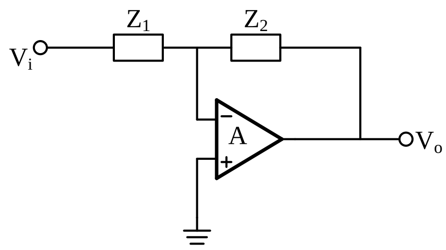
ACTIVE - RC FILTERS

The basic building block is illustrated below

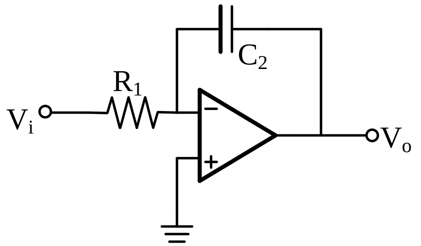


$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\dfrac{-Z_2}{Z_1}}{1 + \dfrac{1}{A}\left(1 + \dfrac{Z_2}{Z_1}\right)}$$

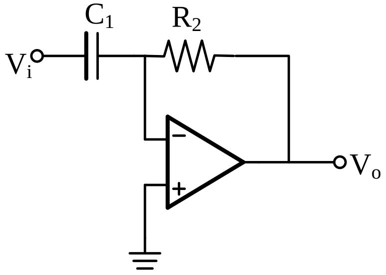
Let us assume that $A \to \infty$, then

$$H(s) = -\frac{Z_2}{Z_1}$$

Next we consider particular cases

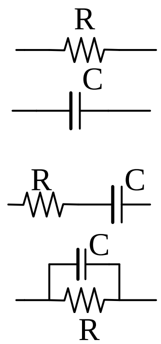


$$H(s) = -\frac{1}{sR_1C_2}$$ Integrator



$$H(s) = -s\,R_2C$$ Differentiator

Z_1 or $Z_F = Z_2$ Can be:

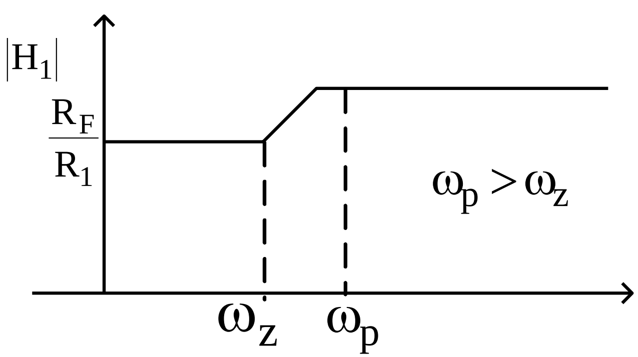


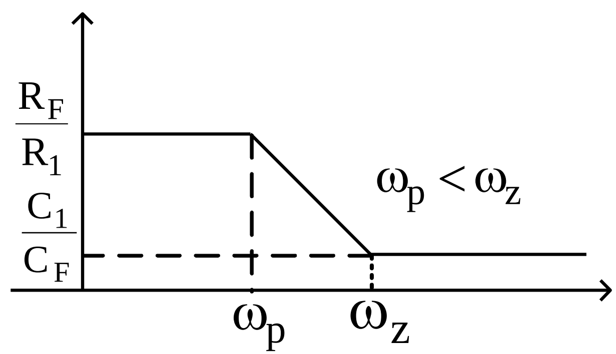
$$\begin{cases} R \\ \dfrac{1}{sC} \\ \dfrac{1+sRC}{sC} \\ \dfrac{R}{1+sRC} \end{cases} \qquad (3)$$

EXAMPLE: Let $Z_1 = \dfrac{R_1}{1+sR_1C_1}$, $Z_F = \dfrac{R_F}{1+sR_FC_F}$

Assuming ideal op amp $A \longrightarrow \infty$. Then using (1)

$$H_1 = \frac{V_{01}}{V_1} = -\frac{R_F/R_1(1+sR_1C_1)}{(1+sR_FC_F)} = -\frac{K_n(1+s/\omega_z)}{(1+s/\omega_p)} \qquad (4)$$





Particular cases are easily derived from (3) and (4)

— Integrator: $C_1 \to 0 \quad , \quad R_F \to \infty$

$$H_1 \cong -\frac{R_F}{R_1}\frac{1}{sR_F C_F} = -\frac{1}{sC_F R_1}$$

— Differentiator ; $R_1 \to \infty \quad , \quad C_F \to 0$

$$H_1 \cong -\frac{R_F}{R_1} sR_1 C_1 = -sR_F C_1$$

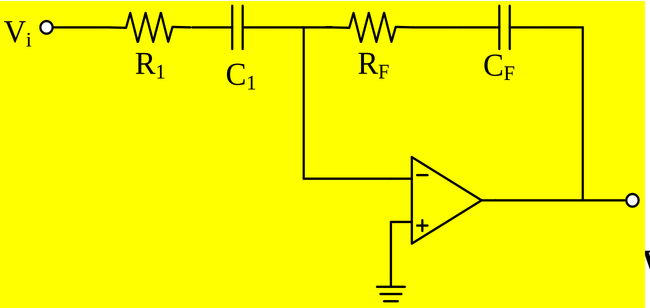
— Low-Pass: $C_1 = 0$

$$H_1 = \frac{-\frac{R_F}{R_1}}{1 + sR_F C_F}$$

— High-Pass: $R_1 \to \infty$

$$H_1 \cong -\frac{R_F}{R_1}\frac{sR_1 C_1}{1 + sR_F C_F}$$

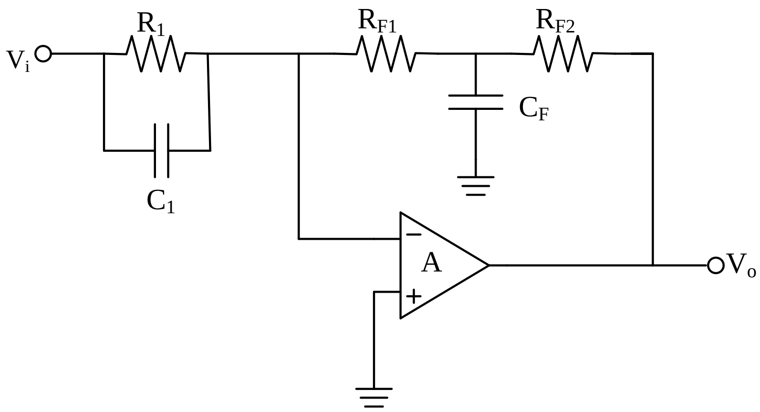
— One pole and one zero



$$\frac{V_o}{V_i} = -\frac{1+sR_FC_F}{sC_F}\frac{sC_1}{1+sR_1C_1} = -\frac{C_1}{C_F}\frac{1+sR_FC_F}{1+sR_1C_1} \qquad (5)$$

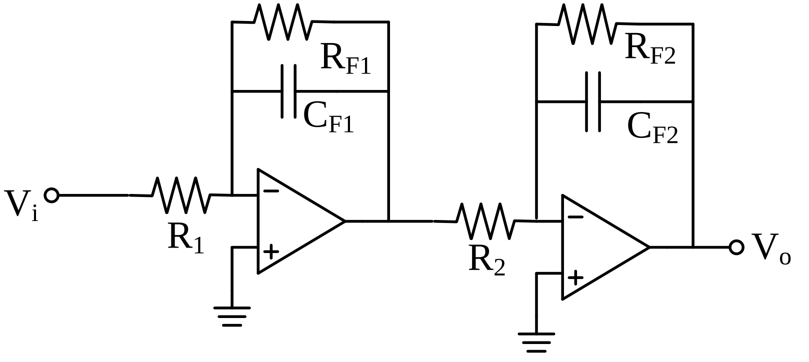
What are the key differences between Eqs. (4) and (5)?

Exercise 1. Obtain the transfer function of the following circuit.



Second-Order Filters Based on a Two-Integrator Loop.

- We can design a second-order filter by cascading two inverters. i.e.



$$\frac{V_o}{v_i} = \frac{-\frac{R_{F1}}{R_1}\left(-\frac{R_{F2}}{R_2}\right)}{(1+sC_{F1}R_{F1})(1+sC_{F2}R_{F2})} = \frac{\frac{R_{F1}}{R_1}\frac{R_{F2}}{R_2}}{s^2C_{F1}R_{F1}C_{F2}R_{F2}+s(C_{F1}R_{F1}+C_{F2}R_{F2})+1} \qquad (6)$$

What are the locations of the poles?

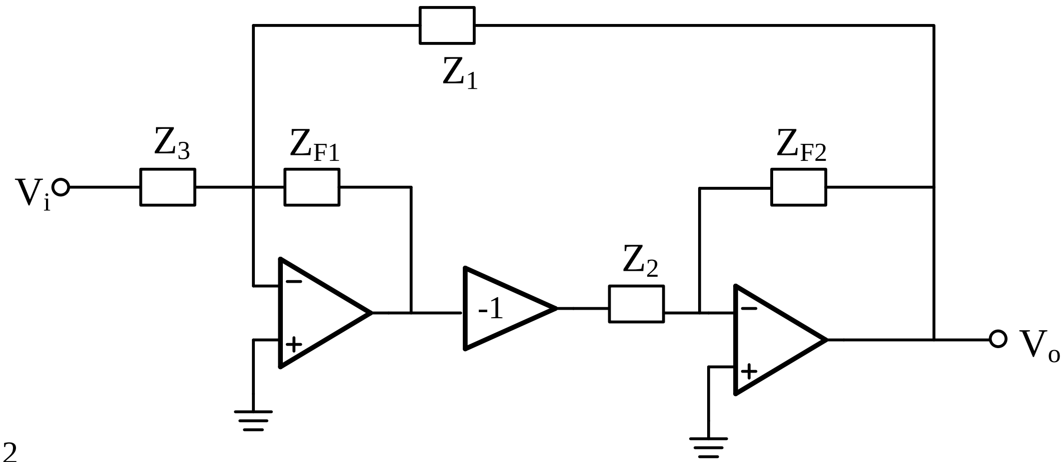
$$s_{p_{1,2}} = \frac{-(C_{F1}R_{F1}+C_{F2}R_{F2}) \pm \sqrt{(C_{F1}R_{F1}+C_{F2}R_{F2})^2 - 4C_{F1}R_{F1}C_{F2}R_{F2}}}{2C_{F1}R_{F1}C_{F2}R_{F2}}$$

To have complex poles it requires that

$$(C_{F1}R_{F1})^2 + (C_{F2}R_{F2})^2 - 2C_{F1}R_{F1}C_{F2}R_{F2} < 0\,?$$

Which it is impossible to satisfy. Therefore, cascading two first-order filter yield a second-order filter with only real poles.

The general form of the second order two-integrator loop has the following topology.



$$H = \frac{-\dfrac{Z_{F1}}{Z_3}\dfrac{Z_{F2}}{Z_2}}{1+\dfrac{Z_{F1}}{Z_1}\dfrac{Z_{F2}}{Z_2}} \qquad (7a)$$

Note the similarity of Eq. (7a) with (2). Also observe that A “-1” needs to be inserted before or after the second inverter to yield a negative feedback loop.

Let us consider the following filter where

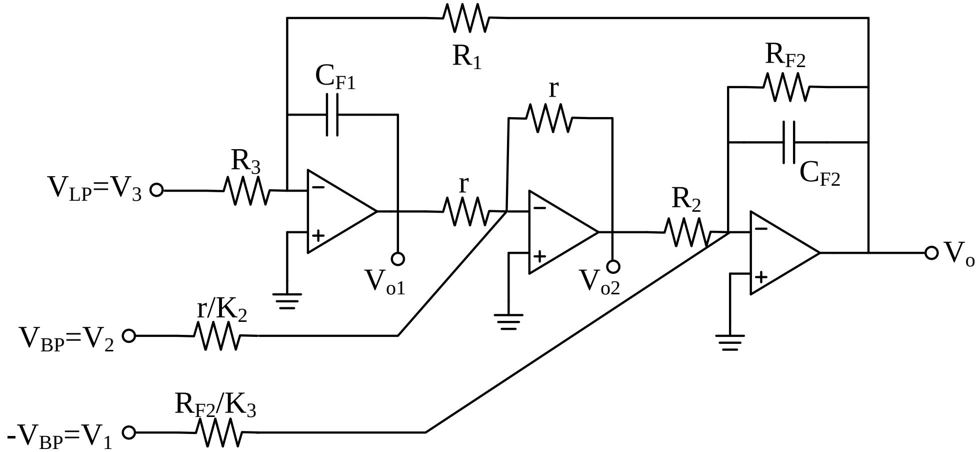
$$Z_3 = R_3,\ Z_1 = R_1,\ Z_2 = R_2,\ Z_{F1} = \frac{1}{sC_{F1}},\ Z_{F2} = \frac{R_{F2}}{1+sC_{F2}R_{F2}}$$

Thus Eq. (7a) yields:

$$H = \frac{-\dfrac{1}{sC_{F1}R_3}\dfrac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})}}{1+\dfrac{1}{sC_{F1}R_1}\dfrac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})}}$$

$$H = \frac{-\dfrac{1}{C_{F1}R_3C_{F2}R_2}}{s^2+\dfrac{s}{C_{F2}R_{F2}}+\dfrac{1}{C_{F1}R_1C_{F2}R_2}} = \frac{-\omega_{o1}^2}{s^2+\dfrac{\omega_o}{Q}s+\omega_o^2}$$

By injecting in different current summing nodes a general biquad filter can be obtained.

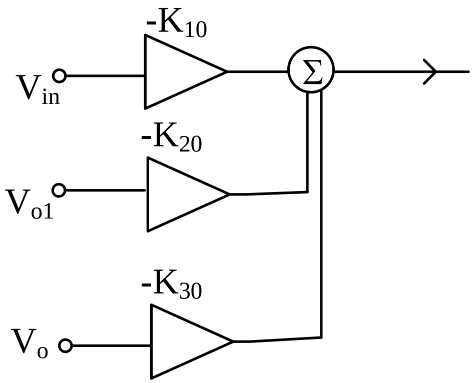


$$V_o = \frac{V_3 \dfrac{1}{sC_{F1}R_3} \dfrac{R_{F2}/R_2}{\left(1+sC_{F2}R_{F2}\right)} + \dfrac{-K_2 \dfrac{R_{F2}}{R_2}}{\left(1+sC_{F2}R_{F2}\right)} - \dfrac{K_3 \dfrac{R_{F2}}{R_{F2}} V_1}{\left(1+sC_{F2}R_{F2}\right)}}{1+\dfrac{1}{sC_{F1}R_1} \dfrac{R_{F2}/R_2}{\left(1+sC_{F2}R_{F2}\right)}}$$

$$V_o = \frac{V_3 \dfrac{1}{C_{F1}R_3C_{F2}R_2} + V_2 sC_{F1}R_1K_2 - V_3 sC_1R_1K_3}{s^2 + \dfrac{s}{C_{F2}R_{F2}} + \dfrac{1}{C_{F1}R_1C_{F2}R_2}}$$

Exercise 2. Obtain the expressions of V_{o1} and V_{o2}.

More general biquad expressions and topologies can be obtained by adding a summer.

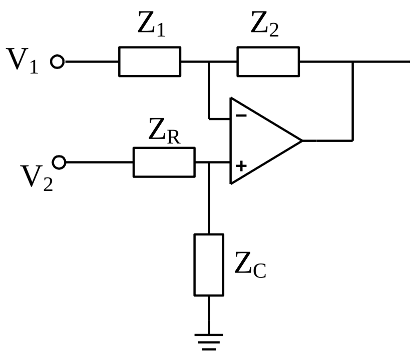


$$V_{oT} = -K_{10}V_{in} - K_{20}V_{o1} - K_{30}V_o$$

Exercise 3. Draw an active-RC topology of the block diagram show above.

Exercise 4 a) For only $V_1 \neq 0$ obtain V_o and V_{o1} when instead of the resistor R_{F2}/K_3 a capacitor $K_4\, C_{F2}$ is used. b) For only $V_3 \neq 0$ obtain V_{o1} when the resistor R_3 is replaced by a capacitor $K_{HP}C_{F1}$.

By using also the positive input of the op amp other useful filters can be obtained.

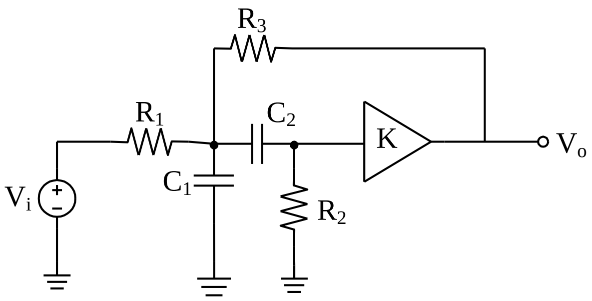


$$V_o = \frac{-\frac{Z_2}{Z_1} V_1 + \frac{Z_R}{Z_R + Z_C} V_2 \left(1 + \frac{Z_2}{Z_1}\right)}{1 + \frac{1}{A}\left(1 + \frac{Z_2}{Z_1}\right)}$$

Example. Phase shifter $Z_2=R_2=R_1$, $Z_1=R_1$, $Z_R=R$ $Z_C = \frac{1}{sC}$ and $A \to \infty$ with $V_1 = V_2$

$$\frac{V_o}{V_1} = -1 + \frac{sRC}{1+sRC} \cdot 2 = \frac{-1 - sRC + 2sRC}{1+sRC} = -\frac{1-sRC}{1+sRC}$$

Sallen and Key Bandpass Filter



K is a non-inverting amplifier

Using Nodal Analysis

$$V_1\left(s(C_1+C_2)+\frac{1}{R_1}+\frac{1}{R_3}\right)-sC_2V_2-\frac{V_o}{R_3}=\frac{V_i}{R_1} \qquad (1)$$

$$-\mathrm{V}_1(sC_2) \qquad + \qquad V_2\left(sC_2+\frac{1}{R_2}\right)=0 \qquad (2)$$

$$\mathrm{V_o}=KV_2 \qquad (3)$$

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{K\,\dfrac{s}{R_1C_1}}{s^2+\left[\dfrac{1}{R_2C_2}+\left(1-K+\dfrac{R_3}{R_1}+\dfrac{R_3}{R_2}\right)\dfrac{s}{R_3C_1}+\dfrac{R_1+R_3}{R_1R_3R_2C_1C_2}\right]}$$

A particular case is for $R_1=R_2=R_3=R$, $C_1=C_2=C$

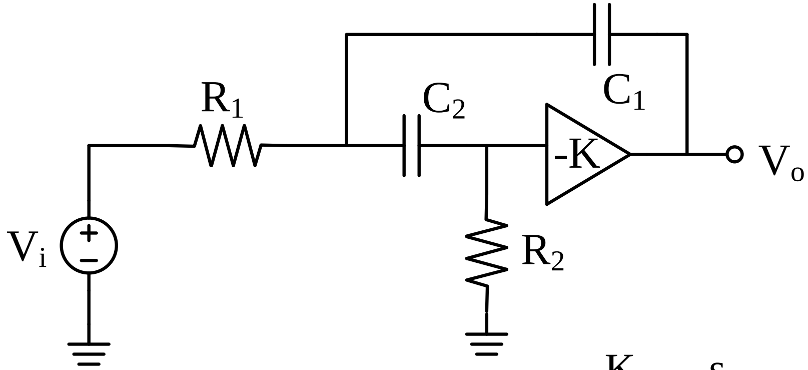
Then

$$\omega_o^2 = \frac{2}{(RC)^2} \quad ; \quad Q = \frac{\sqrt{2}}{4-K}$$

or for a given ω_o and Q

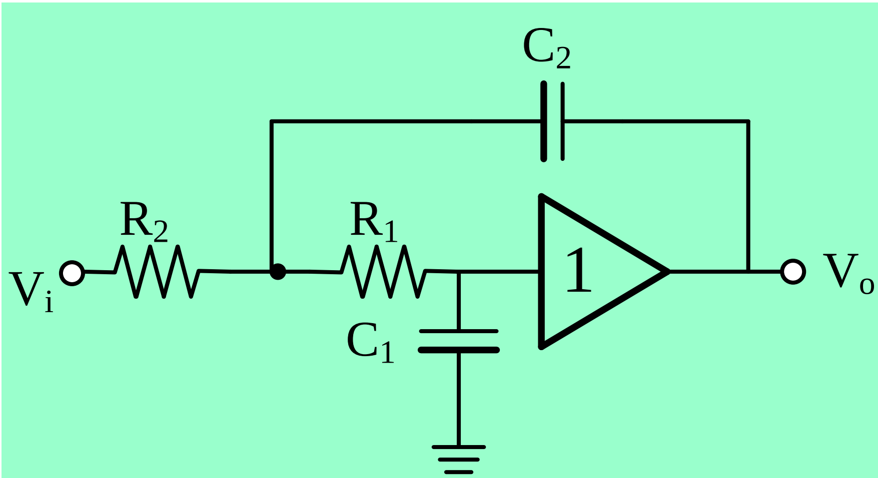
$$RC = \frac{\sqrt{2}}{\omega_o} \quad \text{and} \quad K = 4 - \frac{\sqrt{2}}{Q}$$

Exercise 5. Prove the transfer function is a BP filter of the following circuit



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\dfrac{-K}{K+1}\,\dfrac{s}{R_1C_1}}{s^2 + \dfrac{1}{K+1}\left[\dfrac{R_1+R_2}{R_1R_2C_1} + \dfrac{1}{R_2C_2}\right]s + \dfrac{1}{(K+1)R_1R_2C_1C_2}}$$

In the past before IC fabrication, active filters implementation preferred one op amp structure. One very popular type is the Sallen and Key unity gain implementations.



LP Sallen - Key

$$H_{LP}(s) = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

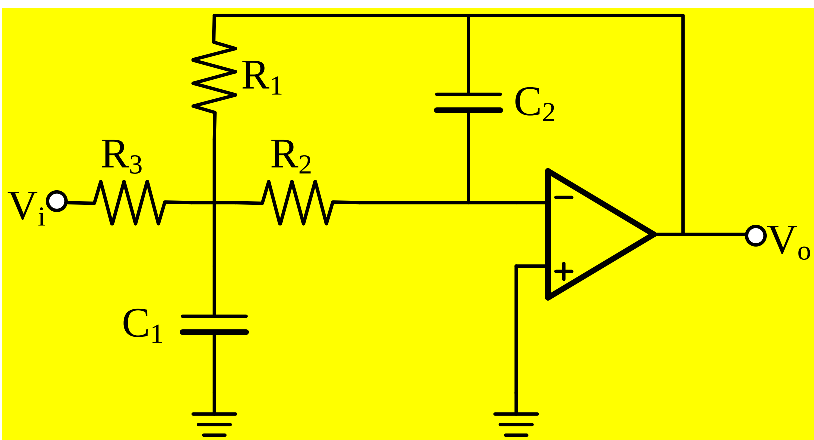
$$R_1 = R_2 = R$$

$$C_1 = C$$

$$C_2 = 4Q^2C$$

$$RC = 1/2\,\omega_o Q$$

One also popular topology is the Rauch Filter

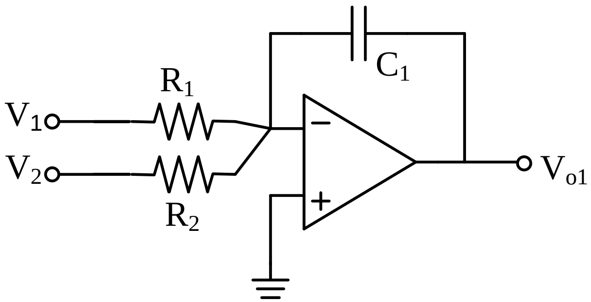


$$H(s) = \frac{\frac{1}{R_2C_2R_3C_1}}{s^2 + \frac{s}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \frac{1}{R_1C_1R_2C_2}}$$

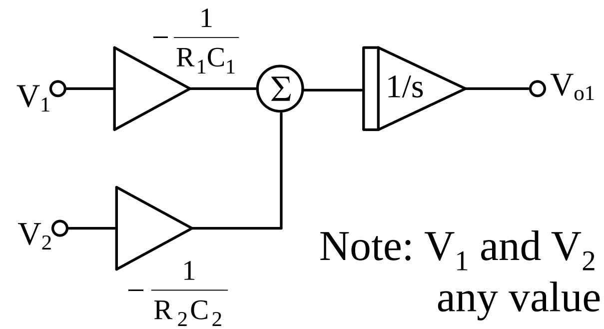
LP Rauch Filer

Another technique for analysis and design based on state-variable uses building blocks.

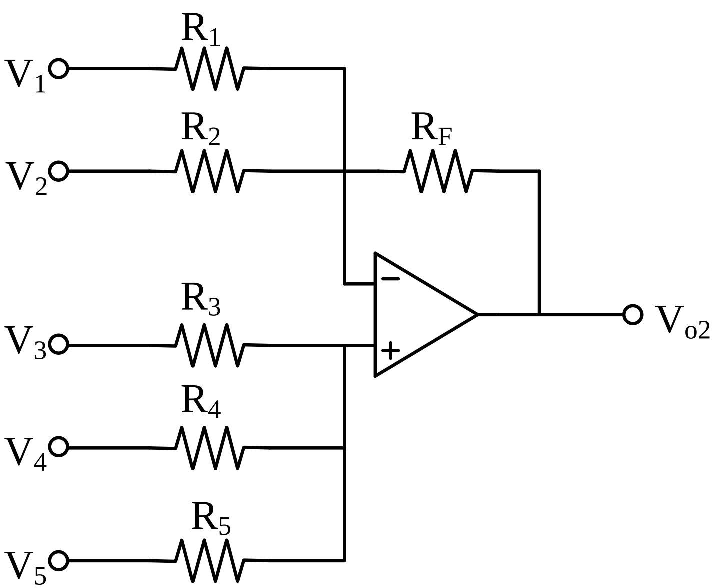
CIRCUIT

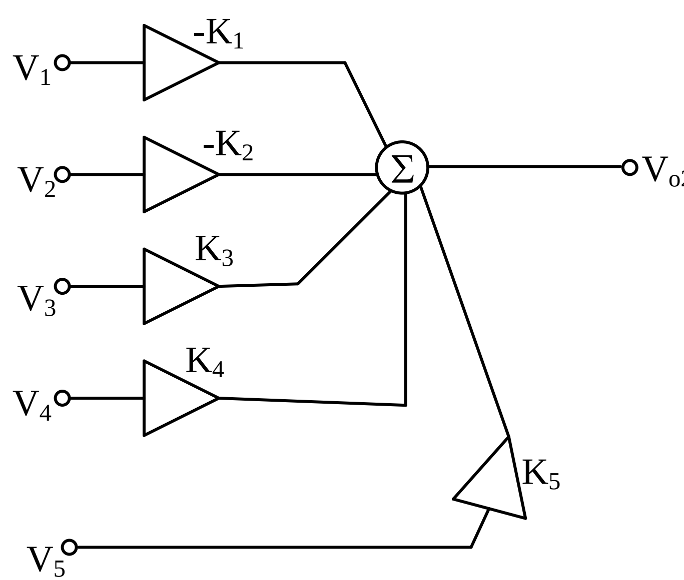


REPRESENTATION

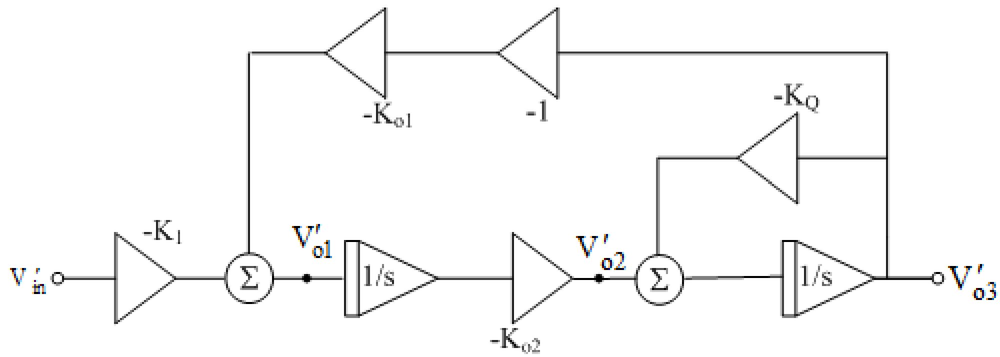


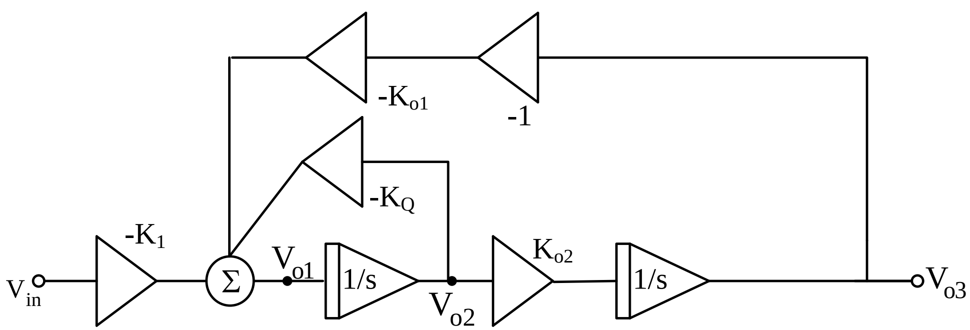
Note: V_1 and V_2 can take any value including V_o.





Let us apply to a two-integrator loop plus Mason's Rule.



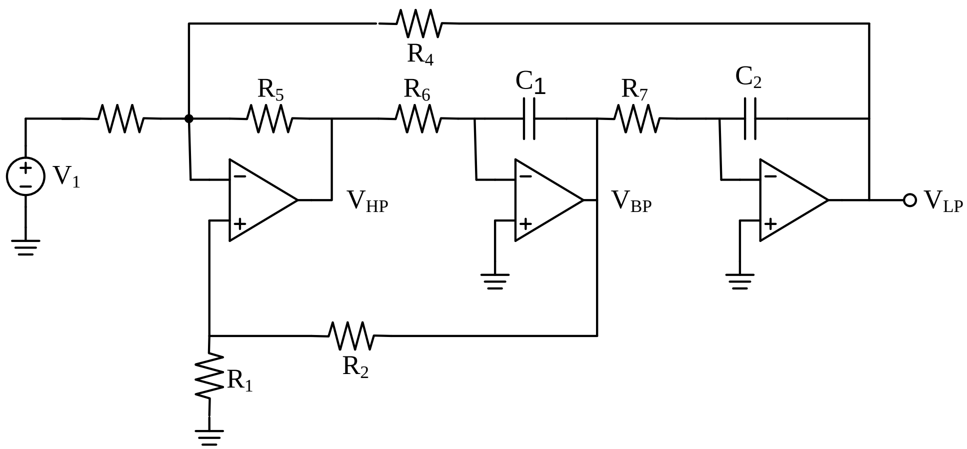


For Second-topology

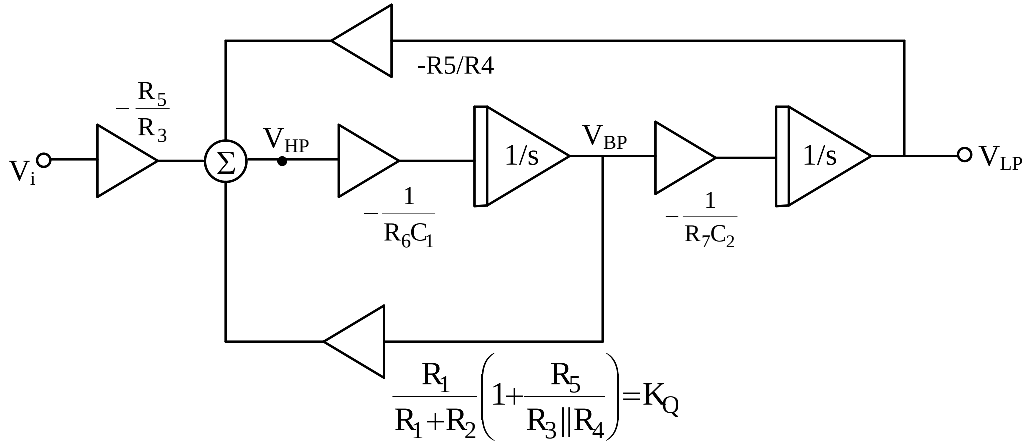
$$V_{o1} = \frac{-K_1 V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1}K_{o2}}{s^2}} = \frac{-K_1 s^2 V_{in}}{s^2 + K_Q s + K_{o1}K_{o2}} \qquad \text{HP}$$

$$V_{o2} = \frac{\frac{-K_1}{s} V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1}K_{o2}}{s^2}} = \frac{-K_1 s V_{in}}{s^2 + K_Q s + K_{o1}K_{o2}} \qquad \text{BP}$$

Next we show that we can go from an Active-RC representation into a block diagram or vice versa.



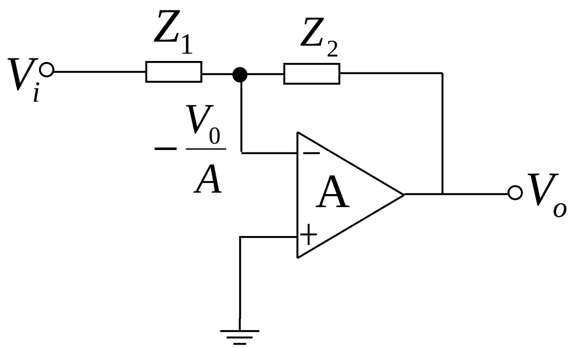
KHN Biquad Filter



$$\frac{V_{HP}}{V_i} = \frac{-\dfrac{R_5}{R_3}}{1 + \dfrac{K_Q}{R_6 C_1}\dfrac{1}{s} + \dfrac{R_5/R_4}{R_6 C_1 R_7 C_2 s^2}} = \frac{-\dfrac{R_5}{R_3}s^2}{s^2 + \dfrac{K_Q}{R_6 C_1}s + \dfrac{R_5/R_4}{R_6 C_1 R_7 C_2}}$$

Non-Ideal Active-RC Integrators

Op Amp Non-Idealities



$$H(s) = \frac{-\dfrac{Z_2}{Z_1}}{1 + \dfrac{1}{A}\left(1 + \dfrac{Z_2}{Z_1}\right)} = -A\beta\,\frac{\dfrac{Z_2}{Z_1}}{1 + A\beta}$$

where

$$\beta = \frac{Z_1}{Z_1 + Z_2}$$

Integrator

Case 1 $\quad Z_1 = R_1\ ;\ Z_2 = \dfrac{1}{sC_2}\ ;\ \dfrac{GB}{s} = A(s)$

$$H(s) \cong \frac{-1}{\dfrac{sRC}{GB}\left(s + \dfrac{1}{RC} + GB\right)} \cong -\frac{1}{sRC(1 + s/GB)}\ ;\ GBRC >> 1$$

$$H(j\omega) = \frac{-1}{\dfrac{-\omega^2 RC}{GB} + j\omega RC}\ ; \qquad \phi = \angle H(j\omega) = -\frac{\pi}{2} - tan^{-1}(\omega/GB)$$

$$\phi = -90^o + \Delta\phi$$

$$\Delta\phi \cong -tan^{-1}\frac{\omega}{GB} \cong -tan^{-1}\frac{1}{\left|A(j\omega)\right|} \quad ; \quad \text{i.e.} \frac{\omega_o}{GB} = \frac{1}{10}$$

$$\Delta\phi \cong -5.7^o$$

$$\left|H(j\omega_o)\right| = \frac{1}{\left|\frac{-\omega_o}{GB} + \frac{j\omega_o}{\omega_o}\right|} = \frac{1}{\sqrt{1+\frac{\omega_o^2}{GB^2}}} = 1 + \Delta_M$$

$$\Delta_M = \frac{1-\sqrt{1+\frac{\omega_0^2}{GB^2}}}{\sqrt{1+\frac{\omega_o^2}{GB^2}}} \qquad \text{i.e.} \quad \frac{\omega_o}{GB} = 0.1$$

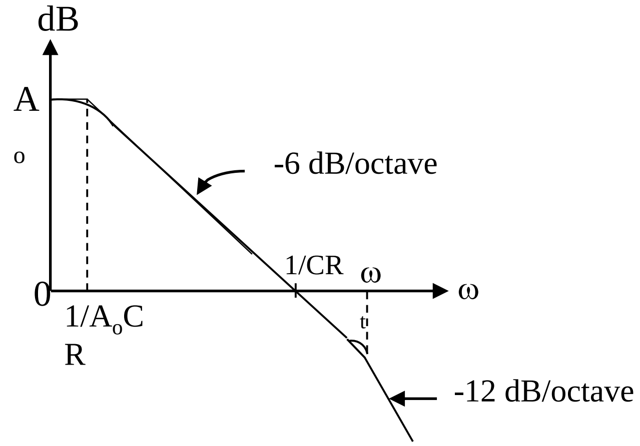
$$\Delta_M \sim 5\% \text{ error}$$

It follows that the ideal -6 dB/octave roll-off expected from an ideal integrator changes to -12 dB/octave at the frequency of the parasitic pole given by

$$s_p = -\left(\omega_t + \frac{1}{CR}\right)$$

which may be approximated by,

$$s = -\omega_t \quad \text{for} \quad \omega_t >> \frac{1}{CR}$$



$$Q_L = -\left(\frac{\omega_t}{\omega}\right) = -\left(\frac{GB}{\omega}\right)$$

In general

$$T(j\omega) = \frac{1}{R(\omega) + jX(\omega)}$$

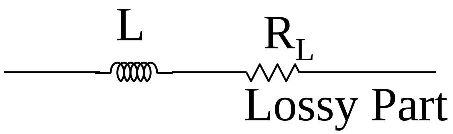
then we define the integrator Q-factor by

$$Q_I = \frac{X(\omega)}{R(\omega)}$$

$$Q_I = -\left(\frac{\omega_t}{\omega}\right) = -\left|A(j\omega)\right|$$

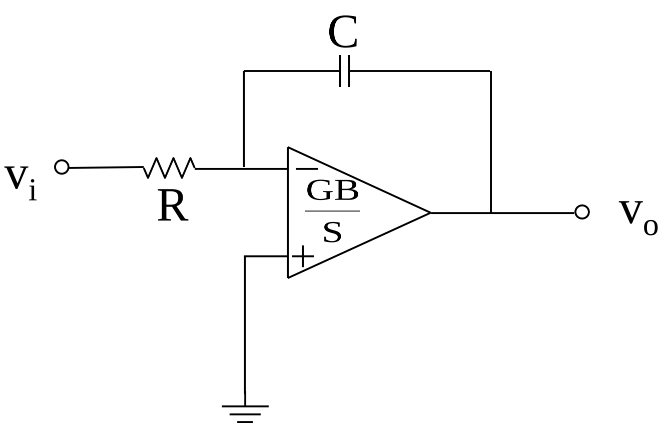
Making an analogy of Q_L of an inductor

$$Q_L = \frac{\omega_o L}{R_L}$$



For an integrator one can obtain

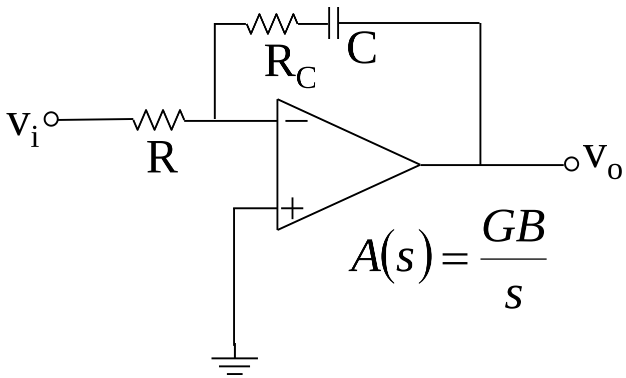
$$Q_I = \frac{\omega RC}{\frac{-\omega^2 RC}{GB}} = -\frac{1}{\frac{\omega}{GB}} = \frac{-1}{\omega} GB = -\left|A(j\omega)\right|$$



Miller Integrator

How can we compensate this degradation of performance?

a)



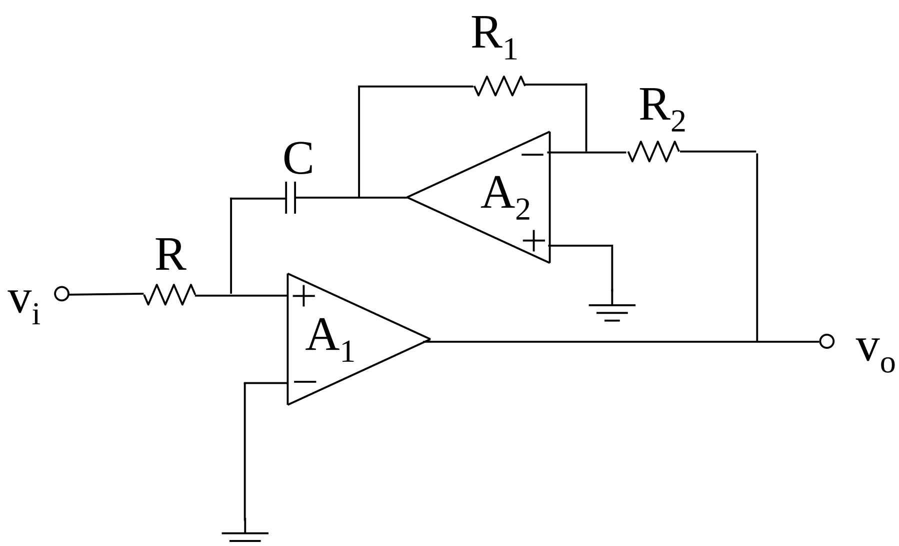
$$A(s) = \frac{GB}{s}$$

If we make

$$R_C = \frac{1}{GB \cdot C}$$

Ideally we obtain

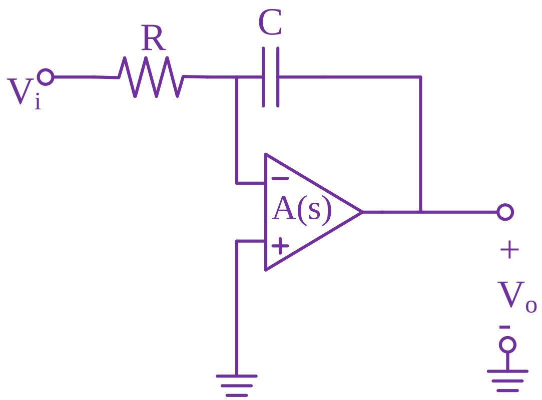
$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{RC}$$



This integrator yields a positive

$$Q_I = +|A(j\omega)|$$

ACTIVE – RC INTEGRATOR: Pole Shift and Predistortion



$$\frac{V_o}{V_i} = \left. \frac{-\dfrac{1}{sRC}}{1 + \dfrac{1}{A(s)}\left(1 + \dfrac{1}{sRC}\right)} \right|_{A(s)\to\infty} = -\frac{1}{sRC} \qquad (1a)$$

$$\frac{V_o}{V_i} = \frac{-1}{sRC + \dfrac{1}{A(s)}(sRC + 1)} \qquad (1b)$$

Let $A(s) = \dfrac{A_o \omega_{3dB}}{s + \omega_{3dB}}$; where A_o is the DC gain and ω_{3dB} the dominant pole in open loop.

Then (1b) becomes

$$\frac{V_o}{V_i} = \frac{-\dfrac{A_o \omega_{3dB}}{RC}}{s^2 + s\left(A_o \omega_{3dB} + \omega_{3dB} + \dfrac{1}{RC}\right) + \dfrac{\omega_{3dB}}{RC}} \cong \frac{-\dfrac{A_o \omega_{3dB}}{RC}}{s^2 + sA_o \omega_{3dB} + \dfrac{\omega_{3dB}}{RC}} \qquad (2)$$

Let $GB = A_o \omega_{3dB}$

G. Daryanani, “Principles of Active Network Synthesis and Design,” John Wiley and Sons, 1976.

The roots of the denominator are

$$P_{1,2} = -\frac{GB}{2} \pm \frac{GB}{2}\left[1 - \frac{4\omega_{3dB}}{(GB)^2 RC}\right]^{1/2} \qquad (3a)$$

Using the approximation $(1-X)^{1/2} \cong 1 - X/2$ for $X<<1$, then

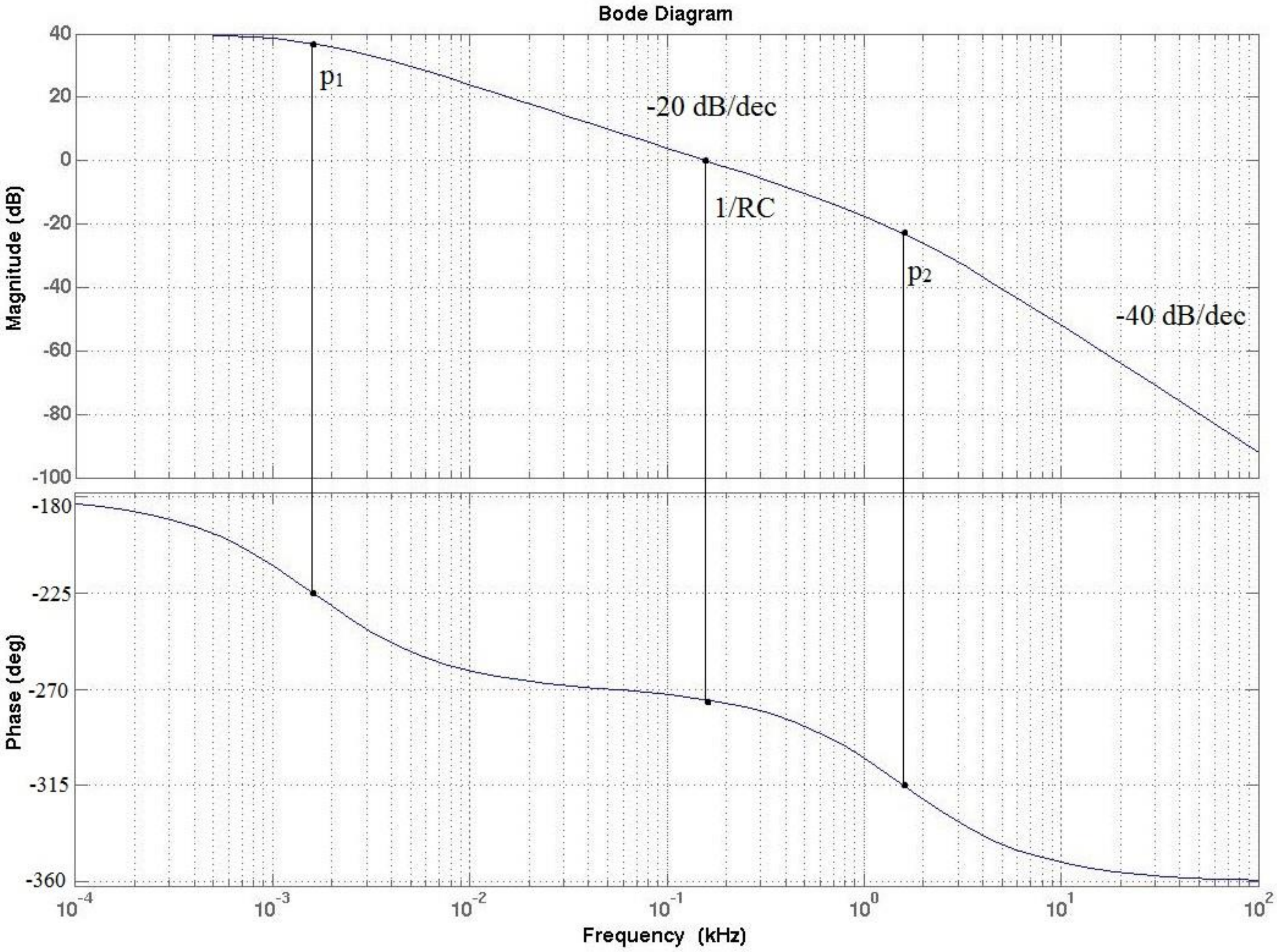
$$P_{1,2} = -\frac{GB}{2} \pm \frac{GB}{2}\left[1 - \frac{2\omega_{3dB}}{(GB)^2 RC}\right] \qquad (3b)$$

Thus the roots yield

$$P_1 = -\frac{1}{A_o RC}$$

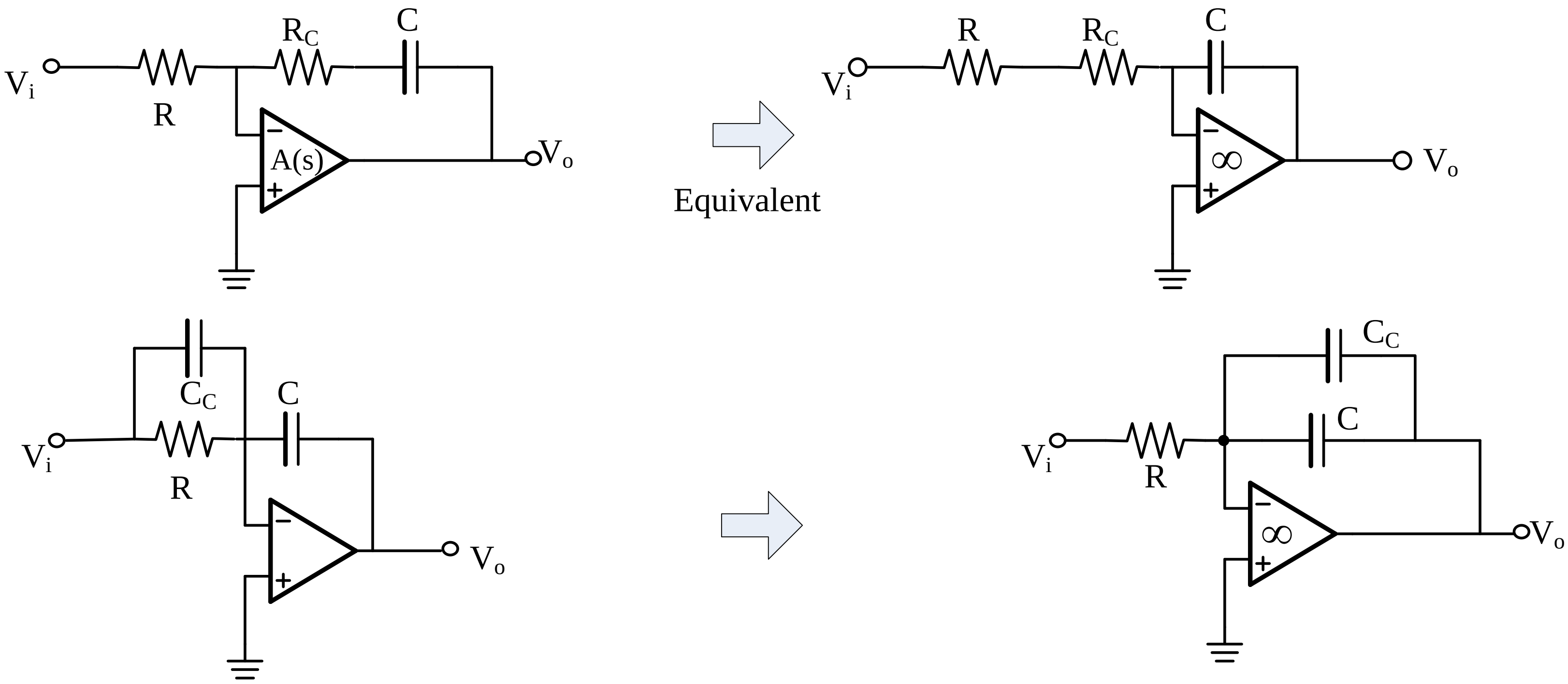
$$P_2 = -GB + \frac{1}{A_o RC} \cong -GB$$

The Bode Plot Looks Like



PREDISTORTION; FREQUENCY COMPENSATION

In order to relax the bandwidth op amp requirement one can use a R_C or C_C on the Miller Integrator. That is

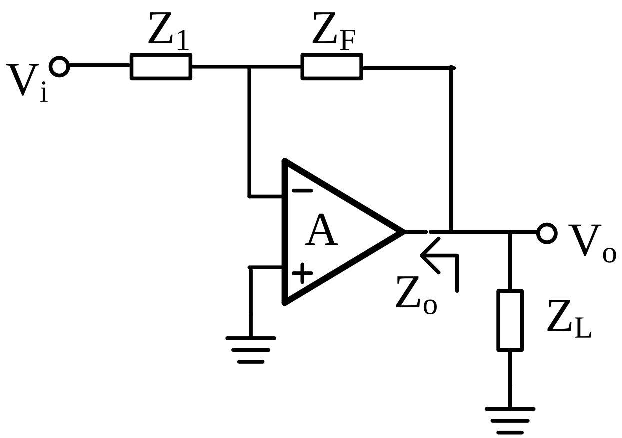


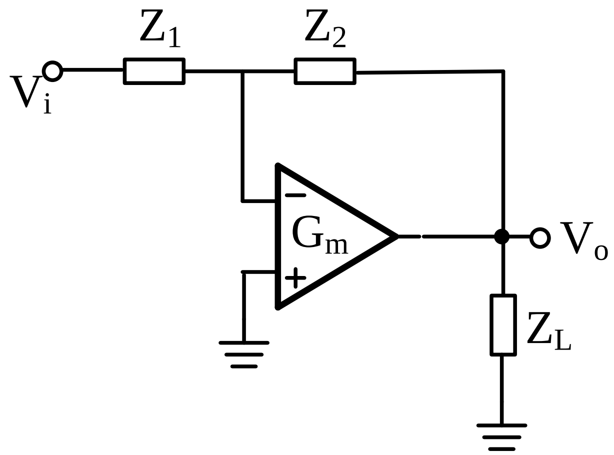
$$\text{Use } R_C\, C = \frac{1}{GB} \quad \text{or} \quad C_C\, R \cong \frac{1}{GB}$$

B. Wu and Y. Chiu, “A 40nm CMOS Derivative-Free IF Active-RC BPF with Programmable Bandwidth and Center Frequency Achieving Over 30dBm IIP3”, IEEE JSSC, Vol. 50, No. 8, pp 1772-1784, August 2015.

Using VCVS vs. VCIS in Active-RC Filters

- The motivation is to use OTA (VCIS) instead of more power hungry Op Amp (VCVS)





For $R_o = Z_o = 0$

$$\frac{V_o}{V_i} = \frac{-\dfrac{Z_F}{Z_1}}{1+\dfrac{1}{A}\left(1+\dfrac{Z_F}{Z_1}\right)}$$

If $A \rightarrow \infty$, then

$$\frac{V_o}{V_i} = -\frac{Z_F}{Z_1}$$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}\,\frac{1-\dfrac{1}{g_m Z_2}}{1+\dfrac{\dfrac{1}{Z_1}+\dfrac{Z_1+Z_2}{Z_1 Z_L}}{g_m}}$$

$$\left.\begin{array}{l} g_m >> \dfrac{1}{Z_2} \\[2ex] g_m >> \dfrac{1}{Z_1}+\dfrac{Z_1+Z_2}{Z_1 Z_L} \end{array}\right\} \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

It can be shown that for equal $A(s) = \dfrac{GB}{s}$, the Tow-Thomas filter has the following deviations

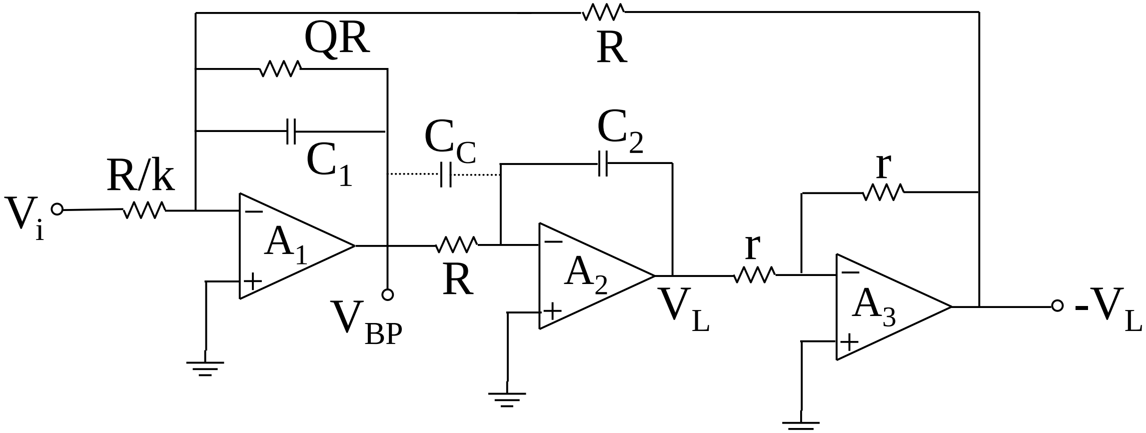
$$Q_a = Q_o \frac{1}{1 - 4Q_o\omega_o / GB}$$

or

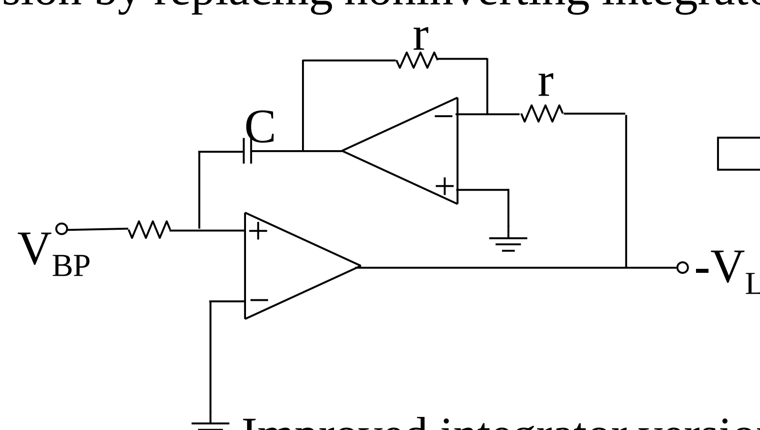
$$\frac{4Q_o\omega_o}{GB} < 1\,; \quad 4Q_o\omega_o < GB$$

and

$$\frac{\Delta\omega_o}{\omega_o} \cong -\frac{2+k}{2}\frac{\omega_o}{GB}$$



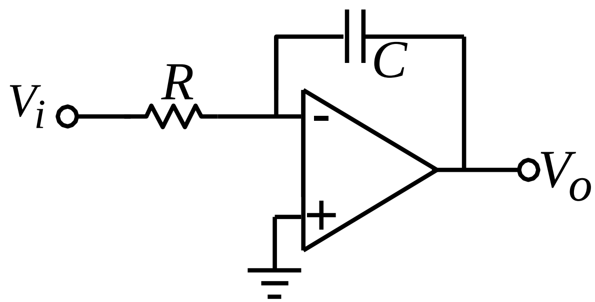
Improved version by replacing noninverting integrator:



$$\Rightarrow \quad Q_a = Q_o \frac{1 - \frac{1}{2}(2+k)\omega_o / GB}{1 + \frac{\omega_o}{GB} + \frac{kQ_o\omega_o^2}{GB^2}}$$

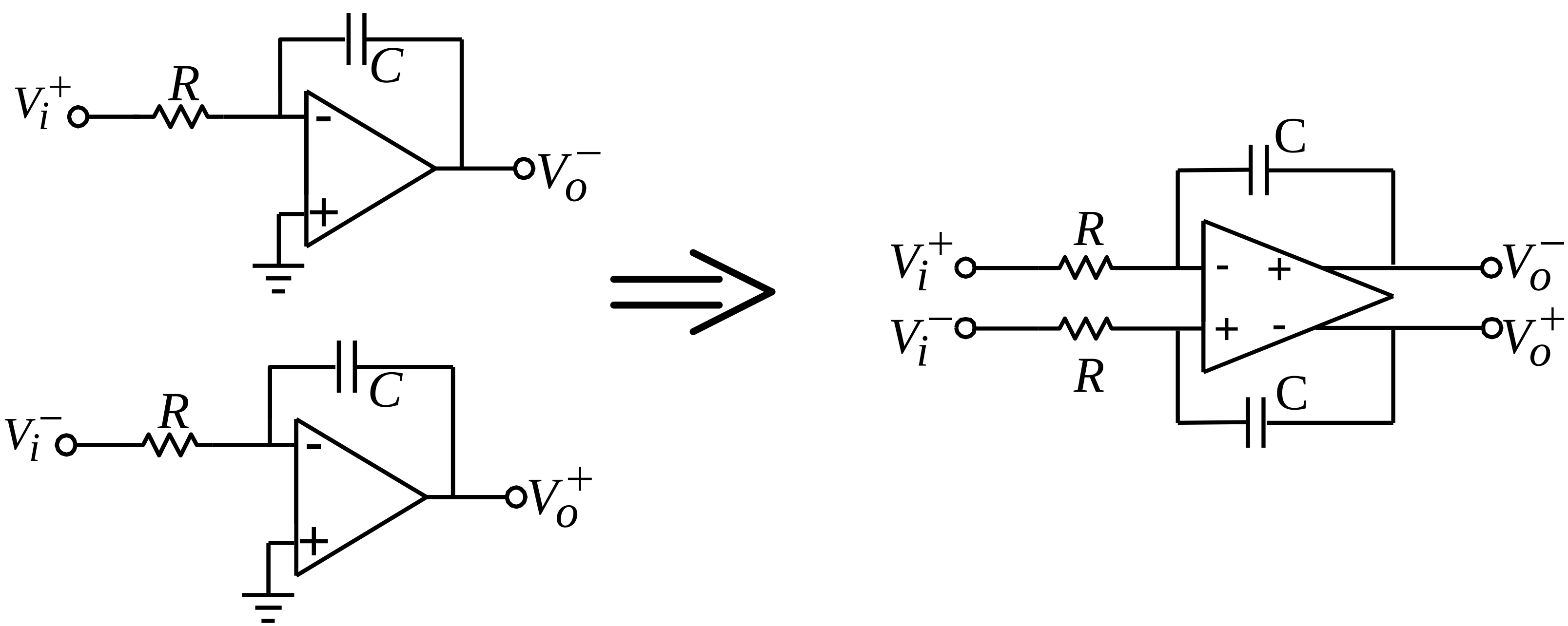
Improved integrator version with positive Q_I

How to generate Fully-Differential Filters based on Single-Ended Version?



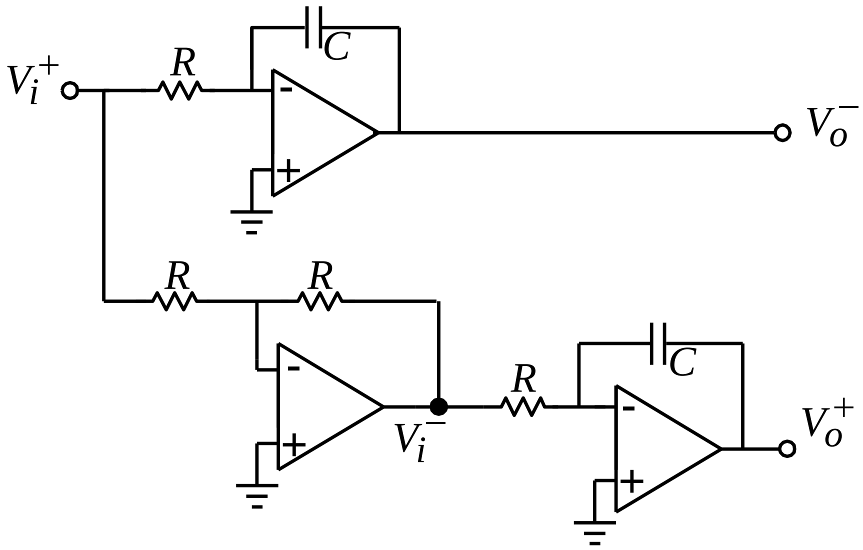
$$\frac{V_o}{V_i} = -\frac{1}{CRs}$$

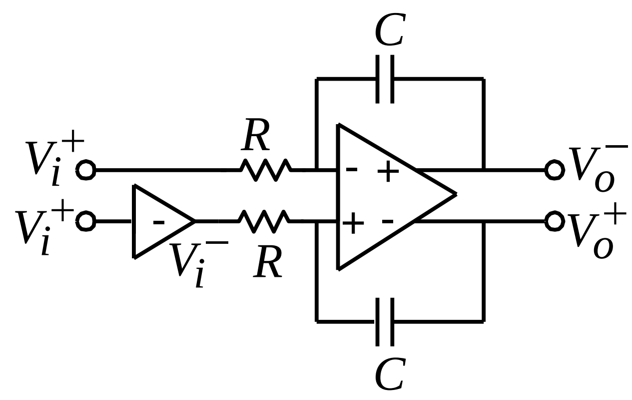
Single Ended



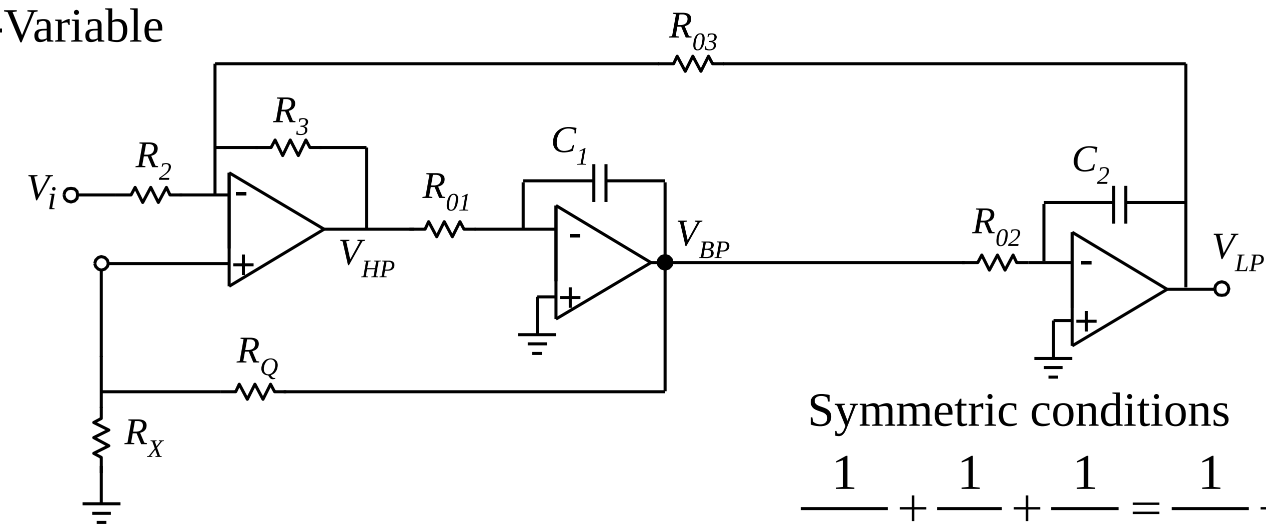
Fully-Differential Version

Particular Case. Assume no V_i^- is Available.





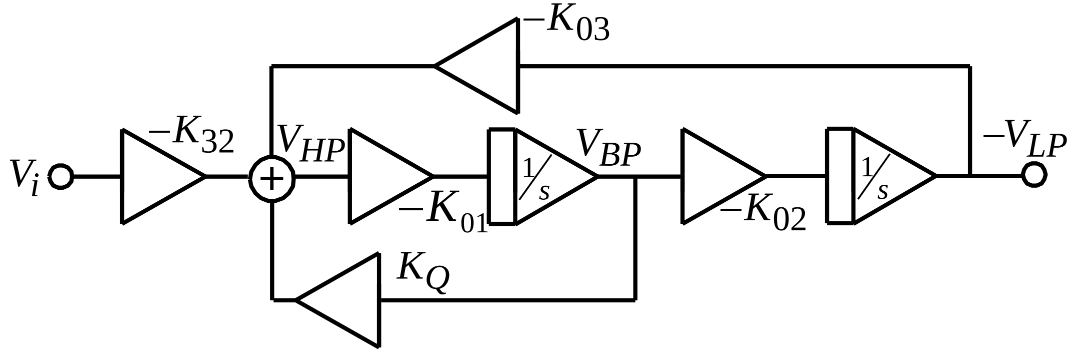
State -Variable Filter



Symmetric conditions

$$\frac{1}{R_{03}} + \frac{1}{R_3} + \frac{1}{R_2} = \frac{1}{R_Q} + \frac{1}{R_X}$$

Read fully balanced - fully symmetric circuits from 607.



$$K_{02} = \frac{1}{R_{02}C_2}, \quad K_{01} = \frac{1}{R_{01}C_1}$$

$$K_{03} = \frac{R_3}{R_{03}}, \quad K_{32} = \frac{R_3}{R_2}$$

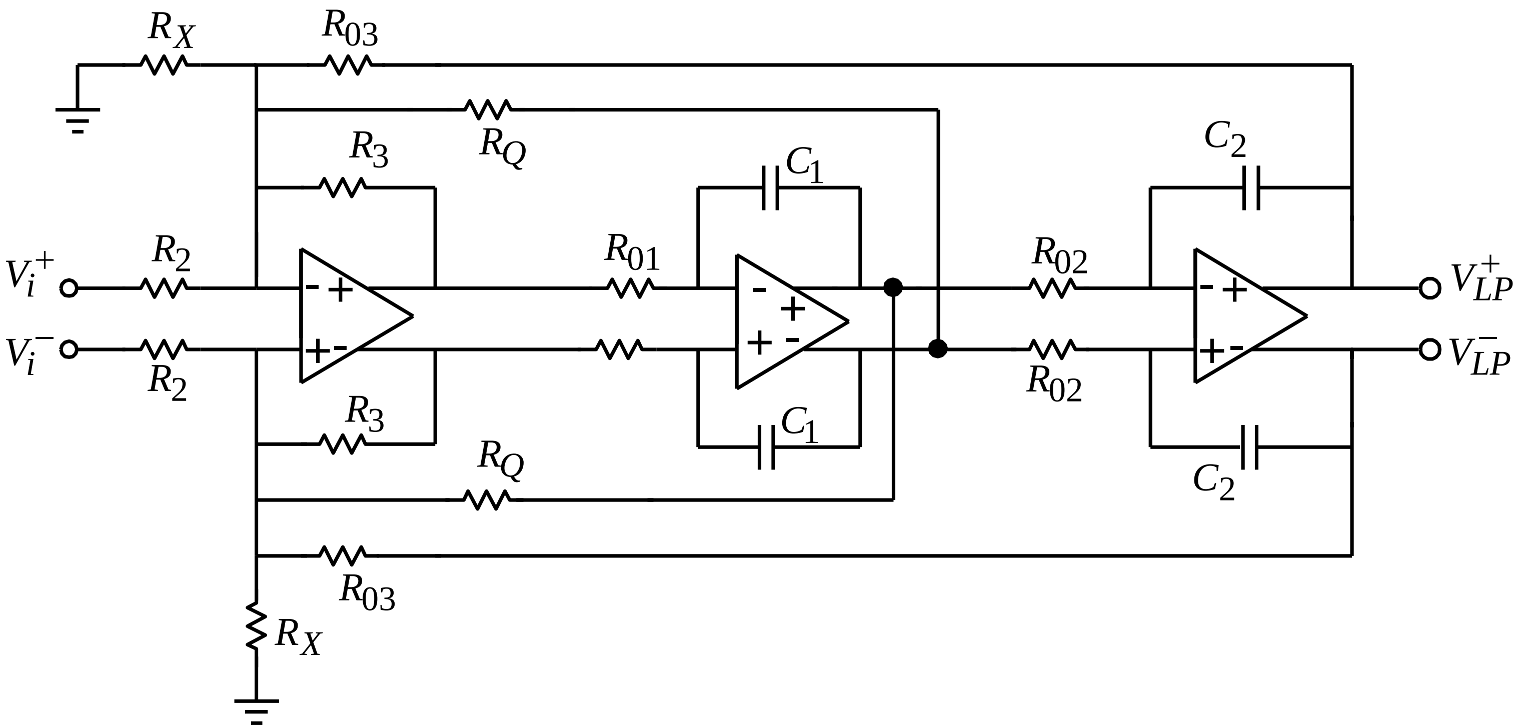
$$K_Q = \frac{R_3}{R_Q}$$

KHN State Variable Two-Integrator Filter

Use Mason's Rule:

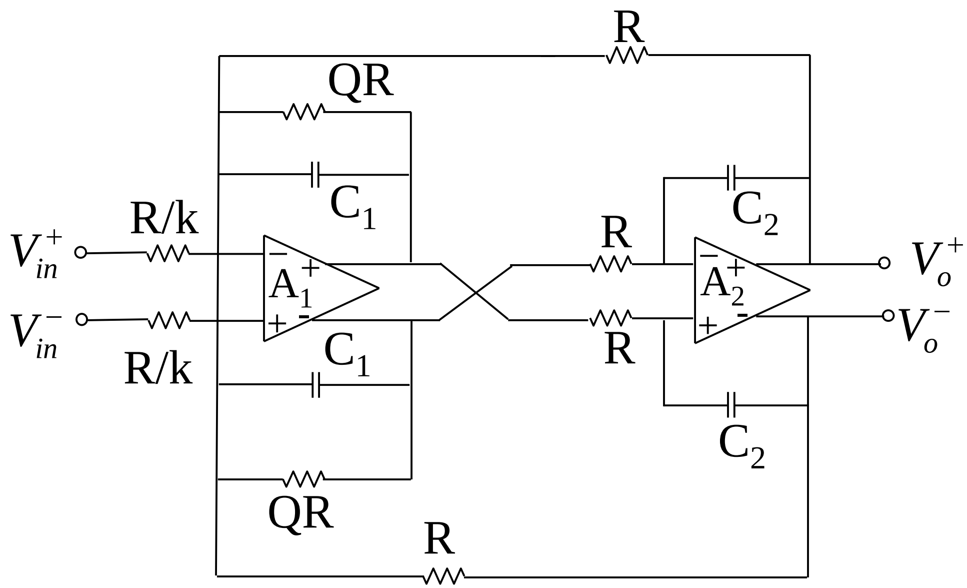
$$\frac{V_{LP}}{V_i} = \frac{-K_{32}K_{01}K_{02}/s^2}{1 + \dfrac{K_{01}K_Q}{s} + \dfrac{K_{01}K_{02}K_{03}}{s^2}} = \frac{-K_{32}K_{01}K_{02}}{s^2 + K_{01}K_Q s + K_{01}K_{02}K_{03}}$$

Next we consider the fully-differential version of the KHN filter.

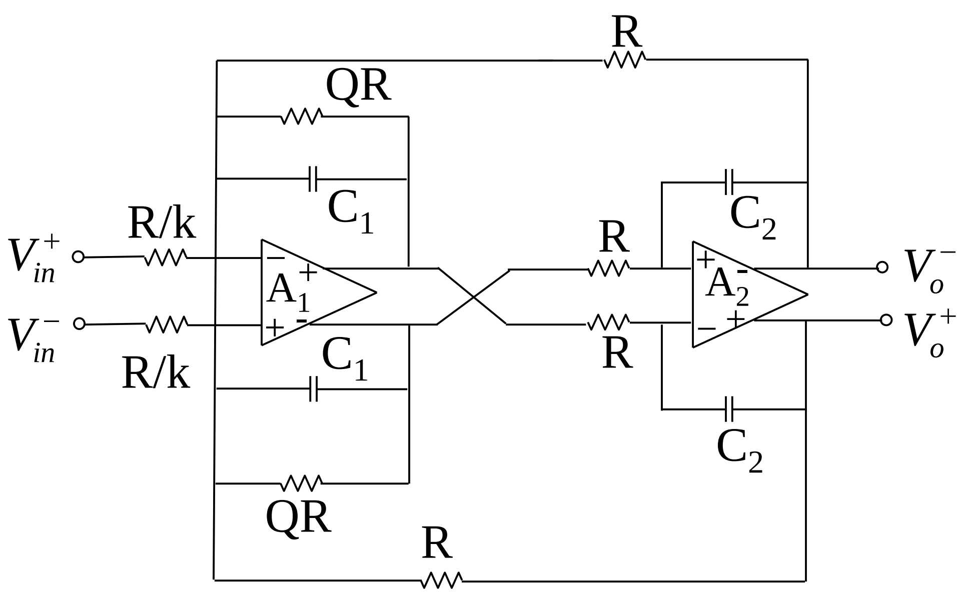


KHN Fully-Differential Version

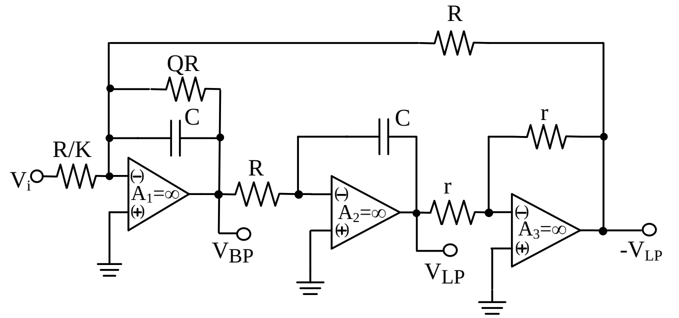
How can we take advantage of improved combination of ± Q_I in fully differential versions?



SAME!



Effects of Non-Ideal Op Amps on the Tow-Thomas Biquad



When $A_i\,(i=1,2,3)$ are finite, the denominator becomes of the transfer function yields:

$$D(s)=s^2+s\frac{\omega_o}{Q}\frac{1+\frac{2Q+1}{A_1}+\frac{1+Q}{A_2}+\frac{3Q+1}{A_1A_2}}{1+\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_1A_2}}+\omega_o^2\frac{\frac{1}{1+\frac{2}{A_3}}+\frac{2}{A_1A_2}+\frac{1}{QA_2}+\frac{1}{QA_1A_2}}{1+\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_1A_2}}$$

Let $A_i = \frac{GB_i}{s}$, $i = 1, 2, 3$

Furthermore assume the range of interest $\omega >> \frac{GB_i}{A_{oi}}$ and $\frac{\omega_o}{GB_i} << 1,\ Q >> 1.$ Then D(s) becomes:

$$D(s) = \frac{1}{\omega_o}\left(\frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} + 2\frac{\omega_o}{GB_3}\right)s^3 + \left(1 + 2\frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2}\right)s^2 + \frac{\omega_o}{Q}\left(1 + \frac{\omega_o}{GB_2}\right)s + \omega_o^2$$

$$D(s) = \left(s^2 + \frac{\omega_{oa}}{Q_a}s + \omega_{oa}^2\right)\left(\frac{s}{\omega_o}\right)\left(\frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} + 2\frac{\omega_o}{GB_3}\right)$$

Thus

$$\omega_{oa} \triangleq \omega_o(1 + \Delta_\omega)$$

or $\Delta_\omega = \frac{\omega_{oa} - \omega_o}{\omega_o}$

for $\Delta_\omega << 1$ and $Q_a >> 1$, then

$$\Delta_\omega = -\frac{\omega_o}{GB_1} - \frac{1}{2}\frac{\omega_o}{GB_2}\bigg|_{GB_1 = GB_2 = GB} = -\frac{3}{2}\left(\frac{\omega_o}{GB}\right)$$

$$\frac{Q_a}{Q} \cong \frac{1}{1 - Q\left(\frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} + 2\frac{\omega_o}{GB_3}\right)}$$

For equal $GB_1 = GB_2 = GB_3$

$$Q_a = \frac{Q}{1 - 4Q(\omega_o / GB)}$$

$$Q_a \cong Q\left(1 + 4Q\left(\frac{\omega_o}{GB}\right)\right) \quad , \quad \text{for } 4Q\left(\frac{\omega_o}{GB}\right) << 1$$

Note that for a stable filter

$4Q\ \omega_o / GB < 1$

or

$$Q < \frac{GB}{4\omega_o}$$

KEY FILTER PARAMETERS IN ACTIVE-RC FILTERS

- Dynamic Range
- Signal-To-Noise Ratio
- Total Output Noise
- Noise Power Spectral Density
- Total Area

Resistor and Capacitors can be expressed as:

$$R_\ell = r_\ell R \quad , \quad C_\ell = c_\ell C$$

where r_ℓ and c_ℓ are the normalized filter values.

The resistor power dissipation for a sinusoidal input yields

$$P_R(f) = \sum_\ell \frac{|V_i H_{i\ell}|^2}{2R_\ell} = \frac{|V_i|^2}{2R} \sum_\ell \frac{|H_{i\ell}(f)|^2}{r_\ell} \qquad (1)$$

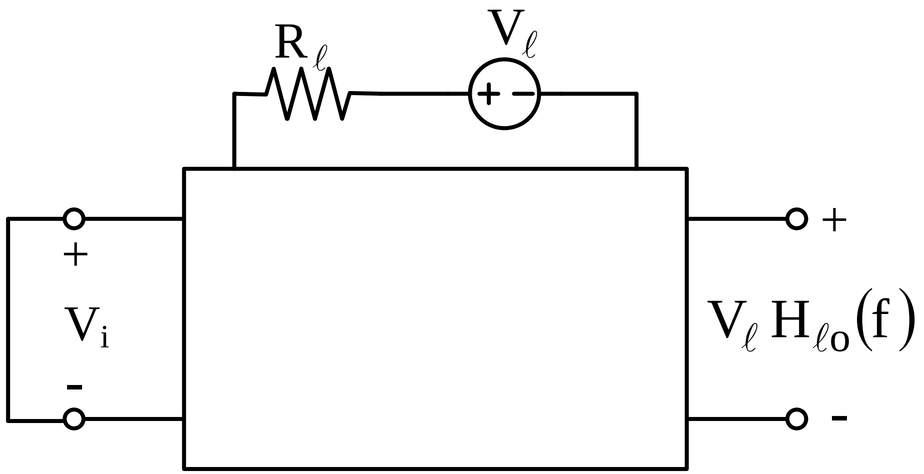
Where $H_{i\ell}(f)$ is the transfer function from the input to the terminals of resistor R_ℓ

Reference. L. oth et all, "General Results for Resistive Noise in Active RC and MOSFET-C Filters", *IEEE Trans on Circuits and Systems II*, Vol. 42, No. 12, pp. 785-793, December 1995.

Focusing on the noise resistor, the power spectral density is given by

$$S_R(f) = \sum_{\ell} 4kTR_{\ell} \left| H_{\ell o}(f) \right|^2 = 4kTR \sum_{\ell} r_{\ell} \left| H_{\ell o}(f) \right|^2 \tag{2}$$

The definition of $H_{\ell o}(f)$ is pictorially shown below:



Thus, the total output noise (mean squared value) due to the resistors become

$$N_R = \int_o^{\infty} S_R(f)\, df$$

In practice the upper limit of the integration is limited to a useful practical value.

The signal-to-ratio for a given V_i and frequency f is given by

$$SNR = \frac{|V_i H(f)|^2}{2N_R} \tag{3}$$

and

$\max_f P_R(f) \le P_{R,max}$

where $P_{R,max}$ is the maximum specified power dissipation in the resistors. Then

$$DR = \frac{|V_i|^2_{max} \max_f |H(f)|^2}{2N_R} \tag{4}$$

Let us consider a second - order BP filer example

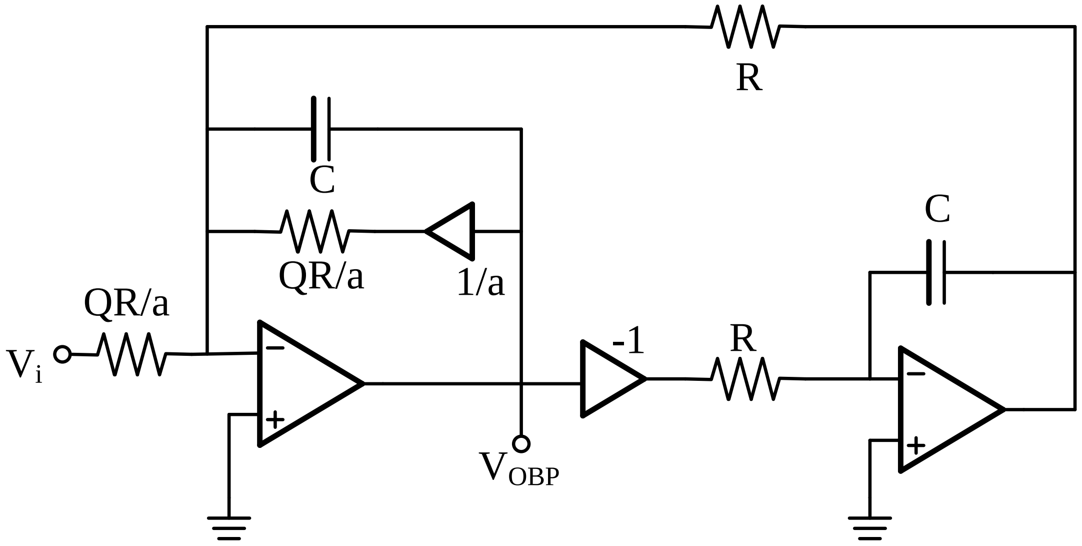
$$H_{BP}(s) = \frac{\frac{\omega_c}{Q} s}{s^2 + \frac{\omega_c}{Q} s + \omega_c^2}$$

Using the following notation the above $H_{BP}(s)$ yields

$$H_{BP}(f) = \frac{Q^{-1} f_c (jf)}{(jf)^2 + Q^{-1} f_c (jf) + f_c^2}$$

$$P_R(f_c) = \frac{|V_i|^2}{R} a \left(a + Q^{-1}\right)$$

For the biquad shown below



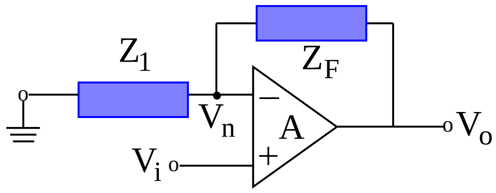
and

$$N_R = \frac{2kT}{C} \frac{\left(1 + Q / a\right)}{a}$$

$a \uparrow \Rightarrow N_R \downarrow \Rightarrow a\, V_{OBP}$ limited by linearity and by resistor power dissipation which is proportional to $(a)^2$.

Fully Differential Fully Balanced Circuits

What is the problem with single-input / single-output?



$$V_n = \frac{V_o Z_1}{Z_1 + Z_F}$$

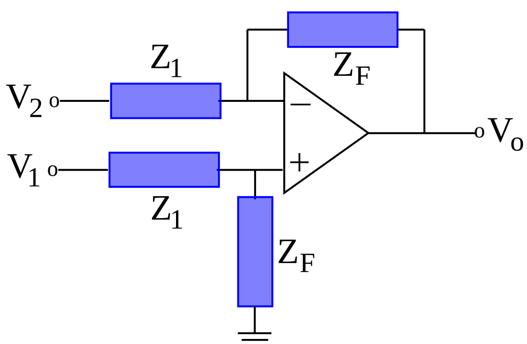
$$V_1 - V_n = \frac{V_o}{A}\Bigg|_{A \to \infty} = 0$$

$$\text{For} \quad V_i = V_{id} + V_{icm}$$

$$V_o = \left(1 + \frac{Z_F}{Z_1}\right)(V_{id} + V_{icm})$$

No elimination of common-mode signal.

How to solve this problem?



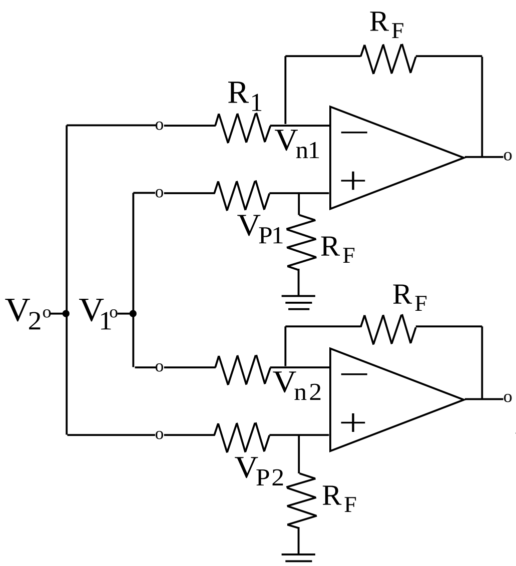
$$\text{For} \quad V_i = V_{id} + V_{icm} = (V_1 - V_2) + \frac{(V_1 + V_2)}{2}$$

$$V_o = \frac{Z_F}{Z_1}(V_1 - V_2)$$

No common-mode output.

How to obtain a fully differential circuit?
We will discuss two potential approaches

Approach 1



$$V_{o1} = -\frac{R_F}{R_1}(V_2 - V_1)$$

$$= \frac{R_F}{R_1}(V_1 - V_2)$$

$$V_{o2} = +\frac{R_F}{R_1}(V_2 - V_1)$$

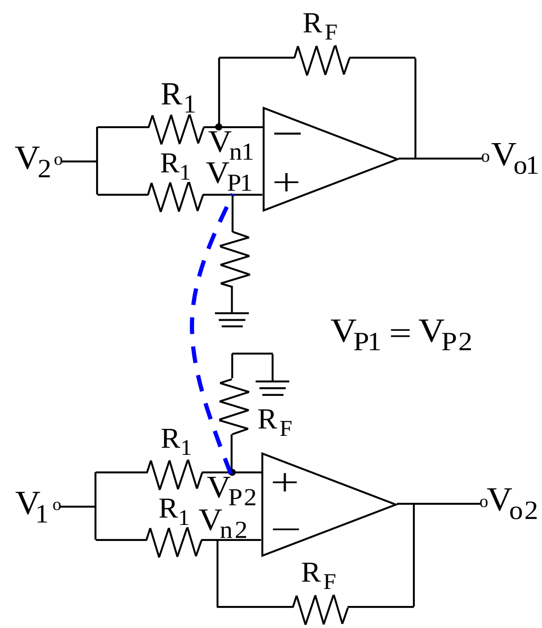
$$V_{o1} - V_{o2} = \frac{R_F}{R_1}(V_1 - V_2 - V_2 + V_1) =$$

$$V_{oD} = \frac{2R_F}{R_1}(V_1 - V_2)$$

conditions $V_{n2} = V_{P2} \quad ; \quad V_{n1} = V_{P1}$

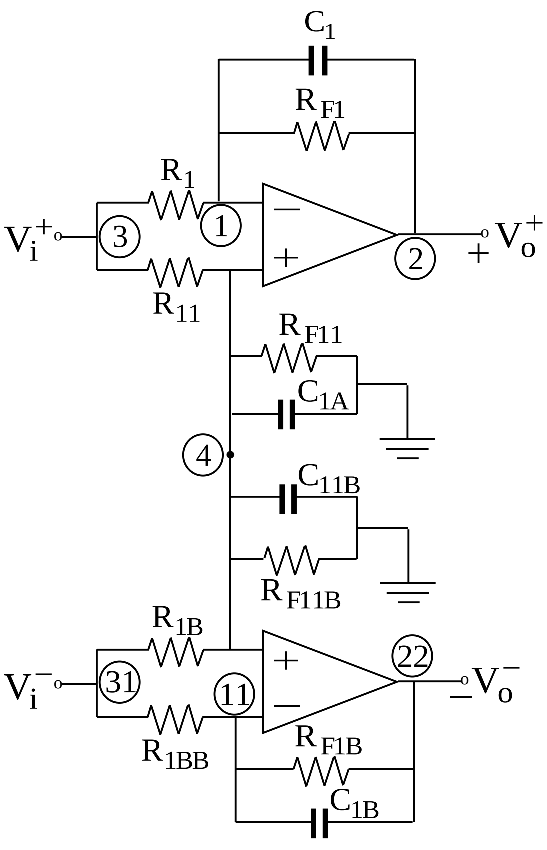
Remark: sensitive to CM signals

Approach 2

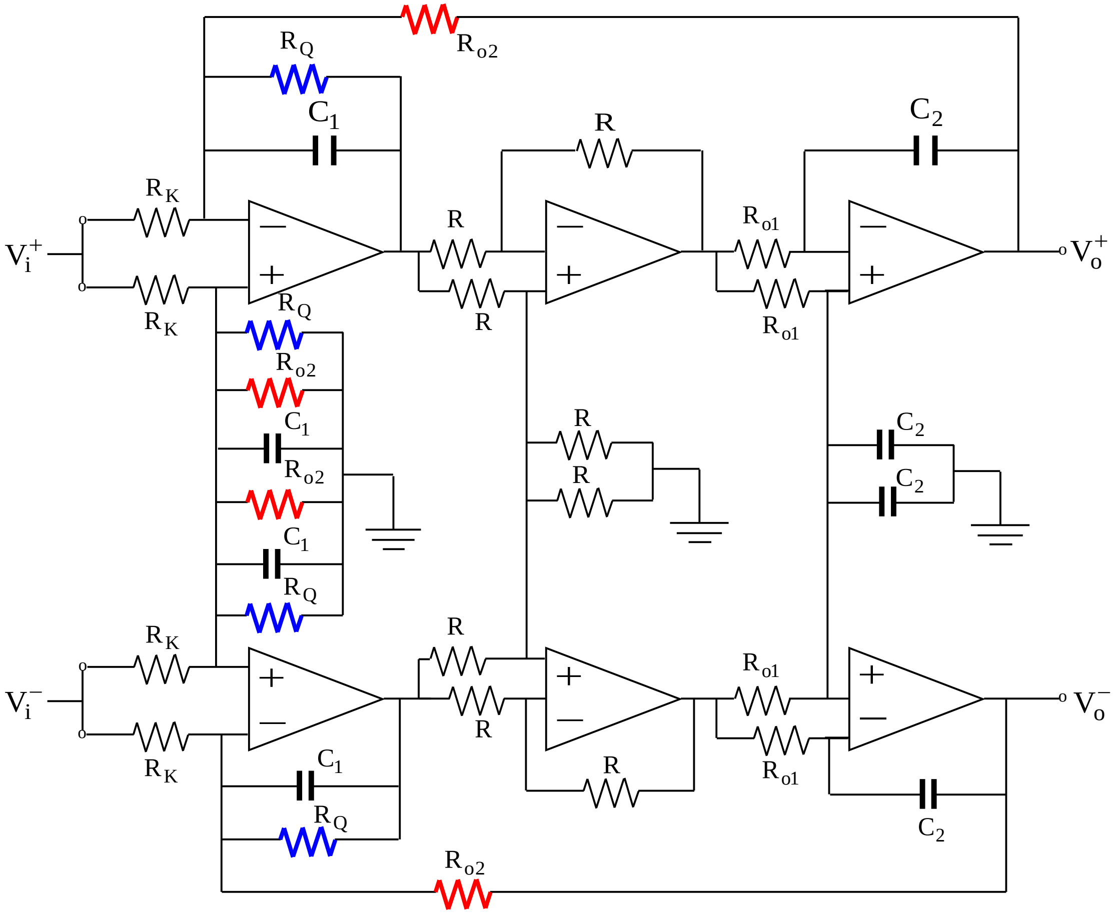


Remark:
More robust to reject common-mode signals

```
First-Order FB Low Pass with Op Amp
*
.subckt opamp non inv out
rin non inv 100K
egain    1    0    (non, inv)   200K
ropen    1    2    2K
copen  2   0    15.9155u
eout   3   0    (2, 0)   1
rout   3    out   50
.ends
*vin       3    31    ac    1.0
vin     31  0   ac   1.0
x1      4      1      2     opamp
x2      4      11     22    opamp
R1      3      1      1K
R11     3      4      1K
R1B     31     4      1K
R1BB 31        11     1K
RF1     2      1      1K
RF1B 22        11     1K
RF11  4        0      1K
RF11B          4      0       1K
C1      2      1      0.159155u
C1B    22      11     0.159155u
C1A    4       0      0.159155u
C11B 4         0      0.159155u
rdummy  3    31    1
.ac   dec   10    10Hz   10KHz
.probe
.end
```



Fully Balanced T-T Active-RC Implementation



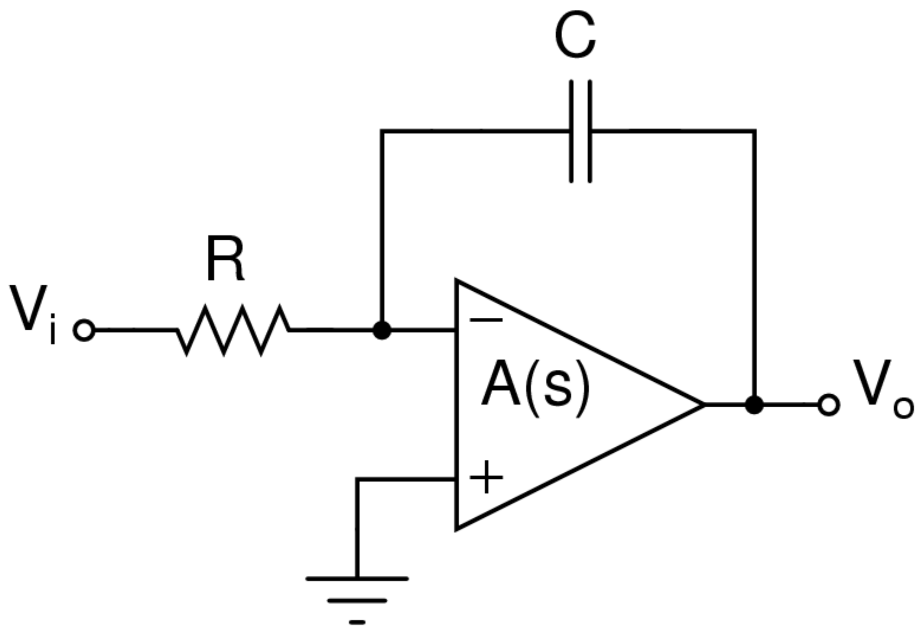
622 Active Filters

By Edgar Sánchez-Sinencio

Texas A&M University

Introduction to Matlab and Simulink For Filter Design

Example 1: Ideal Integrator



R = 1KΩ   C = 0.159mF

Bode Plot: Ideal Integrator (Matlab)

s=tf('s');

R=1e3; %Resistor Value

C=0.159e-3; %Capacitor Value

hs=1/(R*C*s); %hs= Vo(s)/Vi(s)

figure(1)

bode(hs) %Create Bode Plo

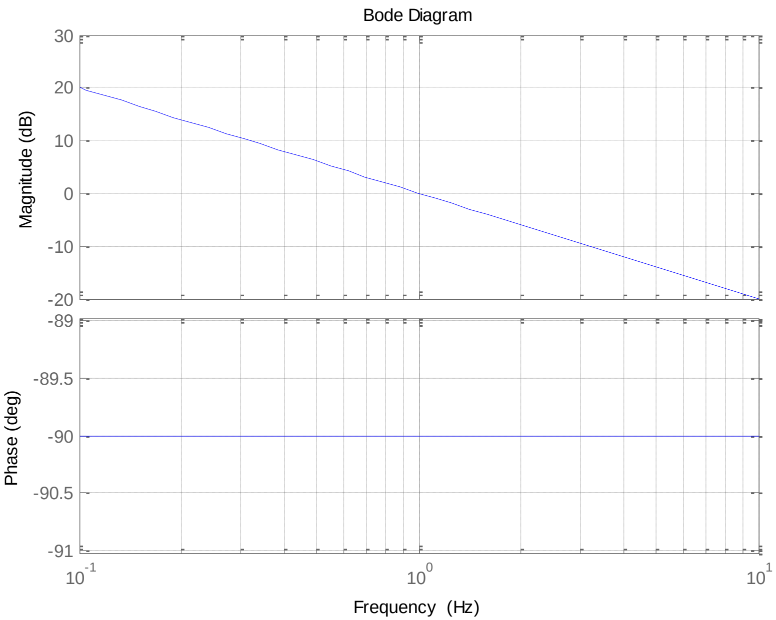
grid minor %Add grid to plo

H= gcr; %change X-axis units

h.AxesGrid.Xunits = 'Hz'; %Set units to Hz

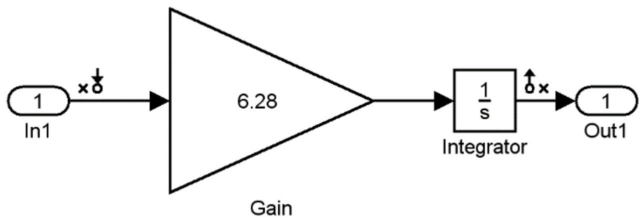
pole(hs); %calculates hs poles

zero(hs); %calculates hs zeros

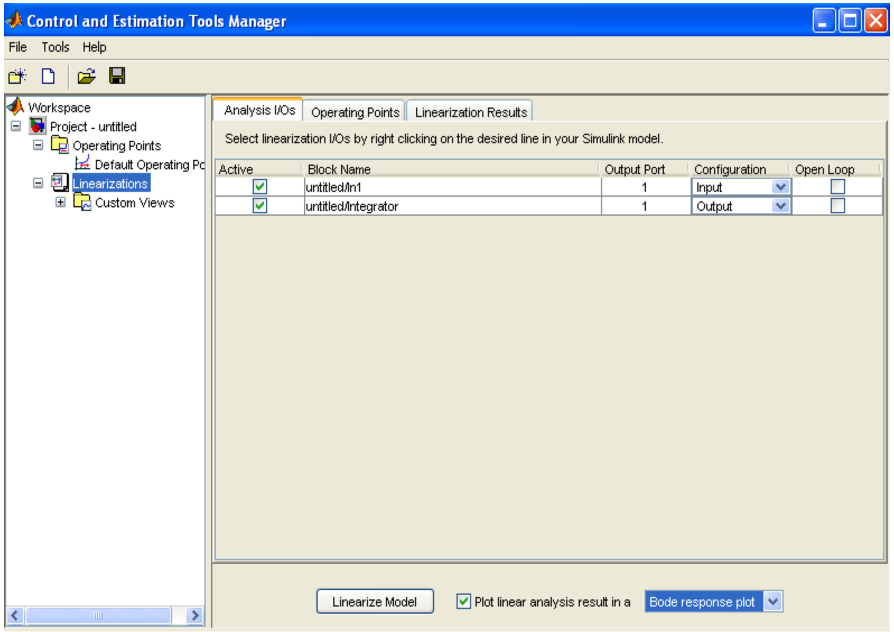


Bode Plot: Ideal Integrator (Simulink)

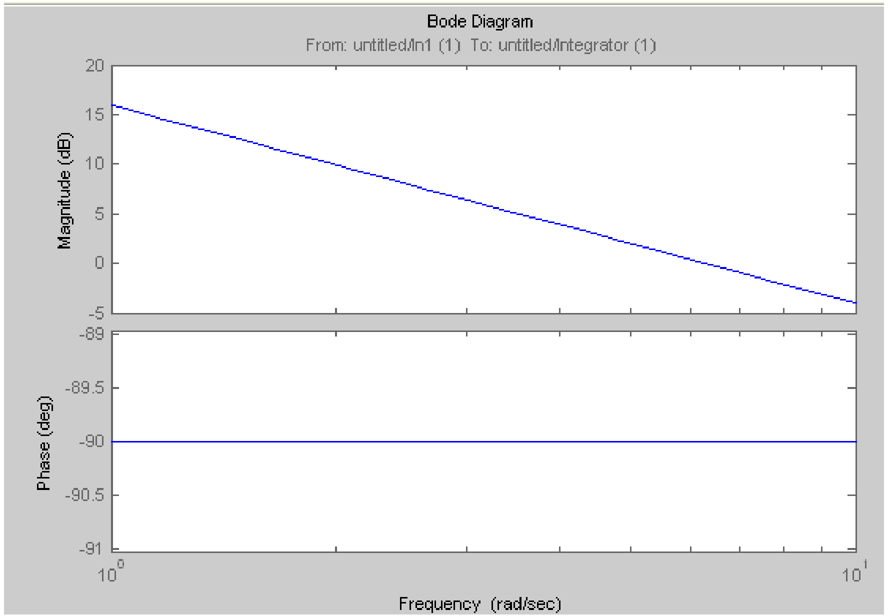
1) Create Model using Gain, Integrator, and In/Out blocks



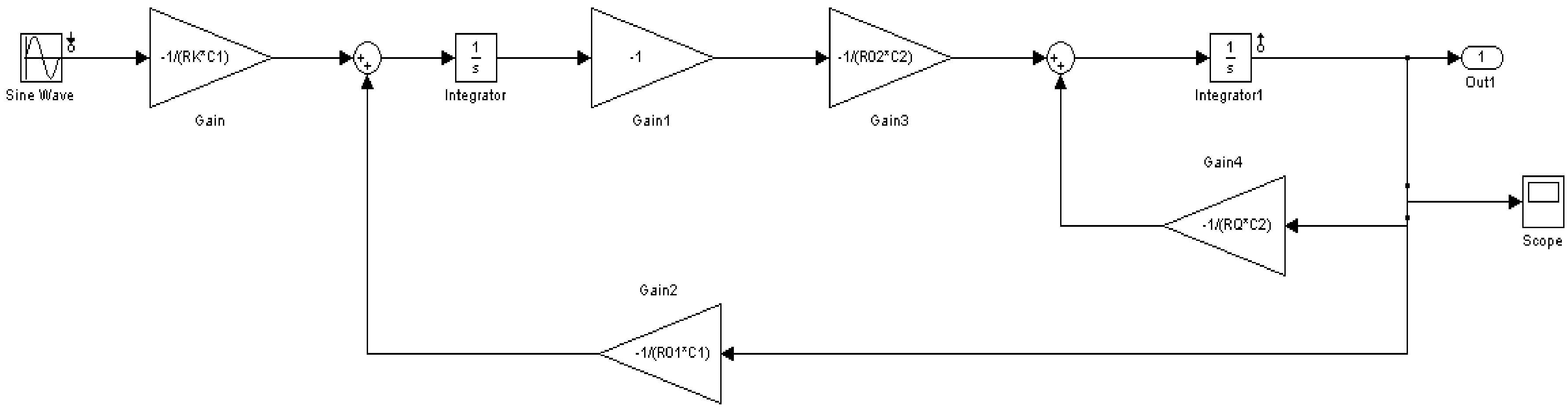
2) Go to: Tools => Control Design=> Linear Analysis



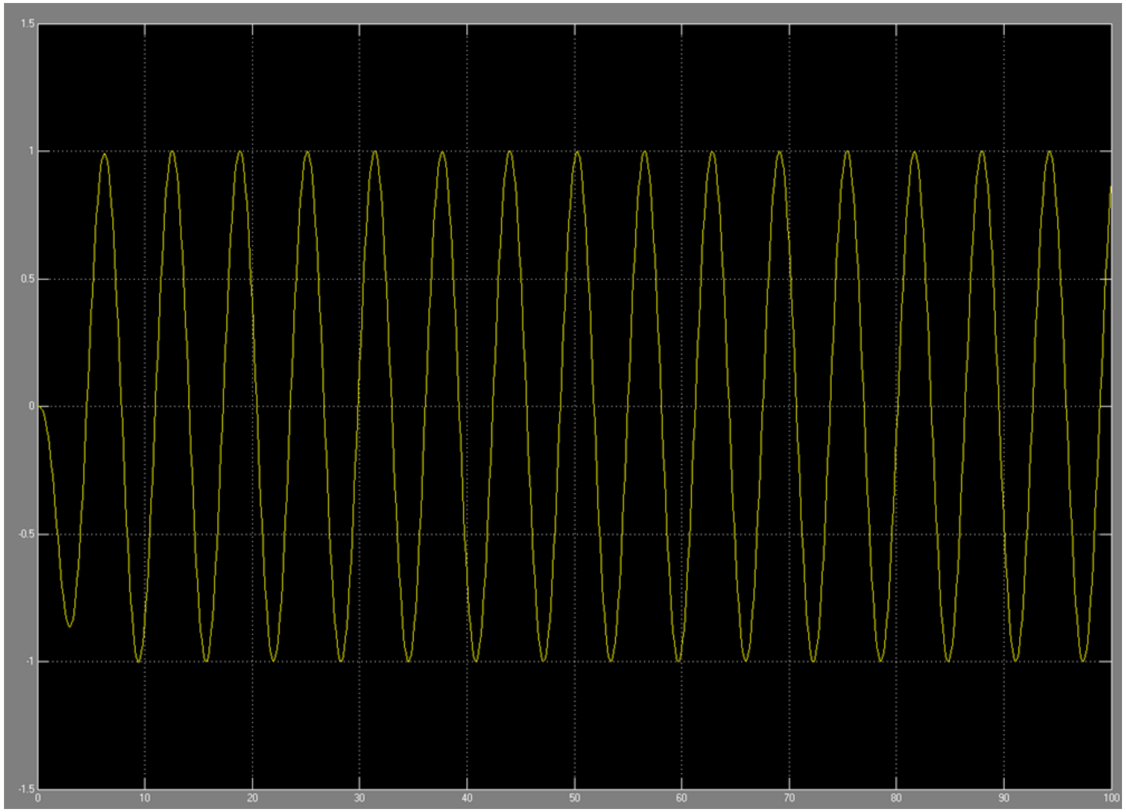
3) Then press: Linearize model



Tow-Thomas Biquad (Simulink)

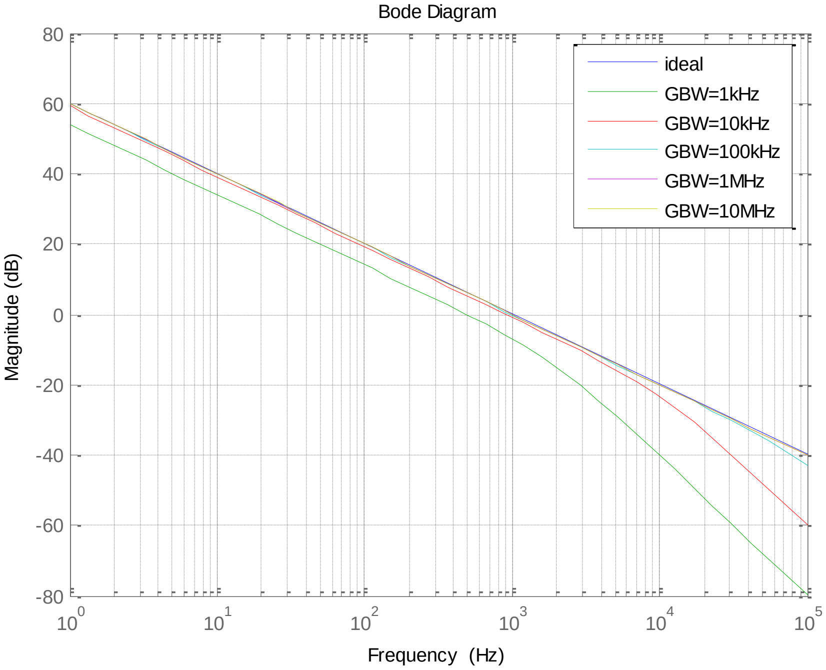


Output Waveform (Scope)



Integrator Non-ideal amplifier

```
clear
clc
s=tf('s');
R=1;                          %Resistor Value
C=0.159e-3;                   %Capacitor Value
hs1=-1/(R*C*s);               %hs= Vo(s)/Vi(s)
figure(1)
bodemag(hs1)
hold on
f=1e3;
for i=1:5;
GBW=2*pi*f;
A=GBW/s;
Beta=R/(R+1/(s*C));
hs2=-1/(R*C*s)*1/(1+1/(A*Beta));
hold on
bodemag(hs2,{2*pi*1,2*pi*1e5})
f=10*f;
end
grid minor                    %Add grid to plot
h= gcr;                       %change X-axis units
h.AxesGrid.Xunits = 'Hz'; %Set units to Hz
legend('ideal', 'GBW=1kHz','GBW=10kHz', 'GBW=100kHz','GBW=1MHz', 'GBW=10MHz',1)
```



Filter Approximation: Low-Pass Butterworth

- E.g.: Use Matlab to find the numerator b and denominator a coefficients for a third-order Butterworth low-pass filter prototype with normalized cutoff frequency.

```
[z,p,k]=buttap(3);   % To get gain and poles
[b,a]=zp2tf(z,p,k);  %To get b and a coefficients
H=tf([b],[a]);  %to generate transfer function
figure(1)
bode(H);        %Bode plot
grid minor;
figure(2)
pzmap(H);       %plot poles and zeros
grid minor;
```

The squared magnitude of a low-pass butterworth filter is given by:

$$|H(\omega)|^2 = \frac{1}{1 + (\omega/\omega_0)^{2n}}$$

Results:

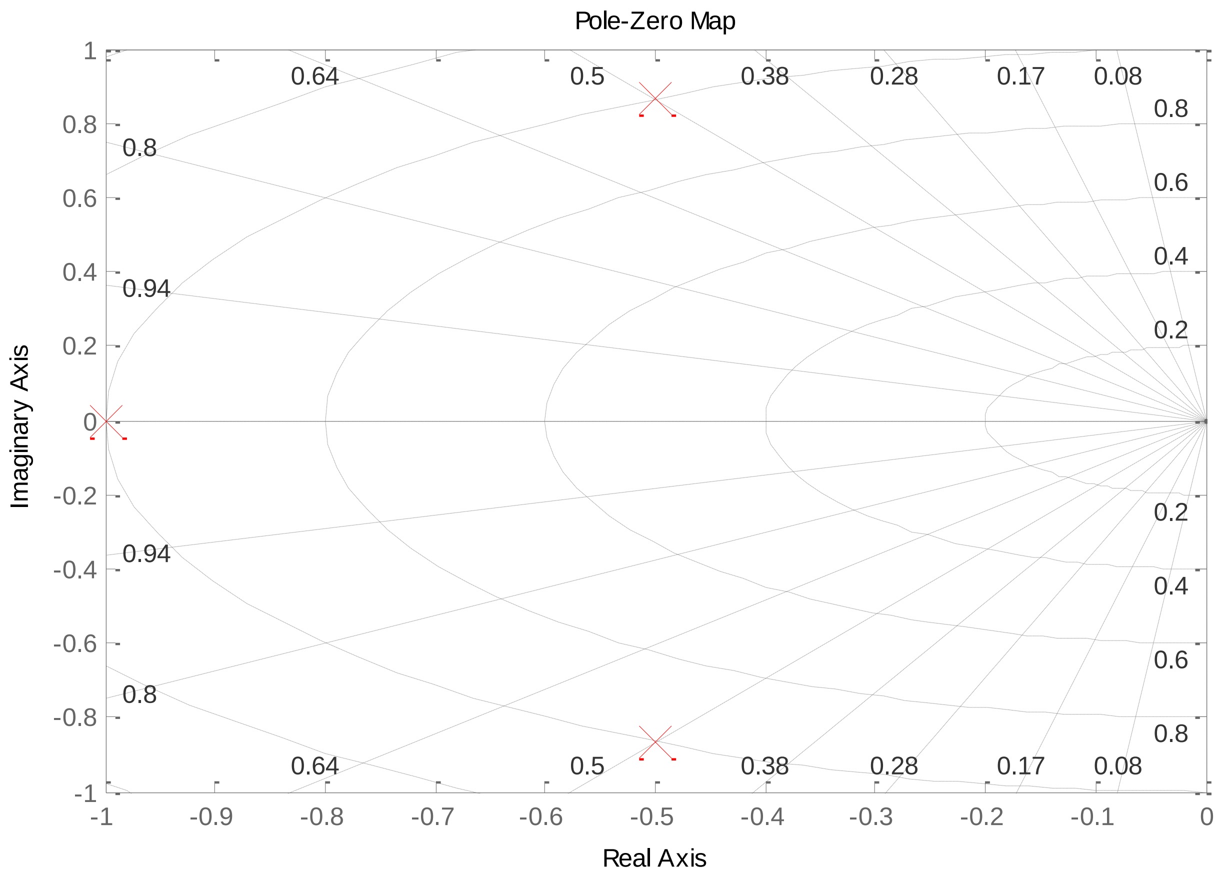
b = 0 0 0 1

a = 1 2 2 1

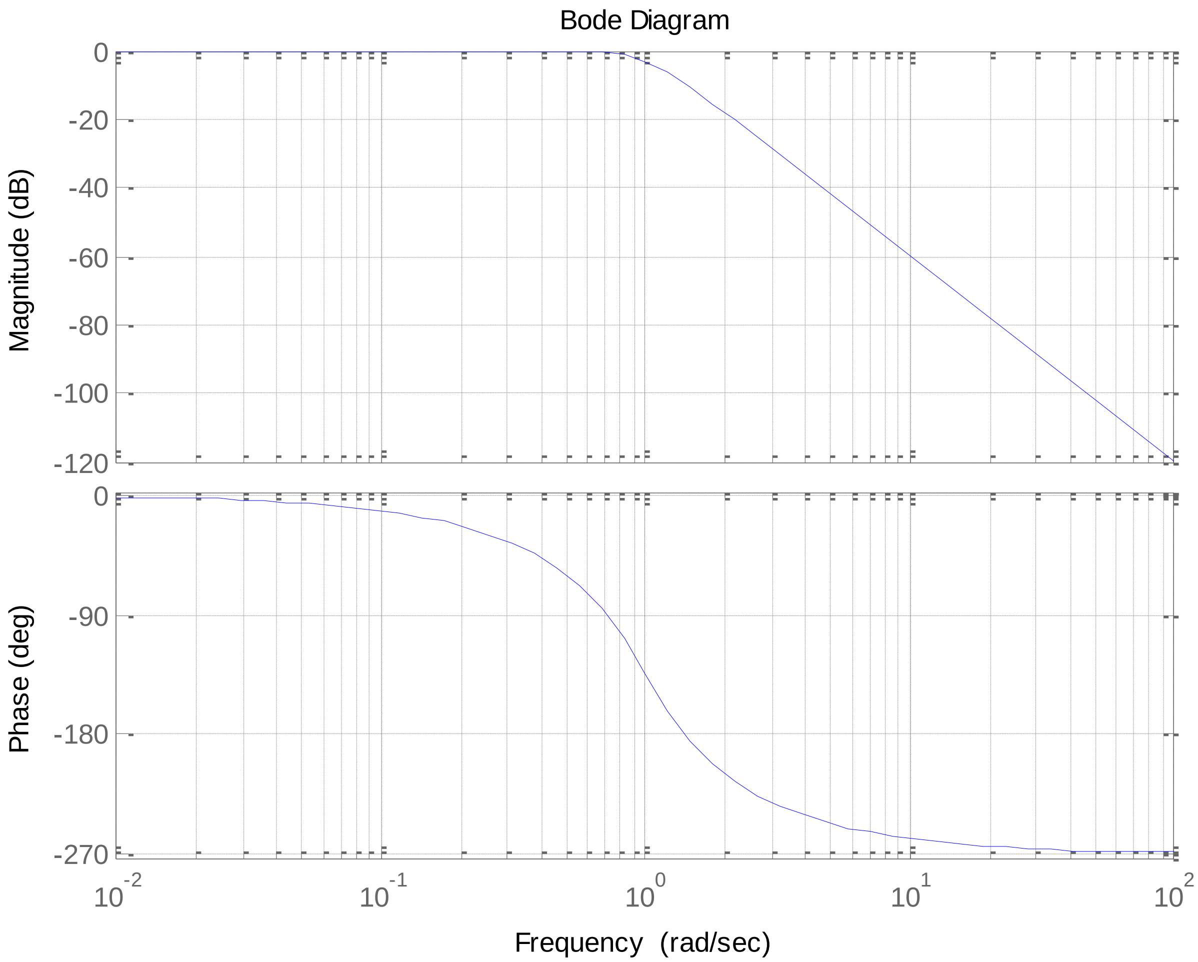
Thus,

$$H(s) = \frac{1}{s^3+2s^2+2s+1}$$

Pole-zero plot



Bode Plot



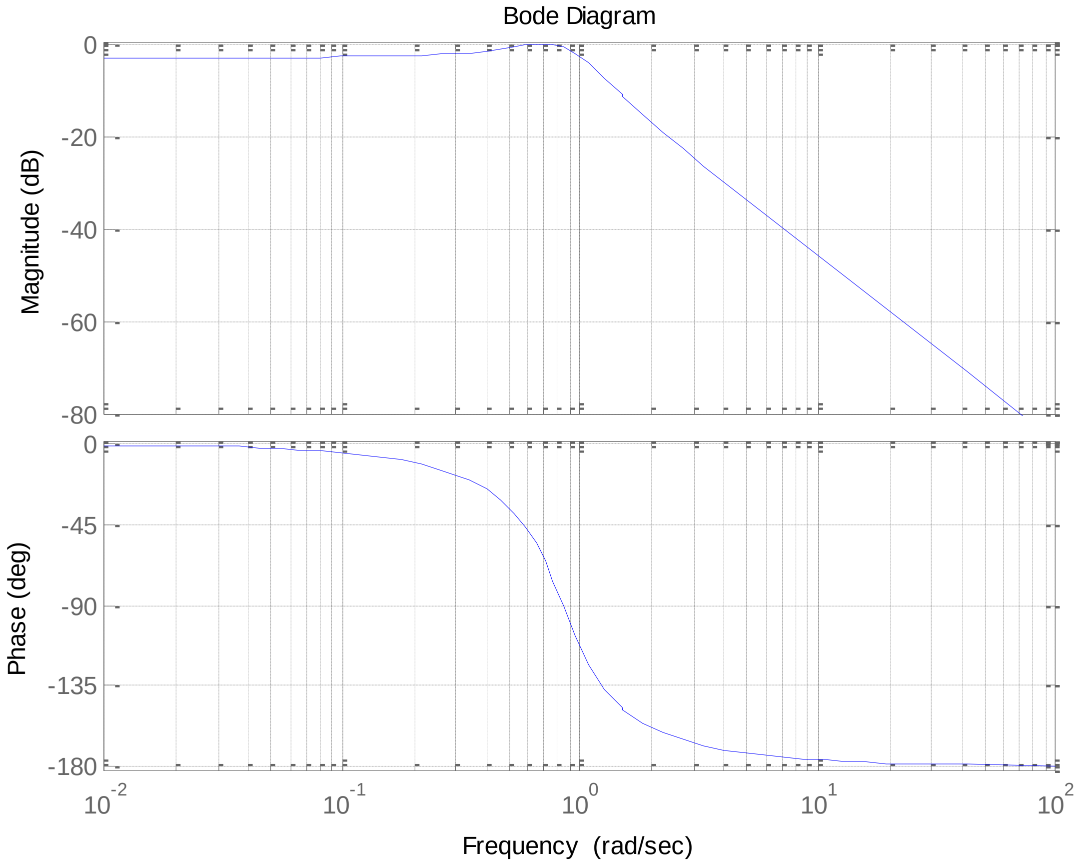
Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
-100
-120
Phase (deg)
0
-90
-180
-270
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

Low-pass Chebyshev Filter

- Use the Matlab cheb1ap function to design a second order Type I Chebyshev low-pass filter with 3dB ripple in the pass band

```
w=0:0.05:400; % Define range to plot
[z,p,k]=cheb1ap(2,3);
[b,a]=zp2tf(z,p,k); % Convert zeros and poles of G(s) to polynomial form
bode(b,a)
grid minor;
```

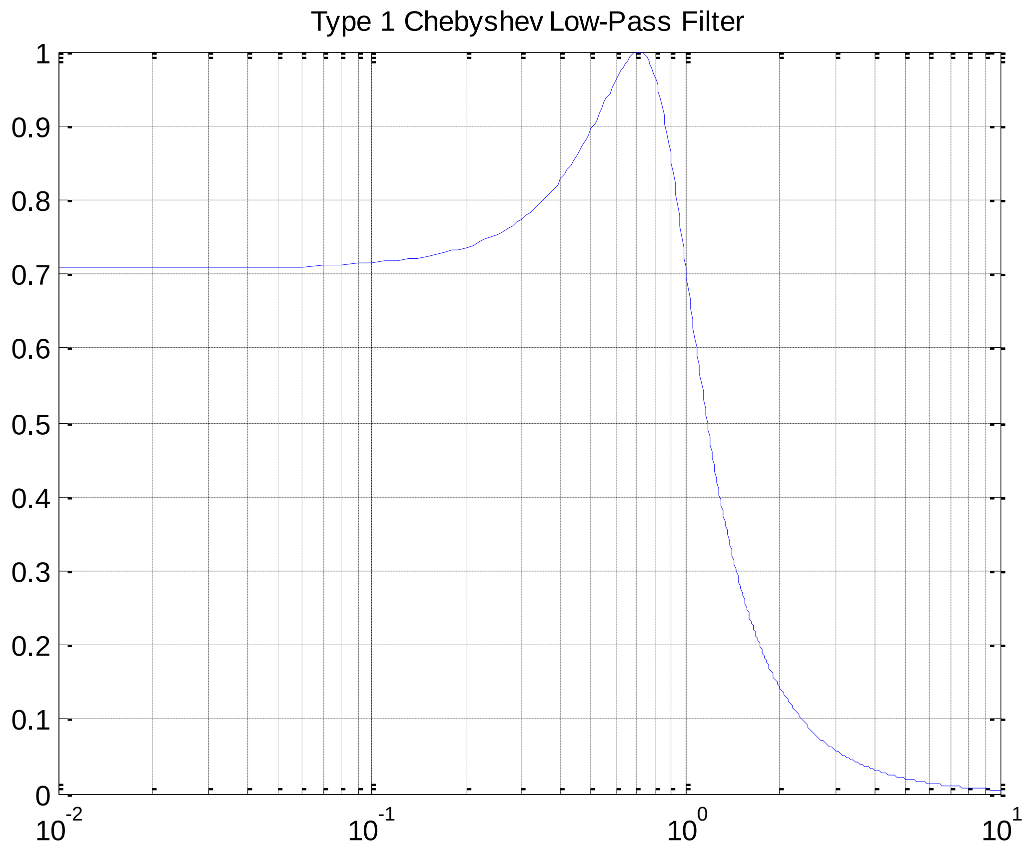
Low-pass Chebyshev Filter



Low-pass Chebyshev Filter

```
% Another way to write the code!
w=0:0.01:10;
[z,p,k]=cheb1ap(2,3);
[b,a]=zp2tf(z,p,k);
Gs=freqs(b,a,w);
xlabel('Frequency in rad/s');
ylabel('Magnitude of G(s)');
semilogx(w,abs(Gs));
title('Type 1 Chebyshev Low-Pass Filter');
Grid;
```

Low-pass Chebyshev Filter

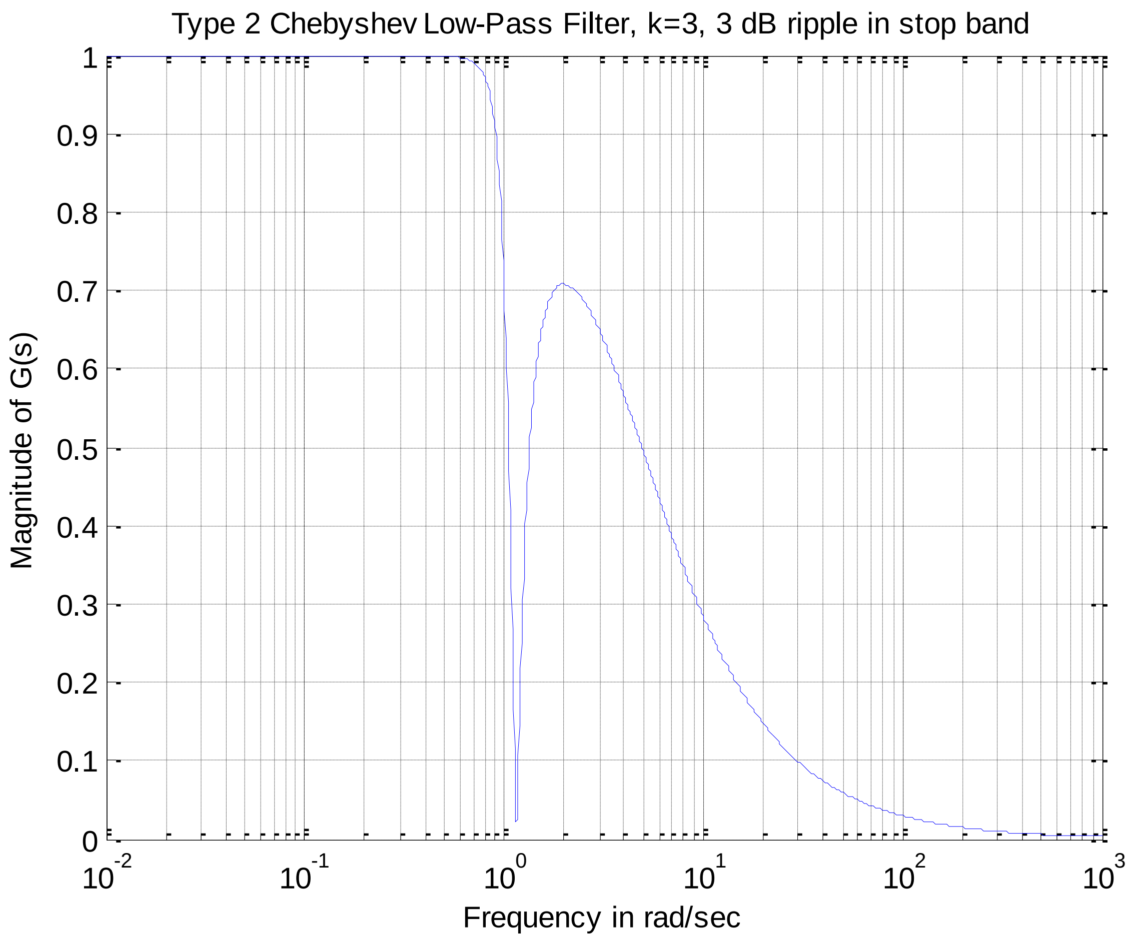


Inverse Chebyshev

- Using the Matlab cheb2ap function, design a third order Type II Chebyshev analog filter with 3dB ripple in the stop band.

```
w=0:0.01:1000;
[z,p,k]=cheb2ap(3,3);
[b,a]=zp2tf(z,p,k); Gs=freqs(b,a,w);
semilogx(w,abs(Gs));
xlabel('Frequency in rad/sec');
ylabel('Magnitude of G(s)');
title('Type 2 Chebyshev Low-Pass Filter, k=3, 3 dB ripple in stop band');
grid
```

Inverse Chebyshev

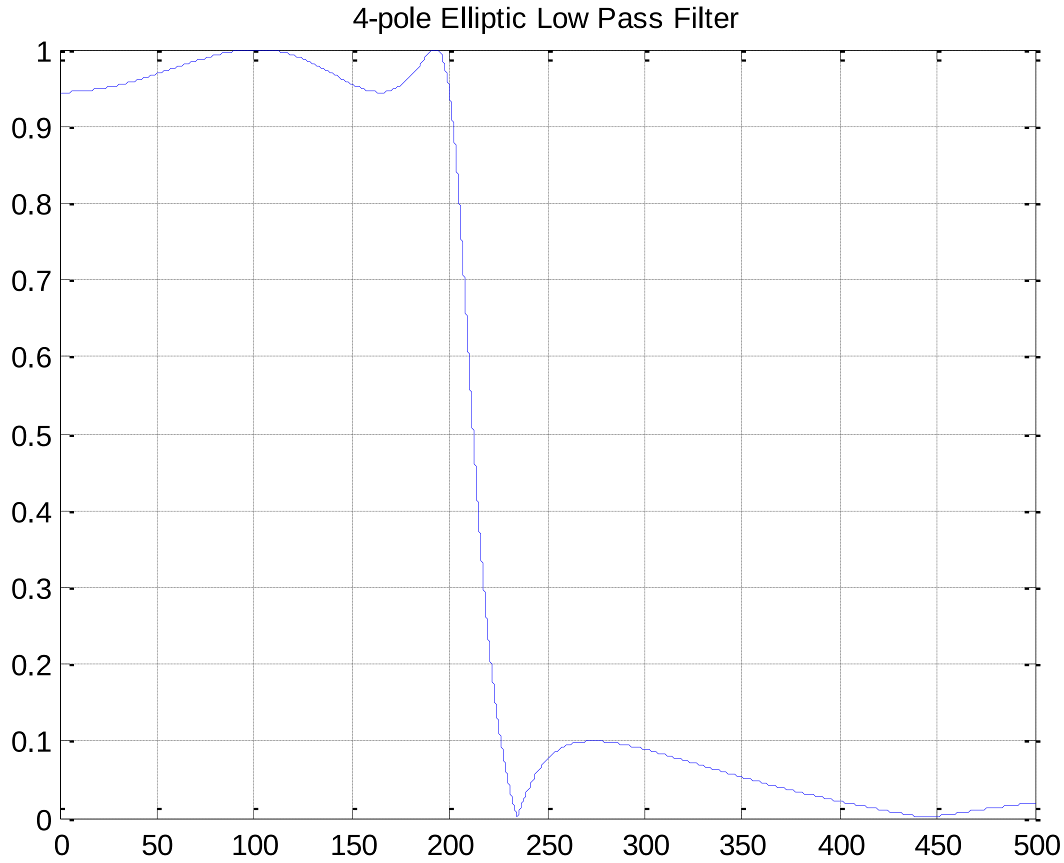


Elliptic Low-Pass Filter

- Use Matlab to design a four pole elliptic analog low-pass filter with 0.5dB maximum ripple in the pass-band and 20dB minimum attenuation in the stop-band with cutoff frequency at 200 rad/s.

```
w=0: 0.05: 500;
[z,p,k]=ellip(4, 0.5, 20, 200, 's');
[b,a]=zp2tf(z,p,k);
Gs=freqs(b,a,w);
plot(w,abs(Gs))
title('4-pole Elliptic Low Pass Filter');
grid
```

Elliptic Low-Pass Filter



Transformation Methods

- Transformation methods have been developed where a low pass filter can be converted to another type of filter by simply transforming the complex variable s.

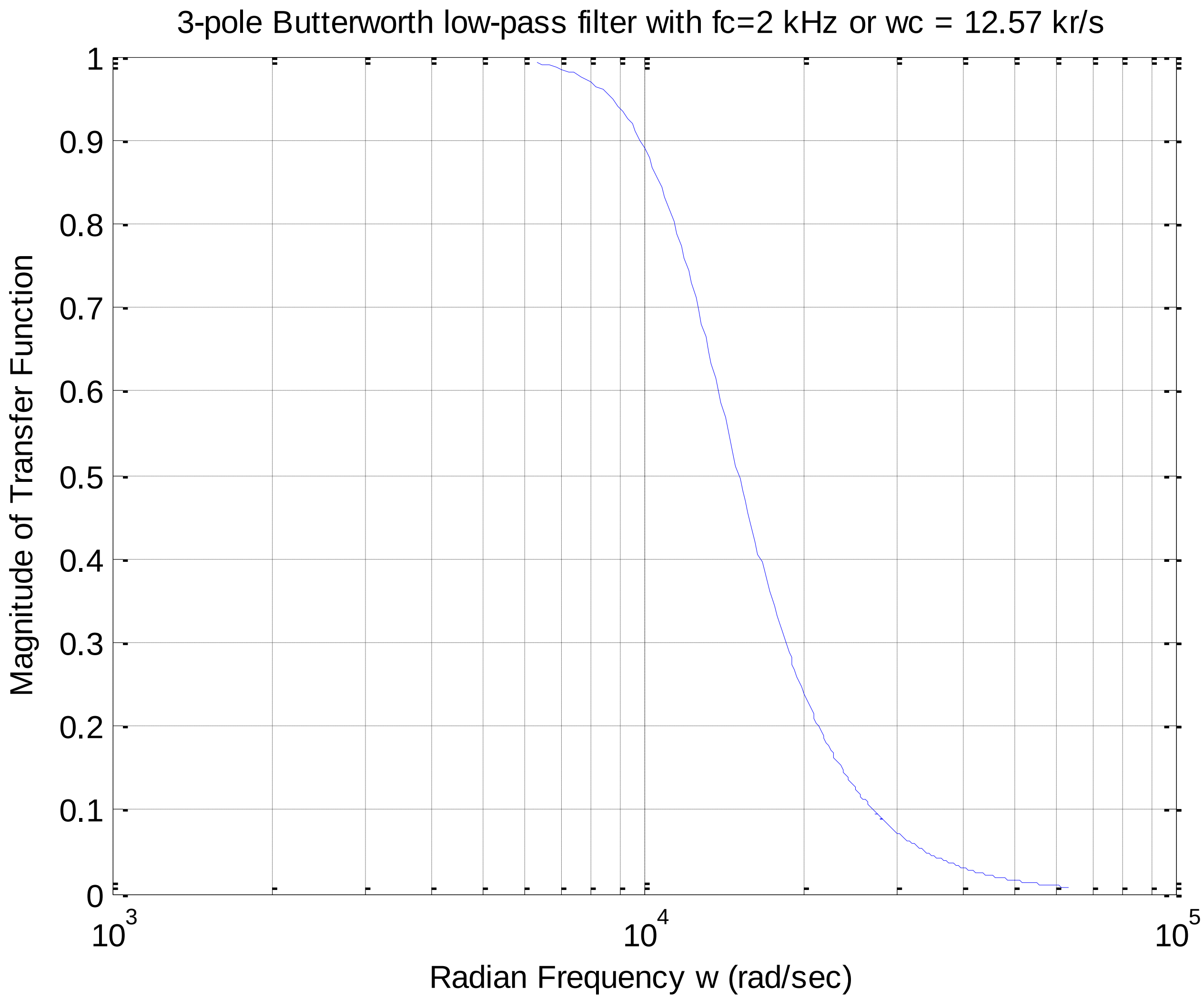
- Matlab lp2lp, lp2hp, lp2bp, and lp2bs functions can be used to transform a low pass filter with normalized cutoff frequency, to another low-pass filter with any other specified frequency, or to a high pass filter, or to a band-pass filter, or to a band elimination filter, respectively.

LPF with normalized cutoff frequency, to another LPF with any other specified frequency

- Use the MATLAB **buttap** and **lp2lp** functions to find the transfer function of a third-order Butterworth low-pass filter with cutoff frequency fc=2kHz.

```
% Design 3 pole Butterworth low-pass filter (wcn=1 rad/s)
[z,p,k]=buttap(3);
[b,a]=zp2tf(z,p,k);            % Compute num, den coefficients of this filter
(wcn=1rad/s)
f=1000:1500/50:10000;   % Define frequency range to plot
w=2*pi*f;                      % Convert to rads/sec
fc=2000;                       % Define actual cutoff frequency at 2 KHz
wc=2*pi*fc;                    % Convert desired cutoff frequency to rads/sec
[bn,an]=lp2lp(b,a,wc);     % Compute num, den of filter with fc = 2 kHz
Gsn=freqs(bn,an,w);        % Compute transfer function of filter with fc = 2 kHz
semilogx(w,abs(Gsn));
grid;
xlabel('Radian Frequency w (rad/sec)')
ylabel('Magnitude of Transfer Function')
title('3-pole Butterworth low-pass filter with fc=2 kHz or wc = 12.57 kr/s')
```

LPF with normalized cutoff frequency, to another LPF with any other specified frequency

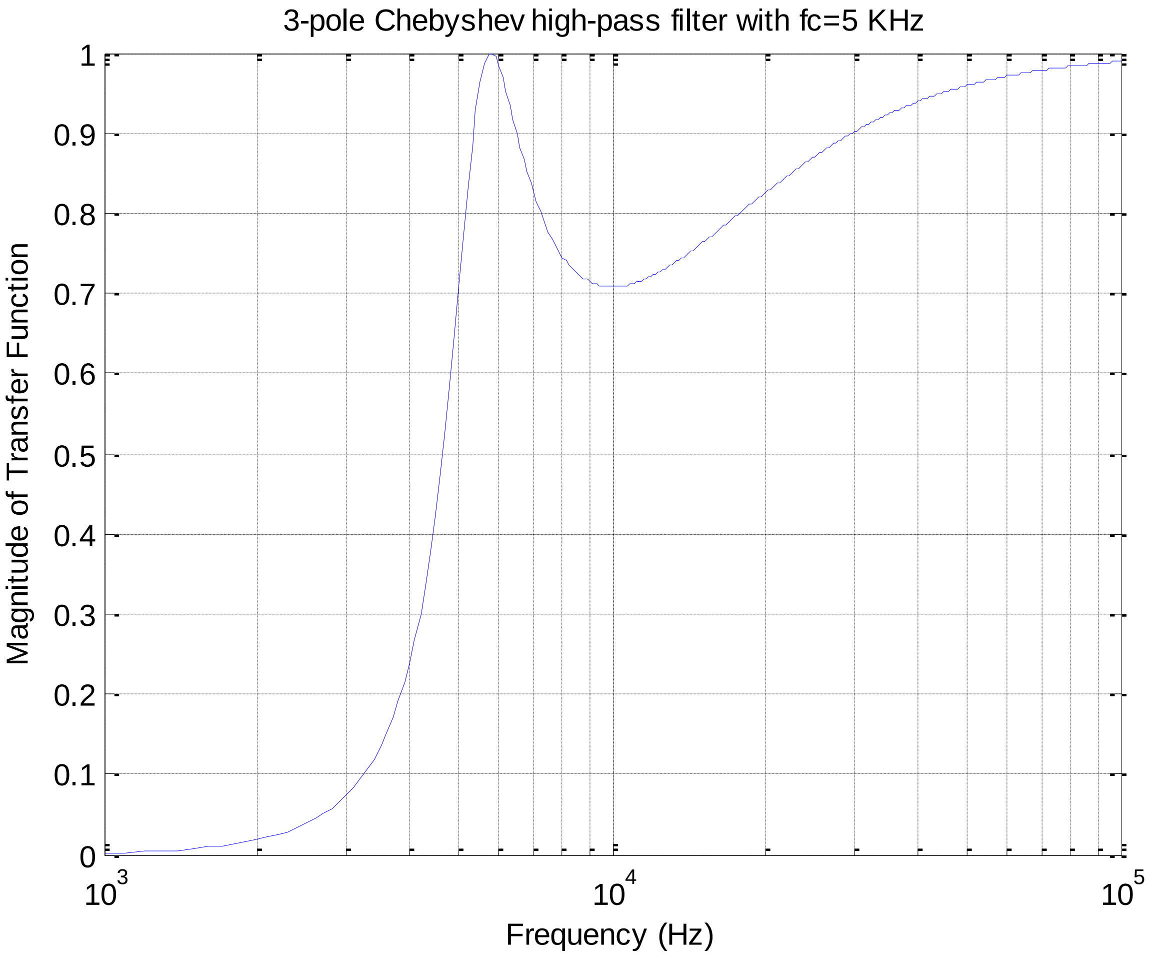


High-Pass Filter

- Use the MATLAB commands **cheb1ap** and **lp2hp** to find the transfer function of a 3-pole Chebyshev high-pass analog filter with cutoff frequency fc = 5KHz.

```
% Design 3 pole Type 1 Chebyshev low-pass filter, wcn=1 rad/s
[z,p,k]=cheb1ap(3,3);
[b,a]=zp2tf(z,p,k);              % Compute num, den coef. with wcn=1 rad/s
f=1000:100:100000;           % Define frequency range to plot
fc=5000;                              % Define actual cutoff frequency at 5 KHz
wc=2*pi*fc;                          % Convert desired cutoff frequency to rads/sec
[bn,an]=lp2hp(b,a,wc);       % Compute num, den of high-pass filter with fc =5KHz
Gsn=freqs(bn,an,2*pi*f);   % Compute and plot transfer function of filter with fc = 5 KHz
semilogx(f,abs(Gsn));
grid;
xlabel('Frequency (Hz)');
ylabel('Magnitude of Transfer Function')
title('3-pole Type 1 Chebyshev high-pass filter with fc=5 KHz ')
```

High-Pass Filter

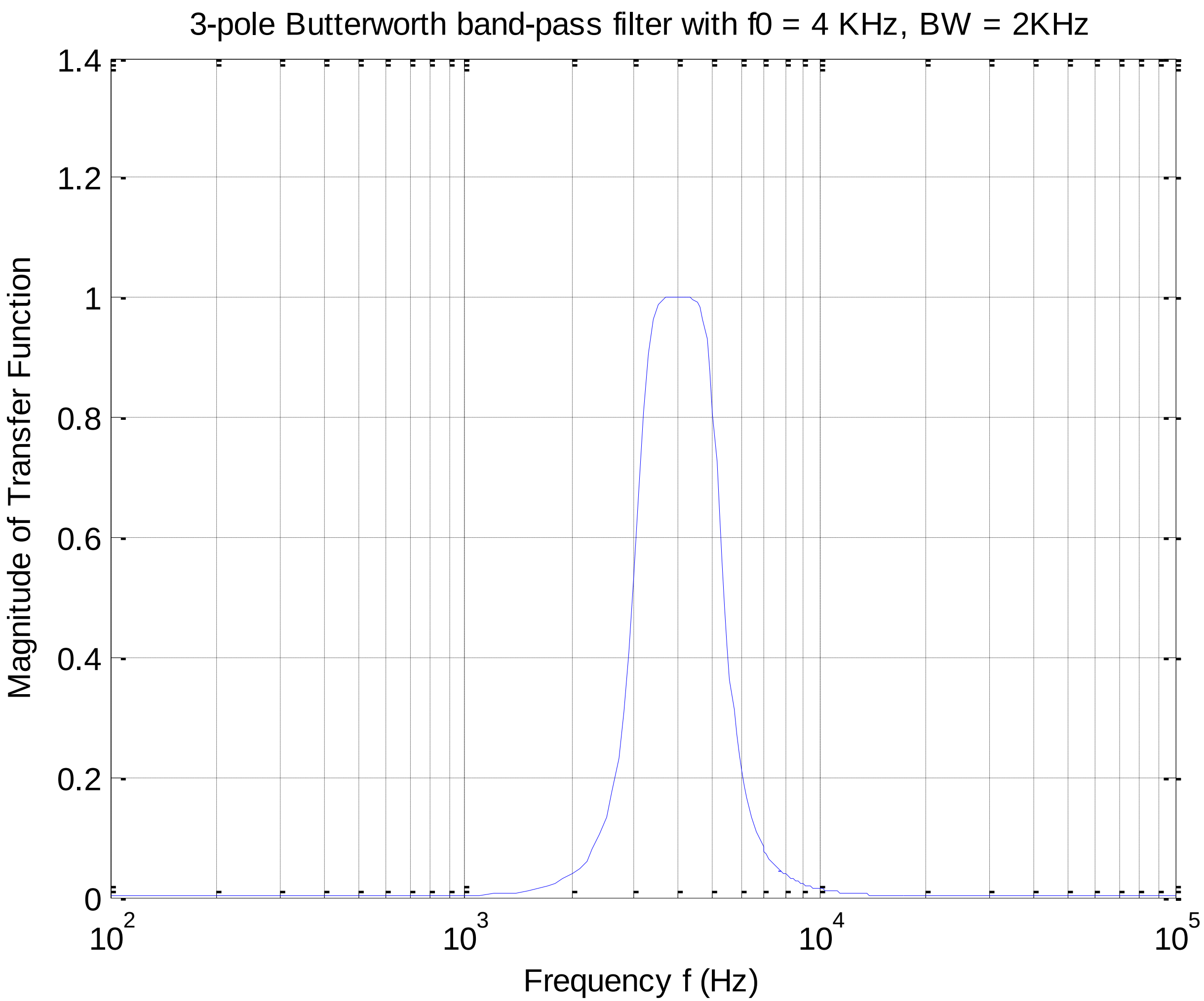


Band-Pass Filter

- Use the MATLAB functions **buttap** and **lp2bp** to find the transfer function of a 3-pole Butterworth analog band-pass filter with the pass band frequency centered at fo = 4kHz , and bandwidth BW =2KHz.

```
[z,p,k]=buttap(3);       % Design 3 pole Butterworth low-pass filter with wcn=1 rad/s
[b,a]=zp2tf(z,p,k);      % Compute numerator and denominator coefficients for wcn=1 rad/s
f=100:100:100000;    % Define frequency range to plot
f0=4000;                  % Define centered frequency at 4 KHz
W0=2*pi*f0;              % Convert desired centered frequency to rads/s
fbw=2000;                % Define bandwidth
Bw=2*pi*fbw;              % Convert desired bandwidth to rads/s
[bn,an]=lp2bp(b,a,W0,Bw);        % Compute num, den of band-pass filter
% Compute and plot the magnitude of the transfer function of the band-pass filter
Gsn=freqs(bn,an,2*pi*f);
semilogx(f,abs(Gsn));
grid;
xlabel('Frequency f (Hz)');
ylabel('Magnitude of Transfer Function');
title('3-pole Butterworth band-pass filter with f0 = 4 KHz, BW = 2KHz')
```

Band-Pass Filter

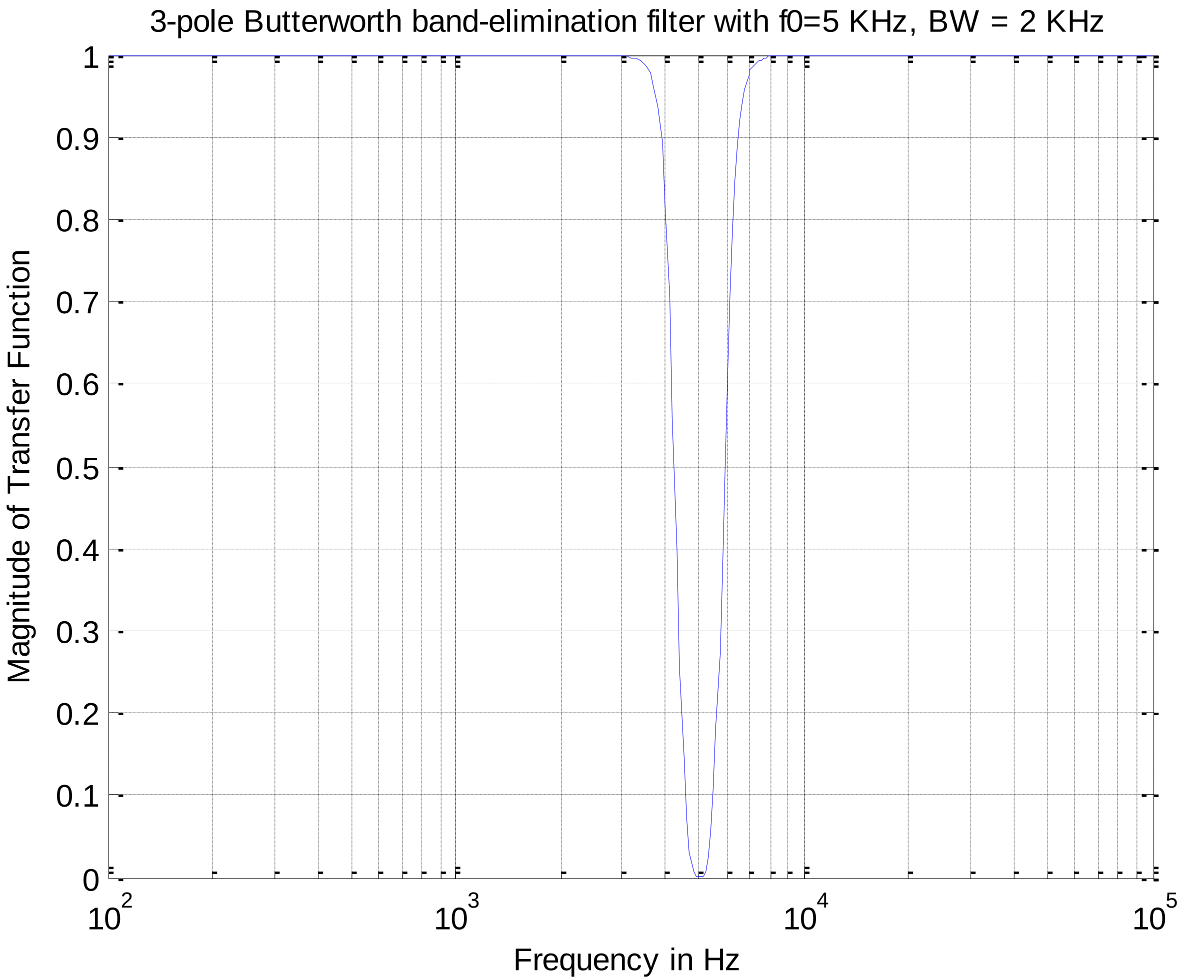


Band-Elimination (band-stop) Filter

- Use the MATLAB functions **buttap** and **lp2bs** to find the transfer function of a 3-pole Butterworth band-elimination (band-stop) filter with the stop band frequency centered at fo = 5 kHz , and bandwidth BW = 2kHz.

```
[z,p,k]=buttap(3);            % Design 3-pole Butterworth low-pass filter, wcn = 1 r/s
[b,a]=zp2tf(z,p,k);           % Compute num, den coefficients of this filter, wcn=1 r/s
f=100:100:100000;             % Define frequency range to plot
f0=5000;                      % Define centered frequency at 5 kHz
W0=2*pi*f0;                   % Convert centered frequency to r/s
fbw=2000;                     % Define bandwidth
Bw=2*pi*fbw;                  % Convert bandwidth to r/s
% Compute numerator and denominator coefficients of desired band stop filter
[bn,an]=lp2bs(b,a,W0,Bw);
% Compute and plot magnitude of the transfer function of the band stop filter
Gsn=freqs(bn,an,2*pi*f);
semilogx(f,abs(Gsn));
grid;
xlabel('Frequency in Hz'); ylabel('Magnitude of Transfer Function');
title('3-pole Butterworth band-elimination filter with f0=5 KHz, BW = 2 KHz')
```

Band-Elimination (band-stop) Filter



How to find the minimum order to meet the filter specifications ?

The following functions in Matlab can help you to find the minimum order required to meet the filter specifications:

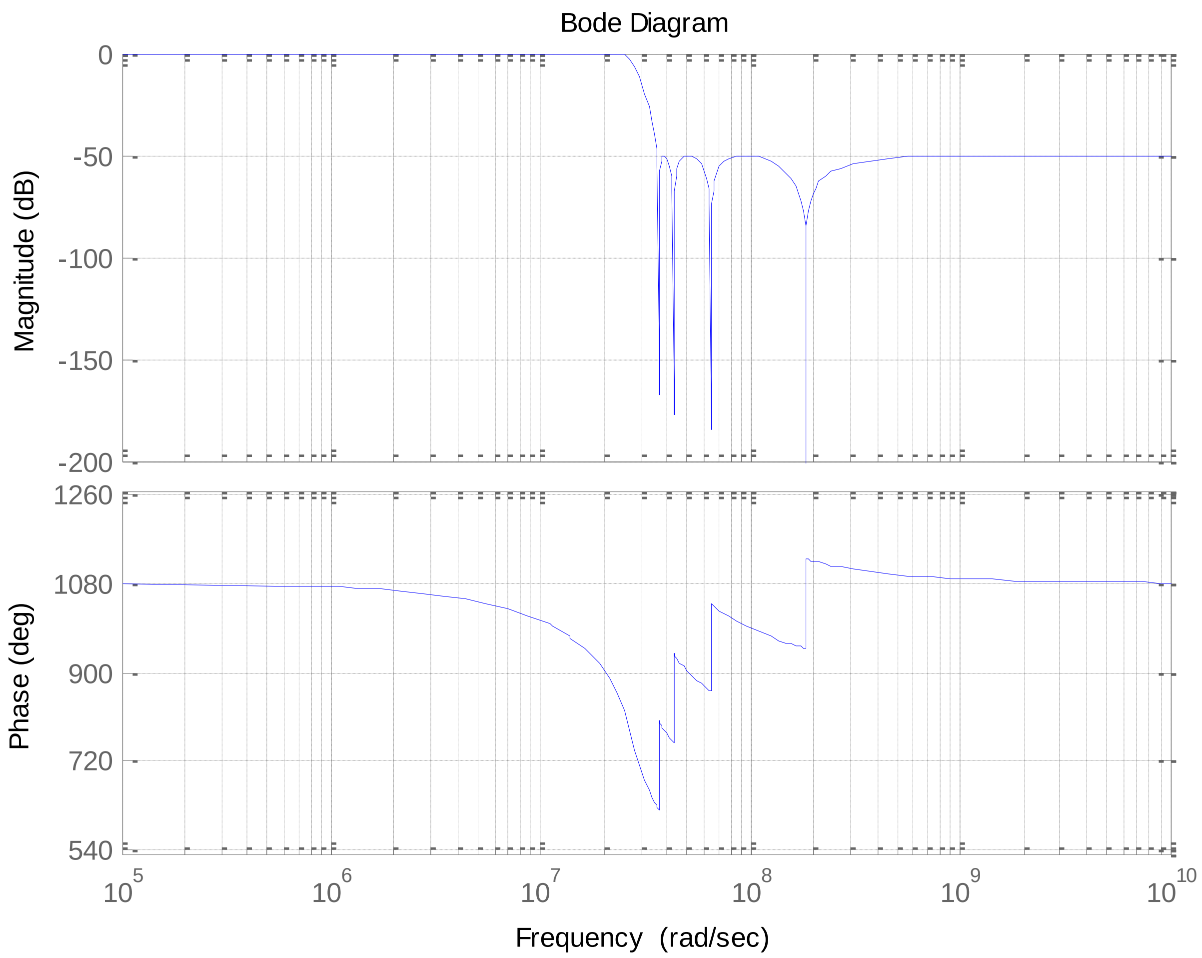
- Buttord for butterworth
- Cheb1ord for chebyshev
- Ellipord for elliptic
- Cheb2ord for inverse chebyshev

Calculating the order and cutoff frequency of a inverse chebyshev filter

- Design a 4MHz Inverse Chebyshev approximation with Ap gain at passband corner. The stop band is 5.75MHz with -50dB gain at stop band.

```
clear all;
Fp = 4e6; Wp=2*pi*Fp;
Fs=1.4375*Fp; Ws=2*pi*Fs;
Fplot = 20*Fs;
f = 1e6:Fplot/2e3:Fplot ;
w = 2*pi*f;
Ap = 1;
As = 50;
% Cheb2ord helps you find the order and wn (n and Wn) that
%you can pass to cheby2 command.
[n, Wn] =   cheb2ord(Wp, Ws, Ap, As, 's');
[z, p, k]  = cheby2(n, As, Wn, 'low', 's');
[num, den] = cheby2(n, As, Wn, 'low', 's');
bode(num, den)
```

Bode Plot



Bode Diagram
Magnitude (dB)
0
-50
-100
-150
-200
Phase (deg)
1260
1080
900
720
540
10^5
10^6
10^7
10^8
10^9
10^{10}
Frequency (rad/sec)

References

[1] S. T. Karris, “Signals and Systems with Matlab Computing and Simulink Modeling,” Fifth Edition. Orchard Publications

[2] Matlab Help Files