Texas A&M University
Department of Electrical and Computer Engineering

ECEN 622: Active Network Synthesis
Homework #2, Fall 2016

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1.i) Obtain the transfer function of circuit shown below assuming ideal Op Amp

![Circuit Diagram](image)

Figure P1.1.

We start by finding the T-network equivalent impedance given by $Z_{eq} = Z_1 + Z_2 + Z_1Z_2/Z_3$. With this we can simply write the gain as an inverting amplifier.

$$H(s) = \frac{Z_F}{Z_{eq}} = \frac{Z_F}{Z_1 + Z_2 + \frac{Z_1Z_2}{Z_3}}$$  \hspace{1cm} (P1.1)

1.ii) Let $Z_1 = kR_1$, $Z_2 = (1-k)R_1$, $Z_F = R_1$, and $Z_3 = 1/C_3 s$. Identify the filter type and sketch the Bode Plot

By assuming the impedances indicated by the instructions we obtain the following expression

$$H(s) = \frac{1}{1 + kR_1C_3(1-k)s}$$  \hspace{1cm} (P1.2)

This expression shows the behavior of a single pole low pass filter, which pole position depends on the value of $k$, where its limits are $0 < k < 1$, given that if $k$ is outside of those limits $R_1$ or $R_2$ will become negative and that is not possible for passive components.

To sketch the bode plot of (P1.2) we assume $C_3 = R_1 = 1$. The following figure shows the behavior of (P1.2) for different values of $k$, where the expected behavior is appreciated.

![Bode Plot](image)

Figure P1.2 Bode plot for (P1.2) with $Z_3 = 1/C_3 s$
1.iii) Same as before but not $Z_3$ is the input impedance shown below with $A_2$ non ideal.

![Figure P1.3](image)

From circuit analysis we find that the equivalent input impedance for figure P1.3 is given by

$$Z_3 = \frac{R_3 R_5 + \left( \frac{1}{1 + A(s)} \right) (R_4 R_5 + R_3 R_4 R_5 C s)}{R_3 + R_3 C (R_5 + R_4) s} \quad \text{(P1.3)}$$

From (P1.2) if we were to assume an ideal op amp ($A(s) \rightarrow \infty$) the input impedance will be the parallel combination of $R_5$ and $C$, with a contribution by $R_4$ as the frequency increases.

Now to find the transfer function we reevaluate (P1.1) with $Z_3$ as the expression in (P1.3) due to the length of the equation we write the transfer function without substituting $A(s)$, obtaining the following expression.

$$H(s) = \frac{R_3 R_5 + \left( \frac{1}{1 + A(s)} \right) (R_4 R_5 + X_3 s)}{k^2 R_3 R_4 - k R_4 R_3 - R_3 R_5 + \left( \frac{1}{1 + A(s)} \right) (k R_3 R_4 - k R_4 R_3 + X_3)}$$

$$+ \left[ k^2 X_1 (R_4 + R_3) - R_4 R_3 + \left( \frac{1}{1 + A(s)} \right) (k X_2 - k X_2 - X_3) \right]$$

Where $X_1 = R_1 R_3 C$, $X_2 = R_4 R_5 C$, $X_3 = R_3 R_4 R_5 C$. Following the same procedure as before we assume all component values as 1 and substitute $A(s) = GB/s$ to find an expression in terms of GB, k and s.

$$H(s) = \frac{GB + 2s + s^2}{[k^2 GB - kGB - GB] + [2k^2 + 2k^2 - 2kGB - 2k - 2]s + [3k^3 - 3k - 1]s^2} \quad \text{(P1.4)}$$

The following figures show the bode plot for (P1.4) for different GB and k.
(P1.4) shows a behavior like a notch filter where the k controls the selectivity and GB controls the attenuation.
2) Obtain the \( H(s) \) using Butterworth, inverse Chebyshev and elliptic approximations that meet the following specs: \( A_{\text{max}} = 0.25 \text{dB}, \ A_{\text{min}} = 18 \text{dB}, \) and \( \omega_s = 1.4 \text{M rad/s}, \ \omega_p = 1 \text{M rad/s}.\)

In order to obtain the approximations needed we use the following MATLAB code:

```matlab
% Butterworth
[nbut,Wn] = buttord(Wp,Ws,Rp,Rs,'s'); % Returns the minimum order and Cutoff Frequency
[zbut,pbut,kbut] = butter(nbut,Wn,'s'); % Find the poles and zeros for the transfer function
Hbut = zpk(zbut,pbut,kbut); % Creates the Transfer Function

% Inverse Chev (type II)
[Nch2,Wchs] = cheb2ord(Wp,Ws,Rp,Rs,'s'); % Returns the minimum order and Cutoff Frequency
[z,p,k] = cheby2(Nch2,Rs,Wchs,'s'); % Find the poles and zeros for the transfer
Hch2 = zpk(z,p,k); % Creates the Transfer Function

% Elliptic
[Nel,Wel] = ellipord(Wp,Ws,Rp,Rs,'s'); % Returns the minimum order and Cutoff Frequency
[z,p,k] = ellip(Nel,Rp,Rs,Wel,'s'); % Find the poles and zeros for the transfer
Hel = zpk(z,p,k); % Creates the Transfer Function
```

The functions `buttord`, `cheb2ord`, and `ellipord` find the minimum filter order that complies with the specifications given. In the code \( \text{Rp} = A_{\text{max}}, \ \text{Rs} = A_{\text{min}}. \)
The previous figures show the behavior of the designed filters and the markers show that the restriction for passband, stopband, and the maximum gain is achieved. The abrupt change in phase is due to the closeness of the zeros in the transfer function.

The Butterworth filters is of 11th order and is compromised by 5 second-order filters cascaded with one extra first order filter.

$$H_{Butter}(s) = \left( \frac{1.16e6}{s + 1.16e6} \right) \left( \frac{1.347e12}{s^2 + (2.227e6)s + 1.1347e12} \right) \left( \frac{1}{s^2 + (1.952e6)s + 1.1347e12} \right) \left( \frac{1}{s^2 + (1.52e6)s + 1.1347e12} \right)$$

The inverse Chebyshev filter is of 5th order and is compromised by 2 second-order filters cascaded with one extra first order filter.

$$H_{Chebyshev}(s) = \left( \frac{8.683e5}{s + 2.356e6} \right) \left( \frac{s^2 + 2.07e12}{s^2 + (3.956e5)s + 1.508e12} \right) \left( \frac{s^2 + 5.41e12}{s^2 + (1.883e6)s + 2.742e12} \right)$$

The elliptic filter is of 4th order and is compromised by 2 second-order filters cascaded.

$$H_{Elliptic}(s) = 0.12589 \left( \frac{s^2 + 1.425e12}{s^2 + (1.188e6)s + 8.841e11} \right) \left( \frac{s^2 + 5.265e12}{s^2 + (1.697e5)s + 1.099e12} \right)$$

A table is provided next summarizing the minimum and maximum Q, in addition with a settling time measurement for 1% for a step input.

<table>
<thead>
<tr>
<th>MEASUREMENT</th>
<th>BUTTERWORTH</th>
<th>INVERSE CHEBYSHEV</th>
<th>ELLIPTIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_MIN</td>
<td>0.5211</td>
<td>0.8793</td>
<td>0.7914</td>
</tr>
<tr>
<td>Q_MAX</td>
<td>3.513</td>
<td>3.1479</td>
<td>6.1775</td>
</tr>
<tr>
<td>1% SETTLING TIME</td>
<td>23.756µs</td>
<td>17.942µs</td>
<td>32.936µs</td>
</tr>
</tbody>
</table>
3) Design a LP second-order filter using the Tow-Thomas topology with \( H_{LP}(0) = 1 \), \( \omega_0 = 2\pi \times 10^6 \) rad/s, and \( Q = 2 \).

i) Assume \( A(s) \to \infty \)

![Figure P3.1 Tow-Thomas Low Pass Topology](image)

Figure P3.1 shows the schematic for a Tow-Thomas low pass filter topology, for which we have the following expression for its transfer function.

\[
H(s) = -\frac{\left(\frac{R_3}{R_4}\right)\left(\frac{1}{R_2R_3C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \frac{1}{R_2R_3C_1C_2}} = \frac{H_0\omega_0^2}{s^2 + \frac{\omega_0^2}{Q} s + \omega_0^2}
\]

From which we can find the component values needed by making \( R_2 = R_3 = R_4 = R \), and \( C_1 = C_2 = C \), thus we can now express the coefficients as

\[
\omega_0 = \frac{1}{RC}, \quad Q = \frac{1}{\omega_0 R}, \quad R_1 = QR, \quad H_{LP}(0) = 1
\]

Now we find the component values to be

\[
R_1 = 15.914k\Omega \approx 16k\Omega \quad C = 20pF \quad R = 7.957k\Omega \approx 8k\Omega
\]

We will use Simulink to represent the system as block diagram, simulate the filter and later add non-idealities.

![Figure P3.2 Block Diagram Representation.](image)

In the following figure we show the bodeplot for the filter with the component values that we found.
We see the expected behavior of the ideal low pass filter with the peaking at the cutoff frequency due to the Q value.

**ii) Assume** $A(s) = GB/s$, with $GB = 16 \times 10^6 \times 2\pi \text{rad/s}$

The Tow-Thomas topology is composed by an inverting amplifier, a lossy integrator and a lossless integrator.

In order to consider the non idealities caused by the lossless integrator we substitute the ideal integrator block $(1/s)$ by

$$H_{\text{int}}(s) = \frac{-1}{GBs^2 + s}$$

The inverting amplifier block is replaced for a transfer function that represents the non-idealities of the op amp with

$$H_{\text{inv}}(s) = \frac{H_{\text{ideal}}}{1 + \frac{1}{A(s)} \frac{H_{\text{ideal}}}{A(s)}} = \frac{1}{1 + \left(\frac{2s}{GB}\right)}$$

Now for the lossy integrator we can consider the parallel combination of $R_1$ and $C_1$ as a single impedance $Z_F$ and consider the expression found in the previous homework for the non-ideal summing amplifier

$$V_{\text{lossy}} = \frac{-1}{1 + \frac{Z_F}{Z_{\text{total}}}} \left[V_i Z_F / Z_4 + V_o Z_F / Z_3\right]$$

Where $Z_F = (R_1) \left(\frac{1}{C_1 s}\right)$, and $Z_{\text{total}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_1} + C_1 s$

This notation allows us to isolate the ideal gain for each input while still consider the non-idealities of the op amp.
The following figure show the bode plot for the filter considering non-ideal op amps.

**Figure P3.5 Bode plot for Tow-Thomas Low Pass Filter with Non-Idealities**

In the same fashion that occurred in the KHN low pass filter the gain is almost doubled and the phase also suffer from the error. The following table summarizes the comparison of results from these simulations.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Ideal Op Amp</th>
<th>Non-ideal Op Amp</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude @ wo</td>
<td>5.95dB</td>
<td>11.7dB</td>
<td>+93%</td>
</tr>
<tr>
<td>Phase @ wo</td>
<td>-270°</td>
<td>-287°</td>
<td>+6.3%</td>
</tr>
</tbody>
</table>

iii) Modify the integrators to cancel the non-idealities

For this section we will apply the same technique that we used in the previous homework to add a series resistor to the feedback capacitor in the lossless and lossy integrator.

By placing a $R_c$ resistor in series with the feedback capacitor we create a zero that can cancel the parasitic pole created by the op amp, by defining the value $R_c = \frac{1}{G_{BC}}$
This leads the transfer function to be

\[ H(s) = \frac{1}{RCs (1 + \frac{1}{GBRC})} \text{ in which } \frac{1}{GBRC} \approx 0 \]

The following figure shows the bode plot of the three systems designed so far, the low pass filter with ideal op amps, the one with non-ideal op amps and the compensated integrators.

In figure P3.7 we can see that the Compensated response is closest to the ideal response, however since the inverting amplifier doesn’t have any kind of compensation to reduce the non-idealities of its op amp we still see an error although is small.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Ideal Op Amp</th>
<th>Non-ideal Op Amp</th>
<th>Error %</th>
<th>Compensated</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude @ wo</td>
<td>5.95dB</td>
<td>11.7dB</td>
<td>+93%</td>
<td>8.38dB</td>
<td>32%</td>
</tr>
<tr>
<td>Phase @ wo</td>
<td>-270°</td>
<td>-287°</td>
<td>+6.3%</td>
<td>-280°</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

The simulations and errors reported so far were done by using GB = 16\(w_o\), as we know from the previous homework if we increase the GB we can reduce the error, for this reason the following table shows the minimum GB needed for 1% error in magnitude for each of the systems we decided in this section.
<table>
<thead>
<tr>
<th>Measurement</th>
<th>Ideal Op Amp</th>
<th>Non-ideal Op Amp</th>
<th>Compensated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude @ wo</td>
<td>5.95dB</td>
<td>6.03</td>
<td>5.99dB</td>
</tr>
<tr>
<td>Phase @ wo</td>
<td>-270°</td>
<td>-270°</td>
<td>-272°</td>
</tr>
<tr>
<td>Minimum GB for 1% error</td>
<td>-</td>
<td>800 * wo</td>
<td>31 * wo</td>
</tr>
</tbody>
</table>

The Compensated integrators need a much lower GB to reduce the error than the non-compensated does, although the design can be more complex when adding the compensation components. Overall it is a good trade-off, as the power needed to achieve such GB is considerable smaller in comparison if we didn’t add the compensation.