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HW1

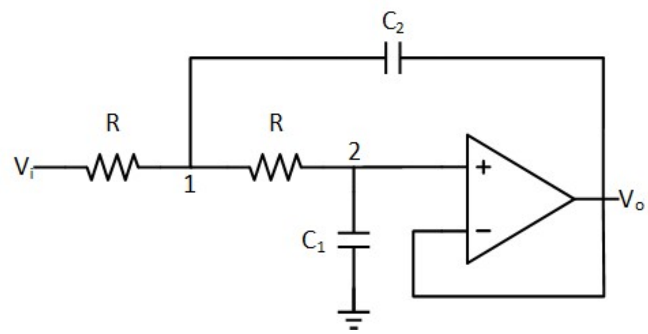
ECEN 622

9/21/2017

Problem 1

Part A

The figure below shows the schematic of the Sallen-Key lowpass filter.



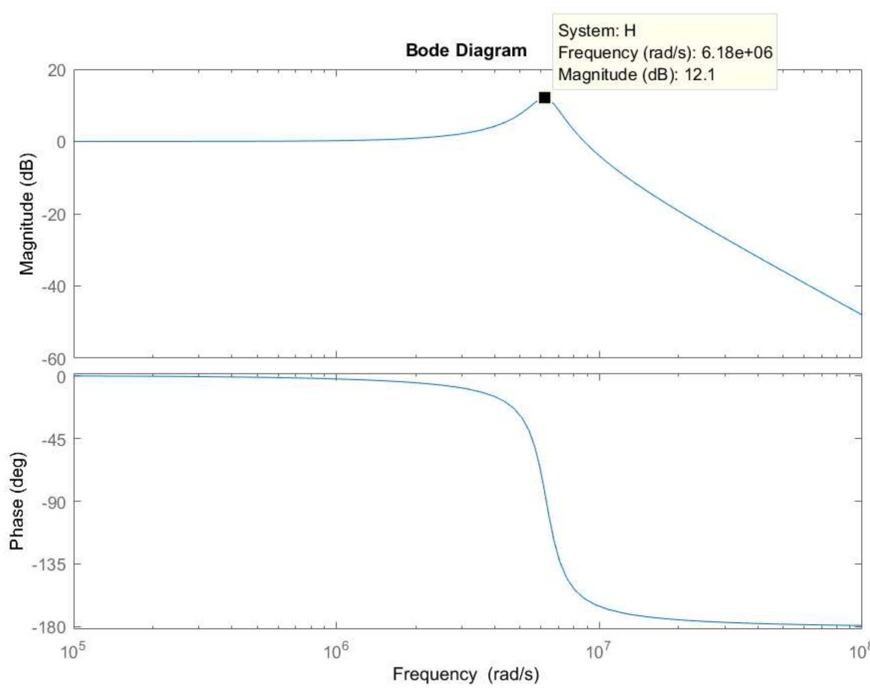
From the lecture notes, if the resistances in the Sallen-Key lowpass filter are equal, we can obtain the following:

$$C_2 = 4Q^2C_1, R = \frac{1}{2\omega_o Q C_1}$$

Therefore, we set C_1 to a reasonable value of 1pF. Therefore, we find the following:

$$C_2 = 4 \cdot 4^2 \cdot 1pF = 64pF, R = \frac{1}{2 \cdot 2\pi \cdot 10^6 \cdot 4 \cdot 1pF} = 19.9k\Omega$$

The figure below shows the magnitude and phase of the output plotted against the input frequency.



As expected, the gain magnitude at ω_o is equal to 12.1dB which matches the value of 4 set for Q. In addition, the frequency ω_o is $2\pi \cdot 10^6 rad/s$, which matches the desired value

Part B

First, the transfer function of the filter with a finite opamp gain is found. First, the gain of the amplifier can be found as:

$$V_o = \frac{A}{A+1} V_2$$

Next, we write the nodal equations for the filter:

$$V_2 + V_2 sRC_1 - V_1 = 0$$
$$2V_1 + V_1 sRC_2 - V_o sRC_2 - V_2 - V_{in} = 0$$

If we use these three equations to solve for V_o/V_{in}, we obtain the following:

$$\frac{V_o}{V_{in}}(s) = \frac{A}{1 + (1-A)RC_2 s + 2RC_1 s + R^2 C_1 C_2 s^2}$$

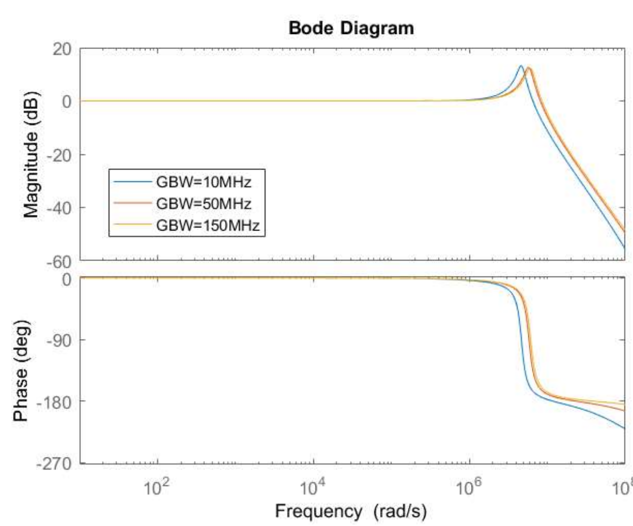
If the amplifier is ideal and A is 1, then the expression above matches the expression in the lecture notes. Next, we assume the amplifier gain is given by:

$$A = \frac{GB}{s + \omega_p}$$

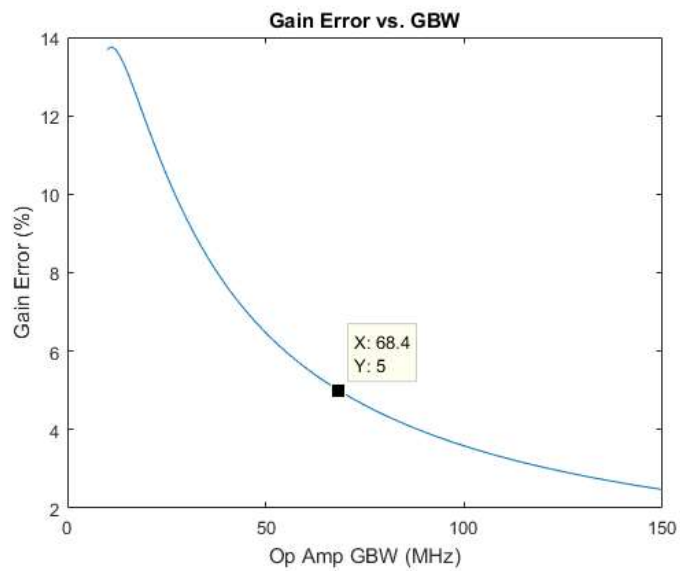
Substituting this into the above expression provides:

$$\frac{V_o}{V_{in}}(s) = \frac{GB}{(\omega_p + GB) + s\left(1 + \omega_p RC_2 + 2RC_1(\omega_p + GB)\right) + s^2\left(RC_2 + 2RC_1 + R^2 C_1 C_2(\omega_p + GB)\right) + s^3 R^2 C_1 C_2}$$

Therefore, the finite GBW of the op-amp creates a third-order response. This effectively shifts the value of ω_o downwards as well as increases the Q of the filter. This is clearly seen in the figure below, which shows the magnitude and phase response for three different GBW values. With a GBW of 10MHz, the natural frequency of the filter is clearly shifted to a lower value while the peak gain of the circuit is increased to 13.2dB.

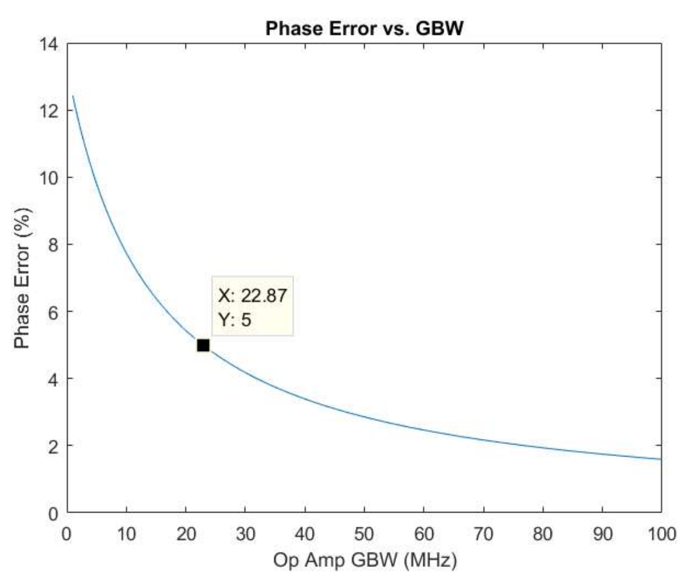


In order to quantify the required performance for the op-amp, the GBW of the op-amp is swept over a wide range and the gain error (in percent) at ω_o is calculated. This is shown in the figure below:



In order to achieve a gain error of less than 5%, the op-amp GBW must be higher than 68.4MHz. This is due to the fact that a low value for the GBW leads to an increase in the effective Q of the filter. As a result, the gain error for low GBW values is quite small.

Next, the phase error at ω_{3dB} is measured to quantify the phase performance for a wide range of op-amp GBW:

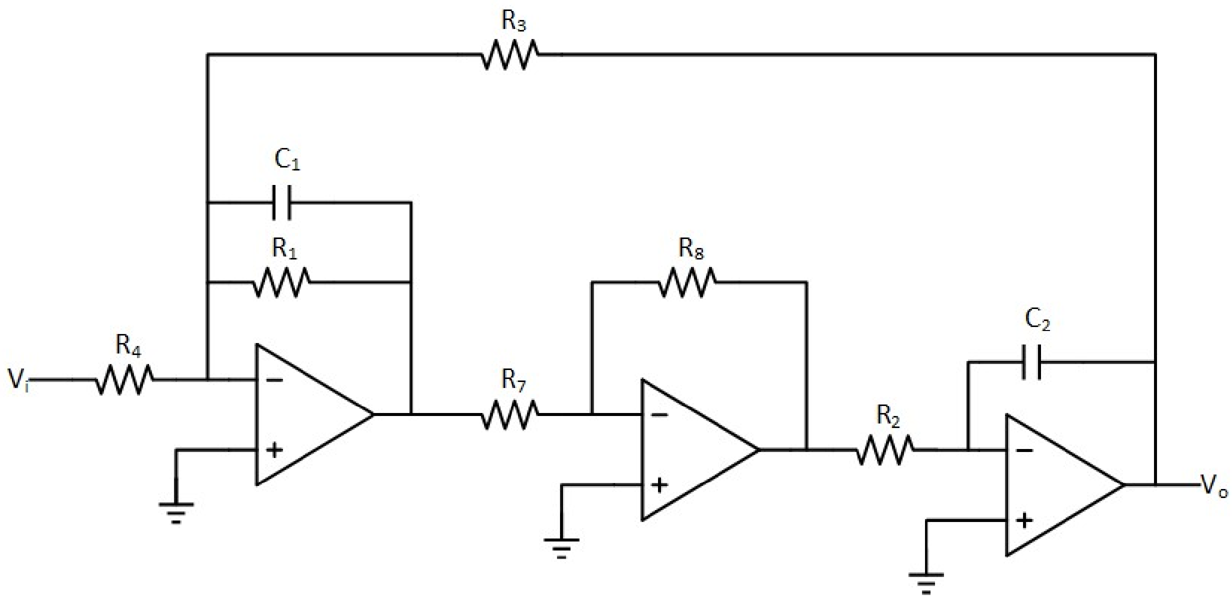


In order to achieve a phase error less than 5%, a GBW of 22.87MHz is required. This indicates that the GBW requirement for a 5% phase error is actually less demanding than the GBW requirement for a 5% peak gain error.

Problem 2

Part A

The figure below shows the Tow-Thomas biquad used to implement the LP filter:



The forward gain from the input to the output is:

$$FG(s) = -\frac{R_1}{R_4} \cdot \frac{1}{1 + \frac{s}{R_1 C_1}} \cdot \frac{R_8}{R_7} \cdot \frac{1}{R_2 C_2} \cdot \frac{1}{s}$$

The gain of the single loop is:

$$LG = -\frac{R_3}{R_4} \cdot \frac{1}{1 + sR_1 C_1} \cdot \frac{R_8}{R_7} \cdot \frac{1}{R_2 C_2} \cdot \frac{1}{s}$$

Therefore, the total transfer function is:

$$H(s) = -\frac{1}{R_4 R_2 C_1 C_2} \cdot \frac{1}{s^2 + \frac{1}{R_1 C_1} s + \frac{1}{R_2 R_3 C_1 C_2}}$$

Therefore:

$$\omega_o = \frac{1}{\sqrt{R^2 C_1 C_2}}, Q = \sqrt{\frac{C_1}{C_2}}$$

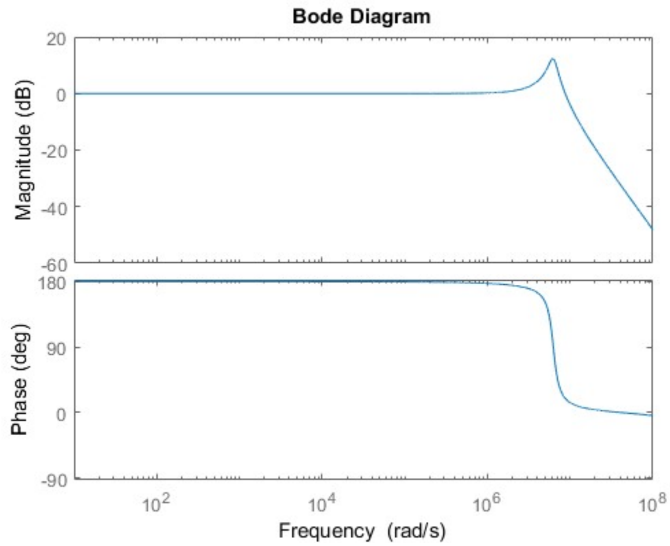
We set C_2 to 1pF which means that for a Q of 4, C_1 must be 16pF. We can then find the required resistance value:

$$2\pi \cdot 10^6 = \frac{1}{R} \cdot \frac{1}{\sqrt{1pF \cdot 16pF}} \rightarrow R = 39.79k\Omega$$

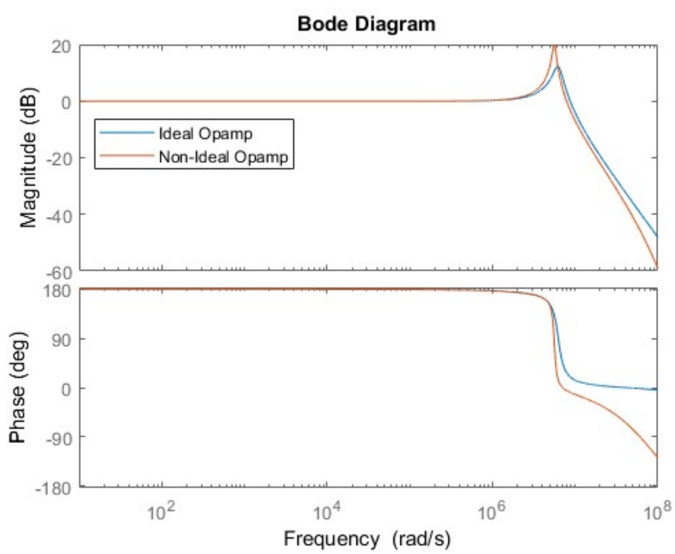
Therefore, all component values have been determined.

Part B

First, the filter was simulated using ideal op amps with infinite gain-bandwidth. The figure below shows the transfer function's magnitude and phase.



Next, the op amp model was replaced with a non-ideal op-amp with a gain-bandwidth 20 times higher than the natural frequency of the filter. The figure below shows the function's magnitude and phase. As in the previous problem, the Q of the filter increases which, in turn, increases the peak gain magnitude seen in the output response. In addition to this, the natural frequency of the filter is shifted downwards.



Part C

The table below shows the magnitude, phase, and group delay for the two cases shown in the previous section.

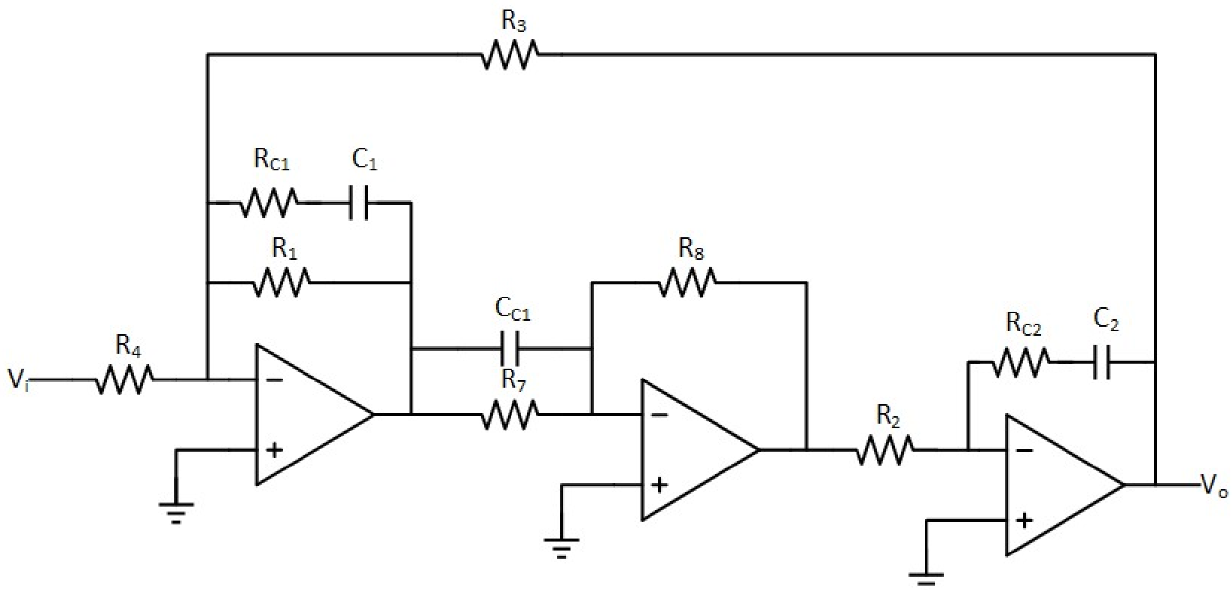
Simulation	Gain (at ω_o)	Gain Error (at ω_o)	Phase (at ω_o)	Phase Error (at ω_o)	Group Delay
Ideal	4	0dB	89.6°	0°	1.309us

Non-Ideal	3.6597	0.7723dB	13.5°	76.1°	3.608us

Clearly, the finite GBW of the op-amps creates a very large phase error at the natural frequency of the filter. As a result, the performance of the filter is severely degraded. In addition to this, the finite GBW leads to increased group delay in the filter.

Part D

Lastly, passive compensation was used to attempt to mitigate the effects of the finite GBW on the filter's performance. The figure below shows all changes:



First, a compensation resistor is added to the lossy integrator at the input. The finite GBW of the amplifier creates a parasitic pole in the lossy integrator at a frequency approximately equal to the GBW of the amplifier. Therefore, a compensation resistor is added in series with the feedback capacitor. The lossy integrator then has, assuming an ideal op-amp, the transfer function:

$$H(s) = \frac{R_1 R_{c1} s C_1 + R_1}{s C_1 (R_1 + R_{c1}) + 1}$$

Therefore, a zero is created at $\omega_z = \frac{1}{C_1 R_{c1}}$. R_{C1} is chosen to place the zero at the op-amp's GBW. Therefore:

$$R_{c1} = \frac{1}{GBW \cdot C_1} = 497.35\Omega$$

The pole of the lossy integrator should remain in the same location, so the value of R_1 is adjusted to $39.292k\Omega$. As a result, R_4 and R_3 are adjusted to the same values to maintain the same DC gain.

Next, a compensation capacitor is added to the inverter. The finite GBW of the op-amp creates a pole at half the GBW frequency. Therefore, C_{C1} is added in parallel to R_7 to create a zero to cancel this pole. The value of C_{C1} is determined by:

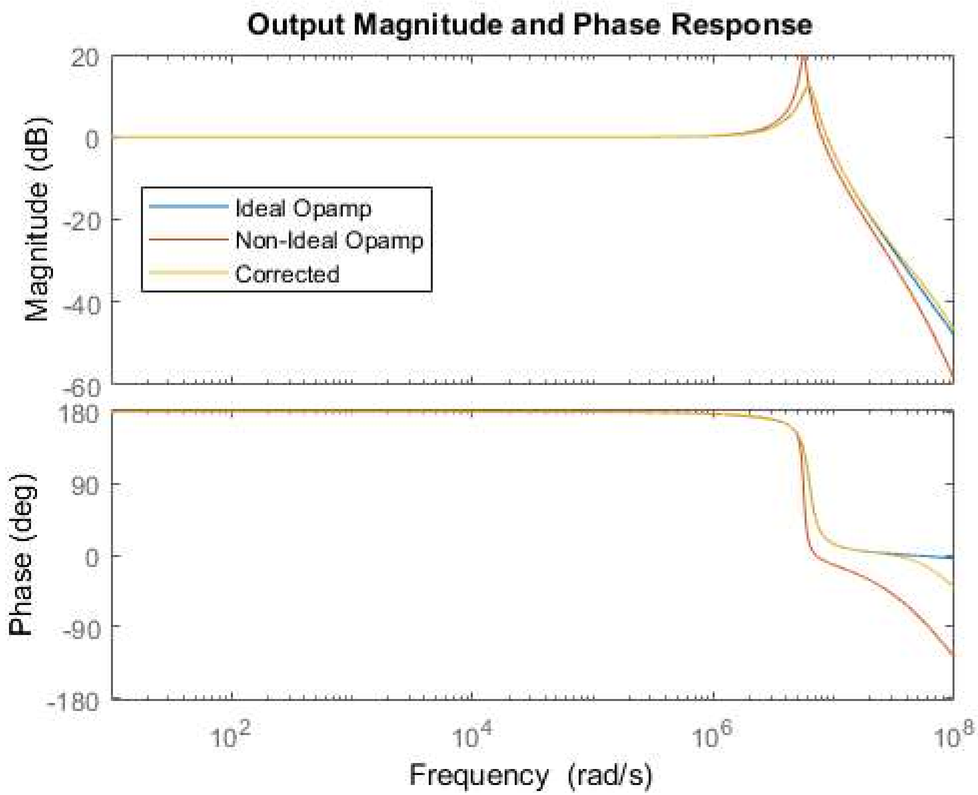
$$\frac{1}{R_7 C_{C1}} = \frac{GB}{2} \rightarrow C_{C1} = \frac{1}{39.79k\Omega \cdot \pi \cdot 10^6 \cdot 20} = 400fF$$

Lastly, the lossless integrator is compensated using a series resistance with the feedback capacitor. Its value is determined by (according to the lecture notes):

$$\frac{1}{R_{C2} C_2} = GB \rightarrow R_{C2} = \frac{1}{1pF \cdot 20 \cdot 2\pi \cdot 10^6} = 7.86k\Omega$$

The value of R_2 is adjusted to 31.83kΩ in order to ensure the integrator gain remains constant.

The filter was once again simulated with these modifications. The results can be seen in the figure below:

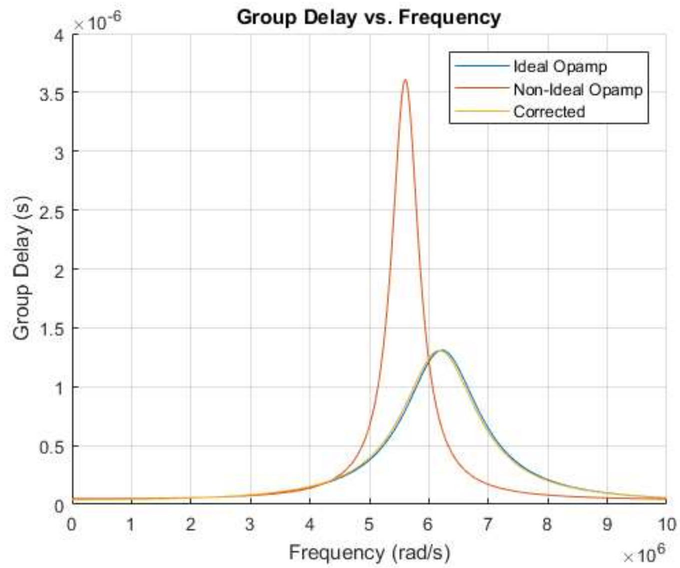


As seen in the figure above, the compensation of the filter corrects the frequency response errors introduced by the finite GBW of the op-amps. The gain and phase of the compensated filter match very closely to the gain and phase of the ideal filter. This is further confirmed in the comparison table below:

Simulation	Gain (at ω_o)	Gain Error (at ω_o)	Phase (at ω_o)	Phase Error (at ω_o)	Group Delay
Ideal	4	0dB	89.6°	0°	1.309us
Non-Ideal	3.6597	0.7723dB	13.5°	76.1°	3.608us
Compensated	4.0410	0.0886dB	87°	2.6°	1.307us

The table above indicates that the gain error and phase error are reduced to very small values after using the compensation techniques discussed above. In addition to this, the group delay of the compensated filter is very close to the ideal group delay. This indicates that passive compensation is a powerful tool that can help break the tradeoff between filter accuracy and power consumption.

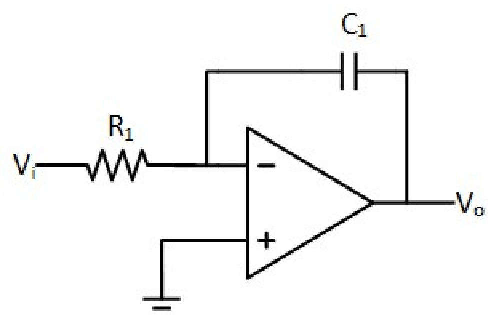
The figure below shows the group delay of the three simulations:



Problem 3

Part A

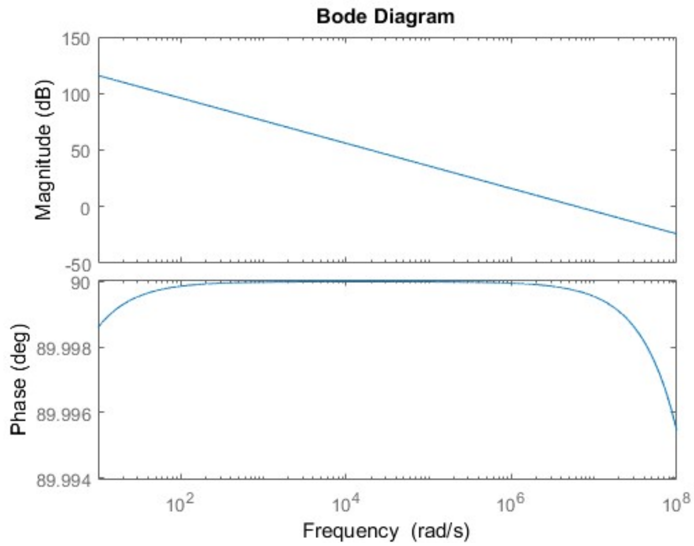
The figure below shows the RC Miller integrator used:



A nominal value of 1pF for C_1 is chosen. R_1 can therefore be calculated as:

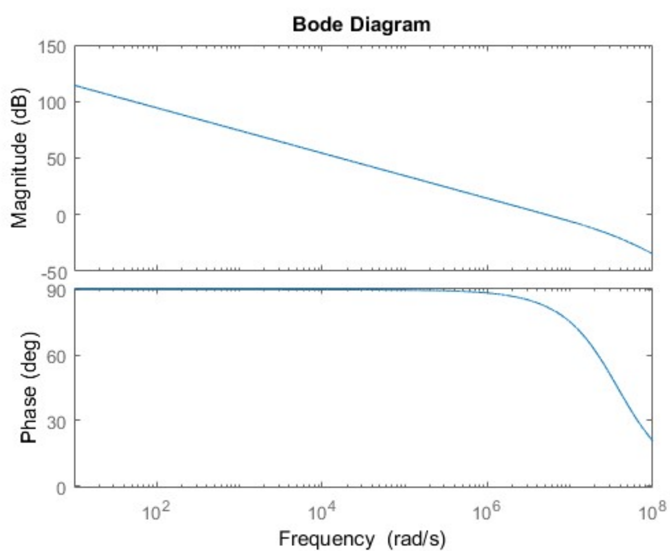
$$R_1 = \frac{1}{C_1 \omega_o} = \frac{1}{1pF \cdot 2\pi \cdot 10^6 rad/s} = 159.15k\Omega$$

The transfer function's magnitude and phase is plotted in the figure below for an ideal op-amp. As expected, the integrator exhibits a high DC gain with a pole at the origin. In addition to this, the phase remains at 90 degrees for all frequencies.



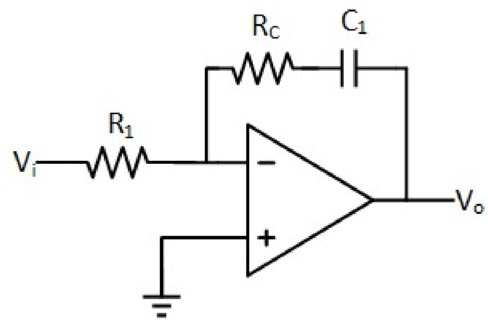
Part B

Next, the GBW of the op-amp was set to a value five times higher than the cutoff frequency of the integrator. The resulting magnitude and phase plot is shown in the figure below. As expected, the finite GBW of the op-amp introduces a parasitic pole at the GBW frequency. This, in turn, adds another -20dB/dec to the integrator's output magnitude response. In addition to this, the parasitic pole also reduces the phase at the output. Therefore, the output phase at the cutoff frequency of the integrator is no longer 90 degrees, it is now 80.54 degrees. Therefore, the finite GBW has introduced excess phase in the integrator.



Part C

Lastly, a compensation resistor was added in series with the feedback capacitor to obtain an ideal integrator transfer function. The compensation capacitor adds a zero in the filter transfer function that cancels the effect of the op-amp's finite GBW. The schematic for this compensation is shown below:

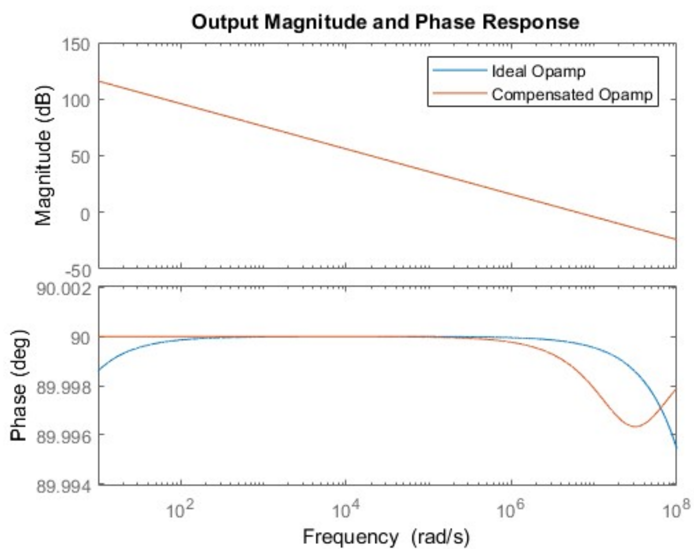


The required value for R_C can be calculated as follows:

$$\frac{1}{C_1 R_C} = GB \rightarrow R_C = \frac{1}{1pF \cdot 2\pi \cdot 5 \cdot 10^6} = 31.83k\Omega$$

The value for R_1 must be adjusted to 127.32kΩ to ensure the cutoff frequency of the filter remains constant.

The figure below shows the magnitude and phase of the filter transfer function for the ideal op-amp and for the non-ideal op-amp with compensation:



The parasitic pole due to the finite GBW has been cancelled and the magnitude and phase response of the compensated integrator tracks very closely with the ideal case. The phase remains very close to 90 degrees at the cutoff frequency.

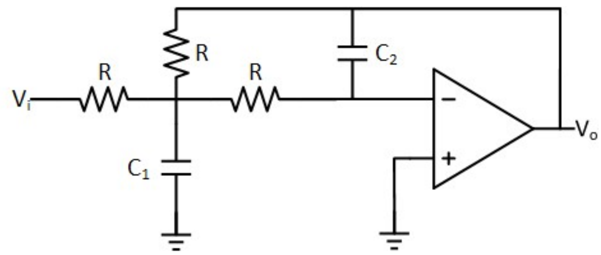
The results from these simulations are summarized in the table below. Clearly, the compensation is able to correct the effects of the finite GBW and a response that is very close to the ideal case is obtained.

Simulation	Gain at ω_o	Phase at ω_o
Ideal	0dB	90°
Non-Ideal	-1.7dB	80.54°
Compensated	0.035dB	90°

Bonus

Part A

The figure below shows the Rauch filter used in this problem.



According to the lecture notes, the transfer function is:

$$H(s) = \frac{\frac{1}{R^2C_1C_2}}{s^2 + \frac{s}{C_1}\cdot\frac{3}{R} + \frac{1}{R^2C_1C_2}}$$

Therefore:

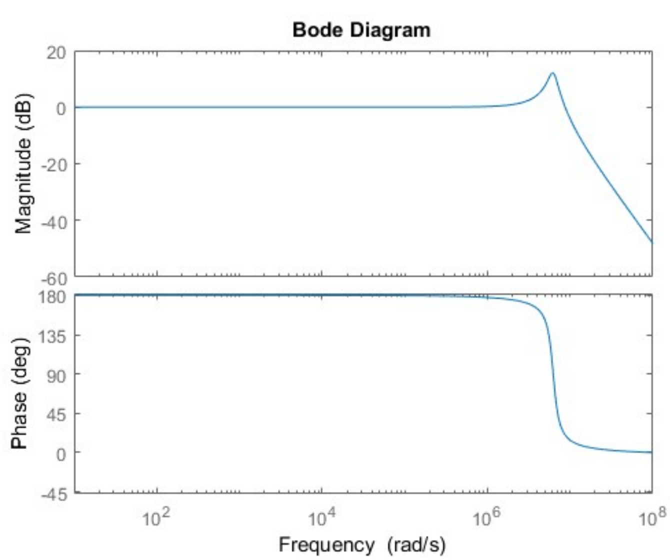
$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}}, Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

After setting R=10kΩ, we can solve these equations simultaneously to set:

$$\omega_o = 2\pi \cdot 10^6 rad/s, Q = 4$$

We obtain C_1=190.94pF and C_2=1.326pF.

The magnitude and phase for this filter is plotted in the figure below. The peak gain matches the expected 12dB for a Q value of 4 and occurs at the desired natural frequency of 1MHz. This indicates that the component values chosen above are correct. The magnitude and phase response also matches the magnitude and phase of the Sallen-Key filter designed in Problem 1.

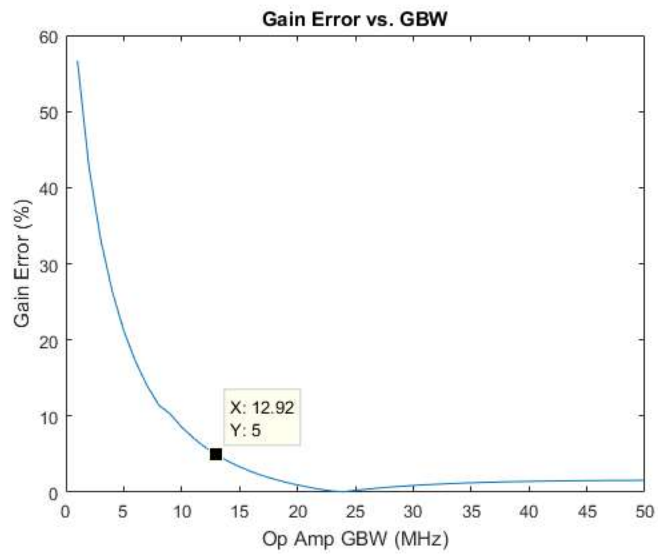


Part B

Next, we assume that the amplifier used in the Rach filter has a gain given by:

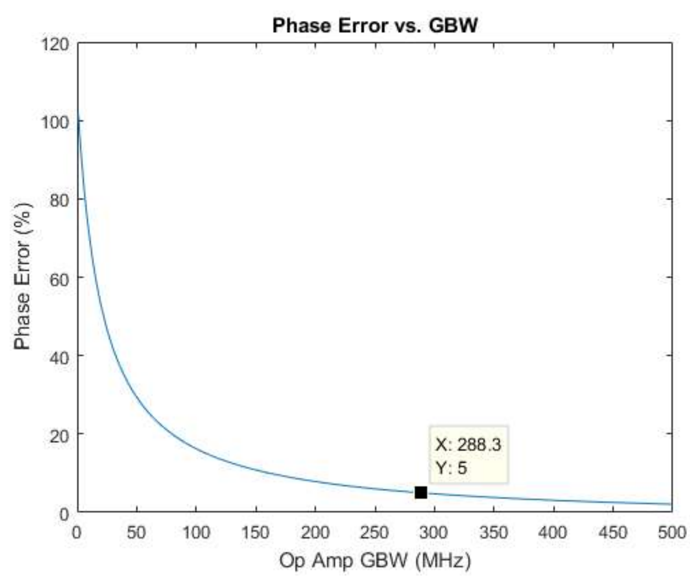
$$A = \frac{GB}{s + \omega_p}$$

Therefore, the non-ideal op-amp will introduce gain and phase errors into the filter due to its finite GBW. In order to quantify the required GBW to obtain a 5% phase or gain error, the GBW is swept. The peak gain error at the natural frequency is then plotted and can be found below for a wide range of op-amp GBWs:



The peak gain reaches a relative error of 5% at an op-amp GBW of 12.92MHz. It is interesting to note that the Rauch filter reaches the 5% gain error at a much lower GBW frequency than the Sallen-Key filter, which reached this error at a GBW of 68.4MHz.

Next, the phase error at the natural frequency was found:



The phase error at the natural frequency reaches an error of 5% at an op-amp GBW of 288.3MHz. This is much higher than the GBW of 22.87MHz required in Sallen-Key filter. It is interesting to note that the Sallen-Key filter requires a higher GBW for the gain requirement, but a lower GBW for the phase requirement. Therefore, the Rauch filter may be used where the gain accuracy is most important while the Sallen-Key filter may be used where the phase accuracy is most important. One important consideration in the selection of these filters is also the large capacitor ratio required in the Rauch filter to obtain a large Q-value. This means that the C_1 capacitor will most likely be very large for large Q values which may make this filter difficult to integrate on an integrated circuit.