Oscillators: Fundamentals and Implementations

- Oscillator is a system that is able to produce independently bounded and permanent oscillations of at least one of the variables that describe it.

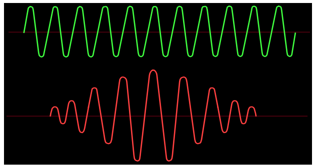
- Types of Oscillators:

- Periodic. This oscillator has an spectrum consisting of a fundamental frequency plus and infinite number of harmonics.
- Pseudo-periodic. The spectrum consists of more than one frequencies not related to each other.
- Chaotic. The spectrum of the response is flat. That is, it contains frequency components of all frequencies.

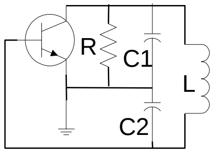
For sinusoidal oscillators a figure of merit is the distortion which is characterized by the Total Harmonic Distortion (THD)

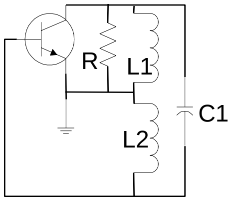
By Edgar Sánchez-Sinencio

SINUSOIDAL OSCILLATORS



There are a number of circuit implementations of sinusoidal oscillators. In traditional communication circuits oscillators use one transistor, R's, C's and sometimes inductors. These filters were often used in RF circuits. Current RF oscillators used either ring oscillators or LC tuned oscillators. The last one consists of differential pairs and inductor(s) and capacitor(s)





In these notes we will first focus more on oscillators based on *state-variable structures* (SVS). We will discuss several implementations and the design procedures.

SINUSOIDAL OSCILLATORS

A sinusoidal oscillator is composed of:

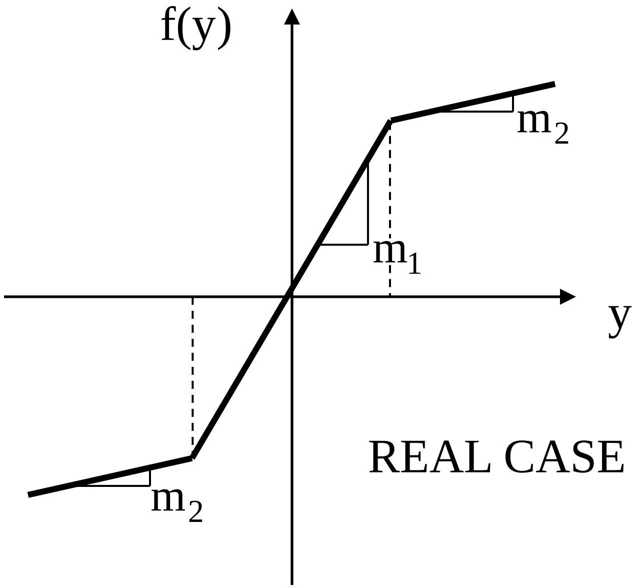
1. A linear circuit that sets the oscillation frequency.
2. An active element that gains power at the frequency of oscillation.
3. A nonlinear mechanism to stabilize the amplitude.

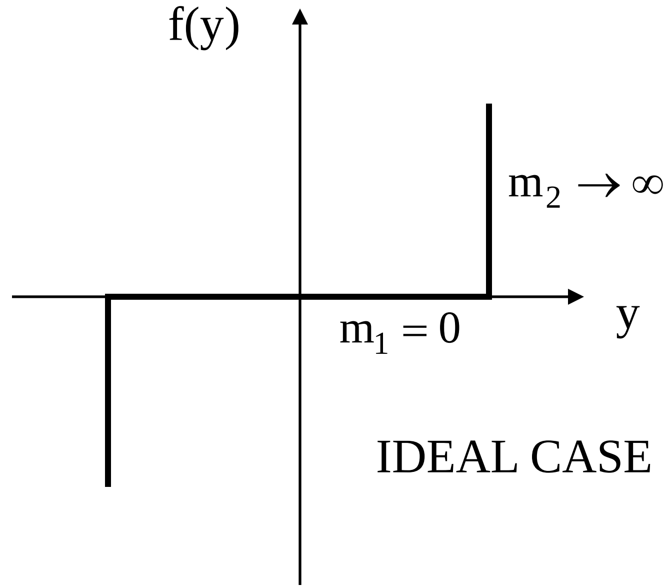
Nonlinear Mechanism

<u>Static</u>, when its input-output characteristic is fixed in time, as in a hard limiting device or an amplifier nonlinear gain.

<u>Dynamic</u>, when its characteristic changes in time as in an AGC.

Examples of Nonlinear (static) Characteristics





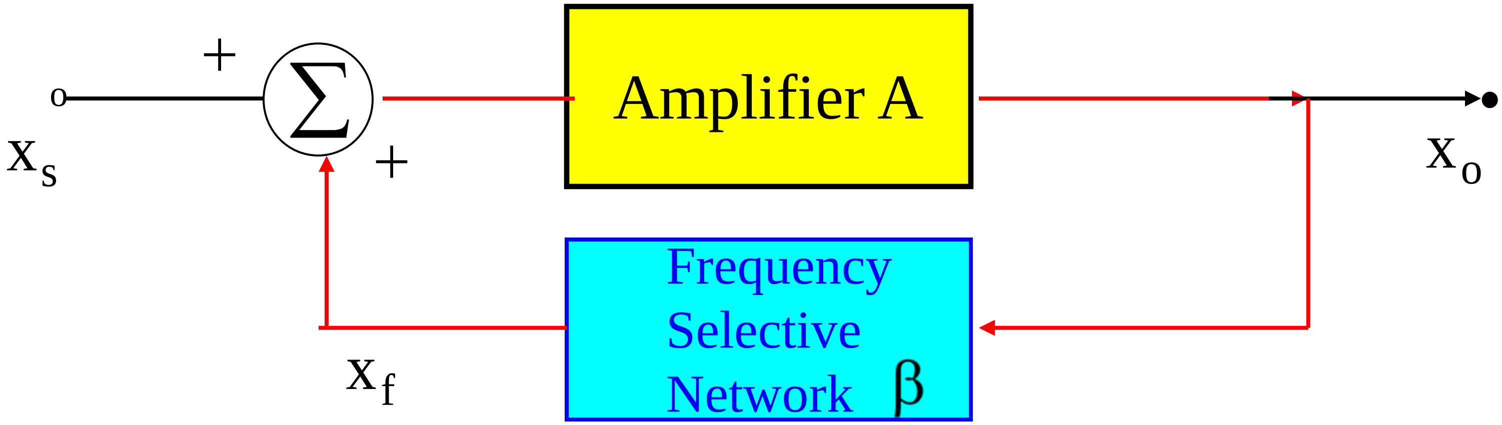
There are a number of topologies yielding sinusoidal oscillators. In our discussion we will first present two popular types in frequency ranges below hundreds of MHz:

i) **Quadrature Oscillator**

ii) **Bandpass - based**

OSCILLATORS

Fundamentals. Linear Aspects: Oscillation conditions.



$$A_f = \frac{x_o}{x_s} = \frac{A}{1-\beta A} = \frac{A(s)}{1-\beta(s)\,A(s)}$$

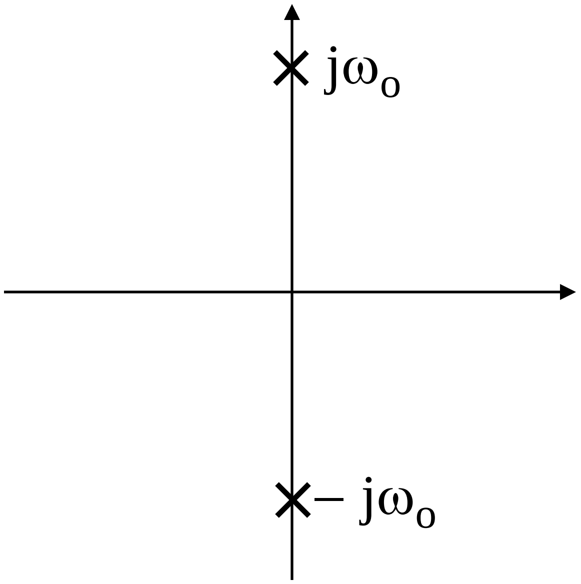
$$D(s) = 1-\beta(s)\,A(s) = 1 - L(s)$$

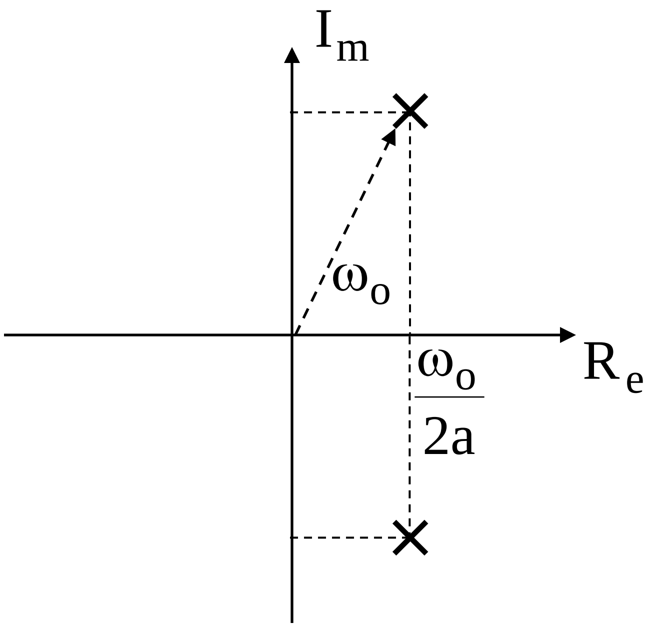
$$L(j\omega_o) \triangleq A(j\omega_o)\,\beta(j\omega_o) = 1$$ **Barkhausen Criteria**

Note that for the circuit to oscillate at one frequency the oscillation criterion should be satisfied at one frequency only; otherwise the resulting waveform will not be a simple sinusoid.

Need of Nonlinear Amplitude Control

Location of poles



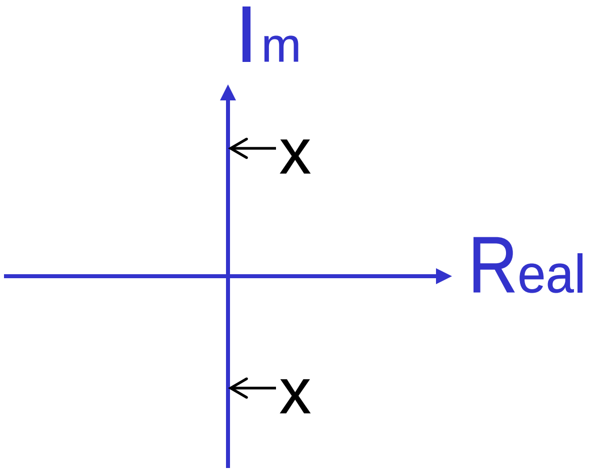


Not practical
why?

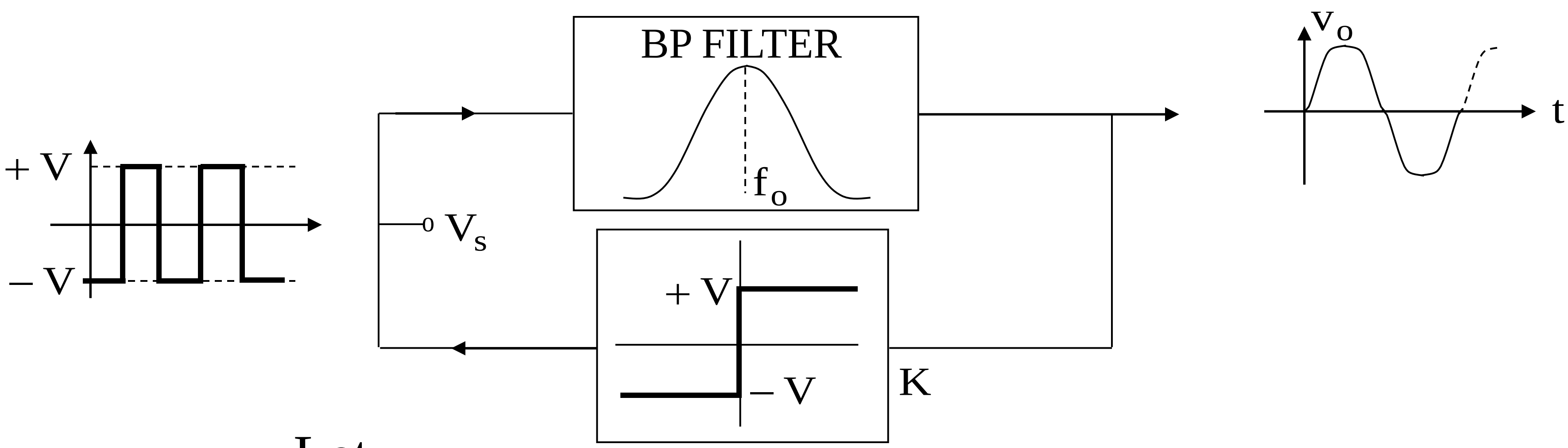
Practical.
How do you push the poles into the $j\omega$ axis?

In the ideal case this is easy to design, but in the real case there a lot of system parameters that can change the poles due to their physical characteristics.

Therefore, the poles should not lie on the imaginary axis. One should design the poles so that they lie in the right half of the s-plane. This creates positive feedback and guarantees that the circuit will oscillate. How far should we place the poles ?



BP Based Oscillator



Let

$$H_{BP}(s) = \frac{K_1 s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

$$D(s) = 1 - K H_{BP}(s)$$

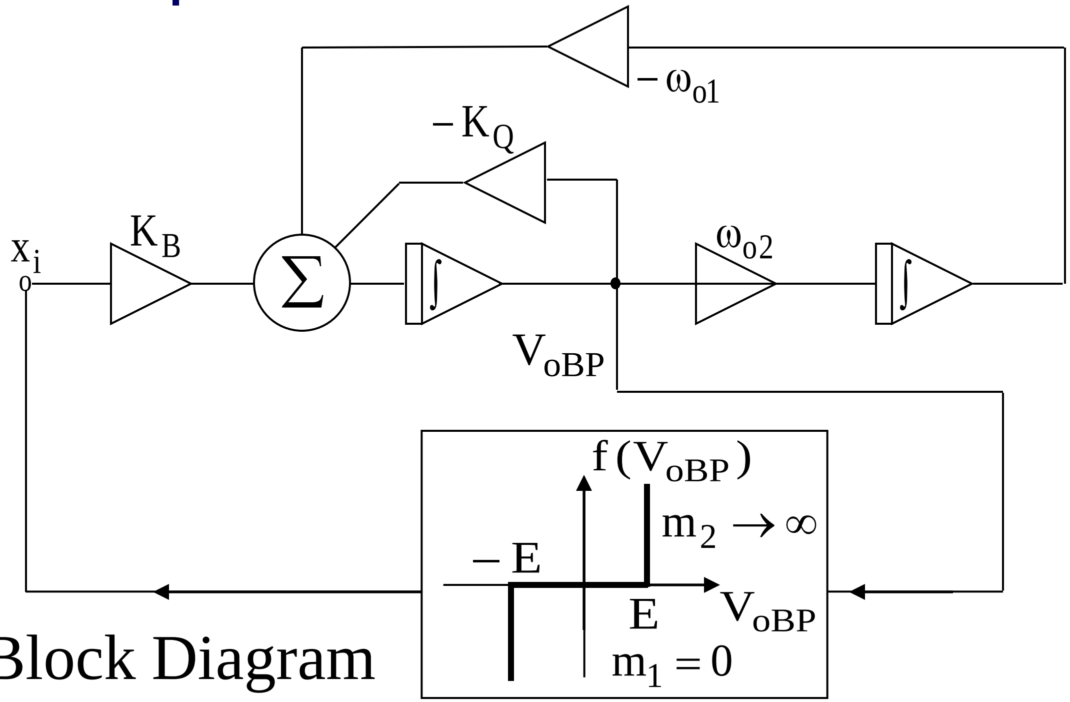
$$\left(s^2 + \frac{\omega_o}{Q} s + \omega_o^2\right) D(s) = s^2 + \frac{\omega_o}{Q} s + \omega_o^2 - K K_1 S$$

Characteristic Equation

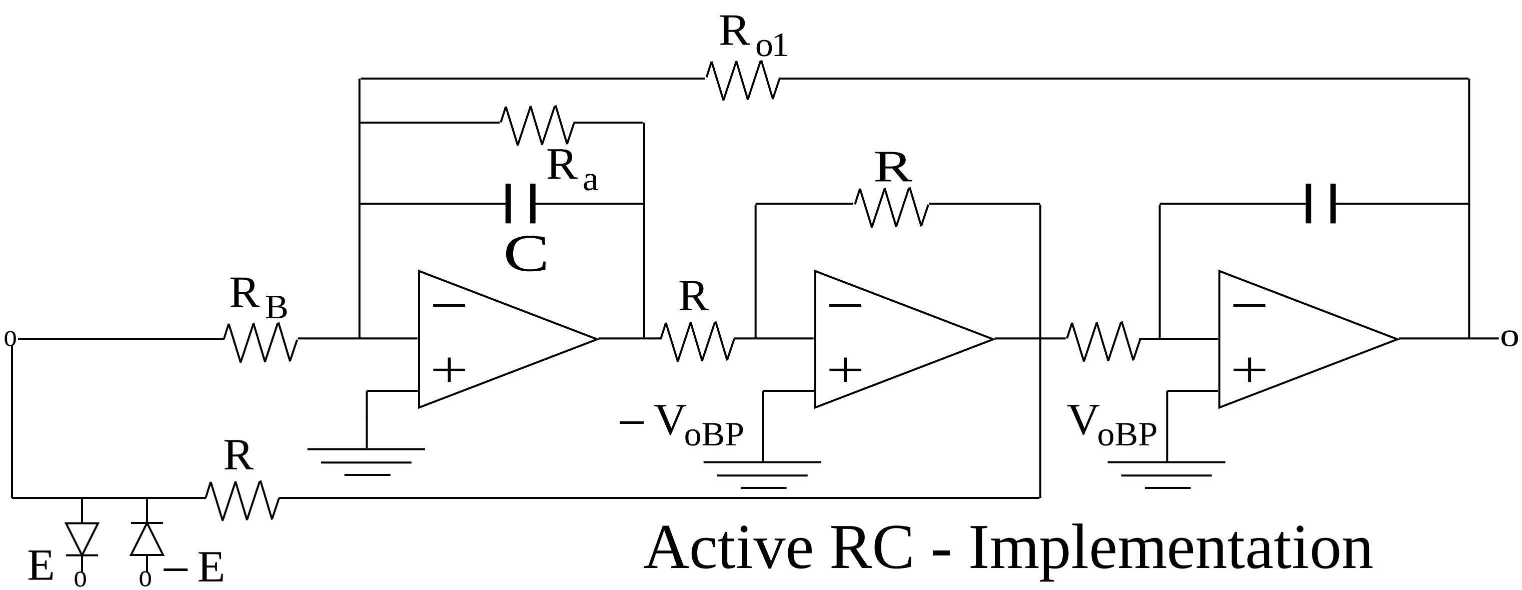
$$s^2 - s\left(K K_1 - \frac{\omega_o}{Q}\right) + \omega_o^2$$

$$K K_1 > \frac{\omega_o}{Q}$$ i.e. positive feedback

A bandpass based Sinusoidal Oscillator

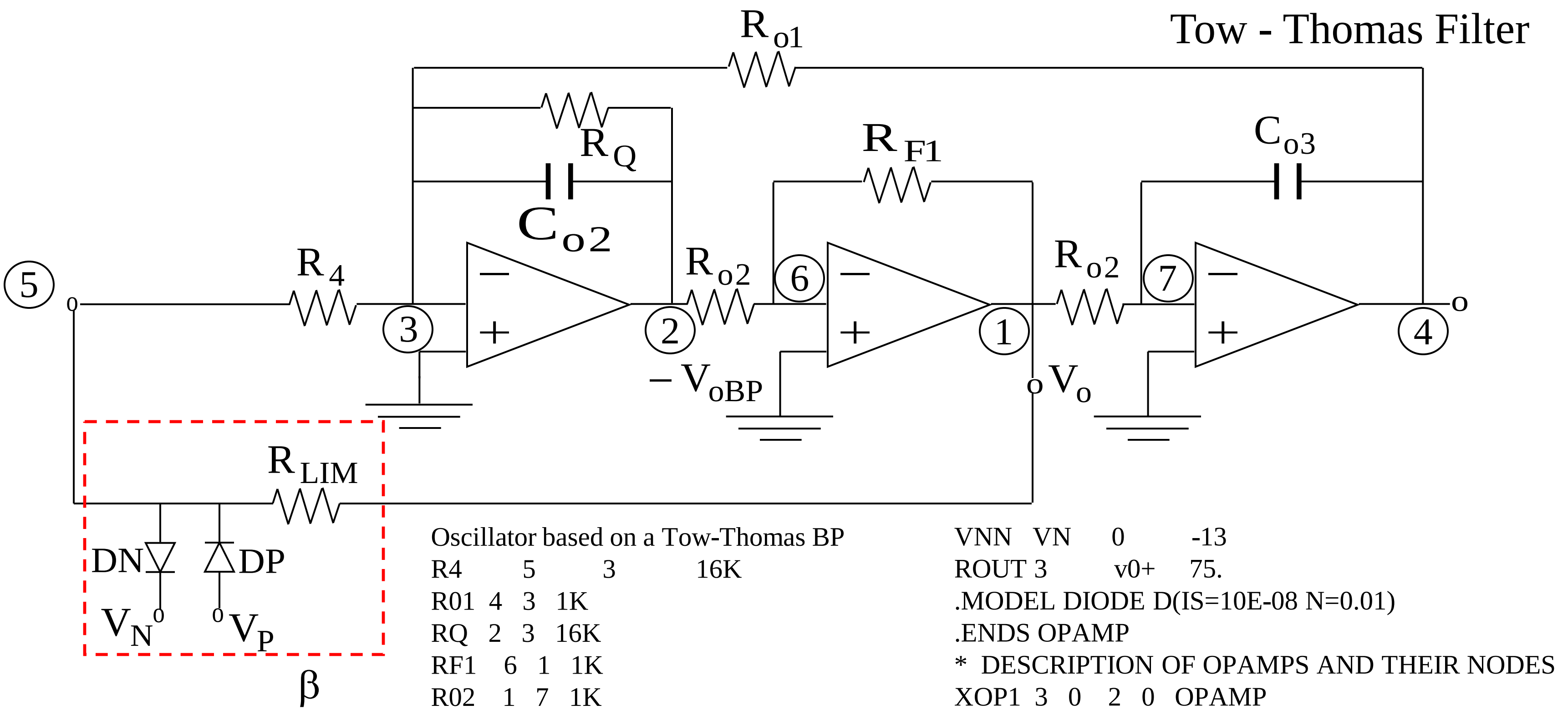


Block Diagram

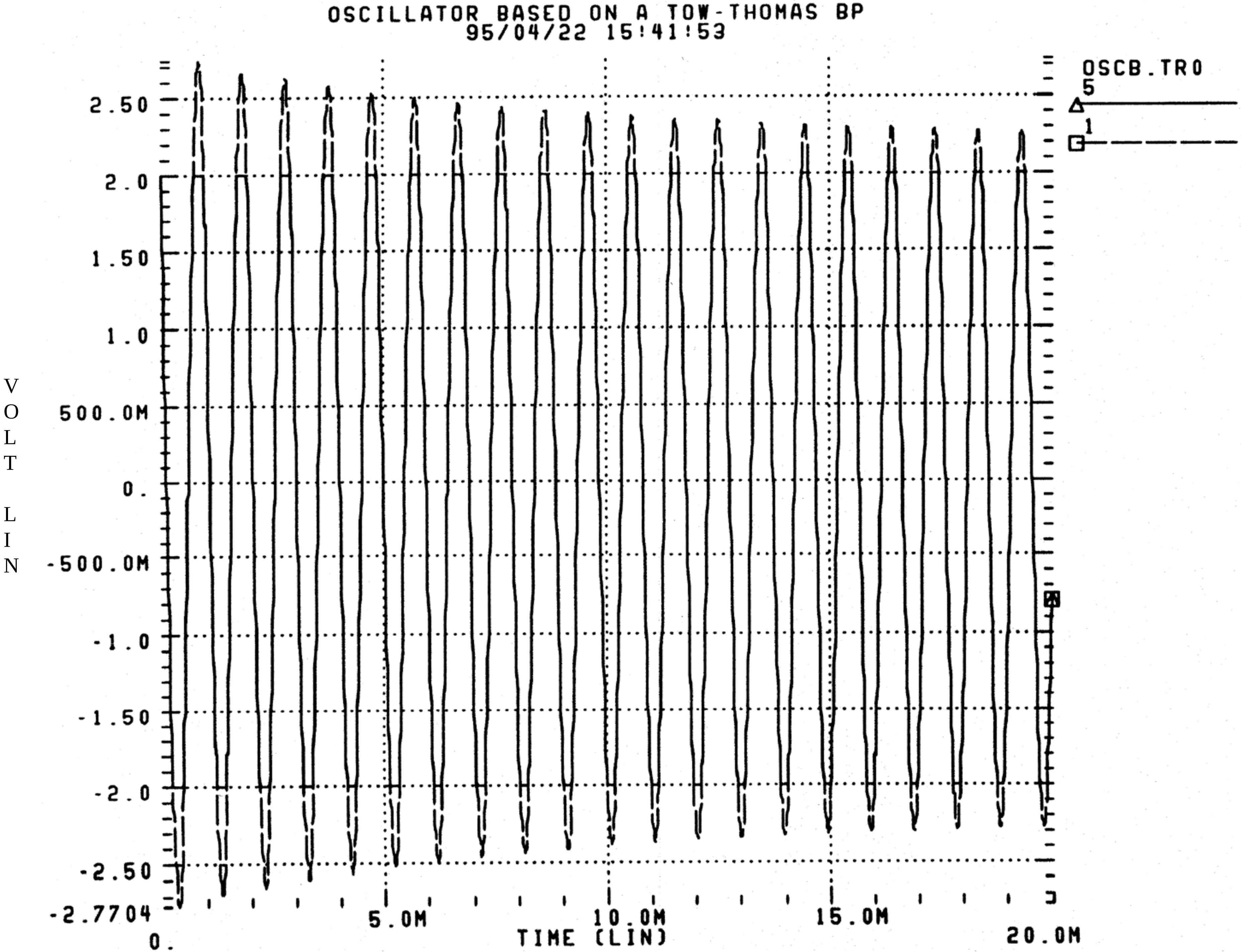


Active RC - Implementation

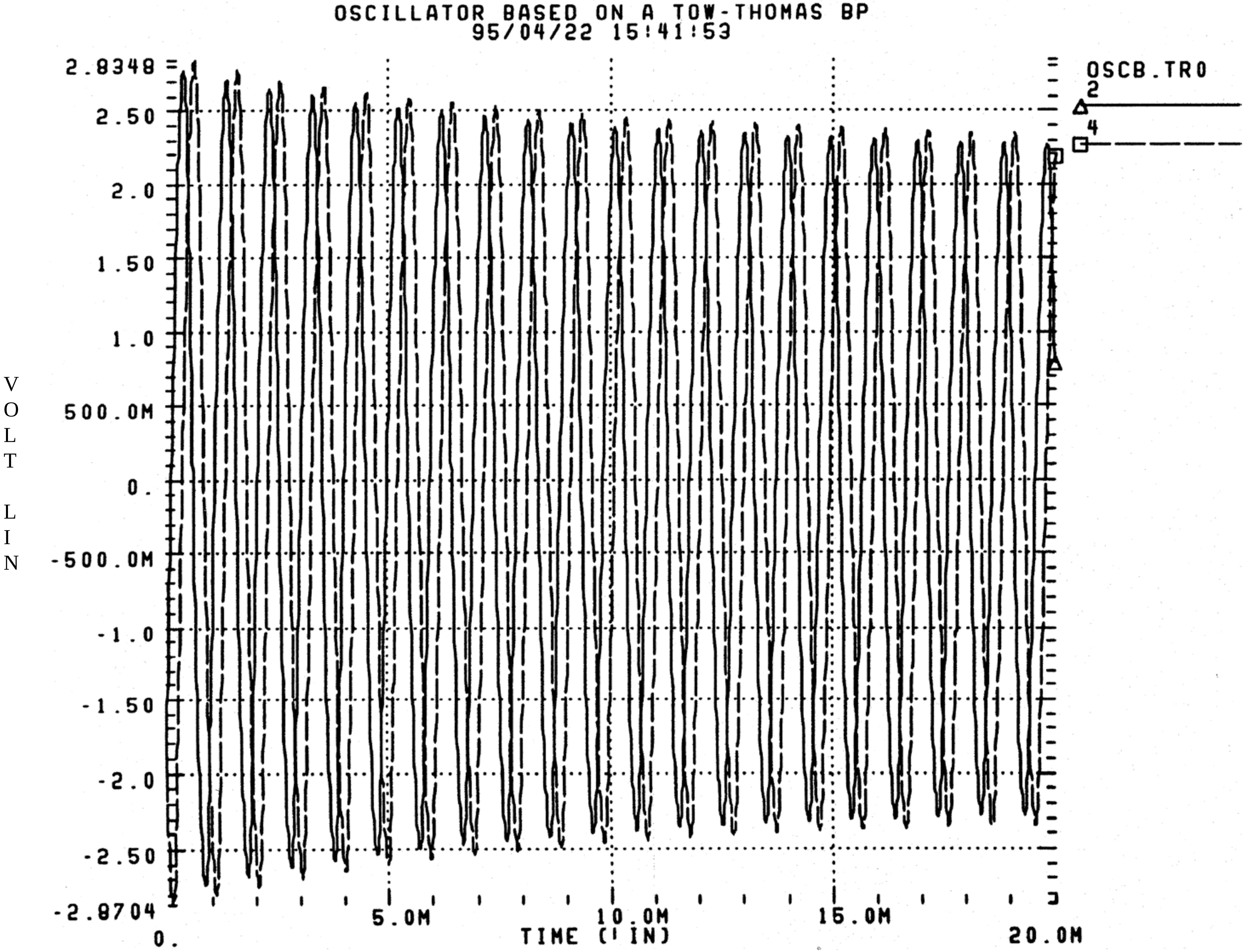
BP Oscillator




```
Oscillator based on a Tow-Thomas BP
R4       5       3       16K
R01  4   3   1K
RQ   2   3   16K
RF1    6   1   1K
R02    1   7   1K
C02    3   2         0.1591549U
R03        2       6            1K
C03    7   4    0.1501549U
.SUBCKT OPAMP vi-   vi+   v0+   v0-
*
*
RIN     vi-    vi+     10E6
EIN     1      0       vi+     vi-     2E05
R3DB  1        2       1K
C3DB  2        0       0.159E-04
EOUT  3       VO-     2        0        1
DP       2      VP      DIODE
DN      VN      2       DIODE
VPP    VP      0       13
VNN   VN     0       -13
ROUT 3       v0+    75.
.MODEL DIODE D(IS=10E-08 N=0.01)
.ENDS OPAMP
*  DESCRIPTION OF OPAMPS AND THEIR NODES
XOP1  3   0    2   0   OPAMP
XOP2    6    0     1     0      OPAMP
XOP3    7    0     4     0      OPAMP
*LIMITER CIRCUIT DESCRIPTION
RLIM       1          5            200
DP          5          VP           DL
DN          VN         5            DL
.MODEL   DL         D (IS=10E-09 N=0.0001)
VNL        VN         0            -2
VPL        VP         0            2
.TRAN      5U         20m          UIC
.FOURIER              57           V(1)       V(2)       V(4)
.IC  V(1)=2  V(2)=-2  V(4)=-2  V(5)=2
.OPTIONS POST
.END
```

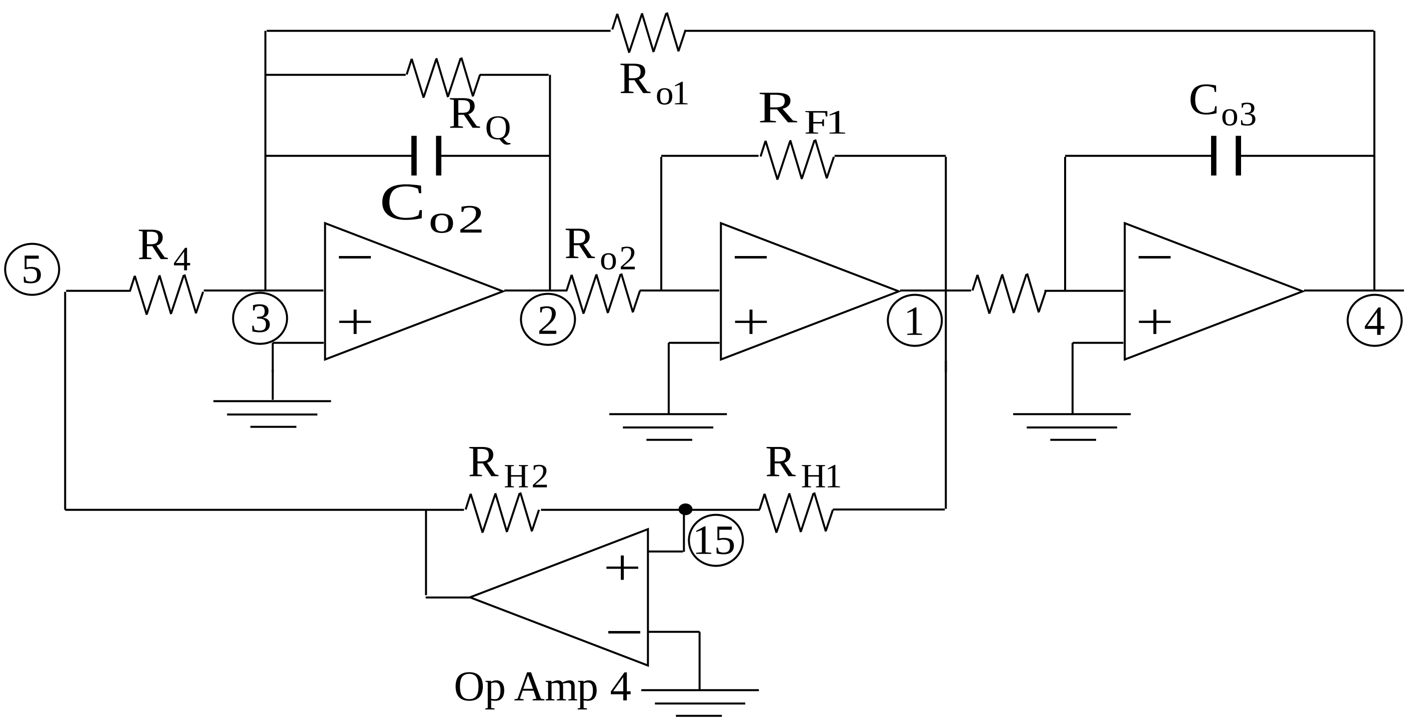


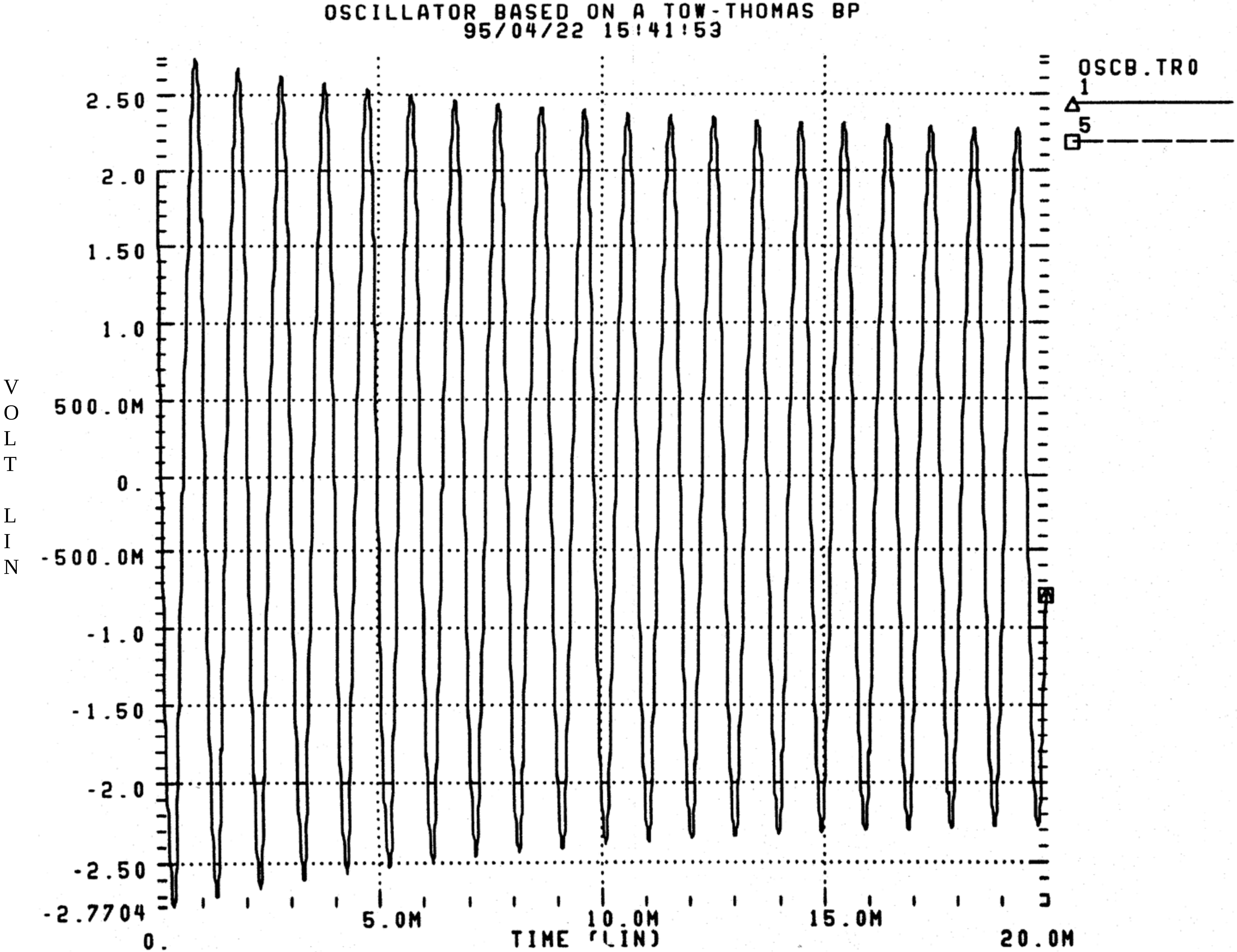
OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
5
1
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.7704
VOLT LIN
0.
5.0M
10.0M
15.0M
20.0M
TIME (LIN)

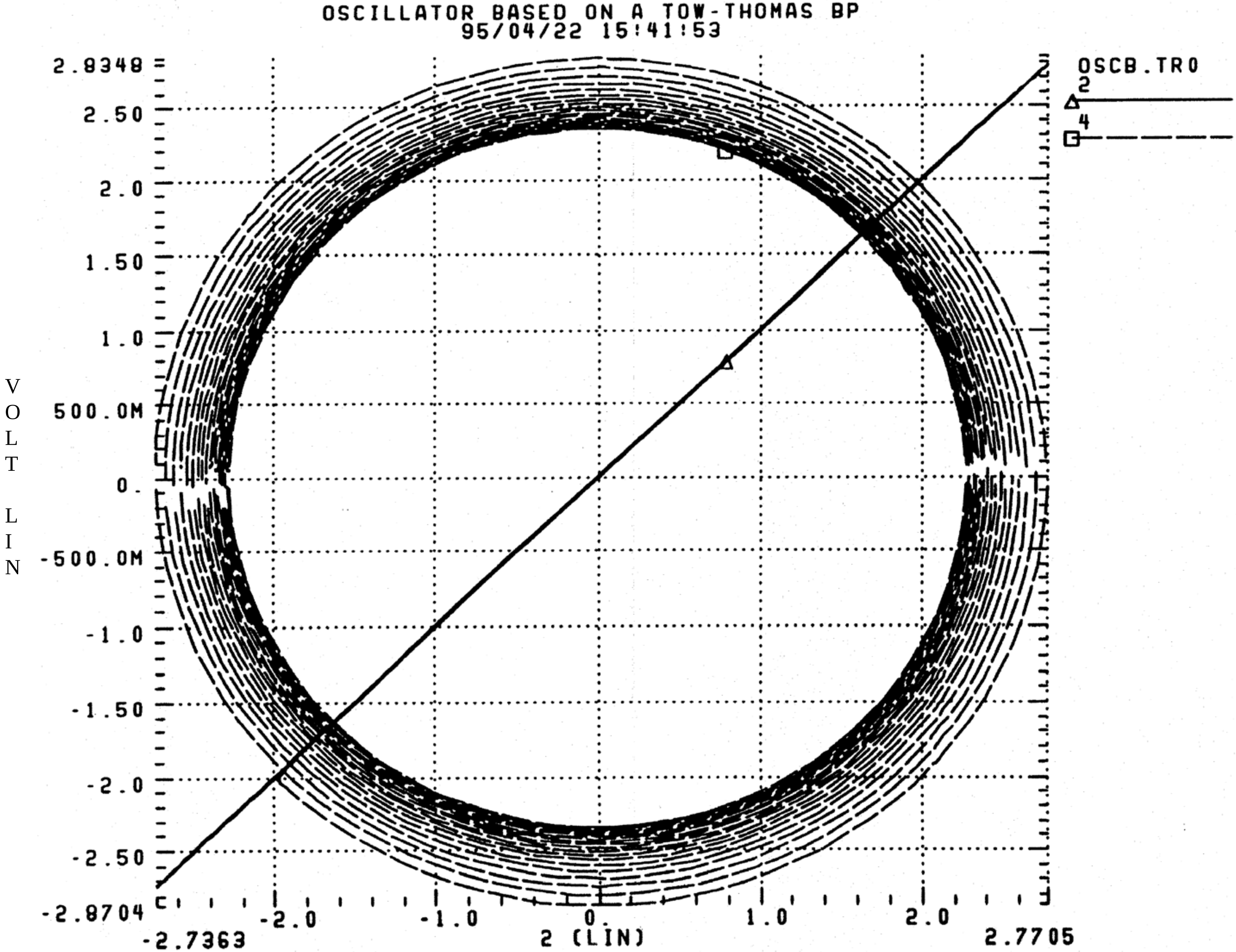

OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
2
4
2.8348
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.8704
VOLT LIN
0.
5.0M
10.0M
15.0M
20.0M
TIME (LIN)

```
Oscillator based on a Tow-Thomas BP
R4      5      3      16K
R01  4   3   1K
RQ   2   3   16K
R     6   1   1K
R     1   7   1K
C02   3   2     0.1591549U
R03     2     6        1K
C03   7   4   0.1501549U
.SUBCKT OPAMP  vi-   vi+   v0+   v0-
*
*
RIN      vi-    vi+    10E6
EIN      1      0      vi+    vi-    2E05
R3DB     1      2      1K
C3DB     2      0      0.159E-04
EOUT     3      V0-    2      0      1
DP       2      VP     DIODE
DN       VN     2      DIODE
```

```
VPP     VP     0      13
VNN     VN     0      -13
ROUT   3      v0+    75.
.MODEL DIODE D(IS=10E-08 N=0.01)
.ENDS OPAMP
*  DESCRIPTION OF OPAMPS AND THEIR NODES
XOP1   3   0   2   0   OPAMP
XOP2     6    0    1    0      OPAMP
XOP3     7    0    4    0      OPAMP
*LIMITER Hysteresis CIRCUIT DESCRIPTION
RH1       1        15       2K
RH2       15       5        13K
XOP4      0        15       5        0        OPAMP
*  DESIRED ANALYSIS RESPONSE
.TRAN      5U        20m        UIC
.FOURIER             57          V(1)    V(2)    V(4)
.IC  V(1)=2  V(2)=-2  V(4)=-2  V(5)=2
.OPTIONS POST
.END
```

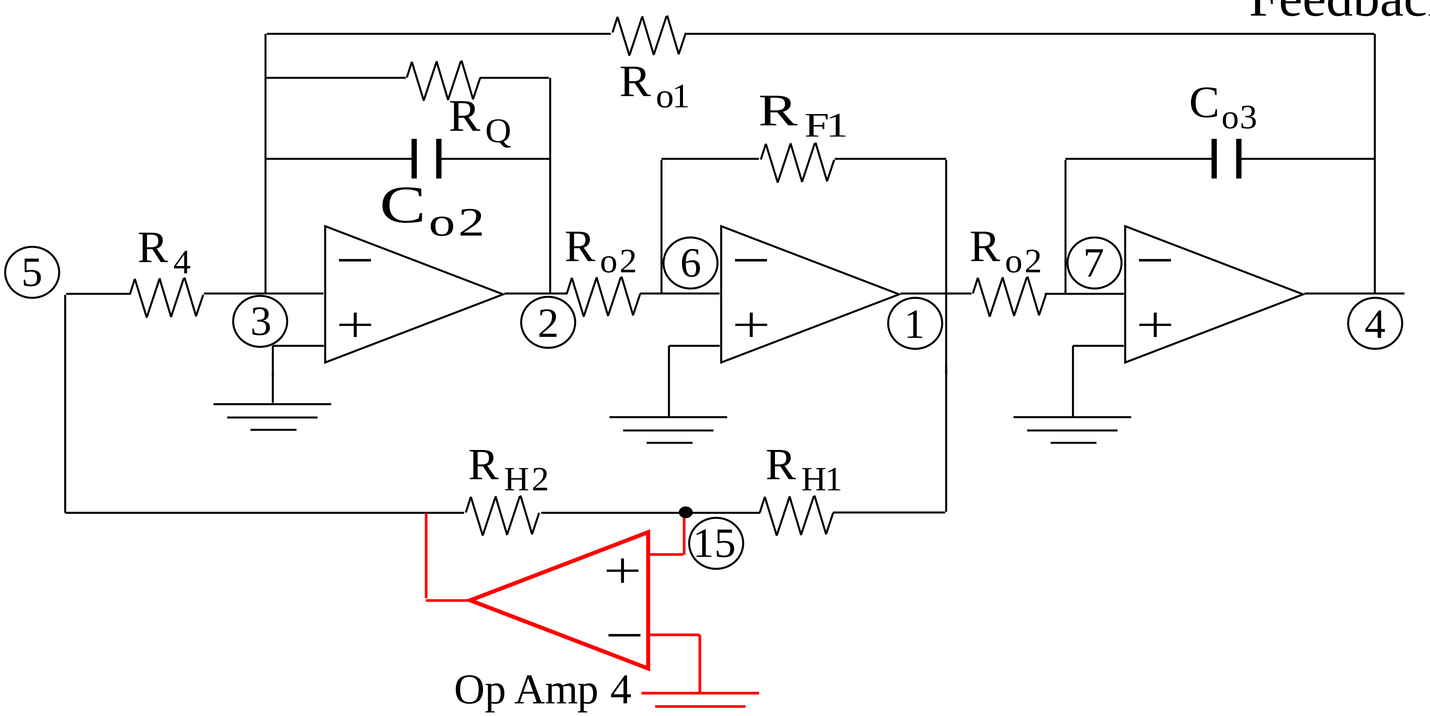


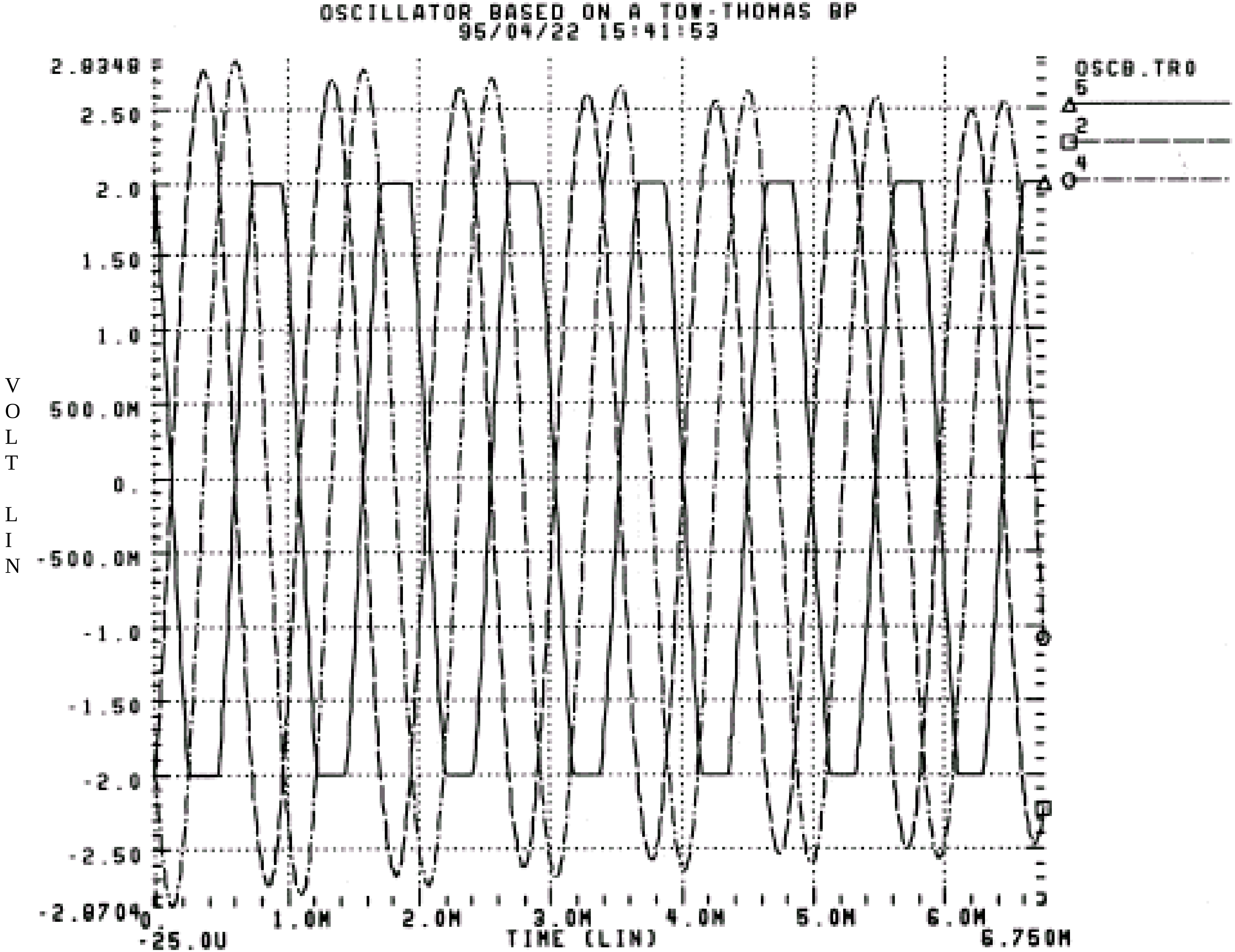

OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
1
5
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.7704
VOLT LIN
0.
5.0M
10.0M
15.0M
20.0M
TIME (LIN)


OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
2
4
2.8348
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.8704
VOLT LIN
-2.0
-1.0
0.
1.0
2.0
-2.7363
2 (LIN)
2.7705

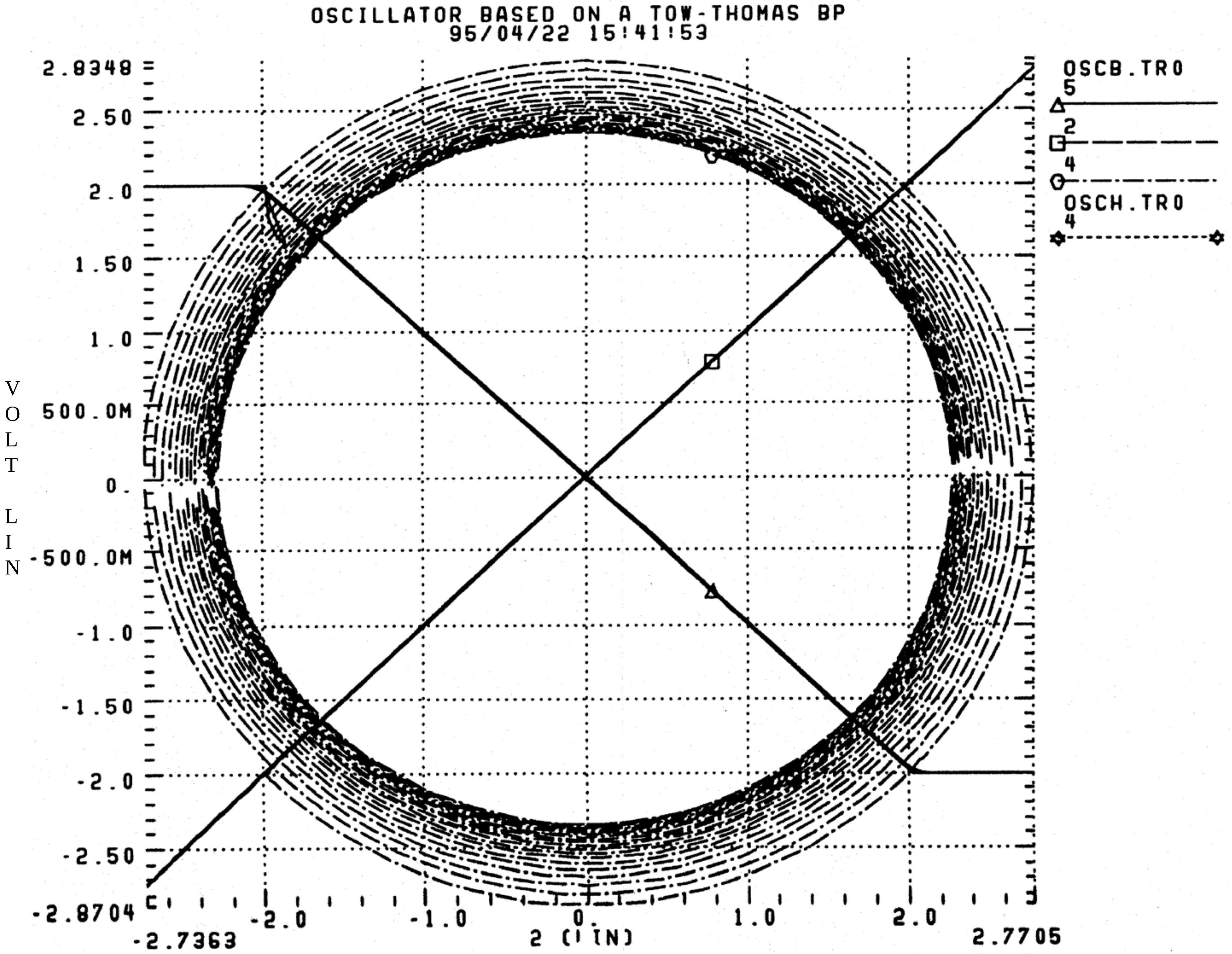
```
Oscillator based on a Tow-Thomas BP
R4       5          3          35K
R01  4   3   1.4K
RQ   1   3   9.6K
R       6   1   1K
R       1   7   1K
C02    3   2       0.1591549U
R03        2       6             1K
C03    7   4   0.1501549U
.SUBCKT OPAMP  vi-    vi+    v0+    v0-
*
*
RIN        vi-     vi+     10E6
EIN        1        0        vi+      vi-      2E05
R3DB    1        2        1K
C3DB    2        0        0.159E-04
EOUT   3        V0-      2        0        1
DP        2        VP      DIODE
DN       VN      2        DIODE
VPP      VP      0       13
VNN     VN      0       -13
ROUT   3       v0+    75.
.MODEL DIODE D(IS=10E-08 N=0.01)
.ENDS OPAMP
*  DESCRIPTION OF OPAMPS AND THEIR NODES
XOP1   3   0    2   0   OPAMP
XOP2     6     0    1    0       OPAMP
XOP3     7     0    4    0       OPAMP
*LIMITER Hysteresis CIRCUIT DESCRIPTION
RH1          1           15          2K
RH2         15          5           13K
XOP4      0           15           5            0          OPAMP
*  DESIRED ANALYSIS RESPONSE
.TRAN       5U           20m           UIC
.FOURIER               57               V(1)    V(2)    V(4)
.IC  V(1)=2  V(2)=-2  V(4)=-2  V(5)=2
.OPTIONS POST
.END
```

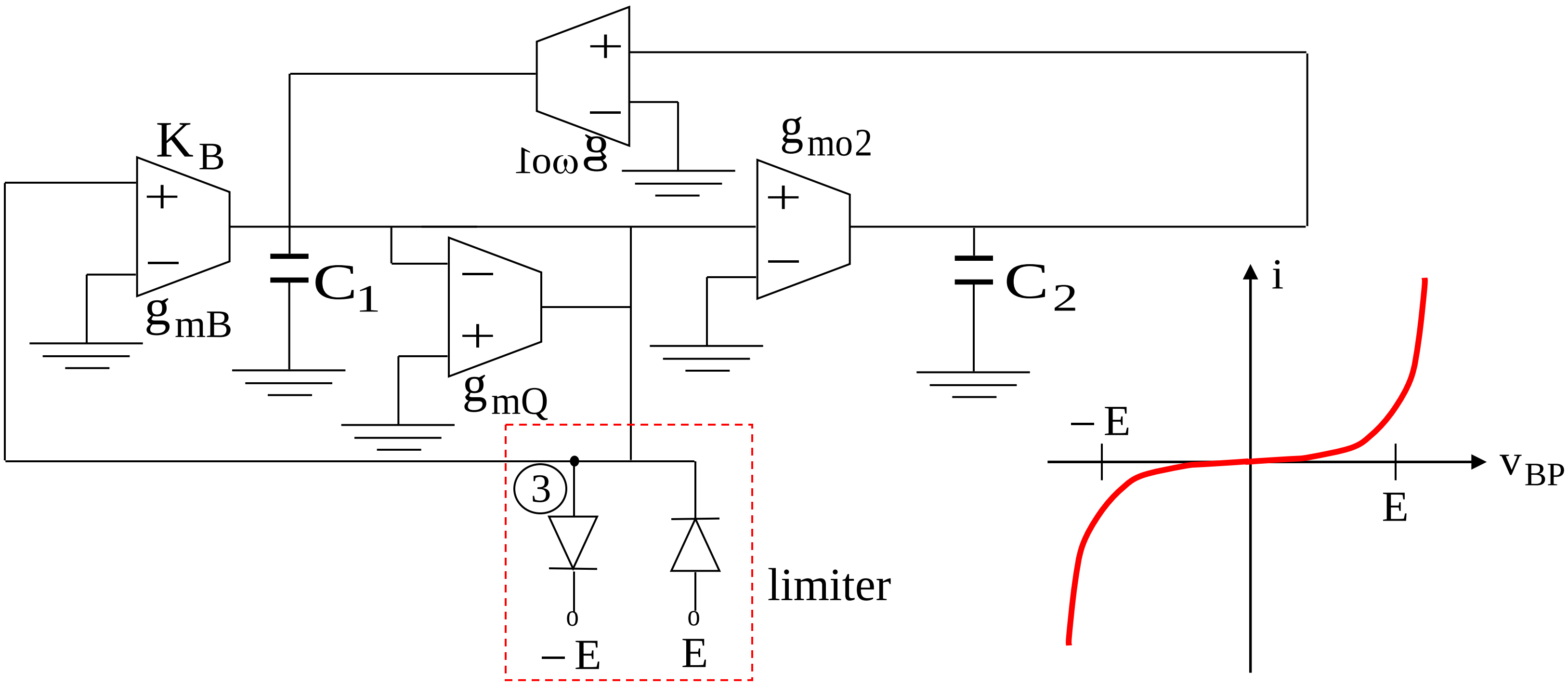
Double Positive Feedback





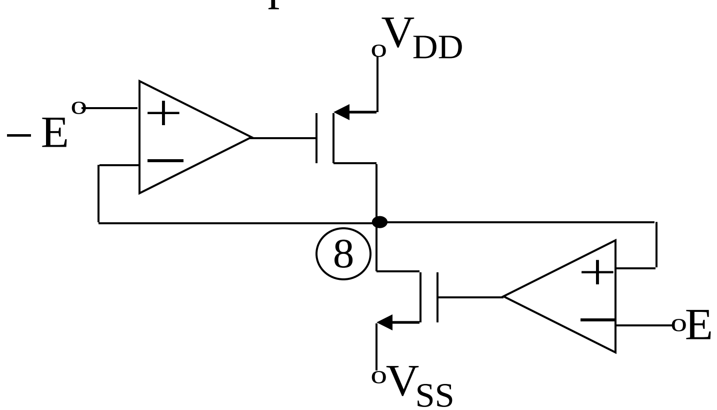
OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
5
2
4
2.8348
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.8704
VOLT LIN
0
-25.0U
1.0M
2.0M
3.0M
4.0M
5.0M
6.0M
6.750M
TIME (LIN)

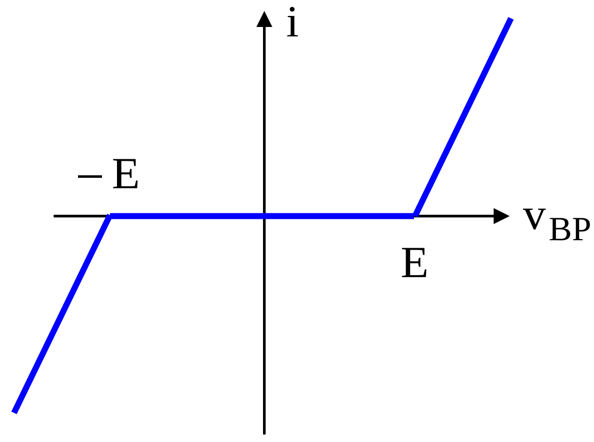

OSCILLATOR BASED ON A TOW-THOMAS BP
95/04/22 15:41:53
OSCB.TR0
5
2
4
OSCH.TR0
4
2.8348
2.50
2.0
1.50
1.0
500.0M
0.
-500.0M
-1.0
-1.50
-2.0
-2.50
-2.8704
VOLT LIN
-2.7363
-2.0
-1.0
0.
1.0
2.0
2.7705
2 (I IN)



OTA - C Bandpass Based Oscillator

Another possible limiter can be:



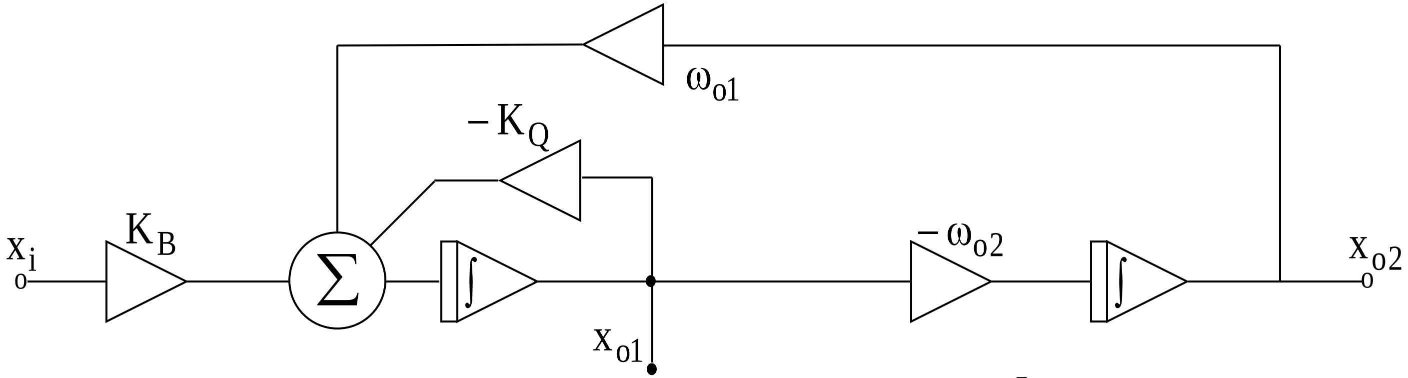


Simulations
Active-Filter Tuned Oscillator

Hints

- First design a band pass filter
  - High Q
  - Gain
- Next decide on type of limiter
- Finally be sure the path from the band pass output through the limiter to the input is positive feedback.

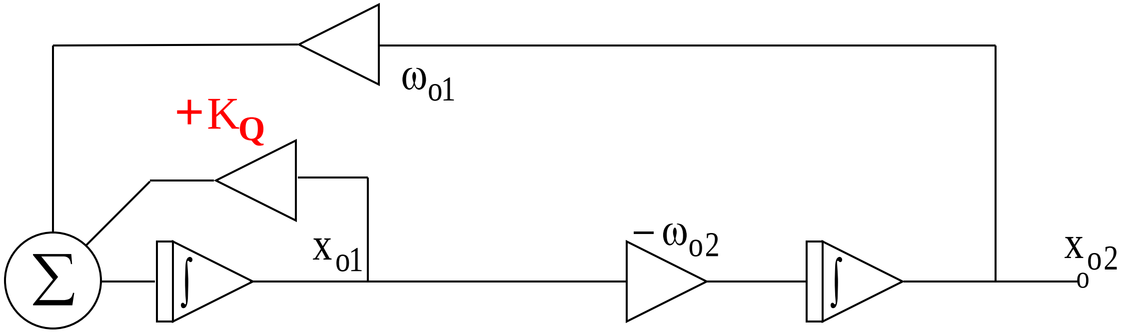
Quadrature Oscillator



Two Integrator Loop Filter

$$H_1(s)=\frac{x_{o1}(s)}{x_i(s)}=\frac{\dfrac{K_B}{s}}{1+\dfrac{K_Q}{s}+\dfrac{\omega_{o1}\cdot\omega_{o2}}{s^2}}=\frac{K_B s}{s^2+K_Q s+\omega_{o1}\omega_{o2}}$$

$H_1(s)$ is a second-order bandpass. The topology of this integrator loop can be modified to yield a *quadrature oscillator*.



Quadrature Oscillator Architecture

Observe that the characteristic equation yields:

$$1-\frac{K_Q}{s}+\frac{\omega_{o1}\omega_{o2}}{s^2}=0$$

or

$$s^2-K_Q s+\omega_{o1}\omega_{o2}=0$$

The roots are placed at

$$s_{1,2}=\frac{K_Q \pm \sqrt{K_Q^2-4\omega_{o1}\omega_{o2}}}{2}$$

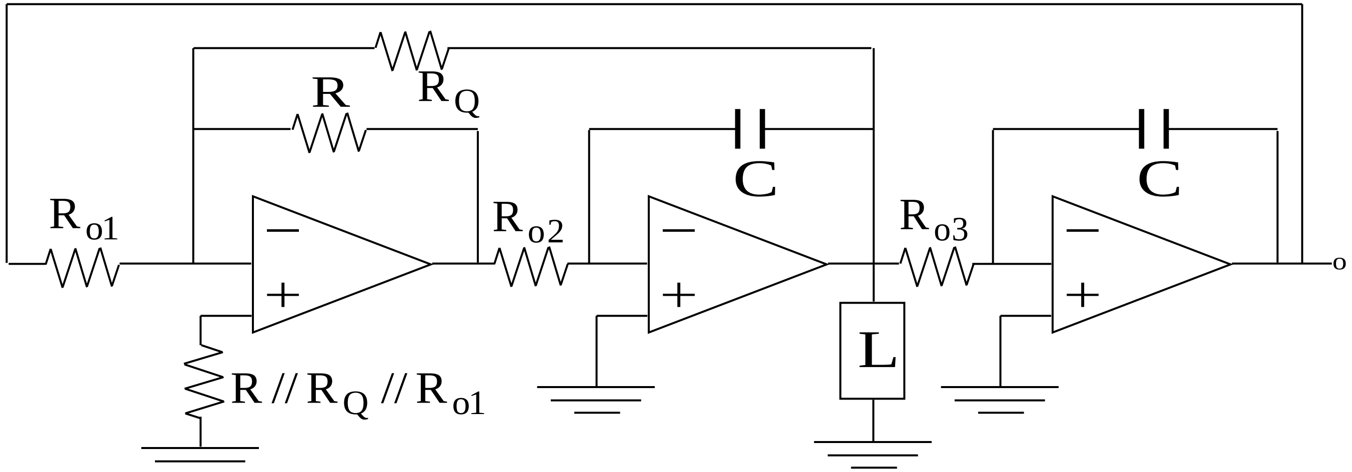
$$=\frac{K_Q}{2}\pm\frac{j}{2}\sqrt{4\omega_{o1}\omega_{o2}-K_Q^2}$$

$$K_Q^2<4\omega_{o1}\omega_{o2}$$

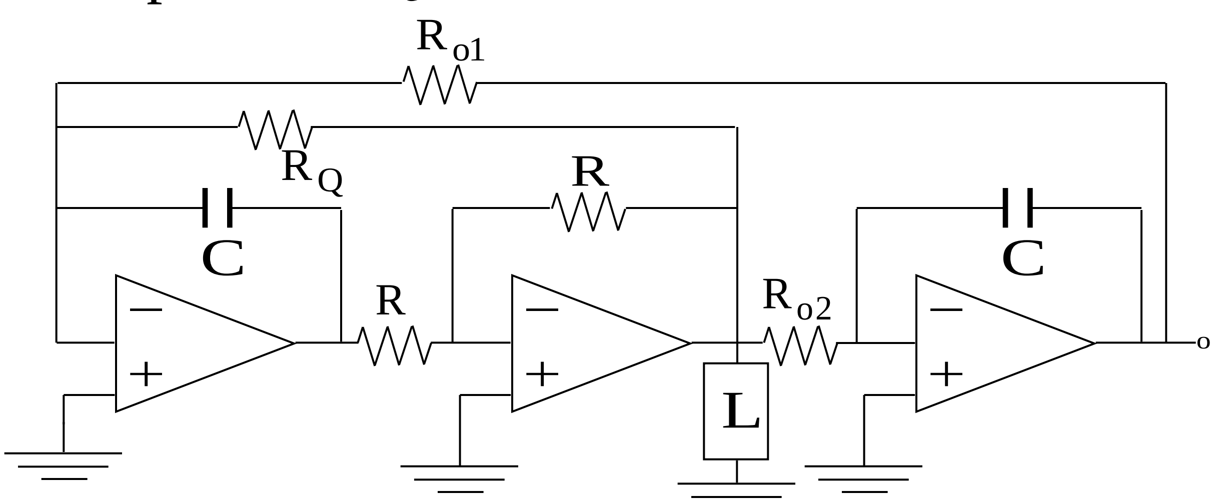
To provide sustained oscillations we need to locate the poles on the $j\omega$ axis, this requires:

i) $K_Q = E$ small positive value

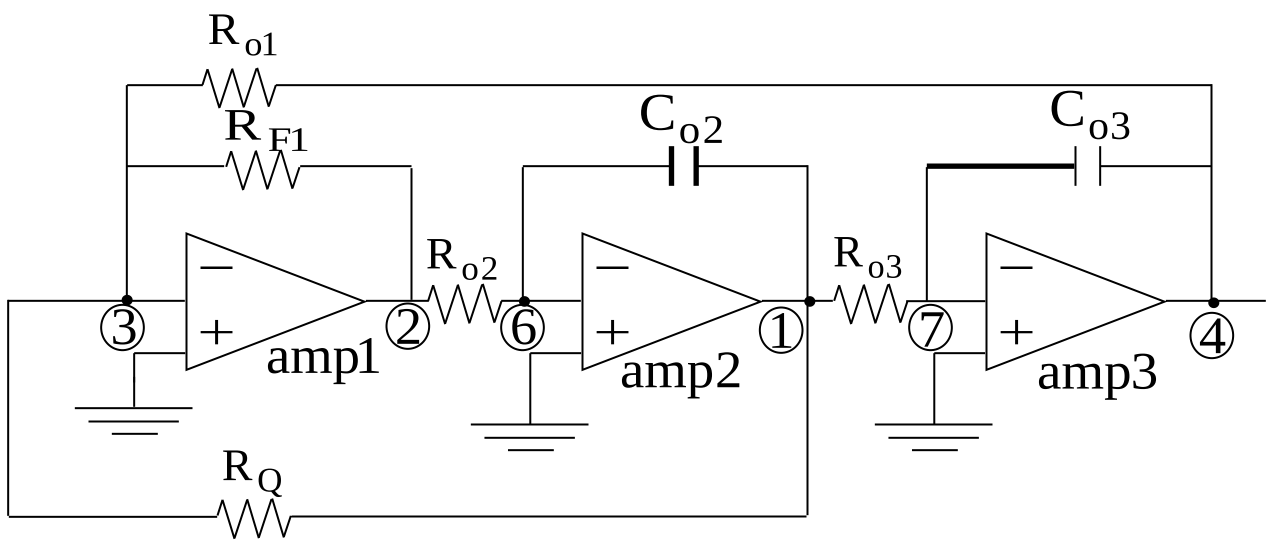
ii) To limit the output by means of a nonlinear gain control.



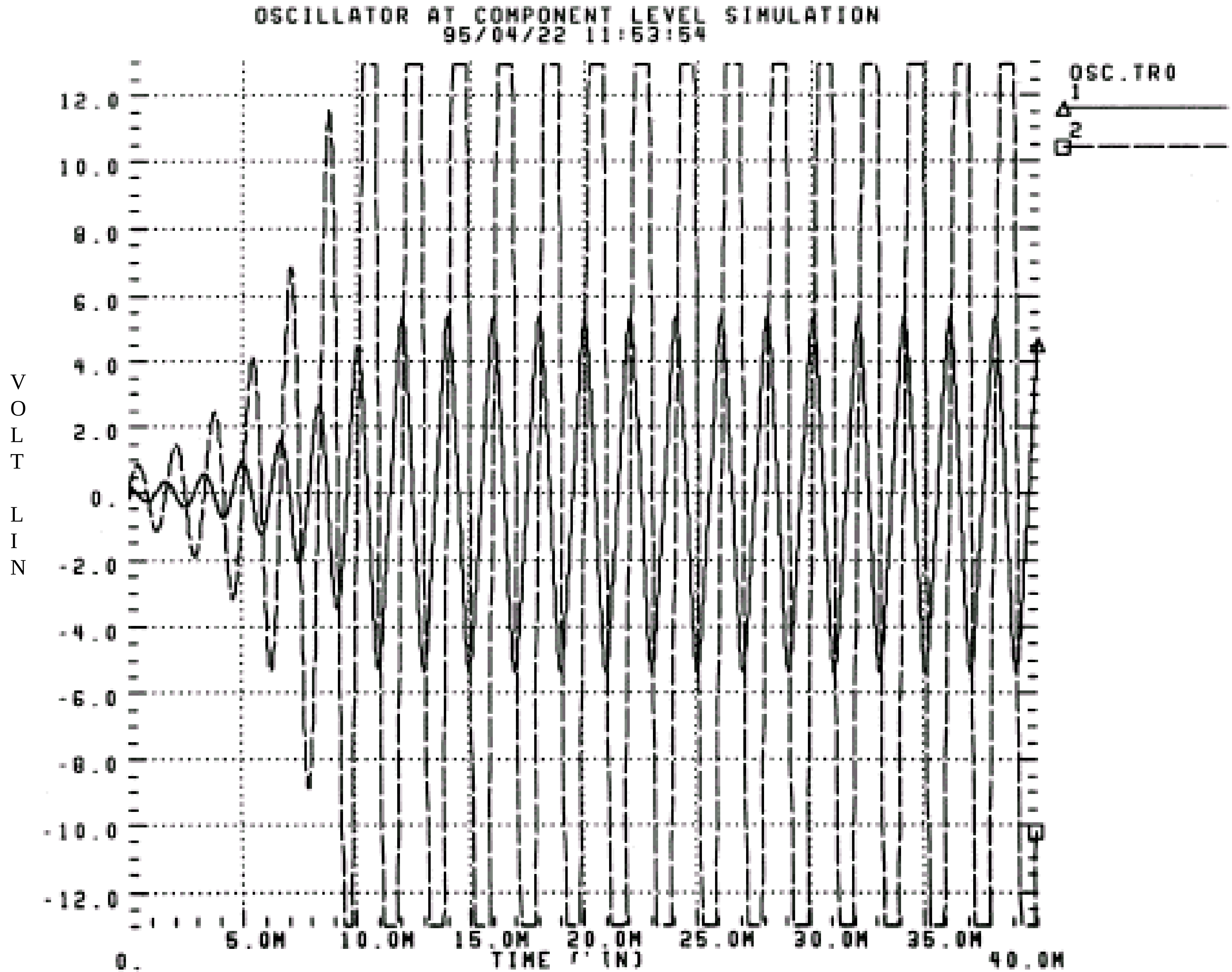
One possible Quadrature Oscillator Structure



```
Oscillator at component level simulation
RO1    4    3    220K
RQ      1    3    510K
RF1     3    2    310K
RO2    2    6    100K
CO2    6    1    0.01U
RO3    1    7    100K
CO3    7    4    0.001U
.SUBCKT OPAMP    vi-     vi+     v0+     v0-
*
*
RIN      vi-    vi+    10E6
BIN      1       0       vi+       vi-       2E05
R3DB   1       2        1K
C3DB   2       0        0.159E-04
BOUT  3     V0-     2        0       1
DP       2     VP      DIODE
DN       VN      2      DIODE
VPP      VP      0      13
VNN    VN      0      -13
ROOT    3      v0+   75.
.MODEL DIODE D (IS=10E-0S N=0.001)
.ENDS OPAMP
*    DESCRIPTION OF OPAMPS AND THEIR NOOES
XOP1         3      0      2      0      OPAMP
XOP2              6      0      1      0      OPAMP
XOP3              7      0      4      0      OPAMP
.TRAN           10U   40m   UIC
.FOURIER      57     V(1)    V(2)     V(4)
.IC                V(1)=.2     V(2)=-.2    V(4)=-.2
.OPTIONS POST
.OP .08
.END
```

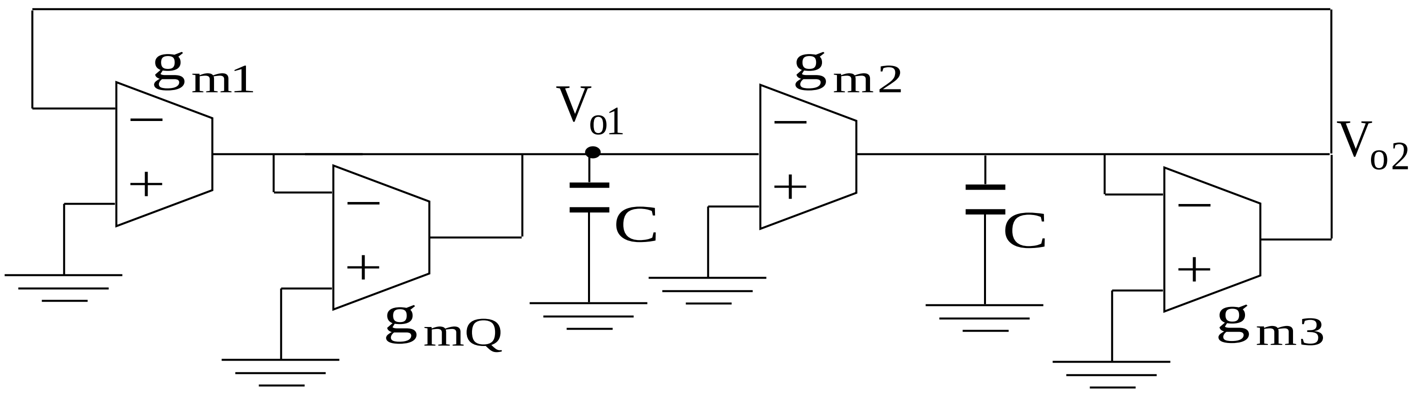


QUADRATURE OSCILLATOR
(NO EXTERNAL LIMITER)



OSCILLATOR AT COMPONENT LEVEL SIMULATION
95/04/22 11:53:54
OSC.TR0
1
2
VOLT LIN
12.0
10.0
8.0
6.0
4.0
2.0
0.
-2.0
-4.0
-6.0
-8.0
-10.0
-12.0
0.
5.0M
10.0M
15.0M
20.0M
25.0M
30.0M
35.0M
40.0M
TIME (LIN)

OTA-C QUADRATURE OSCILLATOR



Structure 1. Note the positive feedback in the main loop

$$V_{o1} = \frac{g_{m1} V_{o2}}{g_{mQ} + sC} \qquad (1)$$

$$V_{o2} = \frac{g_{m2} V_{o1}}{g_{m3} + sC} \qquad (2)$$

C.E

$$1-\frac{g_{m1}}{g_{mQ}+sC}\frac{g_{m2}}{g_{m3}+sC}=0$$

or

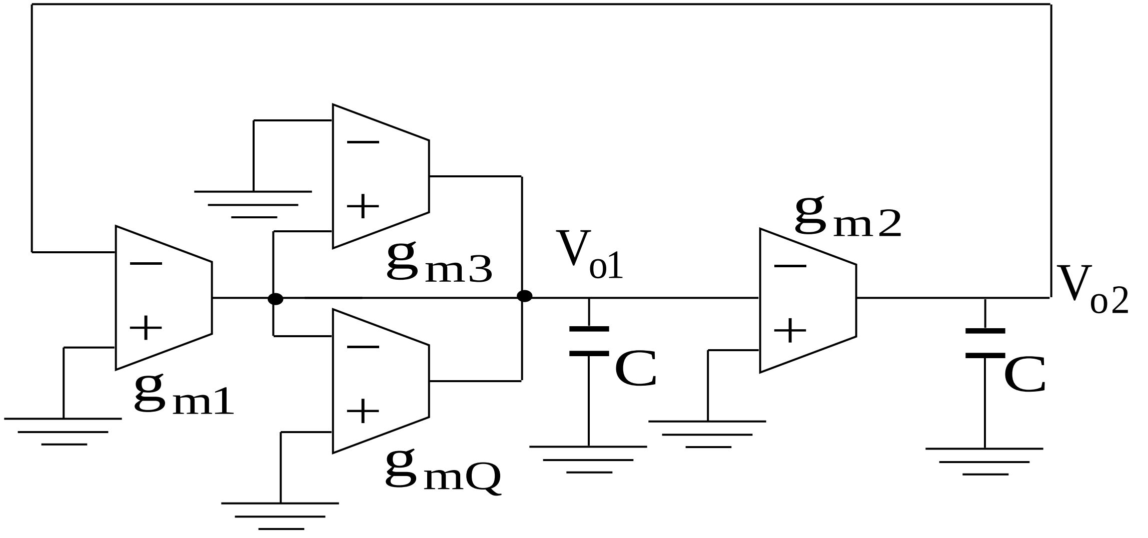
$$s^2 - K_Q s + \omega_{o1}\omega_{o2} = 0$$

Where

$$K_Q = \frac{g_{mQ} - g_{m3}}{C}$$

$$\omega_o^2 = \frac{g_{m1}g_{m2} - g_{mQ}g_{m3}}{C^2}$$

An alternative structure is considered next

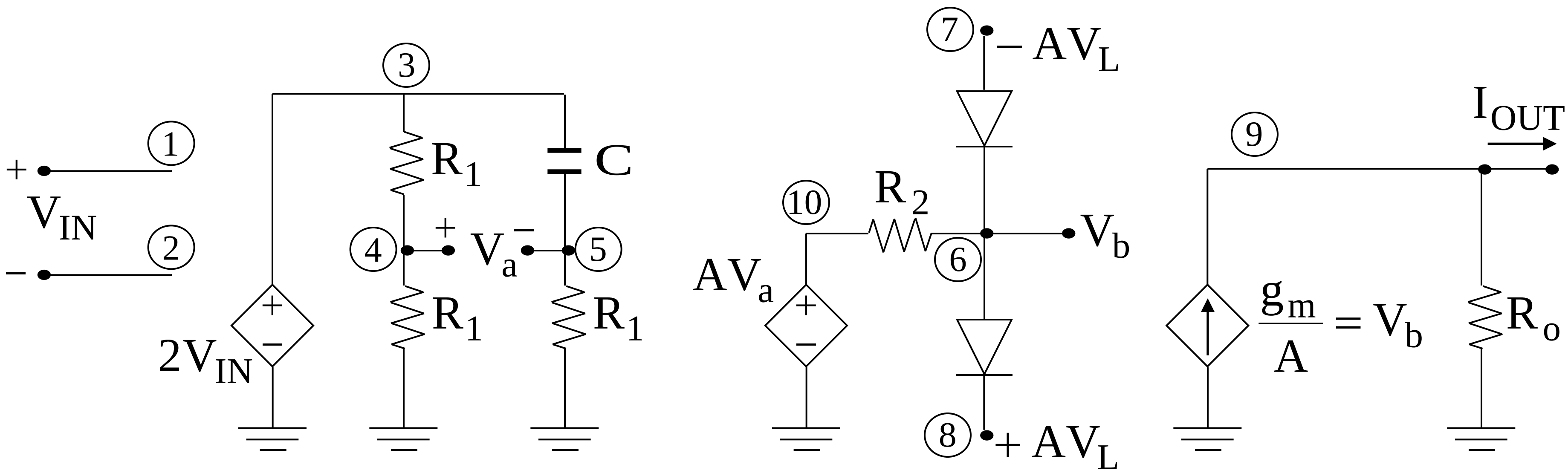


Structure 2. Observe the positive and negative resistance associated with integrator 1.

Where $$\omega_o^2 = \frac{g_{m1} g_{m2}}{C^2}$$

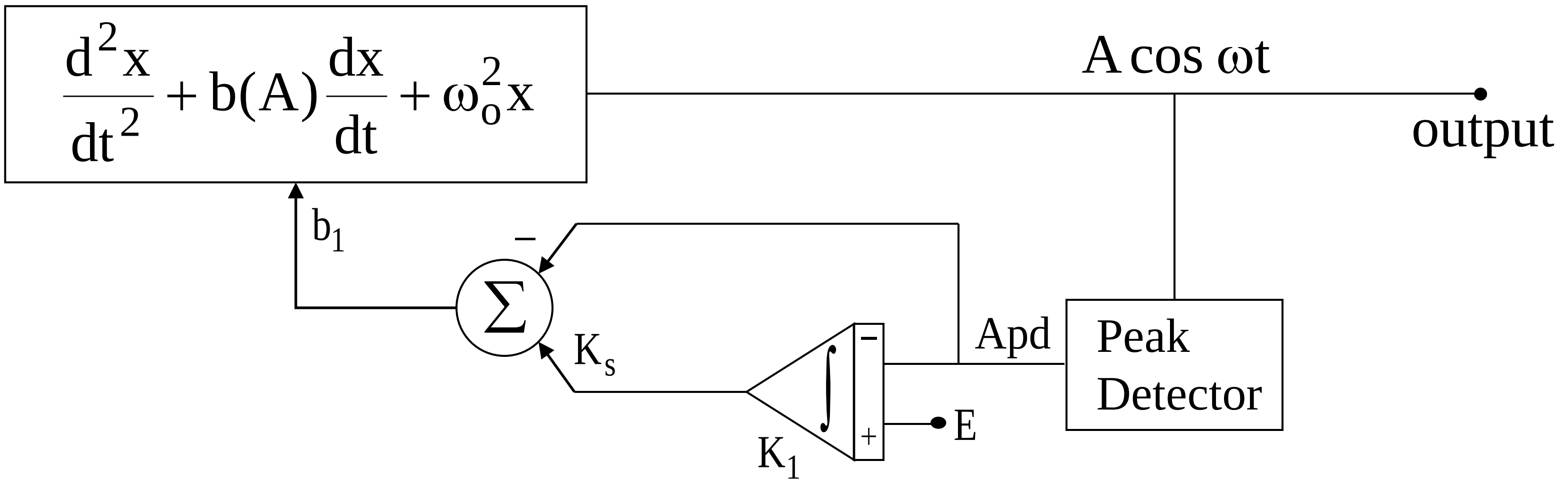
$$K_Q = \frac{g_{mQ} - g_{m3}}{C}$$

This structure is easier to tune. For a realistic design the OTA non-idealities have to be considered.

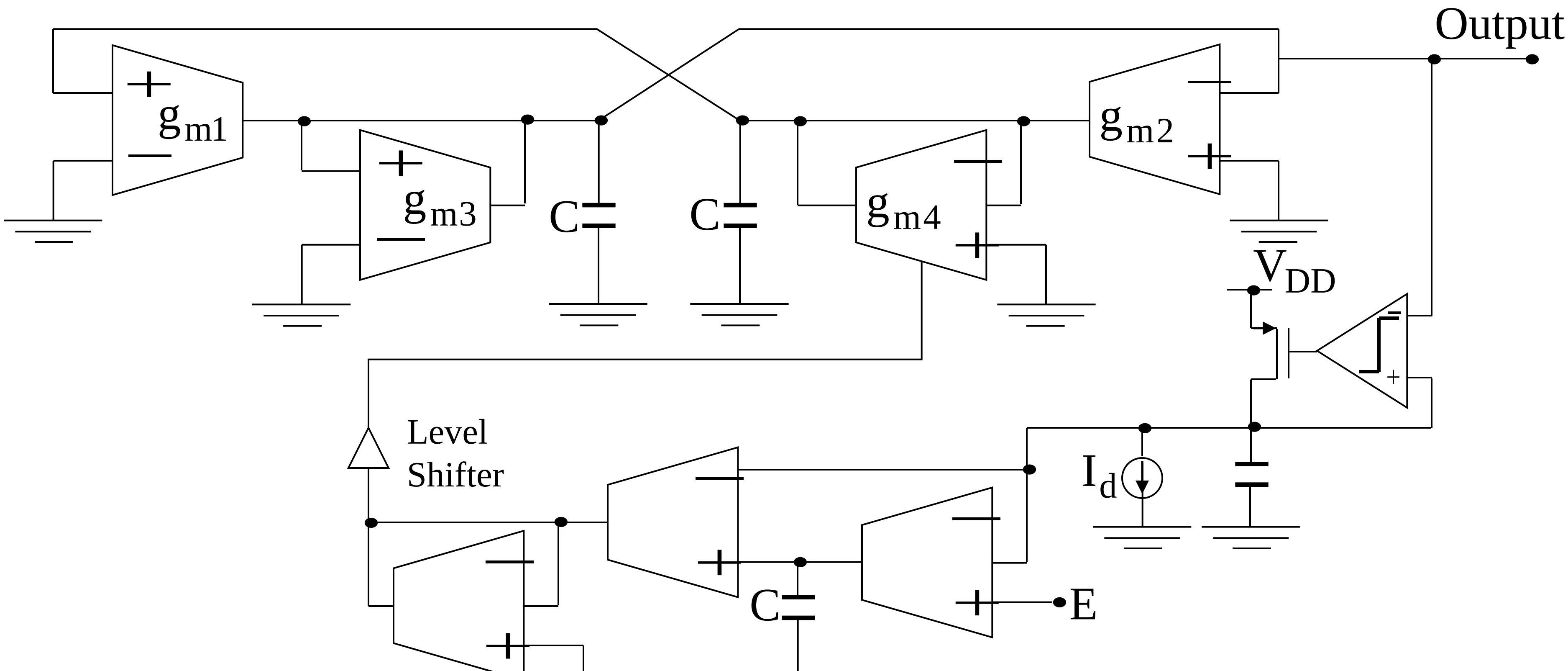


Furthermore amplitude control by external limiters. That is connect the limiters discussed before* at the output of any of the integrators.

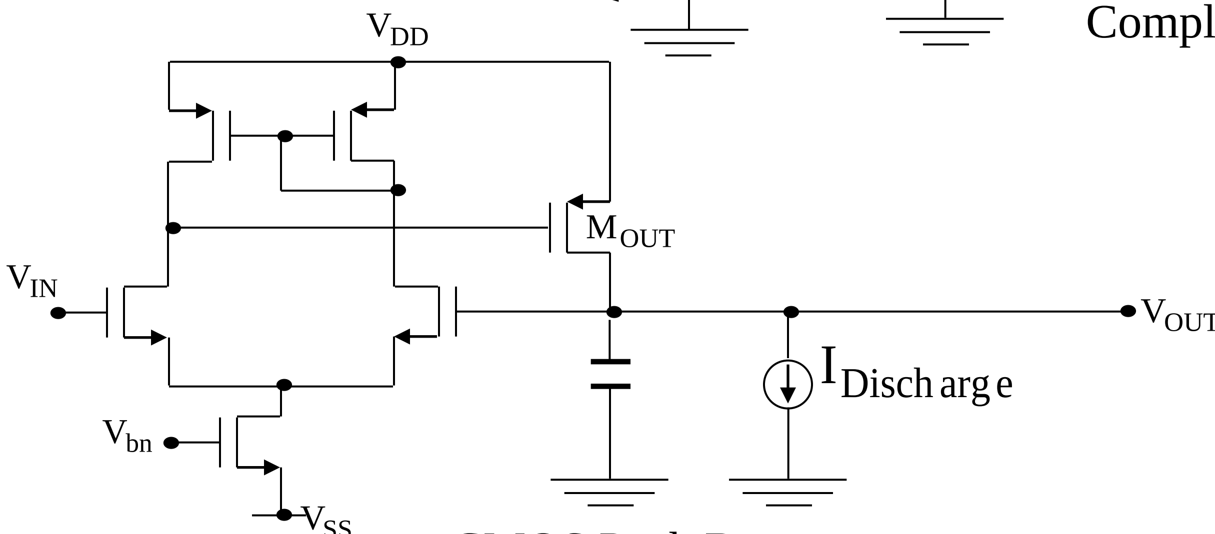
Instead of using limiters, another strategy is using an automatic gain control i.e.,



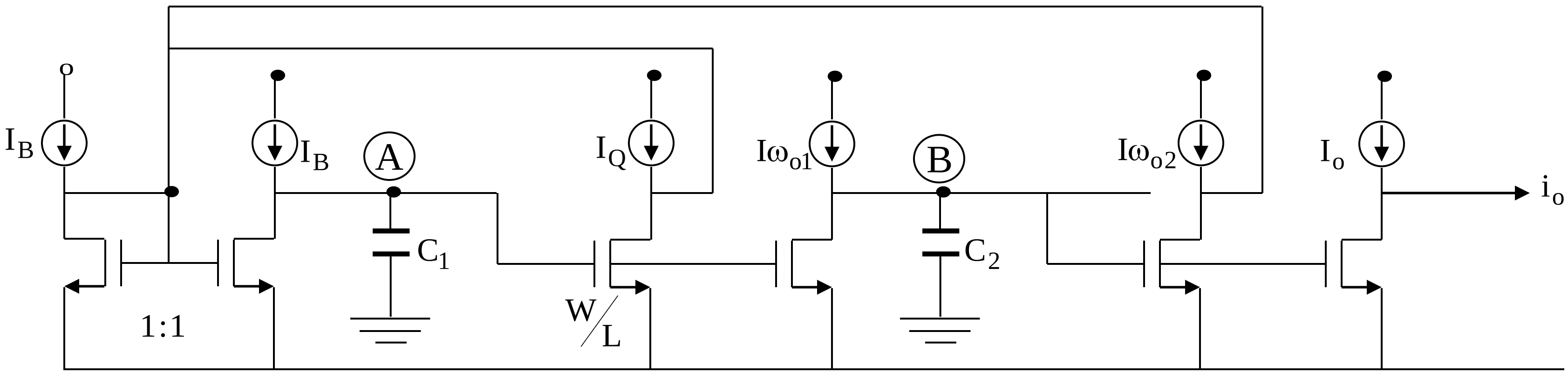
Can you reformulate as an adaptive filter minimizing $(E - Apd)^2$?



Complete AGC 4OTA2c Oscillator



CMOS Peak Detector



- A potential current - mode Quadrature Oscillator

Note that limiters are needed to sustain oscillators. A limiter should be placed at point A or B.

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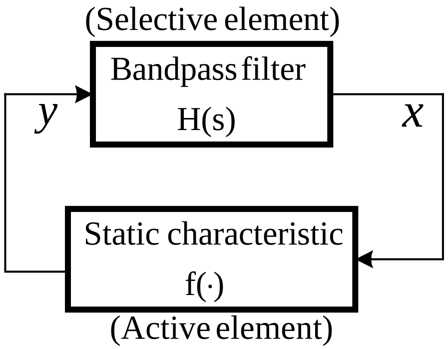
A Low THD Bandpass-Based Oscillator Using Multilevel Hard Limiter

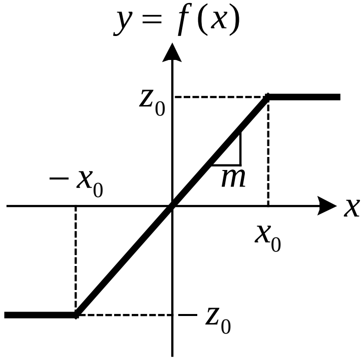
Faramarz Bahmani

Edgar Sanchez-Sinencio

Bandpass Based Oscillator

- Theory
- Filter and comparator are in positive feedback
- Amplitude and frequency of oscillation in decoupled
- Oscillation amplitude is indirectly controlled by the clamping levels $\pm z_0$





Bandpass Based Oscillator

- To analyze this kind of oscillators the nonlinear block needs to be linearized.
  - Describing Function: Assume the input to the nonlinear block is a pure sinusoidal, DF is defined as the ratio of the amplitude of the first harmonic at the output to the input amplitude A_0.

$$N(A_0) = \frac{a}{A_0} = \begin{cases} \dfrac{2m}{\pi}\left[\sin^{-1}\left(\dfrac{x_0}{A_0}\right) + \dfrac{x_0}{A_0}\sqrt{1-\left(\dfrac{x_0}{A_0}\right)^2}\,\right] & A_0 > x_0 \\ m & A_0 < x_0 \end{cases}$$

Bandpass Based Oscillator

$$H(s) = \frac{k_0 s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{X(s)}{Y(s)} = \frac{1}{N(A_0)}$$

$$\frac{d^2 x(t)}{dt^2} + \frac{dx(t)}{dt}\left[\frac{\omega_0}{Q} - k_0 N(A_0)\right] + \omega_0^2 x(t) = 0$$

$$A_0 \cong N^{-1}\left(\frac{\omega_0}{k_0 Q}\right)$$

- Thus, oscillation amplitude A_0 is a function of the nonlinearity…

Linearity

- Due to the nature of the nonlinear block, the output signal is rich in harmonic.
- To increase the linearity of the output signal the most obvious way is to increase the Q of the bandpass filter. However, this solution is power hungry and expensive.
- Question: Does the nonlinear block have any effect on linearity?

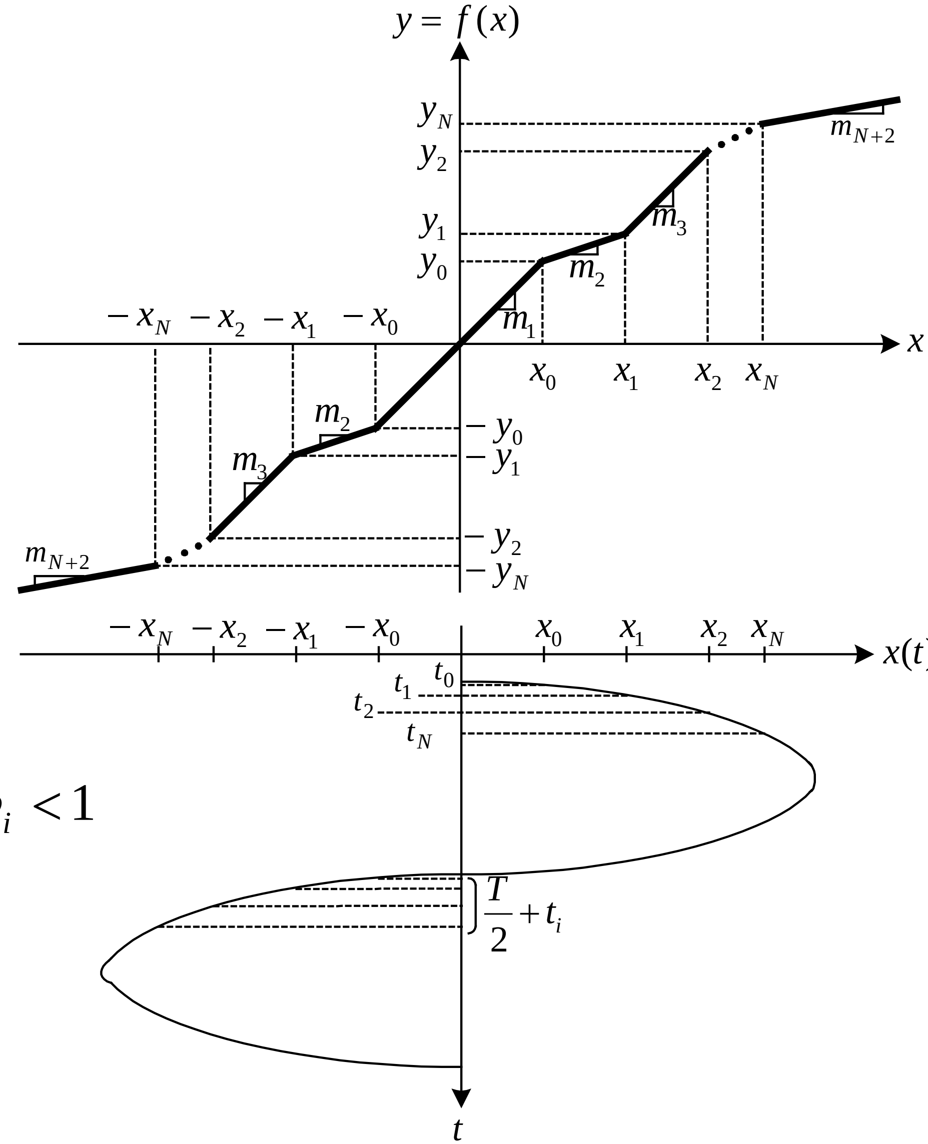
General Case

- N-slope nonlinearity
  - It can be proved:

$$N(A_0) = \frac{1}{2\pi}\sum_{i=1}^{N-1}(m_i - m_{i+1})r(\rho_i)$$

$$r(\rho_i) = \frac{2}{\pi}\left[\sin^{-1}(\rho_i) + \rho_i\sqrt{1-\rho_i^2}\right] \quad ;\rho_i < 1$$

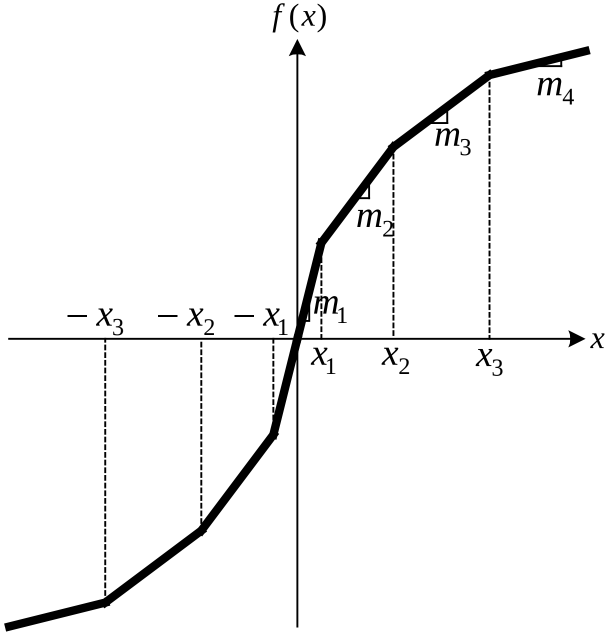
$$\rho_i = x_i / A_0$$



General Case: An Example

- Example: 4-level nonlinearity

$$N(A_0) = \frac{1}{2\pi}\begin{pmatrix} m_3 r(\rho_3) + (m_2 - m_3) r(\rho_2) \\ + (m_1 - m_2) r(\rho_1) \end{pmatrix}$$

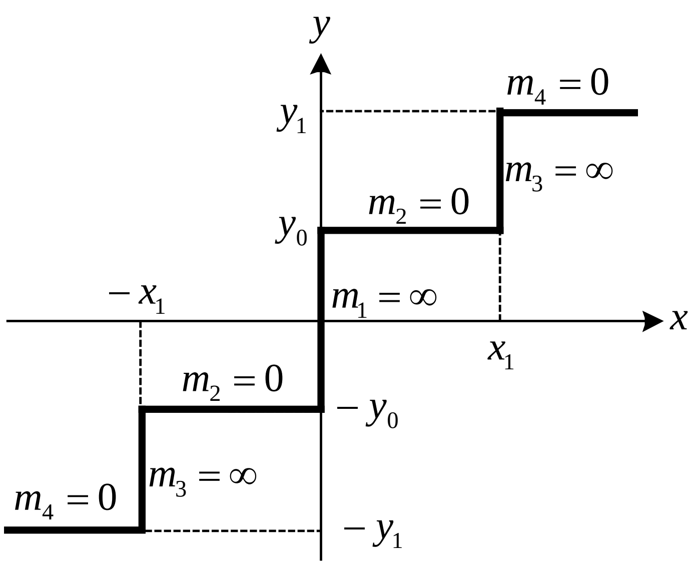


- Assuming $A_0 = x_3$, it can be shown that to have a perfect nonlinear function:

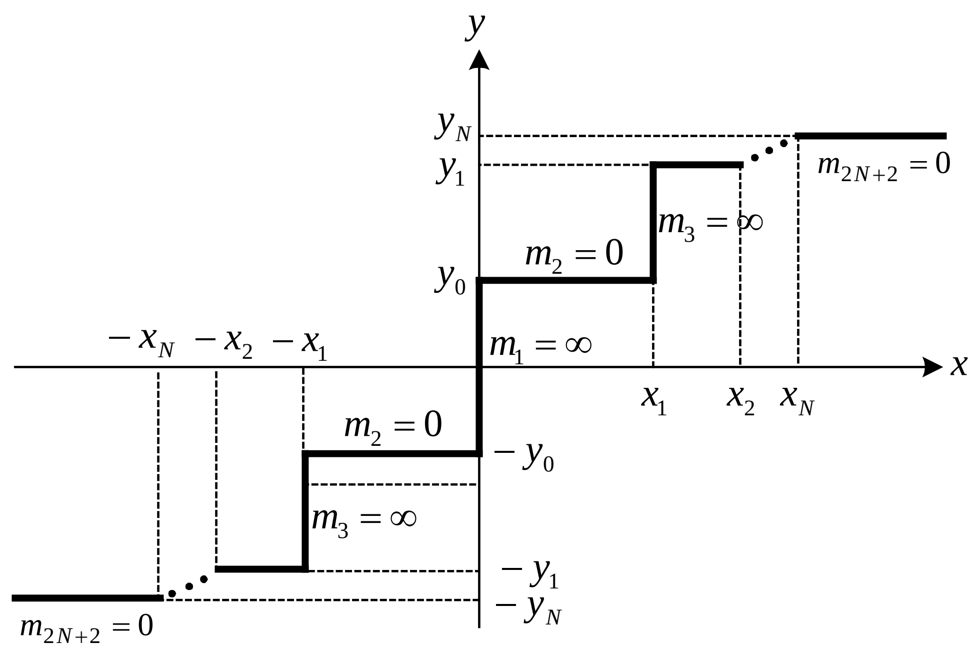
  $m_1 = \infty$, $m_2 = 0$, $m_3 = \infty$ and $m_4 = 0$

General Case: An Example

- Thus optimum nonlinearity functions look like the following figures



Our 4-level example



General N-level

Harmonic Distortion

- Next question: what is the best values of x_i's and y_i's to minimize the THD at the output the nonlinear block?
- Answer: Assuming the input is a sinusoidal the n^{th} Fourier harmonic can be found as

$$a_c(n\omega_0) = a_0(n\omega_0)\left\{1 + \sum_{i=1}^{N-1}\left[y_i\left(1-(-1)^n\right)\cdot\cos\left(\omega_0 t_i\right)\right]\right\},\ n = 3, 5, 7, \cdots$$

- Note that due to symmetry the even order harmonics are zero.

Harmonic Distortion

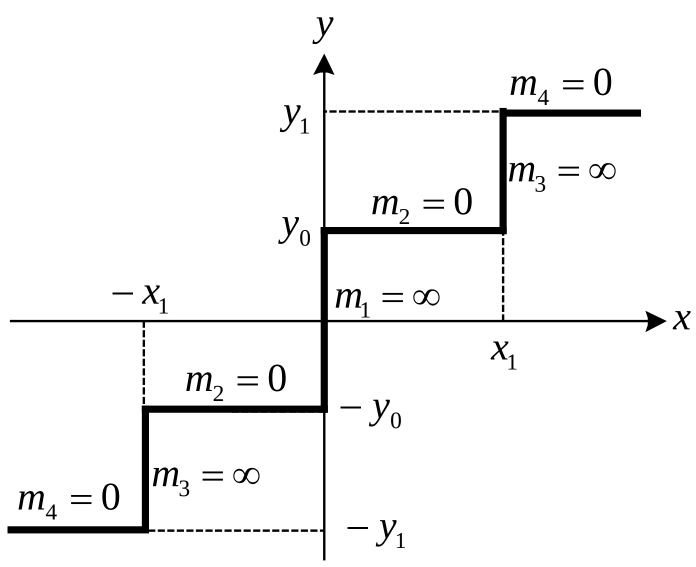
- It can be demonstrated that for specific values of *k* and *n*, the following expressions of t_i and y_i make $a_c(n\omega_0)$ zero

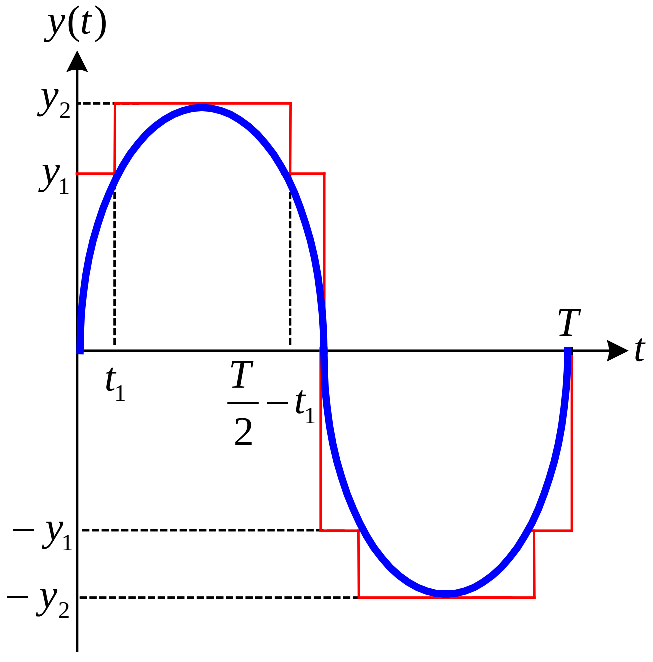
$$t_i = \frac{T}{2^{K+2}} i, \quad i = 1, 2, 3, \cdots$$

$$y_i = \cos\left(\frac{\pi}{2^{K+1}} i\right), \quad i = 1, 2, 3, \cdots$$

Harmonic Distortion: 4-Level Case

- Back to our 4-level example:
  - By applying a sinusoidal to the previously found optimum nonlinear block the following output (Red) can be obtained





Harmonic Distortion: 4-Level Case

- The first three odd harmonic of the output can be expressed as

$$a_c(\omega_0) = y_2\left\{\frac{y_1}{y_2} + \cos\left(\frac{2\pi}{T_1}\right)\right\}a_0(\omega_0)$$

$$a_c(3\omega_0) = y_2\left\{\frac{y_1}{y_2} + \cos\left(\frac{6\pi}{T_1}\right)\right\}a_0(3\omega_0)$$

$$a_c(5\omega_0) = y_2\left\{\frac{y_1}{y_2} + \cos\left(\frac{10\pi}{T_1}\right)\right\}a_0(5\omega_0)$$

Where $a_0(n\omega_0)$ are the nth harmonic of a rectangular wave at f_0 and amplitude of 1.

Harmonic Distortion: 4-Level Case

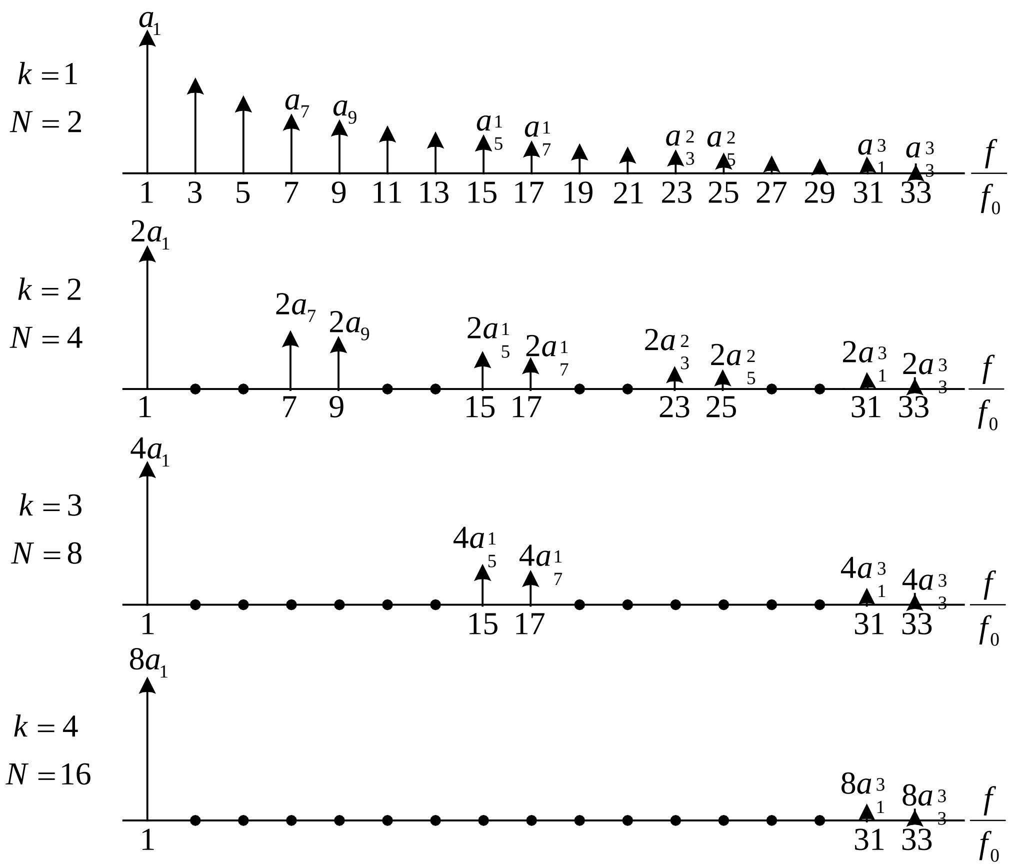
- It can be proven that for the following parameters the $a_c(3\omega_0)$ and $a_c(5\omega_0)$ become zero:

$$\frac{y_2}{y_1} = \sqrt{2}$$

$$t_1 = \frac{T}{8}$$

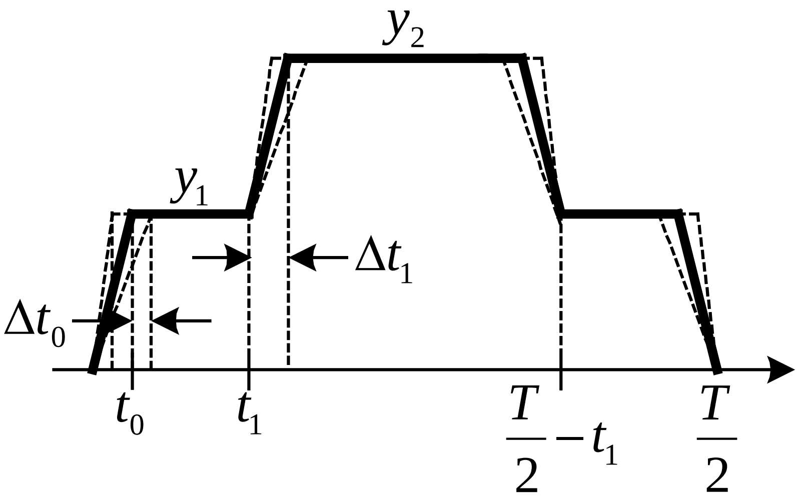
Harmonic Distortion: N-Level Case

- For the general N-level nonlinearity function and for specific values of N and K it can be shown that the first odd harmonics, based on the following figure, become zero.



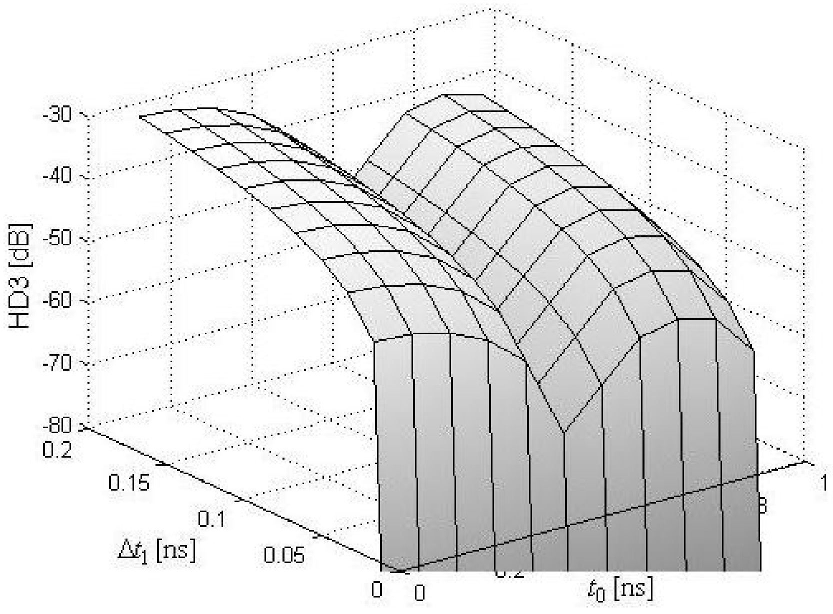
Nonidealities

- Non-idealities:



$$\hat{a}_c(n\omega_0) = \frac{\left(1-(-1)^n\right)\mathrm{sinc}\left(\frac{n\omega_0 t_0}{2}\right)}{n\pi}\left[\begin{array}{l} y_1 \cos\left(\frac{n\omega_0 t_0}{2}\right) + \\ (y_2 - y_1)\cos\left(n\omega_0\left(t_1 + \frac{\Delta t_1}{2}\right)\right) \end{array}\right]$$

Nonidealities

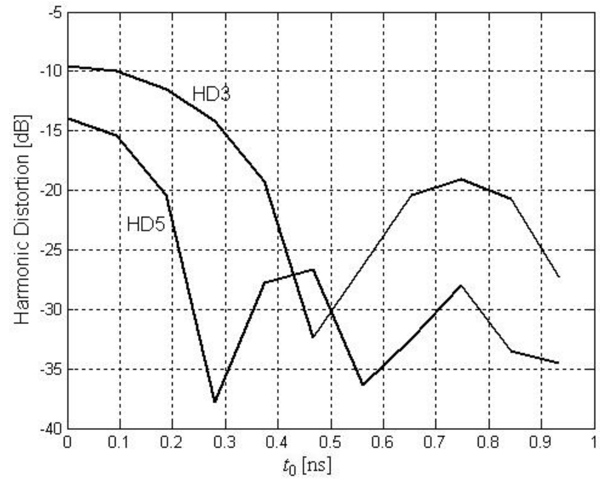


To be added

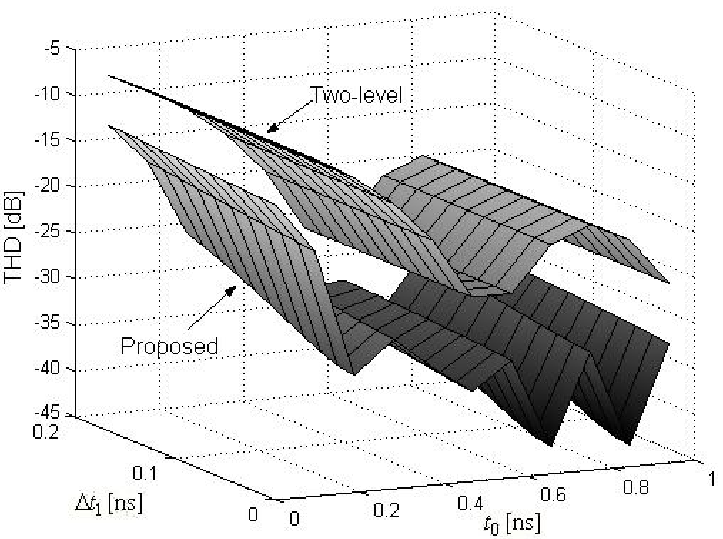
- HD3 and HD5 of the 4-level nonlinearity. Note that at $t_0=\Delta t_1=0$, HD3=HD5=$-\infty$

Nonidealities

- HD3 and HD5 for a 2-level (comparator)

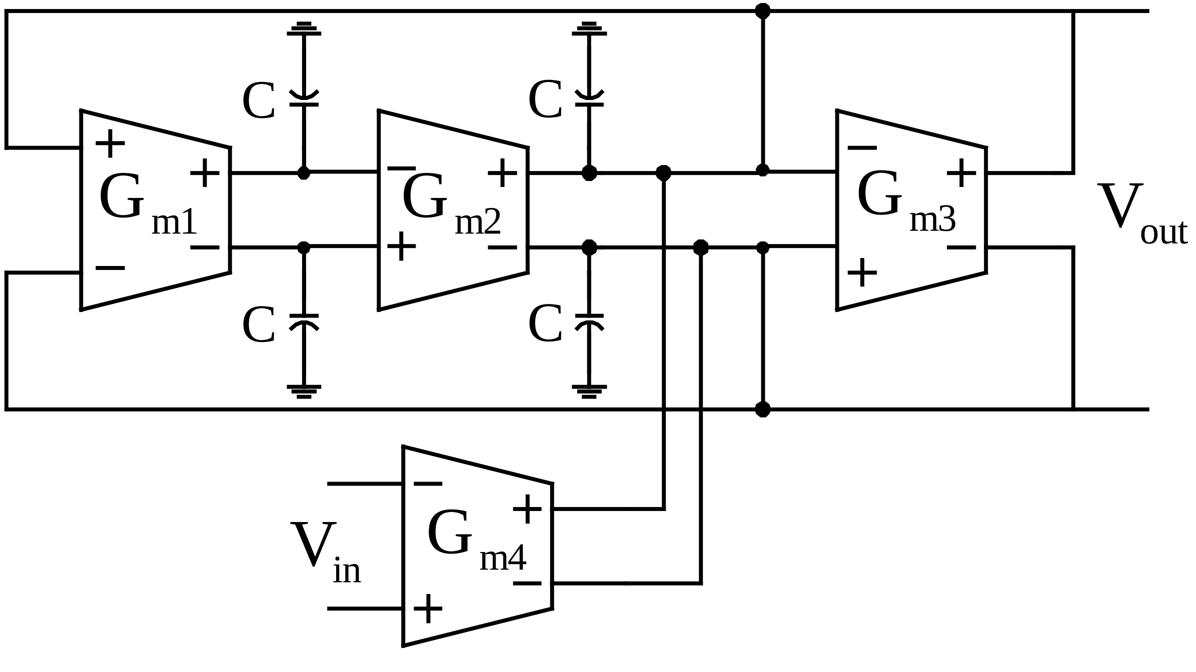


- Total THD:
  - At least 20dB improvement in 4-level nonlinearity



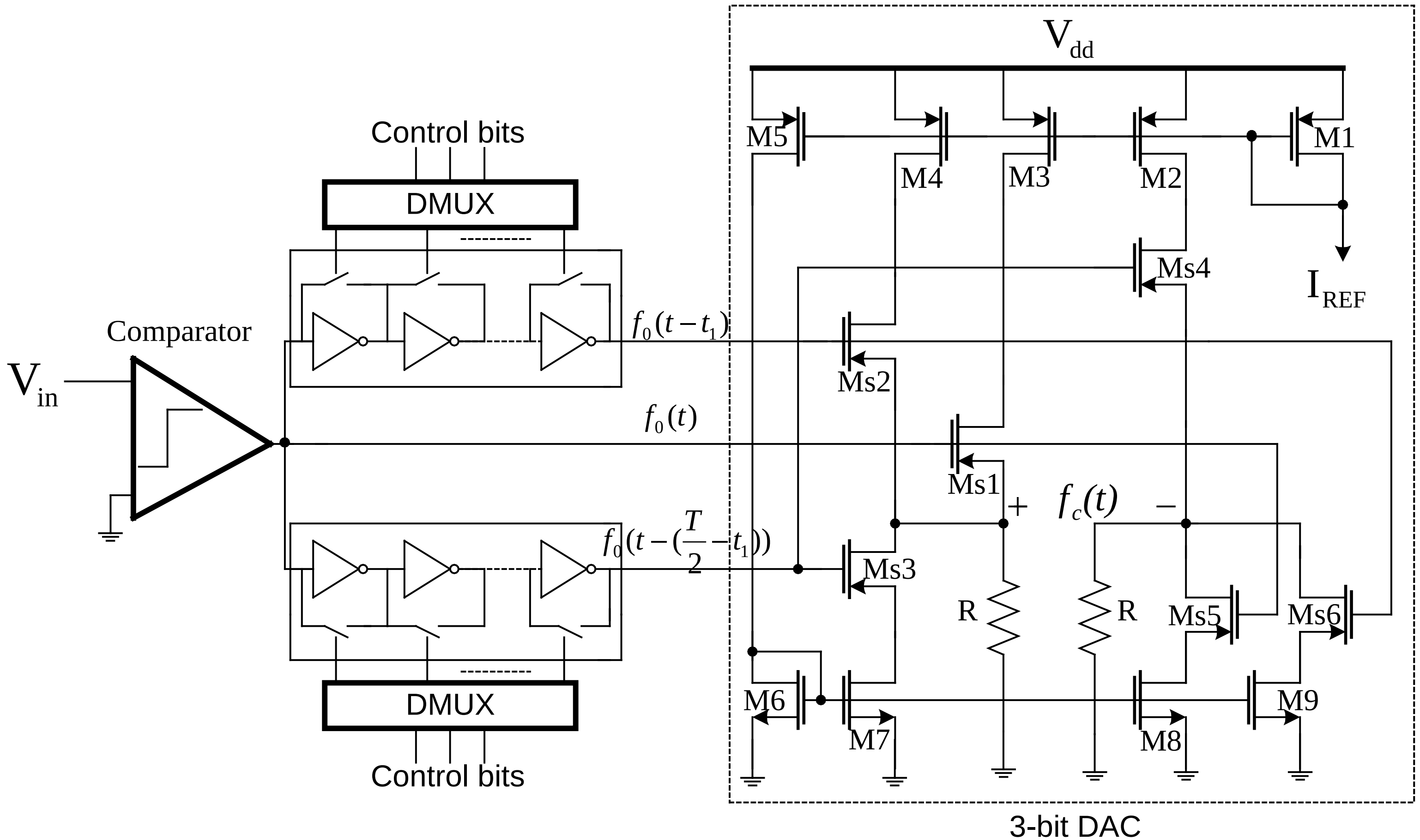
Implementation

- Bandpass filter: A simple Gm-C biquad with a Q of 15



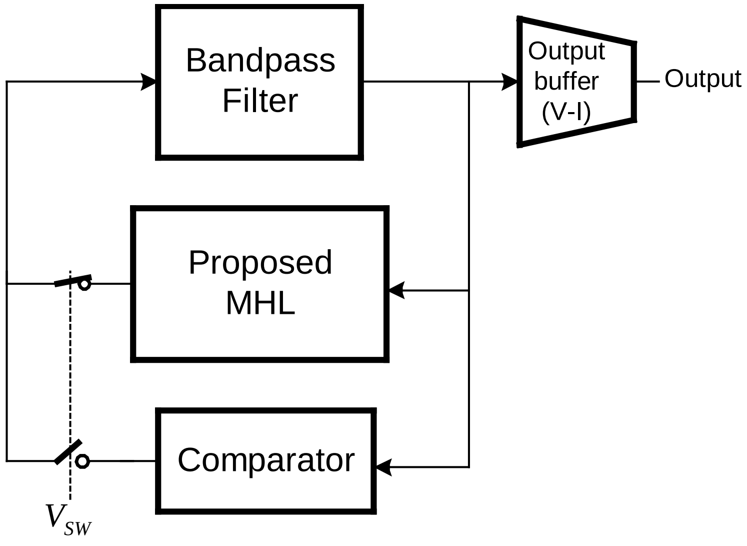
Implementation

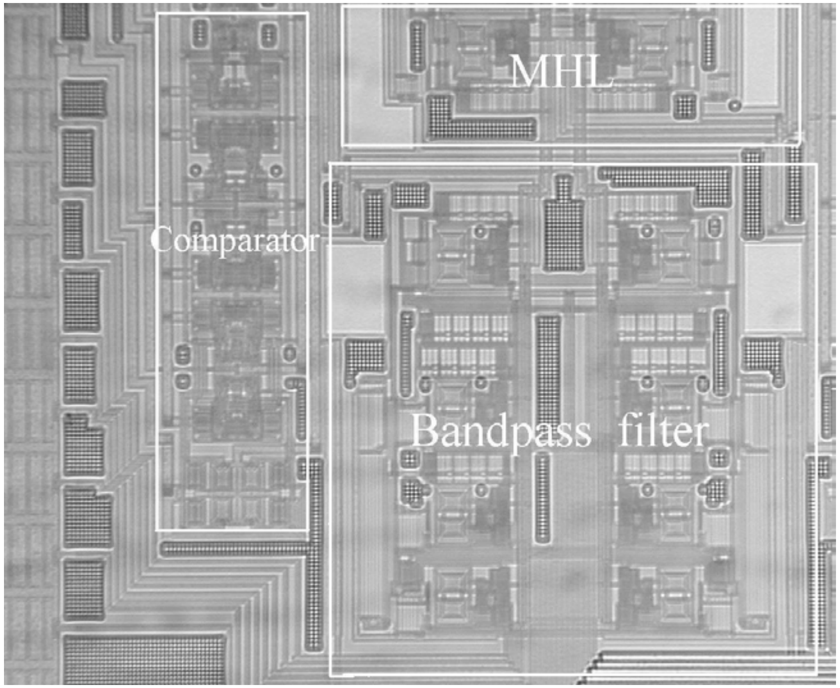
- Implementation of the nonlinear block



Implementation

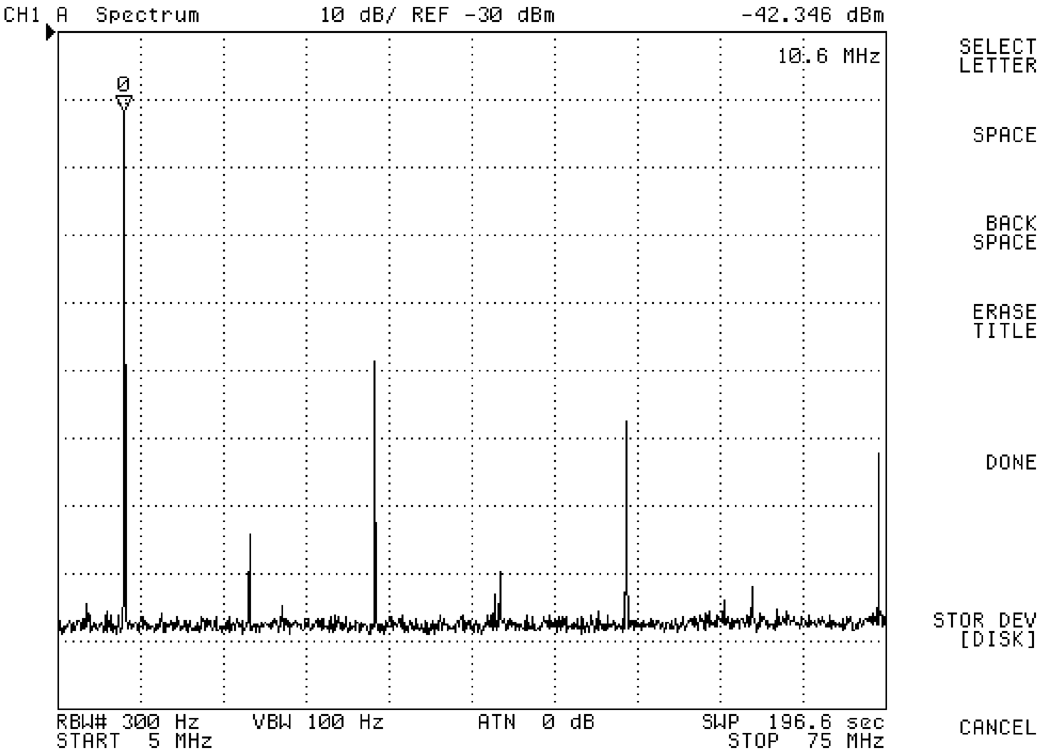
- Silicon





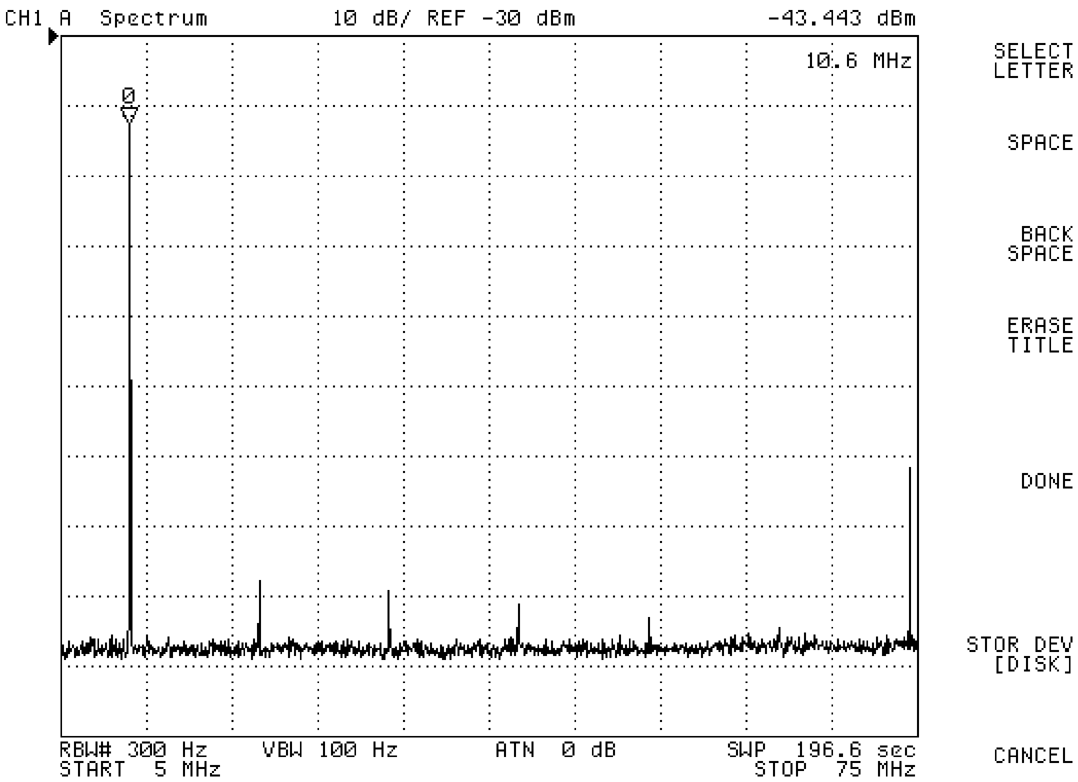
Measurement

- Oscillator with a 2-level (comparator) nonlinearity block



Measurement

- Oscillator with the proposed 4-level nonlinearity block



Measurement

- THD: Theory and measurement