# ECEN 457 (ESS)

# **Op-Amps Stability and Frequency Compensation Techniques**

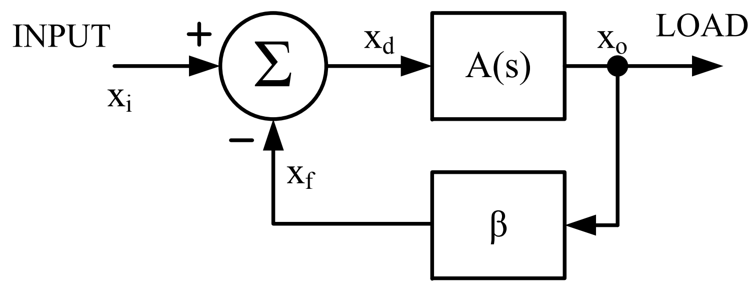
Analog & Mixed-Signal Center
Texas A&M University

Reference*: Sergio Franco, "Design with Operational Amplifiers and Analog Integrated Circuits" 4th Edition , Chapter 8, 2015*

## Stability of Linear Systems

- Harold S. Black, 1927 → Negative feedback concept
- Negative feedback provides:
  - Gain stabilization
  - Reduction of nonlinearity
  - Impedance transformation
- But also brings:
  - Potential stability problems
  - Causes accuracy errors for low dc gain
- Here we will discuss frequency – compensation techniques

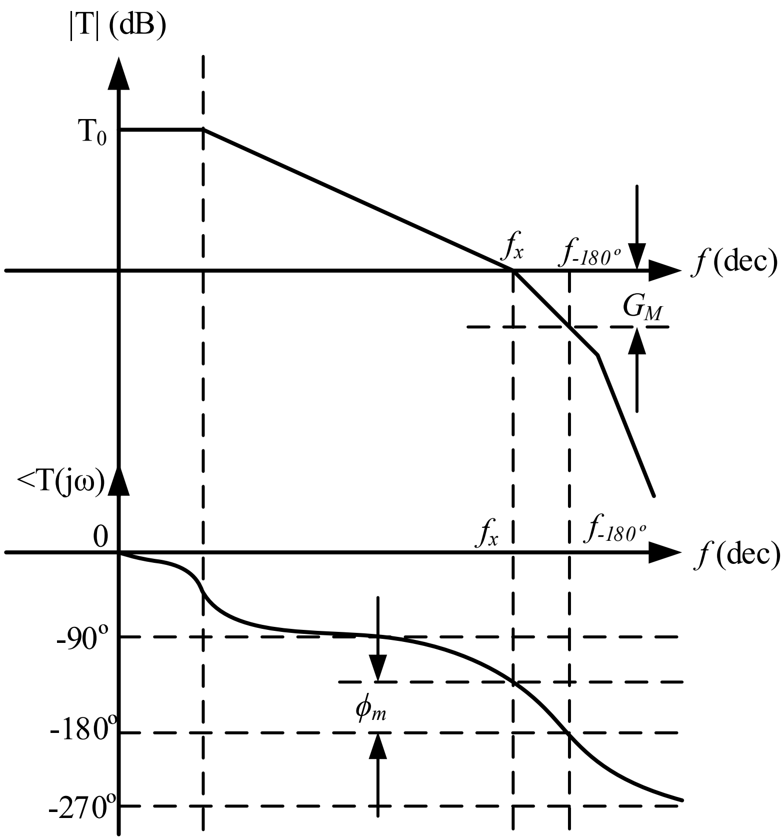
# Stability Problem




$$H_{CL}(s) = \frac{A(s)}{1+\beta A(s)} = \frac{1/\beta}{1+\dfrac{1}{\beta A(s)}} = \frac{A(s)}{1+T(s)}$$

- Feedback forces $x_d$ to become smaller
- It takes time to detect $x_o$ and feedback to the input
- $x_d$ could be overcorrected (diverge and create instability)
- How to find the optimal (practical) $x_d$ will be based on frequency compensation techniques

# Gain Margin & Phase Margin




Conceptual Gain Margin $G_M$
and Phase Margin $\phi_m$

- $G_M$ is the number of dBs by which $|T(j\omega_{-180^\circ})|$ can increase until it becomes 0 dB

$$G_M = 20\log\frac{1}{|T(j\omega_{-180^\circ})|}$$

- Phase margin $\phi_m$ is the number of degrees by which $\angle T(j\omega_x)$ can be reduced until it reaches $-\pi$ (-180º)

$$\phi_m = 180^\circ + \angle T(j\omega_x)$$

or

$$\angle T(j\omega_x) = \phi_m - 180^\circ$$

* $\omega_x$ is the crossover frequency

# Gain Margin & Phase Margin

- At the crossover point,

$$T(j\omega_x) = 1 \cdot \angle T(j\omega_x) = 1 \cdot \angle(\phi_m - 180°) = -e^{j\phi_m}$$

- The non-ideal closed loop transfer function becomes

$$H_{CL}(j\omega_x) = \frac{A(j\omega_x)}{1 + \beta A(j\omega_x)} = \frac{A(j\omega_x)}{1 + T(j\omega_x)} = \frac{A_{ideal}}{1 + 1/T(j\omega_x)}$$

$$= \frac{A_{ideal}}{1 - e^{-j\omega_x}} = \frac{A_{ideal}}{1 - (\cos\phi_m - j\sin\phi_m)}$$

$$\left|H_{CL}(j\omega_x)\right| = \left|A_{Ideal}\right| \frac{1}{\sqrt{(1 - \cos\phi_m)^2 + \sin^2\phi_m}}, \; A_{Ideal} = \frac{1}{\beta}$$

## Gain Margin & Phase Margin

- Observe that different $\phi_m$ yield different errors. i.e.

| $\phi_m$ | $\lvert H_{CL}(j\omega_x)\rvert$ |
|---|---|
| 90º | 0.707 |
| 60º | 1.00 |
| 45º | 1.31 |
| 30º | 1.93 |
| 15º | 3.83 |
| 0º | $\infty$ (oscillatory behavior) |

- In practical systems, $\phi_m = 60º$ is required
- A worst case $\phi_m = 45º$ for a typical lower limit
- For $\phi_m < 60º$, we have

$$\lvert A(j\omega_x)\rvert > \lvert A_{ideal}\rvert$$

indicating a peaked closed-loop response.

# Why a dominant pole is required for a stable amplifier?

- A good Op amp design implies:
  (i) $|\beta A(j\omega)| >> 1$ over as wide a band of frequencies as possible
  (ii) The zeroes of $\beta A(j\omega) - 1 = 0$ must be all in the left-hand plane
  Note that $\beta A(j\omega) < 0$
  These two conditions often conflict with each other. These trade-offs should be carefully considered. Let's consider a practical amplifier characterized with these poles,

$$A(s) = -\frac{A_0}{(1 + s/\alpha_1)(1 + s/\alpha_2)(1 + s/\alpha_3)} = \frac{-A_0\alpha_1\alpha_2\alpha_3}{(s+\alpha_1)(s+\alpha_2)(s+\alpha_3)}$$

The characteristic equation becomes

$$\beta A(s) - 1 = \frac{-\beta A_0\alpha_1\alpha_2\alpha_3}{(s+\alpha_1)(s+\alpha_2)(s+\alpha_3)} - 1 = 0$$

$$(s+\alpha_1)(s+\alpha_2)(s+\alpha_3) + \beta A_0\alpha_1\alpha_2\alpha_3 = 0$$

# Critical Value of $\beta A_0$

$$s^3 + s^2(\alpha_1 + \alpha_2 + \alpha_1) + s(\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3) + \alpha_1\alpha_2\alpha_3(1 + \beta A_0) = 0$$

Note that $\beta A_0$ is the critical parameter that determines the pole locations for a given $\alpha_1$, $\alpha_2$ and $\alpha_3$ ($0 \leq \beta \leq 1$). Furthermore, when $\beta A_0 = 0$, the roots are at $-\alpha_1$, $-\alpha_2$ and $-\alpha_3$. Therefore, for small $\beta A_0$, the roots should be in the left-hand plane (LHP). However, for $\beta A_0 >> 1$, two of the roots might be forced to move to the right-hand plane (RHP). This can be verified by applying Routh's stability criterion. Let us write the polynomial as

$$b_3 s^3 + b_2 s^2 + b_1 s + b_0 = 0$$

In order to have, in the above equation, left half plane roots, all the coefficients must be positive and satisfy

$$b_2 b_1 - b_3 b_0 = 0$$

# Critical Value of $\beta A_0$

The condition for imaginary-axis roots become

$$b_3(j\omega)^3 + b_2(j\omega)^2 + b_1(j\omega) + b_0 = 0$$
$$\left(b_0 - b_2\omega^2\right) + j\omega\left(b_1 - b_3\omega^2\right) = 0$$

Now, for $s = j\omega$ being a root, both real and imaginary parts must be zero. That is,

$$b_0 - b_2\omega^2 = 0, \quad b_1 - b_3\omega^2 = 0 \quad or \quad b_3 b_0 = b_1 b_2$$

Then the two roots are placed at

$$\omega_{p_{2,3}} = \pm j\sqrt{\frac{b_0}{b_2}} = \pm j\sqrt{\frac{b_1}{b_3}}$$

# Critical Value of $\beta A_0$

$$b_0 = \frac{b_2 b_1}{b_3}$$

Then

$$\alpha_1\alpha_2\alpha_3\left[1+(\beta A_0)_C\right] = (\alpha_1+\alpha_2+\alpha_3)(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)$$

Thus, the critical value of $\beta A_0$ becomes

$$(\beta A_0)_C = 2 + \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1}{\alpha_3} + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_3} + \frac{\alpha_3}{\alpha_1} + \frac{\alpha_3}{\alpha_2}$$

Thus when $\beta A_0$ becomes $(\beta A_0)_C$, the amplifier will oscillate at

$$\omega_{OSC} = \sqrt{\frac{b_1}{b_3}} = \sqrt{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}$$

Also when $\beta A_0 > (\beta A_0)_C$, the amplifier has RHP poles, therefore is **unstable**.

# Critical Value of $\beta A_0$

- Let us consider some numerical examples. Let $A_0 = 10^5$,

(i) Three equal poles $\alpha_1=\alpha_2=\alpha_3=10^7$rad/s.

The amplifier oscillates at $\omega_{OSC} = \alpha_1\sqrt{3} = 10^7\sqrt{3}$rad/s

$$\left(\beta A_0\right)_C = 8, \quad \beta_C = 8/A_0 = 8\times 10^{-5}$$

(ii) $\alpha_1 = \dfrac{\alpha_2}{10^4} = \dfrac{\alpha_3}{10^4}$, then the critical loop gains yields

$\left(\beta A_0\right)_C \cong 2\dfrac{\alpha_2}{\alpha_1} = 2\times 10^4$, thus the amplifier is stable if $\beta_C < \dfrac{2\times 10^4}{A_0} = 0.2$

Since $\beta = R_1/\left(R_1+R_2\right), \beta < 0.2$ causes $\left(R_2/R_1\right) > 4$,
which means that for an inverting (non-inverting) configuration, the gain must be greater than -4 (5) to keep the amplifier stable.

# Critical Value of $\beta A_0$

(iii) Let us determine $A_{0C}$ under the most stringent condition $\beta$=1. Then from previous equation,

$$A_0 < 2 + \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1}{\alpha_3} + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_3} + \frac{\alpha_3}{\alpha_1} + \frac{\alpha_3}{\alpha_2}$$

In order to have a large $A_0$, the poles must be widely separated. i.e. $\alpha_1 \ll \alpha_2 \ll \alpha_3$, then the $A_0$ inequality can be approximated as

$$A_0 < \frac{\alpha_2}{\alpha_1} + \frac{\alpha_3}{\alpha_1}$$

To obtain a conservative $A_0$, let $\alpha_2 = \alpha_3$, which yields

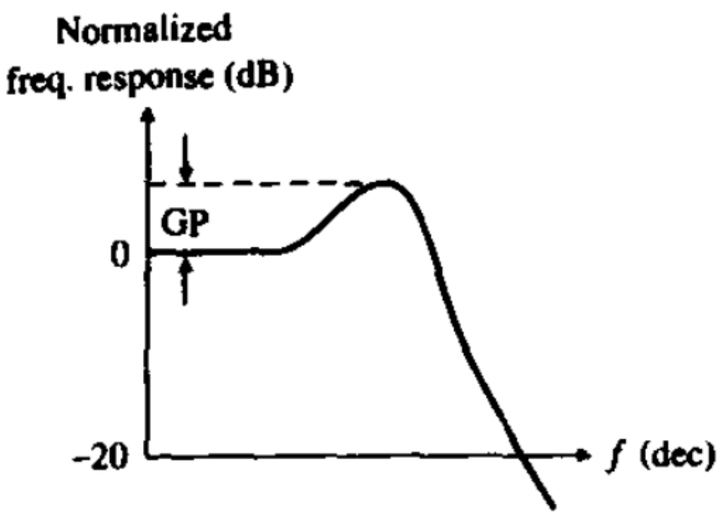
$$A_0 < 2\frac{\alpha_2}{\alpha_1}$$

This inequality bounds the DC gain to provide a stable closed loop configuration.

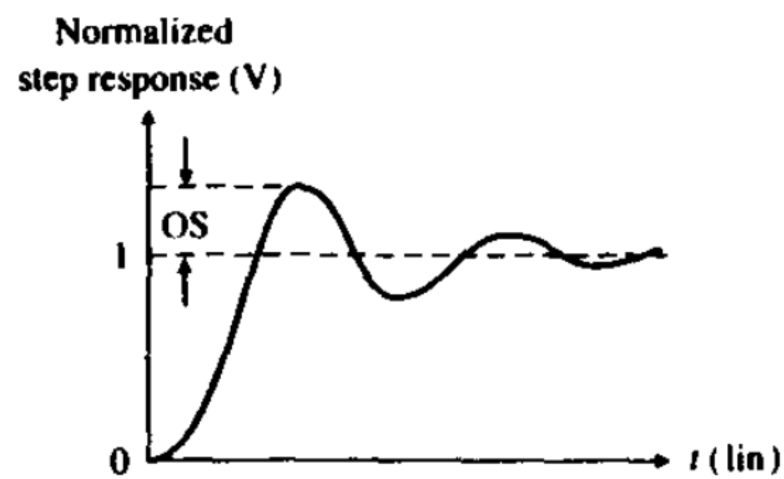
# Peaking and Ringing

- Peaking in the frequency domain usually implies ringing in the time domain
- Normalized second-order all-pole (low pass) system

$$H(s)\Big|_{s=j\omega} = \left.\frac{{\omega_0}^2}{s^2 + \frac{\omega_0}{Q}s + {\omega_0}^2}\right|_{s=j\omega} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2} + j\frac{1}{Q}\cdot\frac{\omega}{\omega_0}}$$




(a)




(b)

# Peaking and Ringing

- ***GP:*** Peak gain – We have the error function,

$$|E(j\omega)| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \frac{1}{Q^2}\cdot\frac{\omega^2}{\omega_0{}^2}}} = \frac{1}{\sqrt{D(\omega)}} \tag{1}$$

To find out the maximum value of $|E(j\omega)|$, calculate the derivative of $D(\omega)$ and make it equal to zero

$$\frac{d}{d\omega}D(\omega) = 2\left(\frac{1}{Q^2}-2\right)\frac{\omega}{\omega_0{}^2} + \frac{4\omega^3}{\omega_0{}^4} = 0 \quad \Rightarrow \quad \omega_*{}^2 = \left(1-\frac{1}{2Q^2}\right)\omega_0{}^2 \quad \text{OR} \quad \omega = 0$$

For $Q > 1/\sqrt{2}$, use $\omega_*$ in Eq. (1) and we get the peak gain

$$GP = \frac{2Q^2}{\sqrt{4Q^2-1}} \geq |E(j0)| = 1 \tag{2}$$

# Peaking and Ringing

- ***OS:*** Overshoot – Inverse Laplace transform

$$\text{s-domain}: \frac{b}{(s+a)^2+b^2} \xrightarrow[\phantom{xx}]{\substack{Inverse\\Laplace}} \text{time-doamin}: e^{-at}\sin(bt)u(t)$$

For a 2nd order all pole error function, the **impulse response** is

$$H(s)\Big|_{s=j\omega} = \frac{\omega_0{}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2} = \frac{\omega_0{}^2}{b}\cdot\frac{b}{(s+a)^2+b^2} \qquad v_{in}(t)=\delta(t)$$

$$a=\frac{\omega_0}{2Q}$$

$$\xrightarrow[\phantom{xx}]{\substack{Inverse\\Laplace}} h(t)=\frac{\omega_0{}^2}{b}e^{-at}\sin(bt)u(t) \qquad b=\sqrt{1-\frac{1}{4Q^2}}\,\omega_0$$

Consider the **normalized step response** of this system,

$$y(t)=\int_0^t h(t)dt = 1-\frac{\omega_0}{b}e^{-at}\sin(bt+\phi), \phi=\tan^{-1}\frac{b}{a} \quad \text{for } v_{in}(t)=u(t)=\int\delta(t)dt \qquad (3)$$

Use the damping factor $\xi$ to represent, $a=\xi\omega_0, b=\sqrt{1-\xi^2}\,\omega_0$

# Peaking and Ringing

Usually, the damping factor ξ=1/2Q is used to characterize a physical 2nd order system. Thus, we rewrite the normalized time-domain equation (3)

$$y(t) = 1 - \frac{\omega_0}{b} e^{-at} \sin(bt + \phi) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right)$$

$$\phi = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} \qquad \phi \approx \frac{\sqrt{1-\xi^2}}{\xi}, \text{ for small } \xi \tag{4}$$

For under damped case **ξ < 1** (Q > 0.5), the overshoot is the peak value of $y(t)$. To find the peak value, we first calculate the derivative of $y(t)$, and make it equal to 0.

$$\frac{d}{d(\omega_0 t)} y(t) = -\frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right) + e^{-\xi\omega_0 t} \cos\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right)$$

# Peaking and Ringing

$$\frac{d}{d(\omega_0 t)} y(t) = 0$$

$$\Rightarrow \quad \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right) = e^{-\xi\omega_0 t} \cos\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right)$$

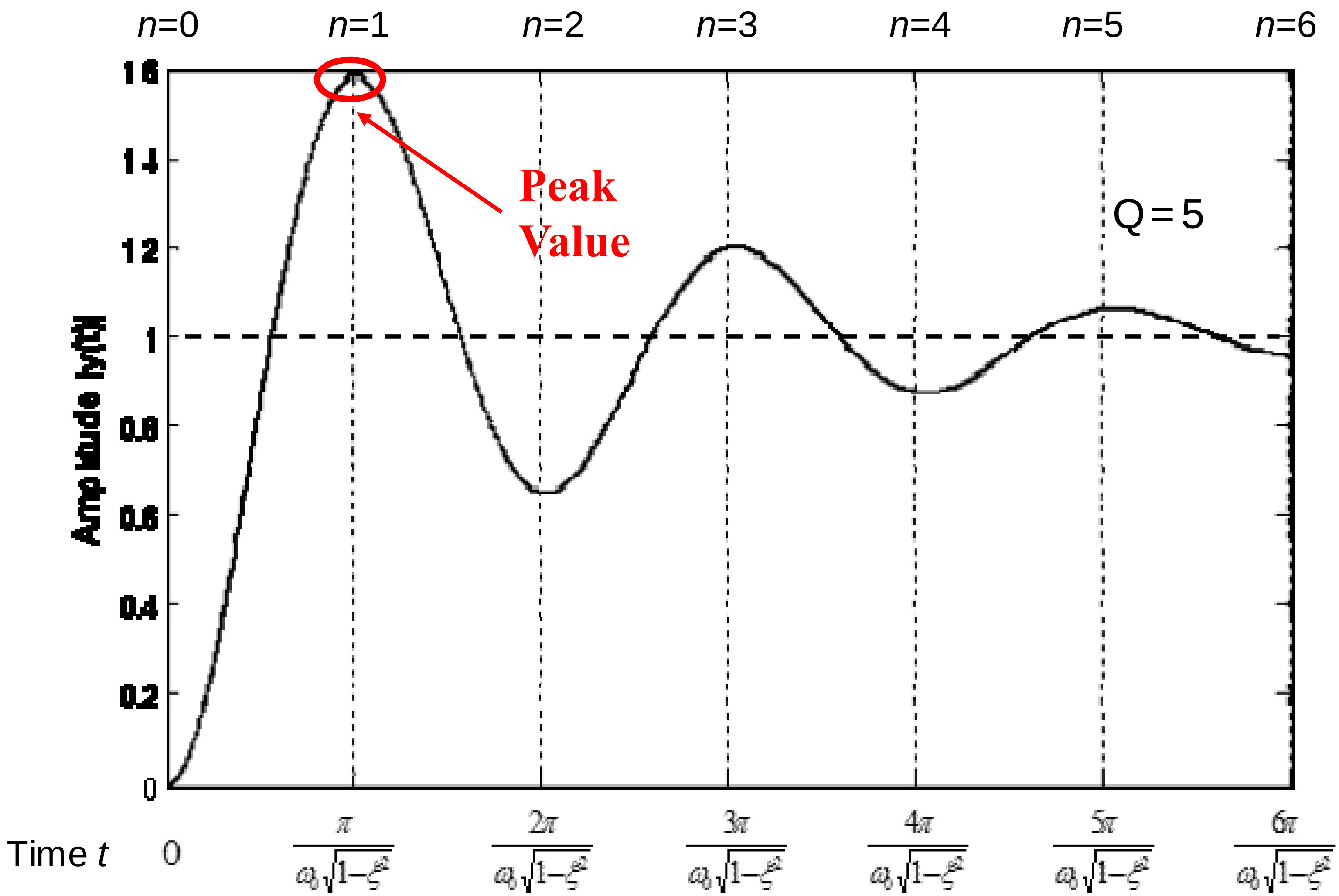
$$\Rightarrow \quad \tan\left(\sqrt{1-\xi^2}\,\omega_0 t + \phi\right) = \frac{\sqrt{1-\xi^2}}{\xi} = \tan\phi$$

To satisfy the equality above, we have

$$\omega_0 t = \frac{n\pi}{\sqrt{1-\xi^2}}, n = 0,1,2,...$$

In other words, the **normalized step response** $y(t)$ in Eq.(3) achieves **extreme values** at time steps of n=0, 1, 2, …

# Peaking and Ringing



# Peaking and Ringing

$$
\begin{array}{lll}
n=0, & \omega_0 t = 0, & y(t)=0 \\
n=1, & \omega_0 t = \dfrac{\pi}{\sqrt{1-\xi^2}}, & y(t) = 1 - \dfrac{1}{\sqrt{1-\xi^2}} e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \sin(\pi+\phi) = 1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \\
n=2, & \omega_0 t = \dfrac{2\pi}{\sqrt{1-\xi^2}}, & y(t) = 1 - \dfrac{1}{\sqrt{1-\xi^2}} e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}} \sin(2\pi+\phi) = 1 - e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}} \\
n=3, & \omega_0 t = \dfrac{3\pi}{\sqrt{1-\xi^2}}, & y(t) = 1 - \dfrac{1}{\sqrt{1-\xi^2}} e^{-\frac{3\pi\xi}{\sqrt{1-\xi^2}}} \sin(3\pi+\phi) = 1 + e^{-\frac{3\pi\xi}{\sqrt{1-\xi^2}}} \\
\cdots & \cdots & \cdots
\end{array}
$$

Therefore, the global peak value of $y(t)$ is achieved when n=1.

Thus, the overshoot is defined as

$$
OS(\%) = 100 \times \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = 100 \times e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \qquad (5)
$$

# Peaking and Ringing

- $\boldsymbol{\phi_m}$**:** Phase Margin

For a 2nd order all-pole error function

$$E(s)=\frac{T(s)}{1+T(s)}=\frac{1}{1+1/T(s)}=\frac{1}{\frac{s^2}{\omega_0^2}+\frac{1}{Q}\frac{s}{\omega_0}+1}$$

Therefore, the loop gain $T(s)$ is given by

$$T(s)=\frac{1}{\frac{s^2}{\omega_0^2}+\frac{1}{Q}\frac{s}{\omega_0}}$$

The cross-over frequency thus can be obtained

$$\left|T(j\omega_x)\right|=\left(\sqrt{\left(\frac{\omega_x{}^2}{\omega_0{}^2}\right)^2+\left(\frac{\omega_x}{\omega_0 Q}\right)^2}\right)^{-1}=1$$

# Peaking and Ringing

Solve the equation and get the crossover frequency,

$$\omega_x = \omega_0\left(\sqrt{\frac{1}{4Q^4}+1}-\frac{1}{2Q^2}\right)^{1/2} = \omega_0\left(\sqrt{4\xi^4+1}-2\xi^2\right)^{1/2}$$

And thus the phase margin is

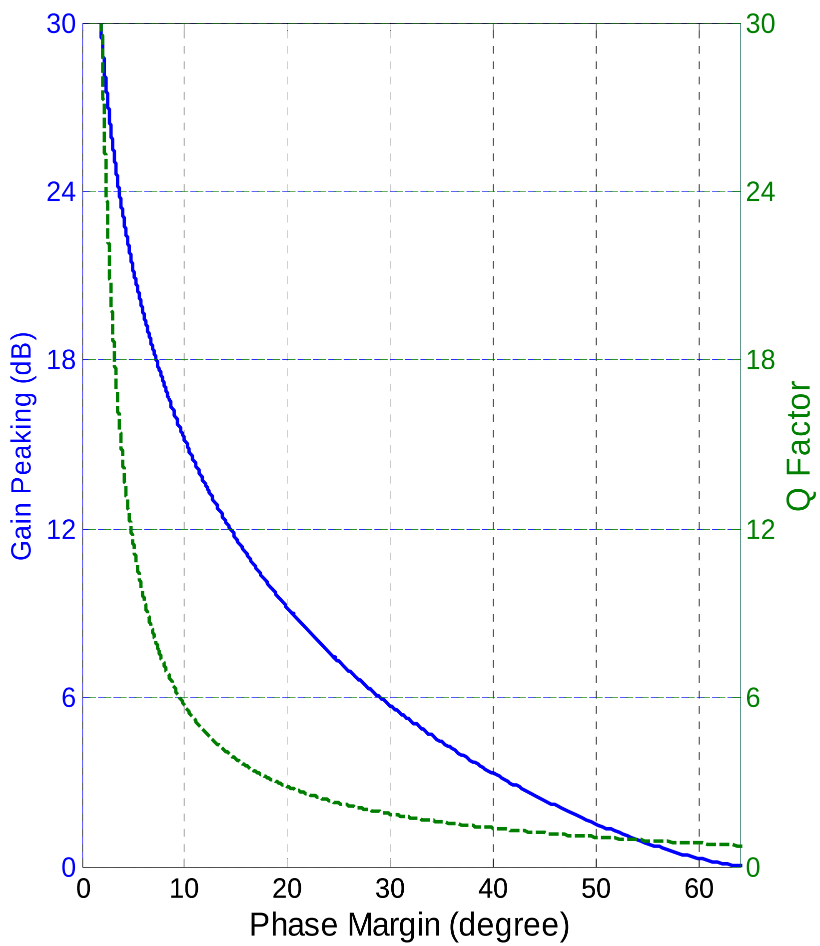
$$\phi_m = 180° + \angle T\left(j\omega_x\right) = \cos^{-1}\left(\sqrt{1+\frac{1}{4Q^4}}-\frac{1}{2Q^2}\right) = \cos^{-1}\left(\sqrt{4\xi^4+1}-2\xi^2\right)$$

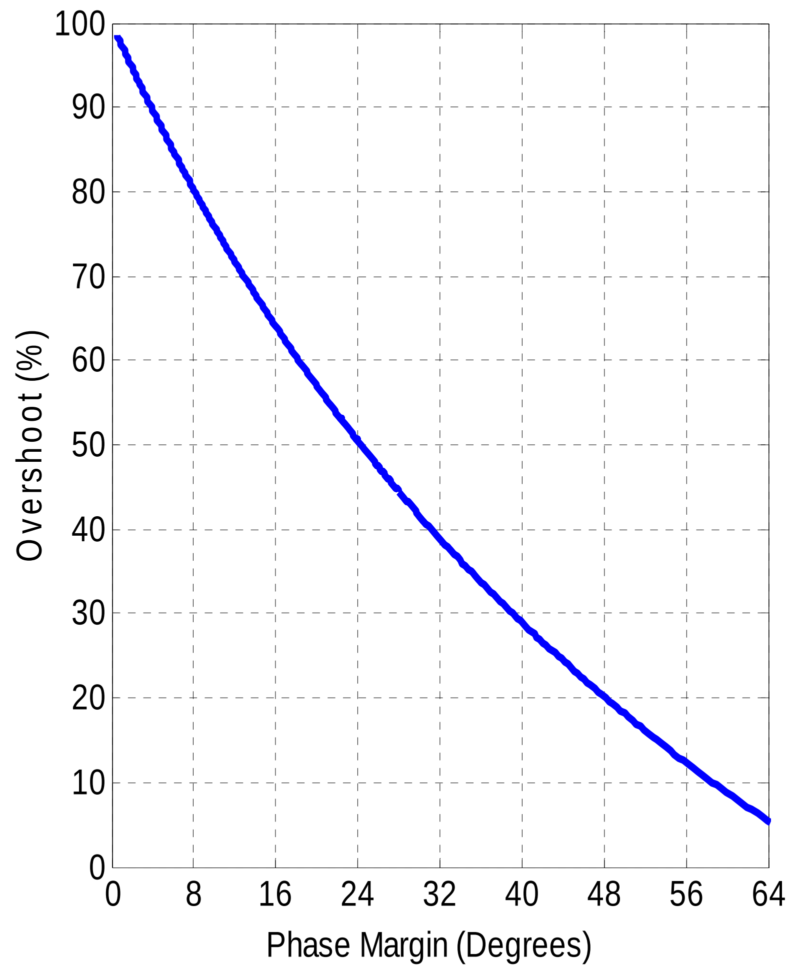
- Study the relationship between phase margin and gain peaking (Eq.2) or overshoot (Eq.5), we have

$$GP(60°) \cong 0.3dB \quad OS(60°) \cong 8.8\% \quad Q \approx 0.82$$

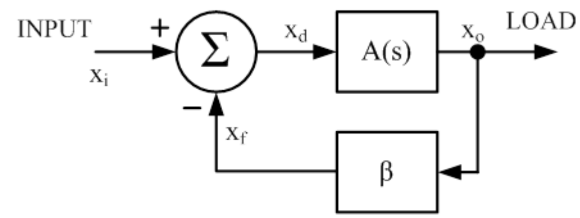
$$GP(45°) \cong 2.4dB \quad OS(45°) \cong 23\% \quad Q \approx 1.18$$

# Peaking and Ringing



# The Rate of Closure (ROC)




- One effective method of assessing stability for minimum phase systems from the magnitude Bode plots is by determining the ROC.

- ***The Rate of Closure (ROC)***
  Determining the ROC is done by observing the slopes of $|A|$ and $|1/\beta|$ at their intersection point (cross-over frequency $f_x$) and deciding the magnitude of their difference.

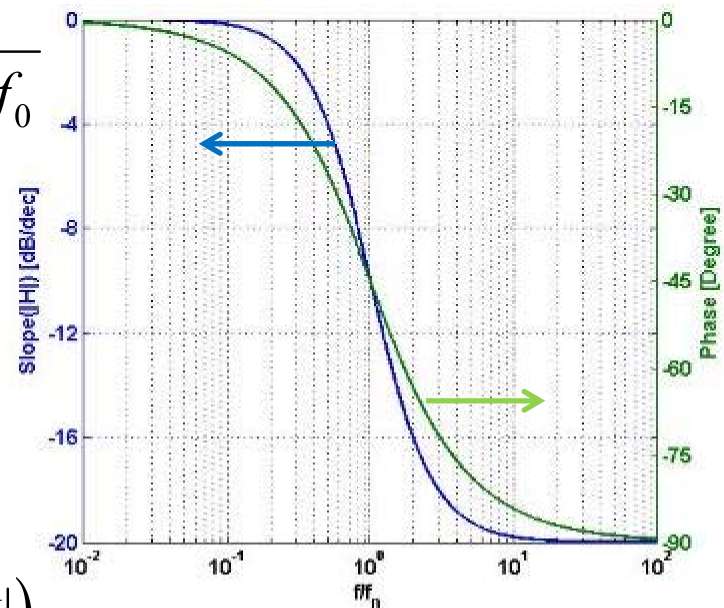
$$ROC = \left|Slope(|A|) - Slope(|1/\beta|)\right|_{f=f_x}$$

The ROC is used to estimate the phase margin and therefore the stability (How ?)

# The Rate of Closure (ROC)

- Observing a single-root transfer function

$$H(jf) = \frac{1}{1 + jf/f_0}$$




$$f \le f_0/10 \quad Slope(|H|) \to 0dB/dec \quad \angle H \to 0°$$

$$f > 10 f_0 \quad Slope(|H|) \to -20dB/dec \quad \angle H \to -90°$$

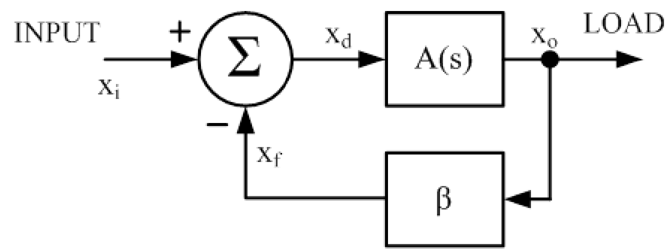
$$f = f_0 \quad Slope(|H|) \to -10dB/dec \quad \angle H \to -45°$$

- ***Empirical Equation***

$$\angle H \cong 4.5 \times Slope(|H|)$$

This correlation holds also if H(s) has more than one root, provided the roots are real negative, and well separated, say, at least a decade apart.

# The Rate of Closure (ROC)




- In a feedback system, suppose both $|A|$ and $|1/\beta|$ have been graphed.

$$\angle T(jf_x) = \angle A(jf_x) - \angle \beta^{-1}(jf_x)$$
$$\cong -4.5 \times ROC$$

Thus, the ROC can be used to estimate the phase margin (Page 4)

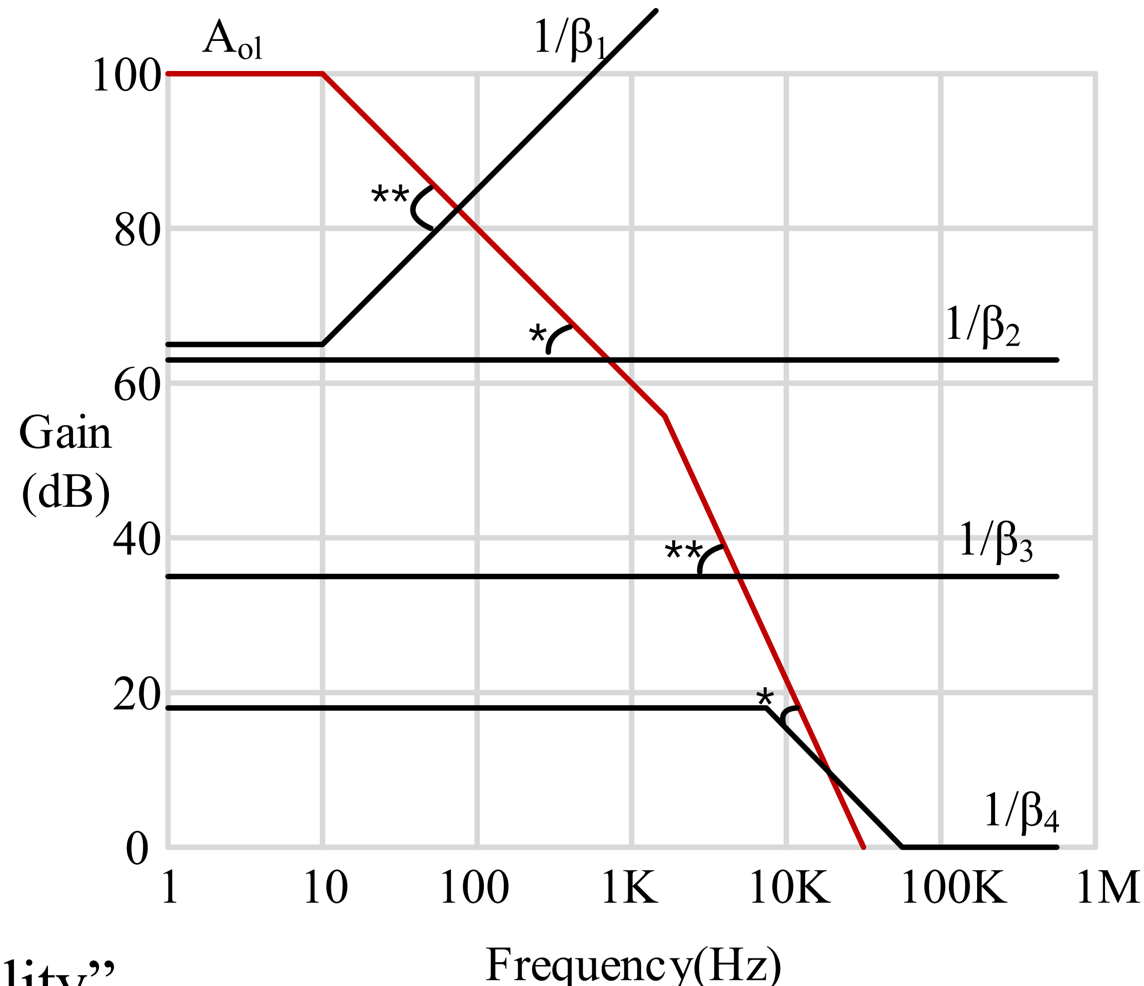
$$\phi_m = 180° + \angle T(jf_x)$$

- Cases

$$ROC \cong 20dB/dec \quad \Rightarrow \quad \phi_m \cong 90°$$
$$ROC \cong 30dB/dec \quad \Rightarrow \quad \phi_m \cong 45°$$
$$ROC \cong 40dB/dec \quad \Rightarrow \quad \phi_m \cong 0°$$
$$ROC > 40dB/dec \quad \Rightarrow \quad \phi_m < 0°$$

# The Rate of Closure (ROC)

$$A(s) = \frac{A_{ol}}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$




* 20dB/dec ROC
→ "Stability"

** 40dB/dec ROC
→ "Marginal Stability"

# Stability in Constant-GBP OpAmp

- Constant-GBP OpAmp (i.e. $A(s)\big|_{s=j\omega} = \dfrac{\omega_t}{j\omega}$ )
  - Unconditionally stable with frequency-independent feedback, or $\angle\beta = 0$. (e.g. in a non-inverting or inverting amplifier, the feedback network contains only resistors)
  - Stable for any $\beta \le 1$.
  - In feedback systems, since now we have

$$\angle T = \angle(A\beta) = \angle A \ , \ \ \angle A \cong -90°$$

    these circuits enjoy

$$\phi_m = 180° + \angle A(jf_x) \cong 180° - 90° \cong 90°$$

  - Typically, due to additional high-order poles in OpAmps,

$$60° \le \phi_m \le 90°$$

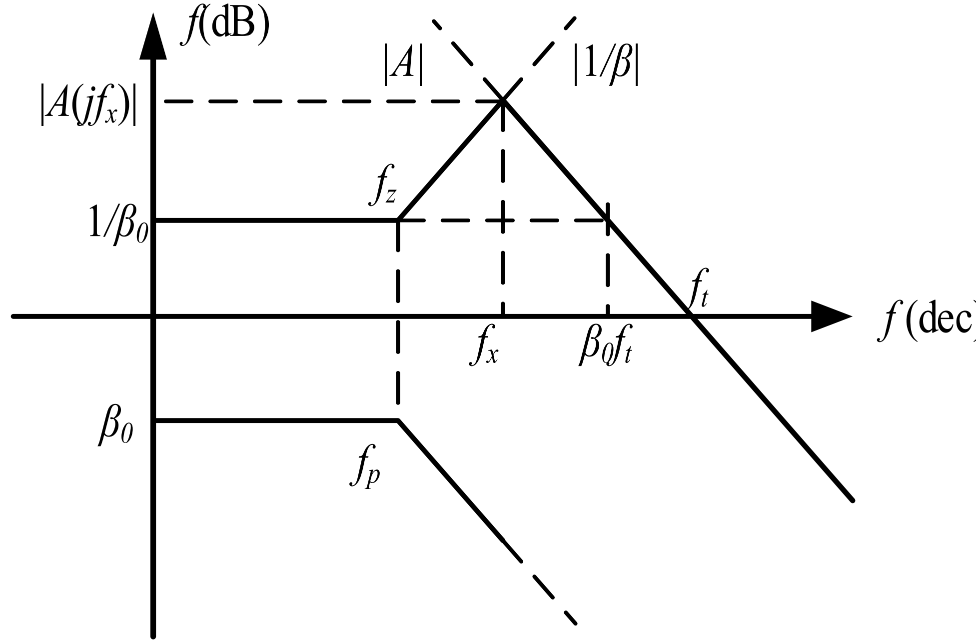
# Feedback Pole

- ***Feedback Pole***
  Feedback network includes reactive elements → Stability may no longer be unconditional

$$\beta(jf) = \frac{\beta_0}{1 + jf / f_p}$$

- A pole $f_p$ (or a zero) of $\beta$ becomes a zero $f_z$ (or a pole) for $1/\beta$.

- For the case $f_z << \beta_0 f_t$




The effect of a pole within the feedback loop

# Feedback Pole

- Examine the error function

$$E(s) = \frac{H_{CL}(s)}{A_{ideal}} = \frac{1}{1+1/T}, \qquad T = A\beta, \quad A_{ideal} = \frac{1}{\beta}$$

Using OpAmp high-frequency approximation: $A(j\omega) = \frac{GB}{j\omega} = \frac{f_t}{jf}$

$$E(s) = \frac{1}{1+\frac{1}{A\beta}} = \frac{1}{1+j\frac{f}{f_t\beta_0} - \frac{f^2}{f_t\beta_0 f_z}}$$

Refer to page 14, and we have $s=j2\pi f$. The peak value of $E(s)$ can be obtained. For Q >> 1, the approximate result is

$$f_x = \sqrt{f_z\beta_0 f_t}, Q = \sqrt{\beta_0 f_t / f_z}$$

# Feedback Pole

- The lower $f_z$ compared to $\beta_0 f_t$, the higher the Q and, hence, the more pronounced the peaking and ringing.

- Derive the phase margin

$$\angle T(jf_x) = \angle A(jf_x) - \angle\left|1/\beta(jf_x)\right| \cong -90^\circ - \tan^{-1}\left(f_x/f_z\right)$$

$$\frac{f_x}{f_z} = \sqrt{\frac{\beta_0 f_t}{f_z}}$$

- As $f_z << \beta_0 f_t$ , $\angle T(jf_x) \cong -180^\circ$ and ROC = 40dB/dec
  The circuit is on the verge of oscillation!

# Differentiator

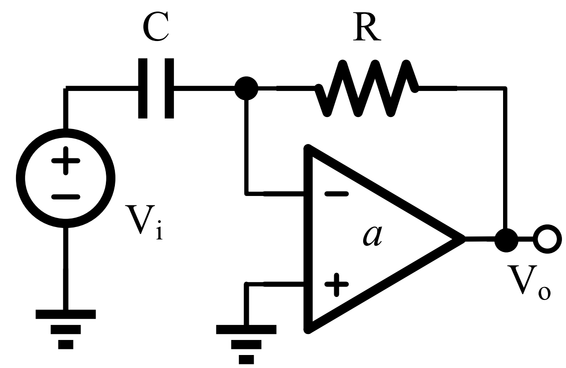
- Feedback pole example: differentiator

- Assume constant-GBP OpAmp

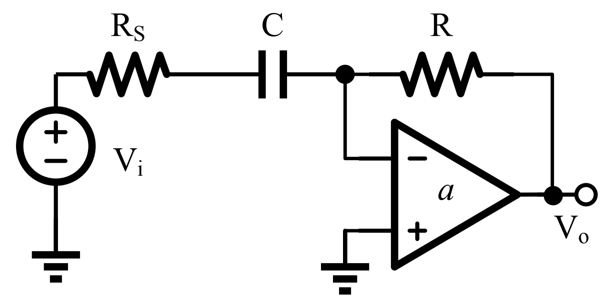
$$A(jf) \cong GB / j\omega \cong f_t / jf$$

$$\beta = \frac{Z_C}{Z_C + R} = \frac{1}{1 + jf / f_z}, Z_C = \frac{1}{j\omega C}$$

- To stabilize the differentiator, add a series resistance Rs.



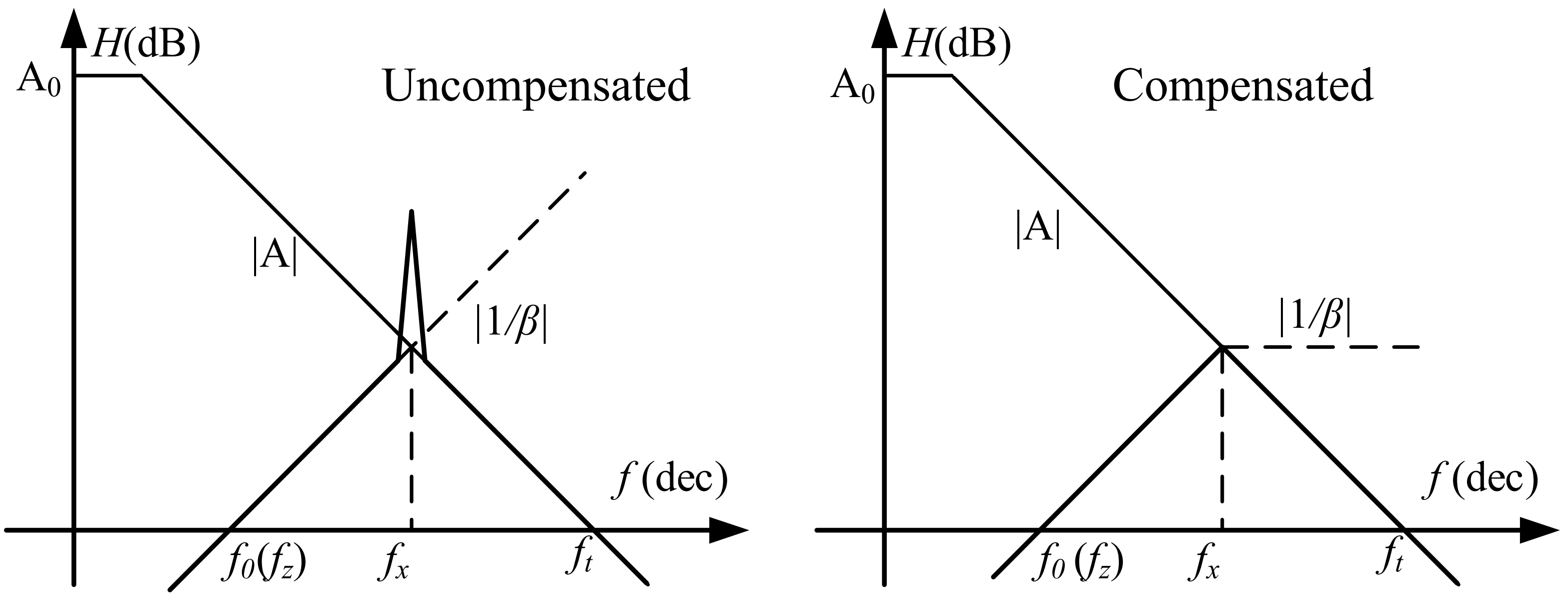

Uncompensated Differentiator




Compensated Differentiator

# Differentiator

- At low frequency, $R_S$ has little effect because $R_S << |1/j\omega C|$
- At high frequency, C acts as a short compared to $R_S$. The feedback network becomes $|1/\beta| = 1 + R/R_S$.



## Differentiator

- Assume $R_S << R_C$, the series resistor $R_S$ introduces an extra pole frequency $f_e$

$$\frac{1}{\beta} = \frac{Z_C + R}{Z_C} \approx \frac{1}{\beta_0} \frac{1 + jf / f_z}{1 + jf / f_e} \qquad Z_C = \frac{1}{j\omega C} + R_S, \beta_0 = 1, f_x = \sqrt{f_t f_z}$$

- Choose $R_S \approx R / \sqrt{f_t / f_z}$, we have $f_e = \sqrt{f_t f_z}$

$$\begin{aligned} \angle T(jf_x) &= \angle A(jf_x) - \angle \left|1/\beta(jf_x)\right| \\ &\cong -90° + \tan^{-1}\left(f_x/f_e\right) - \tan^{-1}\left(f_x/f_z\right) \\ &= -135° \end{aligned}$$

Therefore, ROC = 30 dB/dec, $\phi_m \approx 45°$

# Stray Input Capacitance Compensation

- All practical OpAmps exhibit stray input capacitance. The net capacitance $C_n$ of the inverting input toward ground is

$$C_n = C_d + C_C / 2 + C_{ext}$$

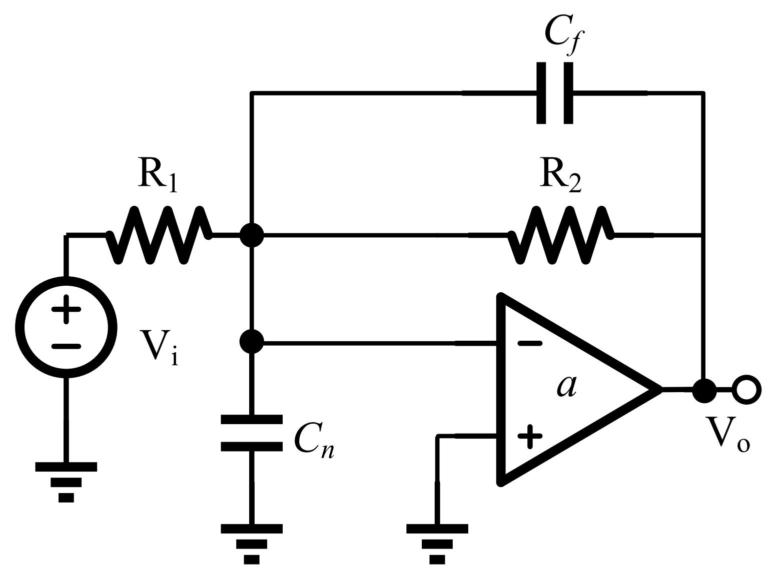
$C_d$ is the diffrential Cap between input pins, $C_c/2$ is the common-mode cap of each input to ground, and $C_{ext}$ is the external parasitic cap.

- In the absence of $C_f$, there's a pole in feedback

$$\frac{1}{\beta} = (1 + R_2 / R_1)\{1 + jf[2\pi(R_1 // R_2)C_n]\}$$

ROC ≈ 40 dB/dec

(See page 29)



# Stray Input Capacitance Compensation

- ***Solution:*** Introduce a feedback capacitance $C_f$ to create feedback phase lead.
- In the presence of $C_f$ we have

$$\frac{1}{\beta} = \left(1 + \frac{R_2}{R_1}\right)\frac{1 + jf/f_z}{1 + jf/f_p}$$

$$f_z = \frac{1}{2\pi(C_n + C_f)(R_1 \| R_2)} \quad , \quad f_p = \frac{1}{2\pi C_f R_2}$$

- To have $\phi_m = 45°$ (i.e. ROC=30 dB/dec):

Make the cross-over frequency exactly at $f_p$

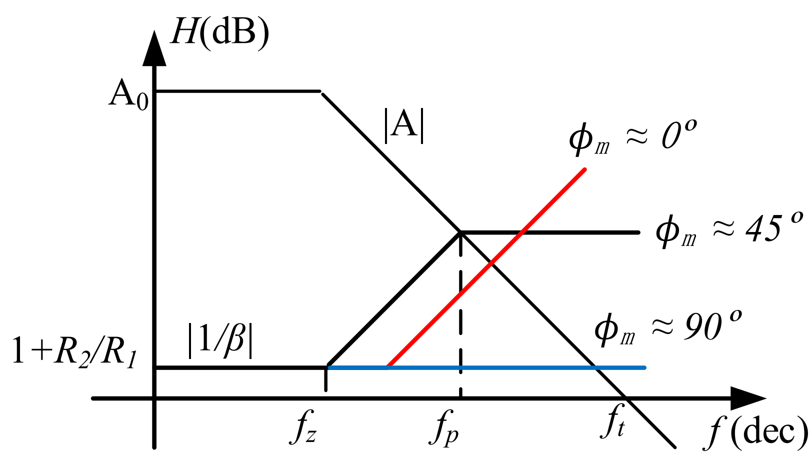
$$|A(jf_p)| = \left|\frac{1}{\beta(jf_p)}\right| \cong \frac{1}{\beta_\infty} \quad \rightarrow |A(jf_p)| \cong 1 + \frac{C_n}{C_f}$$

Since

$$|A(jf_p)| = \frac{f_t}{f_p} \quad \rightarrow \quad \frac{1}{f_p} = 2\pi C_f R_2 = \frac{1}{f_t}\left(1 + \frac{C_n}{C_f}\right)$$

Solve the equation to get $C_f$

$$C_f = \frac{1 + \sqrt{1 + 8\pi R_2 C_n f_t}}{4\pi R_2 f_t}$$



# Stray Input Capacitance Compensation

- To have $\phi_m = 90°$ (i.e. ROC=20 dB/dec):

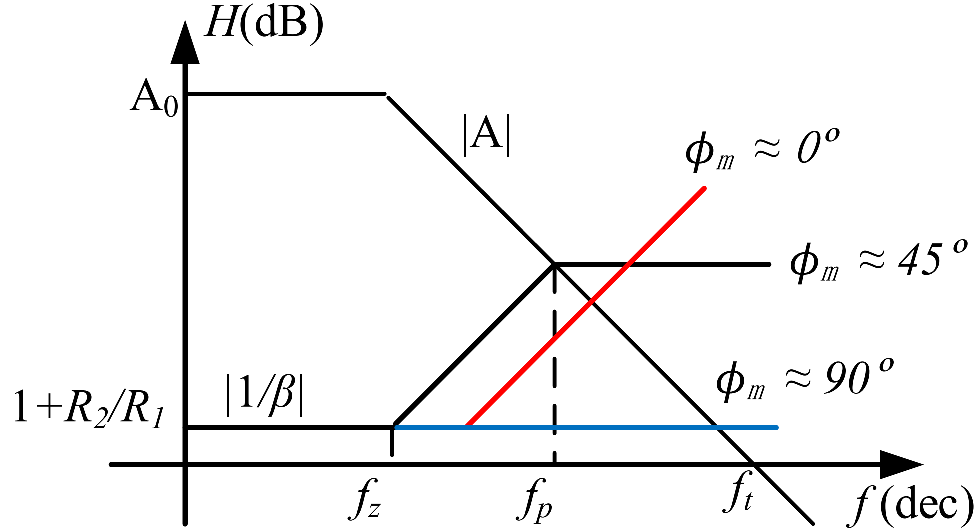
Place $f_p$ exactly on the top of $f_z$ to cause a pole-zero cancellation

$$f_z = f_p$$

$$(C_n + C_f)(R_1 \| R_2) = C_f R_2$$

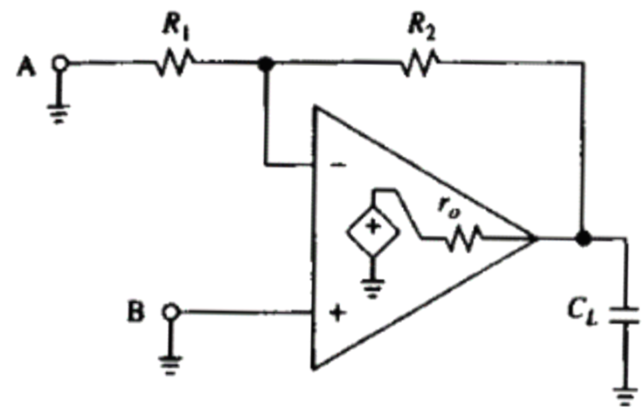
Thus using simple algebra

$$C_f = \frac{R_1}{R_2} C_n \qquad \text{(}\textbf{\textit{Neutral Compensation}}\text{)}$$



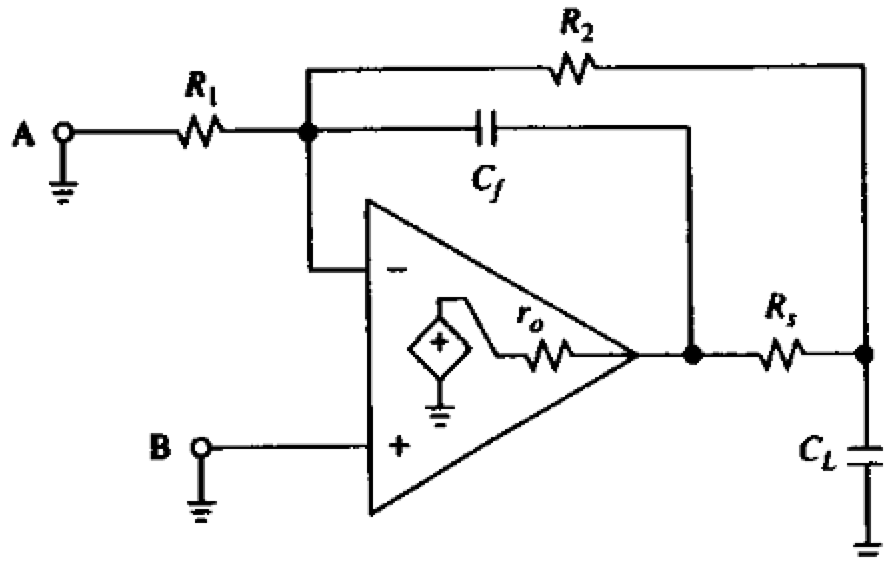
# Capacitive-Load Isolation

- There're applications in which the external load is heavily capacitive.
- Load capacitance $C_L$
  - A new pole is formed with output resistance $r_o$ and $C_L$
  - Ignore loading by the feedback network
  - The loaded gain is $A_{loaded} \cong A\left(1 + jf / f_p\right)^{-1}, f_p = \left(2\pi r_o C_L\right)^{-1}$
  - ROC is increased and thus invite instability




- Solution: Add a small series resistance $R_S$ to decouple the output from $C_L$

# Capacitive-Load Isolation

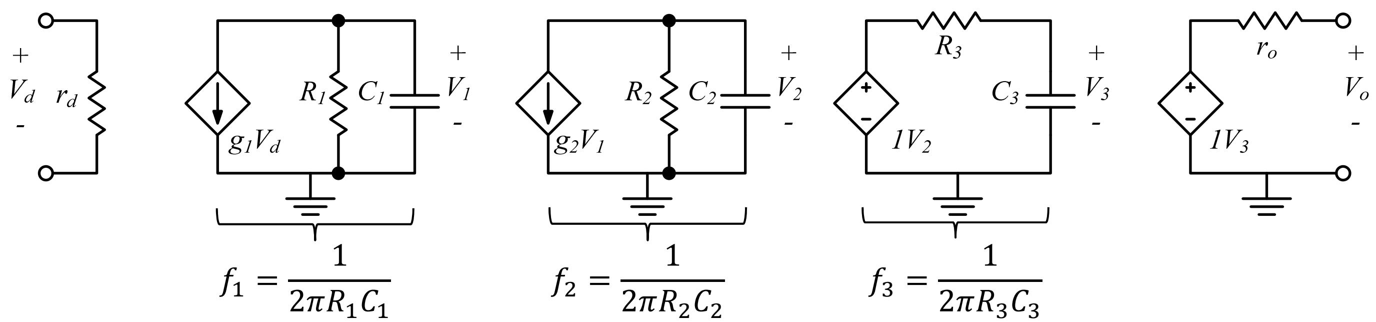



$$R_S = \frac{R_1}{R_2} r_O \qquad C_f = \left(1 + \frac{R_1}{R_2}\right)^2 \left(\frac{r_o}{R_2}\right) C_L$$

# Uncompensated OpAmp

- The poles of the uncompensated OpAmp are located close together thus accumulating about 180° of phase shift before the 0-dB crossover frequency $f_x$.
- Unstable device, thus efforts must be done to stabilize it.
- Example of uncompensated OpAmps is 748 , which is the uncompensated version of 741.
- They can be approximated as a three-pole system

$$a(jf) = \frac{a_0}{(1 + jf/f_1)(1 + jf/f_2)(1 + jf/f_3)}$$




Three-pole OpAmp model

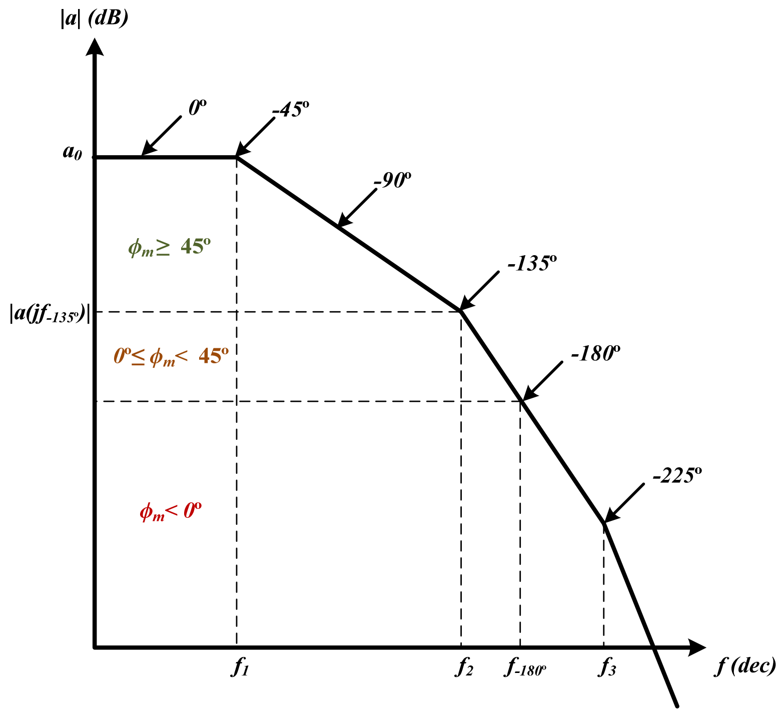
# Stability of Uncompensated OpAmp

- With a frequency-independent feedback (i.e. $1/\beta$ curve is flat) around an uncompensated OpAmp, we have

$$|T| = |a|\beta$$

$|T|$ curve can be visualized as the $|a|$ curve with the $1/\beta$ line is the new 0-dB axis

- For $1/\beta \geq |a(jf_{-135°})|$
  ROC ≤ 30 dB/dec
  $\phi_m \geq 45°$
- For $|a(jf_{-180°})| \leq 1/\beta < |a(jf_{-135°})|$
  30 dB/dec ≤ ROC ≤ 40 dB/dec
  $0° \leq \phi_m \leq 45°$
- For $1/\beta \leq |a(jf_{-180°})|$
  ROC < 40 dB/dec
  $\phi_m < 0°$




Three-pole open-loop response

# Stability of Uncompensated OpAmp

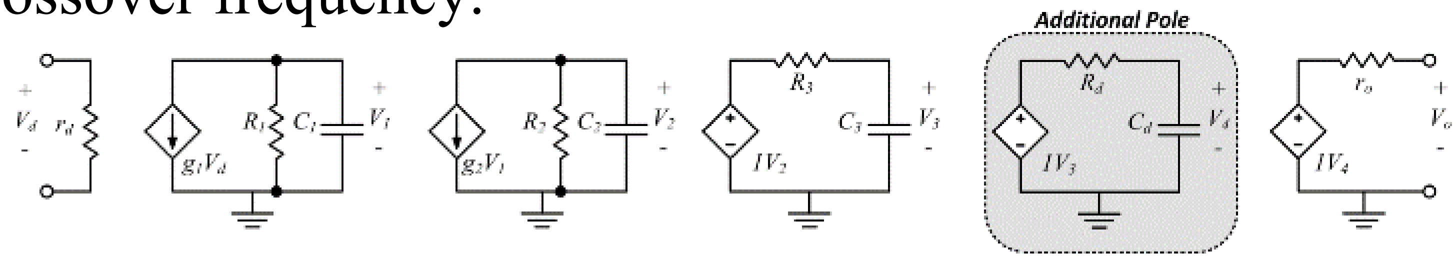
- The uncompensated OpAmp provides only adequate phase margin only in high-gain applications (i.e. high $1/\beta$)
- To provide adequate phase margin in low-gain application, frequency compensation is needed.
  - Internal compensation → Achieved by changing $a(jf)$
  - External compensation → Achieved by changing $\beta(jf)$

# Internal Frequency Compensation

- How to stabilize the circuit by modifying the open loop response $a(jf)$?
  - Dominant-Pole Compensation
  - Shunt-Capacitance Compensation
  - Miller Compensation
  - Pole-Zero Compensation
  - Feedforward Compensation

# Dominant-Pole Compensation

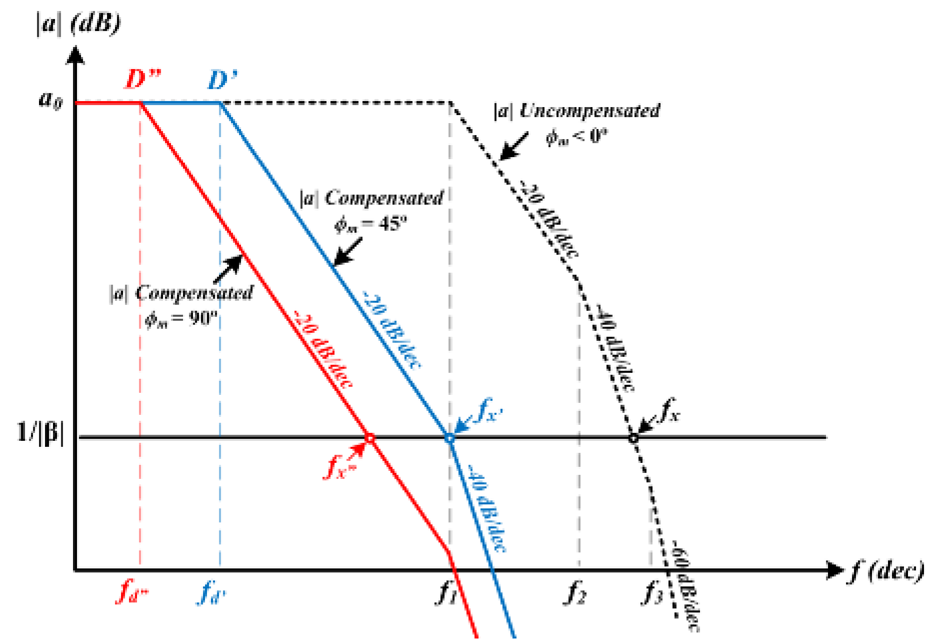
- An additional pole at sufficiently low frequency is created to insure a roll-off rate of -20 dB/dec all the way up to the crossover frequency.




- Cases:
  - $f_{x(new)} = f_1$:

    ROC = 30 dB/dec ➡ $\phi_m = 45°$
  - $f_{x(new)} < f_1$:

    ROC = 20 dB/dec ➡ $\phi_m \cong 90°$
- This technique causes a drastic gain reduction above $f_d$



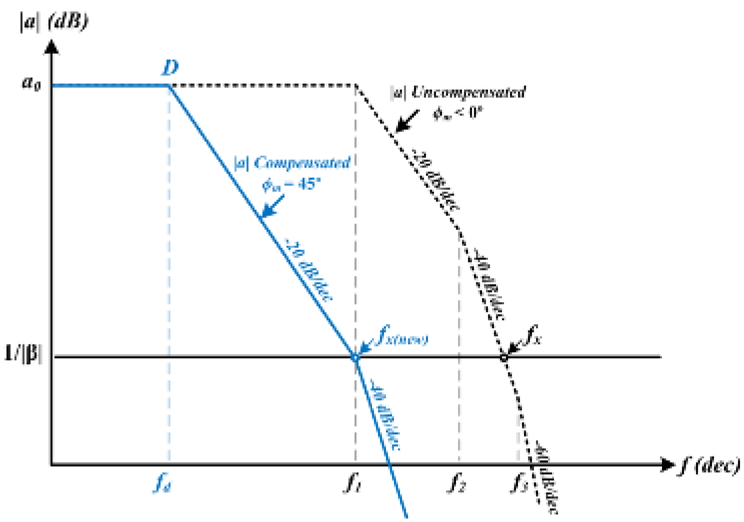
# Dominant-Pole Compensation

- Numerical example:
  - $r_d=\infty$, $r_o=0$
  - $g_1=2$ mA/V, $R_1=100$ kΩ, $g_2=10$ mA/V, $R_2=50$ kΩ
  - $f_1 = 100\ kHz, f_2 = 1\ MHz, f_3 = 10\ MHz$
  - Find the required value of $f_d$ for $\phi_m = 45°$ with $\beta = 1$

For $\phi_m=45^o$, we have

$$f_x = f_1$$

Draw a straight line of slope -20 dB/dec until it intercepts with the DC gain asymptote at point D and get $f_d$.




$$\frac{|a(jf_{x(new)})|}{|a(jf_d)|} = \frac{f_d}{f_{x(new)}}$$

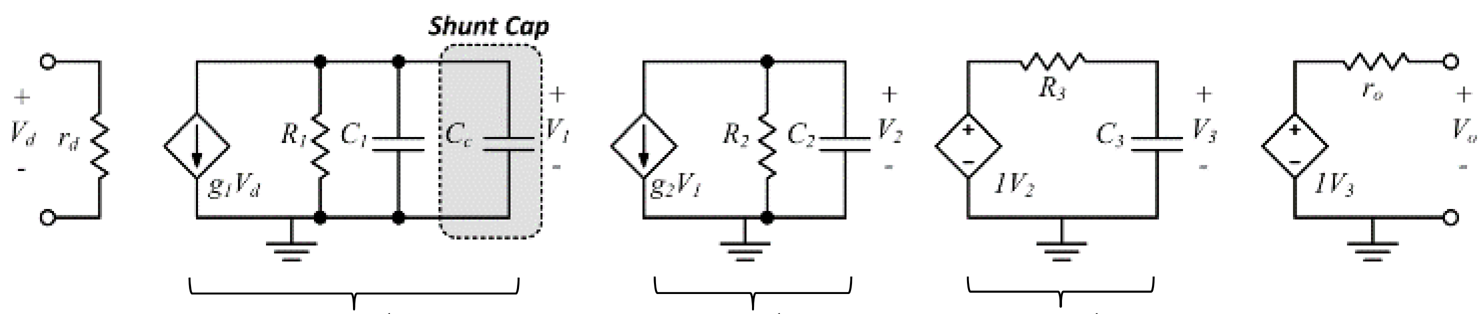
Thus:

$$f_d = \frac{f_{x(new)}}{\beta a_0} = \frac{f_1}{\beta \times (g_1 R_1 g_2 R_2)} = 1\ Hz$$

**Requires EXTREMELY LARGE passive components**

## Shunt-Capacitance Compensation

- The dominant-pole technique adds a fourth pole → Extra cost and less bandwidth.
- This technique rearranges the existing rather than creating a new pole.
- It decreases the first (dominant) pole to sufficiently low frequency to insure a roll-off rate of -20 dB/dec all the way up to the crossover frequency.



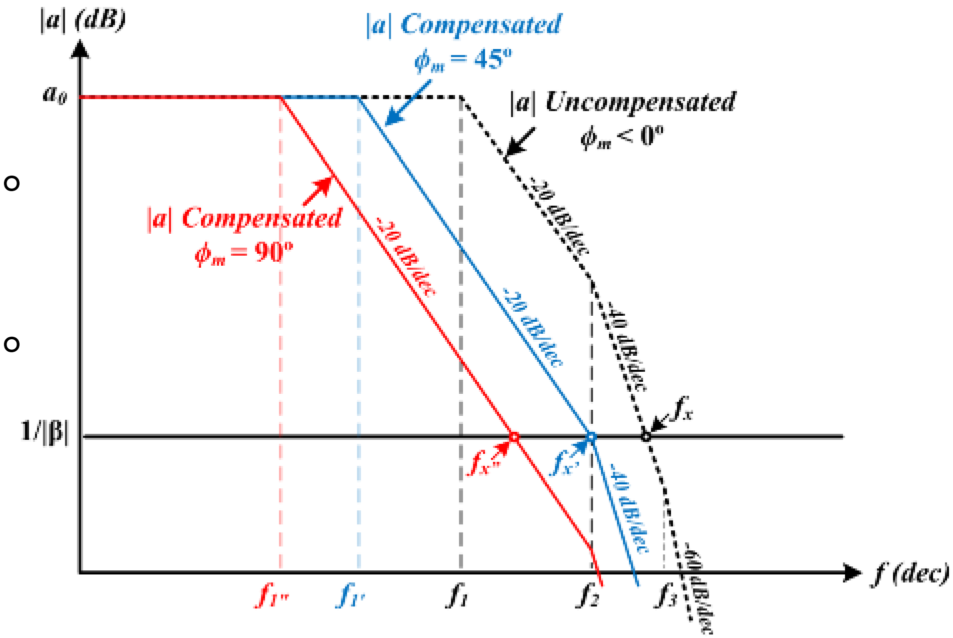

$$f_1 = \frac{1}{2\pi R_1(C_1 + C_c)} \qquad f_2 = \frac{1}{2\pi R_2 C_2} \qquad f_3 = \frac{1}{2\pi R_3 C_3}$$

# Shunt-Capacitance Compensation

- Cases:
  - $f_{x(new)} = f_2$:

    ROC = 30 dB/dec ➡ $\phi_m = 45°$
  - $f_{x(new)} < f_2$:

    ROC = 20 dB/dec ➡ $\phi_m \cong 90°$



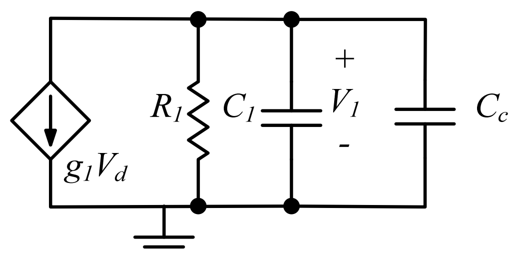

- Since $f_{1(new)}$ is chosen to insure roll-off rate of -20 dB/dec all the way up to the crossover frequency.

Thus:

$$\frac{|a(jf_{x(new)})|}{|a(jf_{1(new)})|} = \frac{f_{1(new)}}{f_x(new)} \quad \Rightarrow \quad f_{1(new)} = \frac{f_x(new)}{\beta a_0}$$

# Shunt-Capacitance Compensation

- The first pole is decreased by adding an extra capacitance to the internal node causing it.

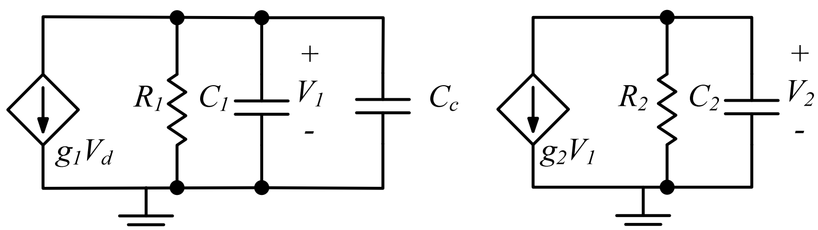



- Given the value of $f_{1(new)}$ from the desired value of $\phi_m$, we can find $C_c$

$$f_{1(new)} = \frac{f_x}{\beta a_0} = \frac{1}{2\pi R_1(C_1 + C_c)} \quad \rightarrow \quad C_c \cong \frac{\beta a_0}{2\pi R_1 f_x}$$

# Shunt-Capacitance Compensation

- Numerical example:
  - $r_d = \infty$, $r_o = 0$
  - $g_1 = 2$ mA/V, $R_1 = 100$ kΩ
  - $g_2 = 10$ mA/V, $R_2 = 50$ kΩ
  - $f_1 = 100\ kHz, f_2 = 1\ MHz, f_3 = 10\ MHz$
  - Find the required value of $C_c$ for $\phi_m = 45°$ with $\beta = 1$




For $\phi_m = 45°$ , $f_x = f_2 = 1\ MHz$

Then, $f_{1(new)} = \frac{f_2}{a_0\beta} = \frac{f_2}{g_1R_1g_2R_2} = 10\ Hz \quad \rightarrow \quad C_c = \frac{1}{2\pi R_1 f_{1(new)}} = 159\ nf$

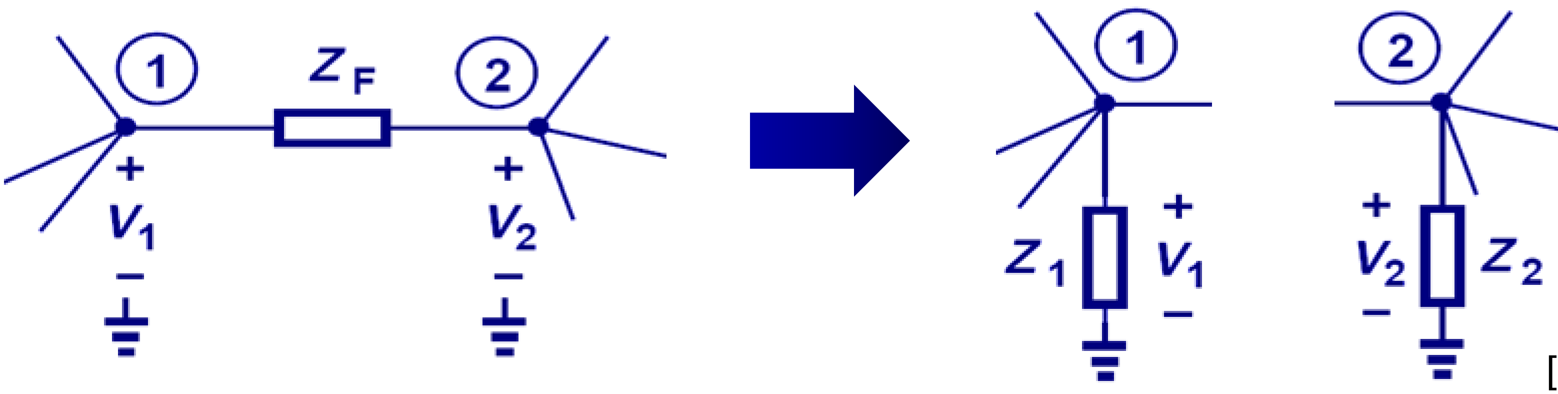
**EXTREMELY LARGE**
**Unsuitable for monolithic fabrication**

# Miller Compensation

## ■ Miller's Theorem

If $A_v$ is the voltage gain from node 1 to 2, then a floating impedance $Z_F$ can be converted to two grounded impedances $Z_1$ and $Z_2$:

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} \Rightarrow Z_1 = Z_F \frac{V_1}{V_1 - V_2} = Z_F \frac{1}{1 - A_v}$$



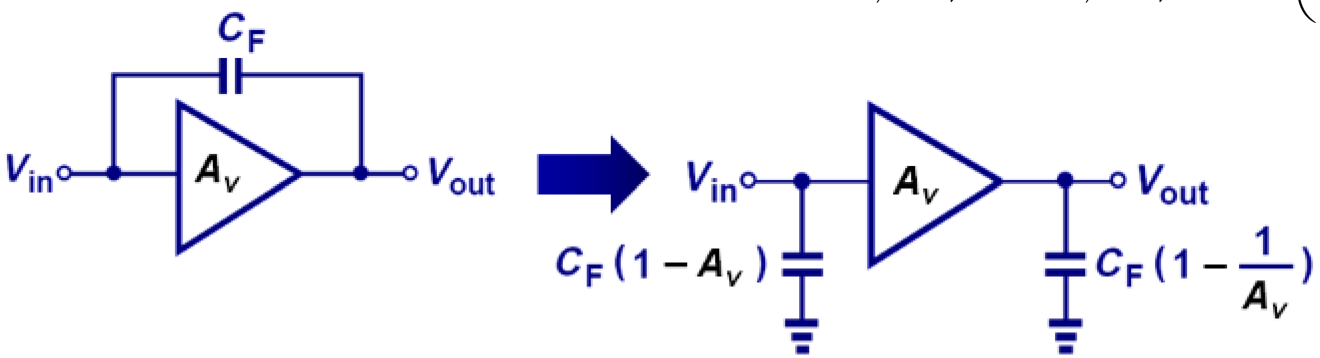
[Liu]

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2} \Rightarrow Z_2 = -Z_F \frac{V_2}{V_1 - V_2} = Z_F \frac{1}{1 - 1/A_v}$$

# Miller Compensation

- Applying Miller's theorem to a floating capacitance connected between the input and output nodes of an amplifier.

$$Z_1 = \frac{Z_F}{1-A_v} = \frac{1/j\omega C_F}{1-A_v} = \frac{1}{j\omega(1-A_v)C_F} \qquad Z_2 = \frac{Z_F}{1-1/A_v} = \frac{1/j\omega C_F}{1-1/A_v} = \frac{1}{j\omega\left(1-1/A_v\right)C_F}$$



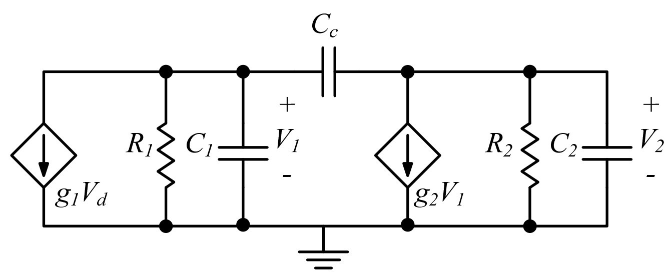

[Liu]

- The floating capacitance is converted to two grounded capacitances at the input and output of the amplifier.

- The capacitance at the input node is larger than the original floating capacitance (Miller multiplication effect)

# Miller Compensation

- This technique places a capacitor $C_c$ in the feedback path of one of the internal stages to take advantage of Miller multiplication of capacitors.




- The reflected capacitances due to $C_c$ and the DC voltage gain between $V_2$ and $V_1$ ($a_2 = -g_2R_2$) yields

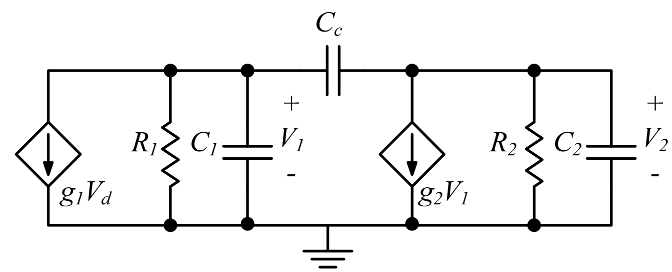
$$C_{1,c} = C_c(1 + g_2R_2) \cong |a_2|C_c \qquad \text{and} \qquad C_{2,c} = C_c\left(1 + \frac{1}{g_2R_2}\right) \cong C_c$$

- A low-frequency dominant pole can be created with a moderate capacitor value.

# Miller Compensation

- Accurate transfer function

$$\frac{V_2}{V_d} \cong g_1 R_1 g_2 R_2 \frac{1 - jf/f_z}{\left(1 - jf/f_{1(new)}\right)\left(1 - jf/f_{2(new)}\right)}$$




- Pole/zero locations

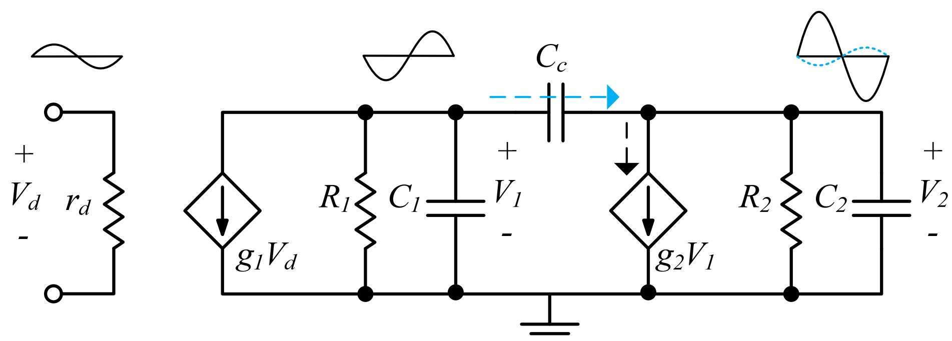
$$\omega_z = \frac{g_2}{C_c}$$ RHP zero

$$\omega_{1(new)} = \frac{1}{R_1 C_1 + g_2 R_2 R_1 C_c + R_2 C_2} \cong \frac{1}{R_1 g_2 R_2 C_c} = \frac{1}{|a_2| C_c R_1}$$ Dominant Pole

$$\omega_{2(new)} = \frac{R_1 C_1 + R_1 g_2 R_2 C_c + R_2 C_2}{R_1 R_2 (C_1 C_c + C_1 C_2 + C_c C_2)} \cong \frac{g_2 C_c}{C_1 C_c + C_1 C_2 + C_c C_2}$$ Second Pole

# Miller Compensation

- ***Right-half plane zero:***
  - The RHP zero is a result of the feedforward path through $C_c$

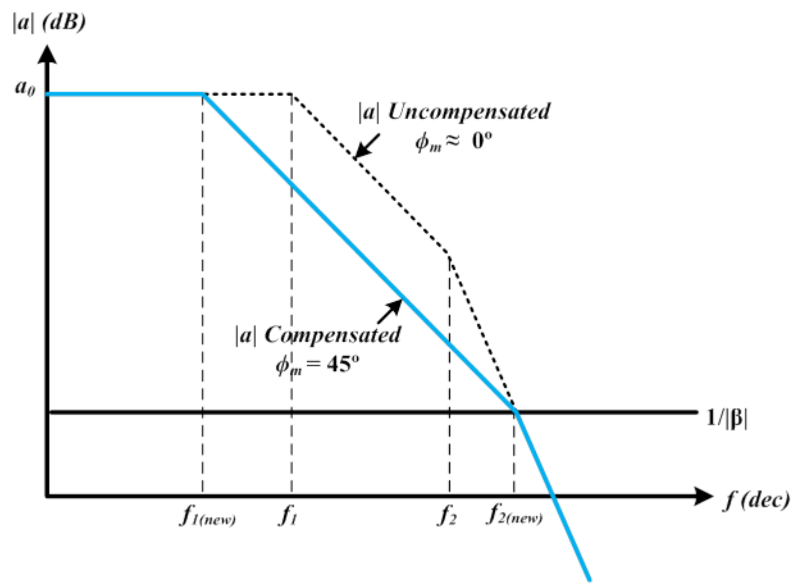



  - The circuit is no longer a minimum-phase system.
  - It introduces excessive phase shift, thus reduces the phase margin.
  - In bipolar OpAmps, it is usually at much higher frequency than the poles $\rightarrow 1 - f/f_z \cong 1$

# Miller Compensation

## ■ *Pole Splitting*

- Increasing $C_c$ lowers $f_{1(new)}$ and raises $f_{2(new)}$
- The shift in $f_2$ eases the amount of shift required by $f_1$ → Higher bandwidth

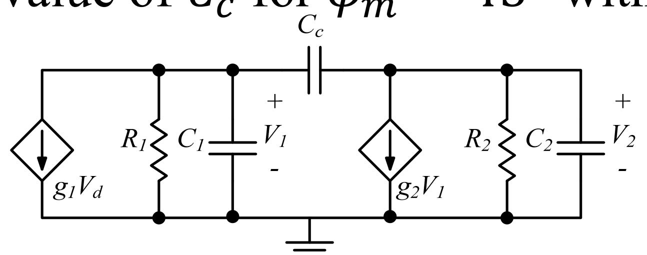



- Increasing $C_c$ above a certain limit makes $f_2$ stops to increase.

$$\omega_{2(new)} = 2\pi f_{2(new)} = \frac{g_2}{C_1 + \frac{C_1 C_2}{C_c} + C_2} \cong \left.\frac{g_2}{C_1 + C_2}\right|_{C_c \gg C_1, C_2}$$

# Miller Compensation

- Numerical example:
  - $r_d=\infty$, $r_o=0$ , $g_1=2$ mA/V, $R_1=100$ kΩ, $g_2=10$ mA/V, $R_2=50$ kΩ
  - $f_1 = 100\ kHz, f_2 = 1\ MHz, f_3 = 10\ MHz$
  - Find the required value of $C_c$ for $\phi_m = 45°$ with $\beta = 1$




From $f_1$ and $f_2$, we can calculate $C_1 = 15.9\ pF$ and $C_2 = 3.18\ pF$

Assume $C_c$ is large $\rightarrow$ $f_{2(new)} = \frac{g_2}{2\pi(C_1+C_2)} = 83.3\ MHz > f_3$

Since $f_{2(new)} > f_3$ $\rightarrow$ $f_3$ is the first non-dominant pole

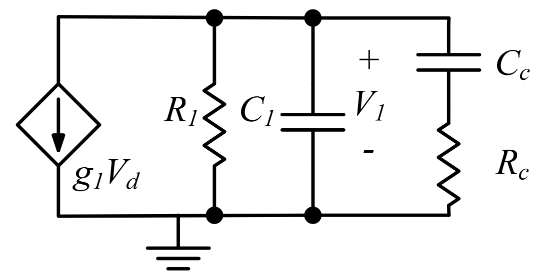
For $\phi_m = 45°$, $f_x = f_3 = 10\ MHz$

Then, $f_{1(new)} = \frac{f_3}{a_0\beta} = \frac{f_3}{g_1R_1g_2R_2} = 100\ Hz$

$$C_c = \frac{1}{2\pi R_1 g_2 R_2 f_{1(new)}} = 31.8\ pF$$

**Much smaller than that of shunt-capacitance compensation**
**Suitable for monolithic fabrication**

# Pole-Zero Compensation




- This technique uses a large compensation capacitor ($C_c \gg C_1$) to lower the first pole $f_1$.
- It also uses a small resistor ($R_c \ll R_1$) to create a zero that cancels the second pole $f_2$.
- The compensated response is then dominated by the lowered first pole up to $f_3$
- Transfer function

$$\frac{V_1}{V_d} = -g_1 R_1 \frac{1 + jf/f_z}{\left(1 + jf/f_{1(new)}\right)\left(1 + jf/f_4\right)}$$

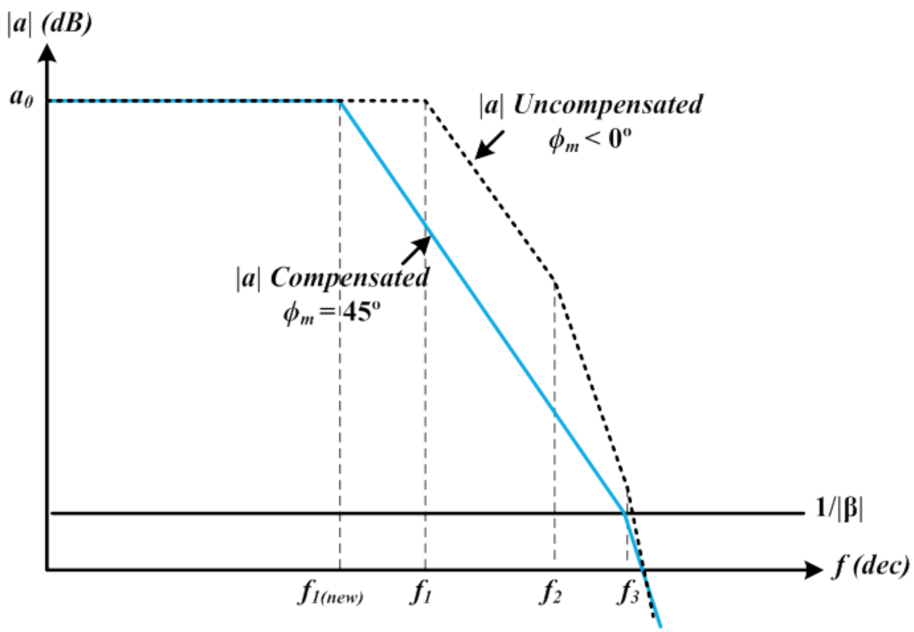
- Pole/zero locations

$$f_{1(new)} \cong \frac{1}{2\pi R_1 C_c} \quad , \qquad f_z = \frac{1}{2\pi R_c C_c} \quad , \qquad f_4 \cong \frac{1}{2\pi R_c C_1}$$

# Pole-Zero Compensation

- $C_c$ and $R_c$ lowers the dominant pole $f_{1(new)} \ll f_1$, creates a zero $f_z$, and creates an additional pole $f_4 \gg f_z$
- Choose $R_c$ such that $f_z$ cancels $f_2$
- The open loop gain now becomes

$$a_{new}(jf) = \frac{a_0}{\left(1 + jf/f_{1(new)}\right)(1 + jf/f_3)(1 + jf/f_4)}$$



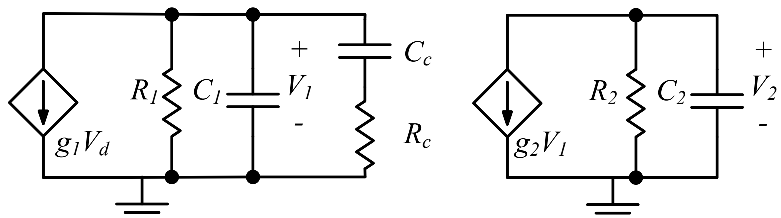

- To have $\phi_m = 45°$, the cross-over frequency should be $f_3$
- Since the compensated response is dominated by $f_{1(new)}$ pole up to $f_3$

$$\frac{\left|a\left(jf_{1(new)}\right)\right|}{|a(jf_3)|} = \frac{a_0}{1/\beta} = \frac{f_3}{f_{1(new)}}$$

- Thus, $f_{1(new)} = f_3/a_0\beta$

# Pole-Zero Compensation

- Numerical example:
  - $r_d=\infty$, $r_o=0$ , $g_1=2$ mA/V, $R_1=100$ kΩ, $g_2=10$ mA/V, $R_2=50$ kΩ
  - $f_1 = 100\ kHz, f_2 = 1\ MHz, f_3 = 10\ MHz$
  - Find the required value of $C_c$ for $\phi_m = 45°$ with $\beta = 1$




From $f_1$ and $f_2$, we can calculate $C_1 = 15.9\ pF$ and $C_2 = 3.18\ pF$

For $\phi_m = 45°$ , $f_x = f_3 = 10\ MHz$

Then, $f_{1(new)} = \frac{f_3}{a_0\beta} = \frac{f_3}{g_1R_1g_2R_2} = 100\ Hz \quad \rightarrow \quad C_c = \frac{1}{2\pi R_1 f_{1(new)}} = 15.9\ nf$

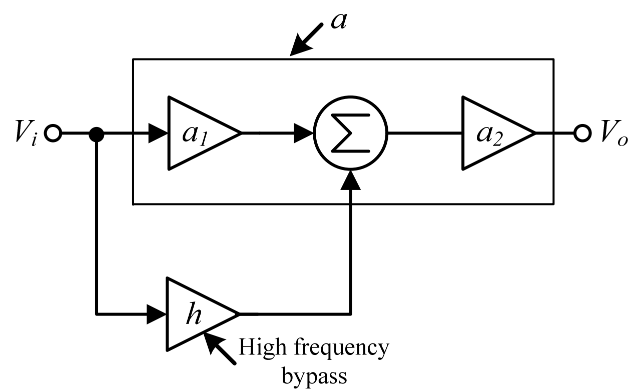
**Relaxed compared to shunt-capacitance compensation, but still LARGE**

$R_c$ is chosen such that $f_z = f_2 \rightarrow \quad R_c = \frac{1}{2\pi C_c f_2} = 10\ \Omega$

Since $f_4 = \frac{1}{2\pi R_c C_1} = 1GHz \gg f_3 \quad \rightarrow \quad$ It will not affect the phase margin

# Feedforward Compensation

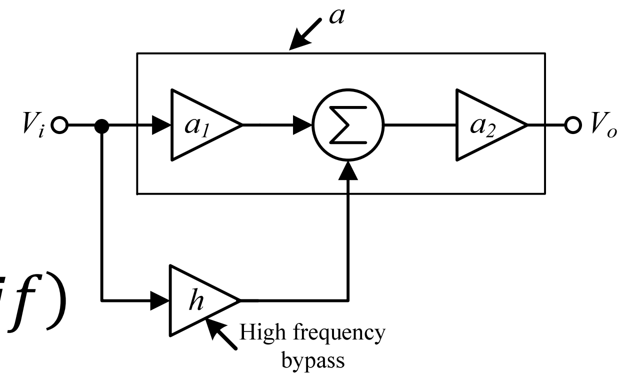
- In multistage amplifiers, usually there is one stage that acts as a bandwidth bottleneck by contributing a substantial amount of phase shift in the vicinity of the cross-over frequency $f_x$
- This technique creates a high-frequency bypass around the bottleneck stage to suppress its phase at $f_x$, thus improving $\phi_m$



# Feedforward Compensation

- The bypass around the bottleneck stage is a high-pass function

$$h(jf) = \frac{jf/f_0}{1+jf/f_0}$$




- The compensated open-loop gain is

$$a_{comp}(jf) = [a_1(jf) + h(jf)]a_2(jf)$$

- ***At low frequency:*** $|h(jf)| \ll |a_1(jf)|$

$$a_{comp}(jf) \cong a_1(jf)a_2(jf) = a(jf)$$

The high low-frequency gain advantage of the uncompensated amplifier still hold.

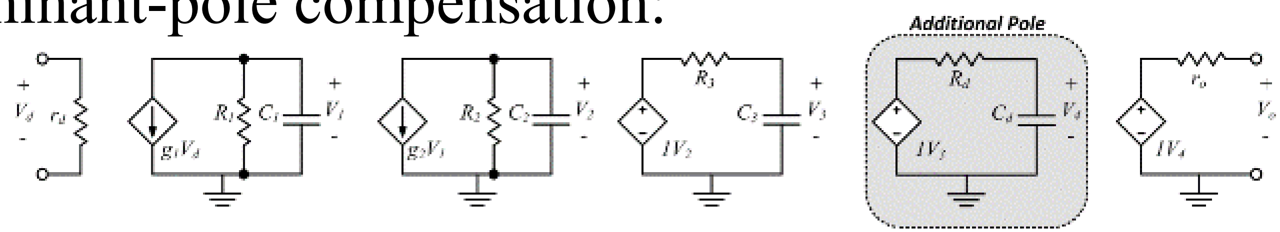
- ***At high frequency:*** $|h(jf)| \gg |a_1(jf)|$

$$a_{comp}(jf) \cong a_2(jf)$$

The dynamics are controlled only by $a_2$ → Wider bandwidth & Lower phase shift

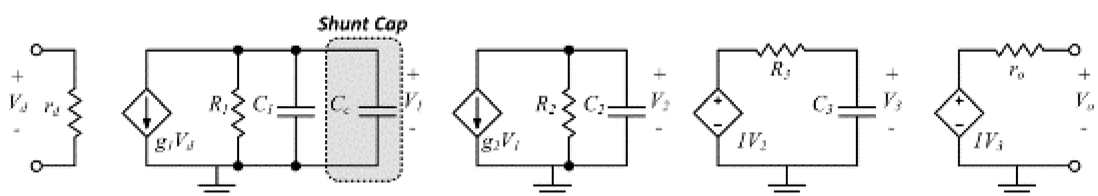
# Summary of Internal Frequency Compensation

- Dominant-pole compensation:




  - It creates an additional pole at sufficiently low frequency.
  - It doesn't take advantage of the existing poles.
  - It suffers from extremely low bandwidth.

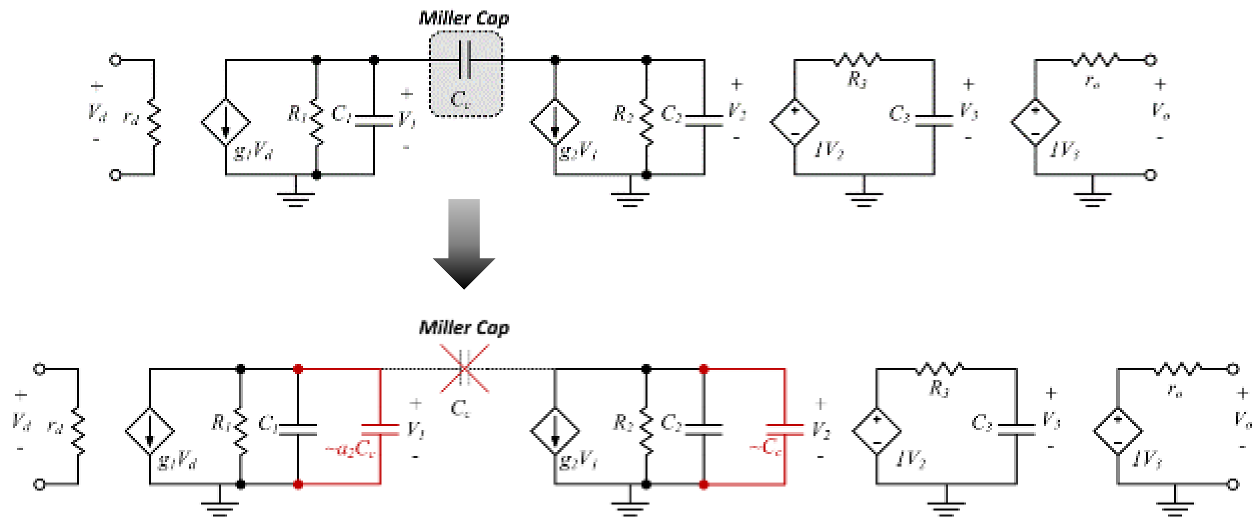
- Shunt-capacitance compensation:




  - It rearranges the existing poles rather than creating an additional pole.
  - It moves the first pole to sufficiently low frequency.
  - The value of the shunt capacitance is extremely large → Extra cost

# Summary of Internal Frequency Compensation

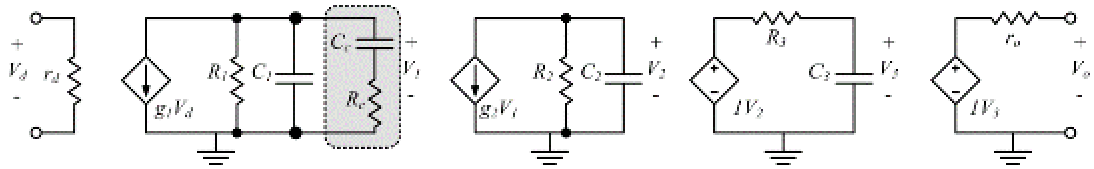
- Miller compensation:




  - It takes advantage of Miller multiplicative effect of capacitors, thus requires moderate capacitance to move the first pole to sufficiently low frequency.
  - It causes pole splitting, where the dominant pole is reduced and the first non-dominant pole is raised in frequency.

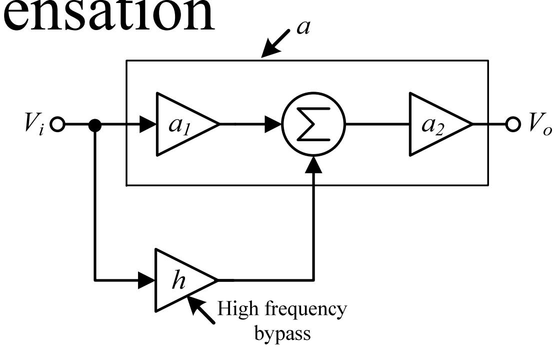
# Summary of Internal Frequency Compensation

- Pole-zero compensation:




  - Similar to shunt-capacitance technique, a large capacitor is used to shift the first pole to sufficiently low frequency.
  - A small resistance is used to create a zero that cancels the first non-dominant pole

- Feedforward Compensation



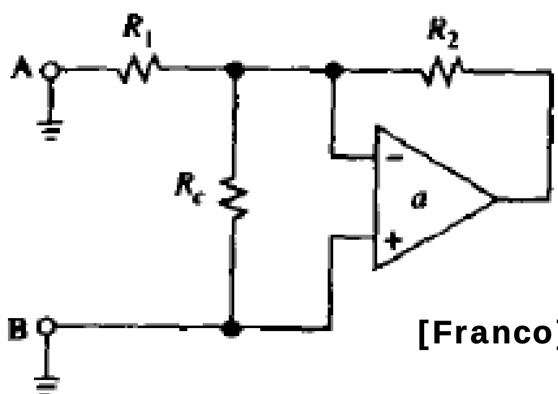

  - It places a high frequency bypass around the bottleneck stage that contributes the most phase shift in the vicinity of $f_x$

## External Frequency Compensation

- How to stabilize the circuit by modifying its feedback factor $\beta$ ?
  - Reducing the Loop Gain
  - Input-Lag Compensation
  - Feedback-Lead Compensation

# Reducing the Loop Gain

- This method shifts |1/β| curve upwards until it intercepts the |a| curve at $f = f_{\phi_m - 180^o}$, where $\phi_m$ is the desired phase margin.
- The shift is obtained by connecting resistance $R_c$ across the inputs.



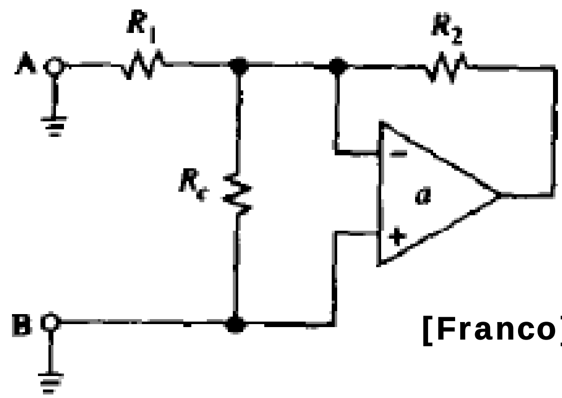

[Franco]

$$\frac{1}{\beta} = 1 + \frac{R_2}{(R_1 \| R_c)} = \overbrace{1 + \frac{R_2}{R_1}}^{\text{Uncompensated}} + \underbrace{\frac{R_2}{R_c}}_{\substack{\text{Shifts the curve upwards} \\ \text{to improve } \phi_m}}$$

# Reducing the Loop Gain

- $R_c$ is chosen to achieve the desired phase margin $\phi_m$:

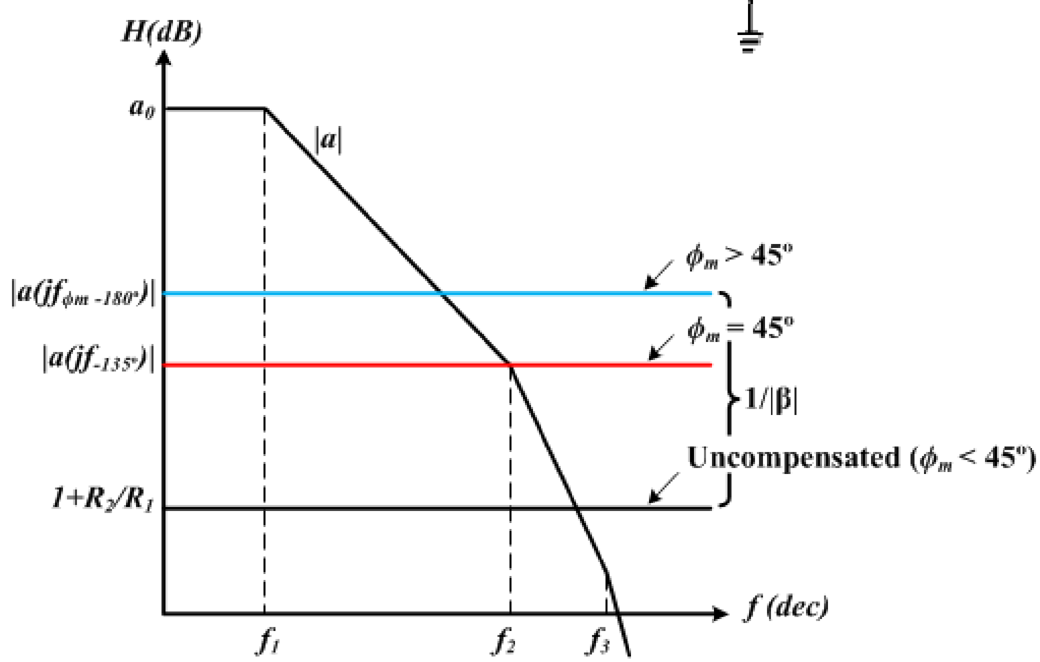
$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c} = \left|a(jf_{\phi_m - 180^\circ})\right|$$




[Franco]

Then,

$$R_c = \frac{R_2}{\left|a(jf_{\phi_m - 180^\circ})\right| - (1 + R_2/R_1)}$$



## Reducing the Loop Gain

- ***Prices that we are paying for stability:***
- ***Gain Error***:

$$H_{CL} = \frac{1}{\beta}\frac{T}{1+T} = \frac{A_{ideal}}{1+1/T}$$

The presence of $R_c$ reduces $T$, thus resulting in a larger gain error.

- ***DC Noise Gain:***

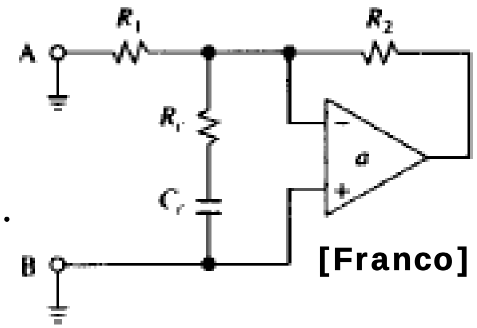
$$H_{CL}(j0) \cong \frac{1}{\beta_0} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}$$

The presence of $R_c$ causes an increased DC-noise gain which may result in an intolerable DC output error.

**THERE'S NO FREE LUNCH !**

# Input-Lag Compensation

- The high DC-noise gain of the previous method can be overcome by placing a capacitance $C_c$ in series with $R_c$.
  - ***High frequencies***:
    - ✓ $C_c$ is short.
    - ✓ $\frac{1}{|\beta|}$ curve is unchanged compared to the previous case.
  - ***Low frequencies***:
    - ✓ $C_c$ is open
    - ✓ $\frac{1}{|\beta|} = 1 + \frac{R_2}{R_1}$, we now have much higher DC loop gain & much lower DC output error.

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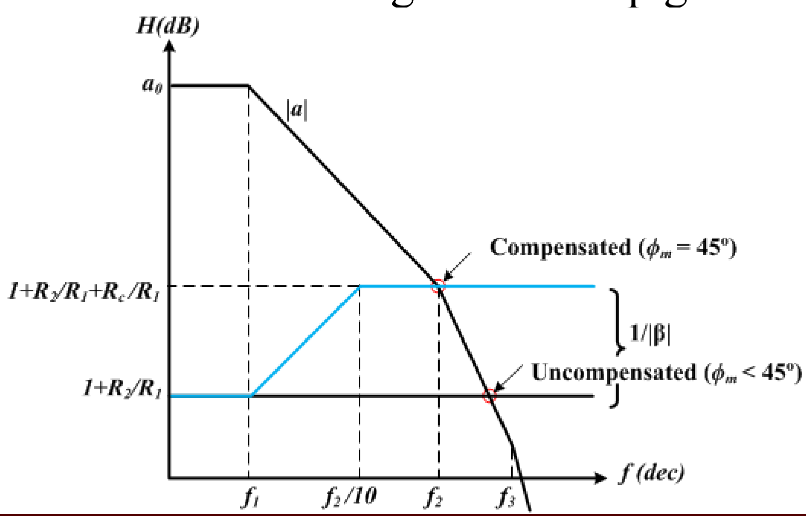
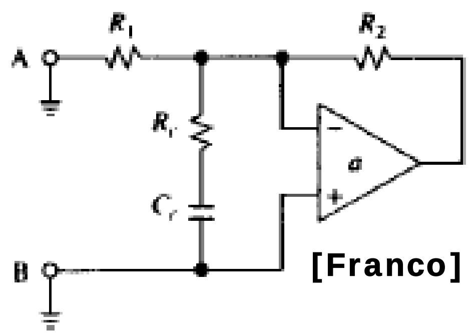


# Input-Lag Compensation

- $R_c$ is chosen to achieve the desired phase margin $\phi_m$:

$$\frac{1}{\beta_\infty} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c} = \left|a(jf_{\phi_m - 180^\circ})\right|$$




Then,

$$R_c = \frac{R_2}{\left|a(jf_{\phi_m - 180^\circ})\right| - (1 + R_2/R_1)}$$

- To avoid degrading $\phi_m$, it is good practice to position the second breakpoint of $|1/\beta|$ curve a decade below $f_{\phi_m - 180^\circ}$.

$$\frac{1}{2\pi C_c R_c} = \frac{1}{10} f_{\phi_m - 180^\circ}.$$

Then,

$$C_c = \frac{5}{\pi R_c f_{\phi_m - 180^\circ}}$$

# Input-Lag Compensation

- ***Advantage(s):***
  - ☺ Lower DC-noise gain due to the presence of $C_c$.
  - ☺ It allows for higher slew rate compared with internal compensation techniques: Op-amp is spared from having to charge/discharge internal compensation capacitance.

- ***Disadvantage(s):***
  - ☹ Long settling tail because of the presence of pole-zero doublet of the feedback network ($|\beta|$).
  - ☹ Increased high-frequency noise in the vicinity of the cross-over frequency.
  - ☹ Low closed-loop differential input impedance ($Z_d$) which may cause high-frequency input loading

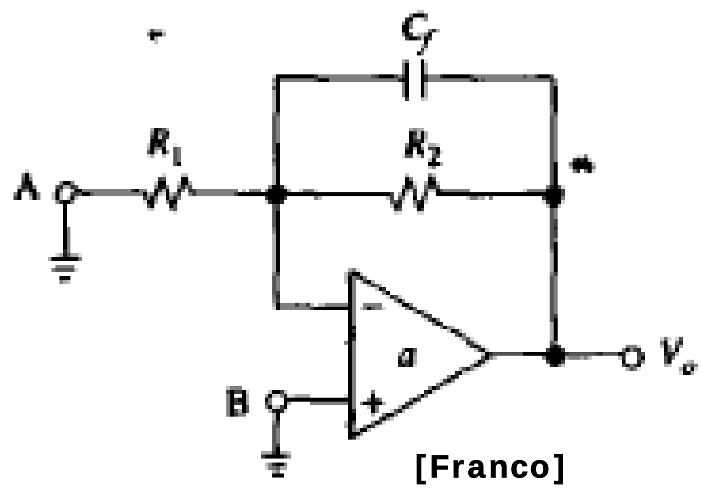
$$Z_d = z_d \| Z_c \quad , \quad Z_c = R_c + 1/sC_c \ll z_d$$

** $z_d$ is the open loop input impedance of the Op-Amp.

# Feedback-Lead Compensation

- This technique uses a feedback capacitance $C_f$ to create phase lead in the feedback path.

- The phase lead is designed to be in the vicinity of the crossover frequency $f_x$ which is were $\phi_m$ is boosted.

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# Feedback-Lead Compensation

## ■ *Analysis:*

$$\frac{1}{\beta} = 1 + \frac{\left(R_2 \| Z_{C_f}\right)}{R_1} = \left(1 + \frac{R_2}{R_1}\right)\frac{1+jf/f_z}{1+jf/f_p}$$

where,

$$f_p = \frac{1}{2\pi C_f R_2} \quad , \quad f_z = \left(1 + \frac{R_2}{R_1}\right) f_p$$

- The phase-lag provided by $\frac{1}{\beta(j\omega)}$ is maximum at $\sqrt{f_z f_p}$.
- The optimum value of $C_f$ , that maximizes the phase margin, is the one that makes this point at the crossover frequency.

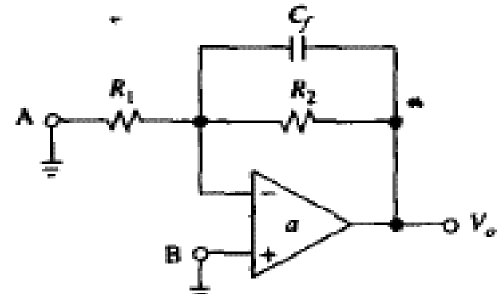
$$f_x = \sqrt{f_z f_p} = f_p\sqrt{1 + \frac{R_2}{R_1}}$$

- The cross-over frequency can be obtained from

$$|a(jf_x)| = \frac{1}{|\beta(jf_x)|} = \sqrt{1 + \frac{R_2}{R_1}}$$

- Having $f_x$, the optimum $C_f$ can be found

$$C_f = \frac{\sqrt{1 + R_2/R_1}}{2\pi R_2 f_x}$$

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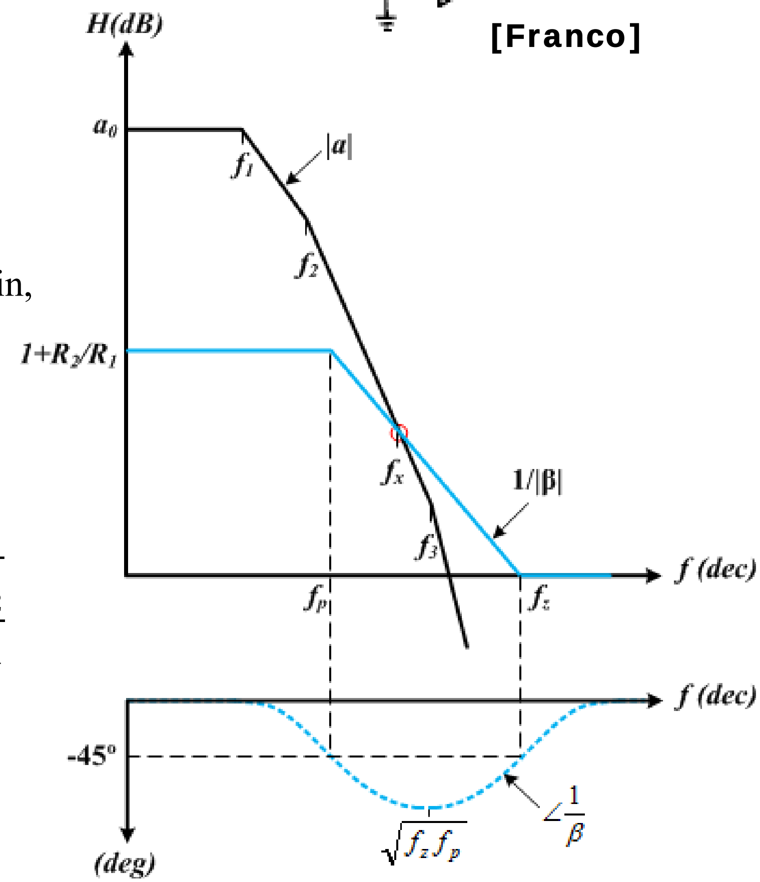


# Feedback-Lead Compensation

## ■ *How much phase margin can we get ?*

- At the geometric mean of $f_p$ and $f_z$, we have

$$\angle\left(\frac{1}{\beta}\right) = 90° - 2\tan^{-1}\left(1 + \frac{R_2}{R_1}\right)$$

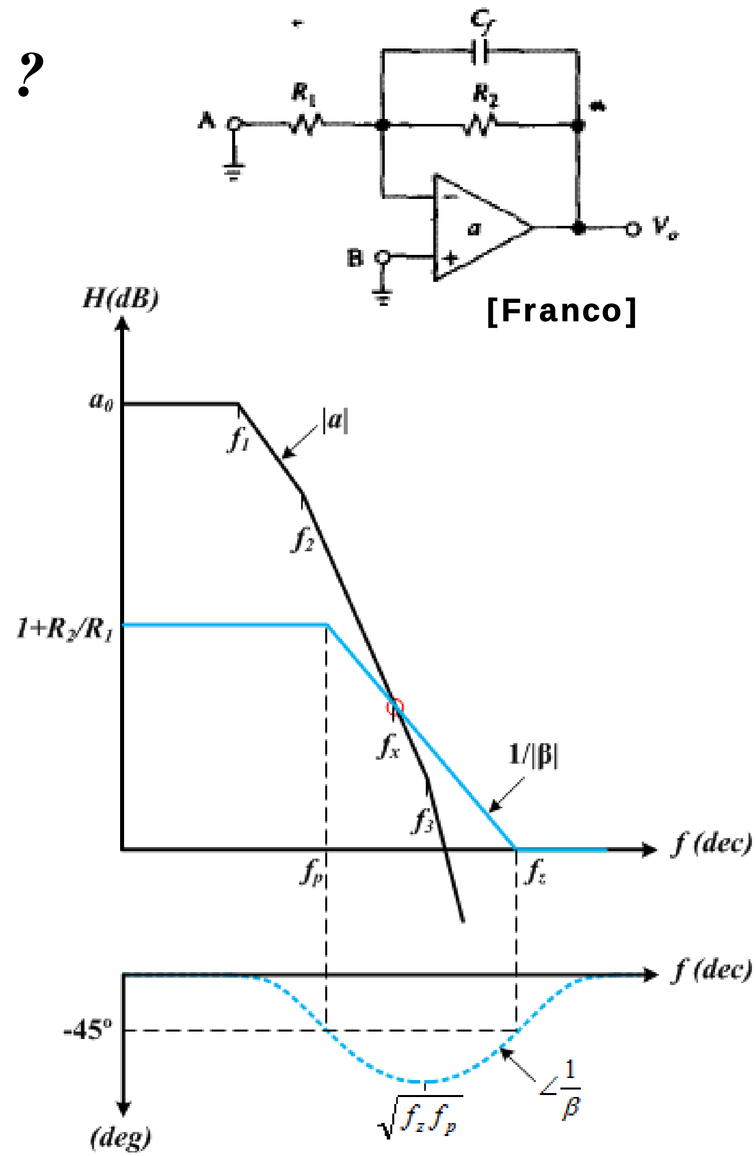
- The larger the value of $1 + R_2/R_1$, the greater the contribution of $1/\beta$ to the phase margin.

- E.g. $1 + R_2/R_1 = 10 \ \rightarrow \angle\big(1/\beta\,(jf_x)\big) = -55°$

Thus,

$$\angle T(jf_x) = \angle a(jf_x) - \angle\left(\frac{1}{\beta(jf_x)}\right) = \angle a(jf_x) + 55°$$

- The phase margin is improved by 55° due to feedback-lead compensation.



# Feedback-Lead Compensation

- ***Advantage(s):***
  - ☺ $C_f$ helps to counteract the effect of the input stray capacitance $C_n$ as we discussed beforehand.
  - ☺ It provides better filtering for internally generated noise.

- ***Disadvantage(s):***
  - ☹ It doesn't have the slew-rate advantage of the input-lag compensation.

# Decompensated OpAmps

- These OpAmps are compensated for unconditional stability only when used with $1/\beta$ above a specified value

$$\frac{1}{\beta} \geq \left(\frac{1}{\beta}\right)_{min}$$

- They provide a constant GBP only for $|a| \geq (1/\beta)_{min}$
- They offer higher GBP and slew rate.
- ***Example***
  - The fully compensated LF356 OpAmp uses $C_c \cong 10\,pF$ to provide $GBP = 5\ MHz$ and $SR = 12\ V/\mu s$ for any $|a| \geq 1$ V/V.
  - The decompensated version of the same OpAmp, LF357, uses $C_c \cong 3\,pF$ and provides $GBP = 20\,MHz$ and $SR = 50\,V/\mu s$ but only for any $|a| \geq 5$ V/V.