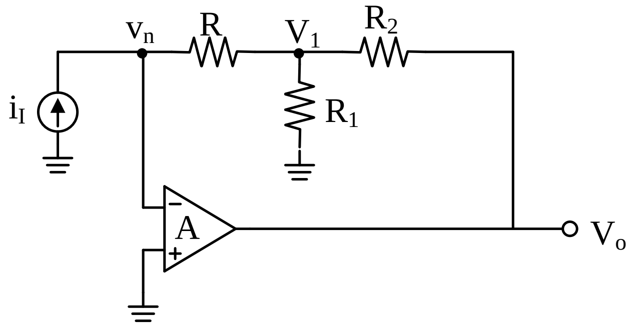
ECEN 457 (ESS)
TAMU

# HIGH SENSITIVITY I-V CONVERTER




$$-Av_n = v_o$$

$$v_n = -\frac{v_o}{A}$$

$$V_1\left(\frac{1}{R}+\frac{1}{R_2}+\frac{1}{R_1}\right)-\frac{v_n}{R}-\frac{v_o}{R_2}=0 \quad (1); \qquad \frac{v_n}{R}-\frac{V_1}{R}=i_I \quad (2)$$

For $A \to \infty$, then $v_n = 0$. Thus

$$V_1\left(\frac{R_2}{R}+1+\frac{R_2}{R_1}\right)=V_o \qquad (1')$$

$$V_1 = -i_I\, R \qquad (2')$$

$(2')$ into $(1')$

$$-i_I\left(R_2+R+\frac{R\,R_2}{R_1}\right)=V_o=k\,Ri_I=-\left(1+\frac{R_2}{R_1}+\frac{R_2}{R}\right)Ri_I$$

$$V_o=-i_I\,\frac{R_2R_1+R\,R_1+R\,R_2}{R_1} \qquad \leftarrow T-\text{Network}$$

$$V_o=-i_IR\,\frac{R_1+R_2+R_2R_1/R}{R_1}$$

$$S_{R_1}^{V_o} = \frac{\partial V_o}{\partial R_1}\frac{R_1}{V_o}$$

$$V_o = -i_I\left(R_2 + R + \frac{R\,R_2}{R_1}\right)$$

$$\frac{\partial V_o}{\partial R_1} = +i_I\,\frac{R\,R_2}{R_1^2} \quad ; \quad \frac{\partial V_o}{\partial R_1}\frac{R_1}{V_o} = i_I\,\frac{R\,R_2}{R_1^2}\,\frac{R_1}{-\,i_I\left(R_2 + R + \frac{R\,R_2}{R_1}\right)}$$

$$S_{R_1}^{V_o} = -\frac{R\,R_2}{R_1}\,\frac{1}{R_2 + R + \frac{R\,R_2}{R_1}}$$

$$S_{R_1}^{V_o} = -\frac{R\,R_2}{R_1R_2 + R_2R + R_1R} = -\frac{R\,R_2 / R_1}{\frac{R_1R_2 + R_2R + R_1R}{R_1}}$$

$$S_{R_1}^{V_o} = \frac{-\,R\,R_2}{R_1R_T}$$

Example: Let $R_T = 1\text{M}\Omega$

$$R_T = \frac{R\,R_2 + R\,R_1 + R_1R_2}{R_1} = R + R_2 + \frac{R\,R_2}{R_1}$$

Assume $R_1 = 100\Omega$

Then $R + R_2 + \dfrac{R\,R_2}{100} = 10^6$

and $R = 100K$ then

$$100K + R_2\left(1 + \frac{100K}{100}\right) = 10^6 \Rightarrow R_2 = \frac{10^6 - 10^5}{1 + 10^3} \sim 10^3 - 10^2$$

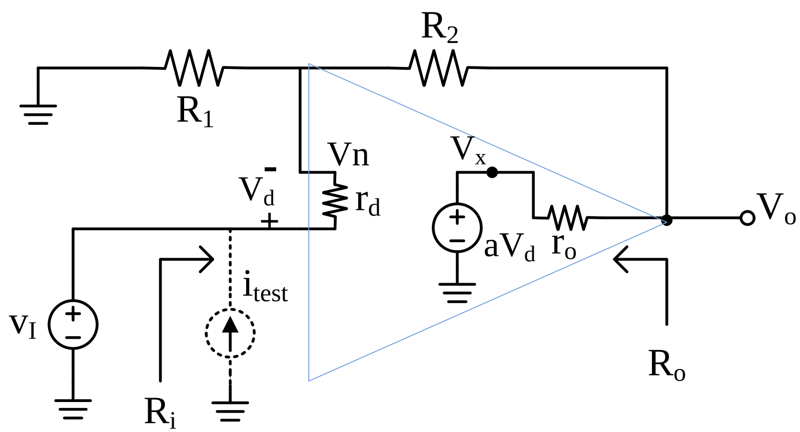
$R_2 \cong 900\Omega$

Thus

$$S_{R_1}^{V_o} = -\frac{R\,R_2}{R_1R_T} = \frac{100K \times 0.9K}{100 \times 10^6} = \frac{-90}{100} = -0.9$$

Effects of Input and Output impedance of the Op Amp.

INVERTING




$$Z_{in} = \frac{v_I}{i_{test}}$$

$$v_n\left[\left(\frac{1}{r_d}\right) + \frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{V_o}{R_2} = \frac{V_I}{r_d} \qquad (1)$$

$$v_d = r_d i_{test} \qquad (2)$$

$$v_n = v_I - v_d = v_I - r_d i_{test} \qquad (4)$$

$$V_o\left(\frac{1}{r_o}+\frac{1}{R_2}\right)-\frac{aV_d}{r_o}-\frac{V_n}{R_2}=0 \qquad (3)$$

$(2)$ into $(3)$

$$V_o\left(\frac{1}{r_o}+\frac{1}{R_2}\right)-\frac{ar_d i_{test}}{r_o}=\frac{V_I-r_d i_{test}}{R_2}$$

$$V_o=+\frac{r_d i_{test} a}{1+\frac{r_o}{R_2}}+\frac{V_I-r_d i_{test}}{1+\frac{R_2}{r_o}} \qquad (3')$$

substituting $(3')$ and $(4)$ into $(1)$

$$(v_I-v_d)\left[\frac{1}{r_d}+\frac{1}{R_1}+\frac{1}{R_2}\right]-\left(\frac{r_d\cdot i_{test}\cdot a}{1+\frac{r_o}{R_2}}+\frac{V_I-r_d\cdot i_{test}}{1+\frac{R_2}{r_o}}\right)\frac{1}{R_2}=\frac{v_I}{r_d}$$

$$(v_I)\left[\cancel{\frac{1}{r_d}}+\frac{1}{R_1}+\frac{1}{R_2}-\cancel{\frac{1}{r_d}}-\frac{1}{1+\frac{R_2}{r_o}}\cdot\frac{1}{R_2}\right]-r_d i_{test}\left[\frac{1}{r_d}+\frac{1}{R_1}+\frac{1}{R_2}+\frac{a}{R_2+r_o}-\frac{1}{1+\frac{R_2}{r_o}}\cdot\frac{1}{R_2}\right]=0$$

$$\frac{v_I}{i_{test}} = \frac{r_d\left[\frac{a}{R_2 + r_o} + \frac{1}{r_d} + \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{1+\frac{R_2}{r_o}} \cdot \frac{1}{R_2}\right]}{\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{1+\frac{R_2}{r_o}} \cdot \frac{1}{R_2}}$$

$$Z_{in} \cong \frac{R_1 R_2}{R_1 + R_2} r_d\left[\frac{a}{R_2 + r_o} + \frac{1}{r_d} + \frac{R_1 + R_2}{R_1 R_2}\right]$$

$$Z_{in} \cong \frac{R_1 \cancel{R_2}}{R_1 + R_2} r_d\left[\frac{a}{\cancel{R_2}}\right] = \frac{r_d a}{1+\frac{R_2}{R_1}} = r_d \frac{a}{1/\beta} = r_d \beta a = r_d T$$

$$Z_{in} \cong \frac{R_1 R_2}{R_1 + R_2}\left[\frac{r_d a}{R_2 + r_o} + 1 + r_d \frac{R_1 + R_2}{R_1 R_2}\right]$$

$$Z_{in} \cong \frac{R_1 R_2}{R_1 + R_2} \frac{a r_d}{R_2 + r_o} + \frac{R_1 R_2}{R_1 + R_2} + r_d$$

$$Z_{in} \cong \frac{R_1}{R_1 + R_2} a r_d + r_d + \frac{R_1 R_2}{R_1 + R_2}$$

$$Z_{in} \cong r_d \left( 1 + \frac{a}{1 + \frac{R_2}{R_1}} \right) = r_d \left( 1 + \frac{a}{1/\beta} \right) = r_d (1 + \beta a)$$

$$Z_{in} = r_d (1 + T)$$

Exercise. Show that

$$\mathrm{R_o} \cong \frac{r_o}{1 + T}$$