Texas A&M University
Department of Electrical and Computer Engineering

ECEN 622: Active Network Synthesis
Homework #2, Fall 2016

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1.i) Obtain the transfer function of circuit shown below assuming ideal Op Amp

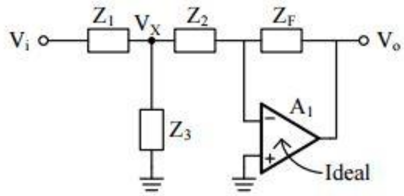


Figure P1.1.

We start by finding the T-network equivalent impedance given by $Z_{eq} = Z_1 + Z_2 + Z_1Z_2/Z_3$. With this we can simply write the gain as an inverting amplifier.

$$H(s) = \frac{Z_F}{Z_{eq}} = \frac{Z_F}{Z_1 + Z_2 + \frac{Z_1Z_2}{Z_3}} \qquad \text{(P1.1)}$$

1.ii) Let $Z_1 = kR_1$, $Z_2 = (1-k)R_1$, $Z_F = R_1$, and $Z_3 = 1/C_3s$. Identify the filter type and sketch the Bode Plot

By assuming the impedances indicated by the instructions we obtain the following expression

$$H(s) = \frac{1}{1 + kR_1C_3(1-k)s} \qquad \text{(P1.2)}$$

This expression shows the behavior of a single pole low pass filter, which pole position depends on the value of k, where its limits are 0 < k < 1, given that the if k is outside of those limits $R_1 or\ R_2$will become negative and that is not possible for passive components.

To sketch the bode plot of (P1.2) we assume $C_3 = R_1 = 1$. The following figure shows the behavior of (P1.2) for different values of k, where the expected behavior is appreciated.

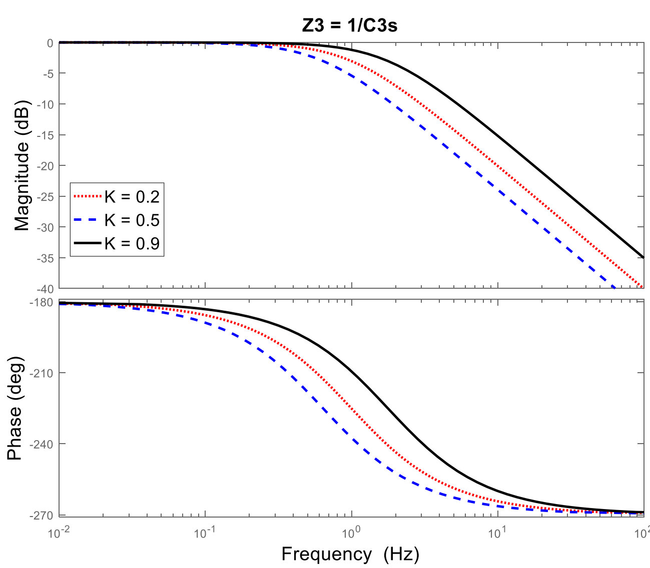


Figure P1.2 Bode plot for (P1.2) with $Z_3 = 1/C_3s$

1.iii) Same as before but not Z_3 is the input impedance shown below with A2 non ideal.

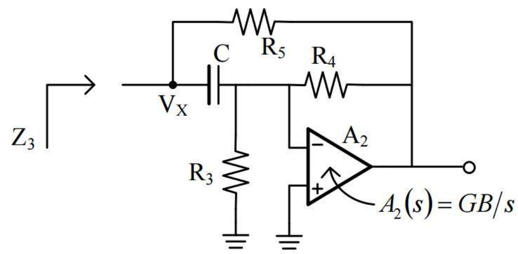


Figure P1.3

From circuit analysis we find that the equivalent input impedance for figure P1.3 is given by

$$Z_3 = \frac{R_3 R_5 + \left[\left(\frac{1}{1+A(s)}\right)(R_4 R_5 + R_3 R_4 R_5 C s)\right]}{R_3 + R_3 C (R_5 + R_4) s} \qquad \text{(P1.3)}$$

From (P1.2) if we were to assume and ideal op amp ($A(s) \to \infty$) the input impedance will be the parallel combination of R_5 and C, with a contribution by R_4 as the frequency increases.

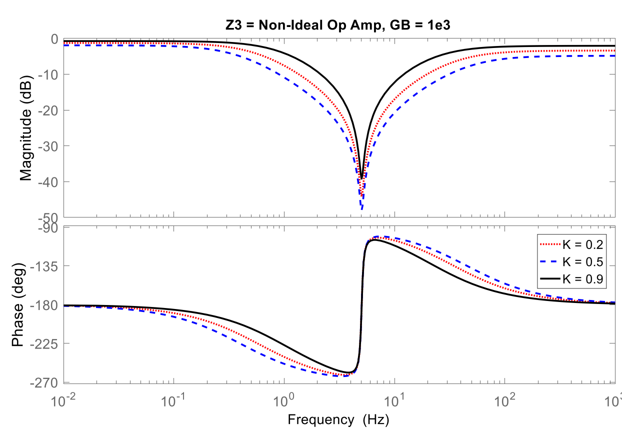
Now to find the transfer function we reevaluate (P1.1) with Z_3 as the expression in (P1.3) due to the length of the equation we write the transfer function without substituting A(s), obtaining the following expression.

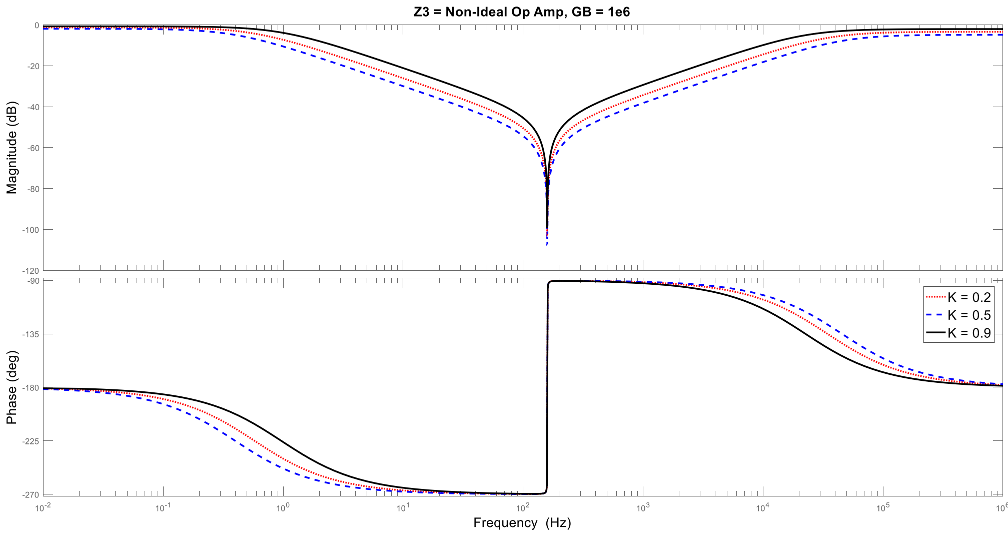
$$H(s) = \frac{R_3 R_5 + \left[\left(\frac{1}{1+A(s)}\right)(R_4 R_5 + X_3 s)\right]}{\left[k^2 R_1 R_3 - k R_1 R_3 - R_3 R_5 + \left(\frac{1}{1+A(s)}\right)(k^2 R_1 R_4 - k R_1 R_4 - R_4 R_5)\right] + \left[k^2 X_1 (R_4 + R_5) - k X_1 (R_4 + R_5) + \left(\frac{1}{1+A(s)}\right)(k^2 X_2 - k X_2 - X_3)\right] s}$$

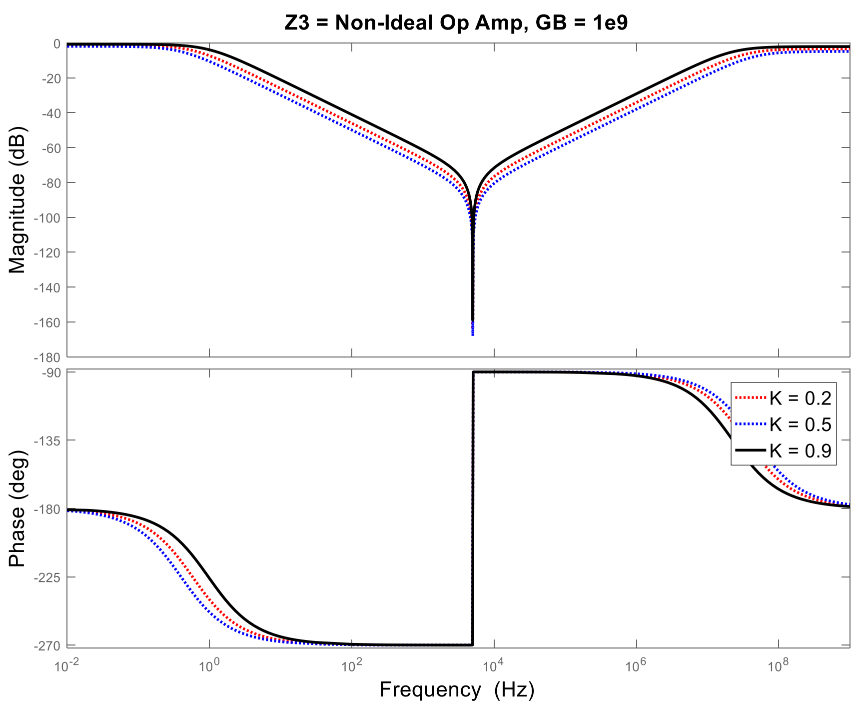
Where $X_1 = R_1 R_3 C$, $X_2 = R_1 R_4 R_5 C$, $X_3 = R_3 R_4 R_5 C$. Following the same procedure as before we assume all component values as 1 and substitute $A(s) = GB/s$ to find an expression in terms of GB, k and s.

$$H(s) = \frac{GB + 2s + s^2}{[k^2 GB - kGB - GB] + [2k^2 + 2k^2 - 2kGB - 2k - 2]s + [3k^3 - 3k - 1]s^2} \qquad \text{(P1.4)}$$

The following figures show the bode plot for (P1.4) for different GB and k.







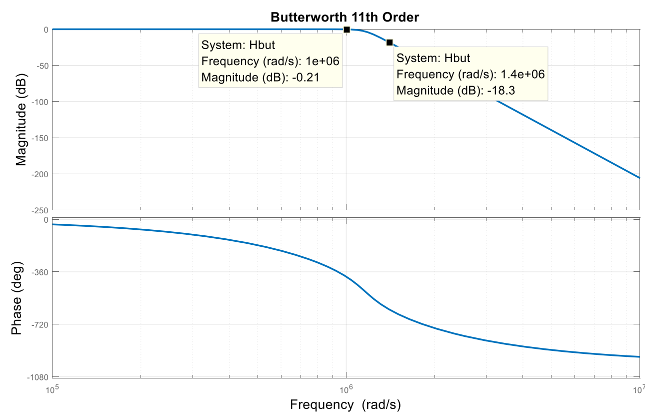
(P1.4) shows a behavior like a notch filter where the k controls the selectivity and GB controls the attenuation.

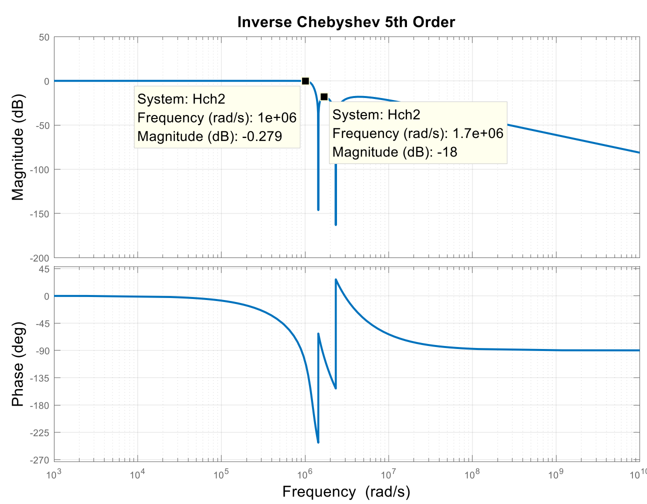
2) Obtain the H(s) using Butterworth, inverse Chebyshev and elliptic approximations that meet the following specs: Amax = 0.25dB, Amin = 18dB, and ws = 1.4M rad/s, wp = 1M rad/s.

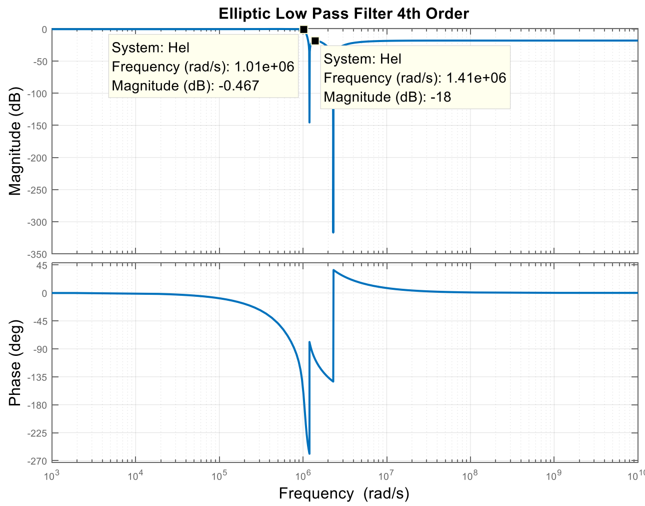
In order to obtain the approximations needed we use the following MATLAB code

```
%Butterworth
[nbut,Wn] = buttord(Wp,Ws,Rp,Rs,'s'); %Returns the minimum order and Cutoff Frequency
[zbut,pbut,kbut] = butter(nbut,Wn,'s'); %Find the poles and zeros for the transfer function
Hbut = zpk(zbut,pbut,kbut); %Creates the Transfer Function


%Inverse Chevi (type II)
[Nch2,Wchs] = cheb2ord(Wp,Ws,Rp,Rs,'s'); %Returns the minimum order and Cutoff Frequency
[z,p,k] = cheby2(Nch2,Rs,Wchs,'s'); %Find the poles and zeros for the transfer
Hch2 = zpk(z,p,k); %Creates the Transfer Function

%Elliptic
[Nel,Wel] = ellipord(Wp,Ws,Rp,Rs,'s'); %Returns the minimum order and Cutoff Frequency
[z,p,k] = ellip(Nel,Rp,Rs,Wel,'s'); %Find the poles and zeros for the transfer
Hel = zpk(z,p,k); %Creates the Transfer Function
```

The functions buttord, cheb2ord, and ellipord finds the minimum filter order that complies with the specifications given. In the code Rp = Amax, Rs = Amin.







The previous figures show the behavior of the designed filters and the markers show that the restriction for passband, stopband, and the maximum gain is achieved. The abrupt change in phase is due to the closeness of the zeros in the transfer function.

The Butterworth filters is of 11^{th} order and is compromised by 5 second-order filters cascaded with one extra first order filter.

$$H_{Butter}(s)=\left(\frac{1.16e6}{s+1.16e6}\right)\left(\frac{1.347e12}{s^2+(2.227e6)s+1.1347e12}\right)\left(\frac{1.347e12}{s^2+(1.952e6)s+1.1347e12}\right)\left(\frac{1.347e12}{s^2+(1.52e6)s+1.1347e12}\right)$$
$$\left(\frac{1.347e12}{s^2+(9.641e5)s+1.1347e12}\right)\left(\frac{1.347e12}{s^2+(3.303e5)s+1.1347e12}\right)$$

The inverse Chebyshev filter is of 5^{th} order and is compromised by 2 second-order filters cascaded with one extra first order filter.

$$H_{Chebyshev}(s)=\left(\frac{8.683e5}{s+2.356e6}\right)\left(\frac{s^2+2.07e12}{s^2+(3.956e5)s+1.508e12}\right)\left(\frac{s^2+5.41e12}{s^2+(1.883e6)s+2.742e12}\right)$$

The elliptic filter is of 4^{th} order and is compromised by 2 second-order filters cascaded.

$$H_{Elliptic}(s)=0.12589\left(\frac{s^2+1.425e12}{s^2+(1.188e6)s+8.841e11}\right)\left(\frac{s^2+5.265e12}{s^2+(1.697e5)s+1.099e12}\right)$$

A table is provide next summarizing the minimum and maximum Q, in addition with a settling time measurement for 1% for a step input.

MEASUREMENT	BUTTERWORTH	INVERSE CHEBYSHEV	ELLIPTIC
Q_{MIN}	0.5211	0.8793	0.7914
Q_{MAX}	3.513	3.1479	6.1775
1% SETTLING TIME	23.756μs	17.942μs	32.936μs

3) Design a LP second-order filter using the Tow-Thomas topology with $H_{LP}(0)=1$, $w_o = 2\pi \times 10^6$ rad/s, and Q = 2.

i) Assume $A(s) \rightarrow \infty$

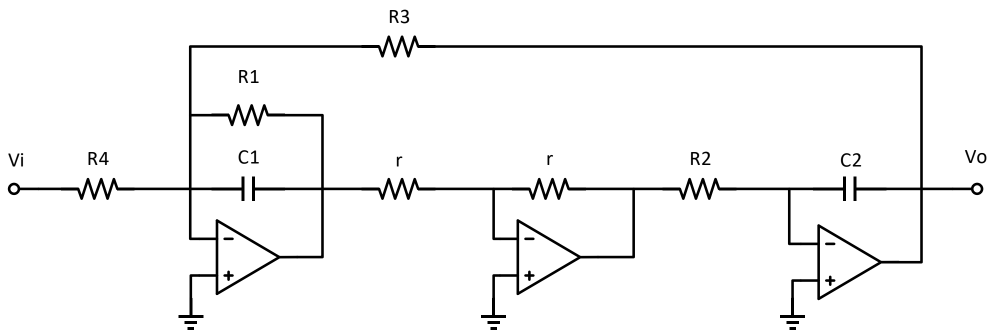


Figure P3.1 Tow-Thomas Low Pass Topology

Figure P3.1 shows the schematic for a Tow-Thomas low pass filter topology, for which we have the following expression for its transfer function.

$$H(s) = -\frac{\left(\frac{R_3}{R_4}\right)\left(\frac{1}{R_2 R_3 C_1 C_2}\right)}{s^2 + s\left(\frac{1}{R_1 C_1}\right) + \frac{1}{R_2 R_3 C_1 C_2}} = \frac{H_o w_o^2}{s^2 + \frac{w_o}{Q}s + w_o^2}$$

From which we can find the component values needed by making $R_2 = R_3 = R_4 = R$, and $C_1 = C_2 = C$, thus we can now express the coefficients as

$$w_o = \frac{1}{RC}, \qquad Q = \frac{1}{w_o R}, \qquad R_1 = QR, \qquad H_{LP}(0) = 1$$

Now we find the component values to be

R1 = 15.914kΩ ≈ 16kΩ C = 20pF R = 7.957kΩ ≈ 8kΩ

We will use Simulink to represent the system as block diagram, simulate the filter and later add non-idealities

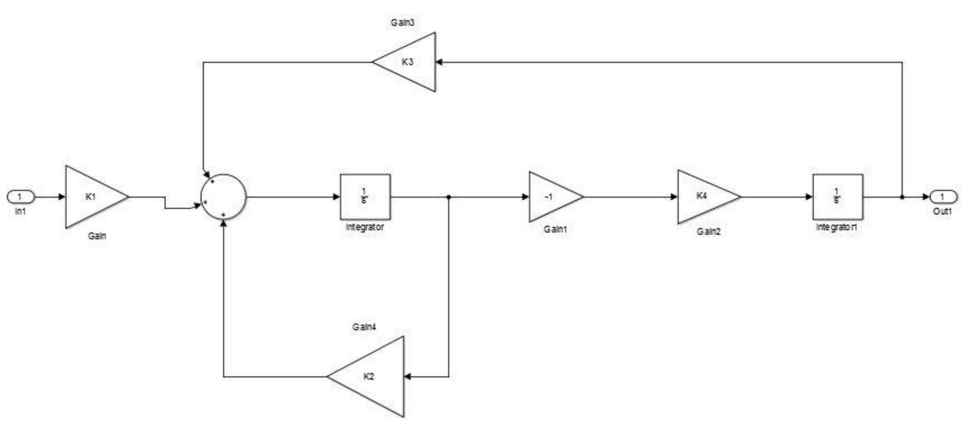


Figure P3.2 Block Diagram Representation.

In the following figure we show the bodeplot for the filter with the component values that we found.

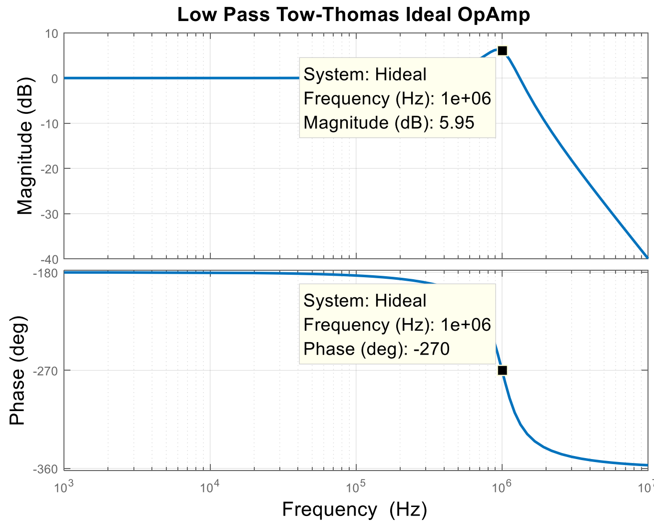


Figure P3.3 Frequency Response for Tow-Thomas Low Pass Filter

We see the expected behavior of the ideal low pass filter with the peaking at the cutoff frequency due to the Q value.

ii) Assume A(s) = GB/s, with $GB = 16x10^6 x2\pi\ rad/s$

The Tow-Thomas topology is composed by an inverting amplifier, a lossy integrator and a lossless integrator.

In order to consider the non idealities caused by the lossless integrator we substitute the ideal integrator block (1/s) by

$$H_{int}(s) = \frac{-1}{GBs^2 + s}$$

The inverting amplifier block is replaced for a transfer function that represents the non-idealities of the op amp with

$$H_{inv}(s) = \frac{H_{ideal}}{1 + \frac{1 + H_{ideal}}{A(s)}} = \frac{1}{1 + \left(\frac{2s}{GB}\right)}$$

Now for the lossy integrator we can consider the parallel combination of R_1 and C_1 as a single impedance Z_F and consider the expression found in the previous homework for the non-ideal summing amplifier

$$V_{o_{lossy}} = \frac{-1}{1+\frac{Z_F Y_{Total}}{GB/s}} [V_i Z_F/Z_4 + V_o Z_F/Z_3\,] \text{ Where } Z_F = (R_1 || \frac{1}{C_1 s}) \text{, and } Y_{total} = \frac{1}{R3} + \frac{1}{R_4} + \frac{1}{R_1} + C_1 s$$

This notation allows us to isolate the ideal gain for each input while still consider the non-idealities of the op amp.

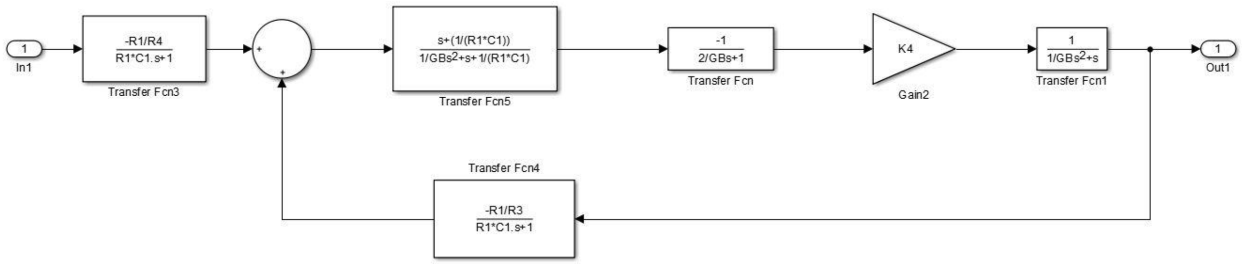


Figure P3.4 Block Diagram Representation with Non idealities.

The following figure show the bode plot for the filter considering non-ideal op amps.

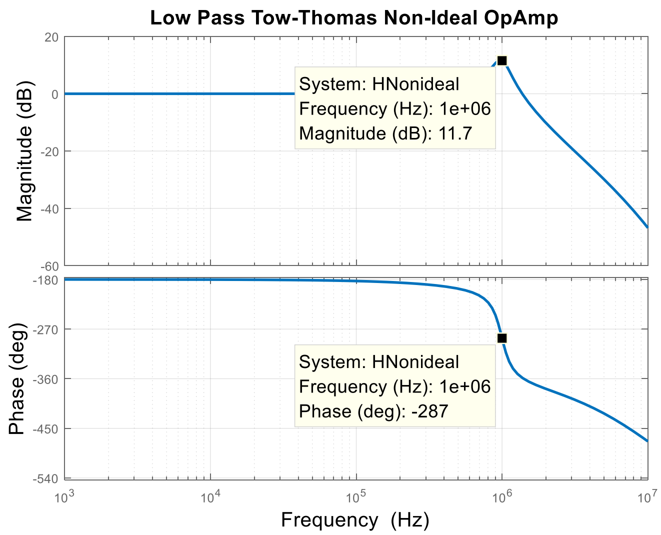


Figure P3.5 Bode plot for Tow-Thomas Low Pass Filter with Non-idealities

In the same fashion that occurred in the KHN low pass filter the gain is almost doubled and the phase also suffer from the error. The following table summarizes the comparison of results from these simulations.

Measurement	*Ideal Op Amp*	*Non-ideal Op Amp*	*Error %*
Magnitude @ wo	5.95dB	11.7dB	+93%
Phase @ wo	-270°	-287°	+6.3%

iii) Modify the integrators to cancel the non-idealities

For this section we will apply the same technique that we used in the previous homework to add a series resistor to the feedback capacitor in the lossless and lossy integrator.

By placing a R_c resistor in series with the feedback capacitor we create a zero that can cancel the parasitic pole created by the op amp, by defining the value $R_c = \frac{1}{GBC}$

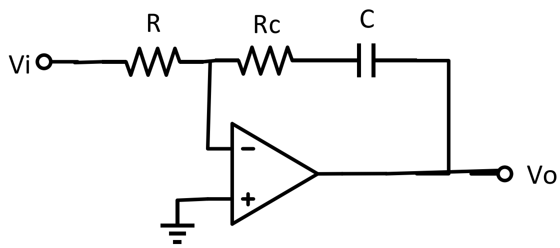


Figure P3.6 Compensated RC Integrator

This leads the transfer function to be

$$H(s) = \frac{1}{RCs\left(1+\frac{1}{GBRC}\right)} \; in\ which \; \frac{1}{GBRC} \approx 0$$

The following figure shows the bode plot of the three systems designed so far, the low pass filter with ideal op amps, the one with non-ideal op amps and the compensated integrators.

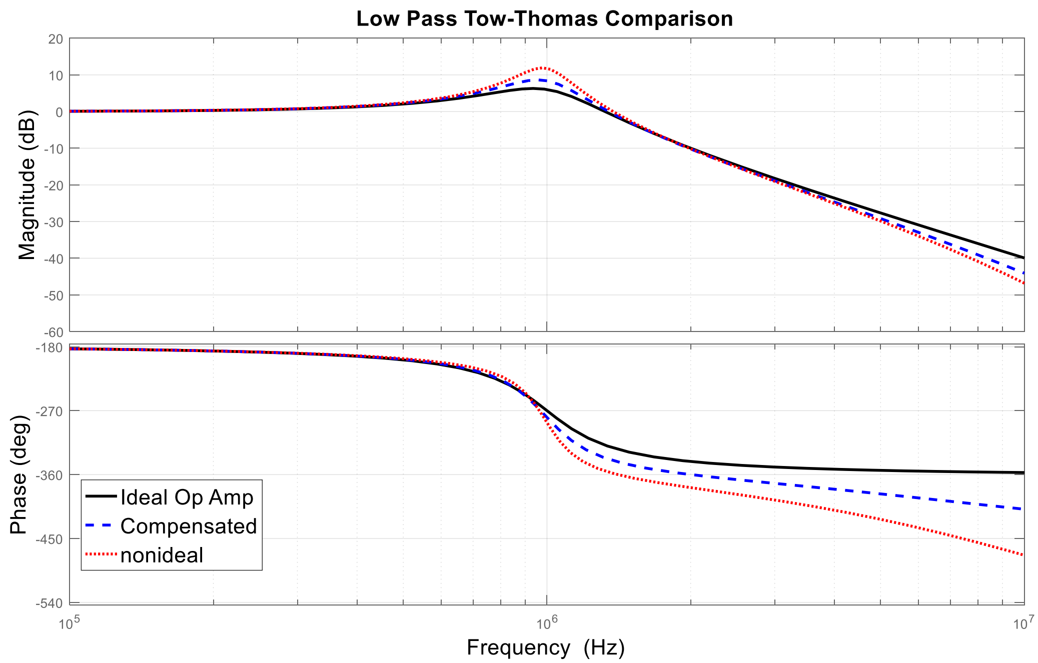


Figure P3.7 Bode Plot Comparison for the three design cases.

In figure P3.7 we can see that the Compensated response is closest to the ideal response, however since the inverting amplifier doesn't have any kind of compensation to reduce the non-idealities of its op amp we still see an error although is small.

Measurement	*Ideal Op Amp*	*Non-ideal Op Amp*	*Error %*	*Compensated*	*Error %*
Magnitude @ wo	5.95dB	11.7dB	+93%	8.38dB	32%
Phase @ wo	-270°	-287°	+6.3%	-280°	3.7%

The simulations and errors reported so far were done by using GB = $16w_o$, as we know from the previous homework if we increase the GB we can reduce the error, for this reason the following table shows the minimum GB needed for 1% error in magnitude for each of the systems we decided in this section.

Measurement	*Ideal Op Amp*	*Non-ideal Op Amp*	*Compensated*
Magnitude @ wo	5.95dB	6.03	5.99dB
Phase @ wo	-270°	-270°	-272°
Minimum GB for 1% error	-	800 * wo	31 * wo

The Compensated integrators need a much lower GB to reduce the error than the non-compensated does, although the design can be more complex when adding the compensation components. Overall it is a good trade-off, as the power needed to achieve such GB is considerable smaller in comparison if we didn't add the compensation.