

# Cardinality-Constrained Texture Filtering

Josiah Manson and Scott Schaefer

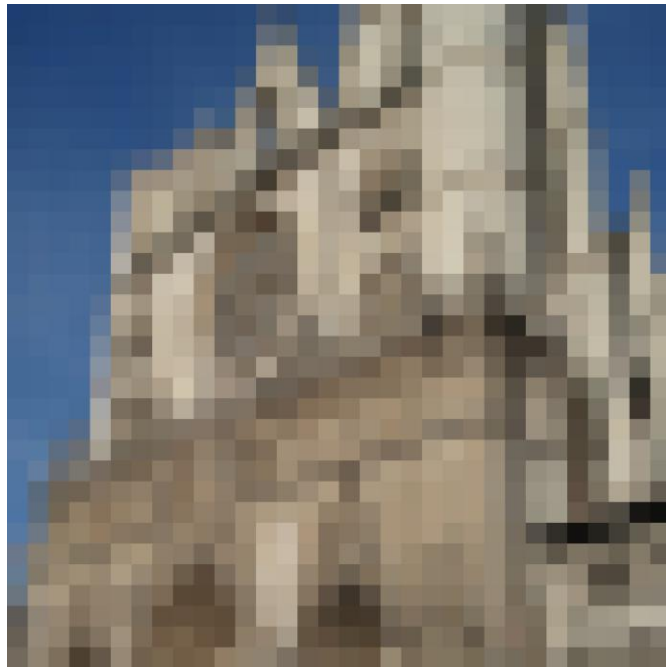
Texas A&M University



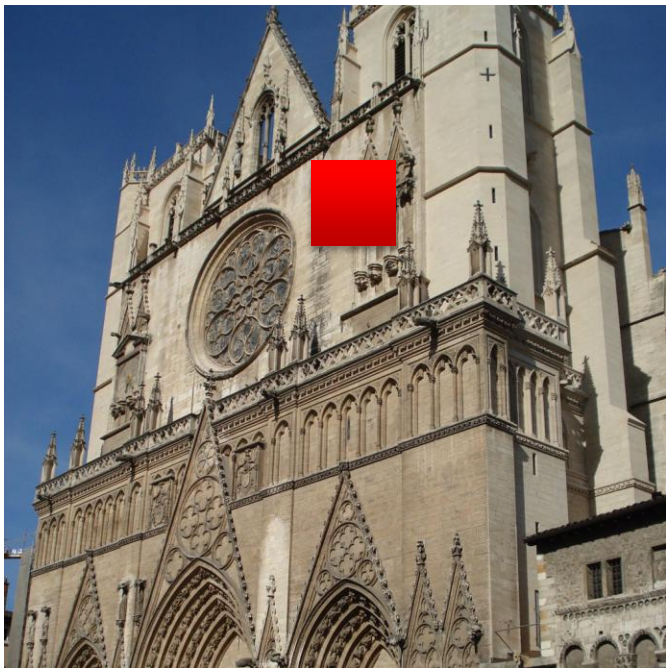
**SIGGRAPH**2013



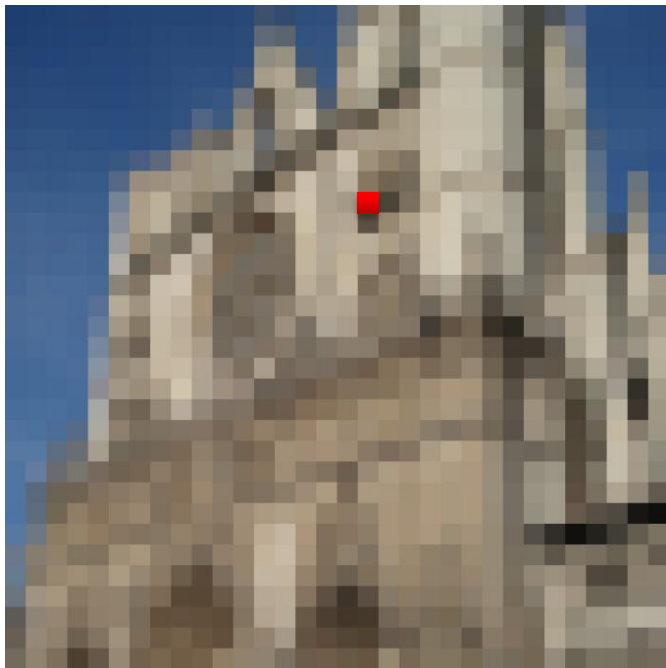
Input



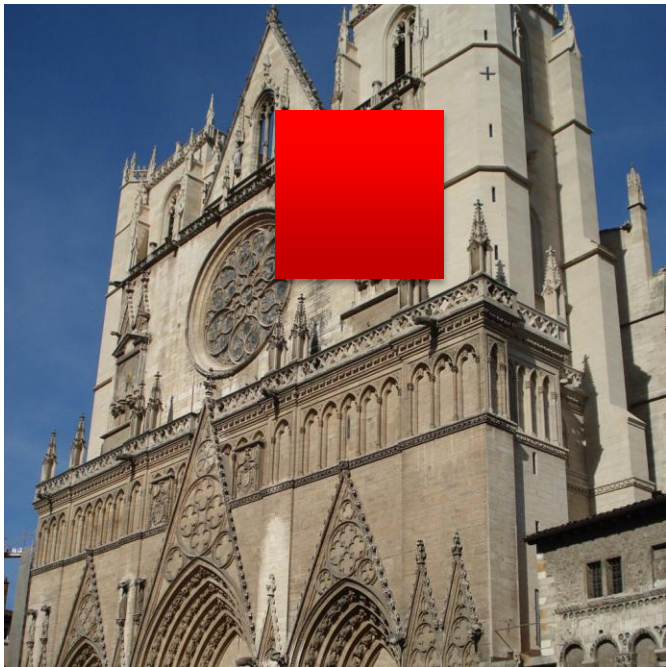
Downsampled



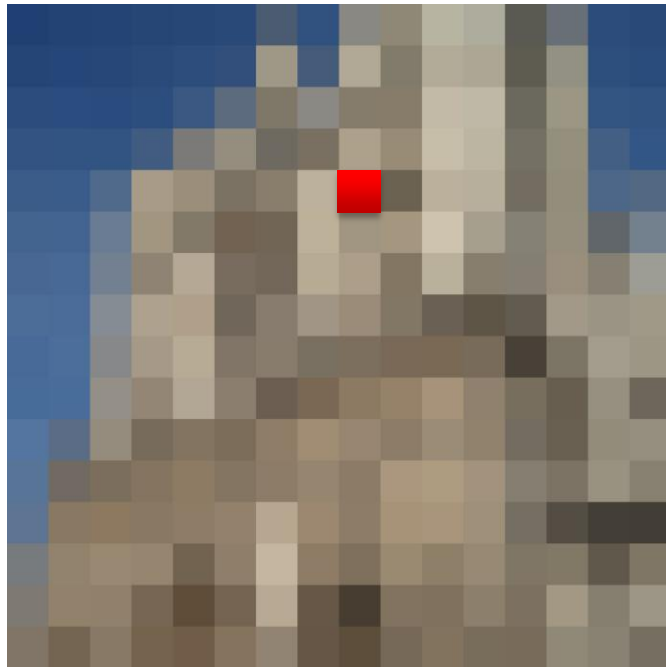
Input



Downsampled

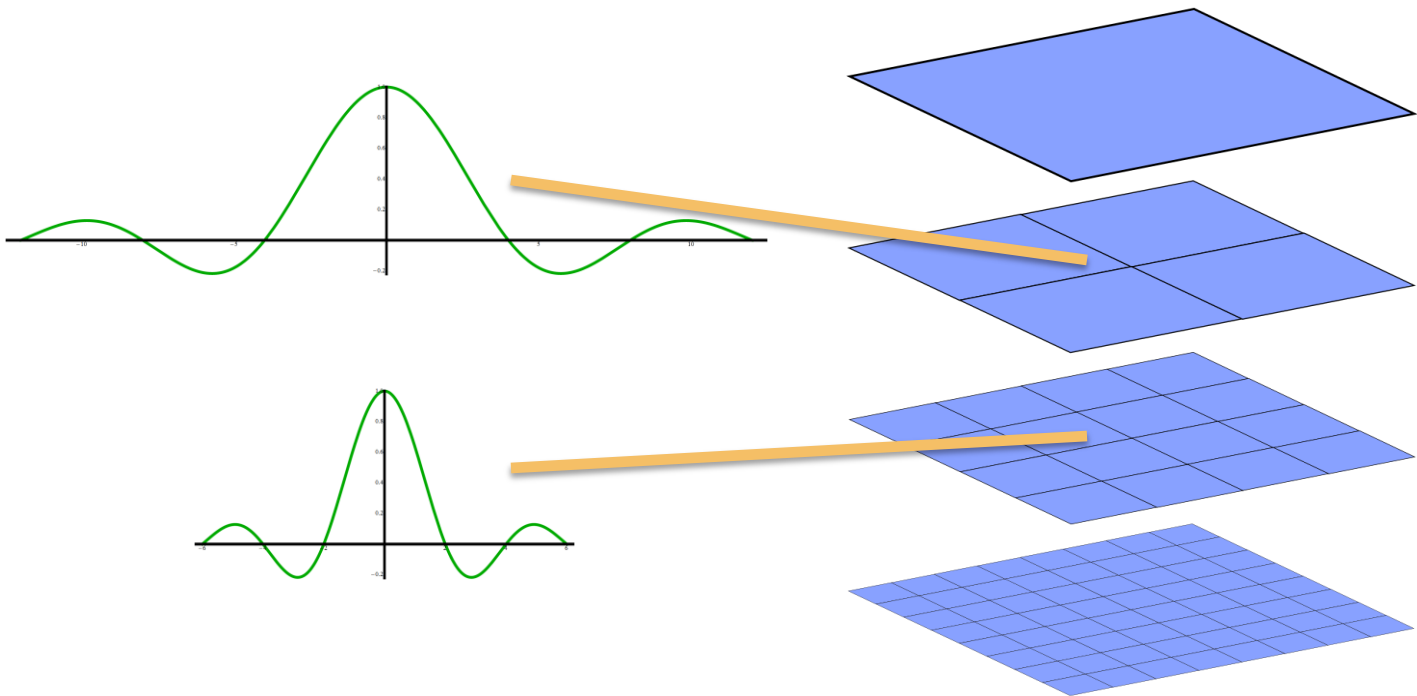


Input

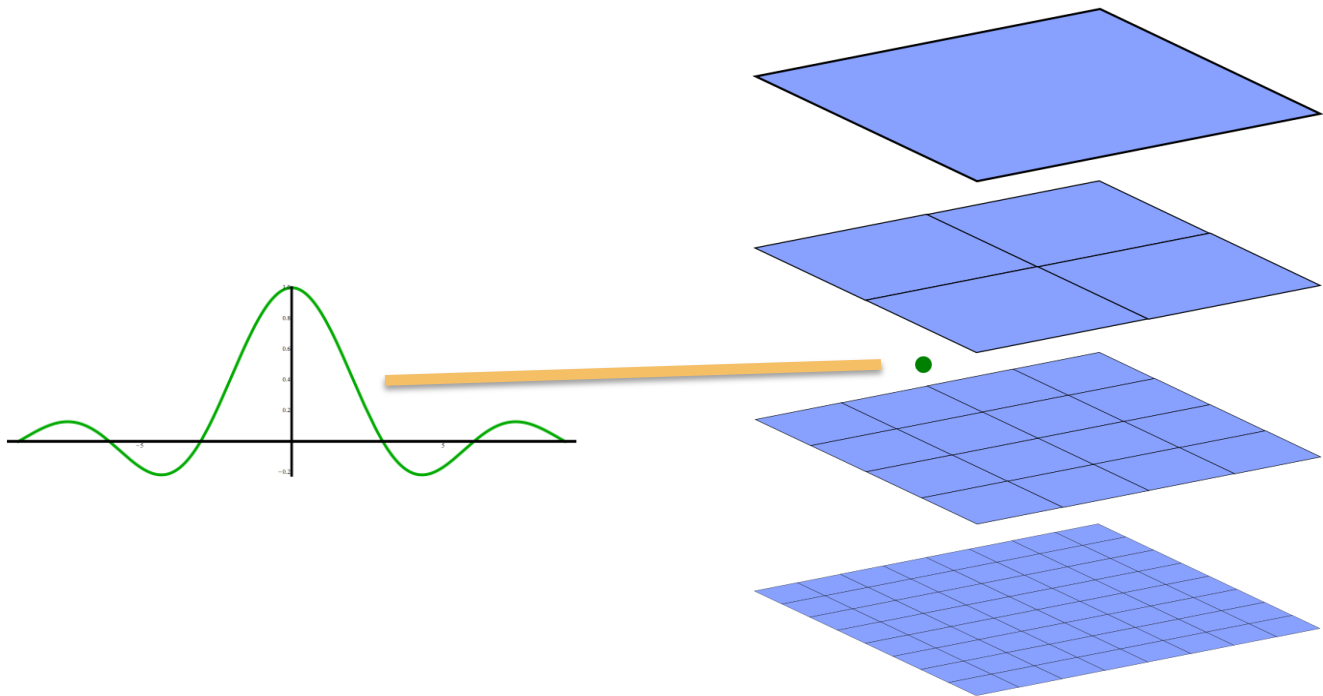


Downsampled

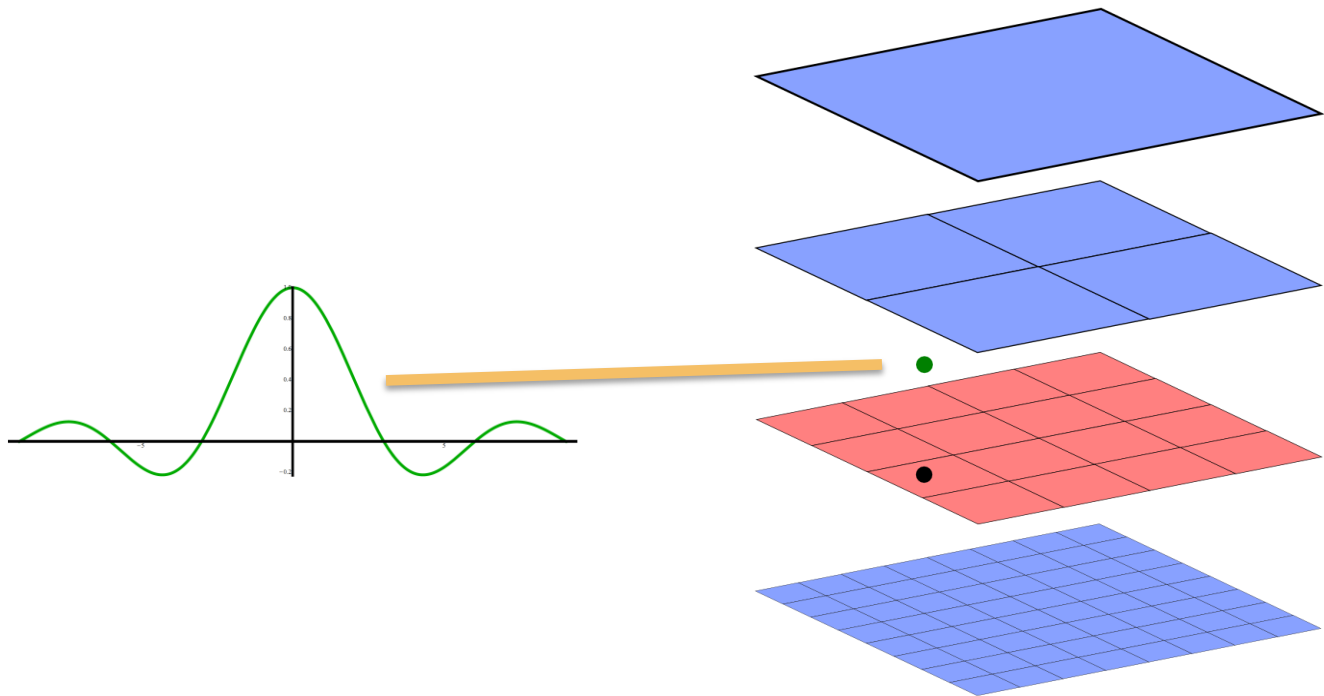
# Mipmapping



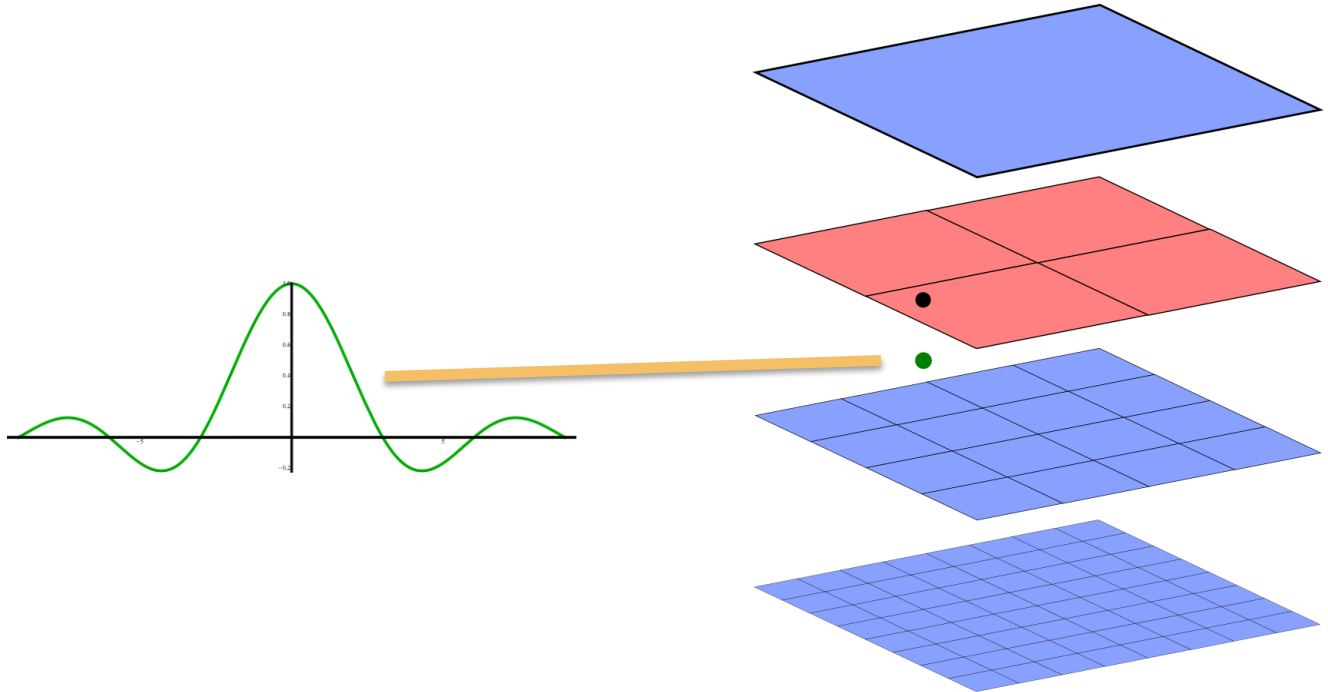
# Mipmapping



# Mipmapping

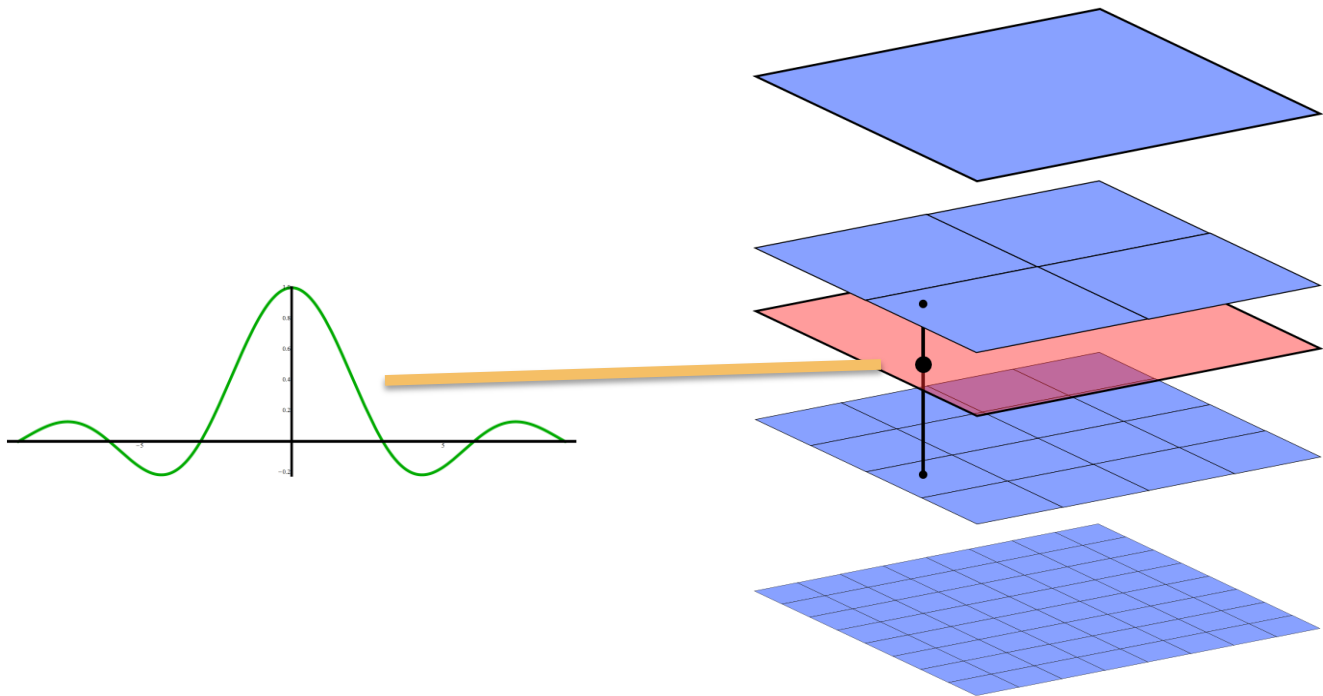


# Mipmapping





# Mipmapping





Input



Exact



Input



Trilinear



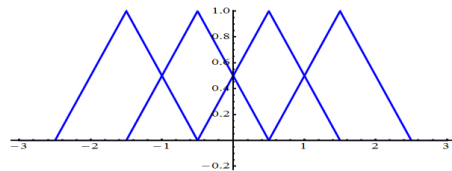
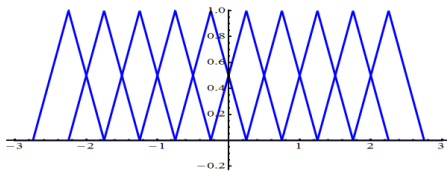
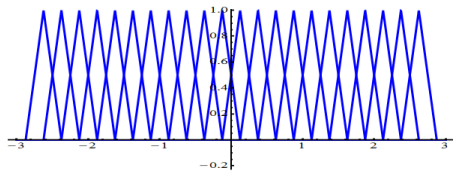
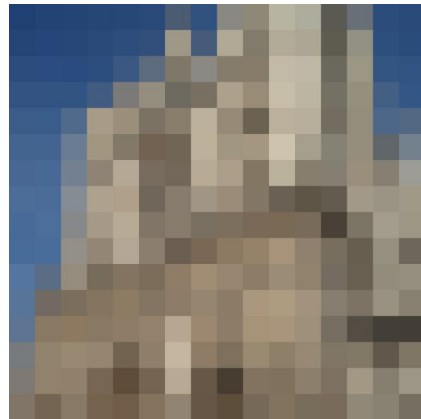
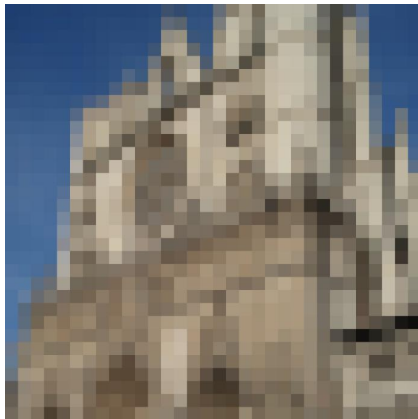
Input



Our Method



# Image Sampling



$$v_i = \iint_{\mathbb{R}^2} I(x) h_i(x) \, dx$$

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$$\iint_{\mathbb{R}^2} I(x) (c_1 h_1(x) + c_2 h_2(x)) \, dx$$

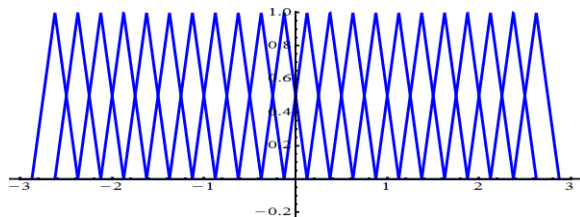
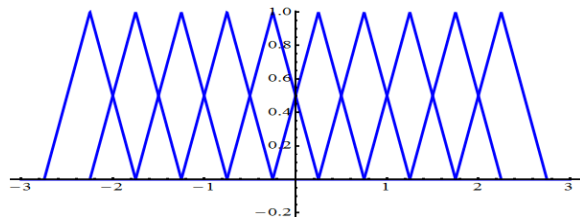
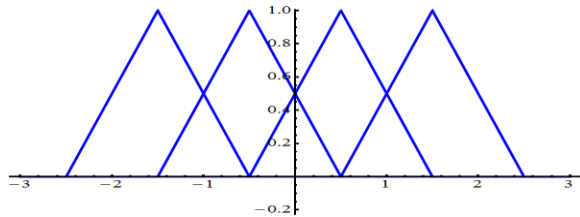
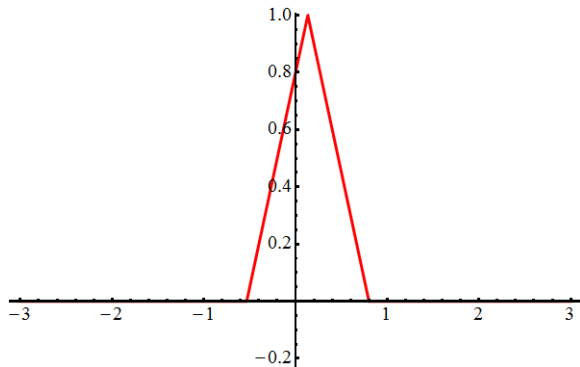
$$v_i = \iint_{\mathbb{R}^2} I(x) h_i(x) dx$$

$$c_1 v_1 + c_2 v_2$$

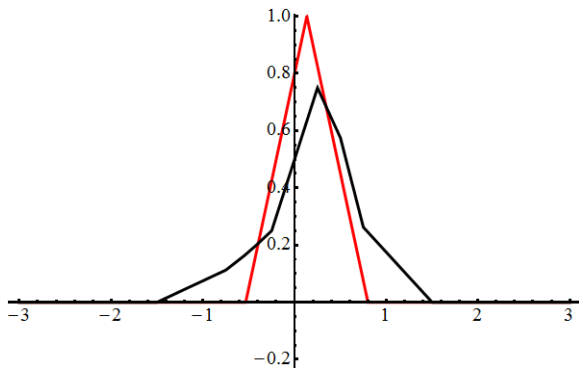
$$c_1 \iint_{\mathbb{R}^2} I(x) h_1(x) dx + c_2 \iint_{\mathbb{R}^2} I(x) h_2(x) dx$$

$$\iint_{\mathbb{R}^2} I(x) (c_1 h_1(x) + c_2 h_2(x)) dx$$

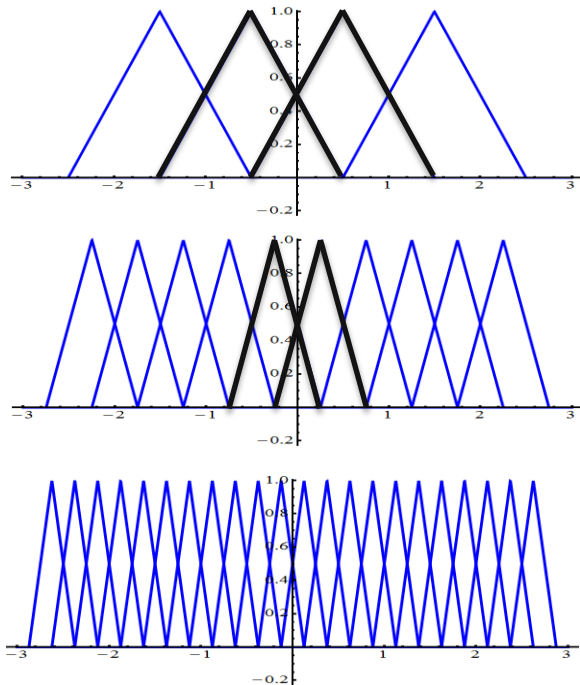
# Filter Approximation



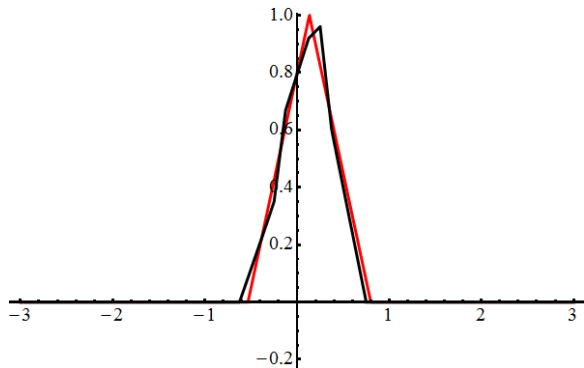
# Filter Approximation



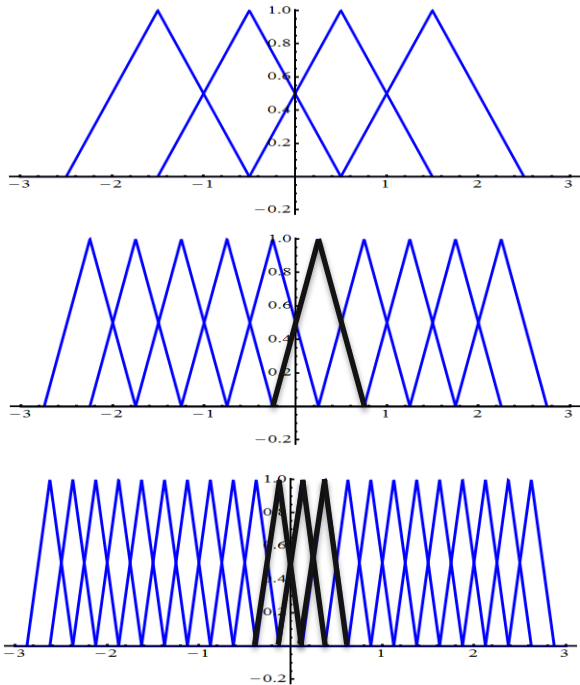
Interpolation of 4 samples



# Filter Approximation

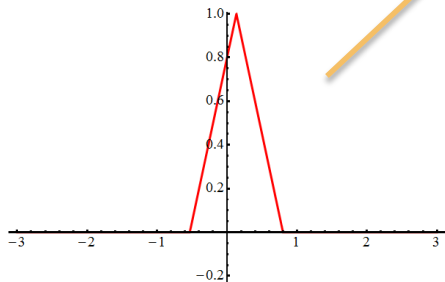


Best fit using 4 samples

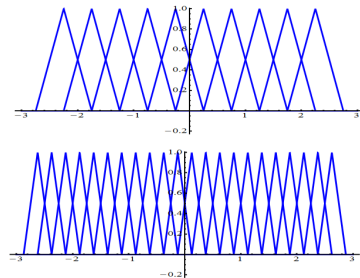
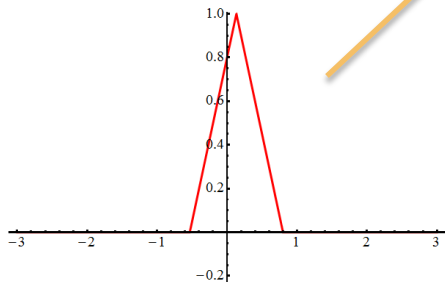


$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

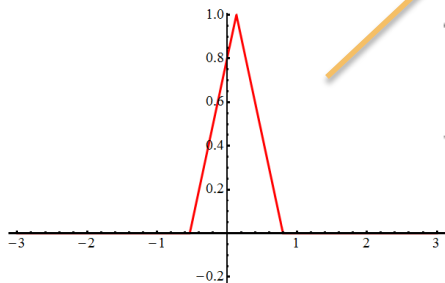


$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

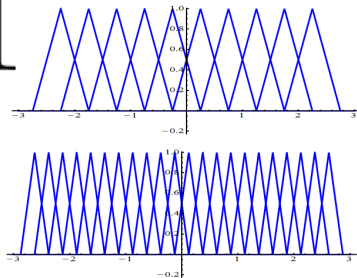




$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

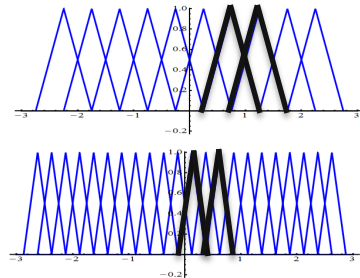
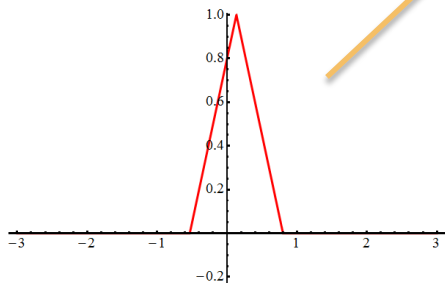


$$\sum c_i = 1$$



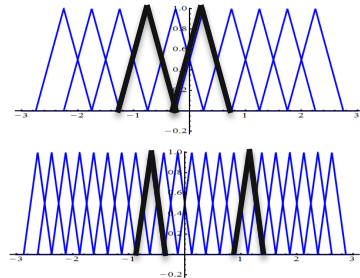
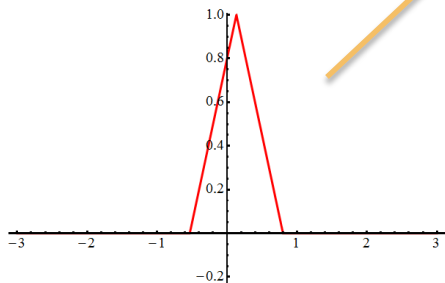
$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

$$|e| = n$$



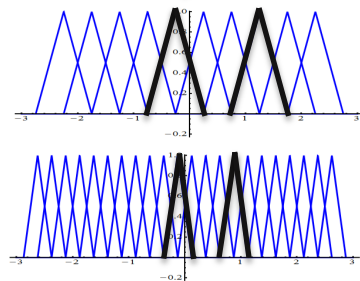
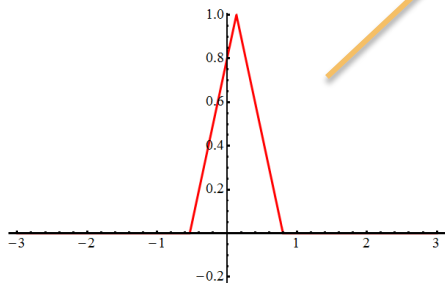
$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

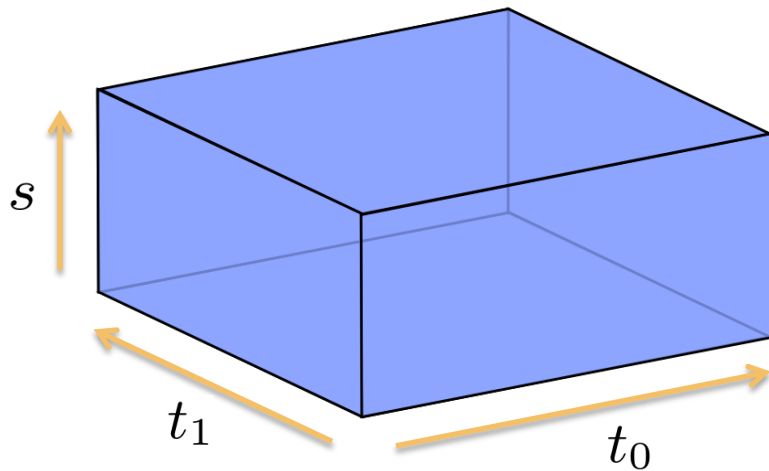
$$|e| = n$$

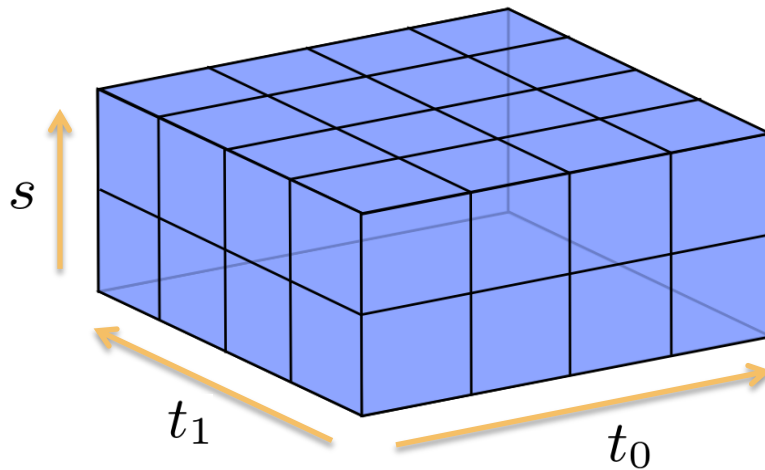


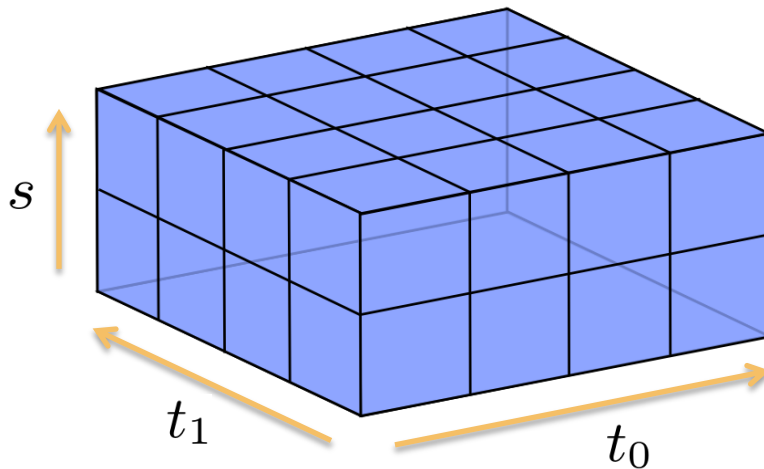
$$\operatorname{argmin}_{c, e \in E} \iint_{\mathbb{R}^2} \left( h_{\hat{s}, \hat{t}}(x) - \sum_{i=1}^n h_{e_i}(x) c_i \right)^2 dx$$

$$|e| = n$$

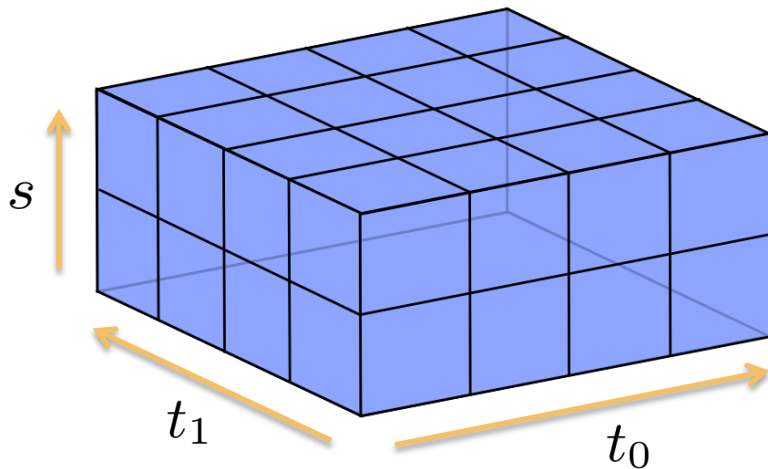








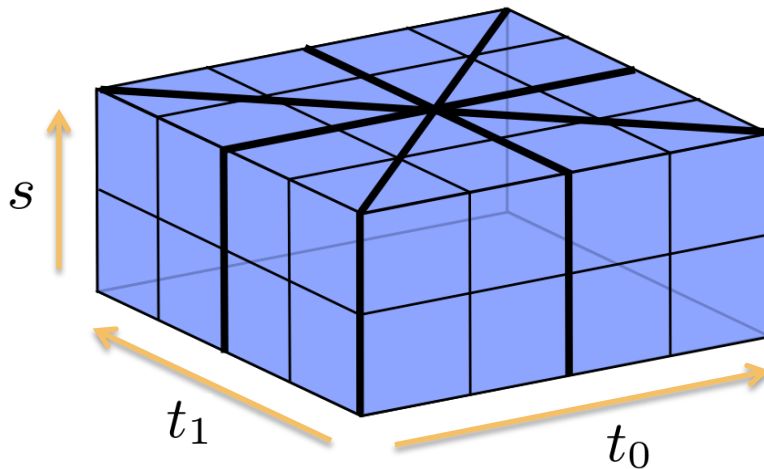
$$c_i(s, t) = \sum_j p_j(s, t) c_{ij}$$



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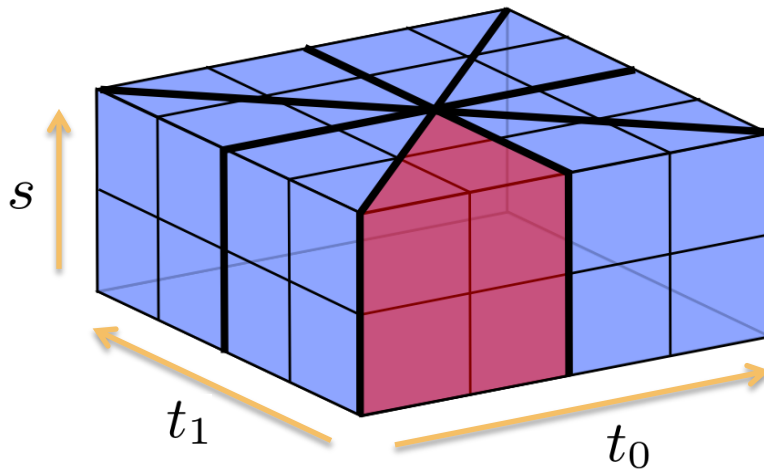
$$p(s, t) = (1, t_0, t_1, s)$$





$$c_i(s, t) = \sum_j p_j(s, t) c_{ij}$$

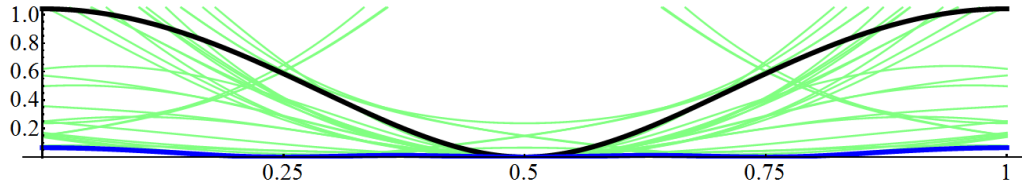
$$p(s, t) = (1, t_0, t_1, s)$$



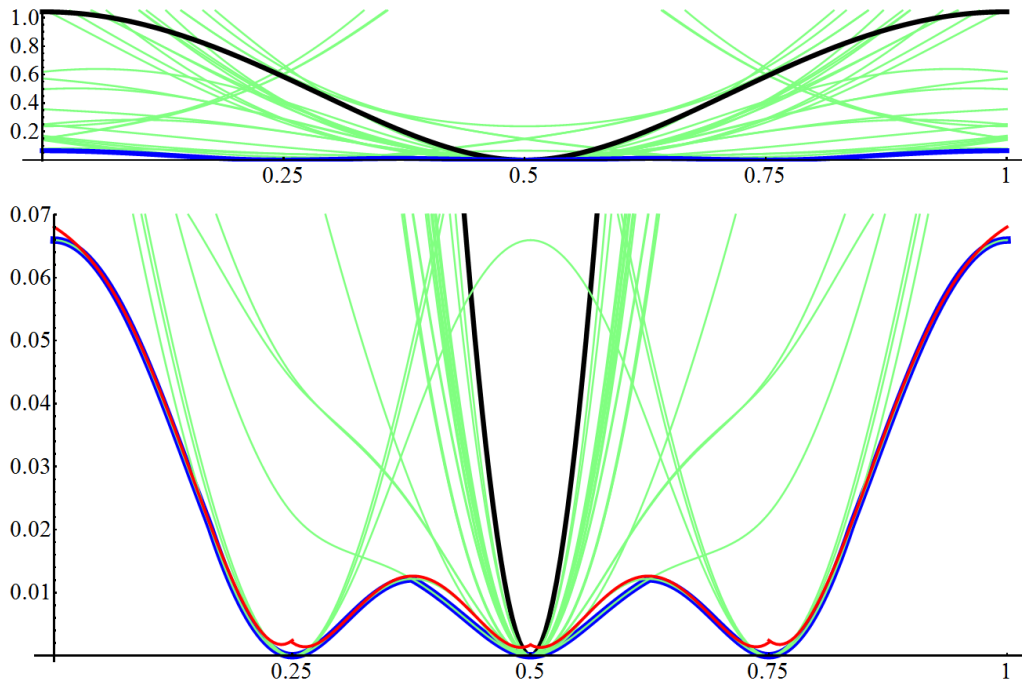
$$c_i(s, t) = \sum_j p_j(s, t) c_{ij}$$

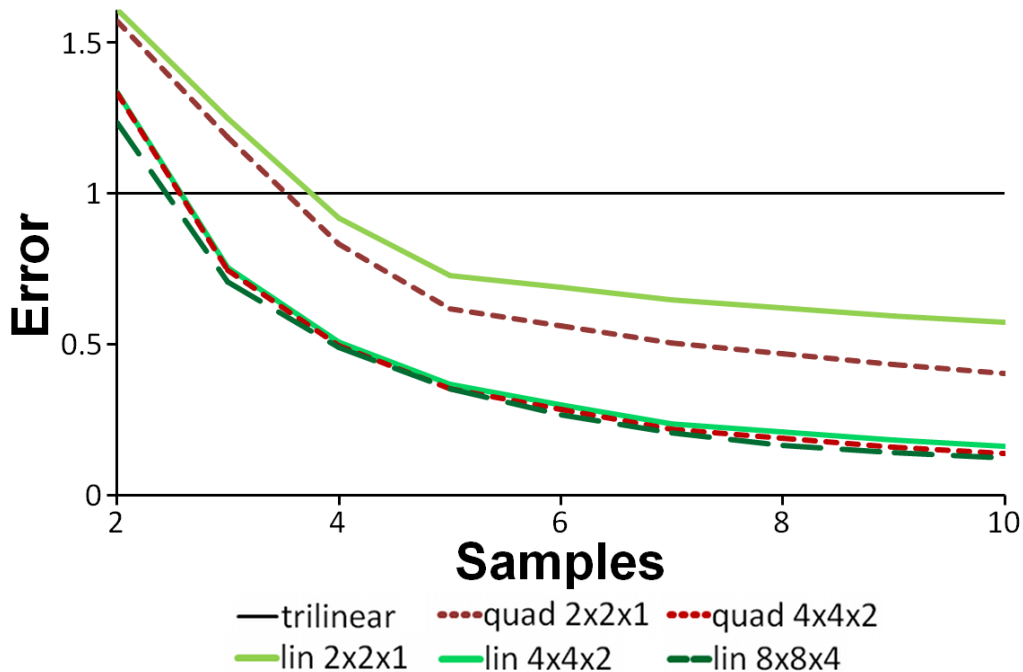
$$p(s, t) = (1, t_0, t_1, s)$$

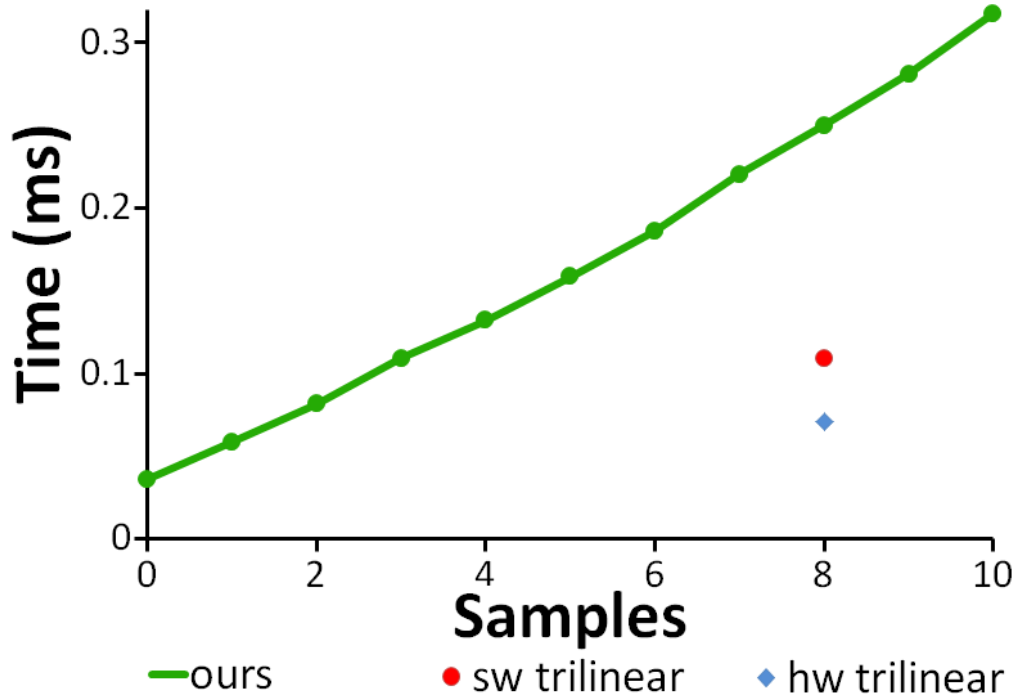
# Fitting Error

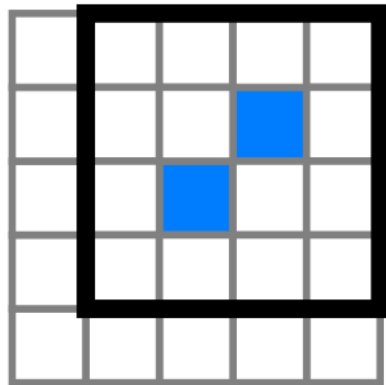
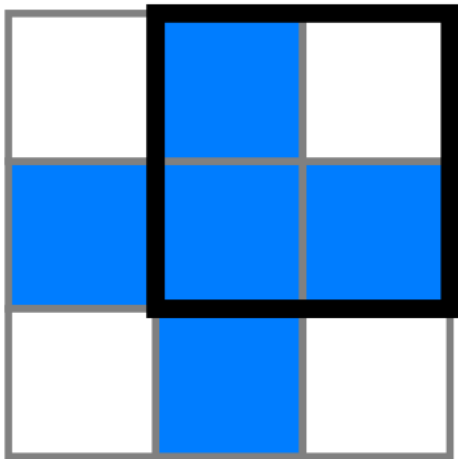
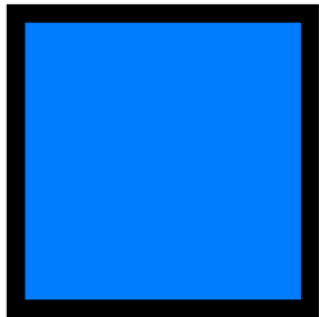


# Fitting Error



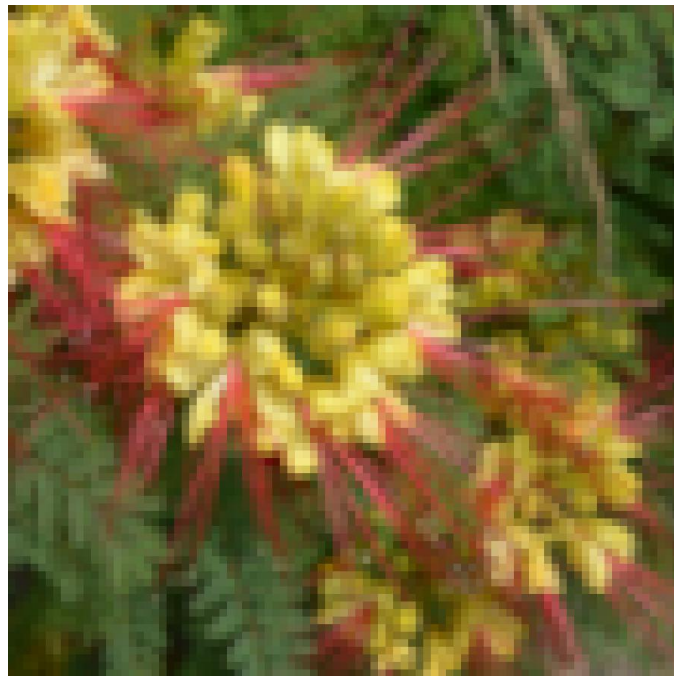








Exact



Trilinear





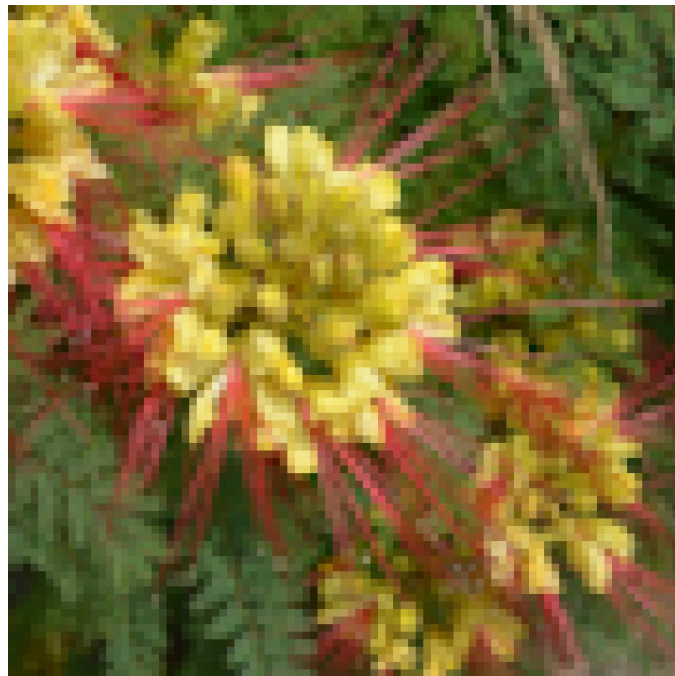
Exact



8 Samples



Exact



4 Samples



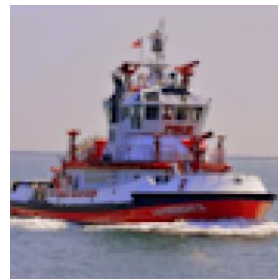
Exact



Trilinear



8 Samples



7 Samples



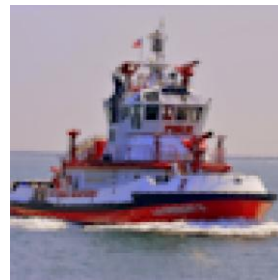
6 Samples



5 Samples

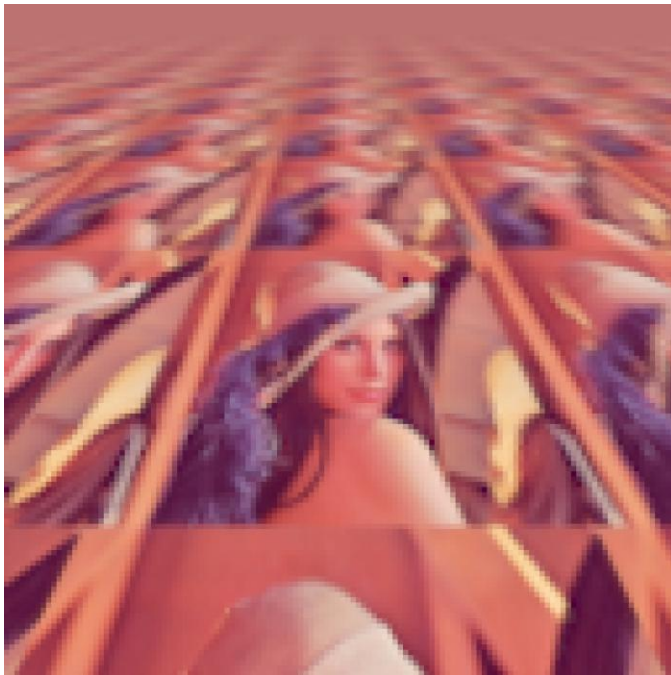


4 Samples



3 Samples

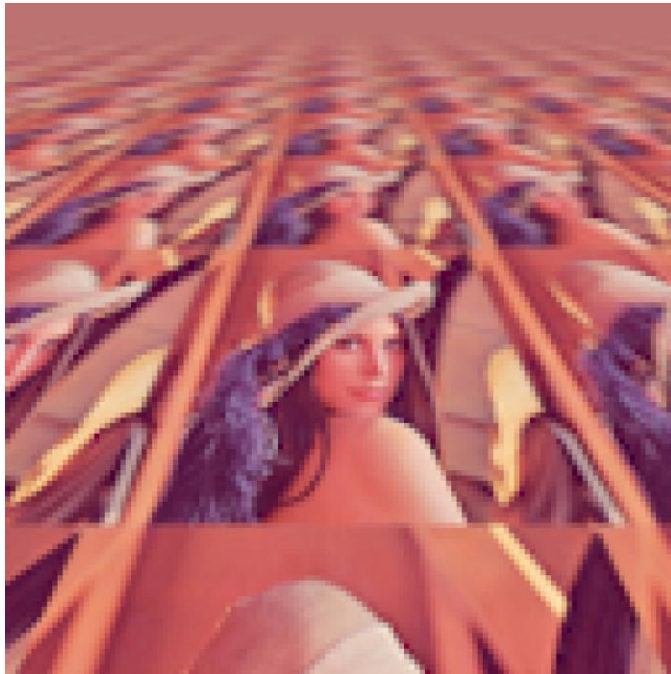
# Isotropic Lánczos 2 Approximation



Trilinear

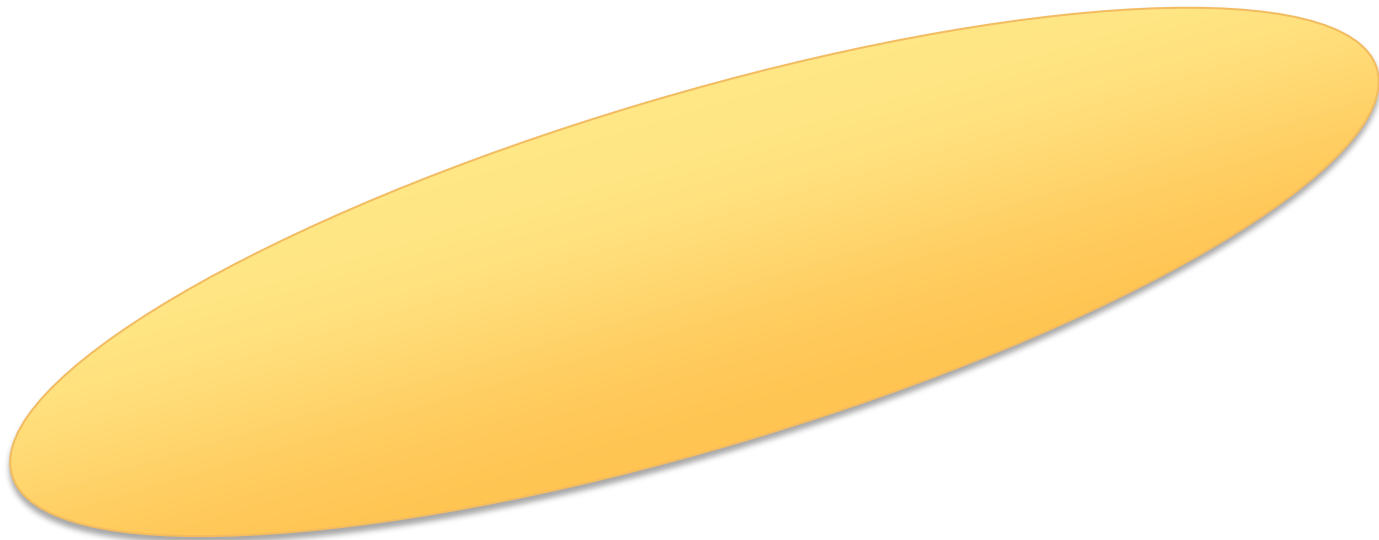


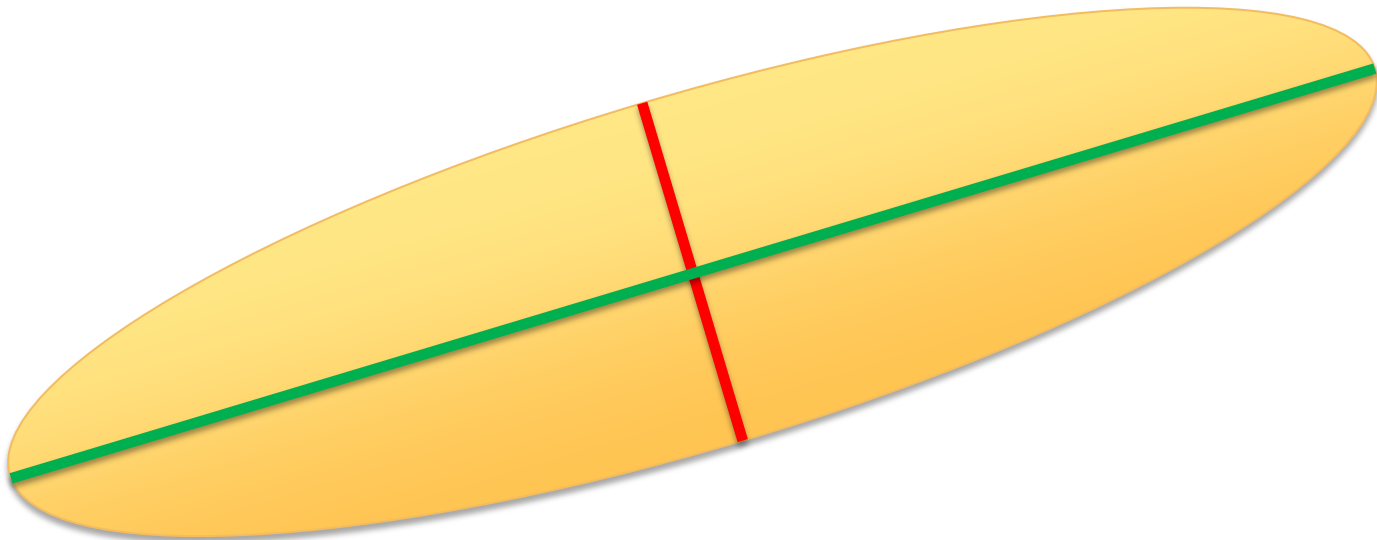
# Isotropic Lánczos 2 Approximation

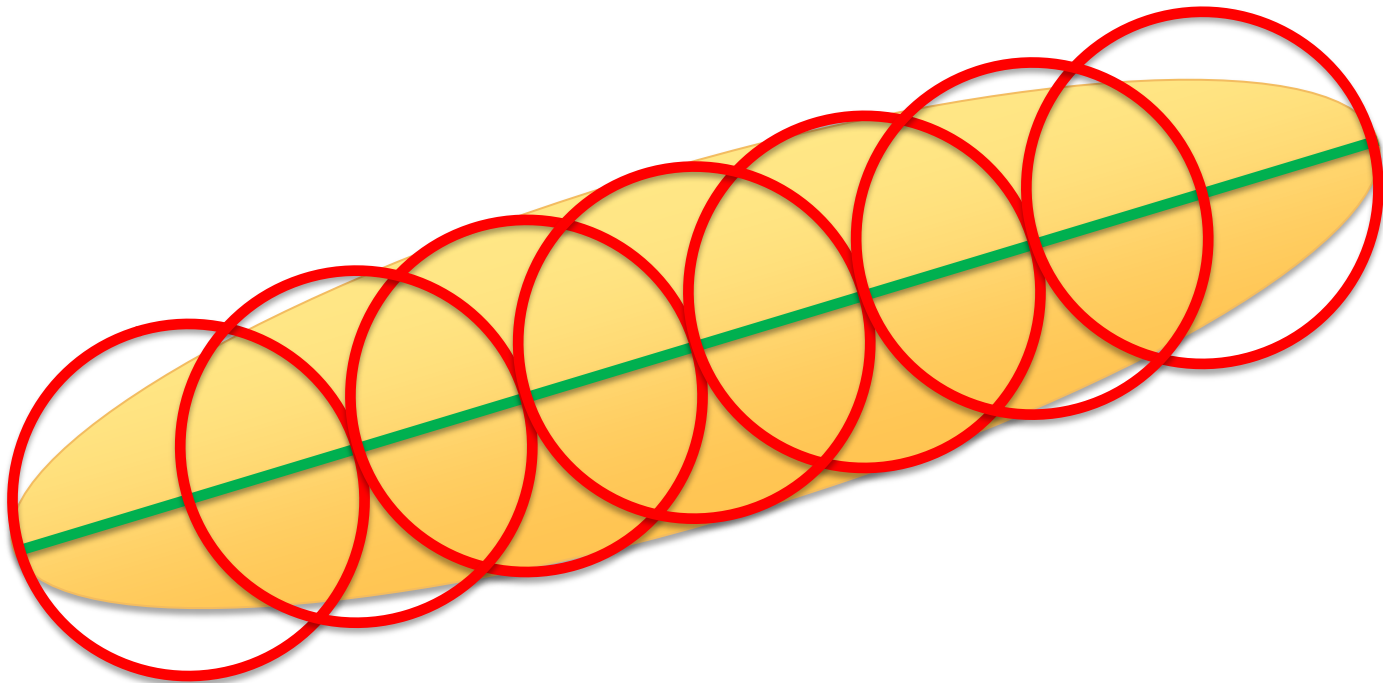


8 Samples

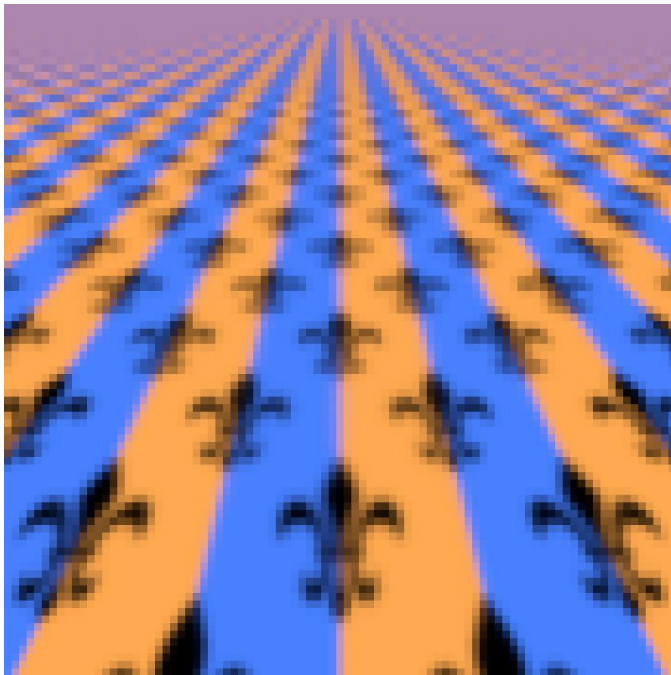
# Anisotropic Filtering



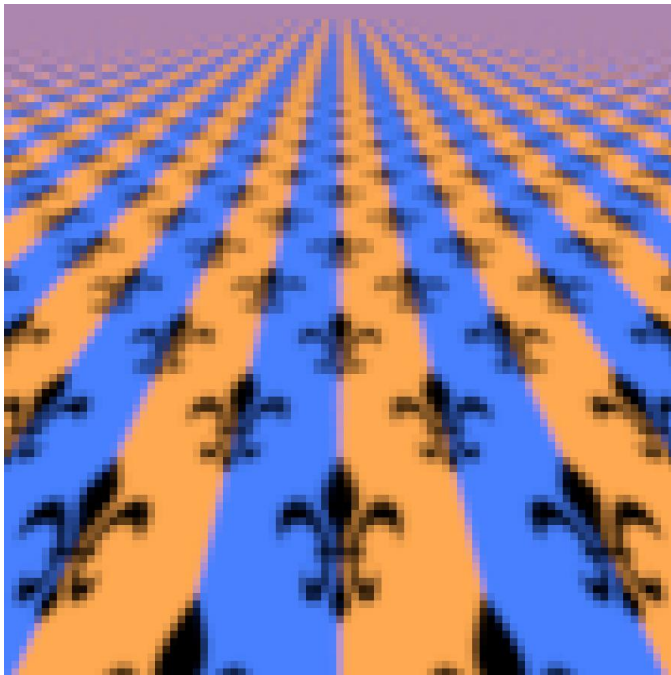




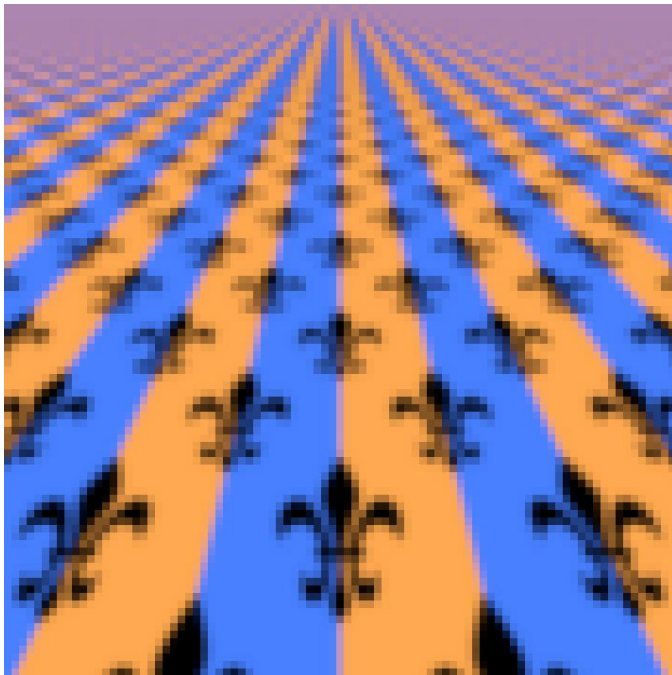




Feline



8 Samples with Feline



Exact

- Higher quality using less bandwidth
  - Scattered reads cost performance
- Approximate filters with few samples
  - Cost is only in preprocessing
- Small lookup tables
  - Easily fits in local memory
- GPU implementation
  - [http://josiahmanson.com/research/cardinality\\_constrained/](http://josiahmanson.com/research/cardinality_constrained/)