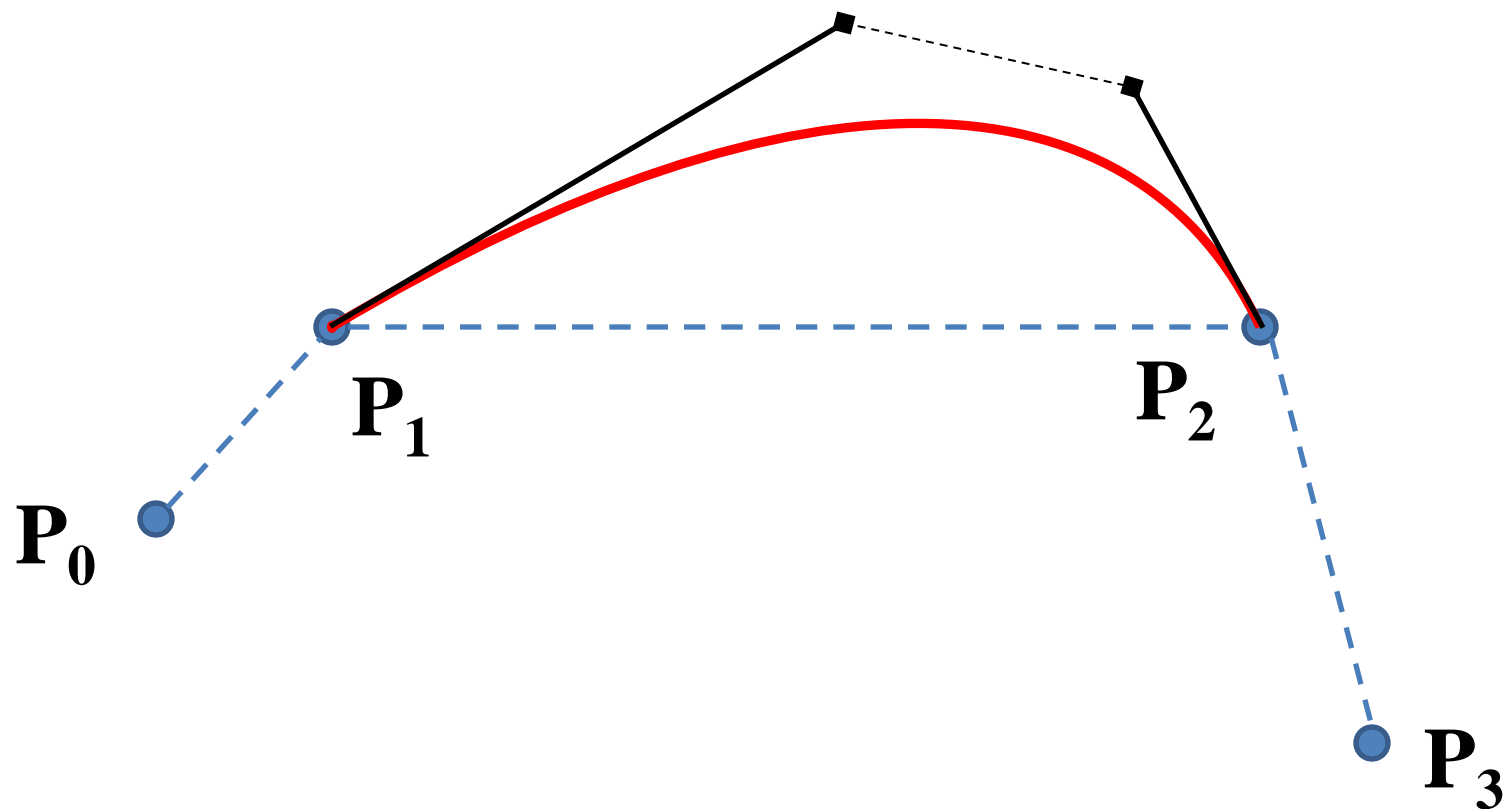
On the Parameterization of Catmull-Rom Curves

CUSPS & SELF-INTERSECTIONS

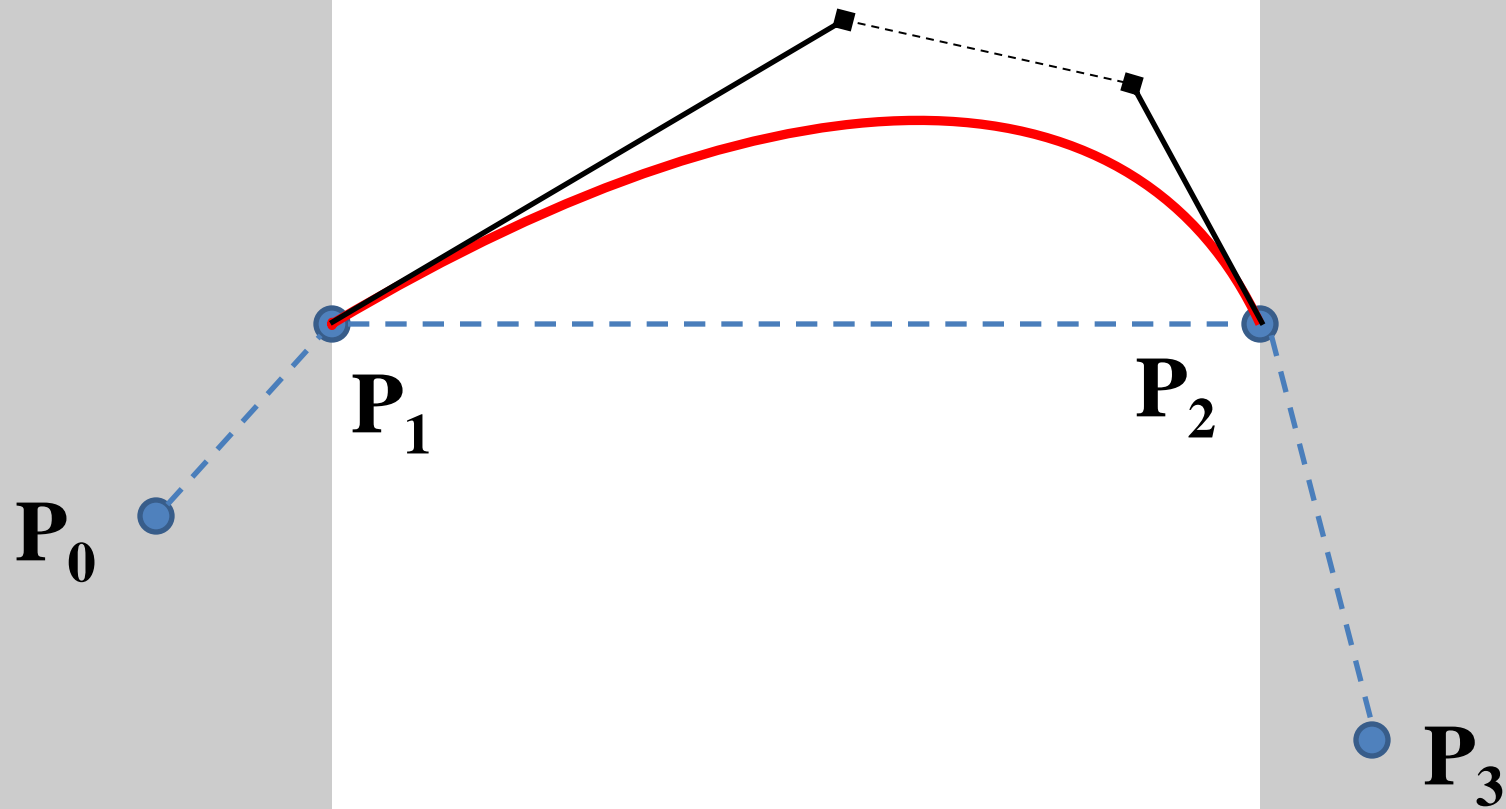
Cusps & Self-Intersections



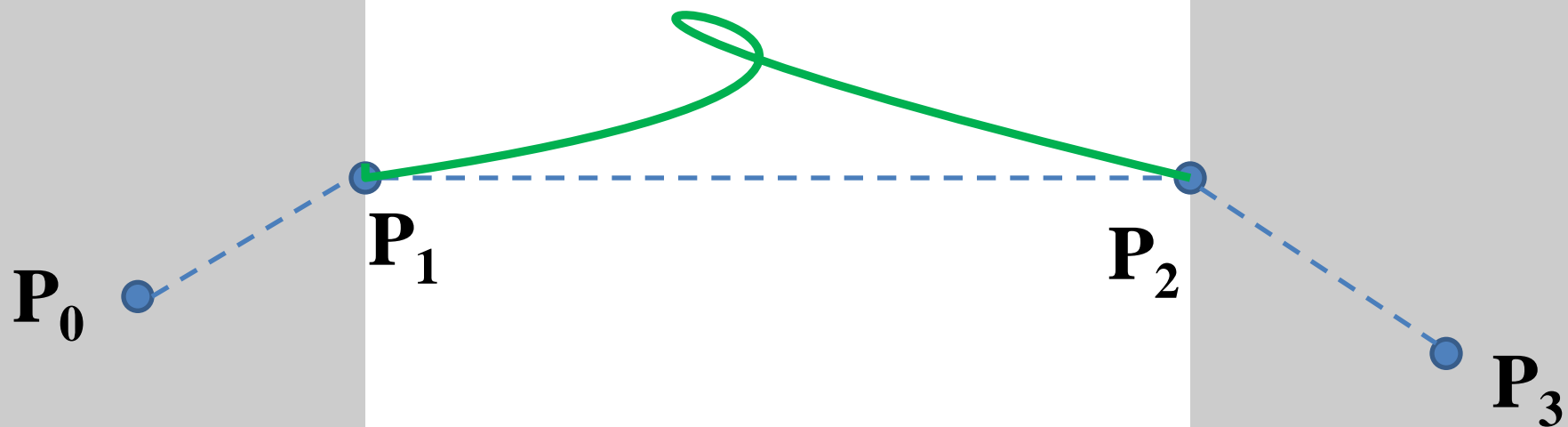
Cusps & Self-Intersections



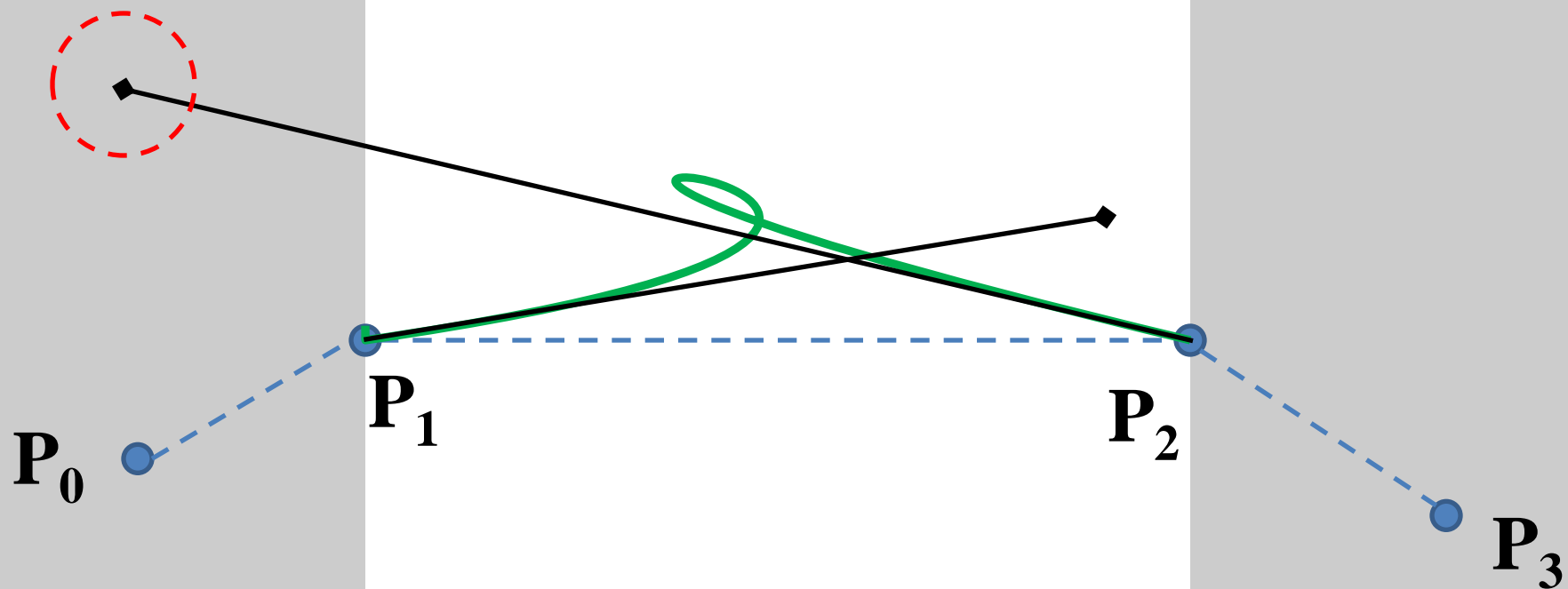
Cusps & Self-Intersections



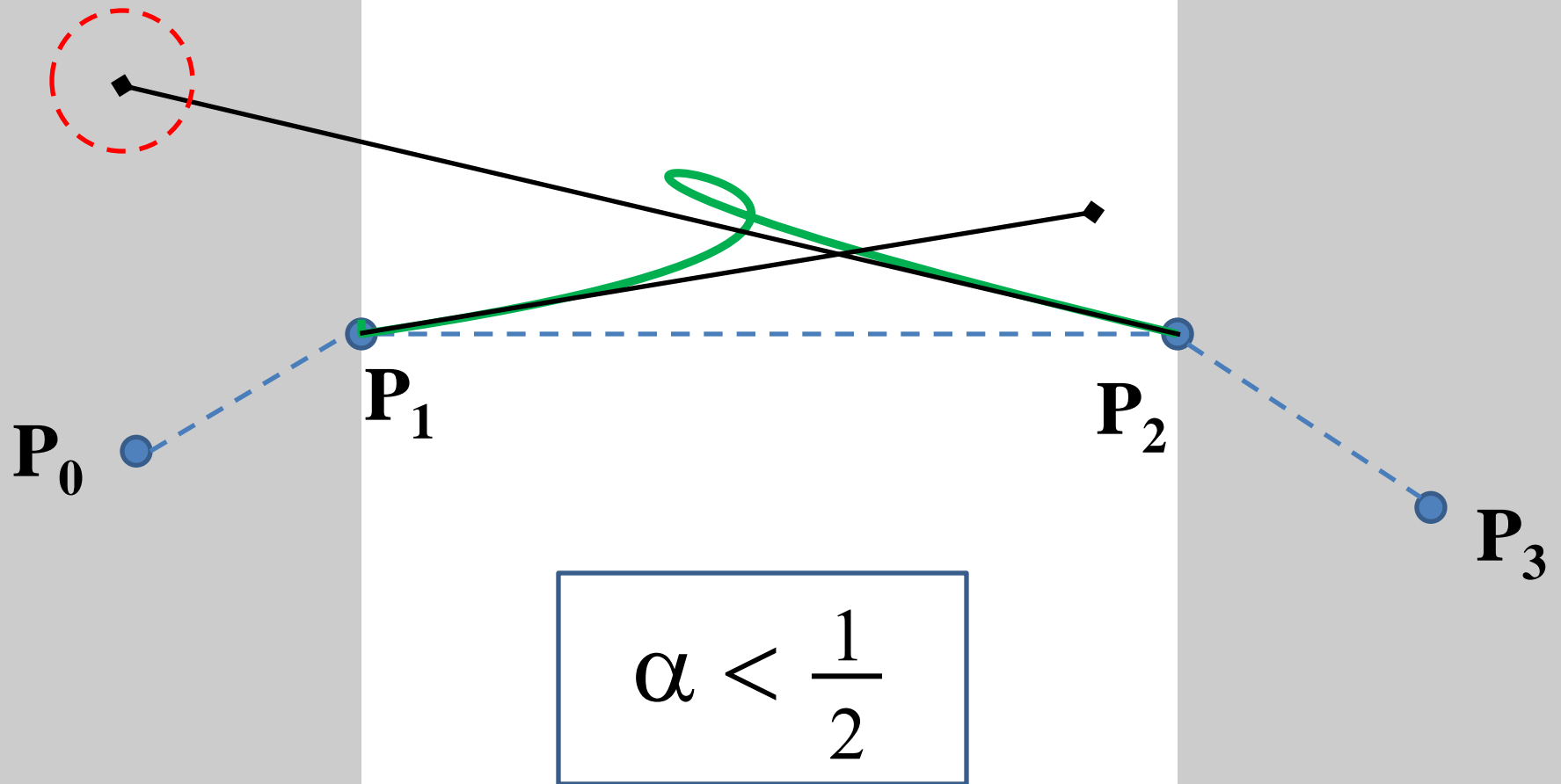
Cusps & Self-Intersections



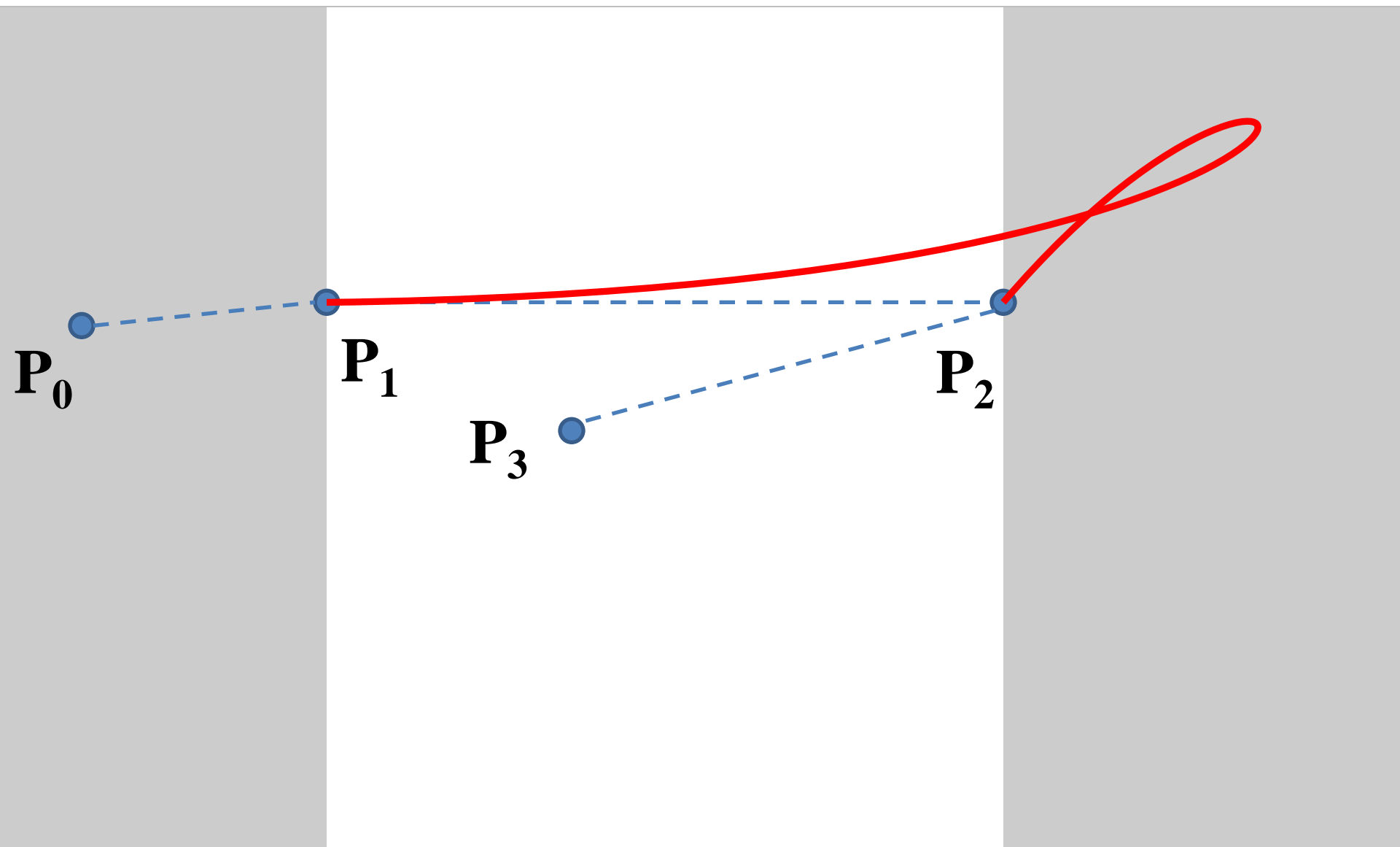
Cusps & Self-Intersections



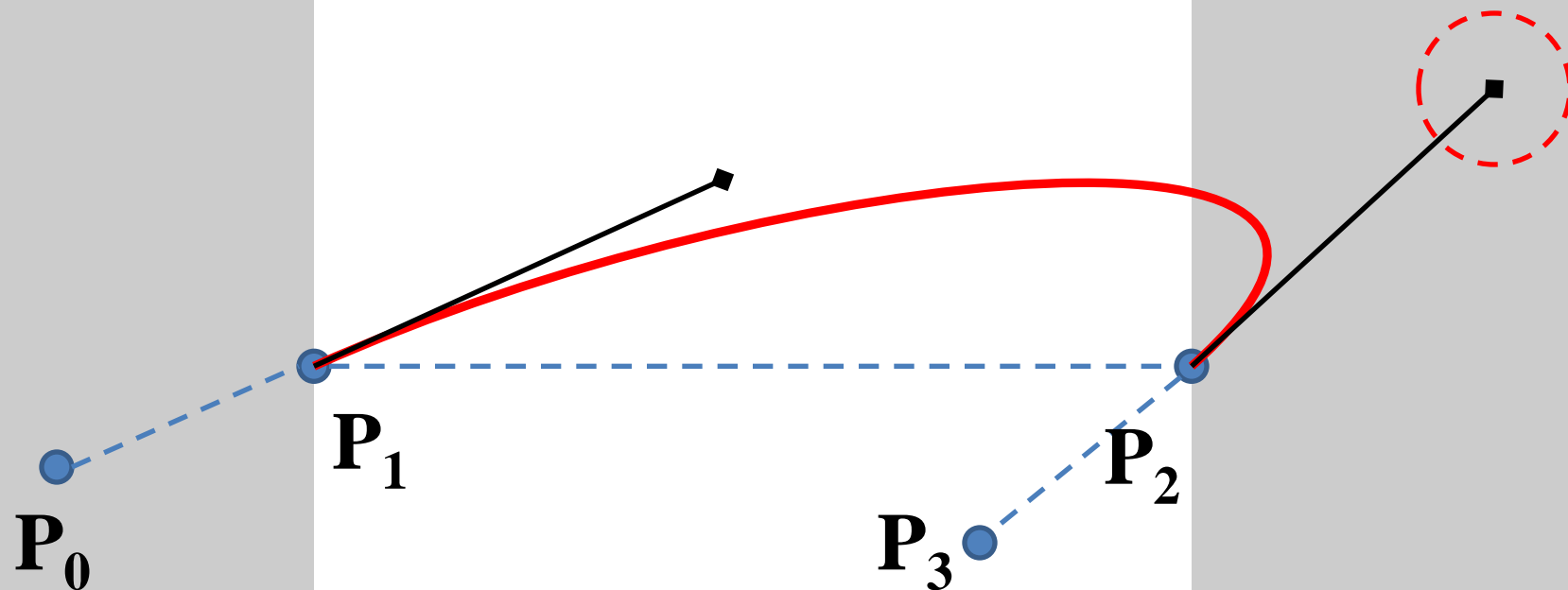
Cusps & Self-Intersections



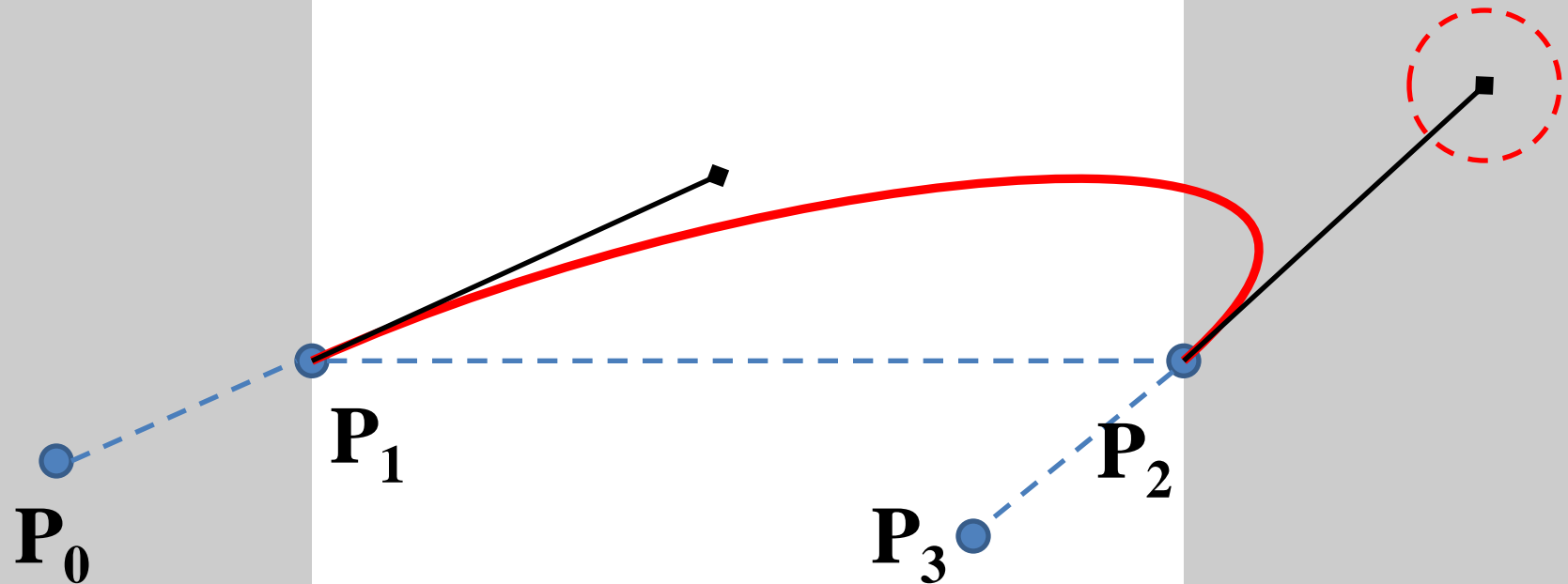
Cusps & Self-Intersections



Cusps & Self-Intersections

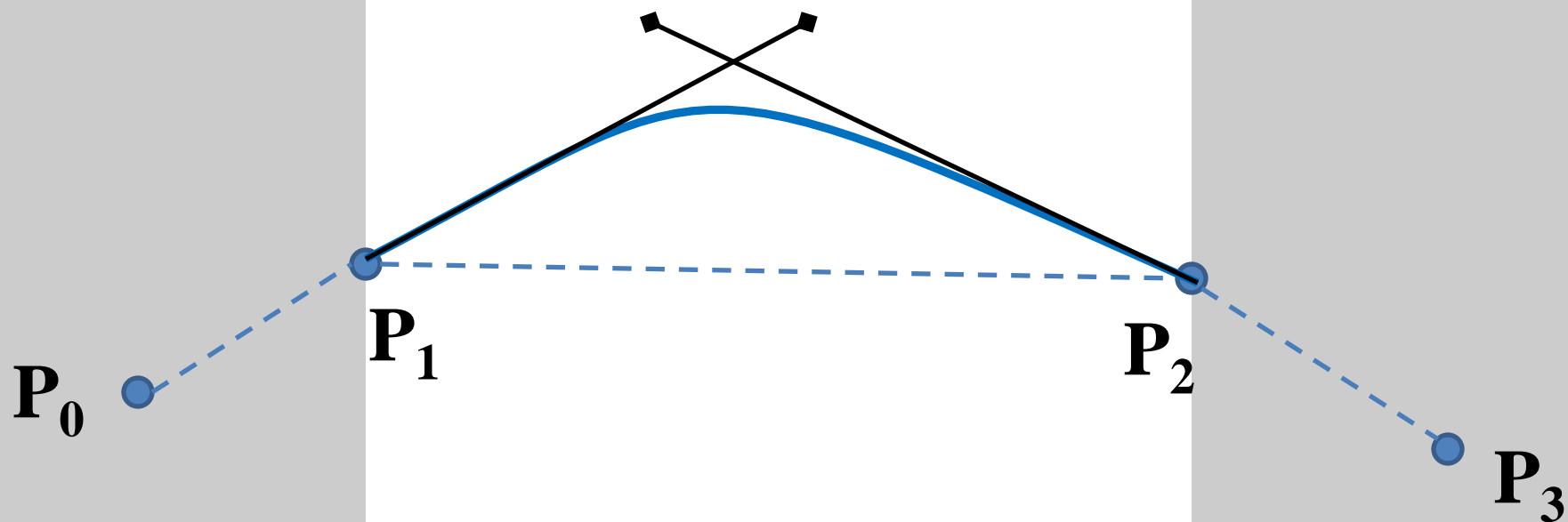


Cusps & Self-Intersections



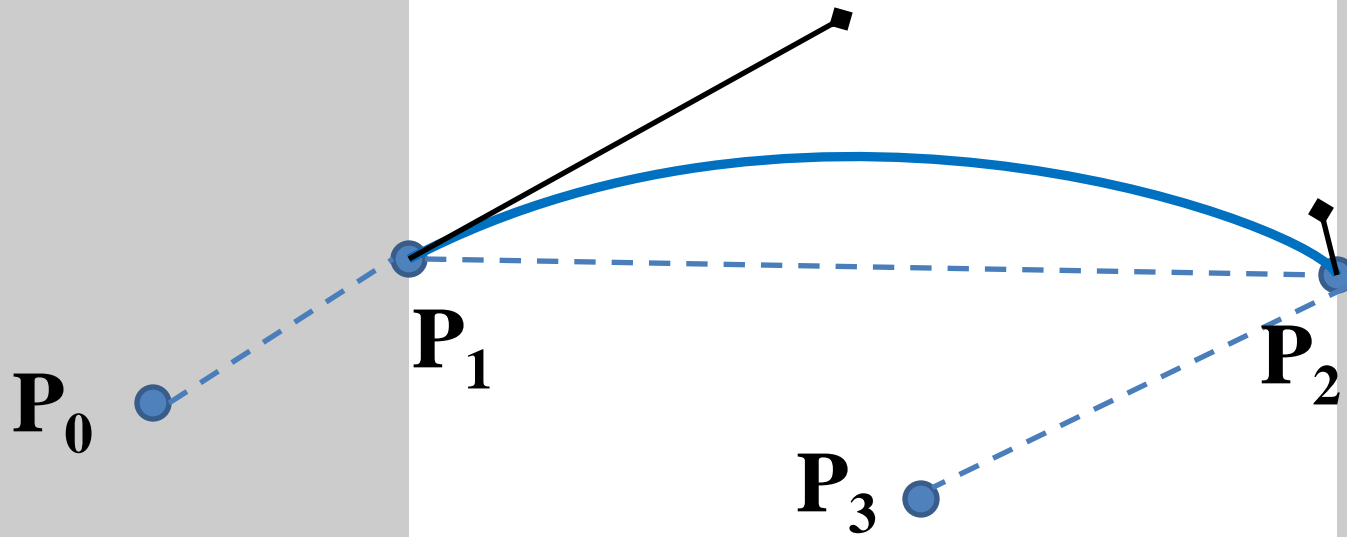
$$\alpha \neq \frac{1}{2}$$

Cusps & Self-Intersections



$$\alpha = \frac{1}{2}$$

Cusps & Self-Intersections

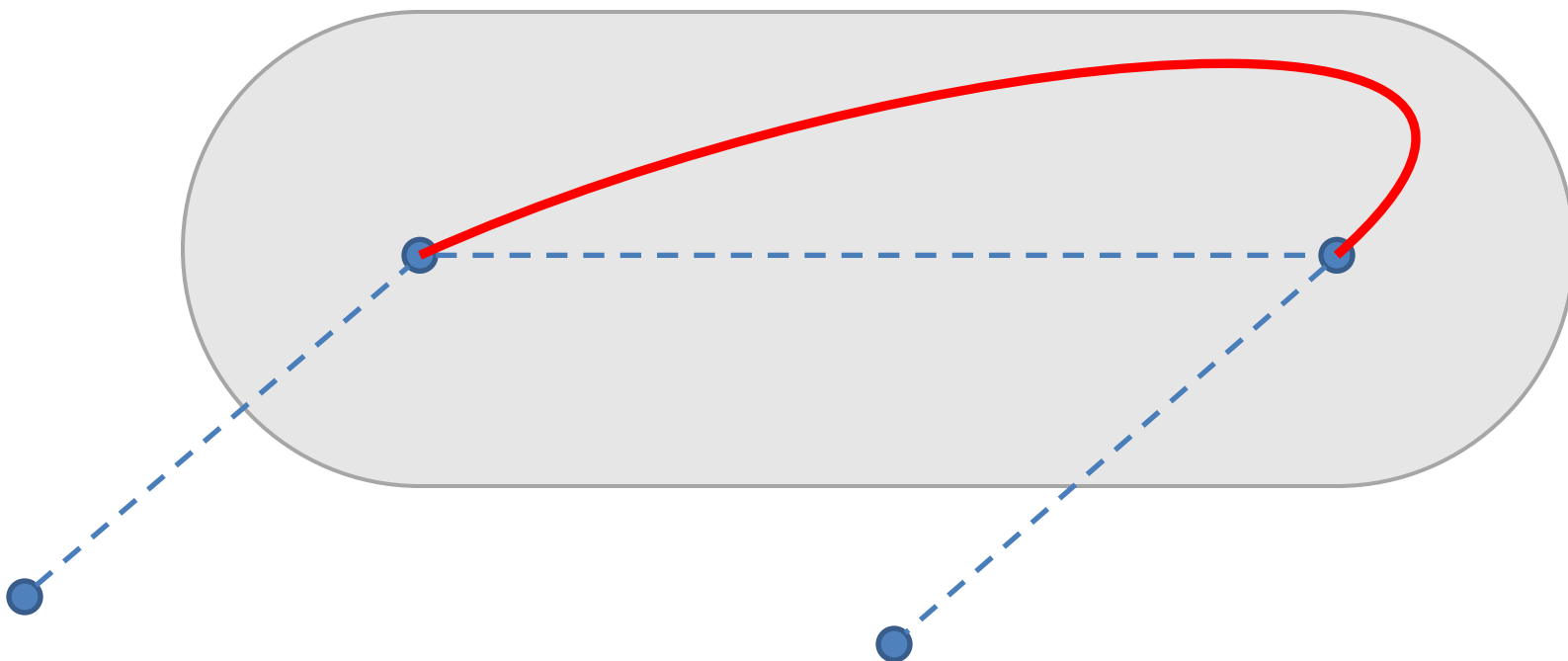


$$\alpha = \frac{1}{2}$$

On the Parameterization of Catmull-Rom Curves

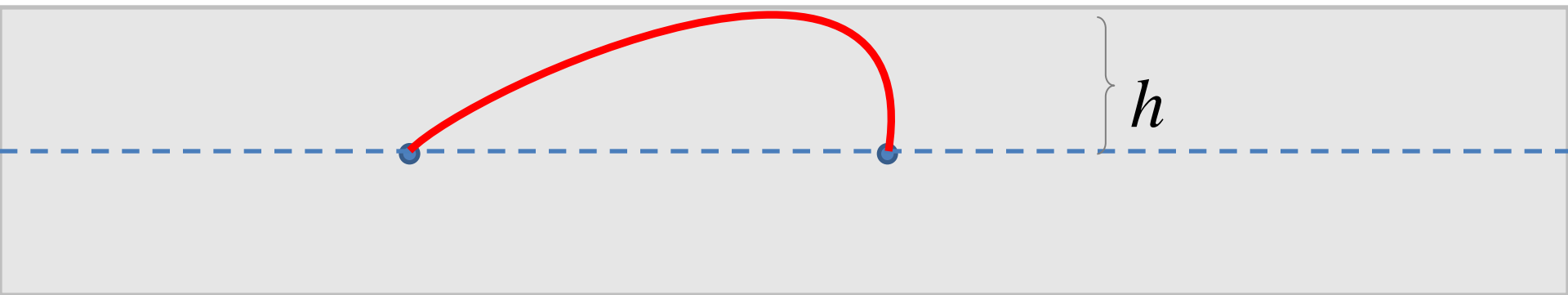
DISTANCE BOUND

Distance Bound

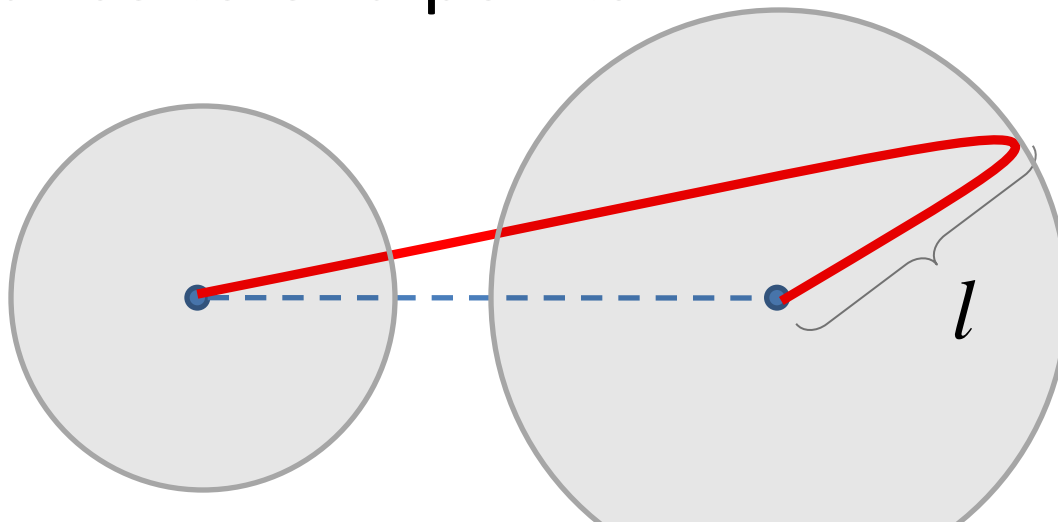


Distance Bound

- Distance to the infinite line

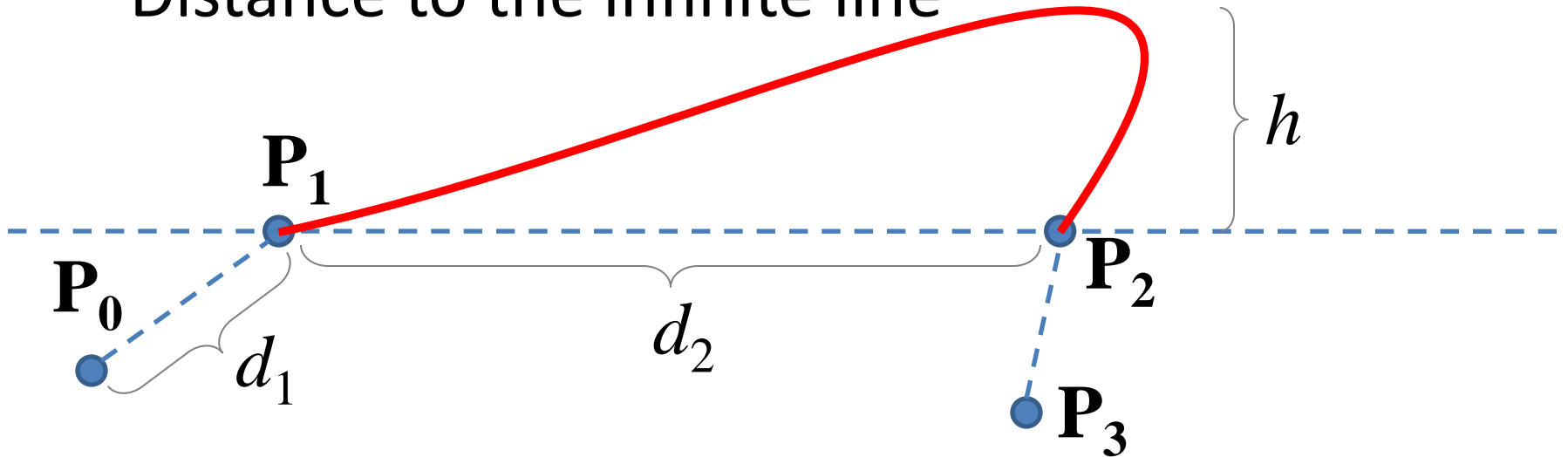


- Distance to end points



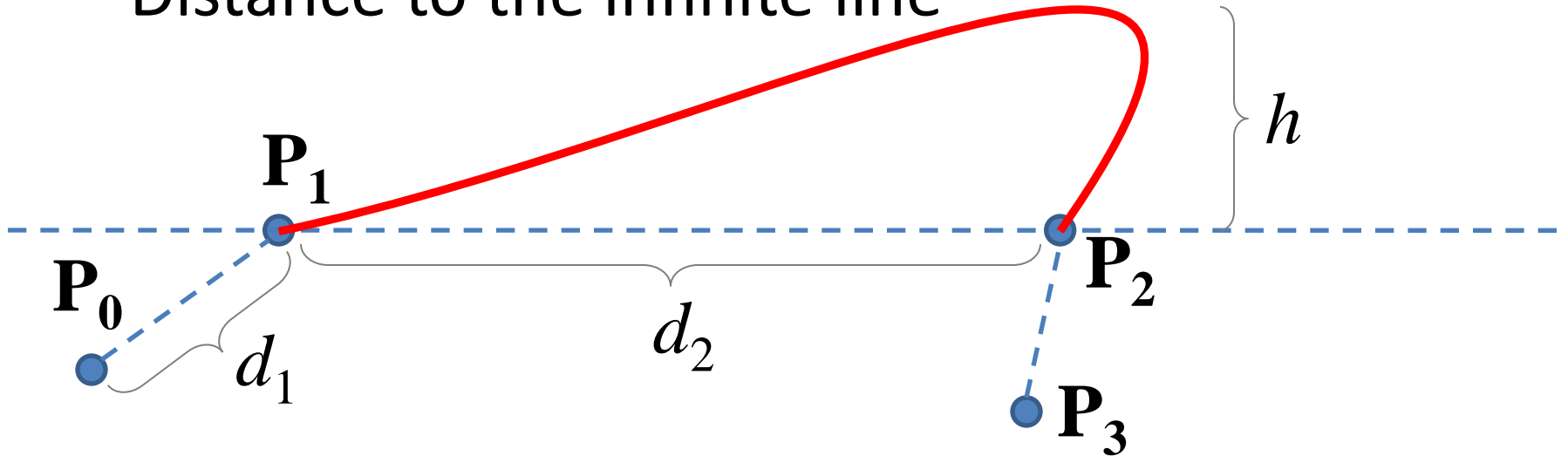
Distance Bound

- Distance to the infinite line



Distance Bound

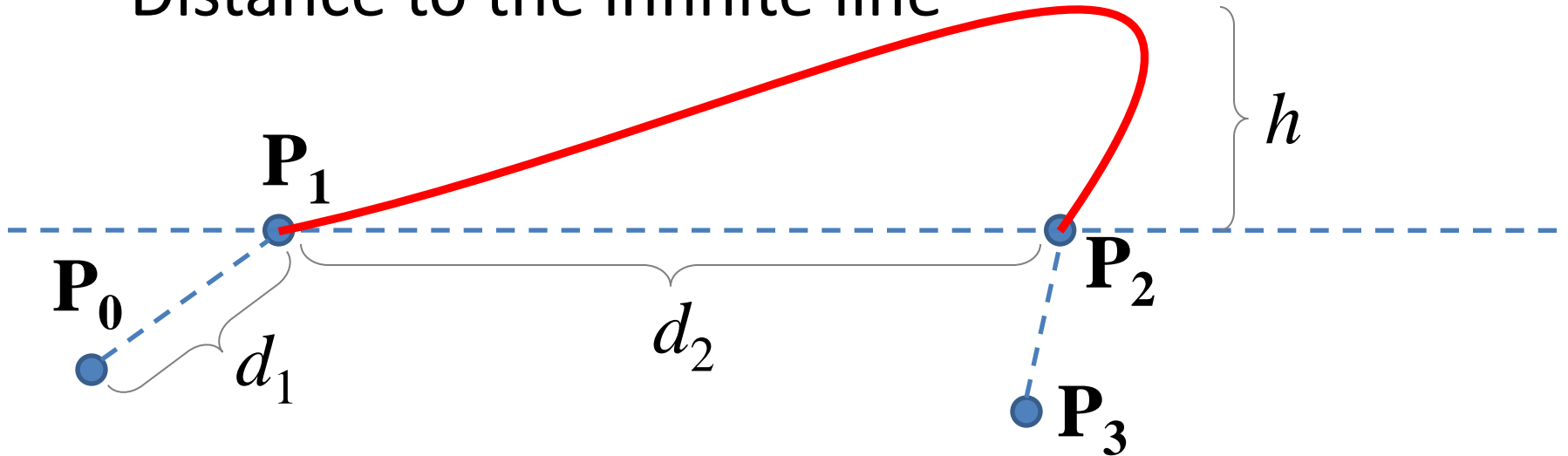
- Distance to the infinite line



$$h \leq d_2 \times A$$

Distance Bound

- Distance to the infinite line

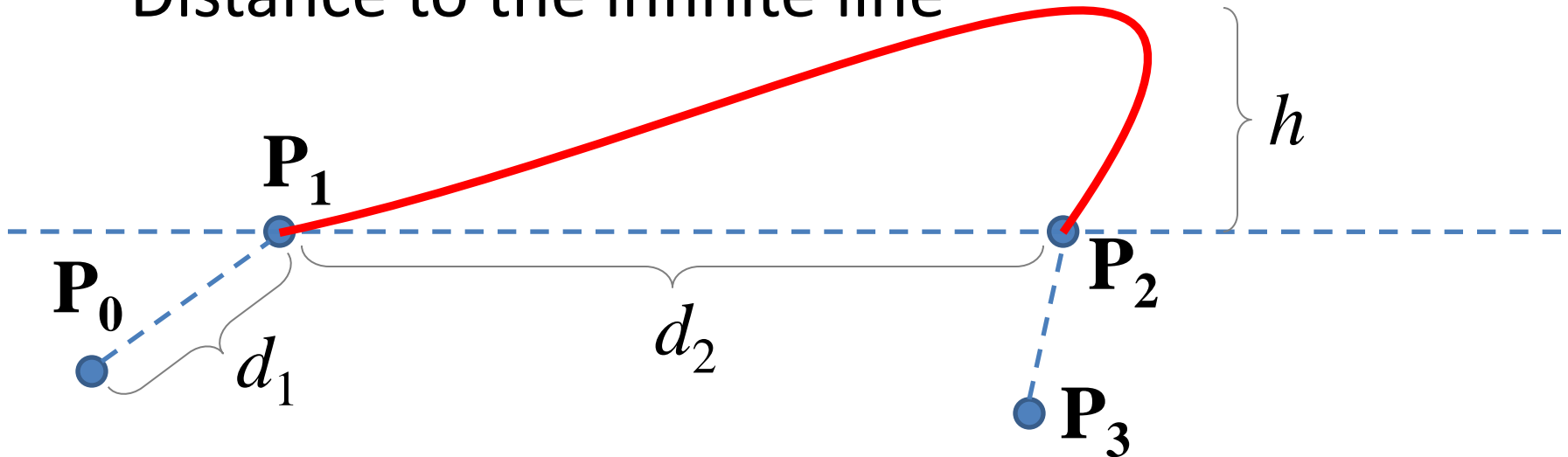


$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

Distance Bound

- Distance to the infinite line



$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$$\alpha < 1/2 \quad h \leq \infty$$

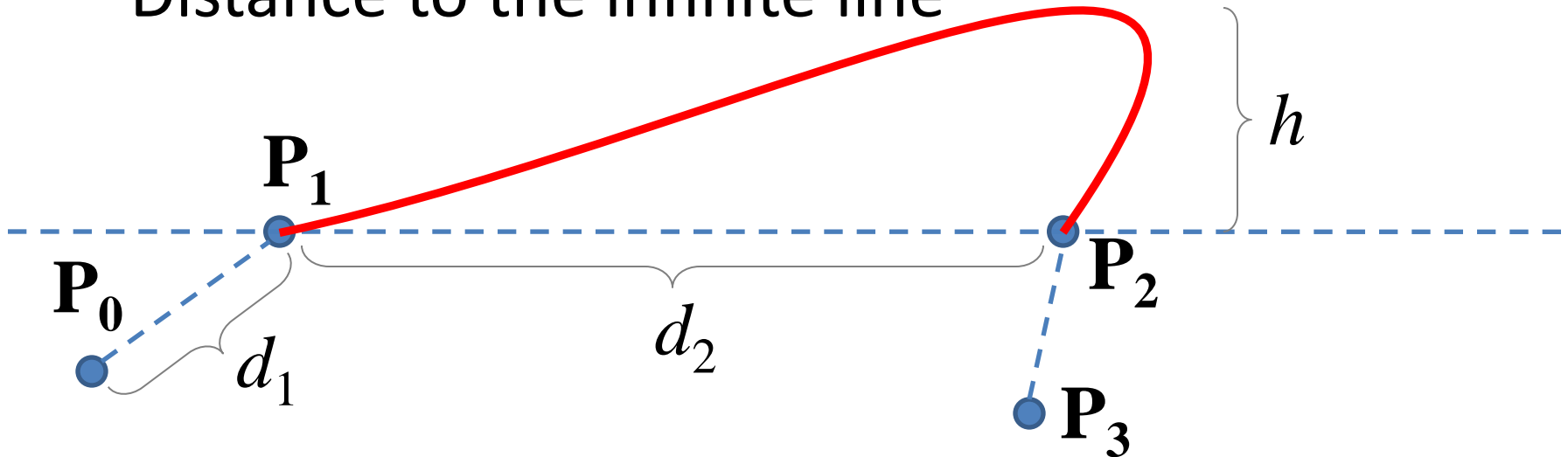
$$\alpha = 1/2 \quad h \leq d_2/4$$

$$\alpha = 2/3 \quad h \leq d_2/8$$

$$\alpha = 1 \quad h \leq d_2/4$$

Distance Bound

- Distance to the infinite line



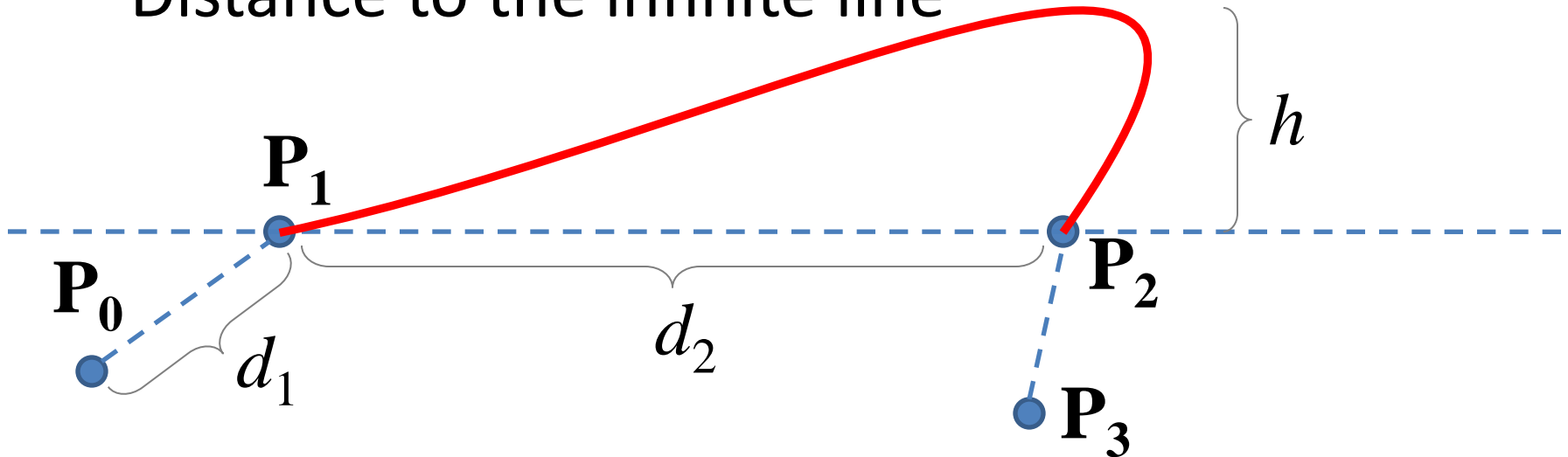
$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$\alpha < 1/2$	$h \leq \infty$
$\alpha = 1/2$	$h \leq d_2/4$
$\alpha = 2/3$	$h \leq d_2/8$
$\alpha = 1$	$h \leq d_2/4$

Distance Bound

- Distance to the infinite line



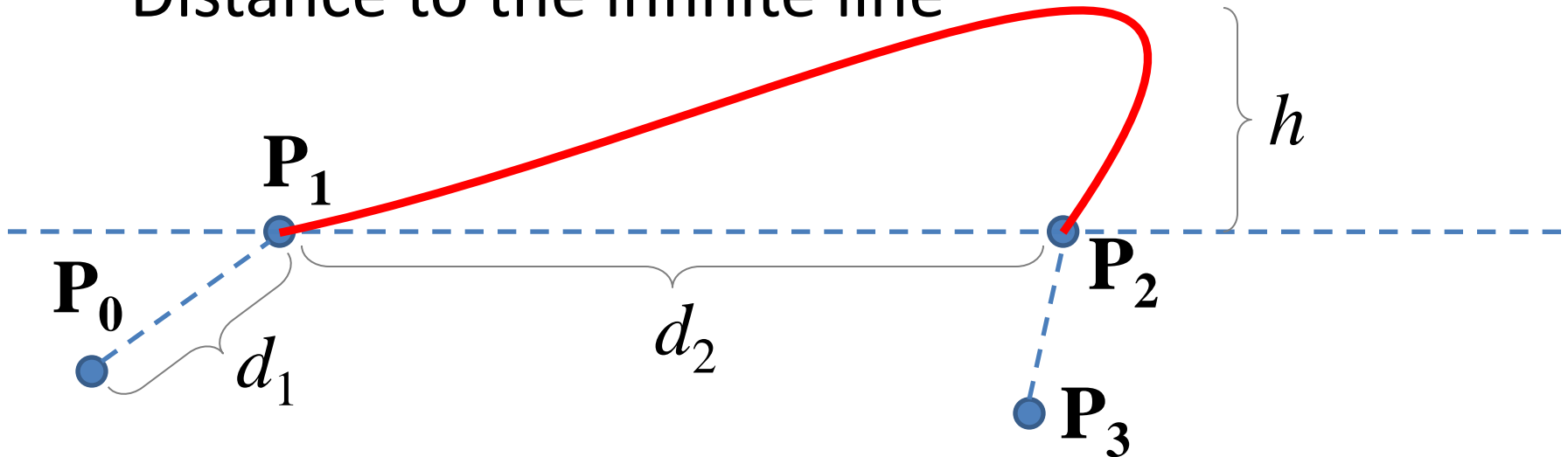
$$r = \frac{d_1}{d_2}$$

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$\alpha < 1/2$	$h \leq \infty$
$\alpha = 1/2$	$h \leq d_2/4$
$\alpha = 2/3$	$h \leq d_2/8$
$\alpha = 1$	$h \leq d_2/4$

Distance Bound

- Distance to the infinite line



$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$$\alpha < 1/2 \quad h \leq \infty$$

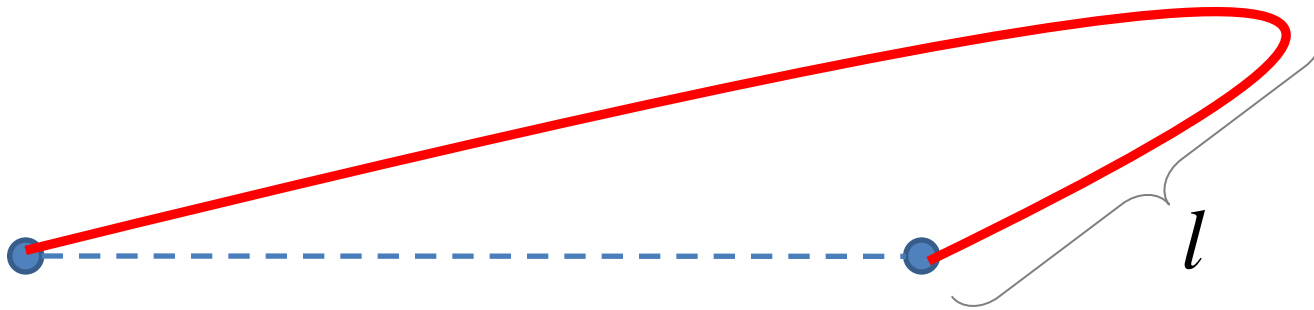
$$\alpha = 1/2 \quad h \leq d_2/4$$

$$\alpha = 2/3 \quad h \leq d_2/8$$

$$\alpha = 1 \quad h \leq d_2/4$$

Distance Bound

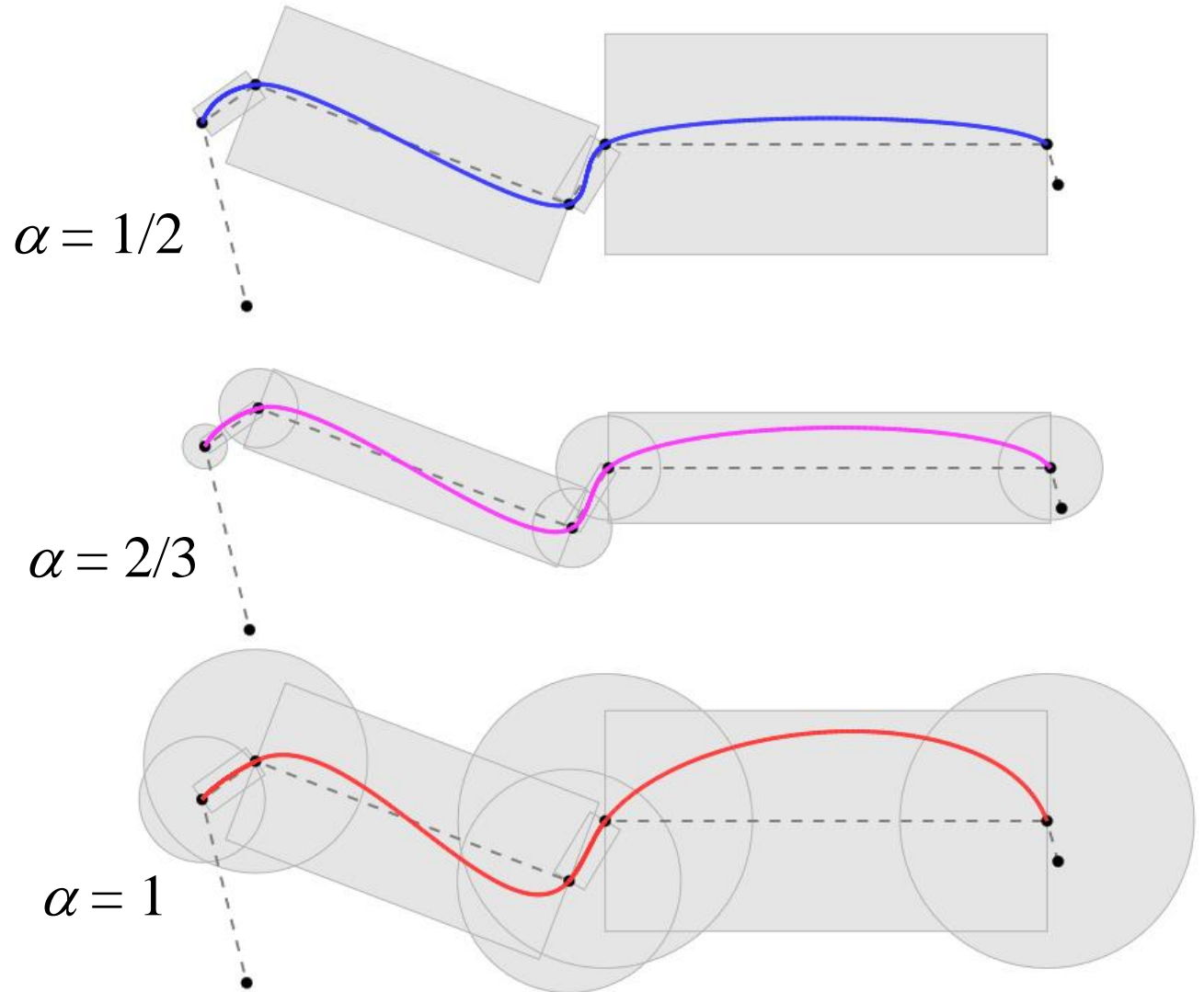
- Distance to the end points



$$l \leq \frac{d_2 \sqrt{r^2 - r^{4\alpha}}}{3 r^\alpha (1 + r^\alpha)}$$

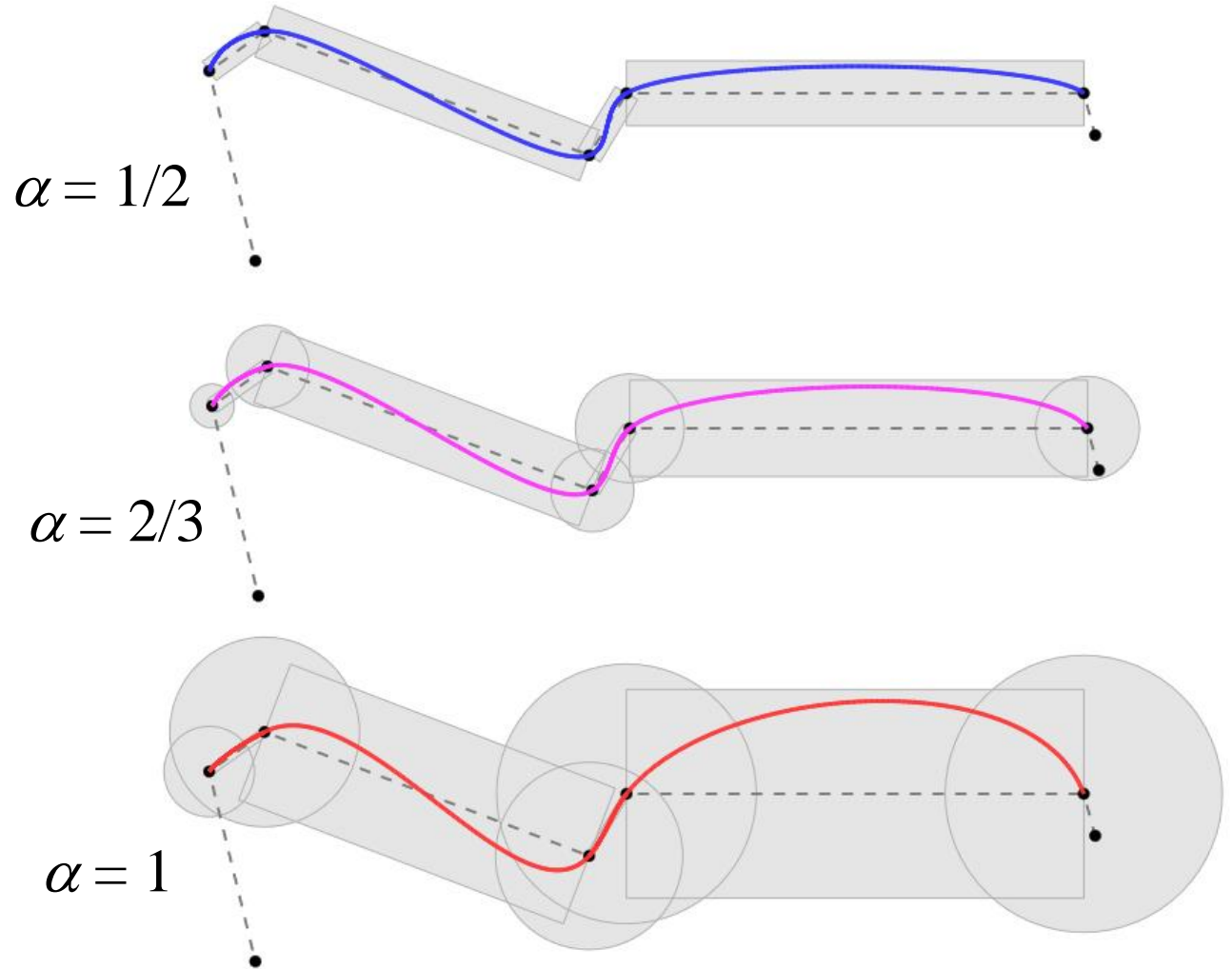
Distance Bound

α only



Distance Bound

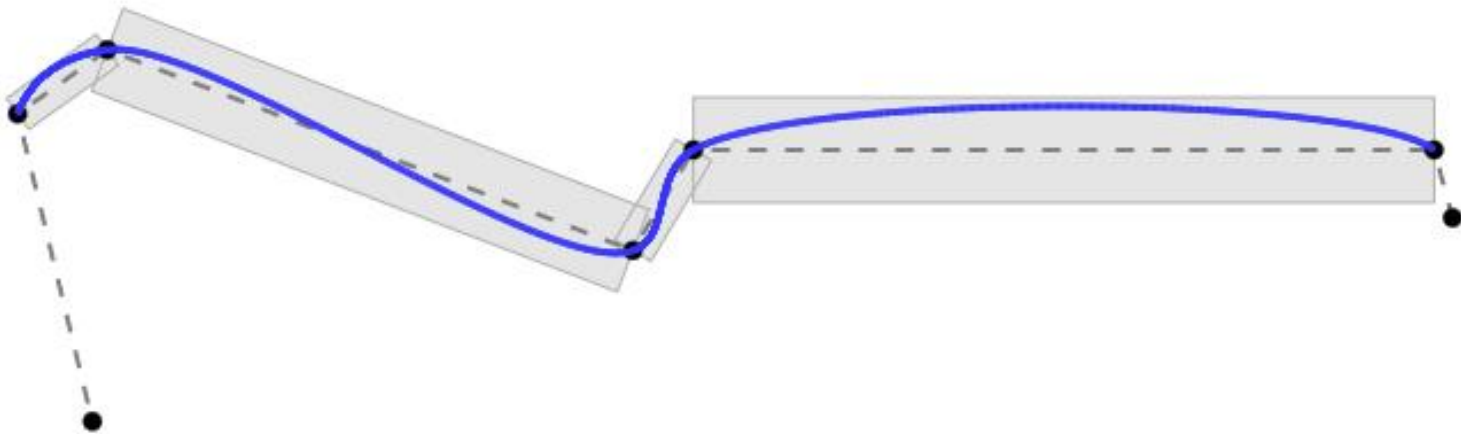
α and r



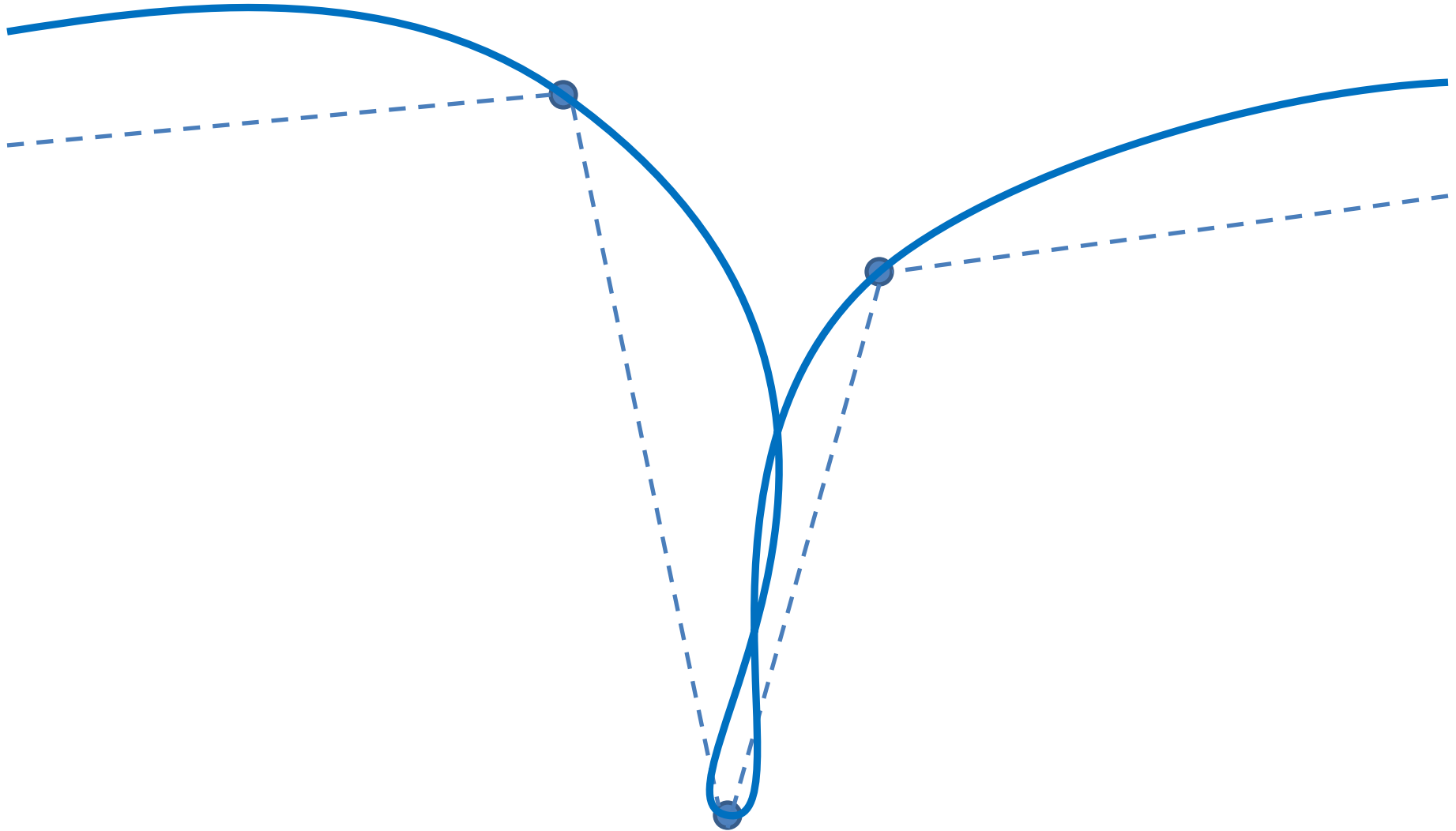
On the Parameterization of Catmull-Rom Curves

INTERSECTION-FREE CURVES

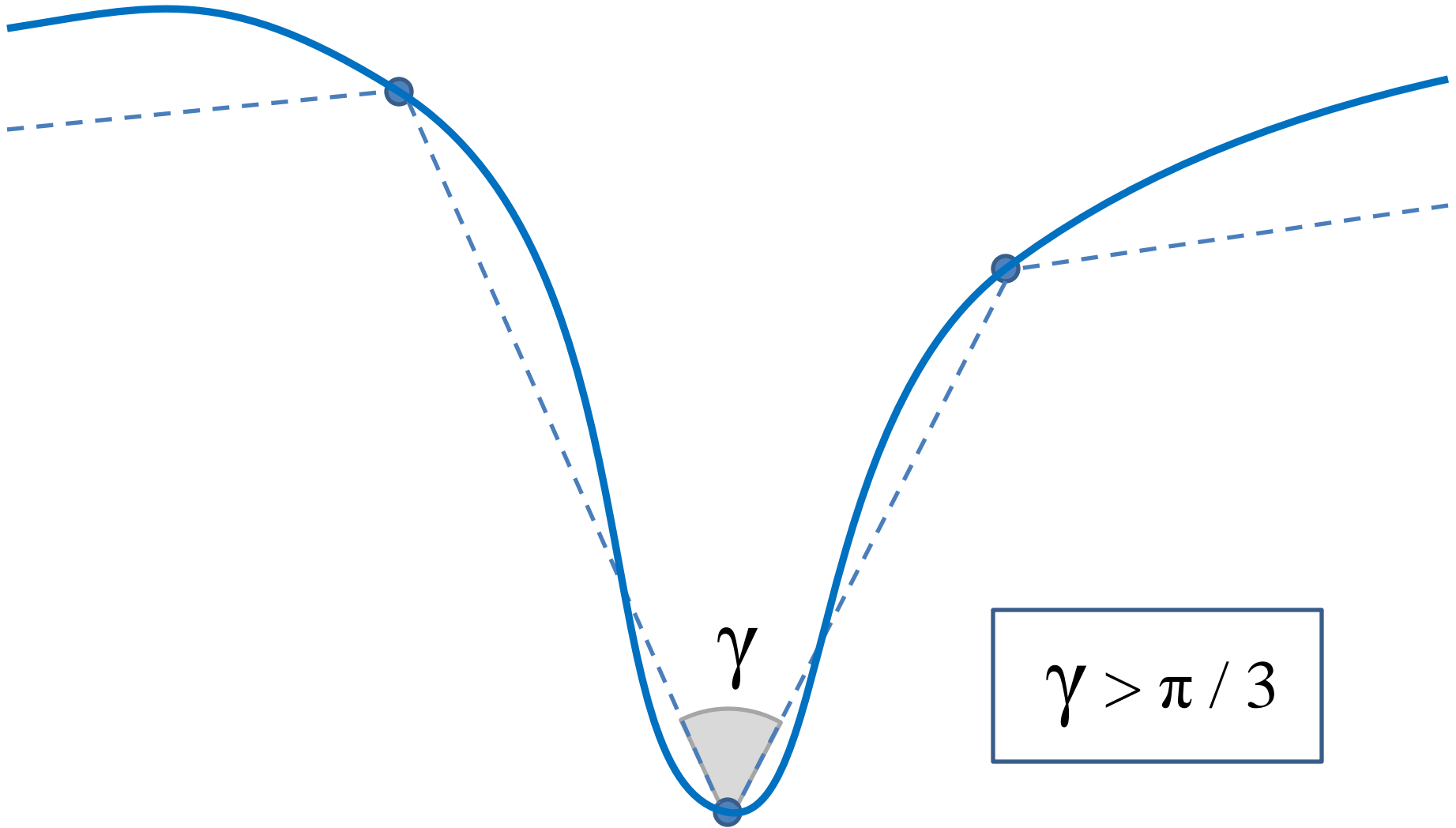
Intersection-Free Curves



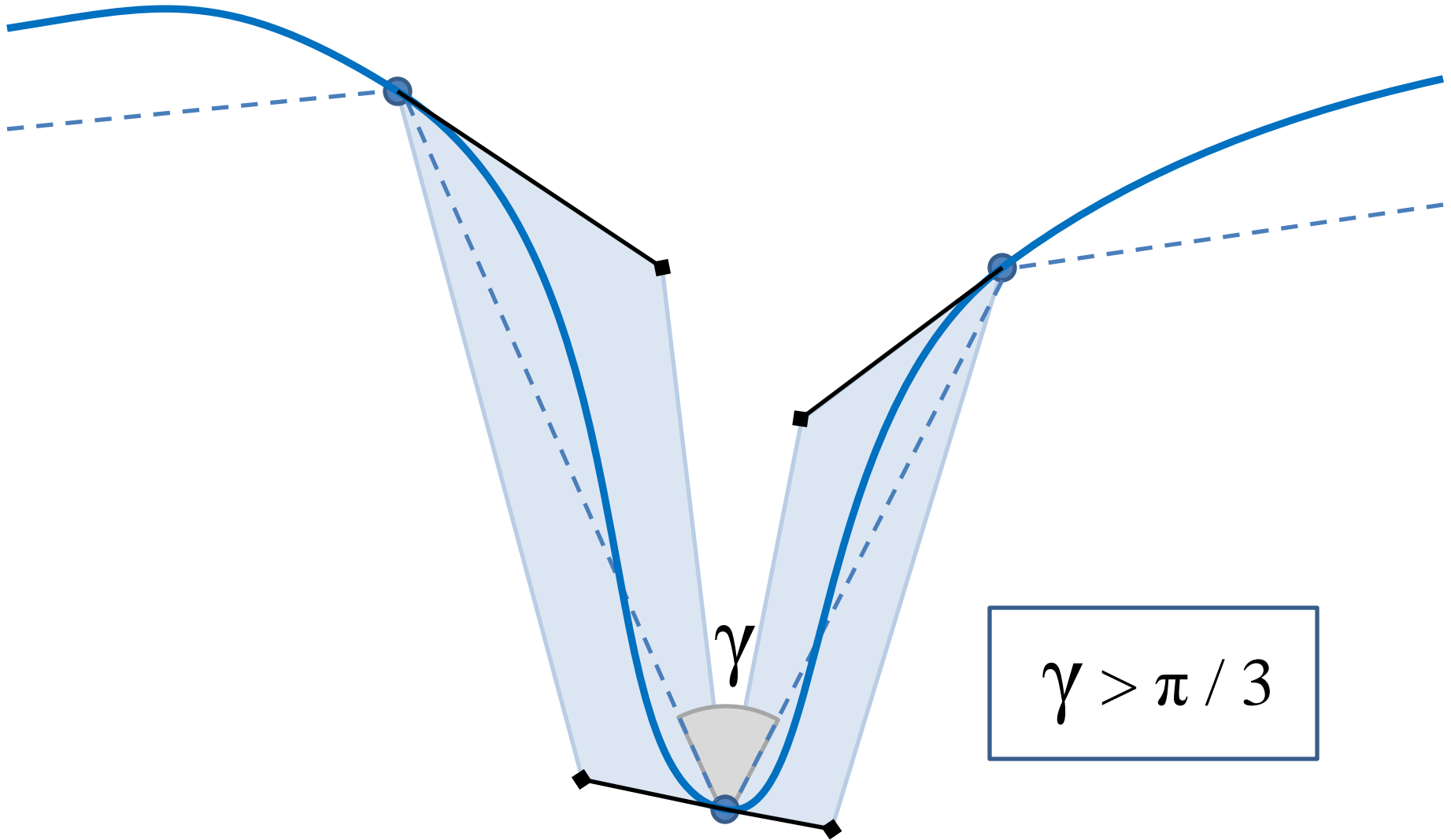
Intersection-Free Curves



Intersection-Free Curves



Intersection-Free Curves



Intersection-Free Curves

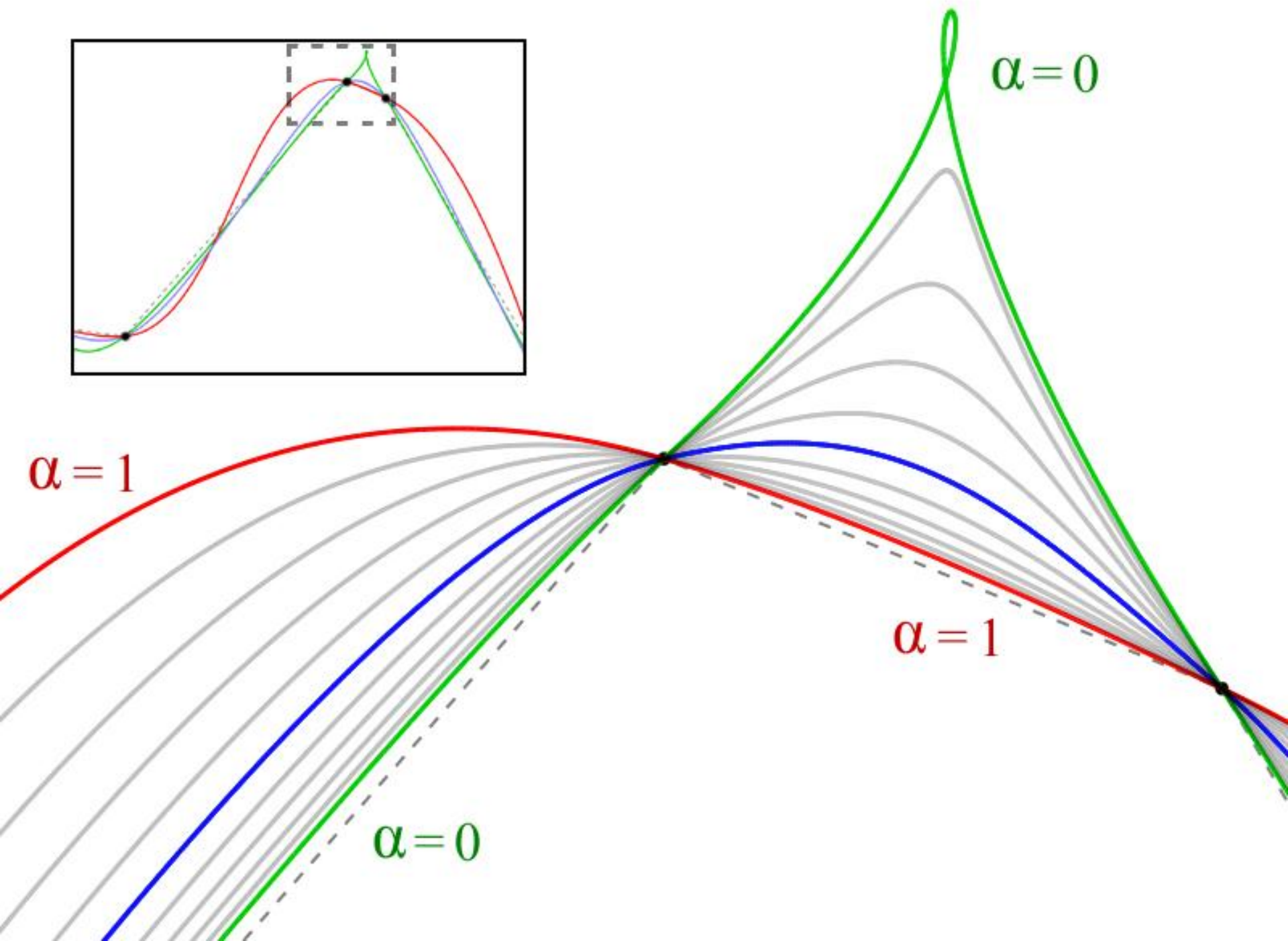
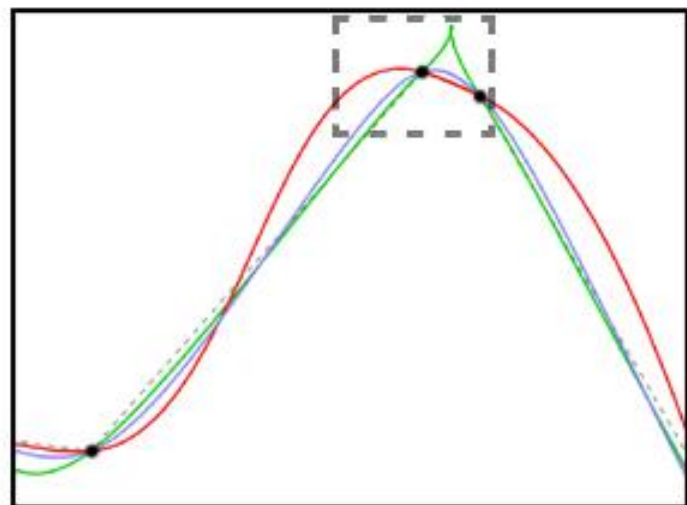
- Avoid self-intersections
 - Centripetal parameterization
- Avoid adjacent segment intersections
 - Control polygon angle $> \pi / 3$
- Avoid non-adjacent segment intersections
 - Bounding box

On the Parameterization of Catmull-Rom Curves

DISCUSSION

Discussion

- Distance to Control Polygon
 - Uniform is closer for longer segments
 - Chordal is closer for shorter segments

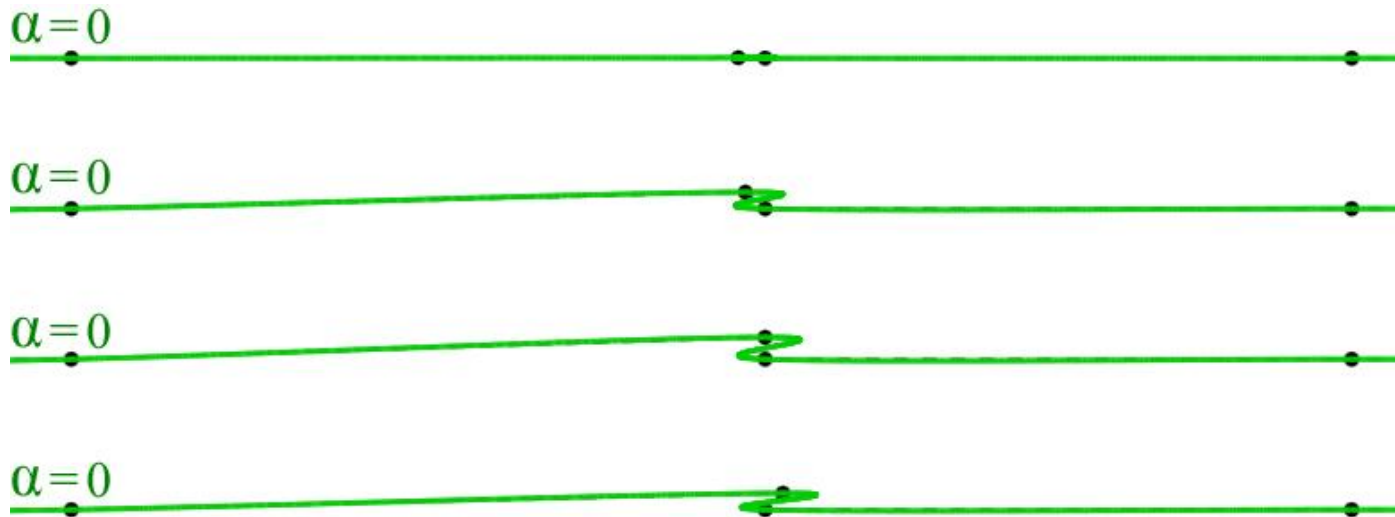


Discussion

- Edge Direction
 - Chordal has extreme sensitivity to short edge directions

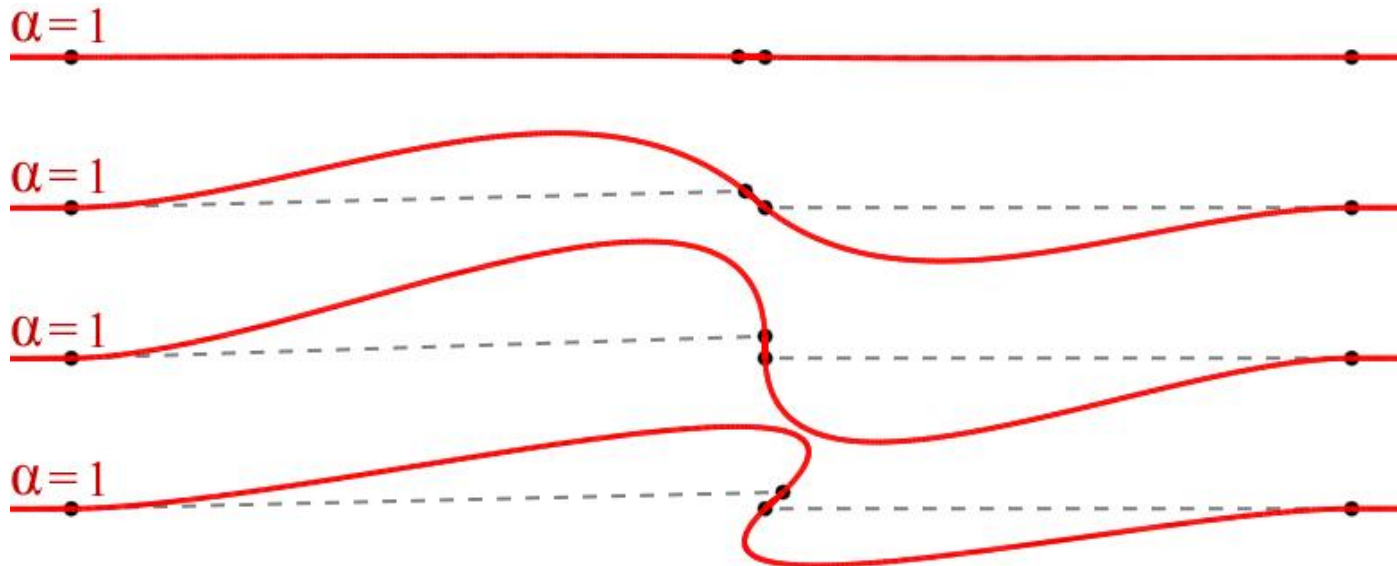
Discussion

- Edge Direction
 - Chordal has extreme sensitivity to short edge directions



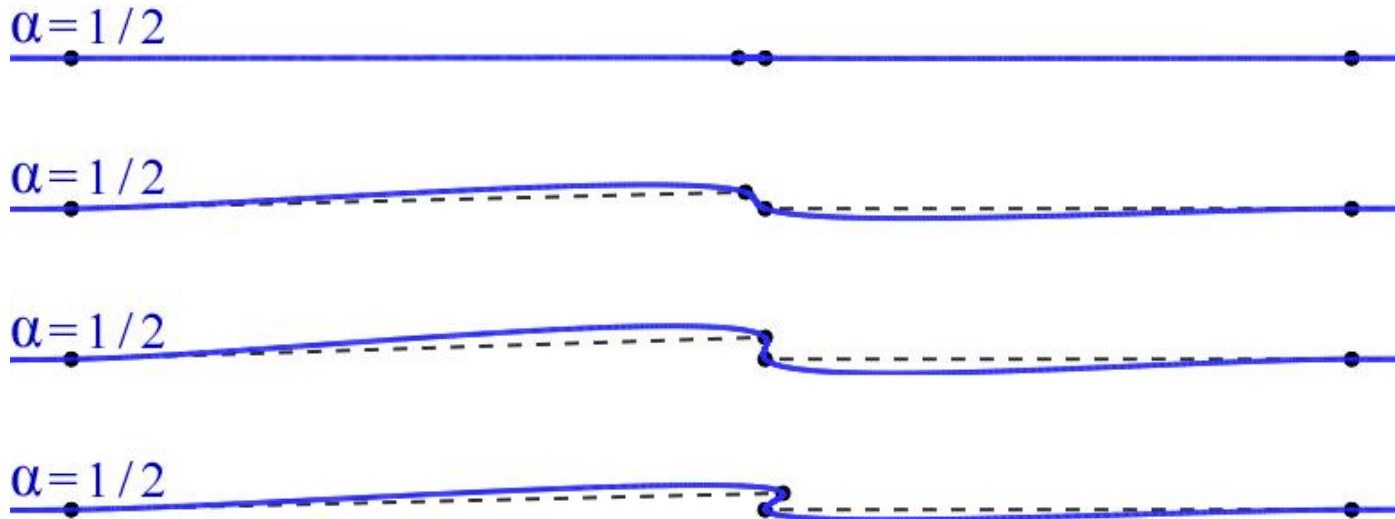
Discussion

- Edge Direction
 - Chordal has extreme sensitivity to short edge directions



Discussion

- Edge Direction
 - Chordal has extreme sensitivity to short edge directions



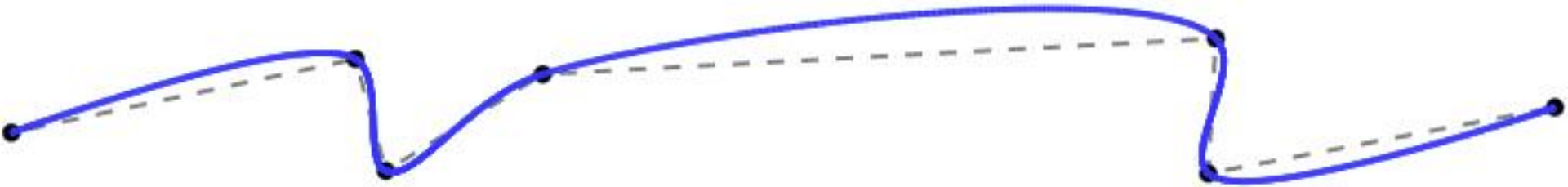
Discussion

- Curvature
 - Centripetal *tends* to have higher curvature at control points.



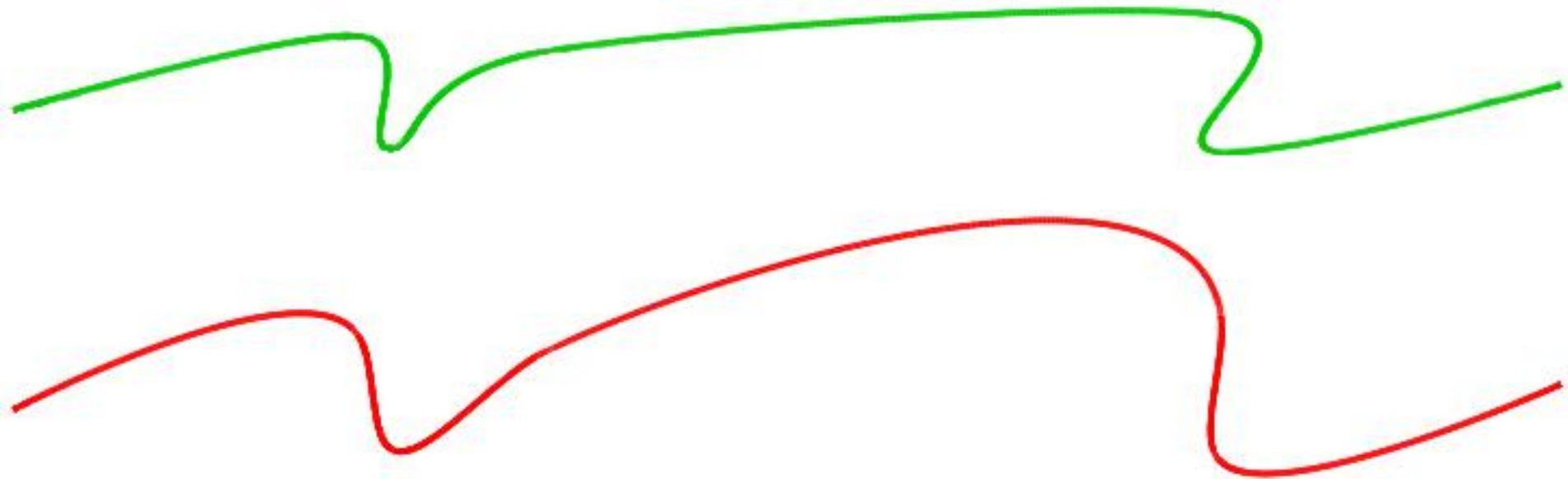
Discussion

- Curvature
 - Centripetal *tends* to have higher curvature at control points.



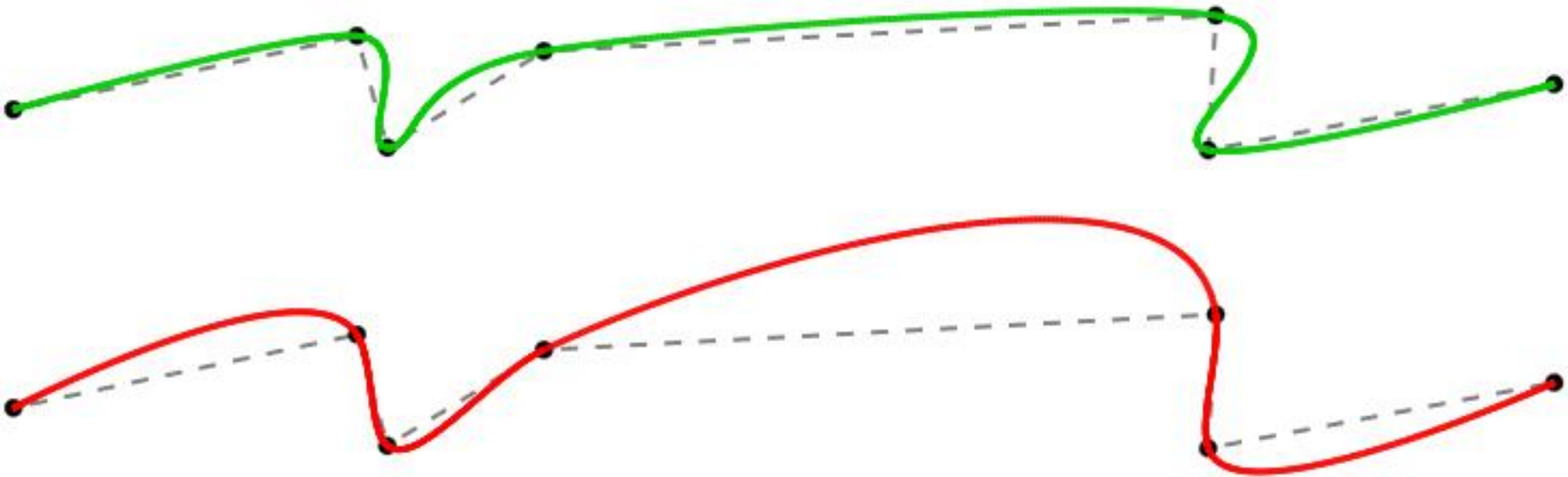
Discussion

- Curvature
 - Centripetal *tends* to have higher curvature at control points.



Discussion

- Curvature
 - Centripetal *tends* to have higher curvature at control points.



Catmull-Rom Curves



Lee Perry-Smith



Lee Perry-Smith

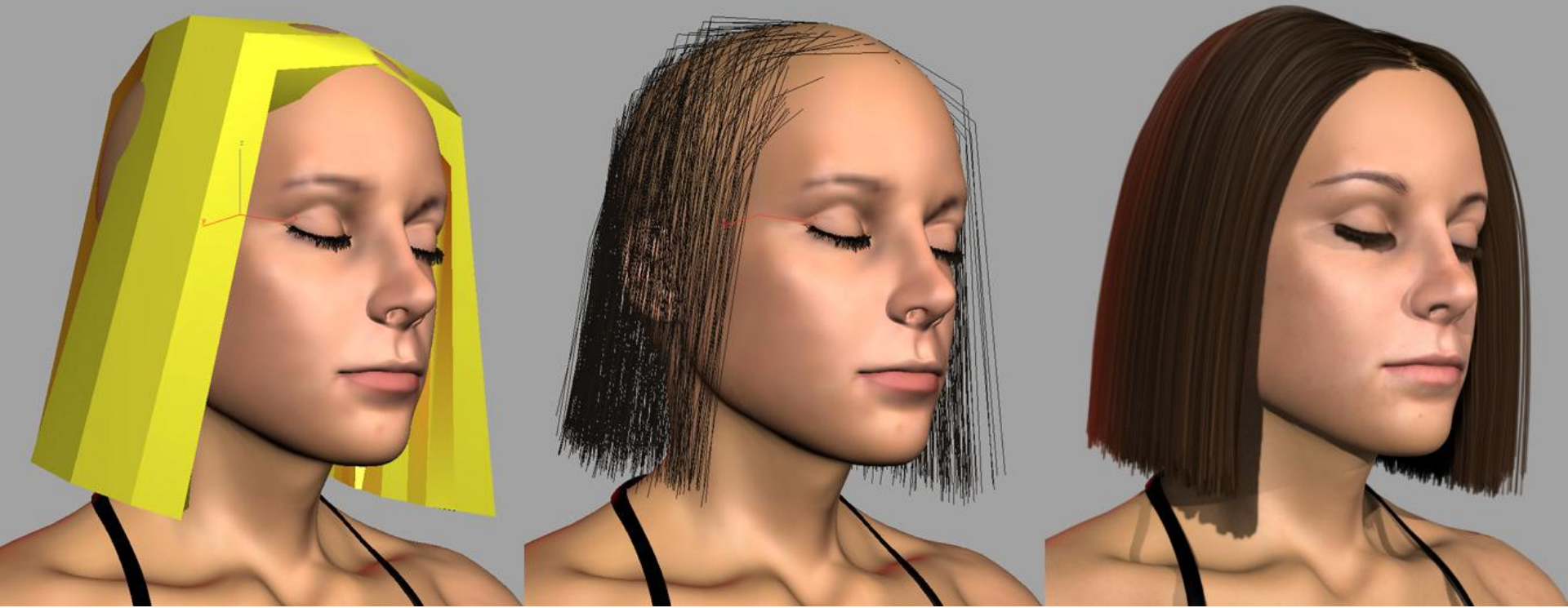


Alexander Tomchuk

Cem Yuksel, Scott Schaefer, John Keyser, "Hair Meshes," Siggraph Asia 2009

HAIRFARM[™]
THE ULTIMATE HAIR PLUG-IN FOR 3DS MAX

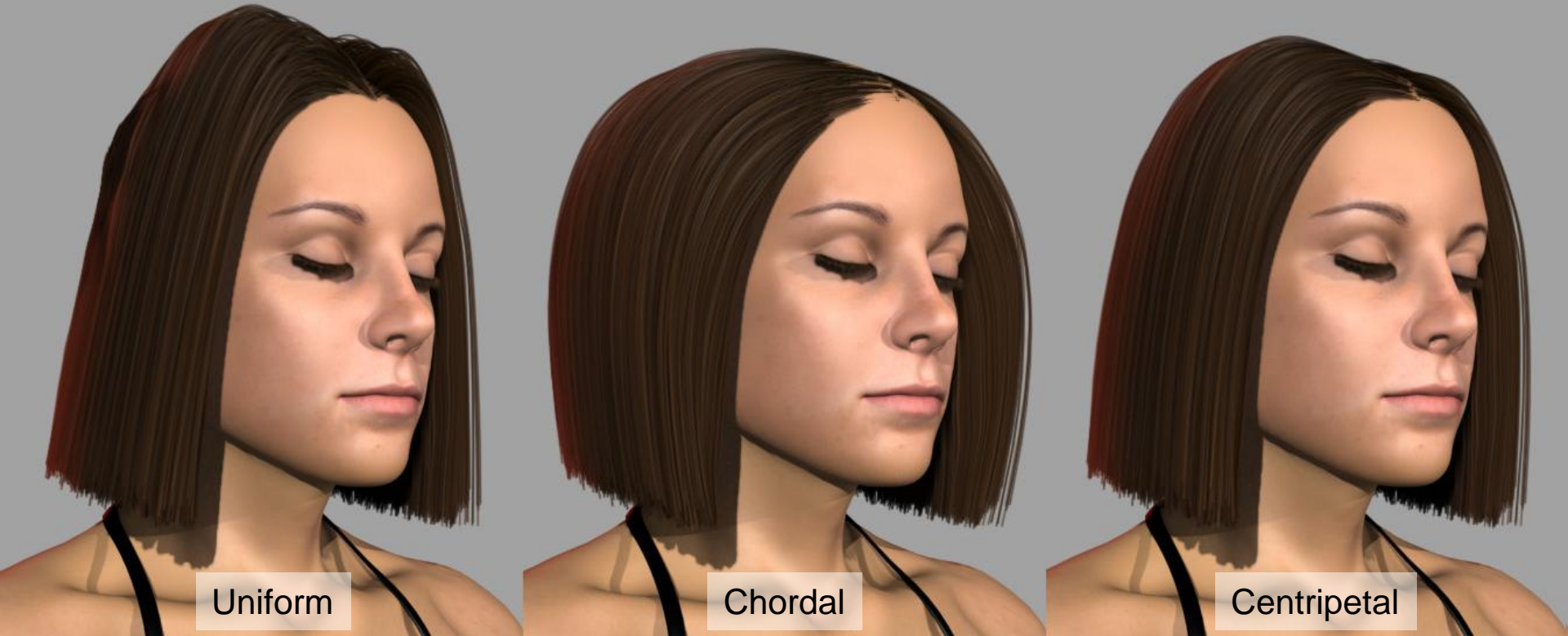
Catmull-Rom Curves



Cem Yuksel, Scott Schaefer, John Keyser, "Hair Meshes," Siggraph Asia 2009

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Catmull-Rom Curves



Cem Yuksel, Scott Schaefer, John Keyser, "Hair Meshes," Siggraph Asia 2009

HAIRFARM[™]
THE ULTIMATE HAIR PLUG-IN FOR 3DS MAX

Summary

- Parameterization of Catmull-Rom curves

$$0 \leq \alpha \leq 1$$

- Cusps and self-intersections
- Distance bound
- Intersection-free curves

- C^1 Catmull-Rom curves only!

On the Parameterization of Catmull-Rom Curves

QUESTIONS?