Texas A&M University
Electrical Engineering Department
ELEN 622 Integrated Active Filter Design Methodologies
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HOMEWORK ASSIGNMENT #2

- **PROBLEM 1**

Obtain at least three different magnitude approximations that satisfy the following specifications:

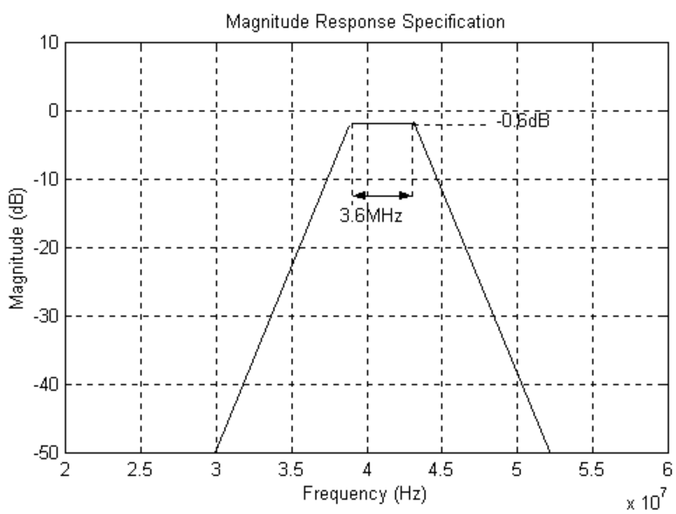

Figure 1

Four different approximations (Butterworth, Chebyshev, Elliptic and Inverse Chebyshev) were obtained for the given specifications using the CAD software FIESTA2. First, the transfer function and main performance plots will be shown for each of the approximations and then, a comparison between the four different approaches will be presented supported by a table and comparison plots.

a) Butterworth Approximation

Transfer Function:

$$\frac{6.801e029\, s^4}{s^8 + 7.504e7\, s^7 + 2.678e17\, s^6 + 1.497e25\, s^5 + 2.67e34\, s^4 + 9.918e41\, s^3 + 1.175e51\, s^2 + 2.181e58\, s + 1.925e67}$$

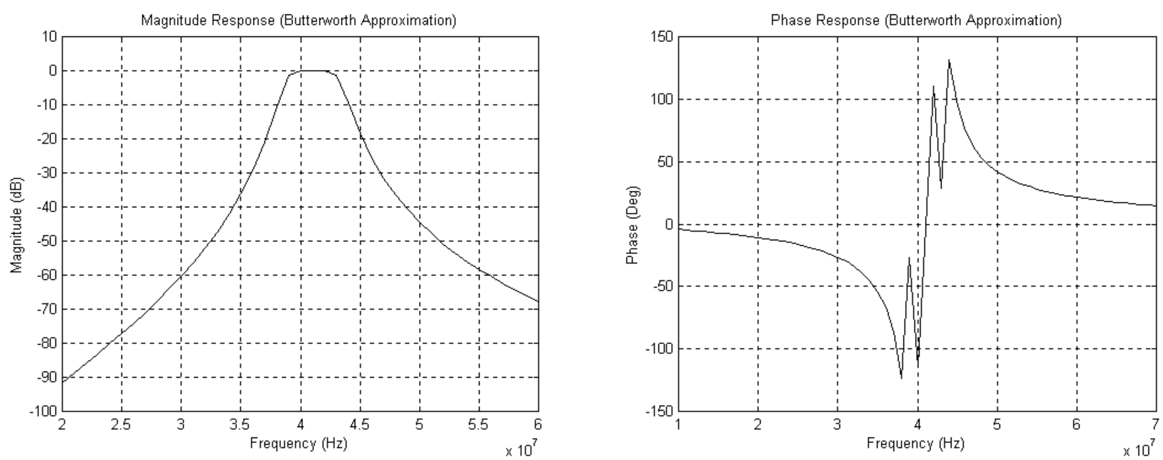

Figure 2

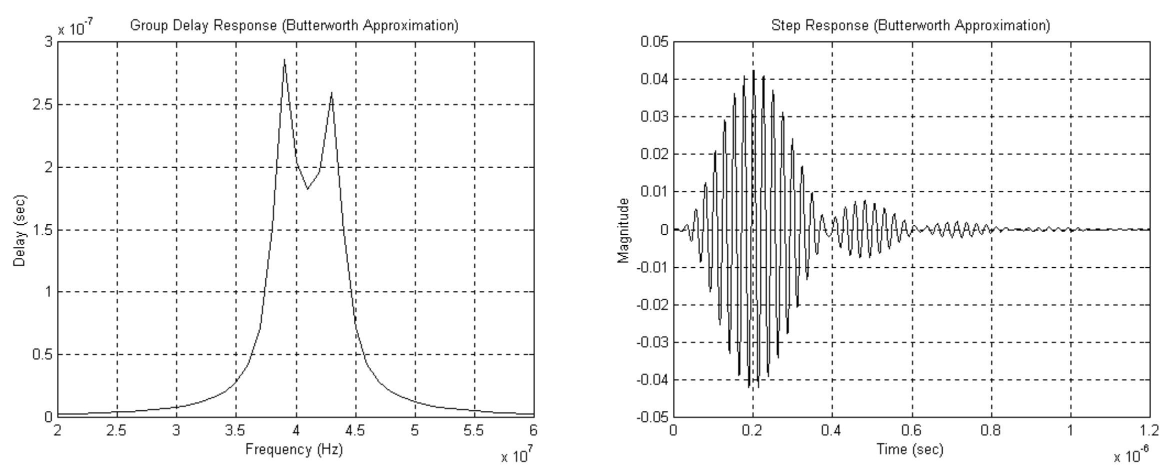

Figure 3

b) Chebyshev Approximation

Transfer Function:

$$\frac{8.501e028\, s^4}{s^8 + 2.561e7\, s^7 + 2.658e17\, s^6 + 5.1e24\, s^5 + 2.643e34\, s^4 + 3.378e41\, s^3 + 1.166e51\, s^2 + 7.442e57\, s + 1.925e67}$$

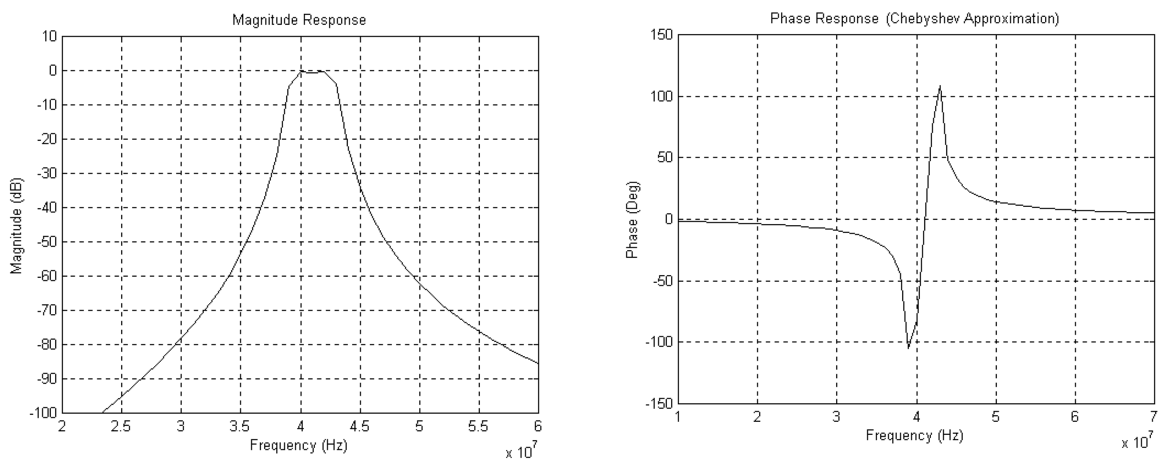

Figure 4

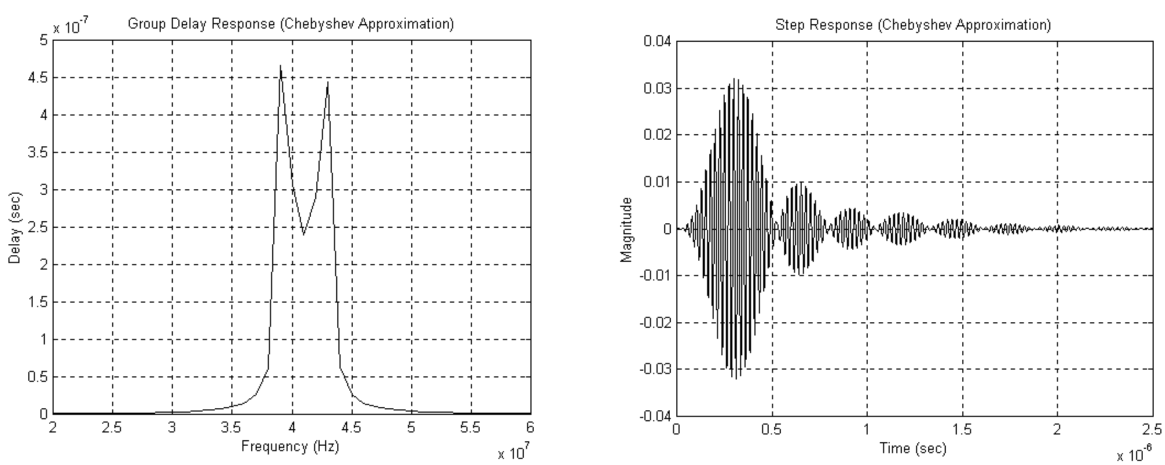

Figure 5

c) Elliptic Approximation

Transfer Function:

$$\frac{3.729e05\ s^5 + 5.701e028\ s^3 + 1.636e039\ s}{s^6 + 2.667e07\ s^5 + 1.994e017\ s^4 + 3.54e024\ s^3 + 1.321e034\ s^2 + 1.17e041\ s + 2.906e050}$$

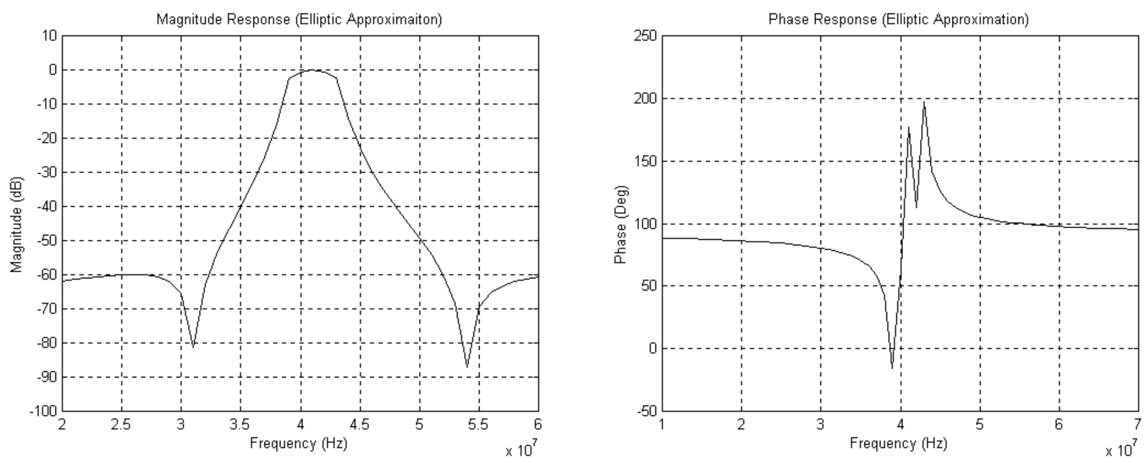

Figure 6

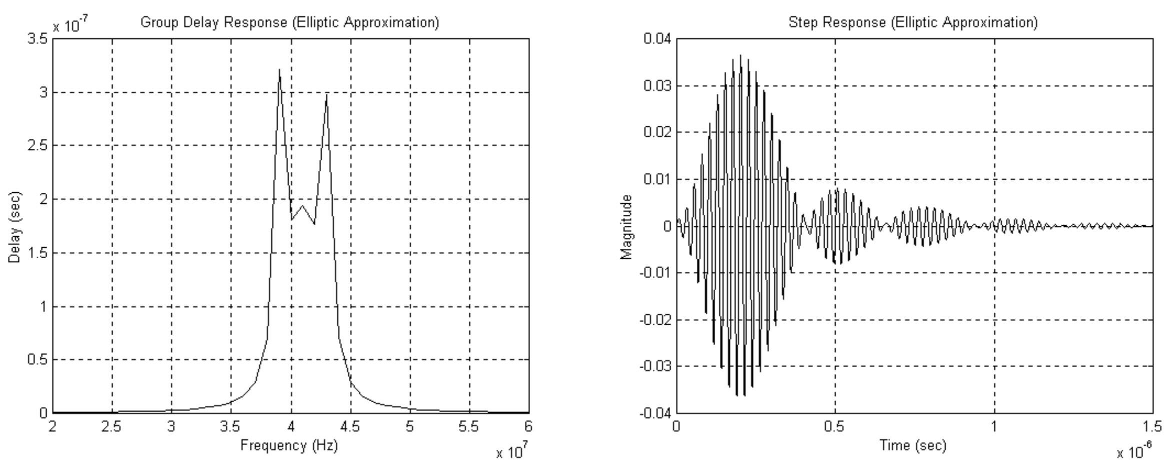

Figure 7

d) Inverse Chebyshev Approximation

Transfer Function:

$$\frac{0.0003719\,s^{8} + 1.443e014\,s^{6} + 1.655e031\,s^{4} + 6.33e047\,s^{2} + 7.158e063}{s^{8} + 7.525e7s^{7} + 2.678e17s^{6} + 1.502e25s^{5} + 2.67e34s^{4} + 9.946e41s^{3} + 1.175e51s^{2} + 2.187e58s + 1.925e67}$$

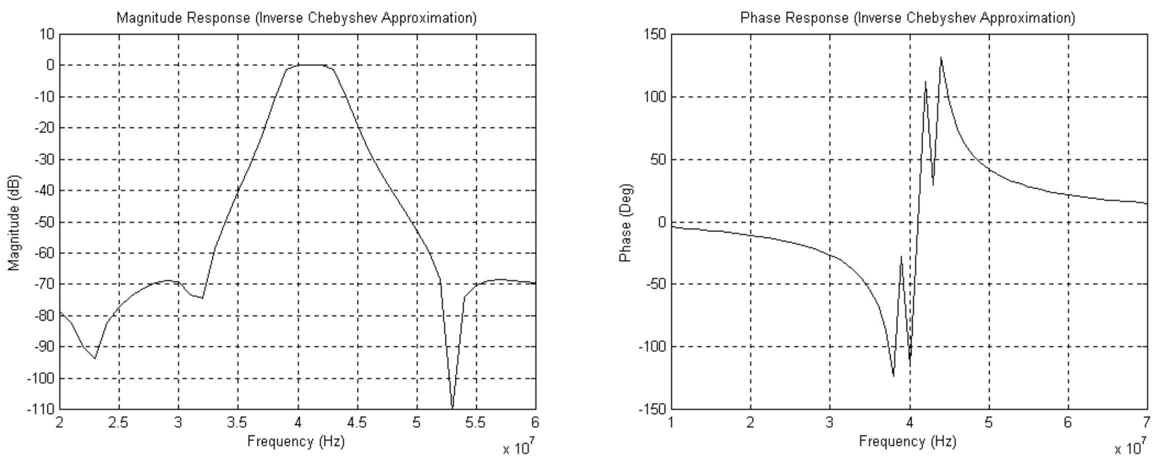

Figure 8

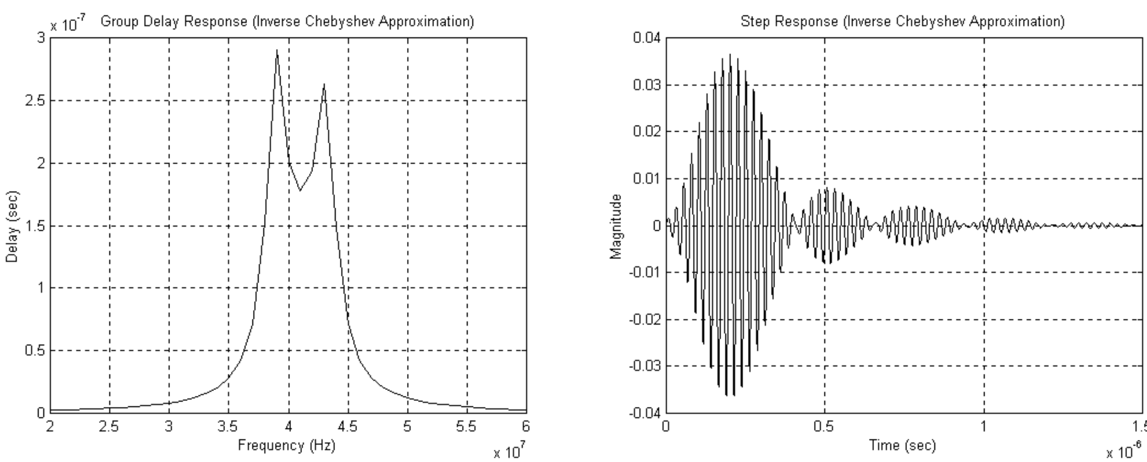

Figure 9

Table 1 shows a comparison between the center frequency, bandwidth and quality factor of each second order block of each one of the four implementations. A graphical comparison of the four different magnitude and group delay responses is shown in figures 10 and 11.

Stage	Butterworth			Chebyshev		
	ω_0	BW	Q	ω_0	BW	Q
1	2.63E+08	2.71E+07	9.702	2.62E+08	9.22E+06	28.428
2	2.52E+08	2.60E+07	9.702	2.53E+08	8.89E+06	28.428
3	2.71E+08	1.16E+07	23.449	2.69E+08	3.92E+06	68.687
4	2.44E+08	1.04E+07	23.449	2.46E+08	3.58E+06	68.687

Stage	Elliptic			Inverse Chebyshev		
	ω_0	BW	Q	ω_0	BW	Q
1	2.57E+08	1.35E+07	19.036613	2.63E+08	2.75E+07	9.56805
2	2.69E+08	6.86E+06	39.191502	2.52E+08	2.63E+07	9.56805
3	2.46E+08	6.28E+06	39.191502	2.71E+08	1.13E+07	24.033671
4	N/A	N/A	N/A	2.44E+08	1.02E+07	2.40E+01

Table 1

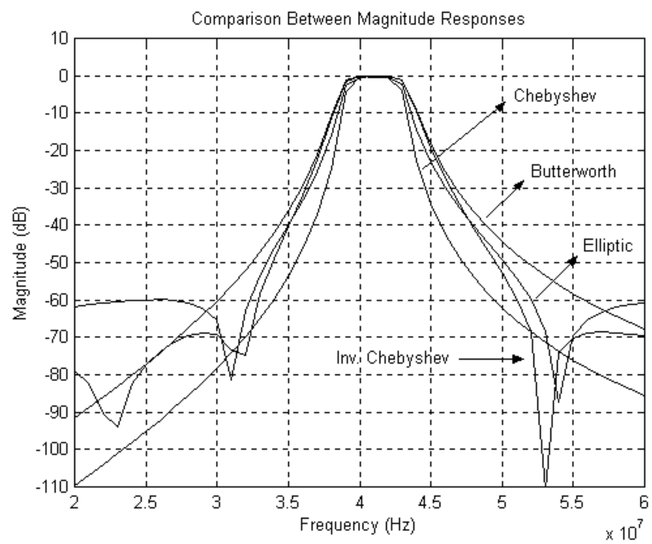

Figure 10

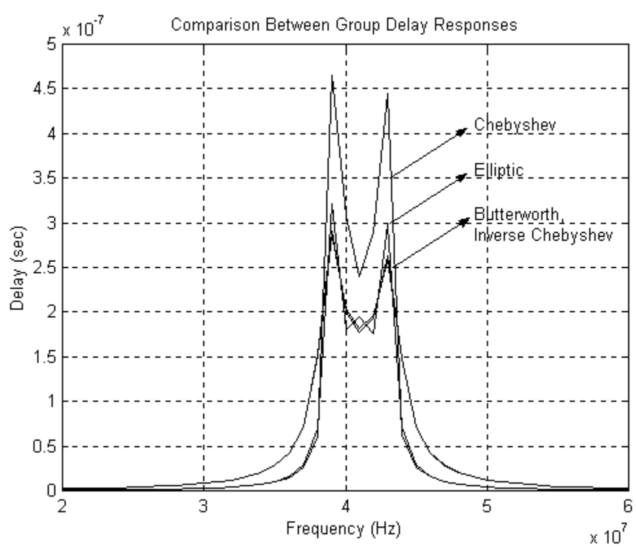

Figure 11

Discussion:

- ✓ The Chebyshev approximation yields the best response in terms of sharpness and out-of-band rejection but has several disadvantages: it requires the highest quality factors in the second order blocks, has the larger settling time and the largest group delay in the pass band.

- ✓ The Butterworth and Inverse Chebyshev approximations are the easiest to implement in terms of the required quality factors. Their in-band group delay response is very similar and smaller than the response of the other two approaches. The out of band rejection of the Inverse Chebyshev approximation is better than the one obtained with the Butterworth but the settling time of the former is slightly larger (around 10%) than the one of the later.

- ✓ The Elliptic approximation requires one stage less (3 in total) than the others. Its in-band group delay is slightly greater than the one of the Butterworth approach and, both, its step response and out-of-band rejection are very similar to the ones of the Inverse Chebyshev approximation. The required quality factors are not the smallest but not as high as the ones demanded by the Chebyshev approach.

- **PROBLEM 2**

Obtain the group delay approximations: Bessel and Equal-Ripple that meet the following specifications:

- **Ripple less or equal to 1%**
- **Group Delay less or equal to 3.2 nsec**
- **Cutoff Frequency = 150MHz**

The approximations obtained with FIESTA 2 are presented and compared below.

a) Bessel Approximation

Transfer Function:

$$\frac{s^4 - 6.667e09s^3 + 2e019s^2 - 3.111e028s + 2.074e037}{s^4 + 6.667e09s^3 + 2e019s^2 + 3.111e028s + 2.074e037}$$

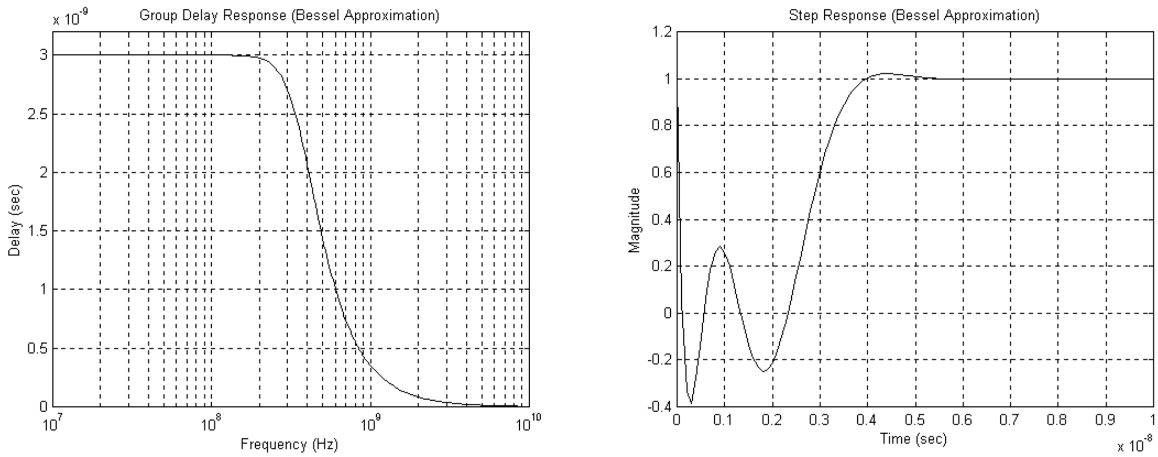

Figure 12

b) Equal Ripple

Transfer Function:

$$\frac{s^6 - 1.481e011s^5 + 5.928e021s^4 - 3.365e031s^3 + 9.344e040s^2 - 1.432e050s + 9.529e058}{s^6 + 1.481e011s^5 + 5.928e021s^4 + 3.365e031s^3 + 9.344e040s^2 + 1.432e050s + 9.529e058}$$

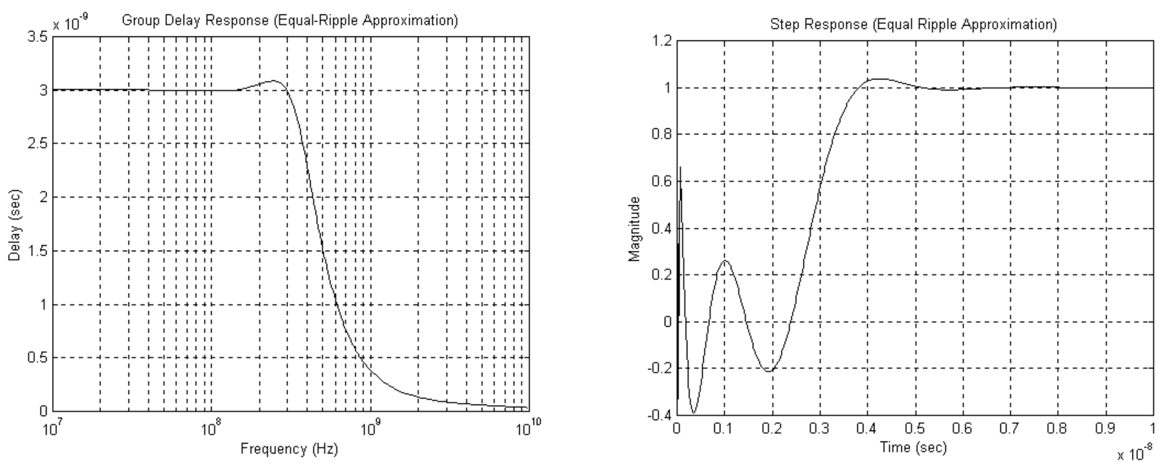

Figure 13

The characteristics of the required second order stages are shown in table 2. The Bessel approximation appears to be better in this case since it uses one stage less, has slightly less ripple and there is practically no difference between the two step responses.

Stage	Bessel		Equal Ripple	
	ω_0	Q	ω_0	Q
1	2.02E+09	0.521935	1.96E+09	0.511379
2	2.26E+09	0.805538	7.09E+10	0.500068
3	N/A	NA	2.22E+09	0.945436

Table 2

- **PROBLEM 3**

a) Design a 2nd order LP Chebyshev for $\omega_0 = 2\pi X10^3$ rad/s and 1dB ripple

Fiesta2 yields the following approximation for the given specifications:

DC GAIN	8.912509e-01
PASS/CENTER FREQUENCY	6.597375e+03 (rad/s)
Q	0.956520
POLE =	-3.448634e+03+j(5.624259e+03) -3.448634e+03+j(-5.624259e+03)
ZERO =	Inf, Inf
TRANSFER FUNCTION	$\dfrac{3.879e007}{s^2 + 6897 \cdot s + 4.353e007}$

Table 3

The corresponding magnitude and group delay responses are shown in figure 14

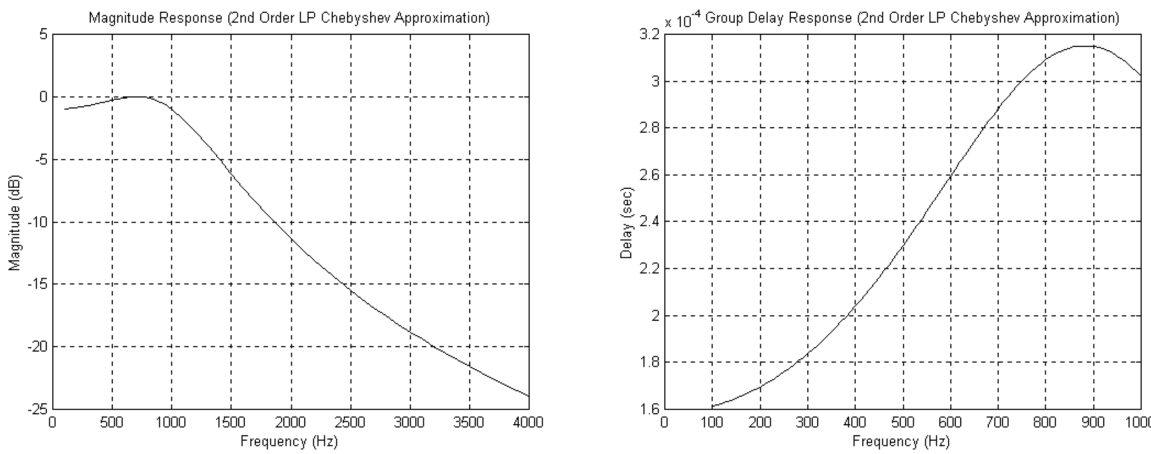

Figure 14

b) Add a phase equalizer to the LP to yield a constant group delay in the range of 0 to 800Hz

After a trial and error process using the Non-Conventional Group Delay Approximation feature of FIESTA2, it was determined that the following specifications yield convergence to an equalizer that satisfactorily compensate the group delay response showed in figure 14:

Predistortion percentage:	40.000000		
Relaxation factor:	0.100000		
Constraints:	Frequency: 100	Min: 0.460e-03 sec	Max: 0.580e-03 sec
	Frequency: 800	Min: 0.300e-03 sec	Max: 0.420e-03 sec

As it can be observed, the intention is to obtain a 0.16ms difference in the delay at 800Hz with respect to the delay at 100Hz. In order to achieve convergence, all of the accuracy parameters had to be relaxed to the maximum and the Min-Max interval for the specified points had to be broadened. The parameters of the obtained 2nd order equalizer are shown in table 4.

DC GAIN	1.000000e+00
PASS/CENTER FREQUENCY	7.741775e+03 (rad/s)
Q	0.500123
POLE =	-7.739873e+03+j(1.715783e+02) -7.739873e+03+j(-1.715783e+02)
ZERO =	7.739873e+03+j(1.715783e+02) 7.739873e+03+j(-1.715783e+02)
TRANSFER FUNCTION	$\frac{s^2 - 1.548e004\,s + 5.994e007}{s^2 + 1.548e004\,s + 5.994e007}$

Table 4

Both, the group delay response of the equalizer and the group delay response of the overall transfer function (LP cascaded with equalizer) are shown in figure 15. Since the equalizer has an (ideally) all pass magnitude response, the overall magnitude response is unaltered with respect of the standalone LP.

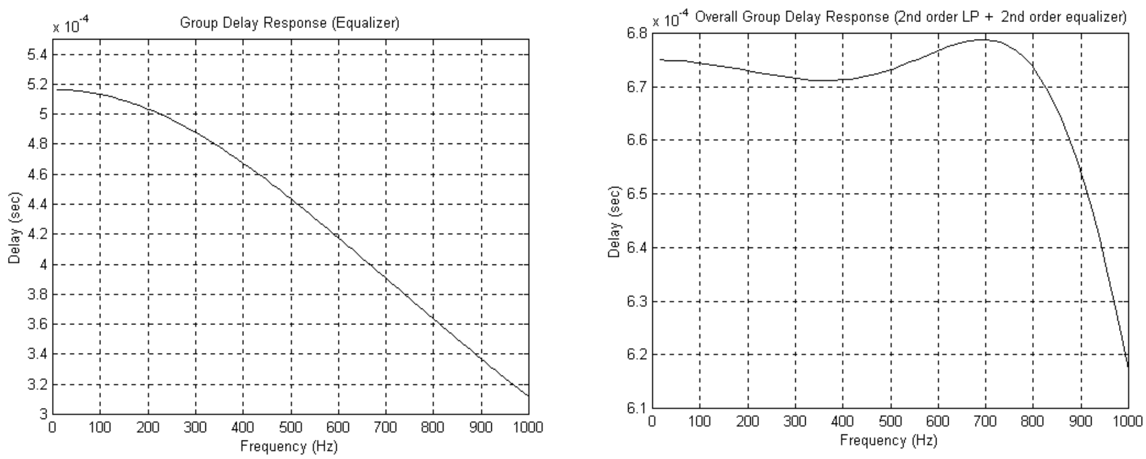

Figure 15

It is important to note on figure 15 (left) that the difference in the delay response of the equalizer from 100Hz to 800Hz is close to 0.16ms as intended. The overall group delay in the 0 to 800Hz range is around 0.675ms with a ripple of less than 0.01 ms, which corresponds to **only 1.5%.**

It is worth to mention that by using the same approximation constraints, but targeting a higher constant group delay (i.e. 0.77ms, 0.87ms, etc.), convergence was also obtained but with a higher ripple (absolute and proportional) in the final response and, in some cases, a higher order for the equalizer.

References:

[1] Course Notes
[2] Fiesta2 Manual