Nonlinear Macromodeling of Amplifiers and Applications to Filter Design.

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Thanks to Heng Zhang for part of the material

OP AMP MACROMODELS

Systems containing a significant number of Op Amps can take a lot of time of simulation when Op Amps are described at the transistor level. For instance a 5^{th} order filter might involve 7 Op Amps and if each Op Amps contains say 12 to 15 transistors, the SPICE analysis of a circuit containing 60 to 75 Transistors can be too long and tricky in particular for time domain simulations. Therefore the use of a macromodel representing the Op Amp behavior reduces the simulation time and the complexity of the analysis.

The simplicity of the analysis of Op Amps containing macromodels is because macromodels can be implemented using SPICE primitive components. Some examples of macromodels are discussed next.

http://www.analog.com/static/imported-files/application_notes/4813614450026940863180101 6AN138.pdf

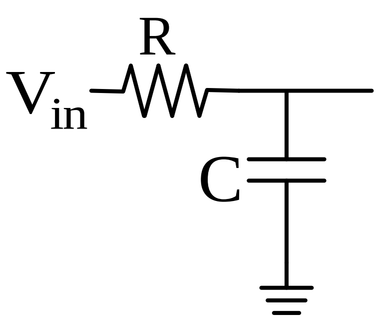
http://www.national.com/analog/amplifiers/spice_models

FUNDAMENTAL ON MACROMODELING USING ONLY PRIMITIVE SPICE COMPONENTS

1. Low Pass First Order

$$H_{LP_1} = \frac{k}{1 + s/\omega_p}$$

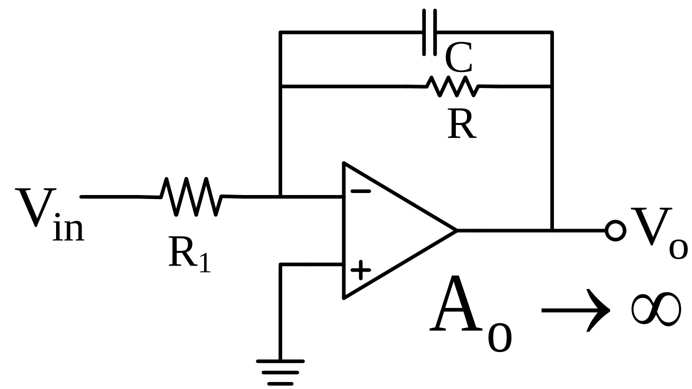
Option 1



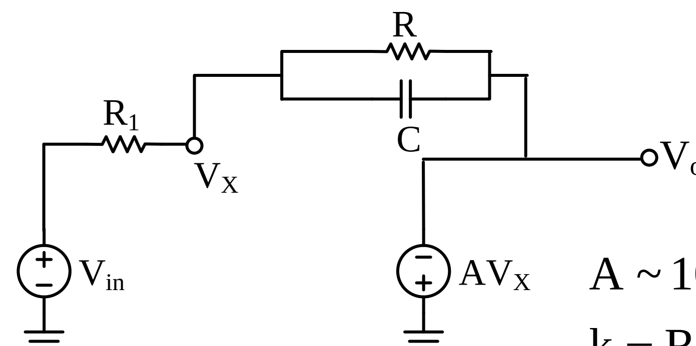
$$\omega_p = \frac{1}{RC}$$

$$k = 1$$

Option 2



$$A_o \rightarrow \infty$$

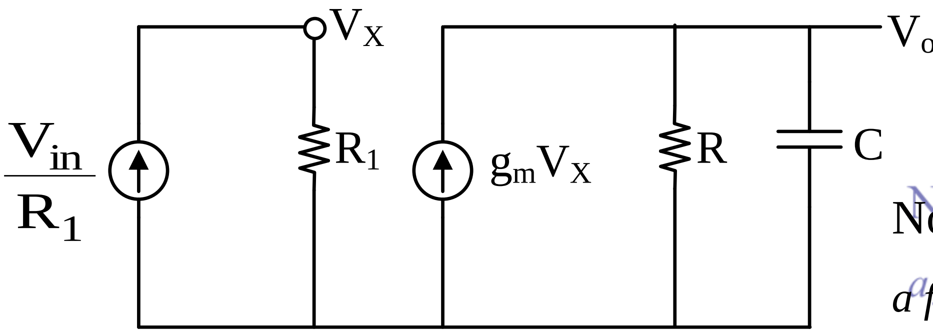


$$A \sim 10^9$$

$$k = R/R_1$$

$$\omega_p = \frac{1}{RC}$$

Option 3



Note.- *If you need to isolate the output use a final VCVS with a gain of one*

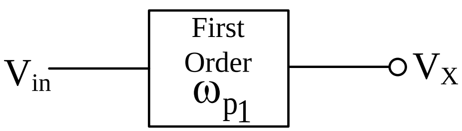
$$k = g_m R \quad ; \quad \omega_p = \frac{1}{RC}$$

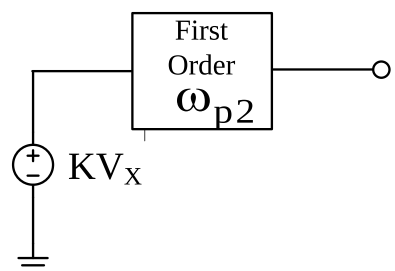
2. Higher Order Low Pass

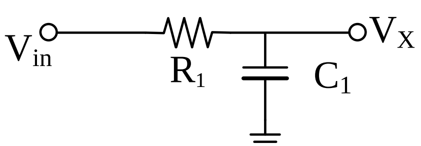
$$H_{LP_2} = \frac{K}{\left(1 + s/\omega_{p_1}\right)\left(1 + s/\omega_{p_2}\right)}$$

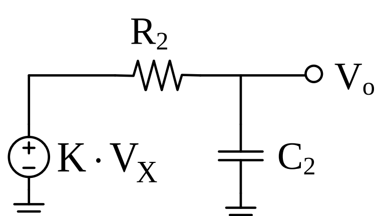
Le us consider a second-order case:

Concept. –







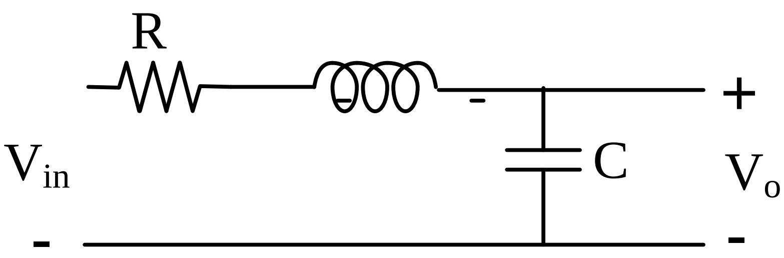


$$\omega_{p_1} = 1/R_1 C_1$$

$$\omega_{p_2} = 1/R_2 C_2$$

$$K = K$$

$$H_{LP_3} = \frac{K_o}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$



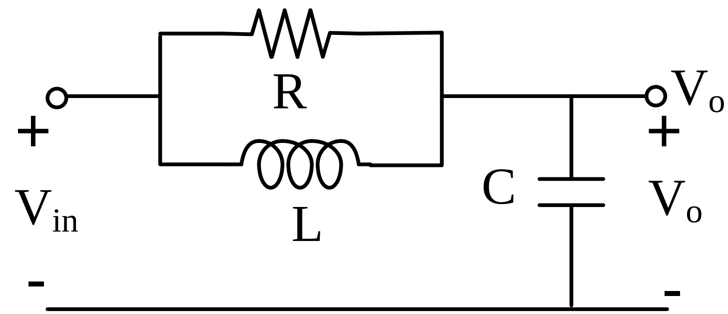
$K_o = 1/LC$; $H_{LP_3}(0) = 1$

$\omega_o^2 = 1/LC$

$\frac{\omega_o}{Q} = \frac{R}{L}$

Resonator (one zero, two complex poles)

$$H_R = \frac{k(1 + s/\omega_z)}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$



$k = 1/LC$

$\omega_z = R/L$

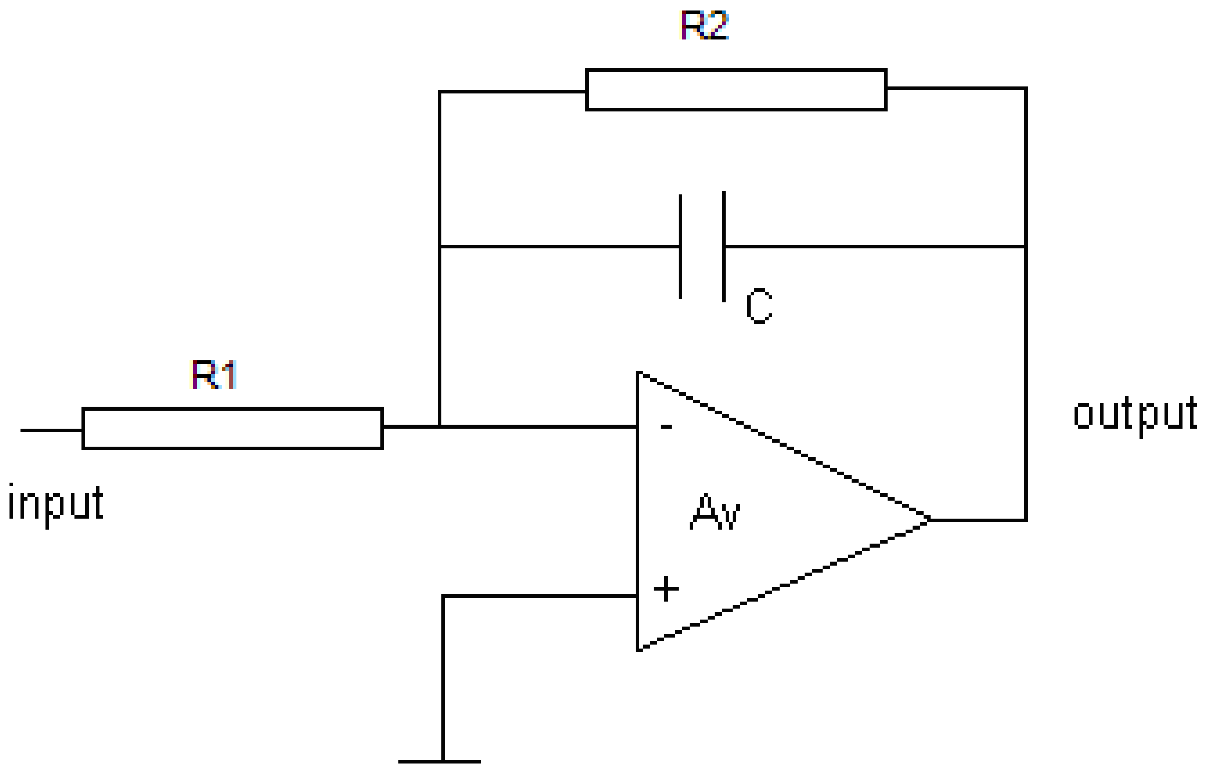
$\omega_o^2 = 1/LC$

$\frac{\omega_o}{Q} = 1/RC$

Active RC Filter Design with Nonlinear Opamp Macromodel

- Design a two stage Miller CMOS Op Amp in 0.35 μm and propose a macromodel containing up to the seventh-harmonic component
- Compare actual transistor model versus the proposed non-linear macromodel
- Use both macromodel and transistor level to design a LP filter with H(o) =10dB, f_{3dB}=5 MHz
- Result comparison

1st order Active-RC LP filter



Filter transfer function with Ideal Opamp

$$H_{LP,ideal}(s) = -\frac{R_2}{R_1}\frac{1}{(1+sR_2C)} \qquad (1)$$

$$H(o) = 10dB \rightarrow \frac{R_2}{R_1} = 10dB = 3.16 \qquad (2)$$

$$f_{3dB}=5\ MHz \rightarrow \frac{1}{R_2C} = 6.28*5M = 31.4Mrad \qquad (3)$$

Choose R1, R2 and C from equations (1) ~ (3). To minimize loading effect, R2 should be large enough. Here we choose R2 = 31.6kΩ, R1 = 10kΩ, and C = 1pF.

Filter transfer function with finite Opamp gain and GBW

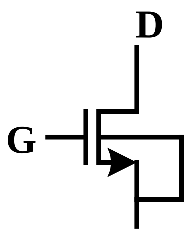
- One pole approximation for Opamp Modeling: Av = GB/s.(it holds when GBW >> f_{3dB} and Av(0) >>1)

$$H_{LP,nonideal}(s) = \frac{-R_2}{R_1(1+\frac{s}{GB})(1+sR_2C)+\frac{s}{GB}R_2}$$

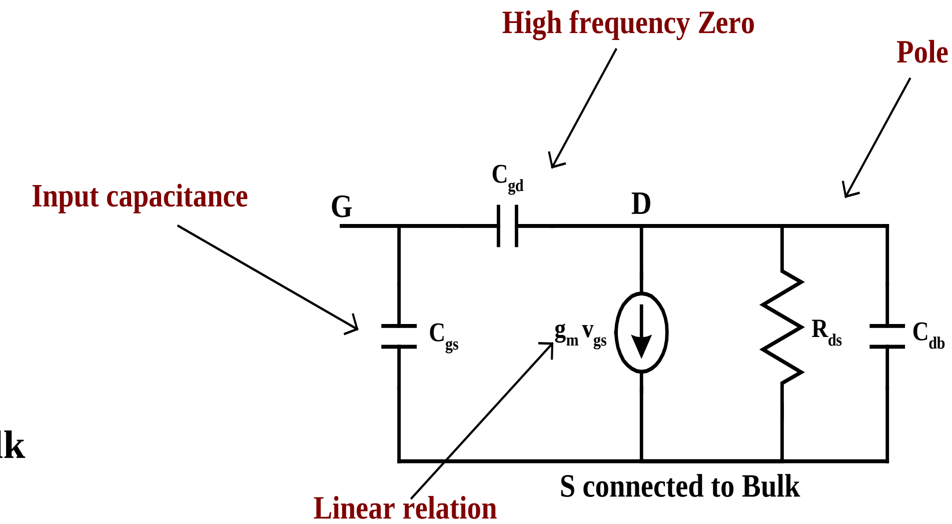
- A two stage Miller Op amp is designed. GBW is chosen ~20 times the f_{3dB} to minimize the finite GBW effect; GBW = 100MHz is also easy to achieve in 0.35μm CMOS technology.

Non-Linear Macro-Model for a source-degenerated OTA

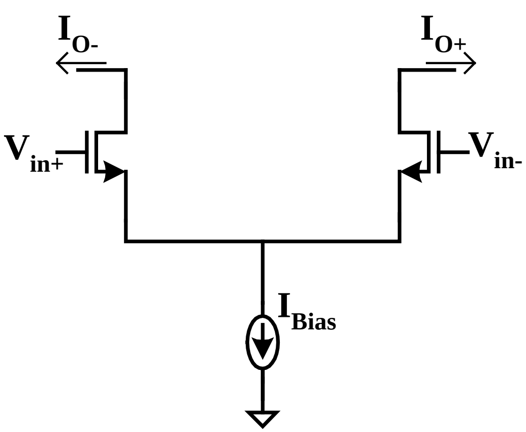
Linear Transistor Model:

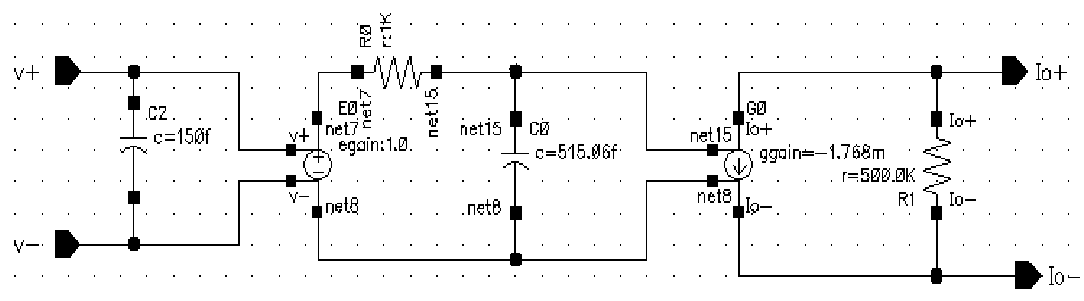


S connected to Bulk



Linear OTA model:

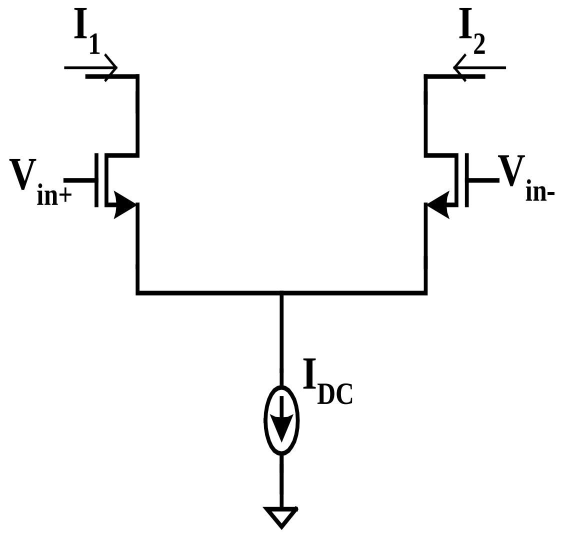




Non-Linear OTA model:

Let: $I_0 = I_1 - I_2,\ I_{DC} = I_1 + I_2$

$$V_{GS1} - V_{GS2} = V_{in}^{+} - V_{in}^{-} = v_d$$



We can easily get:

$$I_0 = v_d \sqrt{\beta I_{DC}} \left(1 - \frac{\beta v_d^2}{4 I_{DC}} \right)^{1/2}$$

Which can be expanded to: $I_0 = \alpha_1 v_d + \alpha_3 v_d^3 + O(v_d^5)$

To determine Odd Harmonic effects for an ideal OTA !!

How to Extract the Coefficients:

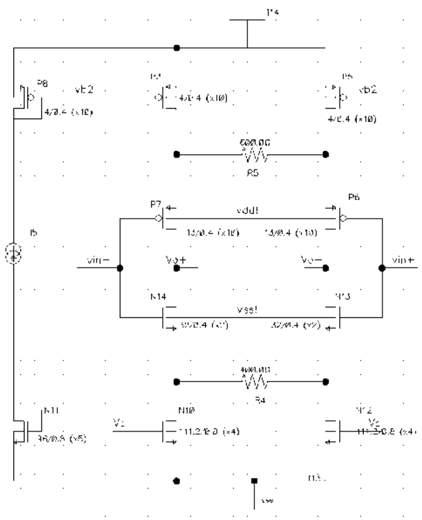
Generally if we have: $i_{out} = a_o + a_1 v_d + a_2 v_d^{\ 2} + a_3 v_d^{\ 3}$

We can extract the coefficients by differentiation, where:

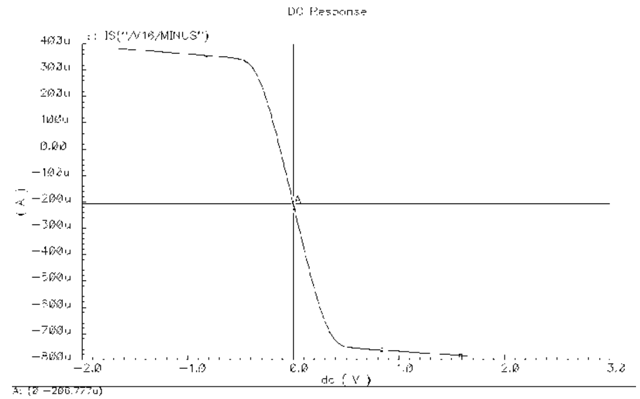
$$At \ \ v_d = 0, \qquad i_{out} = a_o \qquad \frac{d}{dv_d} i_{out} = a_1 \qquad \frac{d^2}{dv_d^{\ 2}} i_{out} = 2a_2 \qquad \frac{d^3}{dv_d^{\ 3}} i_{out} = 6a_3$$

- By Sweeping the input voltage and integrating the output current, we can these coefficients.
- a_2 is ideally zero.
- Getting the first 3 coefficients only is a valid approximation.

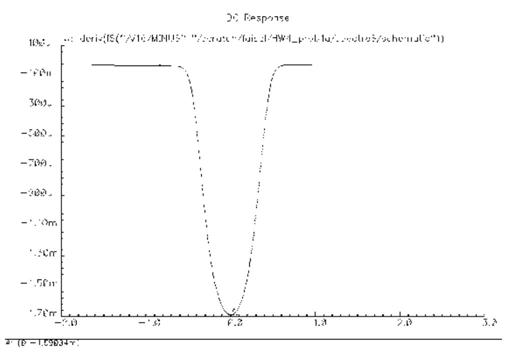
A source degenerated OTA as an example:



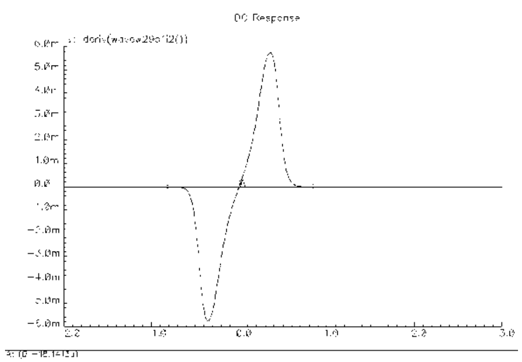
OTA



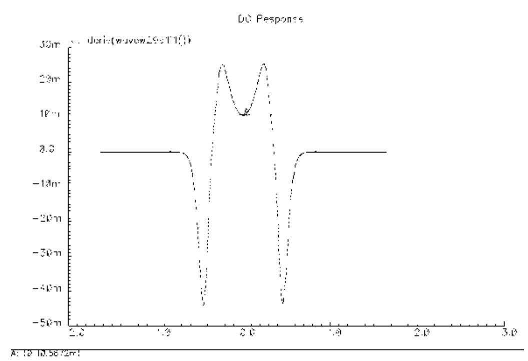
Output current of one branch versus input differential voltage.



1st derivative



2nd derivative



3rd derivative

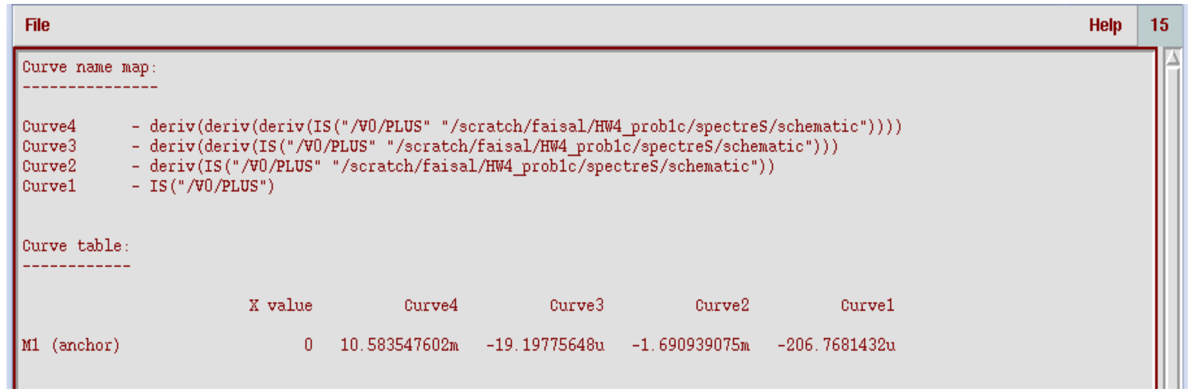
Coefficients:

a_0=206.777 µA

a_1=1.69094mA/v

a_2=9.07µA/v2

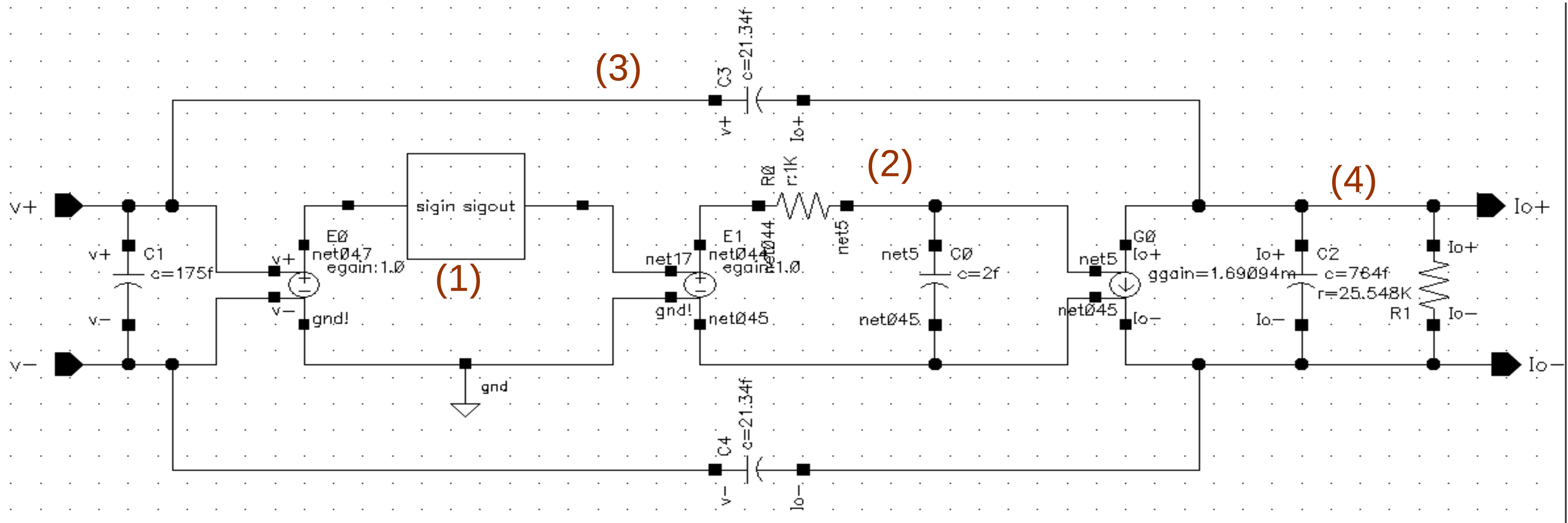
a_3= - 1.764mA/v3



The accuracy of these numbers depends on the number of points used in the DC sweep.

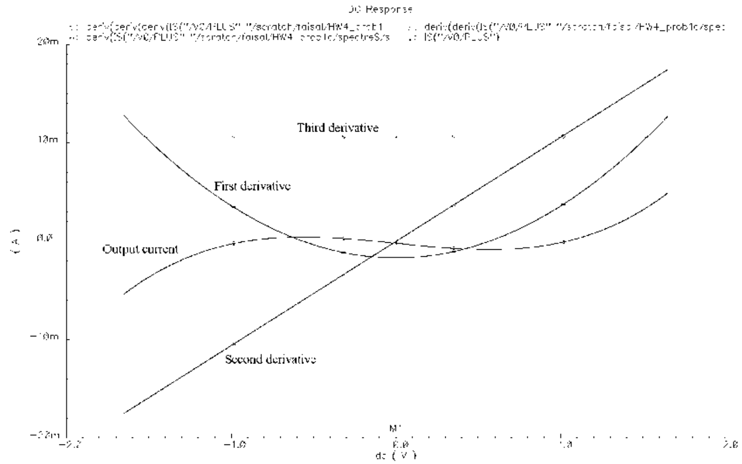
By taking more points, even harmonics reduce to zero.

Macromodel used:



1. Non-linear transfer function.
2. non-dominant pole .
3. Feed-forward path leads to Right half plane zero. (C_{gd} of the driver trans.)
4. Output Resistance and Load Capacitance.

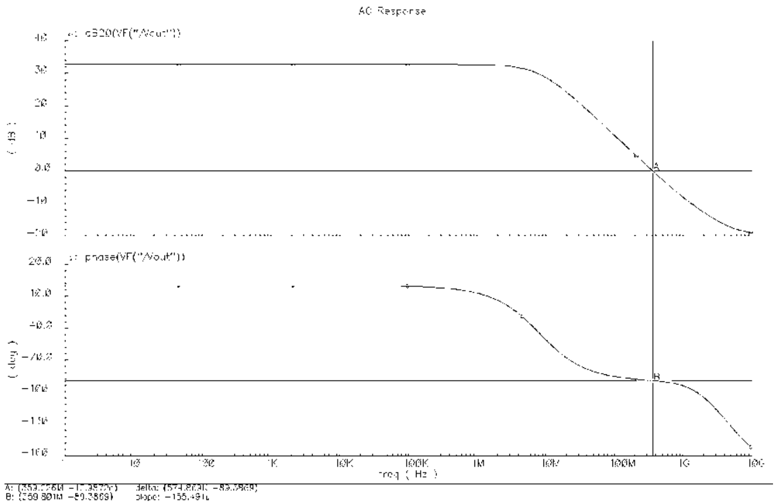
DC sweep of Macromodel:



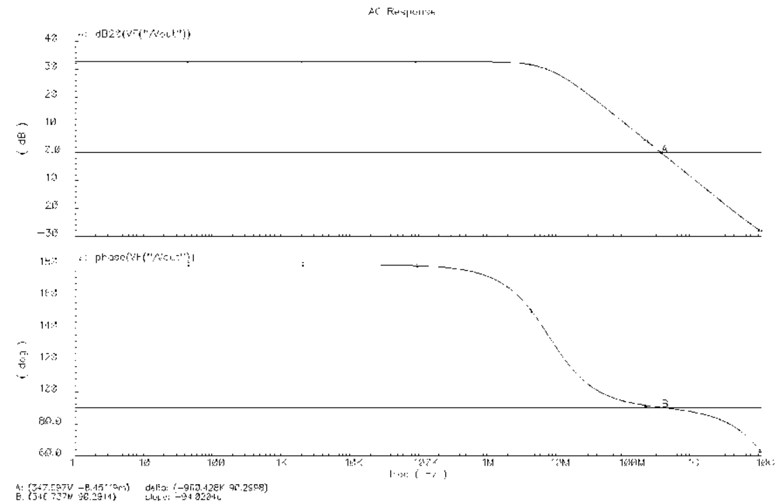
	Transistor level	Macro-model	Percentage of error
a_0 (μA)	206.777	206.768	4.3 m %
a_1 (mA/V)	1.69094	1.69094	0
a_2 ($\mu A/V^2$)	9.07	9.5988	5.5 %
a_3 (mA/V^3)	-1.764	-1.7639	5.66 m %

Changes due to measurement accuracy and number of points

AC response comparison:



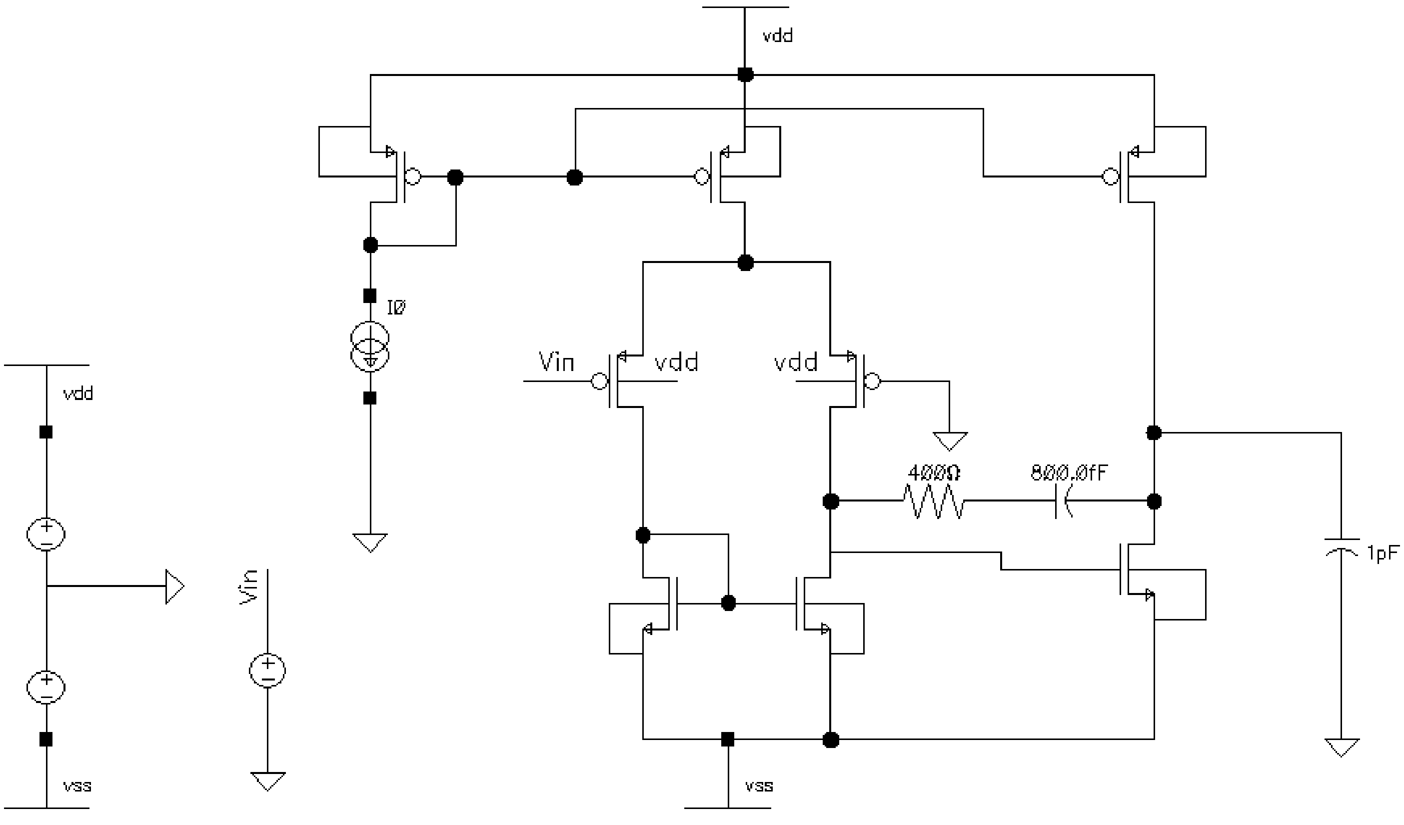
Transistor level



Macro-model

	Transistor level	Macro-model	Percentage of error
DC gain	32.8 dB	32.8 dB	0
w_o	359.226 MHz	347.69 MHz	3.3 %
Phase margin	90.62 ^{0}	89.7 ^{0}	1.015 %

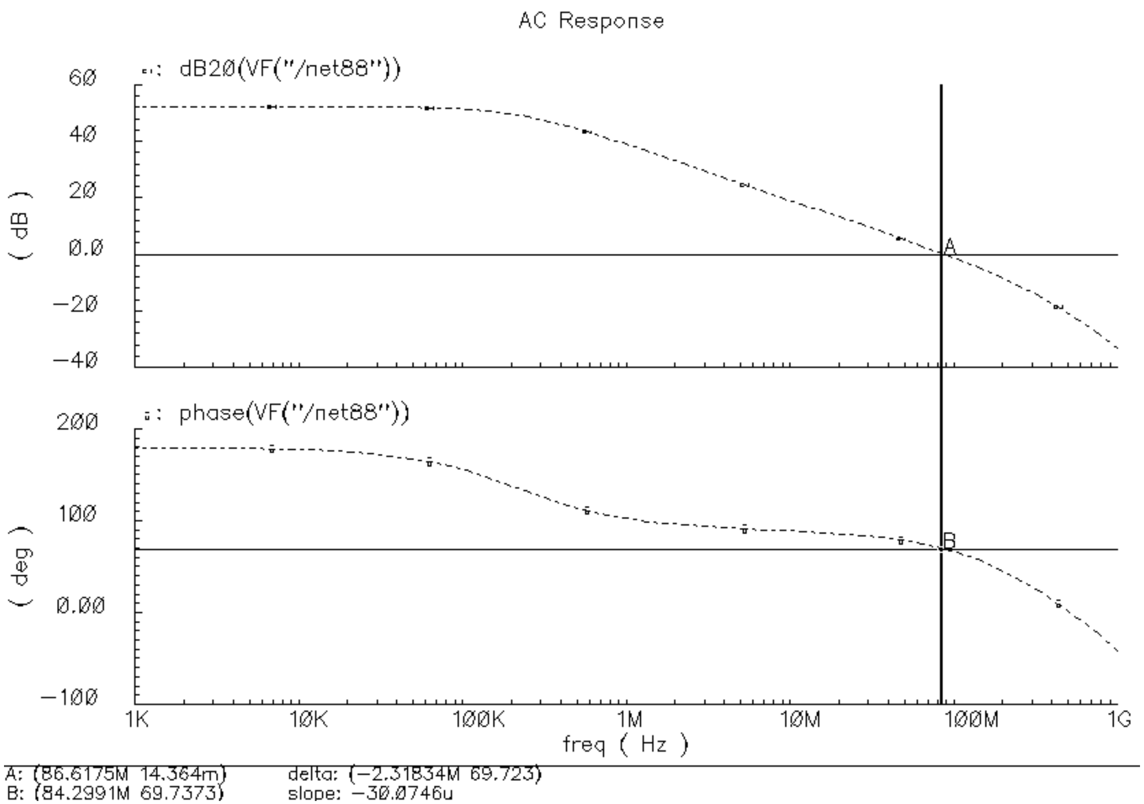
Two stage Miller Amplifier Design



Opamp Design parameters

Power	278uA @ 3V	
1st Stage	PMOS(W/L)	30u/0.4u
	NMOS(W/L)	15u/0.4u
2nd Stage	PMOS(W/L)	120u/0.4u
	NMOS(W/L)	60u/0.4u
Miller Compensation	Cm	800fF
	Rm	400 Ω

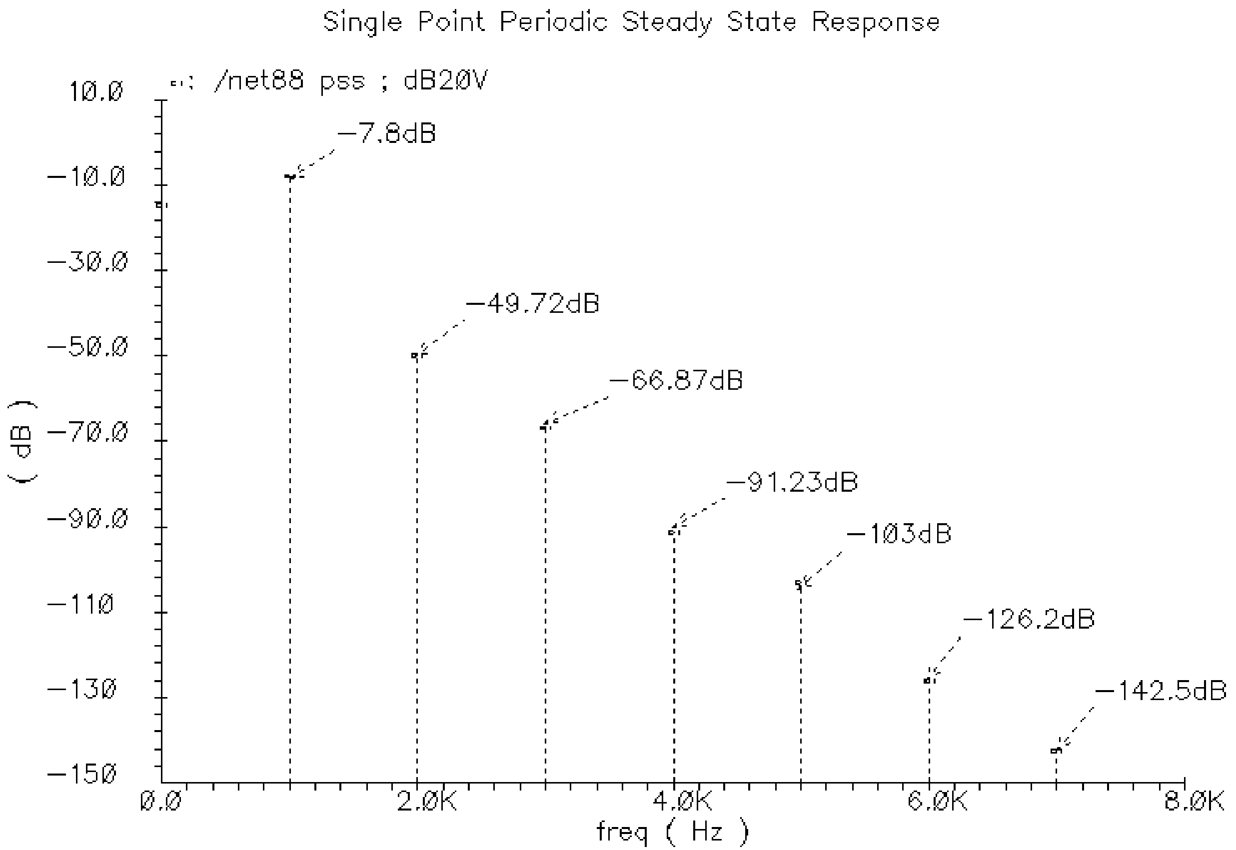
OPAMP Frequency response



- DC Gain: 53 dB, GBW: 86.6 MHz, phase margin: 69.7 deg.
- Dominant pole:154KHz, Second pole: 197MHz

Output Spectrum of Open loop OPAMP

1mVpp input @ 1KHz (THD= -49.2dB)



$$V_{out} = a_o + a_1 v_d + a_2 v_d^2 + a_3 v_d^3 + a_4 v_d^4 + a_5 v_d^5 + a_6 v_d^6 + a_7 v_d^7$$

- a_1~a_7 can be extracted from PSS simulation results:

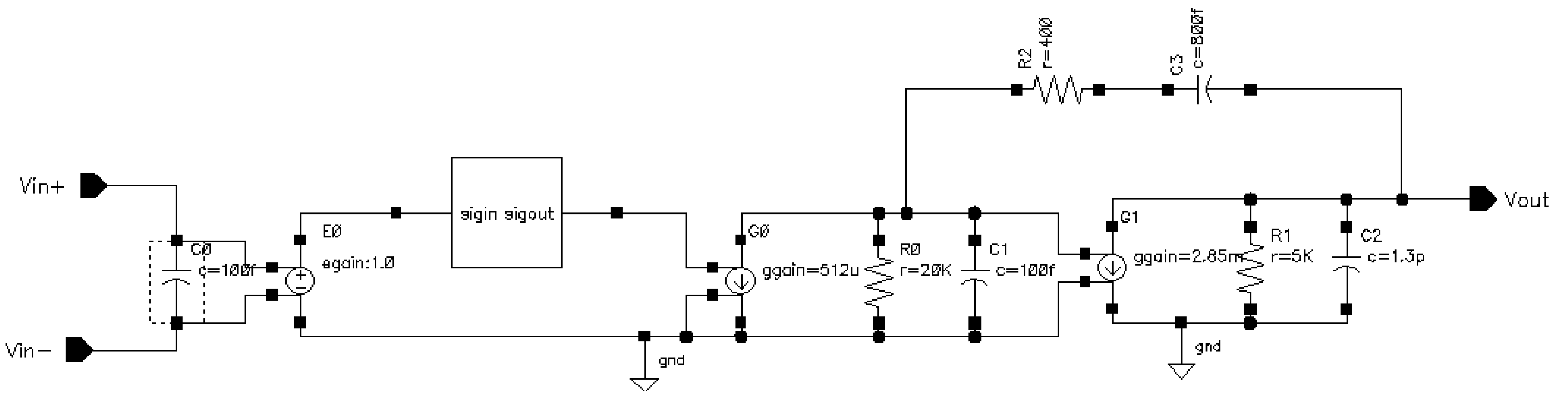
a_1 = DC gain = 450

$$HD_2 = \frac{a_2 A}{2a_1} = 41.92dB \quad \rightarrow a_2 = 2934$$

$$HD_3 = \frac{a_3 A^2}{4a_1} = 59.1dB \quad \rightarrow a_3 = 2.16e6$$

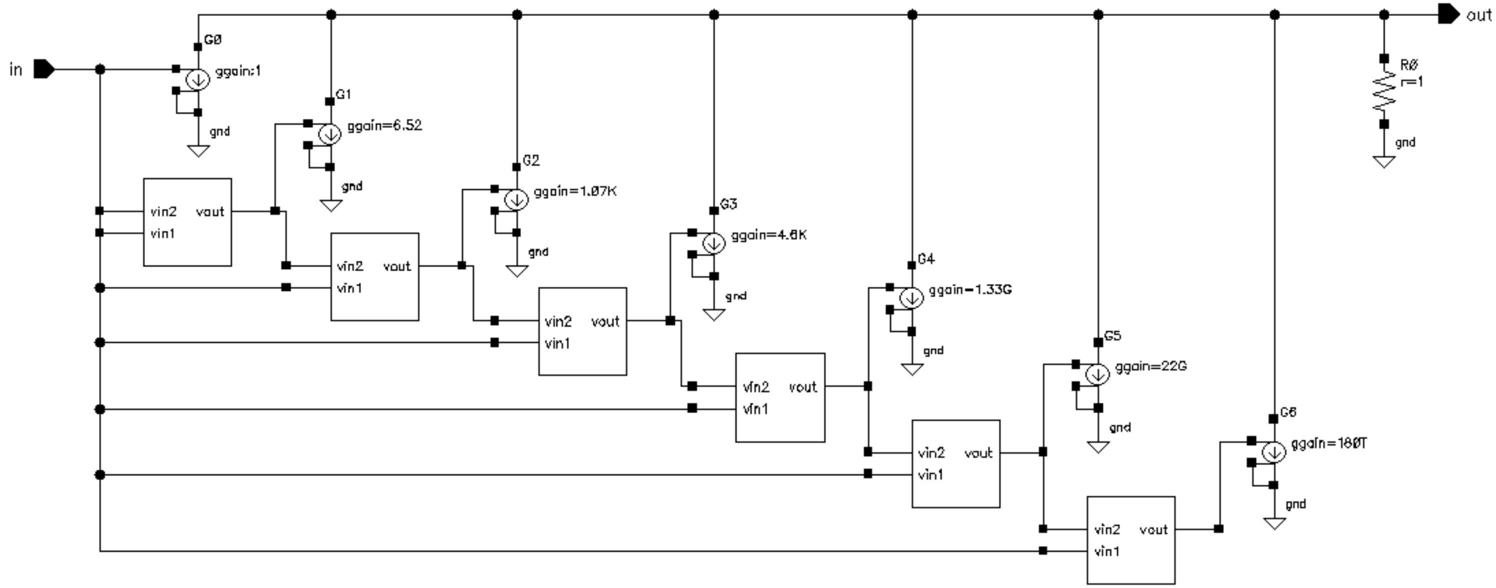
Similarly, we can obtain: a_4 = 6e7, a_5 = 5.6e11, a_6 = 1e13, a_7 = 7e16

Opamp Macro model



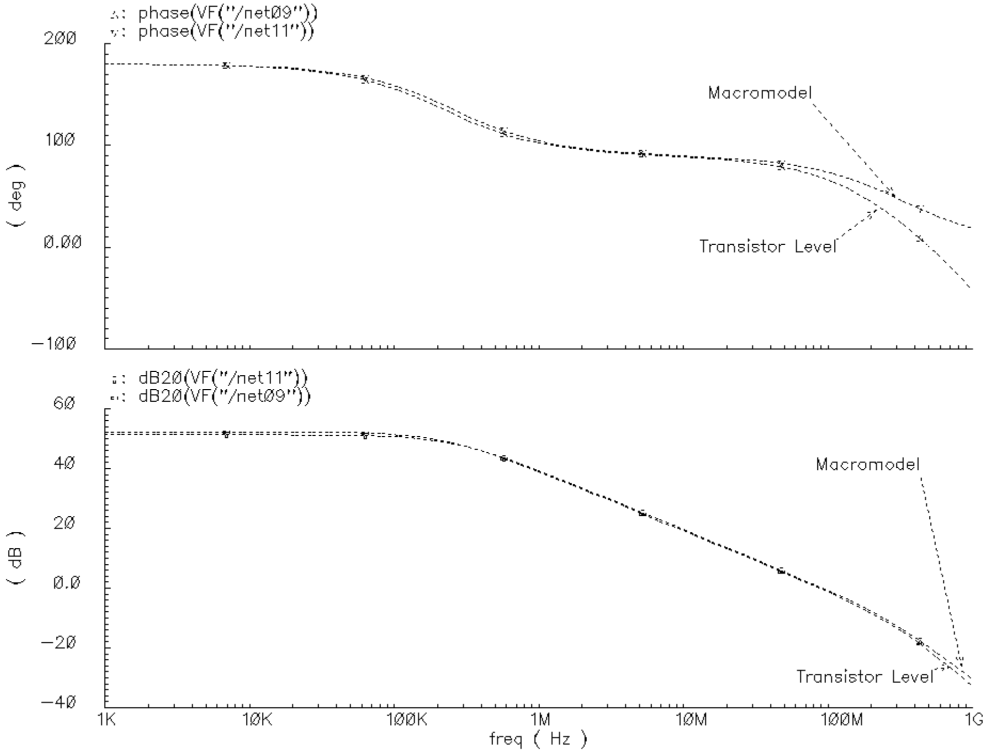
- Modeled: input capacitance, two poles, one RHP zero, nonlinearity, finite output resistance, and capacitance
- Nonlinearity model should be placed before the poles to avoid poles multiplication

Nonlinearity Model



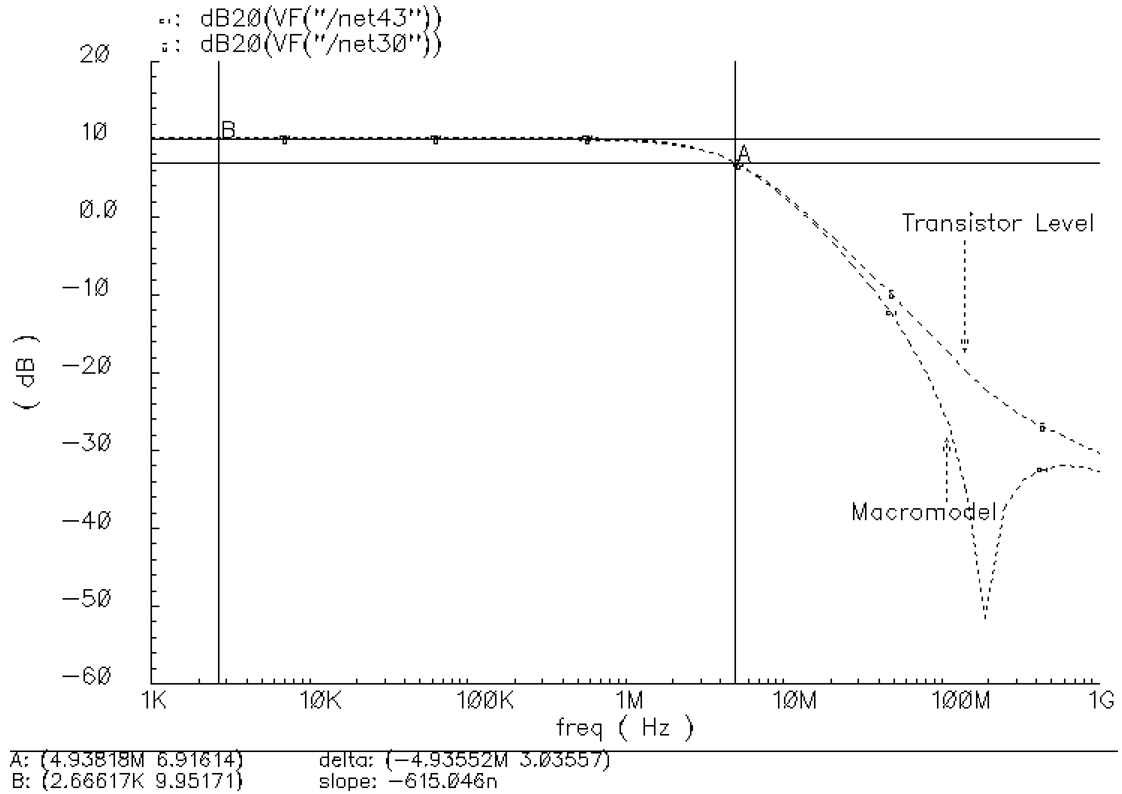
- uses mixer blocks to generate nonlinear terms
- model up to 7th order non-linearity
- set each VCCS Gain as the nonlinear coefficients.
- set the gain for 1st VCCS = gm1 = 512uA/V, gain for 2nd VCCS = gm2 = 2.85mA/V, and scale all the nonlinear coefficients derived above by a_1.

Opamp AC response: Transistor-level vs. Macromodel



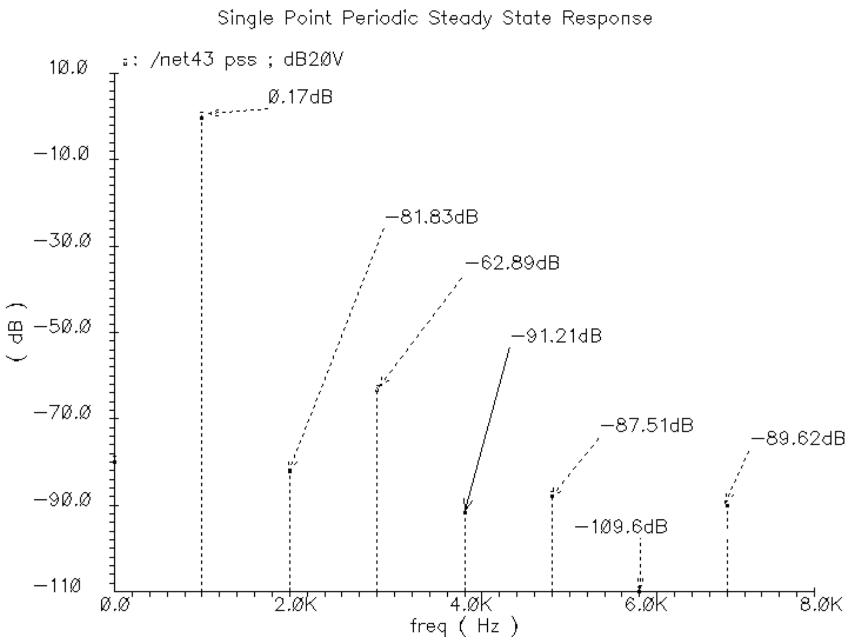
- Macro-model mimic the transistor level very well at frequencies below 10MHz
- discrepancy at higher frequency due to the higher order poles and zeros not modeled in the Macromodel

Filter AC response: Transistor-level vs. Macromodel

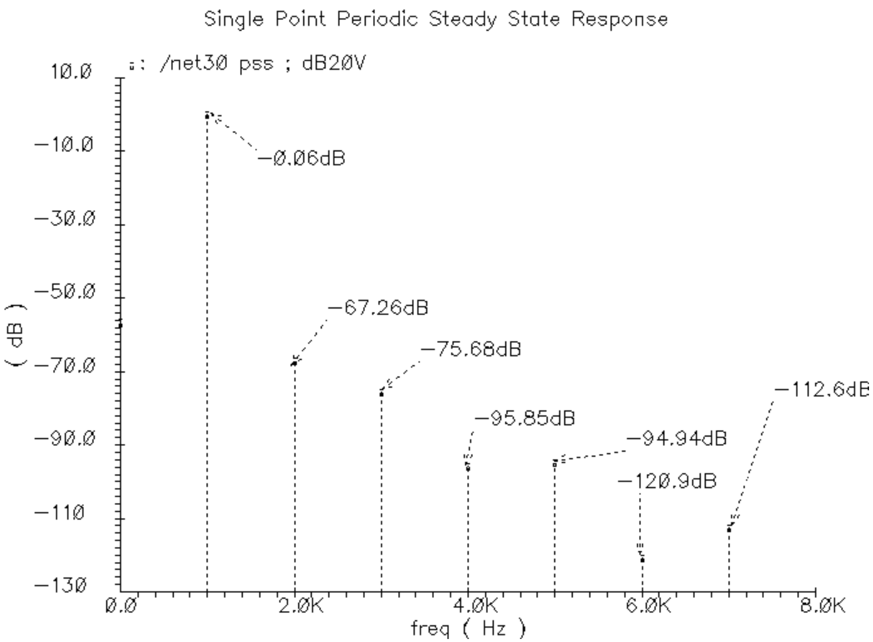


Output Spectrum (0dBm input @ 1KHz)

- Macromodel(THD=-63dB)



- Transistor Level
- (THD = -66.4dB)



Performance Comparison

Table I. Open loop Opamp Performance Comparison

	Transistor Level	Macro-model
-3dB BW	154KHz	180KHz
GBW	86.6MHz	90MHz
DC Gain	53 dB	51.3 dB
Phase Margin	69.7 degree	74.9 degree
THD: -50dBm @ 1KHz	-49.2 dB	-49.6 dB

Table II. LPF Performance Comparison

	Transistor Level	Macro-model
BW of LPF	4.9MHz	4.86MHz
DC Gain of LPF	9.95 dB	10.19 dB
THD: 0dBm @ 1KHz	-66.4 dB	-63dB

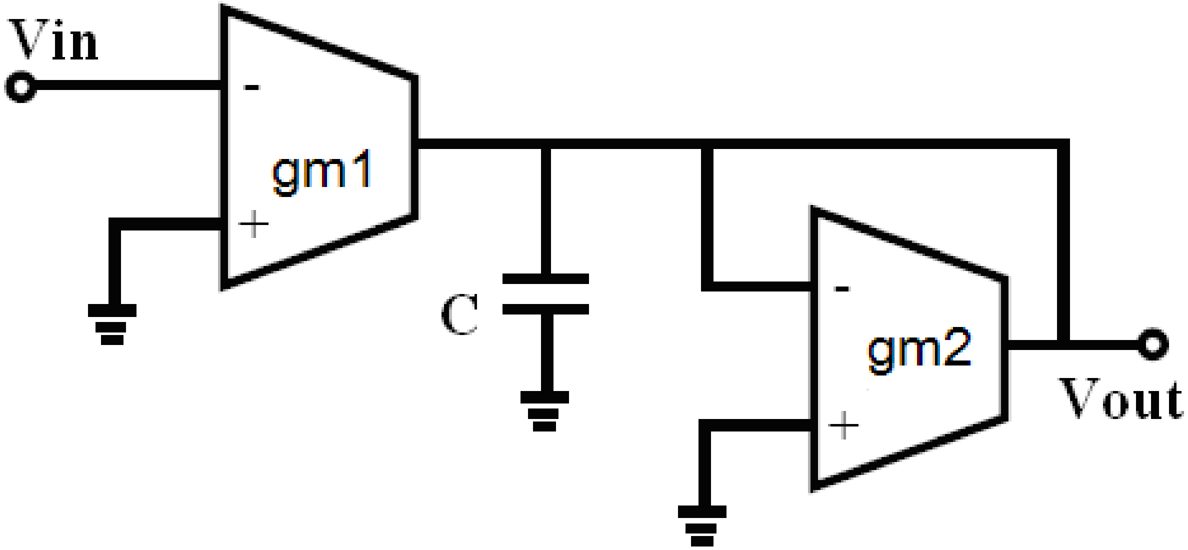
Observation

- THD of the LPF at 0dBm input is better than that of the open loop Opamp with a small input at -50dBm. This is because OPAMP gain is ~50 dB, when configured as a LPF, OPAMP input is attenuated by the feedback loop→ better linearity.
- when keep increasing the input amplitude, the THD of the transistor-level degrades dramatically. This is because large swing activates more nonlinearity and even cause transistors operating out of saturation region; however, the THD of Macro-model doesn’t reflect this because we didn’t implement the limiter block.

Gm-C Filter Design with Nonlinear Opamp Macromodel

- Use a three current mirror Transconductance Amplifier.
- Compare actual transistor model versus the non-linear macromodel
- Use both macromodel and transistor level to design a LP filter with H(o) =10dB, f_{3dB}=5 MHz
- Result Comparison

1st order Gm-C LP filter



Filter transfer function

- With Ideal OTA:

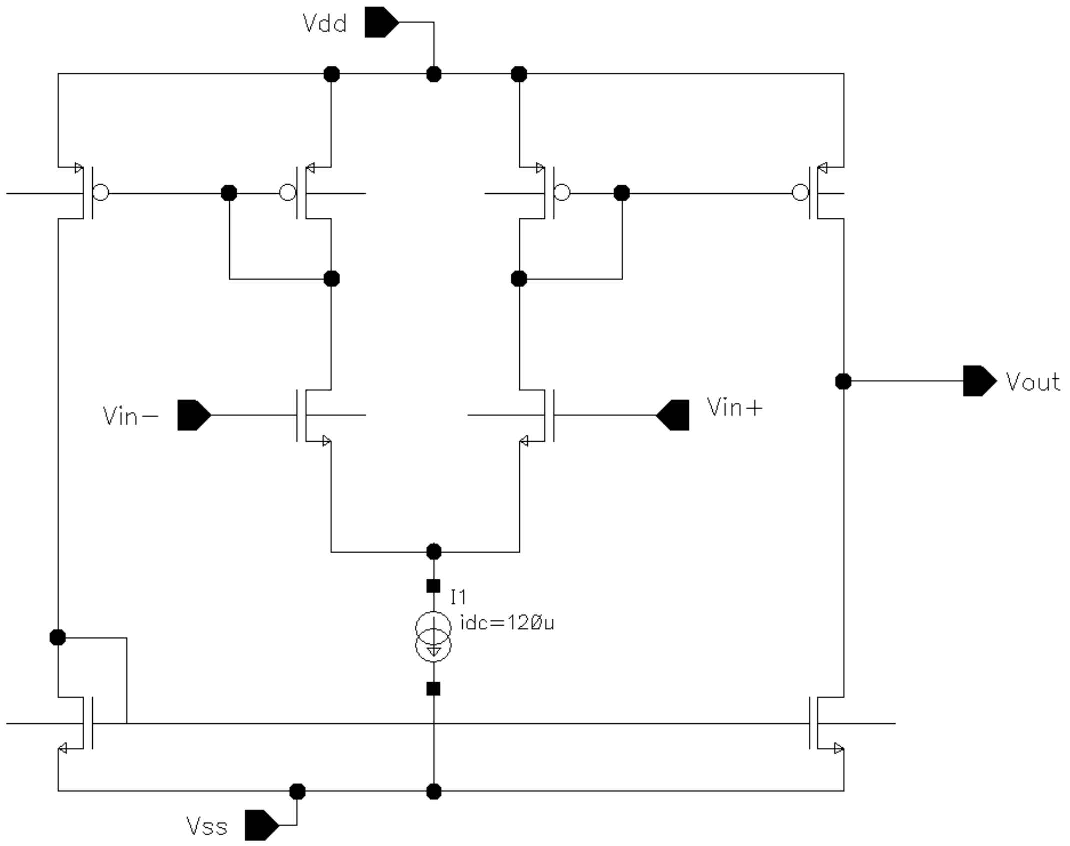
$$H(s)_{ideal} = -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + sC / g_{m2}}$$

H(o) = 10dB → $\frac{g_{m1}}{g_{m2}}$ = 10dB = 3.16

f_{3dB}=5 MHz → $\frac{g_{m2}}{C}$ = 6.28*5M = 31.4Mrad

Output resistance of gm1 should be >> 1/gm2
Choose C = 4pF, →gm2 = 0.126mA/V, gm1 = 0.4mA/V

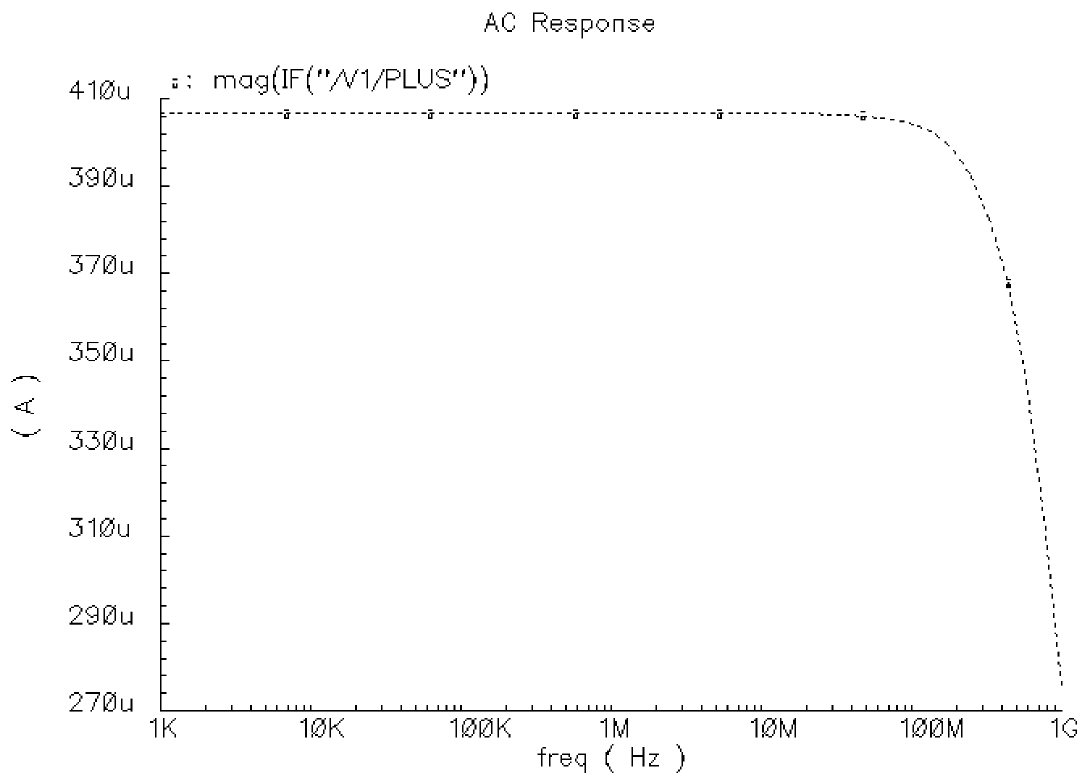
Three Current mirrors OTA Design



OTA Design parameters

Power	240uA @ ± 1.5V
Input NMOS	4u/0.6u
PMOS current mirror	12u/0.4u
NMOS current mirror	1u/0.4u

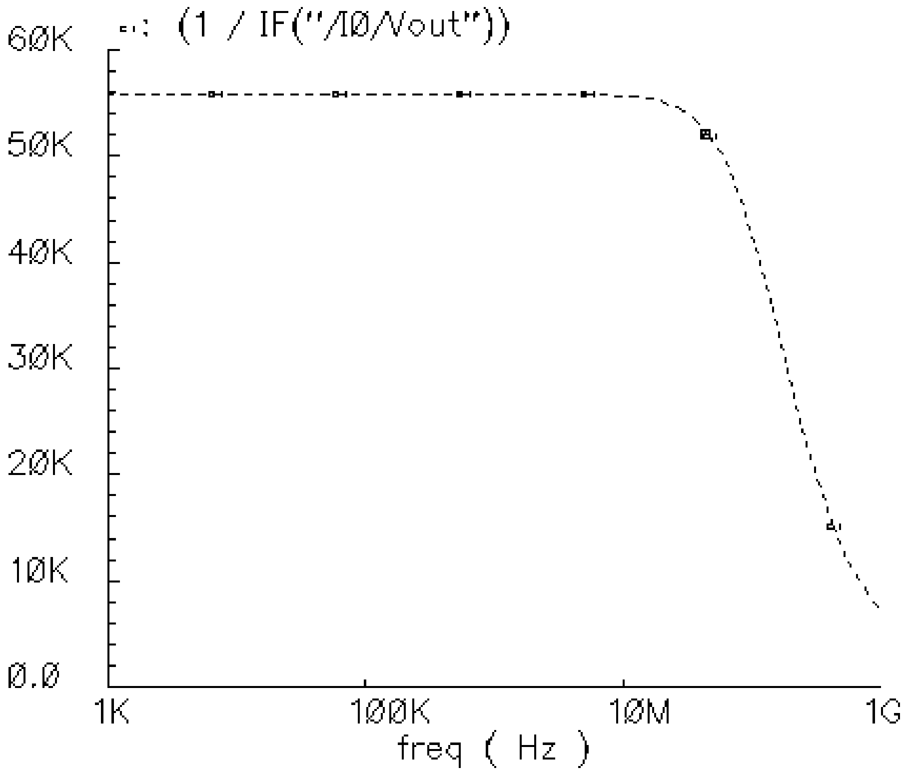
AC simulation of Gm: Transistor Level



gm = 0.4mA/V, which is our desired value

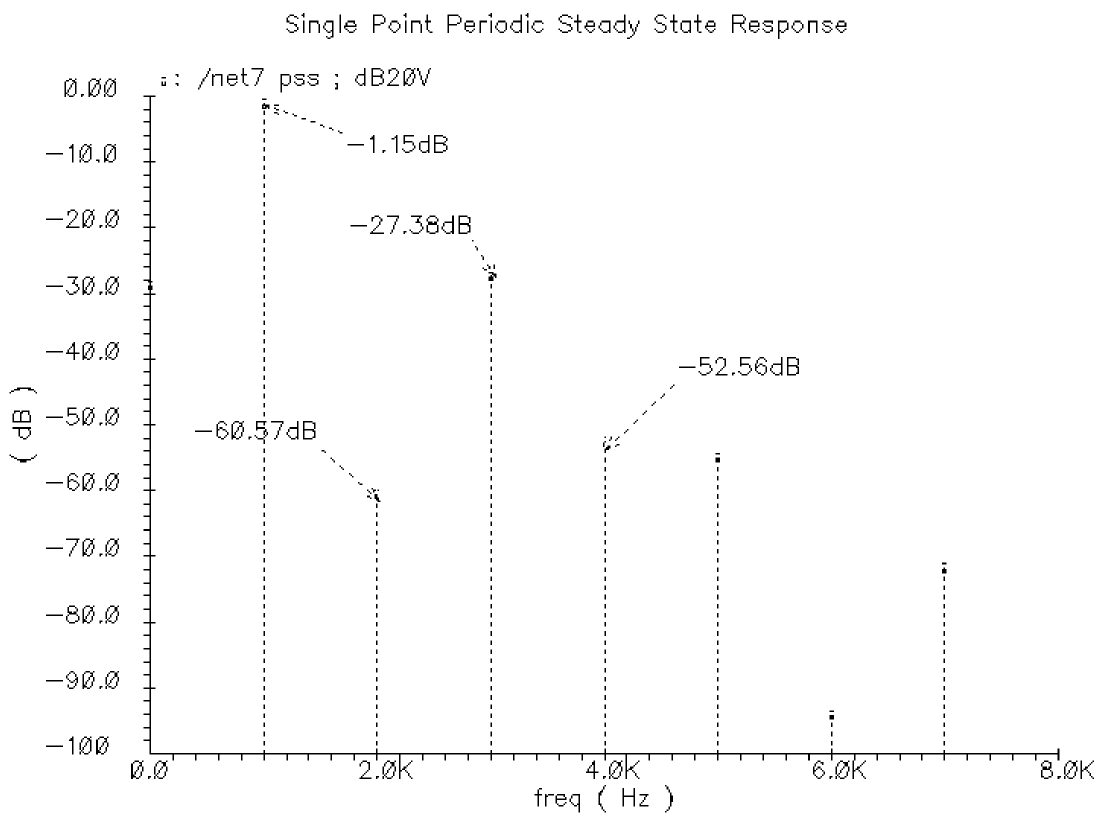
its frequency response is good enough for a LPF with 5MHz cutoff frequency

OTA Output resistance: Transistor Level



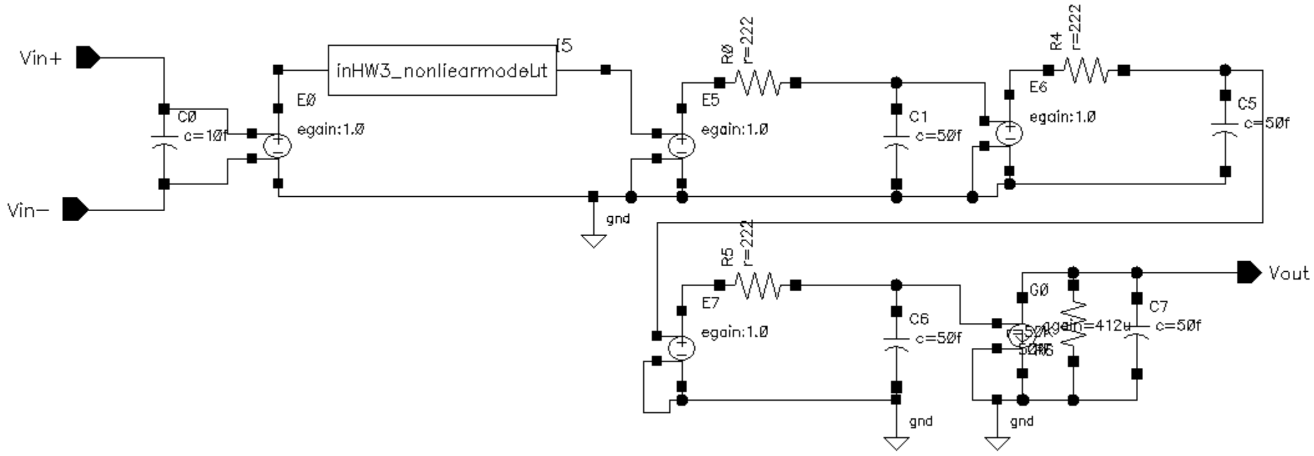
output resistance of the OTA >>1/gm2

Gm-C LPF Output spectrum: Transistor level



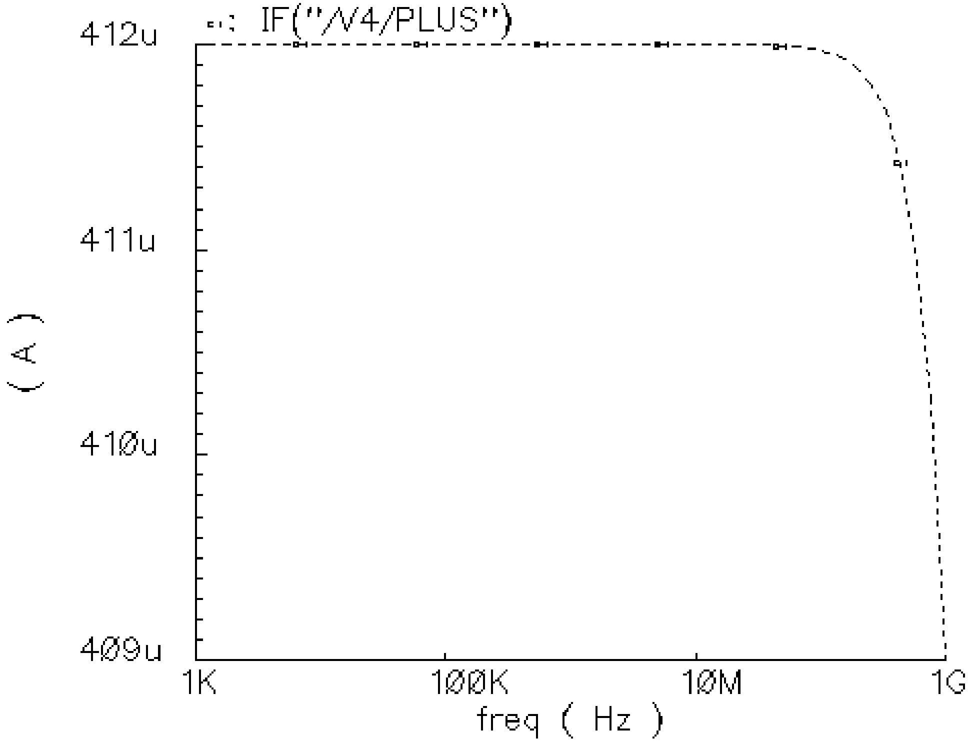
- THD = -26dB for 0dBm input@1kHz

OTA Macro model

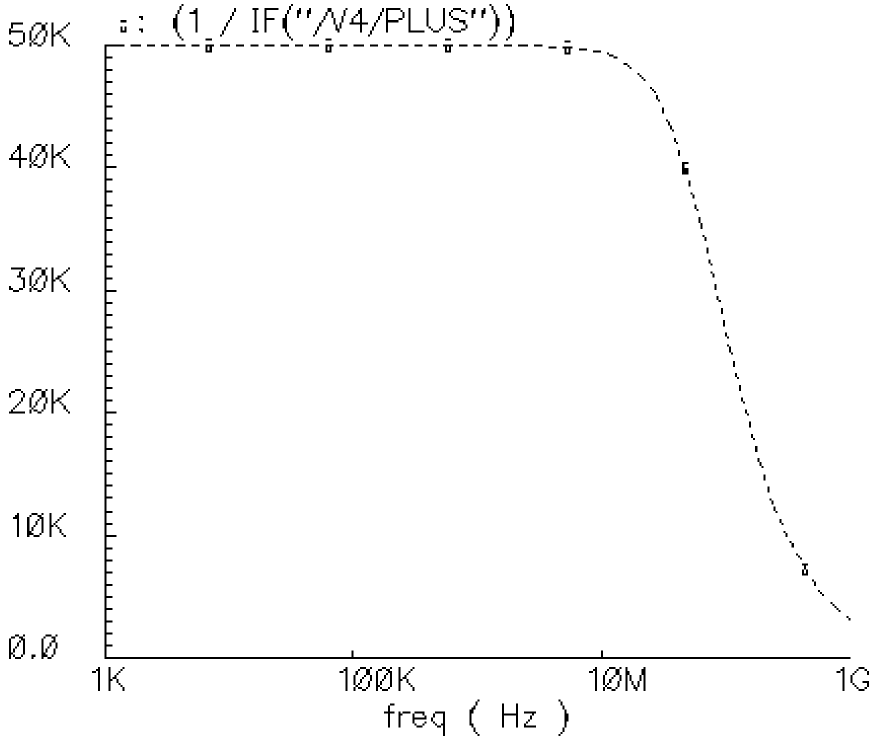


- Since the internal poles and zeros are at much higher frequency than 5MHz, only the important ones are included in the macro-model
- Nonlinearity model is the same as the Opamp in Active-RC filter

AC simulation of Gm: Macro-model

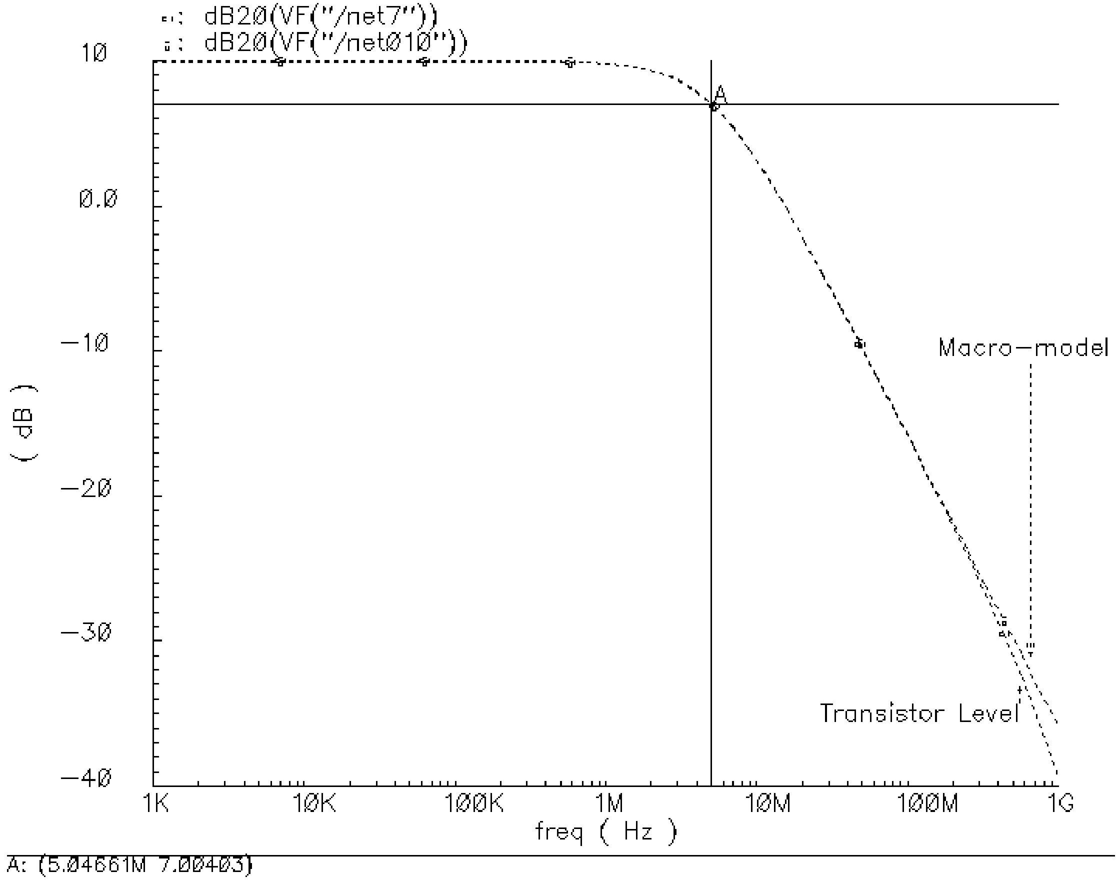


OTA Output resistance: Macro-model

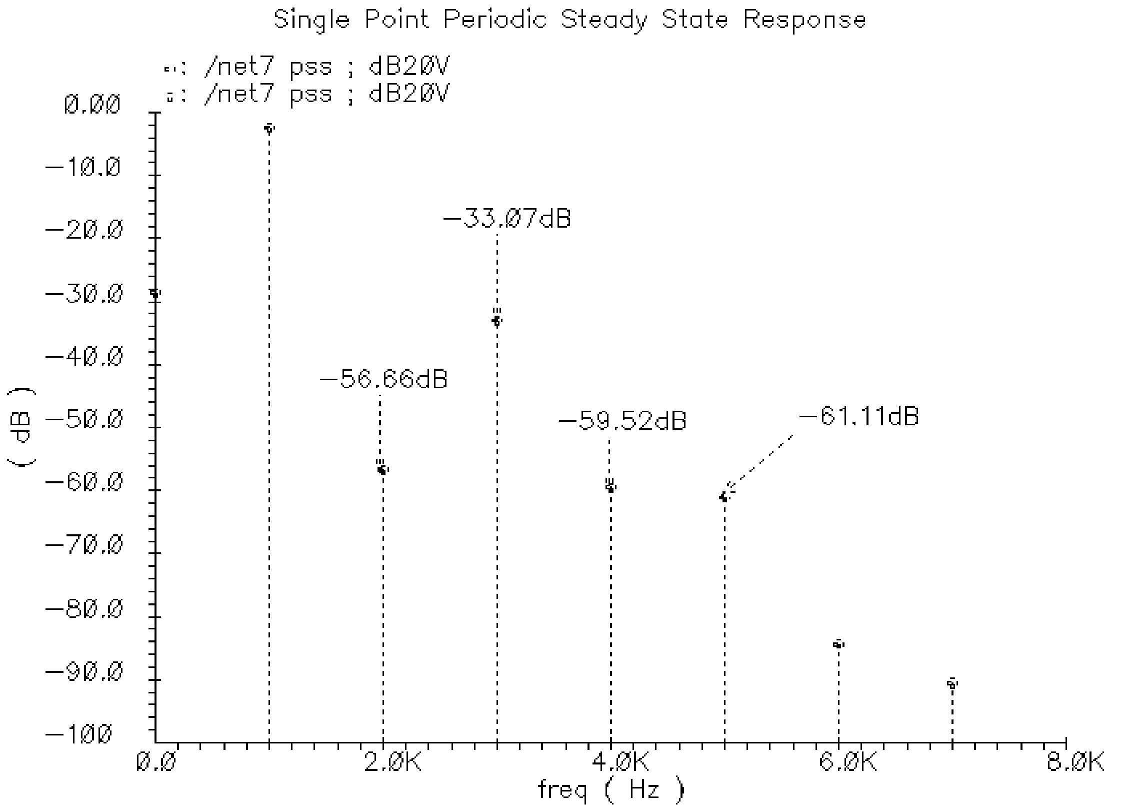


output resistance of the OTA >>1/gm2

Gm-C LPF Frequency response



Gm-C LPF Output spectrum: Macromodel



- THD = -33dB for 0dBm input@1kHz

$\sqrt{Hz}$

Performance Comparison

Table I. Gm-C Filter Performance Comparison

	Transistor Level	Macro-model
Gm	409uA/V	412uA/V
BW of LPF	5.05MHz	5.05MHz
DC Gain of LPF	10 dB	10 dB
THD: 0dBm @ 1KHz	-26 dB	-33dB

Table II. Comparison between Transistor Level Active-RC and Gm-C LPF

	Active RC	Gm-C
DC gain	9.95dB	10dB
BW	4.9MHz	5.05MHz
THD: 0dBm @ 1KHz	-66.4 dB	-26 dB
Noise Level	0.048μV/ @1kHz	0.05μV/ @1kHz
Power	0.83mW	0.72mW

Discussion

- With comparable DC gain, BW, Noise level and Power consumption, Gm-C filter has much worse linearity than Active RC because:
  - Active RC: feedback configuration improves linearity;
  - Gm-C filter: open loop operation, the gm stage sees large signal swing, thus linearization technique is needed, which adds power consumption.
- Active RC is preferable for low frequency applications if linearity is a key issue

Note that SPICE allows you to describe non-linear components:

Nonlinear capacitor and inductors

CXXXXXXXX N+ N- POLY C0 C1 C2 …….<IC=INCOND>

LYYYYYYYY N+ N- POLY L0 L1 L2 ……..< IC=INCOND>

Nonlinear dependent sources E, G, F and H type

A two-dimensional polynomial function is expressed as

f(x1,x2) = po +p1x1 +p2x2 +p3(x1)*2 + p4x1x2 + p5 (x2)*2 + p6(xi)*3
+ p7(x1)*2 x2 + p8x1(x2)*2 + p9(x2)*3

Ename N+ N- <POLY(ndim)> nc1+nc1 -<nc2+ nc2-….>
+ p0<p1<p2….>> <IC=vnc1,nc1-, vc2+,nc2-,…>

E1 3 4 POLY(1) 7 10 10.5 2.0 1.95

Which means $V_{E1} = 10.5 + 2.0V_{7,10} + 1.95(V_{7,10})*2$

References
J. Alvin Connelly and P. Choi,” Macromodeling with Spice”, Prentice Hall, Englewood Cliffs, NJ 07632. 1992