Mesh Denoising via L_0 Minimization

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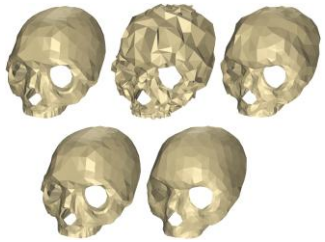
Texas A&M University

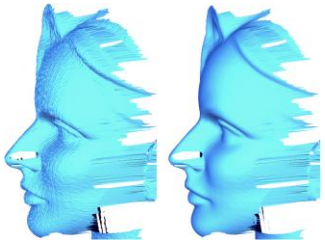
SIGGRAPH2013

Surface Denoising

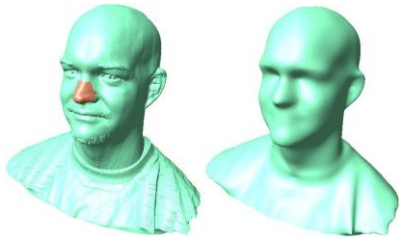
Related Work



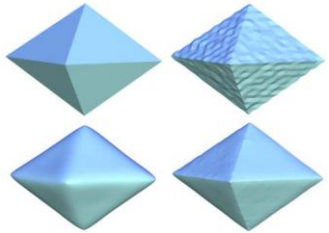
[Vollmer et al. 1999]



[Desbrun et al. 1999]



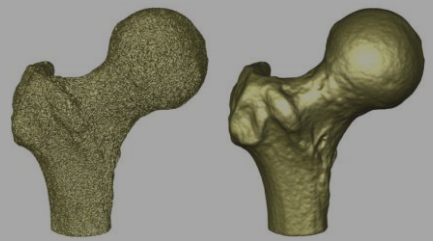
[Kim et al. 2005]



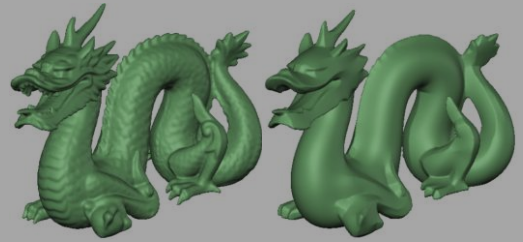
[Nealen et al. 2006]

[Clarenz et al. 2000]

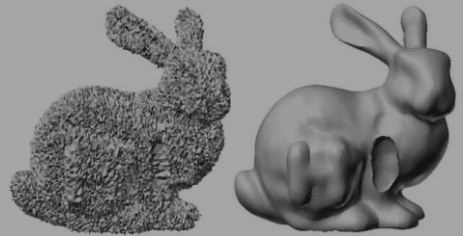
[Bajaj and Xu 2003]



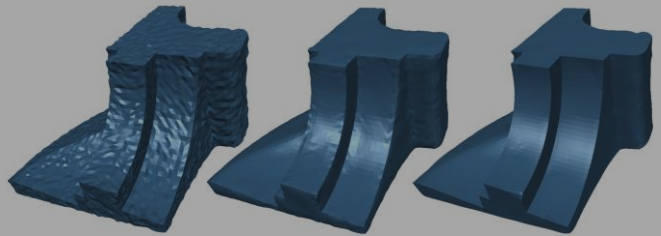
[Hildebrandt et al. 2004]



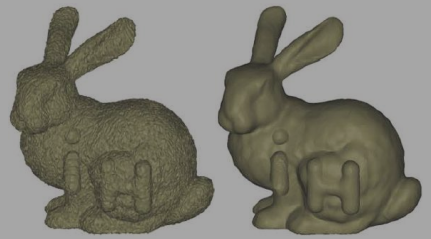
[Tasdizen et al. 2002]



[Yagou et al. 2002]



[Fleishman et al. 2003] [Jones et al. 2003]



[Zheng et al. 2012]

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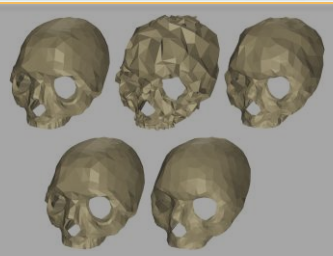

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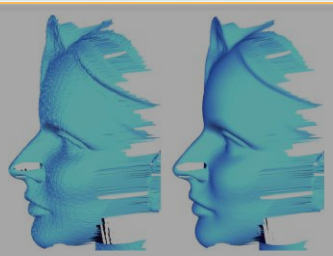

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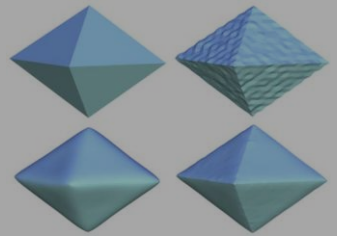

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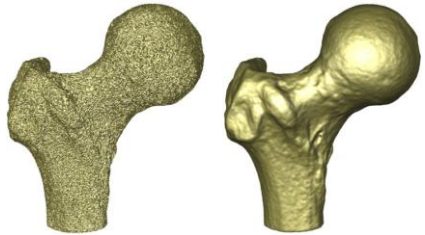

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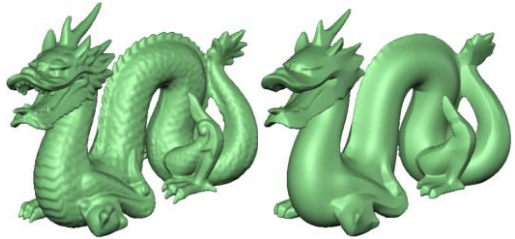

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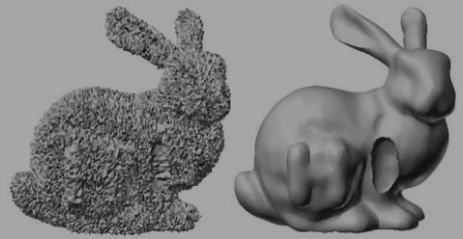

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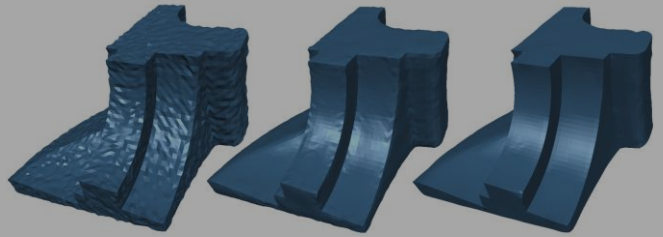

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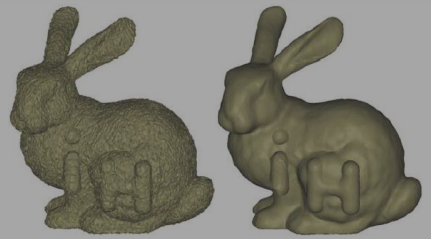

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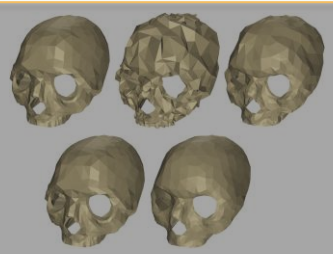
Related Work



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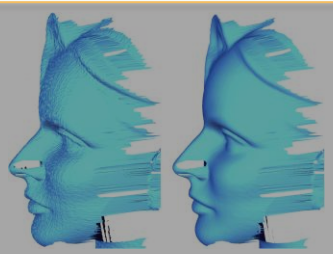

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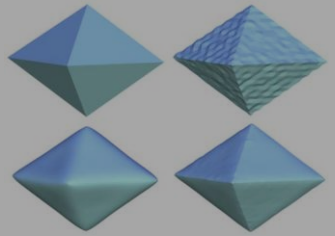

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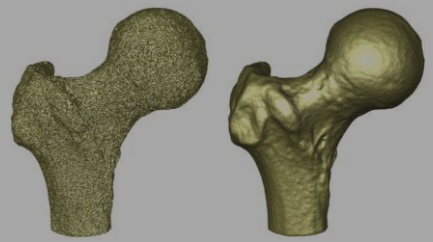

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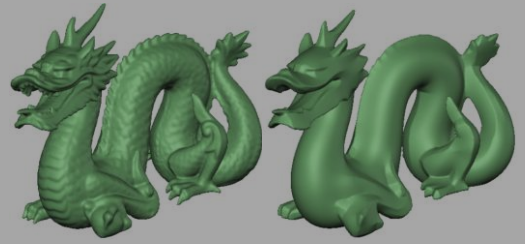

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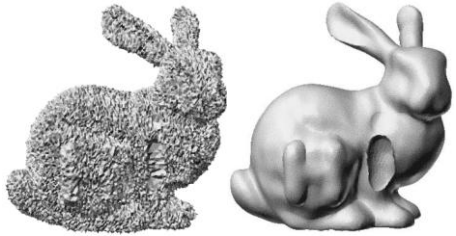

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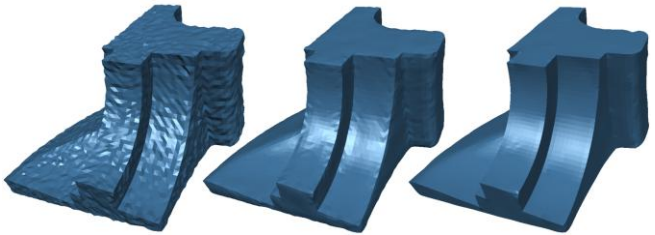

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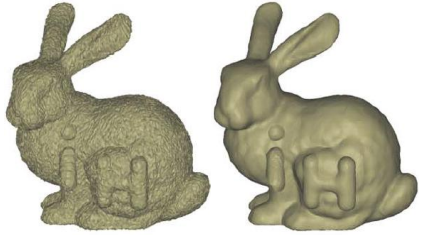


[Zheng et al. 2012]

Motivation

[Xu et al. 2011]

L_0 Norm

$$|v|_0 = \#\{v_i \big| |v_i| \neq 0\}$$

L_0 Minimization for Images

$$\min_c |c - c^*|^2 + \lambda |\nabla c|_0$$

[Xu et al. 2011]

L_0 Minimization for Images

$$\min_{c} |c - c^*|^2 + \lambda |\nabla c|_0$$

$\mu > 1$, for L=0,1,2,…

$$\min_{c,\delta} |c - c^*|^2 + \beta_0 \mu^L |\nabla c - \delta|^2 + \lambda |\delta|_0$$

auxiliary variables

[Xu et al. 2011]

L_0 Minimization for Images

- Fix c , update δ locally

$$\min_{\delta} \beta_0 \mu^L |\nabla c - \delta|^2 + \lambda |\delta|_0$$

- Fix δ , update c globally

$$\min_{c} |c - c^*|^2 + \beta_0 \mu^L |\nabla c - \delta|^2$$

- Increment L

[Xu et al. 2011]

L_0 Minimization for Images

[Xu et al. 2011]

L_0 Minimization for Images

piecewise constant images!

[Xu et al. 2011]

L_0 Minimization for Images

$$\min_c |c - c^*|^2 + \lambda |\nabla c|_0$$

[Xu et al. 2011]

L_0 Minimization for Surfaces

$$\min_p |p - p^*|^2 + \lambda| \quad ? \quad |_0$$

L_0 Minimization for Surfaces

$$\min_{p} |p - p^*|^2 + \lambda |D(p)|_0$$

L_0 Minimization for Surfaces

$$\min_p |p - p^*|^2 + \lambda |D(p)|_0$$

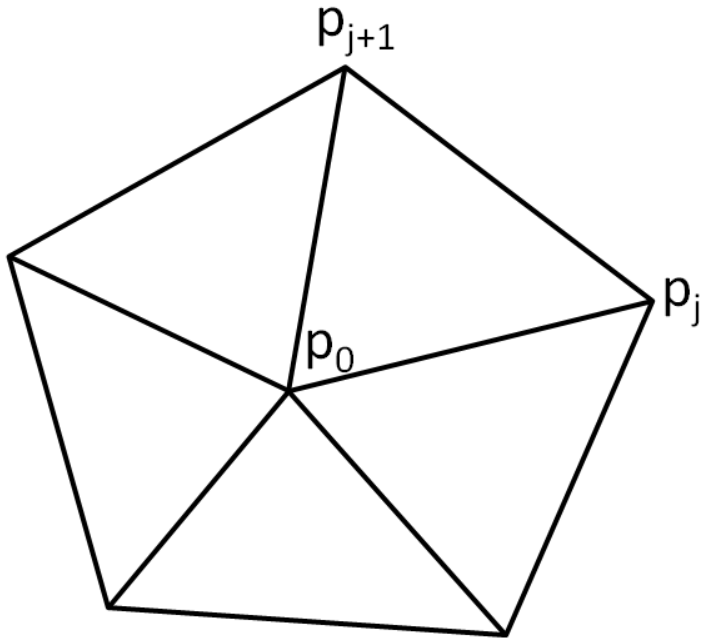
- Properties of $D(\cdot)$
  - a discrete linear operator
  - measure planarity

$$D(p) = \sum_j w_j p_j$$

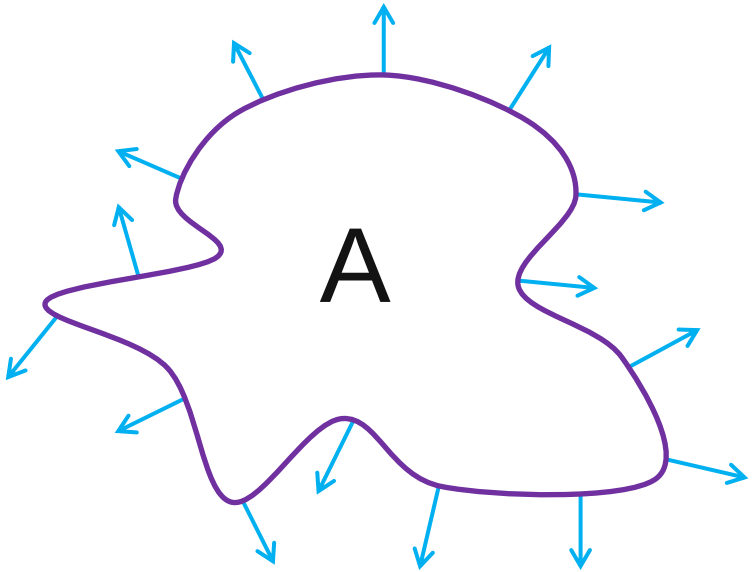
Discrete Differential Operator

- When p_j are planar,

$$\begin{aligned} \sum_j w_j &= 0 \\ \sum_j w_j p_j &= 0 \end{aligned}$$



Discrete Differential Operator



$$\int_A \mathbf{H} dA = \oint_{\partial A} \mathbf{n} ds$$

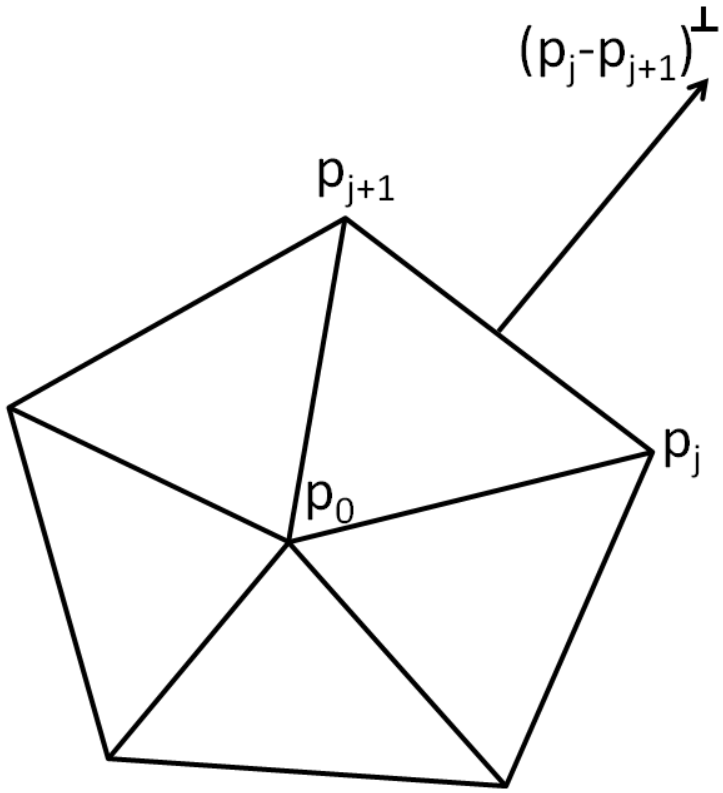
Discrete Differential Operator

- When p_j are planar,

$$\begin{aligned} \sum_j w_j &= 0 \\ \sum_j w_j p_j &= 0 \end{aligned}$$

- Vertex-based cotangent operator

$$\sum_j (p_j - p_{j+1})^{\perp}$$



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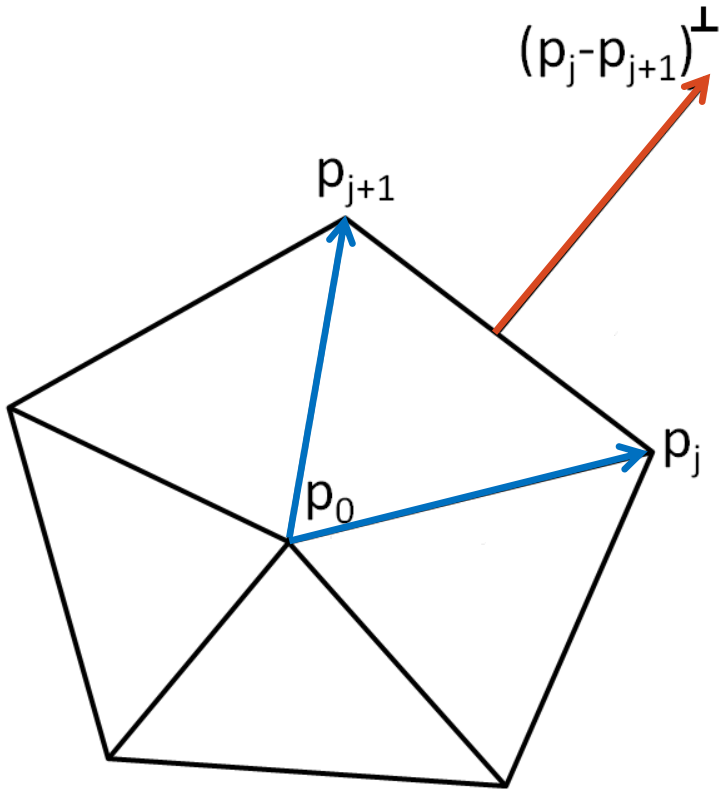

Discrete Differential Operator

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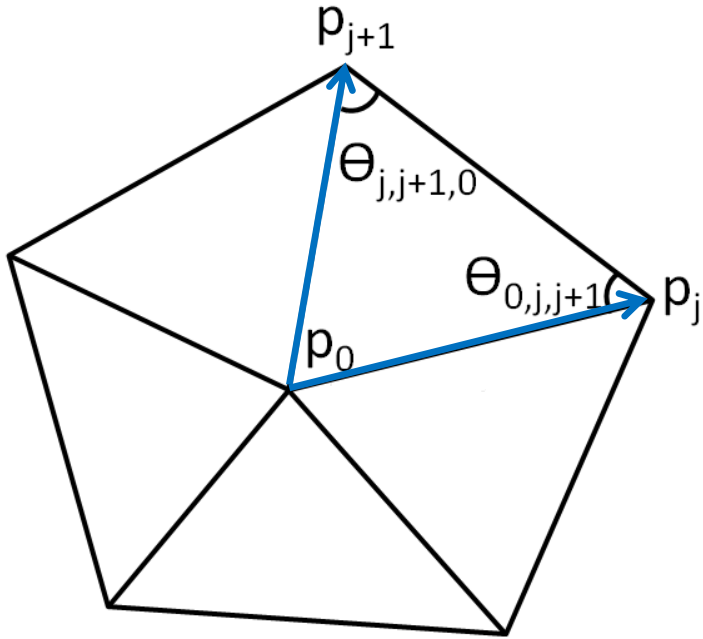
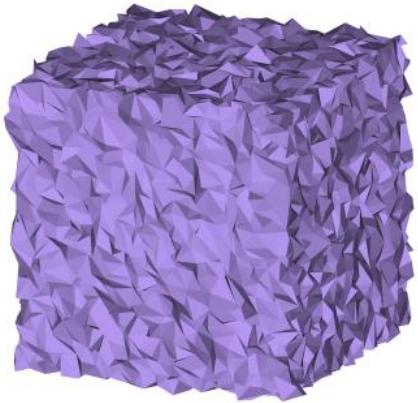
- Vertex-based cotangent operator

$$\sum_j \cot(\theta_{0,j,j+1})(p_{j+1}-p_0)+\cot(\theta_{j,j+1,0})(p_j-p_0)$$

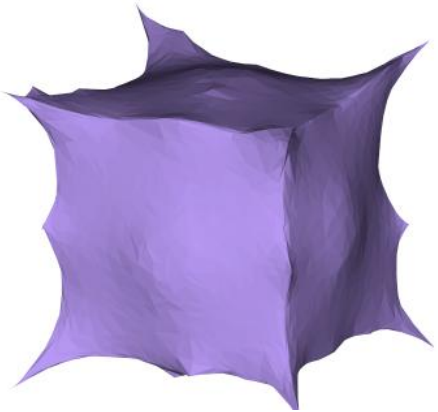
[Pinkall and Polthier 1993]



Discrete Differential Operator



input surface



vertex-based
cotangent operator

Discrete Differential Operator

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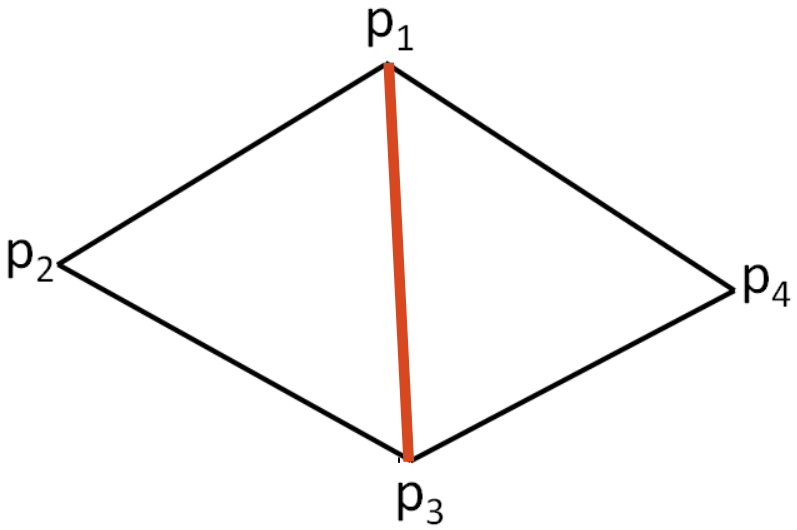

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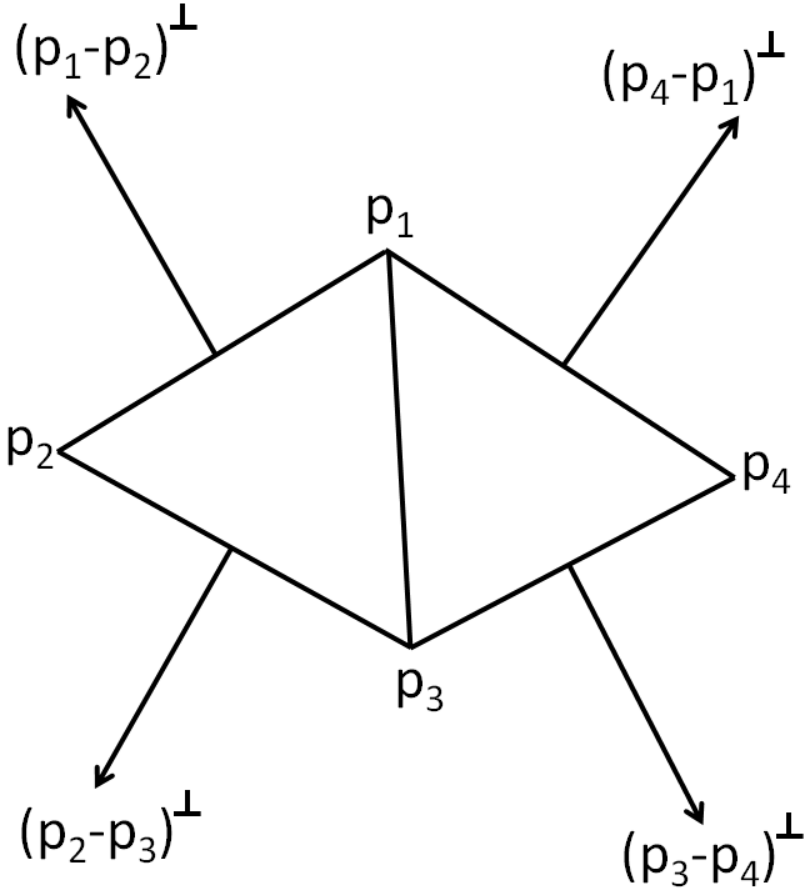

Discrete Differential Operator

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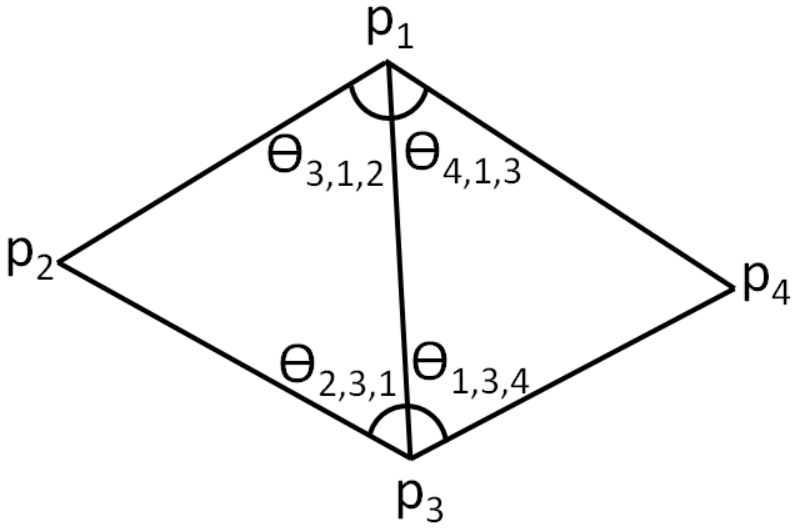
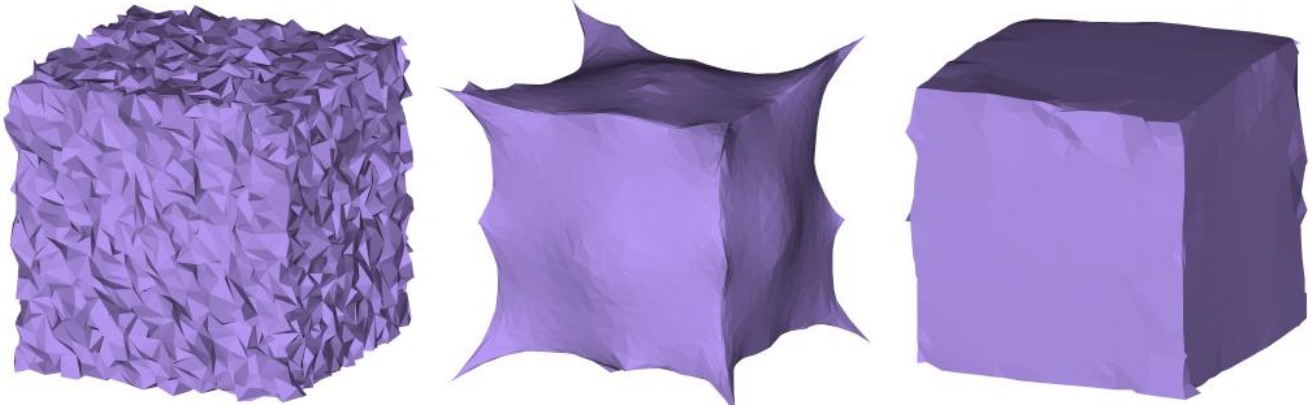
- Edge-based cotangent operator

$$D(e) = \begin{bmatrix} -\cot(\theta_{2,3,1}) - \cot(\theta_{1,3,4}) \\ \cot(\theta_{2,3,1}) + \cot(\theta_{3,1,2}) \\ -\cot(\theta_{3,1,2}) - \cot(\theta_{4,1,3}) \\ \cot(\theta_{1,3,4}) + \cot(\theta_{4,1,3}) \end{bmatrix}^T \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$



[Bergou et al. 2006]

Discrete Differential Operator

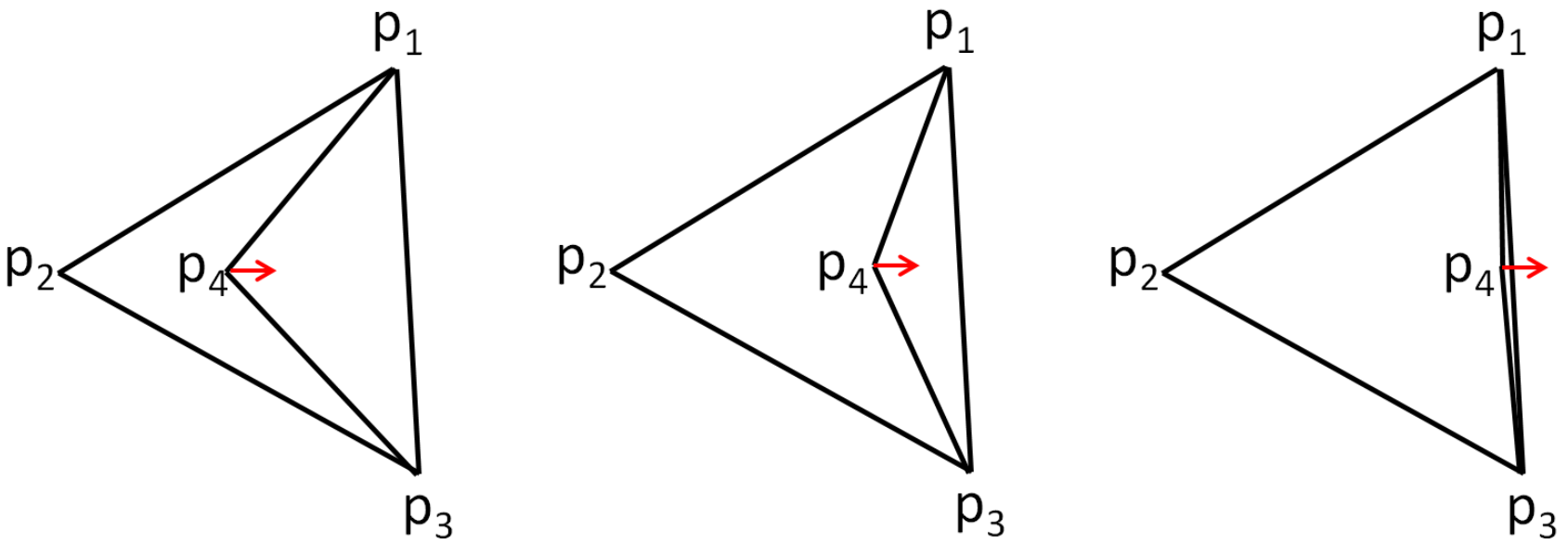


input surface

vertex-based
cotangent operator

cotangent
edge operator

Discrete Differential Operator



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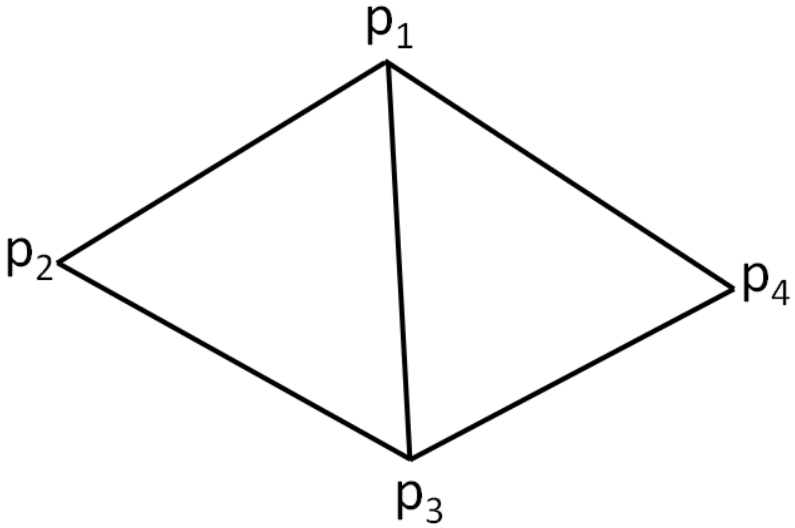
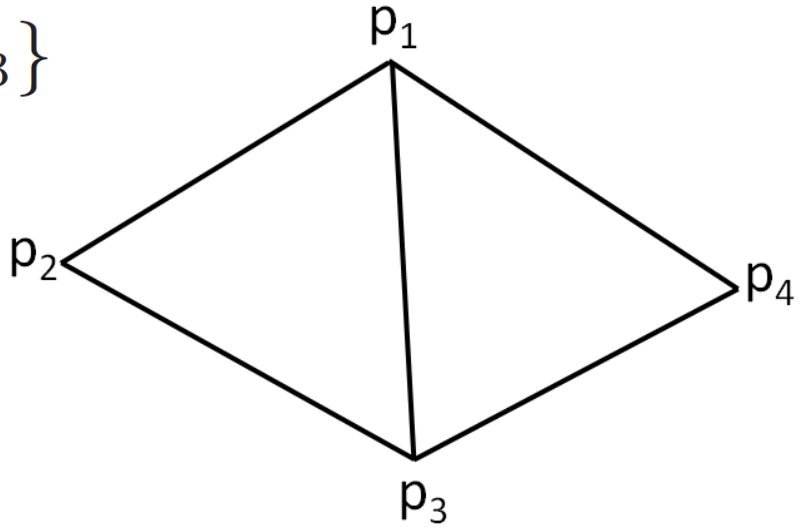


Discrete Differential Operator

- When p_j are planar,

$$\{-\Delta_{2,3,4}, \Delta_{1,3,4}, -\Delta_{1,2,4}, \Delta_{1,2,3}\}$$



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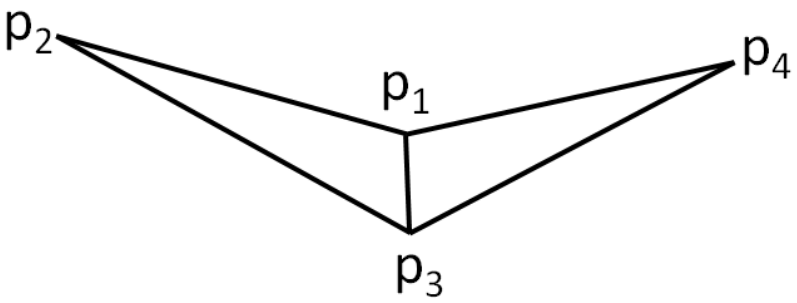

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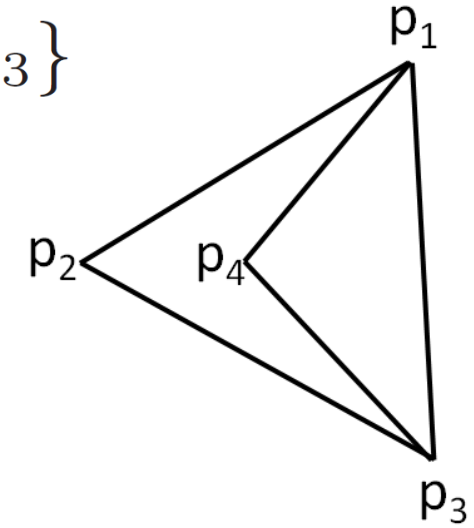

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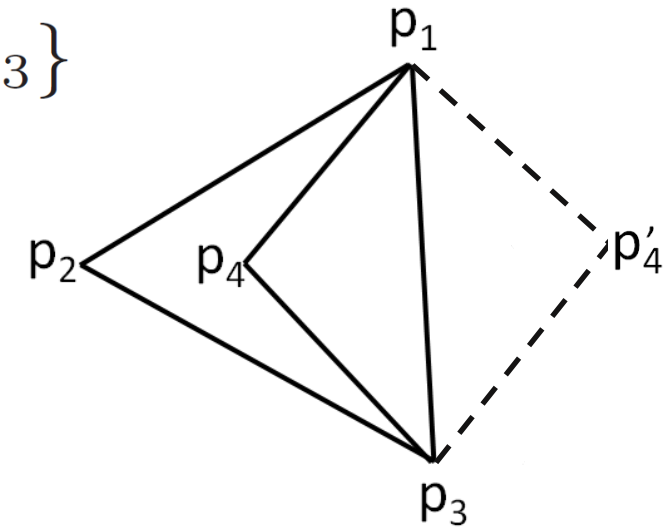
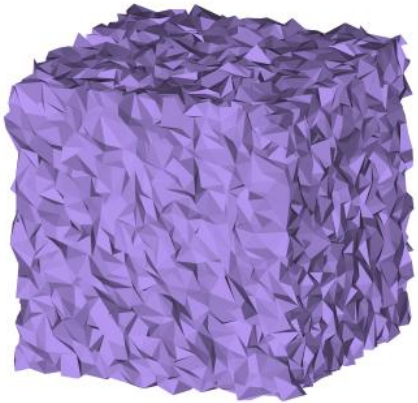
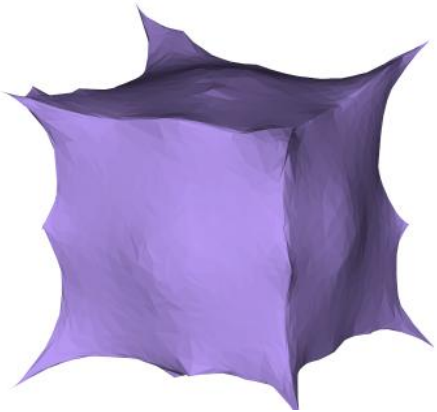


Discrete Differential Operator

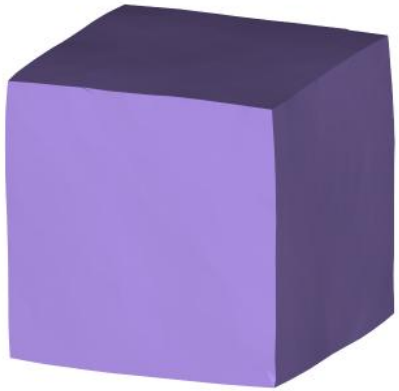


input surface



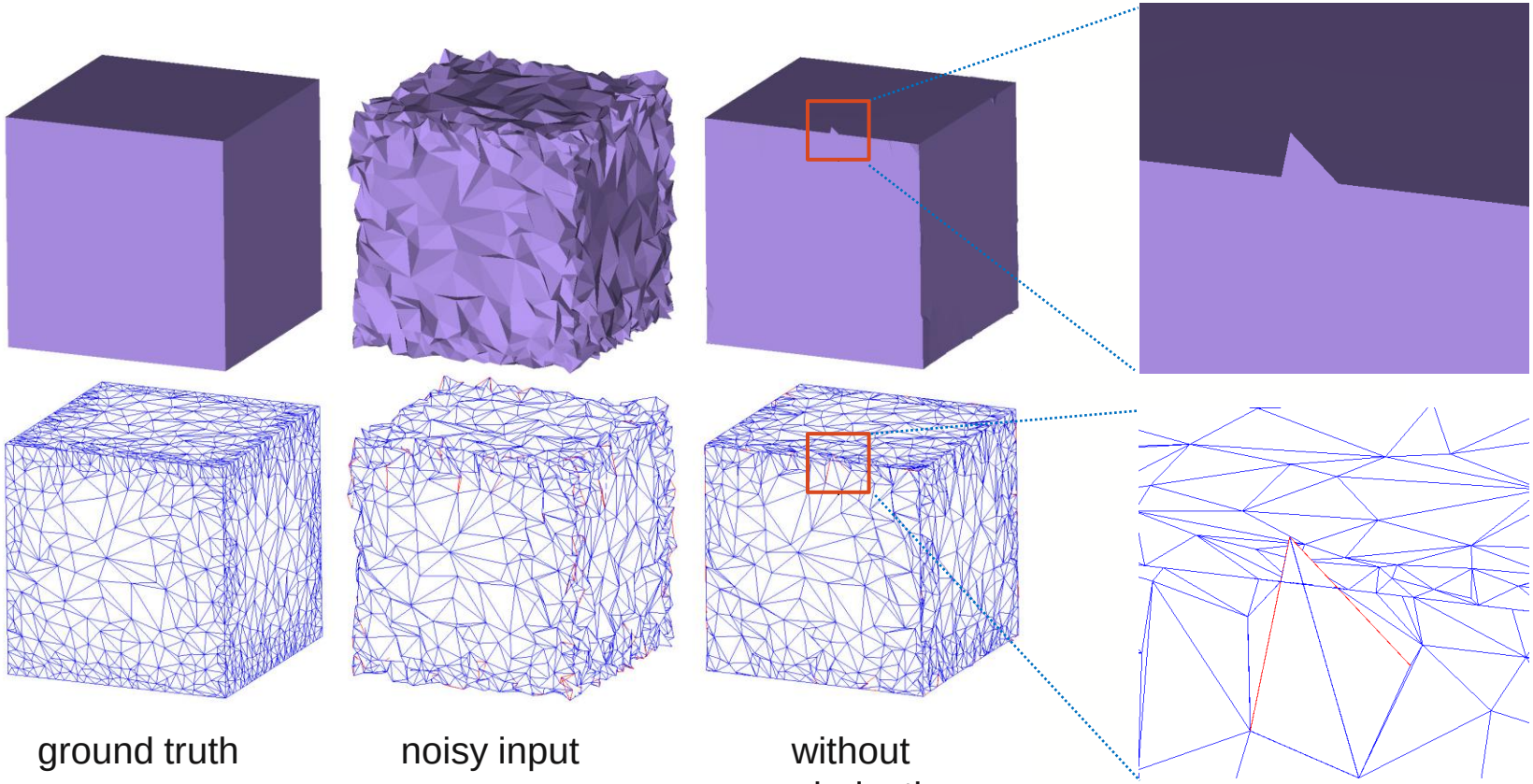
vertex-based
cotangent operator

cotangent
edge operator



area-based
edge operator

Regularization



ground truth

noisy input

without regularization

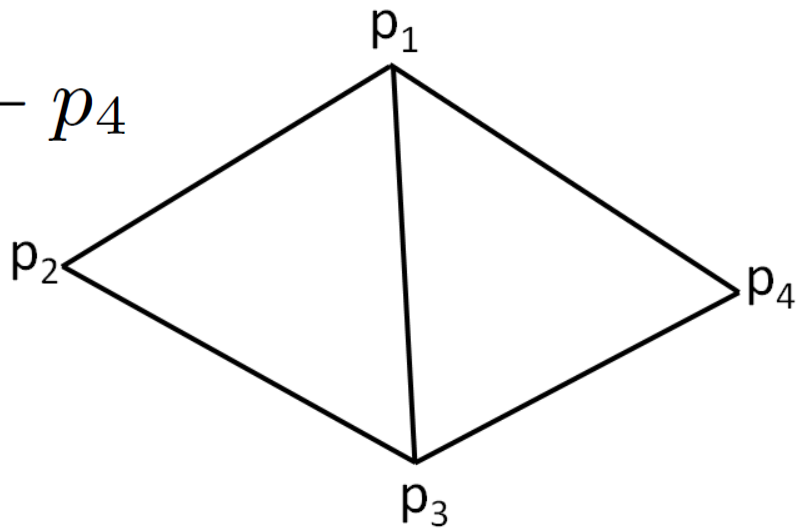
Regularization

$$\min_{p} |p - p^*|^2 + \alpha |R(p)|^2 + \lambda |D(p)|_0$$

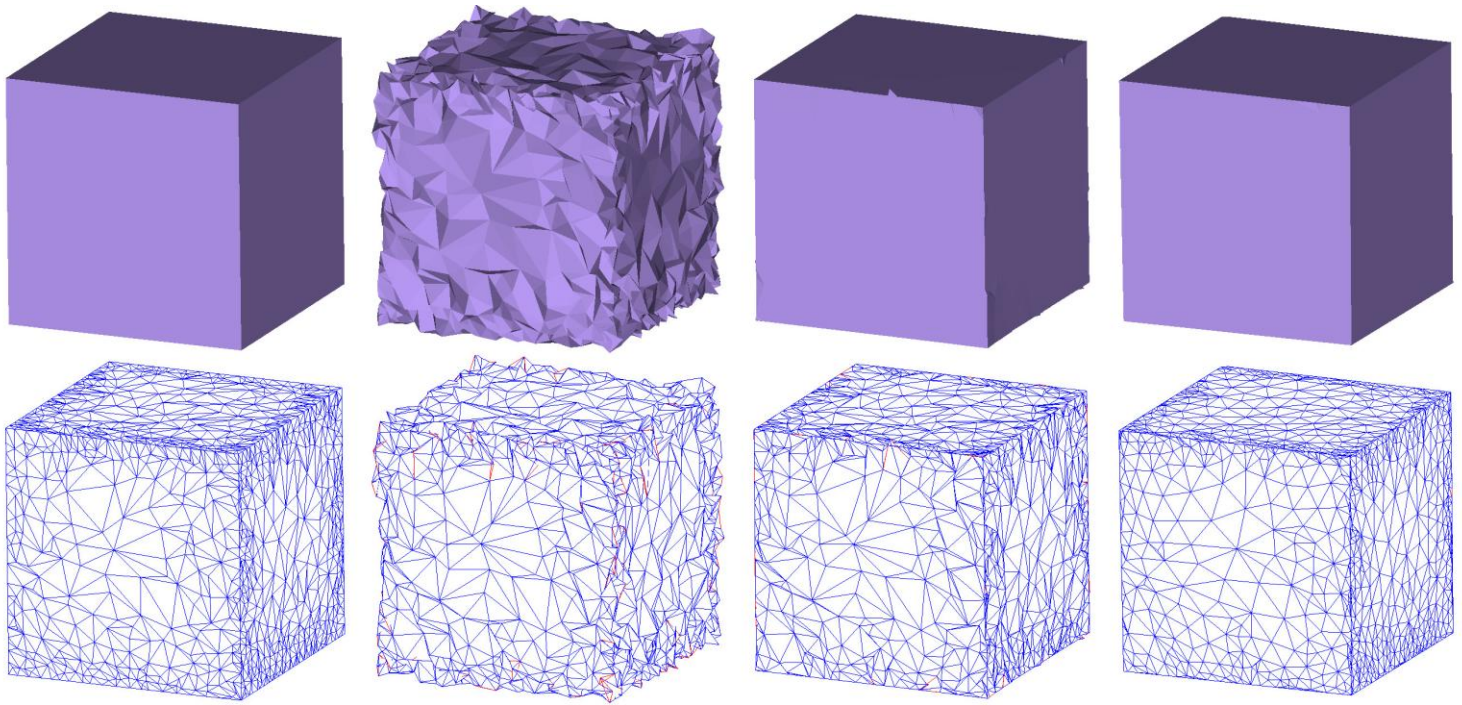
Regularization

$$\min_{p} |p - p^*|^2 + \alpha |R(p)|^2 + \lambda |D(p)|_0$$

$$R(p) = p_1 - p_2 + p_3 - p_4$$



Regularization



ground truth | noisy input | without regularization | with regularization

Optimization

$\mu > 1$, for L=0,1,2,…

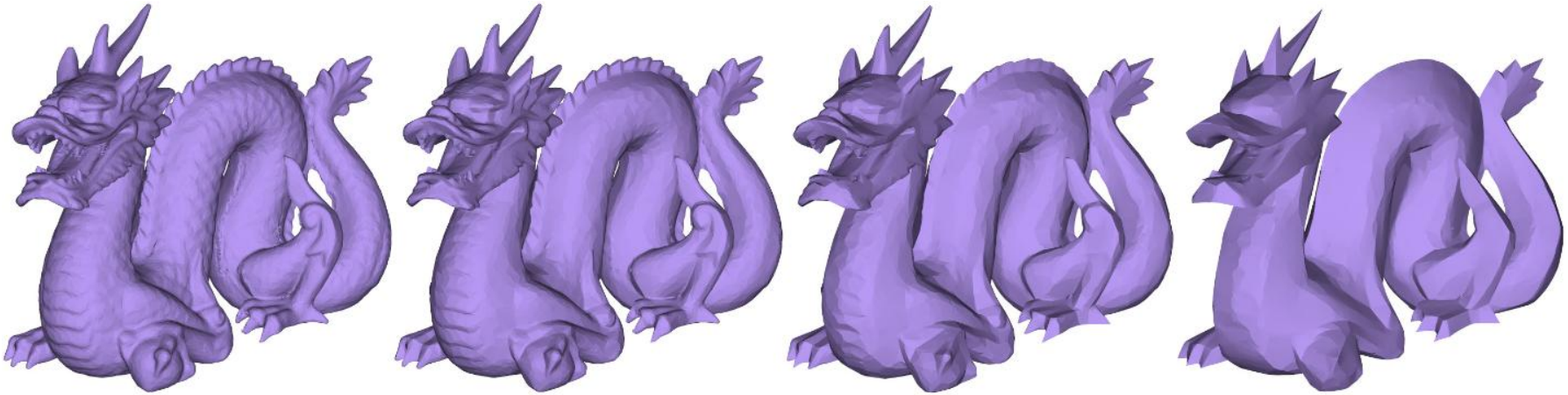
$$\min_{p,\delta} |p - p^*|^2 + \alpha_0 \mu^{-L} |R(p)|^2 + \beta_0 \mu^L |D(p) - \delta|^2 + \lambda |\delta|_0$$

Optimization

$\mu > 1$, for L=0,1,2,…

$$\min_{p,\delta} |p - p^*|^2 + \alpha_0 \mu^{-L} |R(p)|^2 + \beta_0 \mu^{L} |D(p) - \delta|^2 + \lambda |\delta|_0$$

Parameters



input mesh

increase λ

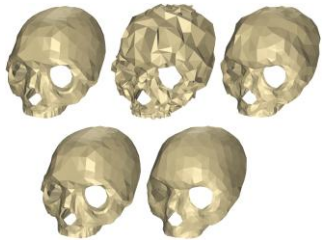
Parameters

ground truth

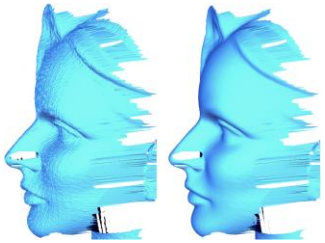
noisy input

decrease μ

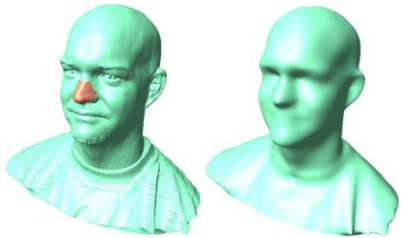
Results



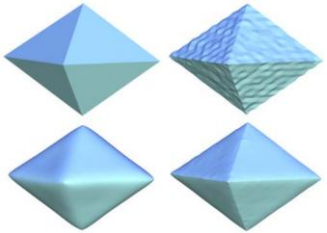
[Vollmer et al. 1999]



[Desbrun et al. 1999]



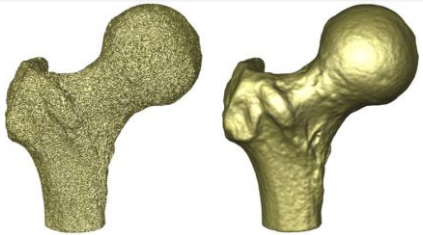
[Kim et al. 2005]



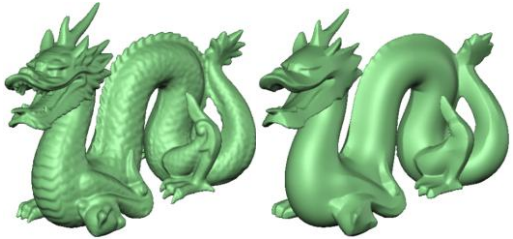
[Nealen et al. 2006]

[Clarenz et al. 2000]

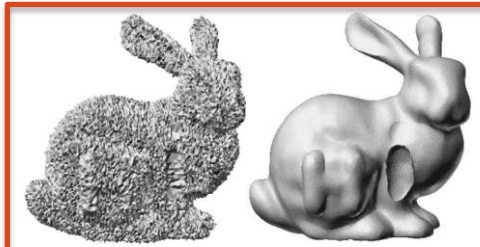
[Bajaj and Xu 2003]



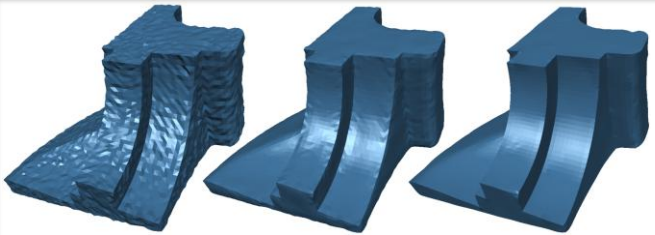
[Hildebrandt et al. 2004]



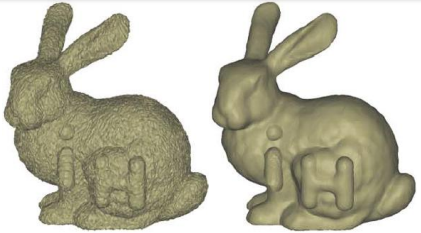
[Tasdizen et al. 2002]



[Yagou et al. 2002]



[Fleishman et al. 2003] [Jones et al. 2003]



[Zheng et al. 2012]

Results

[Vollmer et al. 1999]

[Desbrun et al. 1999]

[Kim et al. 2005]

[Nealen et al. 2006]

[Clarenz et al. 2000]

[Bajaj and Xu 2003]

PMC

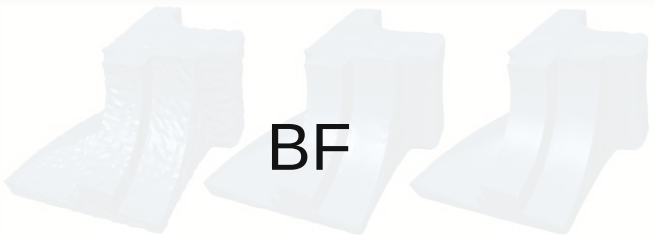
[Hildebrandt et al. 2004]

[Tasdizen et al. 2002]

MF

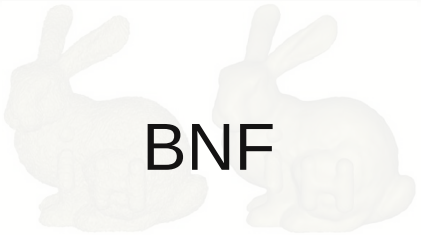
[Yagou et al. 2002]

BF



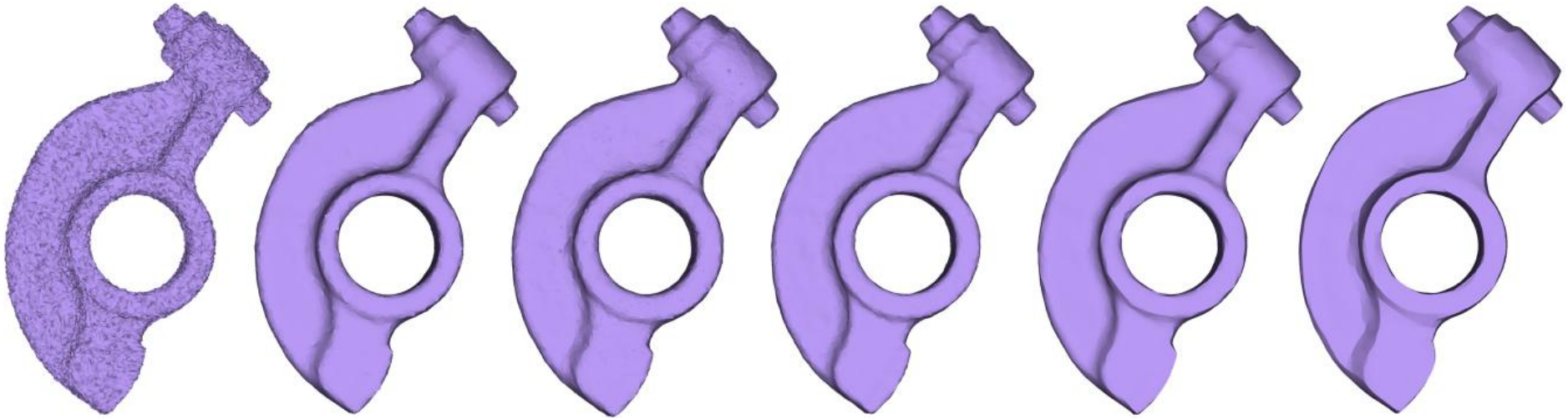
[Fleishman et al. 2003] [Jones et al. 2003]

BNF



[Zheng et al. 2012]

Results

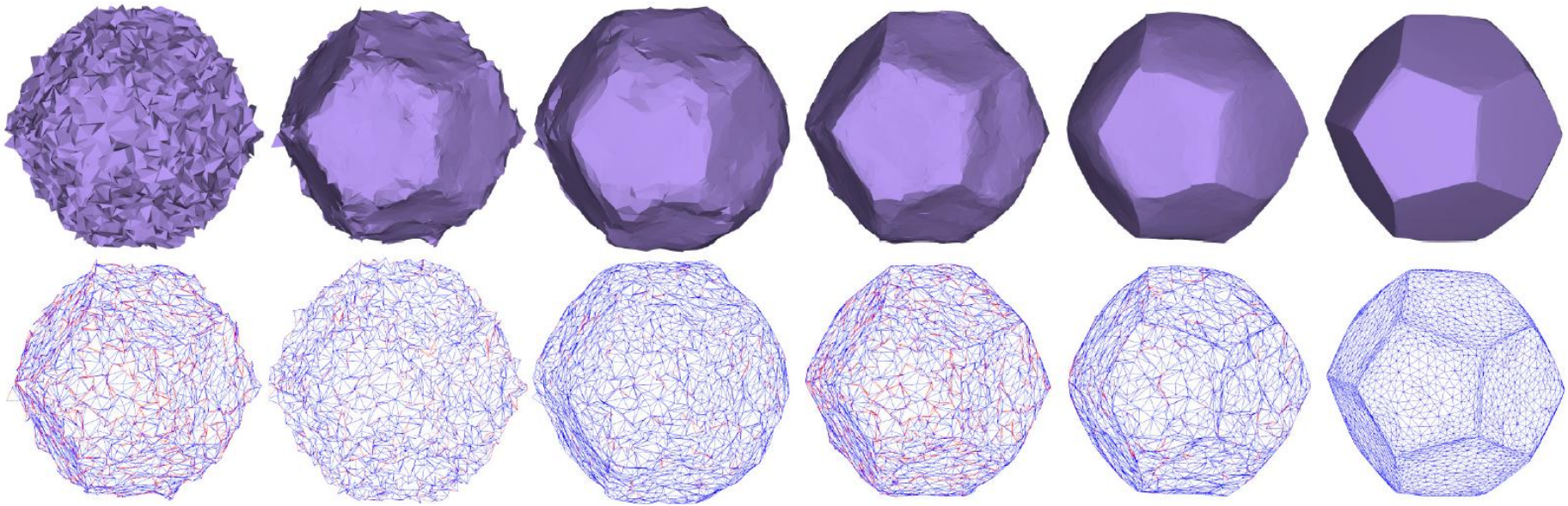


noisy input BF PMC MF BNF our result

Results

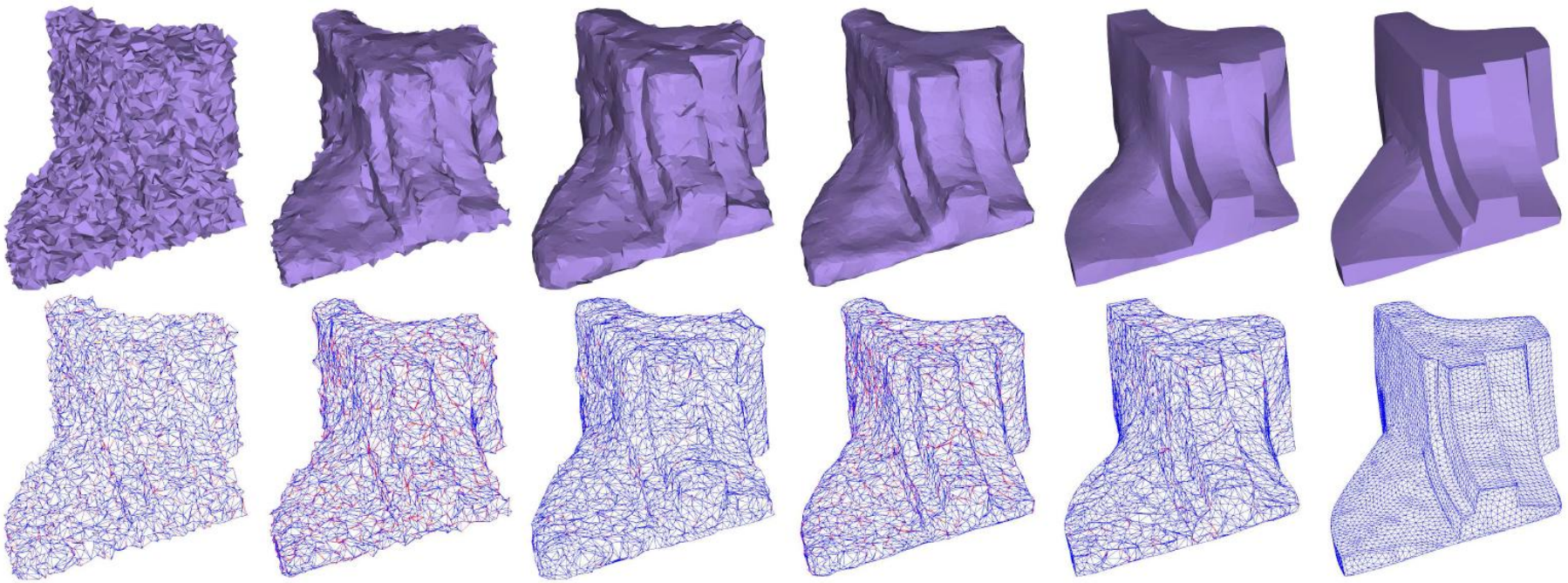
noisy input BF PMC MF BNF our result

Results



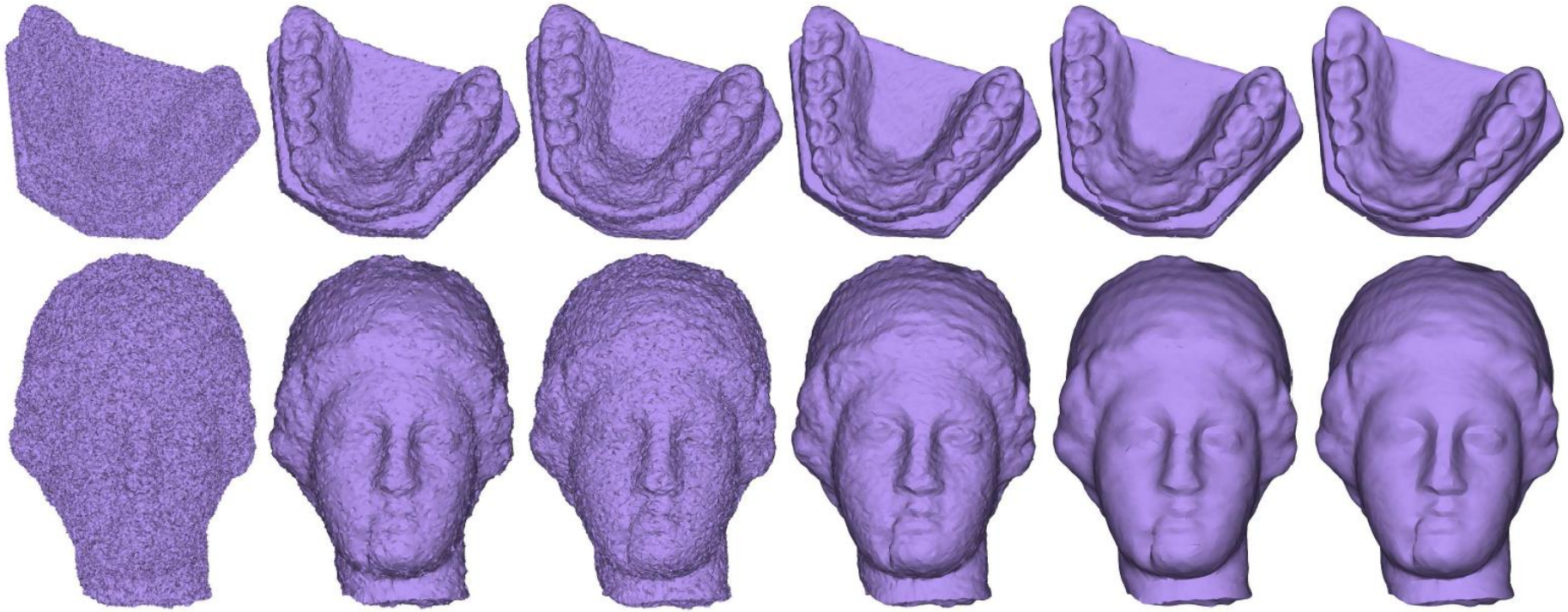
noisy input BF PMC MF BNF our result

Results



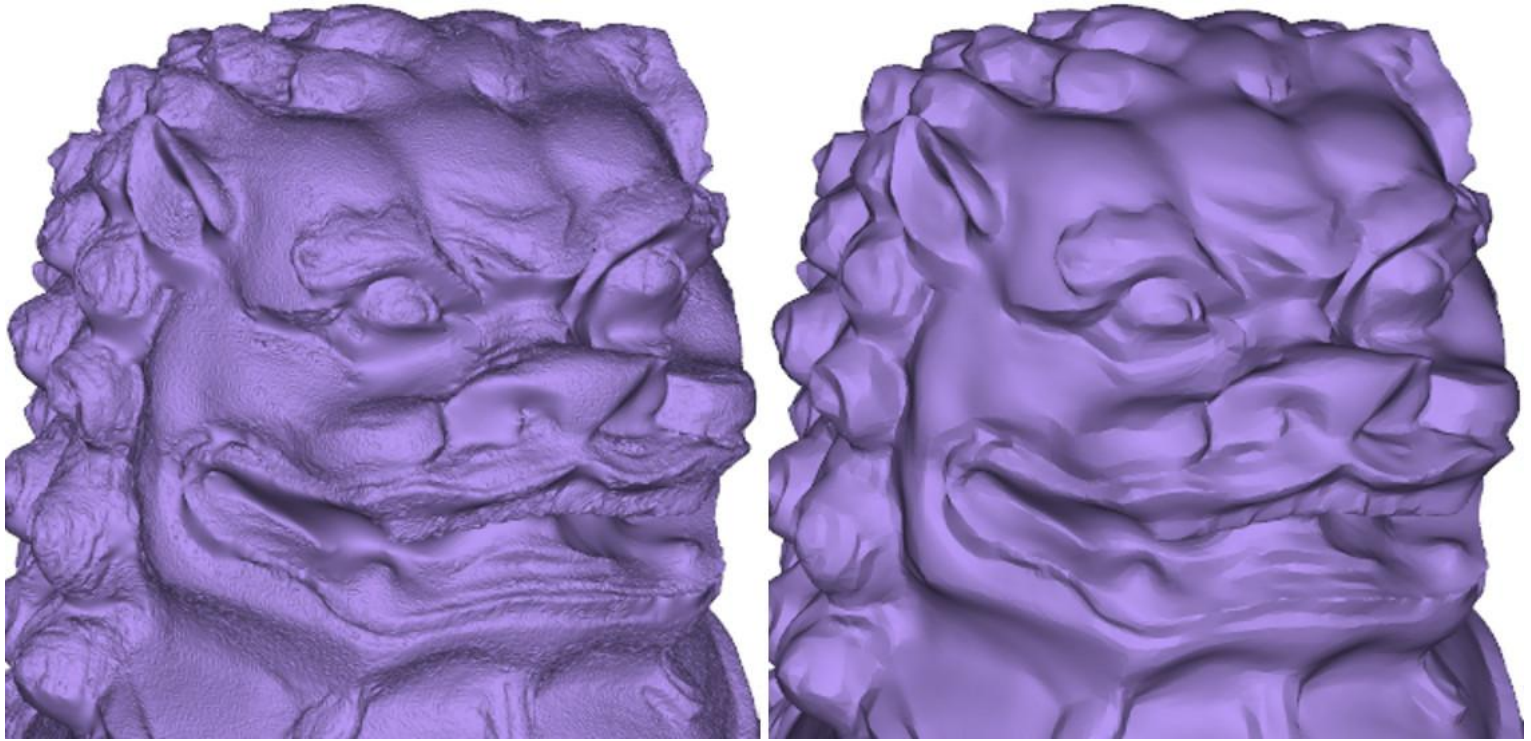
noisy input BF PMC MF BNF our result

Results



noisy input BF PMC MF BNF our result

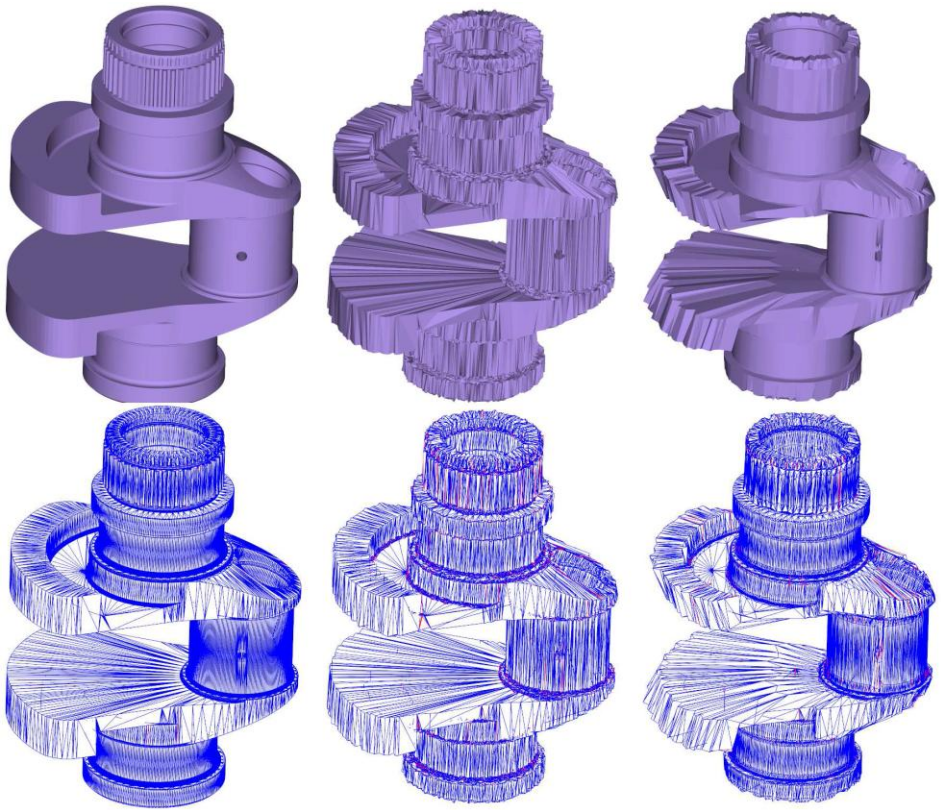
Results



noisy input

our result

A failure case



ground truth noisy input our result