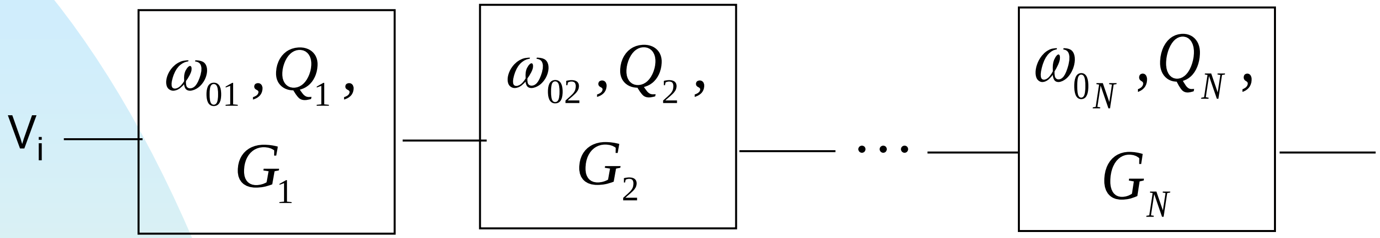
HIGHER-ORDER FILTERS

- Cascade of Biquad Filters
- Follow the Leader Feedback Filters (FLF)

Thanks for some of the material to
David Hernandez Garduño

CASCADE FILTER DESIGN

$$H(s) = \Pi_{i=1}^{N} H_i(s) = H_i(s)\, H_2(s) \cdots H_N(s)$$



Is the order in which sections are connected important?

- Mathematically is irrelevant
- On one hand, to avoid loss of dynamic and to avoid signal clipping in the high-Q sections use:

$$Q_1 < Q_2 < \cdots Q_N$$

- On the other to minimize noise, use

$$Q_1 > Q_2 > \cdots Q_N$$

- What is the optimal ordering to yield the best S/N?

$$H_i(s) = k_i \frac{a_{2i}s^2 + a_{1i}s + a_{oi}}{s^2 + s^{\omega_{oi}}/Q_i + \omega_{oi}^2}$$

$$H(s) = \prod_{i=1}^{n/2} k_i t_i(s)$$ n is even

How do we assign pole-zero pairs?

Proposition 1. Minimize the maximum value of d_i where

$$d_i = \log \frac{G_i}{m_i}$$

, G_i and m_i are the maximum and minimum gain in the frequency range of interest, $\omega_L \le \omega \le \omega_H$. Then determine the optimal sequence of the biquad sections, such that again the maximum number d_i is minimized. The final step is to assign the gain constants (k_i) of the biquads. To yield optimal dynamic range i.e. ,

$$\max\left|V_{oi}(j\omega)\right| = \max\left|V_{o,n/2}(j\omega)\right| = \max\left|V_{out}(j\omega)\right|$$

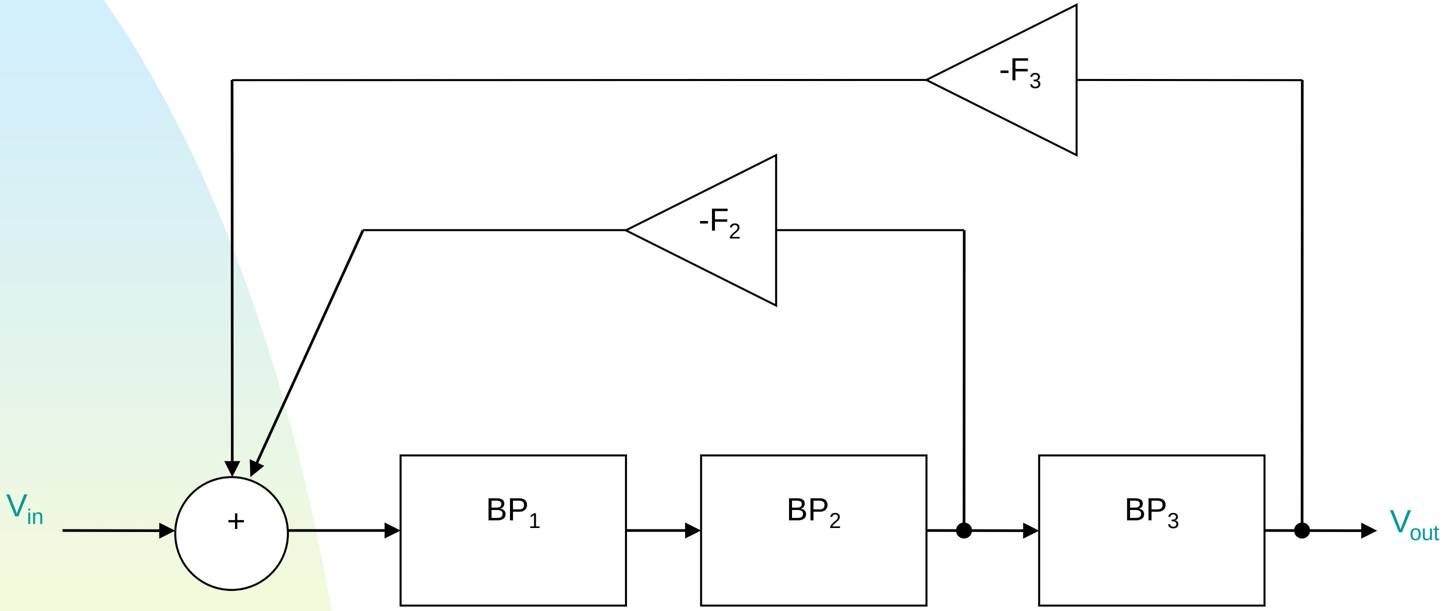
Proposition 2. To jointly optimize signal gain and noise gain. Noise by itself is minimized by assigning the largest possible gain to the first-stage, then the second with the largest gain among the other stages and so on.

Realization of High Order Transfer Functions (N>2)

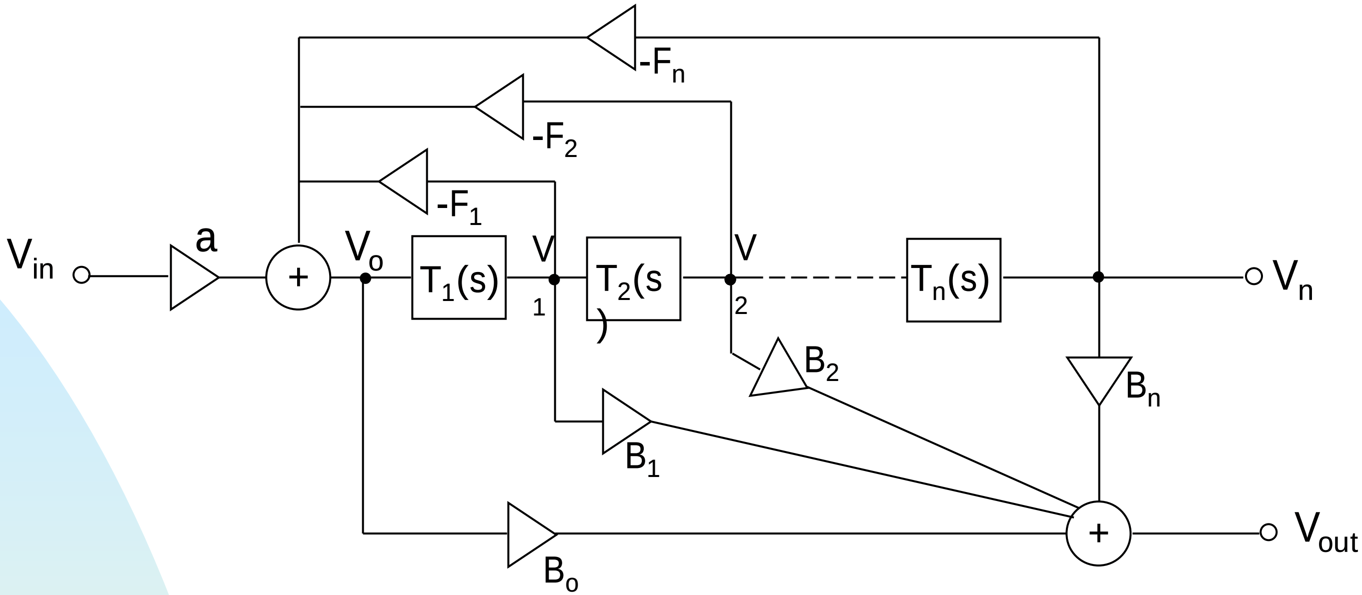
- Cascade of 2nd order sections (one 1st order section if N is odd)
- Leapfrog
- Follow-The-Leader

	Cascade	FLF	Leap-Frog
Sensitivity	High	Medium	Low
Easy to Tune	Medium	Easy	Difficult

Primary Resonator Block



- It provides compensatory internal interactions between the different filter sections through coupling the biquad building blocks.
- F_1 is incorporated in BP_1.



Follow-the-Leader Feedback Filter Topology

$$H_n(s) = \frac{V_n}{V_{in}} = \frac{a \prod_{j=1}^{n} T_j(s)}{1 + \sum_{k=1}^{n} \left[F_k \prod_{j=1}^{k} T_j(s) \right]}$$

$$H(s) = \frac{V_{out}}{V_{in}} = a \frac{B_o + \sum_{k=1}^{n} \left[B_k \prod_{j=1}^{k} T_j(s) \right]}{1 + \sum_{k=1}^{n} \left[F_k \prod_{j=1}^{k} T_j(s) \right]}$$

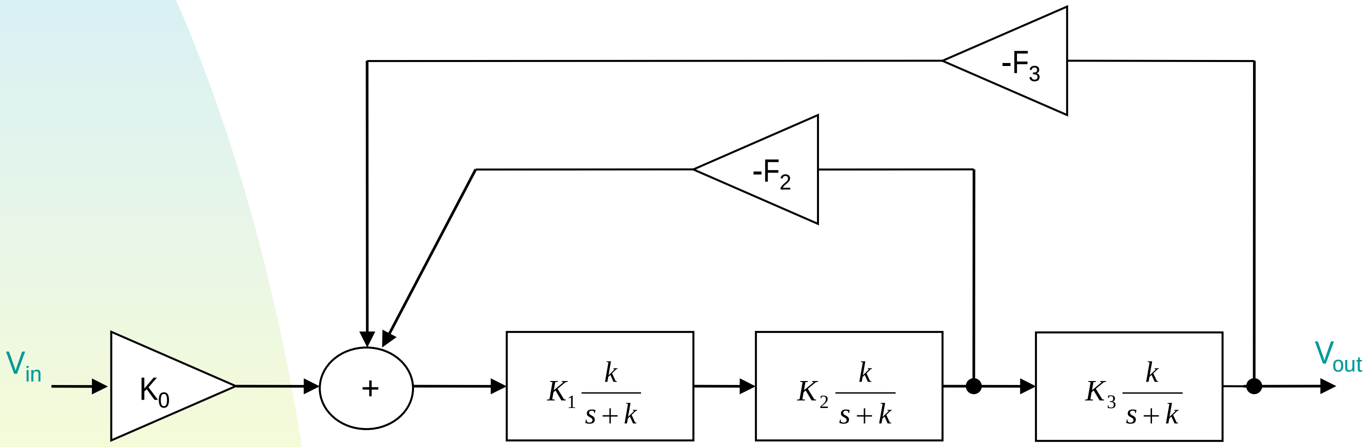
Jump to first page

Design Questions:

- How do we obtain the feedback coefficients F_2 and F_3?
- How do we determine the specifications for each biquadratic section?
    - Q
    - ω_o
    - Gain

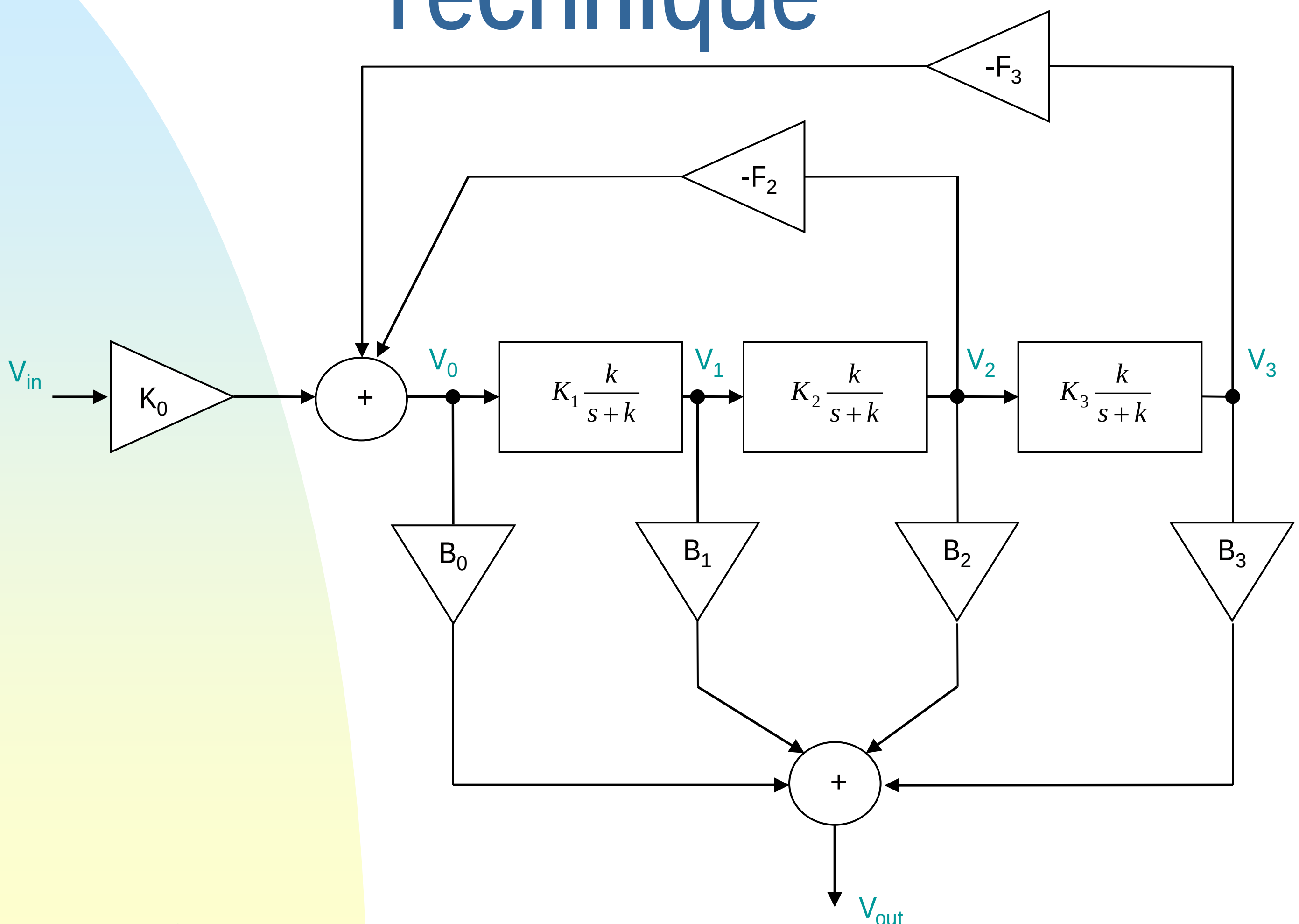
Design Procedure

- Start with the Lowpass equivalent system.



However Elliptic Filters need finite zeros in their lowpass equivalent transfer function.

Implementation of Finite Zeros by the Summation Technique



EXAMPLE

- To design a 6th order bandpass elliptic filter using the Follow-the-Leader (FLF) architecture. The specifications are:

Specification	**Value**
Order	6
Passband	1.9MHz to 2.1MHz
Passband Ripple	0.1dB
Attenuation	≥30dB at 0.6MHz

Design Procedure

Let for now $K_1=K_2=K_3=1$

and

$$T(s)=\frac{k}{s+k} \tag{1}$$

Applying Mason's rule, the complete transfer function is given by:

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=K_0\frac{B_0+B_1T(s)+B_2T^2(s)+B_3T^3(s)}{1+F_2T^2(s)+F_3T^3(s)}$$

$$=K_0\frac{B_0(s+k)^3+B_1k(s+k)^2+B_2k^2(s+k)+B_3k^3}{(s+k)^3+F_2k^2(s+k)+F_3k^3} \tag{2}$$

Design Procedure

- From Matlab or Fiesta, we can obtain the lowpass prototype transfer function of the desired 6th Order Elliptic Filter:

$$H(s) = m\frac{a_0}{b_0}\frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \quad (3)$$

- Equating the denominators of equations (2) and (3), we obtain the following set of equations from which we can solve for k, F_2, and F_3.

$$3k = a_2 \quad (4)$$
$$3k^2 + F_2 k^2 = a_1$$
$$(1 + F_2 + F_3)k^3 = a_0$$

- Also from equations (2) and (3)

$$K_0 = m\frac{a_0}{b_0} \quad (5)$$

Jump to first page

Design Procedure

- To obtain the summation coefficients, we equate the numerators of equations (2) and (3). If H(s) is a bandpass: $B_0=b_3=0$. Then, we obtain the following set of equations from which we can determine B_1, B_2 and B_3.

$$
\begin{aligned}
&B_0 = b_3 = 0 \\
&B_1 k = b_2 \\
&2B_1 k + B_2 k^2 = b_1 \\
&(B_1 + B_2 + B_3)k^3 = b_0
\end{aligned}
\qquad (6)
$$

Designing for Maximum Dynamic Range

- We need to distribute the gains of each section T(s), i.e. K_1, K_2 and K_3 such that we maximize the Dynamic Range.
- The maximum dynamic range will be obtained if the signal spectra at the output of all sections have equal maxima, i.e.

$$V_{out,max}=V_{3,max}=V_{2,max}=V_{1,max}=V_{0,max}$$

Maximizing Dynamic Range

- To make $V_{3,max}$= $V_{out,max}$

$$K_0 \to K_0' = K_0 q \quad (7)$$

where

$$q = \frac{V_{out,\max} \leftarrow prior\ to\ scaling}{V_{3,\max} \leftarrow prior\ to\ scaling} \quad (8)$$

- We also need to adjust the summation coefficients to keep the overall gain:

$$B_i \to B_i' = \frac{B_i}{q} \quad (9)$$

If we assume a flat spectrum for the input, i.e. Vin(ω)=1

$$V_{out,\max} = Max\left|H(\omega)\right| \quad (10)$$

$$V_{3,\max} = Max\left|H_3(\omega)\right|$$

Gain Values Determination

Where

$$H_3(s)=\frac{V_3(s)}{V_{in}(s)}=K_0\frac{k^3}{s^3+a_2s^2+a_1s+a_0} \quad (11)$$

- To obtain K_1, K_2 and K_3 :

$$K_i=\frac{Max\left\{\left|H_3(\omega)\right|\left(\frac{k^2+\omega^2}{\omega^2}\right)^{\frac{4-i}{2}}\right\}}{Max\left\{\left|H_3(\omega)\right|\left(\frac{k^2+\omega^2}{\omega^2}\right)^{\frac{3-i}{2}}\right\}} \quad for\ i=1,2,3. \quad (12)$$

Feedback Coefficients

- The feedback coefficients need to be readjusted to keep the same loop gains:

$$F_2 \to F_2' = \frac{F_2}{K_1 K_2} \qquad (13)$$

$$F_3 \to F_3' = \frac{F_3}{K_1 K_2 K_3}$$

- The summation coefficients also need to be readjusted again:

$$B_1 \to B_1' = B_1 \frac{K_2 K_3}{q} \qquad (14)$$

$$B_2 \to B_2' = B_2 \frac{K_3}{q}$$

$$B_3 \to B_3' = B_3 \frac{1}{q}$$

Summary of Design Procedure

- Obtain from Matlab or Fiesta the lowpass prototype for the desired filter.
- From equations (4), (5) and (6), obtain K_0, the feedback and the summation coefficients.
- To maximize dynamic range, obtain q using equation (8). Recalculate K_0 using equation (7).
- Calculate the gain of each section, i.e. K_1, K_2 and K_3 using equation (12).
- Recalculate the feedback and summation coefficients using equations (13) and (14).
- Finally, apply a lowpass-to-bandpass transformation to obtain the desired bandpass filter specifications:

$$\omega_0 = 2\pi\sqrt{f_L f_U}$$

$$Q_0 = \frac{Q}{k}$$

where Q is the quality factor of the overall filter and Q_0 is that required for each biquad section.

- Note: A Matlab program was written to automate the design procedure for an arbitrary filter specification of order N.

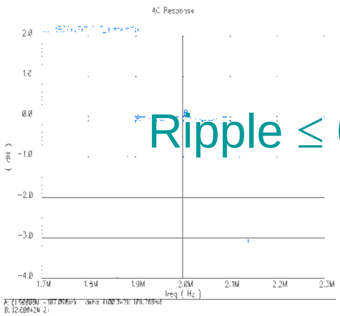
Summary of Results

- For the required specifications, the following values were obtained:

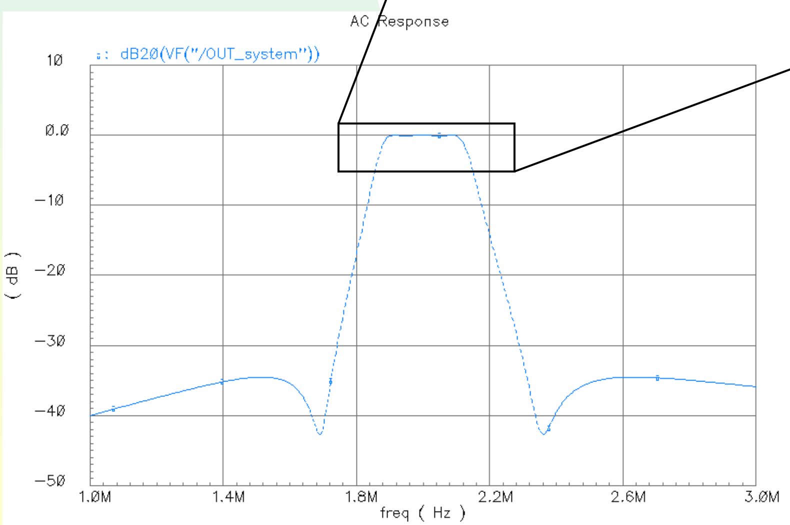
Feedback Coefficients	F2 = 0.657640 F3 = 0.227545
Feedforward Coefficients	B0 = 0 B1 = 0.169792 B2 = -0.216571 B3 = 1.129814
Gain for the input and each biquad stage	K0 = 0.604488 K1 = 2.349 K2 = 2.165 K3 = 2
Center frequency and Q_0 of each biquad stage	f_0 = 1.9975MHz Q_0 = 15.7

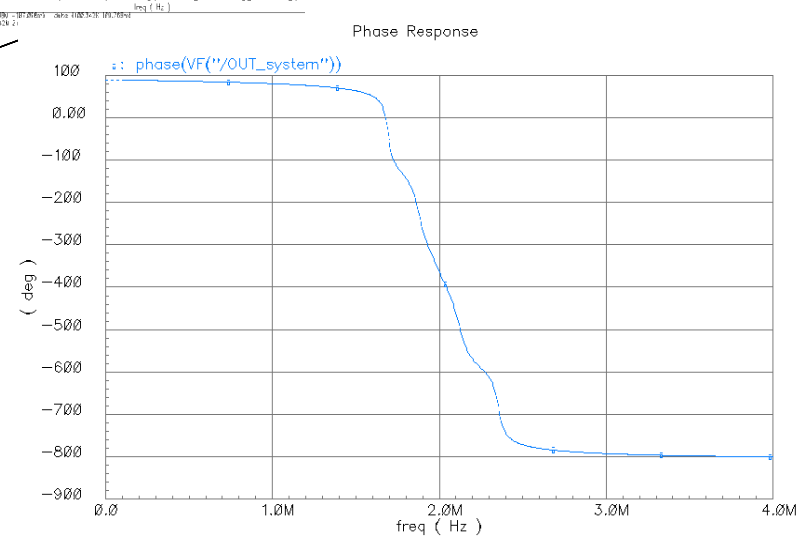
Simulation Results
System Level

- The complete filter was simulated in Cadence at a system-level. The results are shown below:



Ripple ≤ 0.1dB





Magnitude Response

Phase Response

Transistor Level Implementation

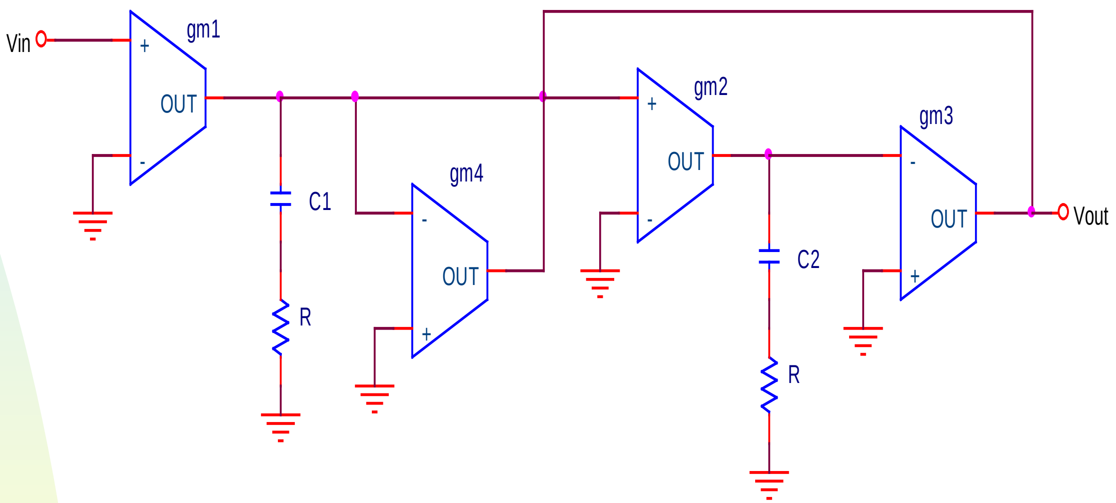
- To implement each biquadratic section, a two-integrator loop biquad OTA-C filter was used.

- Advantages with respect to Active-RC:
  - Easy Tunability by changing the bias currents of the OTAs. (Active-RC needs the use of varactors).
  - Lower Power Consumption and Smaller Area.

- Disadvantages with respect to Active-RC:
  - Smaller Dynamic Range
  - Poorer Linearity

Jump to first page

Transistor Level Implementation of each Biquad Section



$$H(s)=\frac{s\frac{g_{m1}}{C_1}}{s^2+s\frac{g_{m4}}{C_1}+\frac{g_{m2}g_{m3}}{C_2C_1}}$$

Design of Lossless Integrator

- The lossless integrator was designed to have unity gain at f_0=1.9975MHz.

$$\left|H(\omega)\right| = \frac{g_{m2}}{\omega C_2} = 1$$

$$g_{m3} = 376.52\,\mu A/V$$

$$C_2 = 30\,pF$$

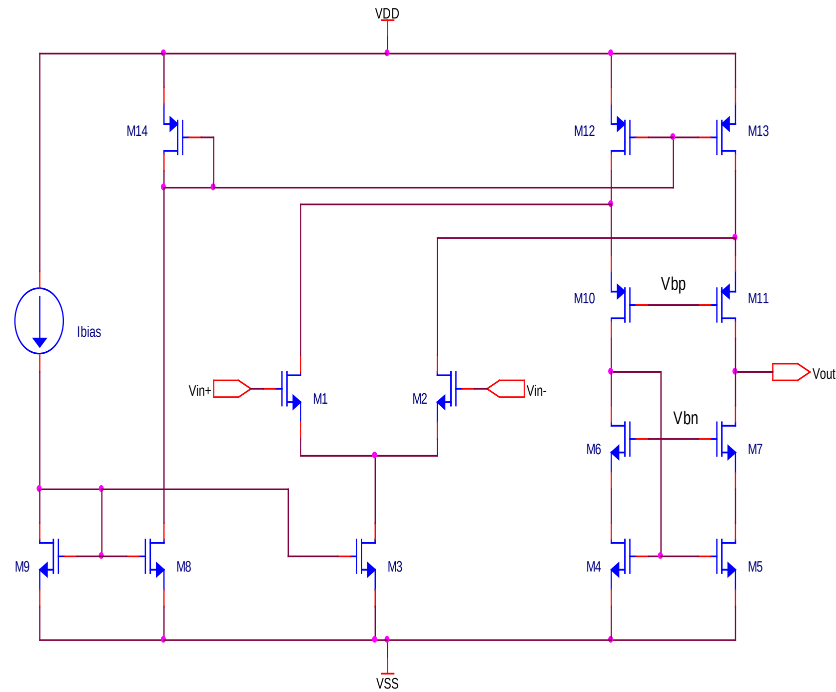
- The following specifications are needed if a 5% variation in Q is allowed:

$$Excess\ \ Phase:\ \ \phi_E \le \frac{1}{2Q}\left(1 - \frac{Q}{Q_a}\right) = 1.5\times10^{-3}\,rad = 0.086°$$

$$DC\ \ Gain:\ \ A_V \ge \frac{2Q}{\frac{Q}{Q_a} - 1} = 602 = 55.58dB$$

Design of Lossless Integrator

- Due to the relatively high DC gain required for the OTA, a folded-cascode topology was used:

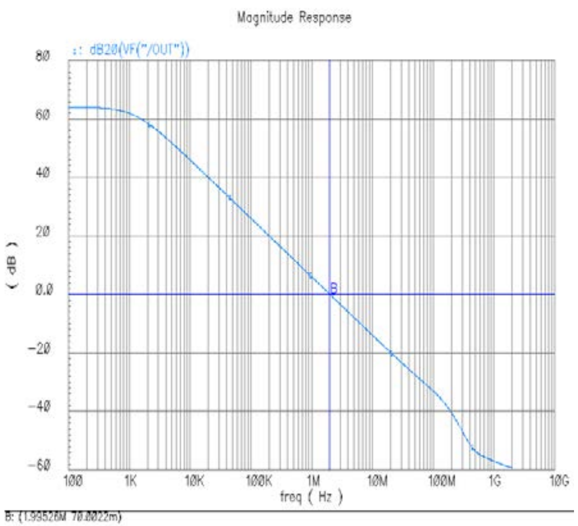


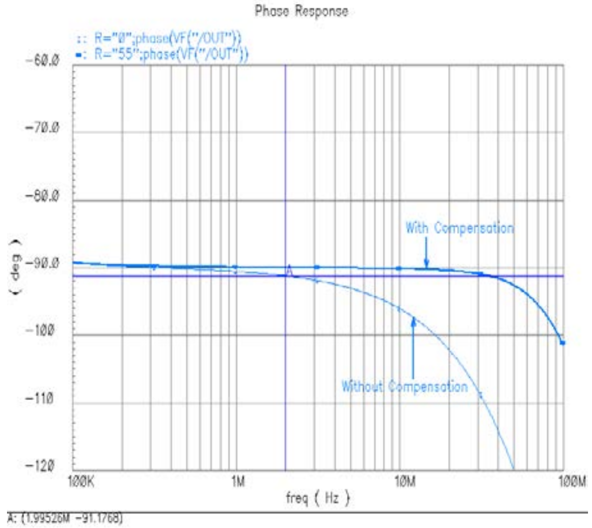
Transconductance	DC Gain	GBW	Bias Current	Power Consumption	Active Area
376.52μA/V	64.1dB	263MHz	40μA	792μW	727μm^2

Jump to first page

Simulation Results of the Lossless Integrator

- The excess phase without any compensation was 1.17°.
- Passive excess phase compensation was used →R=55Ω.





Magnitude Response

Phase Response

Design of Biquadratic Bandpass Filter

- To reuse the designed OTA:

$$g_{m3} = g_{m2} = 376.52\,\mu A/V$$

$$C_1 = C_2 = 30\,pF$$

$$g_{m4} = \frac{C_1}{Q_0}\omega_0 = 23.827\,\mu A/V$$

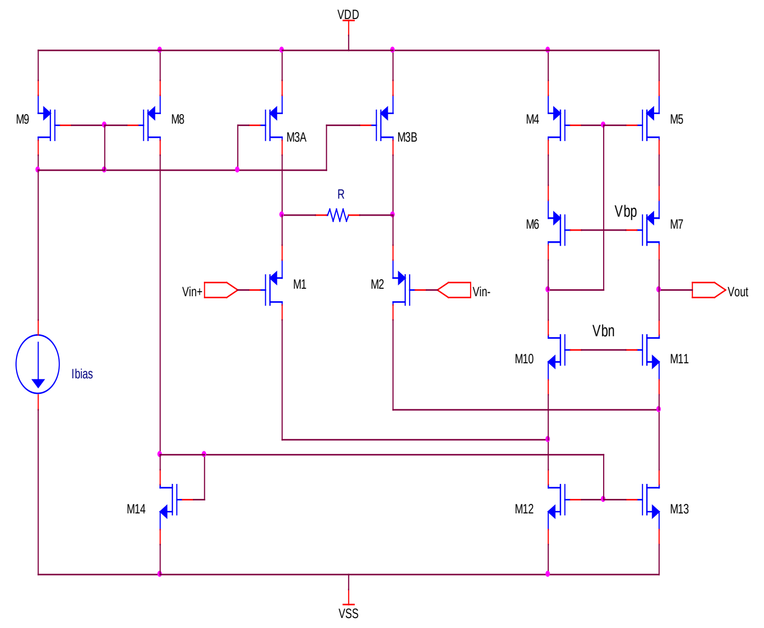
- Transconductance g_{m1} depends on K_i

$$g_{m1} = K_i g_{m4}$$

- For demonstrations purposes, $g_{m1}=g_{m4}$, i.e. K=1

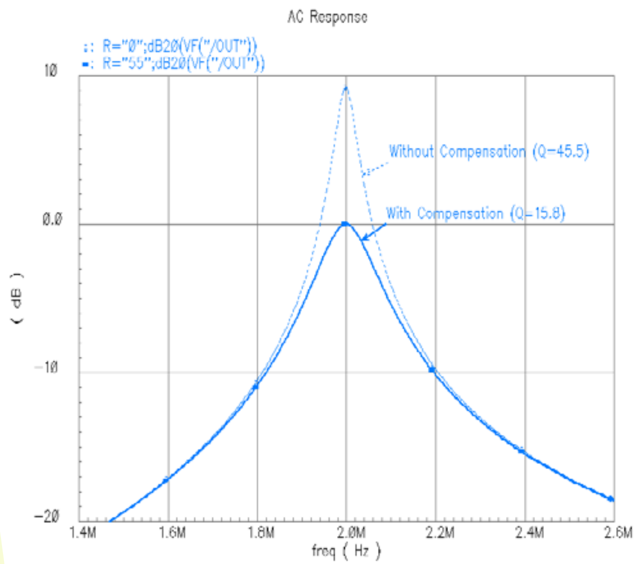
Design of Biquadratic Bandpass Filter

- Due to the relatively small transconductance required, source degeneration was used. Also, a PMOS differential pair was more suitable.

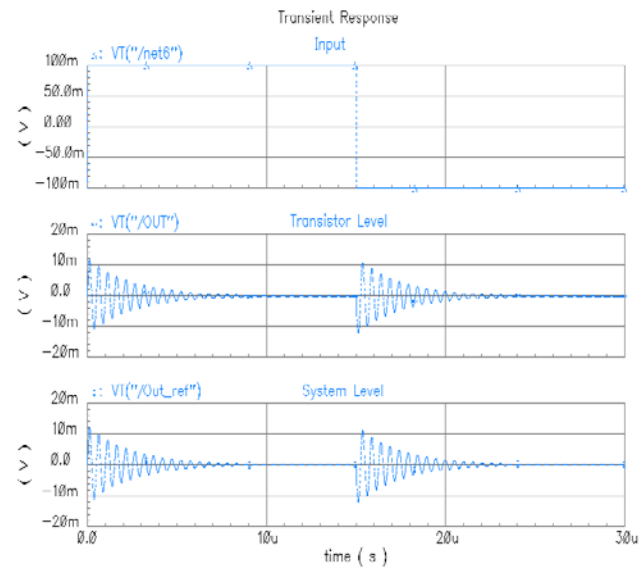


Transconductance	DC Gain	GBW	Bias Current	Power Consumption	Active Area
23.827μA/V	61.15dB	62.3MHz	14μA	277.2μW	820μm^2

Simulation Results of the Biquadratic Bandpass Filter



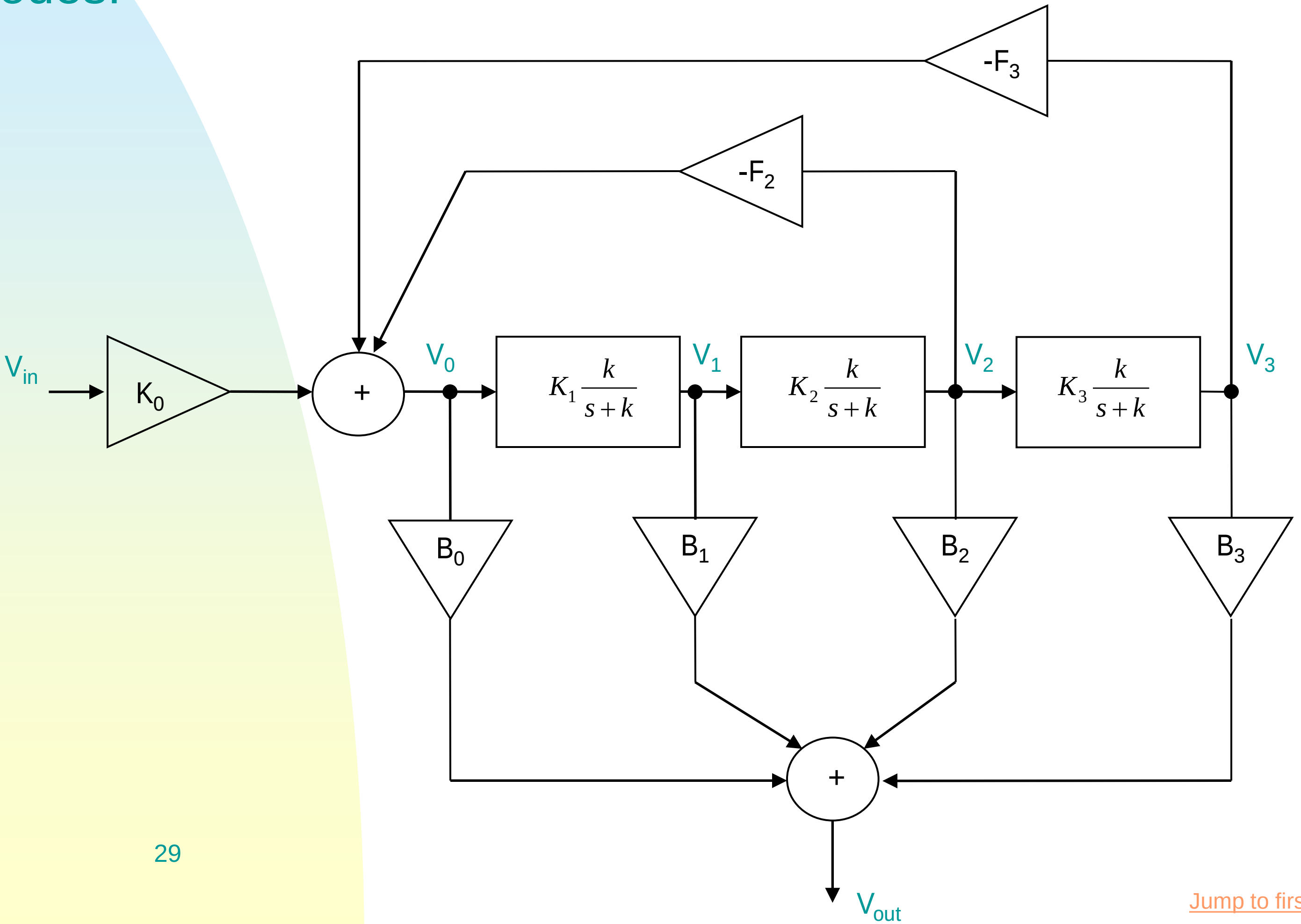
Frequency Response



Step Response

Summation Nodes

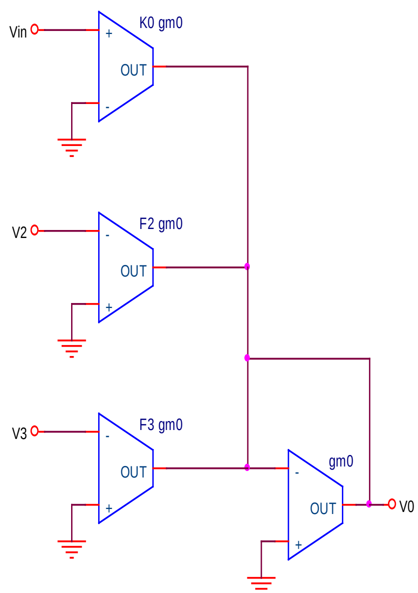
- To complete the transistor-level design, we need two summation nodes:



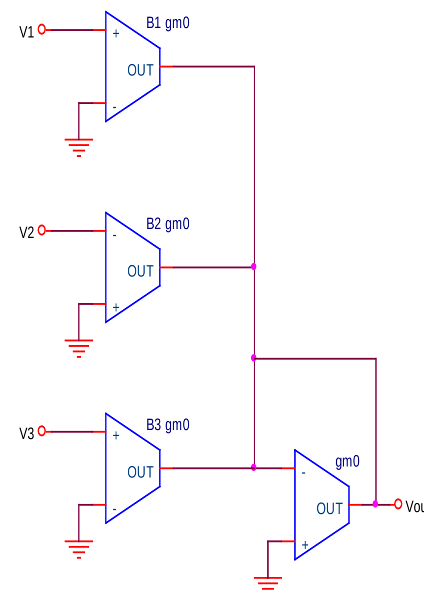
Jump to first page

Summation Nodes

The summation nodes can be implemented with OTAs in the following configurations:



Summation Node for the Feedback Paths



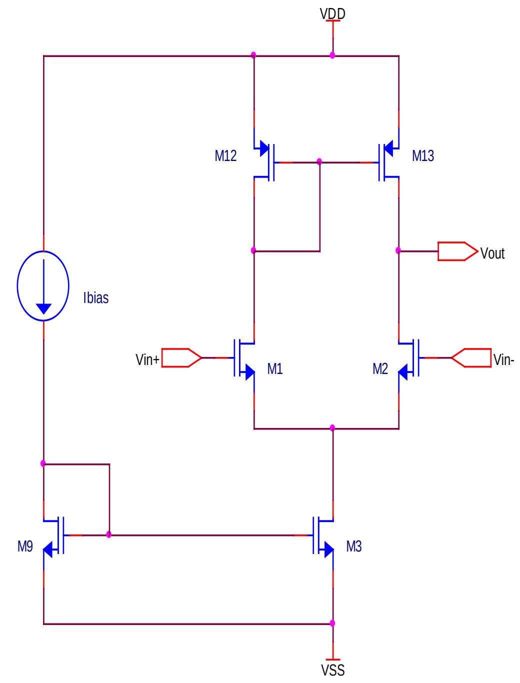
Summation Node at the Output

If g_{m0} is chosen large enough, the output resistance of each OTA does not need to be very high. Excess Phase of OTAs can be a concern.

Jump to first page

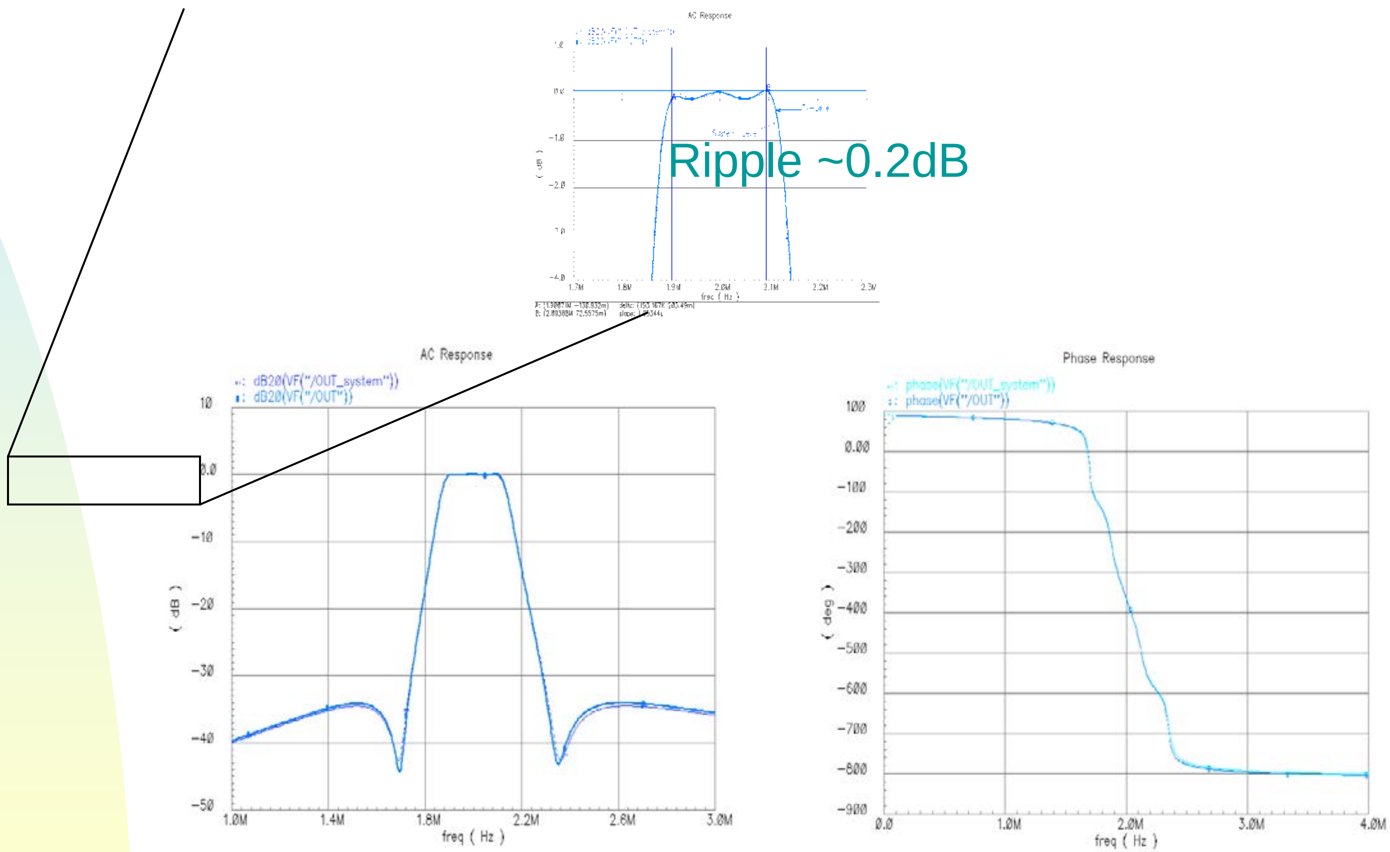
Summation Nodes

- Due to the desired low excess phase introduced by the OTAs, it is more convenient to use a simple differential pair.



Transconductance	Bias Current	Power Consumption	Active Area
300μA/V	40μA	264μW	269μm^2

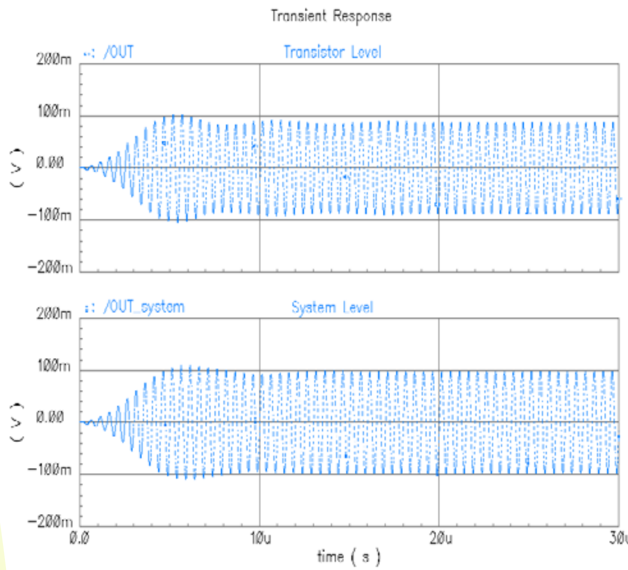
Simulation Results of the Complete FLF Filter (Transistor vs. System Level)

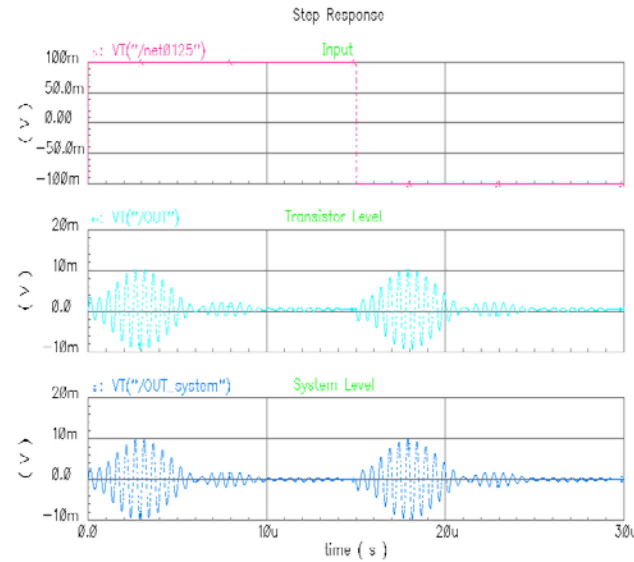


Magnitude Response

Phase Response

Simulation Results of the Complete FLF Filter (Transistor vs. System Level)





Transient Response to a Sinewave

Step Response

Summary of Results

Specification	Value
Passband	1.9MHz to 2.1MHz
Passband Ripple	~0.2dB
Attenuation	≥40dB at 0.6MHz
Power Consumption	8.53mW
Active Area	11,434μm^2
Total Area	~ 211,549μm^2

Problems to be considered

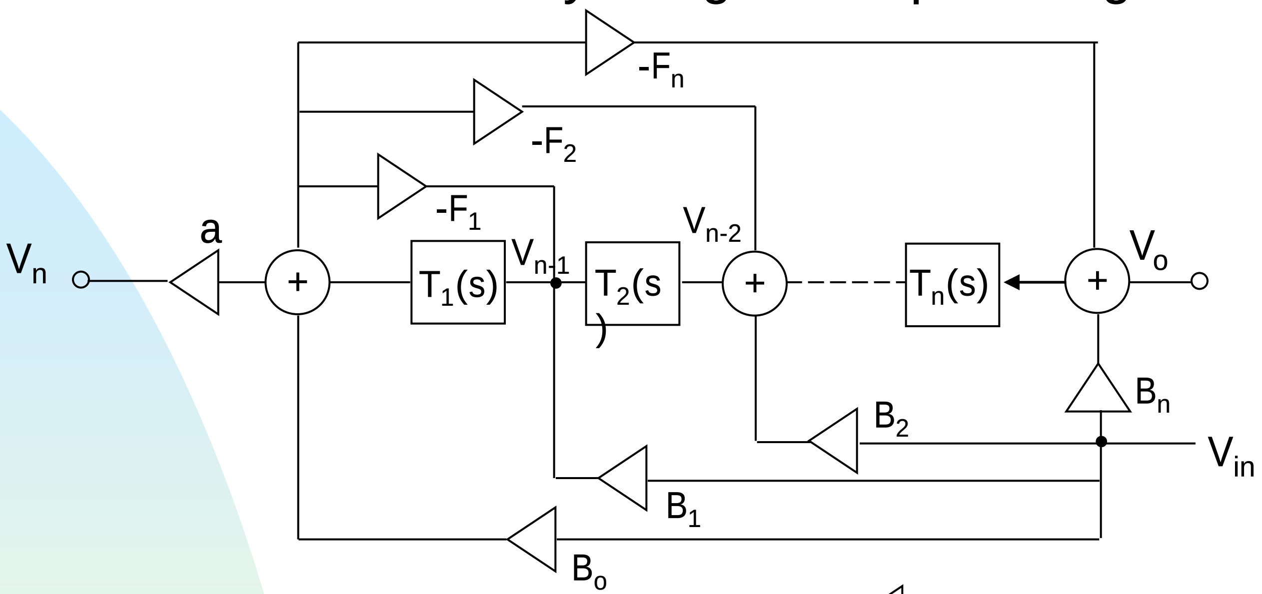
- **Voltage Swing:** The allowable input voltage swing is only 100mV. A small voltage swing is expected, since the OTAs have a small linear range limited by ±VDSAT of the input transistors (in case no linearization technique is used, such as source degeneration or others). Nevertheless, 100mV is too small and is basically because the OTAs with g_m=376.52μA/V use input transistors with a small VDSAT and no linearization technique is being used. I need to redesign these OTAs to increase the linear range.

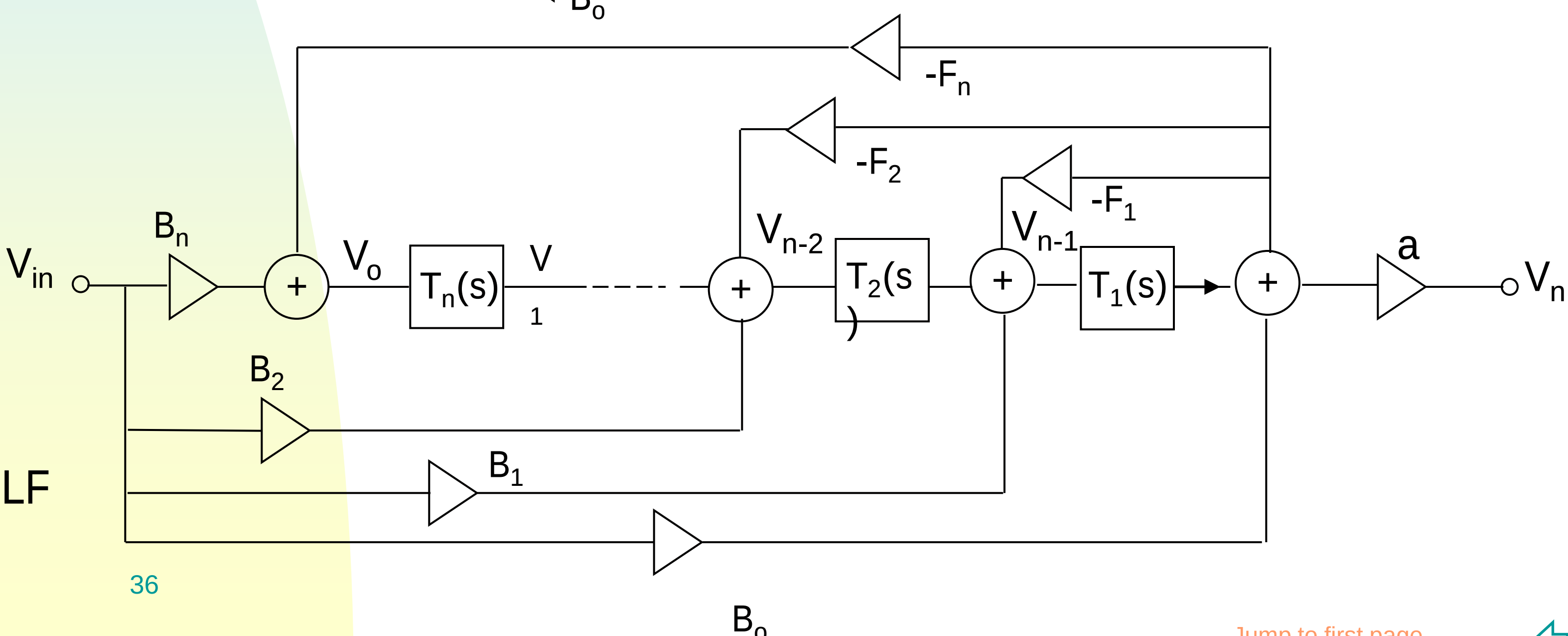
- **Bias Network:** To design the bias network for the folded-cascode OTAs capable of effectively tracking changes of VT due to process variations.

- **Sensitivity and Tunability:** To characterize the complete filter in terms of sensitivity and tunability.

- **Layout**

Another structure for higher-order filters that can be obtained from the FLF is the inverse FLF by using a transposed signal-flow graph.





IFLF

References

[1] Sedra, Brackett. Filter Theory and Design: Active and Passive. Matrix Series in Circuits and Systems. pp. 589-659.

[2] Deliyannis, Sun, Fidler. Continuous-Time Active Filter Design. CRC Press 1999. pp. 151-180.

[3] G. Hurtig, III. The Primary Resonator Block Technique of Filter Synthesis Proc. Int. Filter Symposium, p.84, 1972.

[4] Barbargires. Explicit Design of General High-Order FLF OTA-C Filters. Electronics Letters. 5th August 1999, Vol. 35, No. 16, pp. 1289-1290.

[5] Jie Wu, Ezz I. El-Masry. Synthesis of Follow-the-Leader Feedback Log-Domain Filters. IEEE 1998. 0-7803-5008-1/98. pp. 381-384.

References (cont.)

[6] G. S. Moschytz, “Linear Integrated Networks-Design”, Van Nostrand, Reinhold, New York 1975

[7] R. Schaumann, M.S. Ghaussi, and K.R. Laker, “Design of Analog Filters: Passive, Active RC and Switched Capacitor”, Prentice Hall, Englewood Cliffs, 1990.

[8] A.S. Sedra, and P.O. Brackett, “Filter Theory and Design: Active and Passive”, Matrix, Portland 1978.

[9] W.M. Snelgrove, and A.S. Sedra, “Optimization of Dynamic Range in Cascade Active Filters”, Proc. IEEE ISCAS, pp. 151-155, 1978.