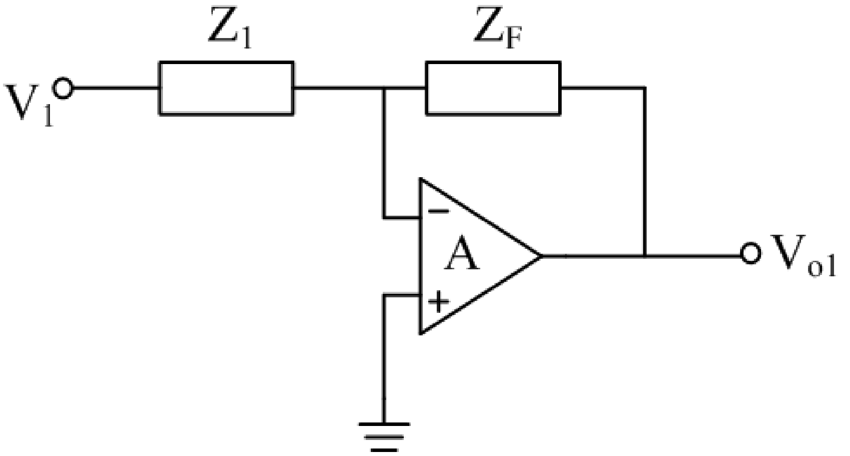
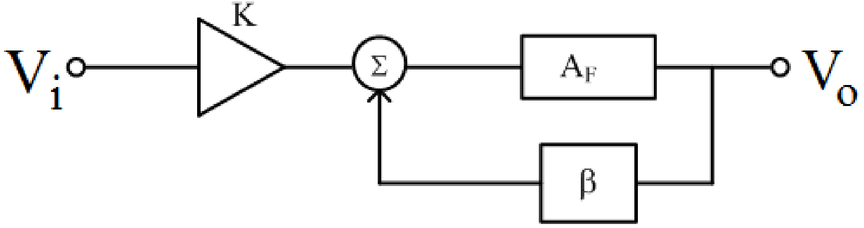
ECEN 457 (ESS)
TAMU

ACTIVE-RC FILTER ARCHITECTURES

- We will discuss first-order and second-order filters based on general inverter configurations.
- This approach will be based on two key building blocks.

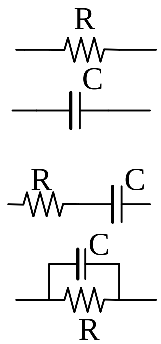


$$\frac{V_{01}}{V_1} = \frac{-\frac{Z_F}{Z_1}}{1+\frac{1}{A}\left(1+\frac{Z_F}{Z_1}\right)} \qquad (1)$$



$$\frac{V_o}{V_i} = \frac{KA_F}{1+\beta A_F} \qquad (2)$$

Z_1 or Z_F Can be:



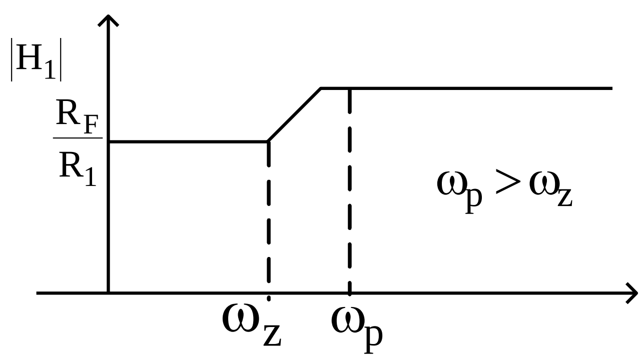
R	R
C	$\dfrac{1}{sC}$
R in series with C	$\dfrac{1 + sRC}{sC}$
R in parallel with C	$\dfrac{R}{1 + sRC}$

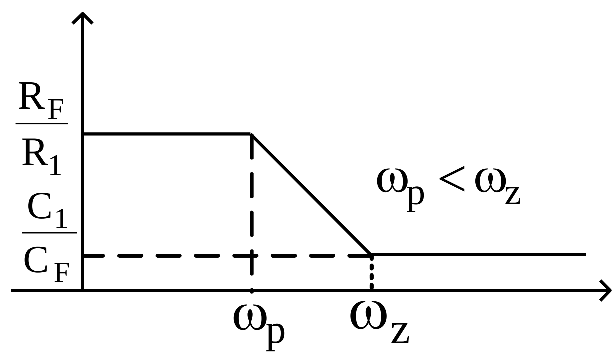
(3)

EXAMPLE: Let $Z_1 = \dfrac{R_1}{1+sR_1C_1}$, $Z_F = \dfrac{R_F}{1+sR_FC_F}$

Assuming ideal op amp $A \rightarrow \infty$. Then using (1)

$$H_1 = \frac{V_{01}}{V_1} = -\frac{R_F/R_1(1+sR_1C_1)}{(1+sR_FC_F)} = -\frac{K_n(1+s/\omega_z)}{(1+s/\omega_p)} \quad (4)$$





Particular cases are easily derived from (3) and (4)

— Integrator: $C_1 \to 0 \quad , \quad R_F \to \infty$

$$H_1 \cong -\frac{R_F}{R_1}\frac{1}{sR_FC_F} = -\frac{1}{sC_FR_1}$$

— Differentiator ; $R_1 \to \infty \quad , \quad C_F \to 0$

$$H_1 \cong -\frac{R_F}{R_1}sR_1C_1 = -sR_FC_1$$

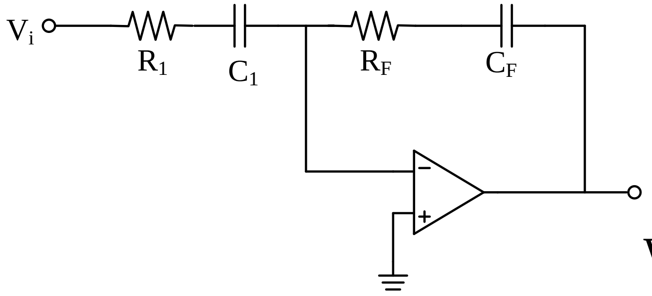
— Low-Pass: $C_1 = 0$

$$H_1 = \frac{-\frac{R_F}{R_1}}{1+sR_FC_F}$$

— High-Pass: $R_1 \to \infty$

$$H_1 \cong -\frac{R_F}{R_1}\frac{sR_1C_1}{1+sR_FC_F}$$

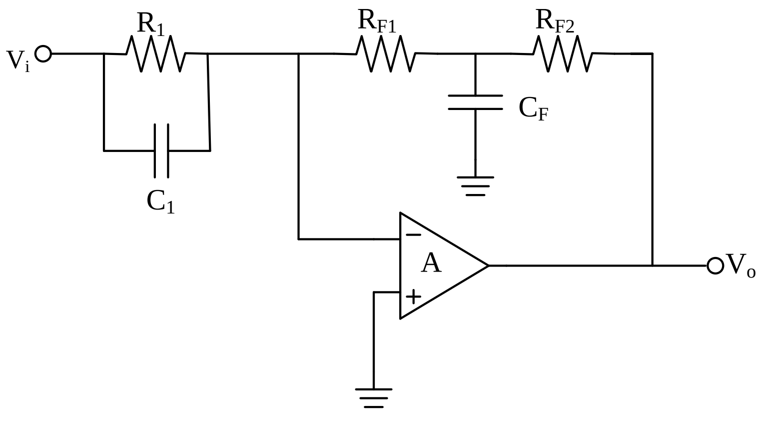
— One pole and one zero



$$\frac{V_o}{V_i} = -\frac{1+sR_FC_F}{sC_F}\frac{sC_1}{1+sR_1C_1} = -\frac{C_1}{C_F}\frac{1+sR_FC_F}{1+sR_1C_1} \qquad (5)$$

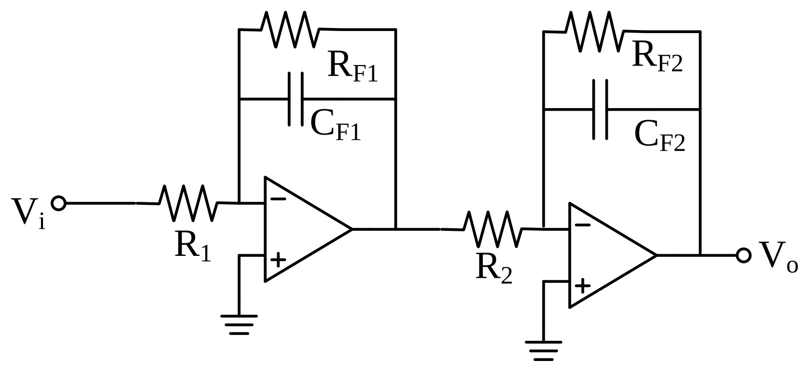
What are the key differences between Eqs. (4) and (5)?

Exercise 1. Obtain the transfer function of the following circuit.



Second-Order Filters Based on a Two-Integrator Loop.

- We can design a second-order filter by cascading two inverters. i.e.



$$\frac{V_o}{v_i} = \frac{-\frac{R_{F1}}{R_1}\left(-\frac{R_{F2}}{R_2}\right)}{(1+sC_{F1}R_{F1})(1+sC_{F2}R_{F2})} = \frac{\frac{R_{F1}}{R_1}\frac{R_{F2}}{R_2}}{s^2C_{F1}R_{F1}C_{F2}R_{F2}+s(C_{F1}R_{F1}+C_{F2}R_{F2})+1} \qquad (6)$$

What are the locations of the poles?

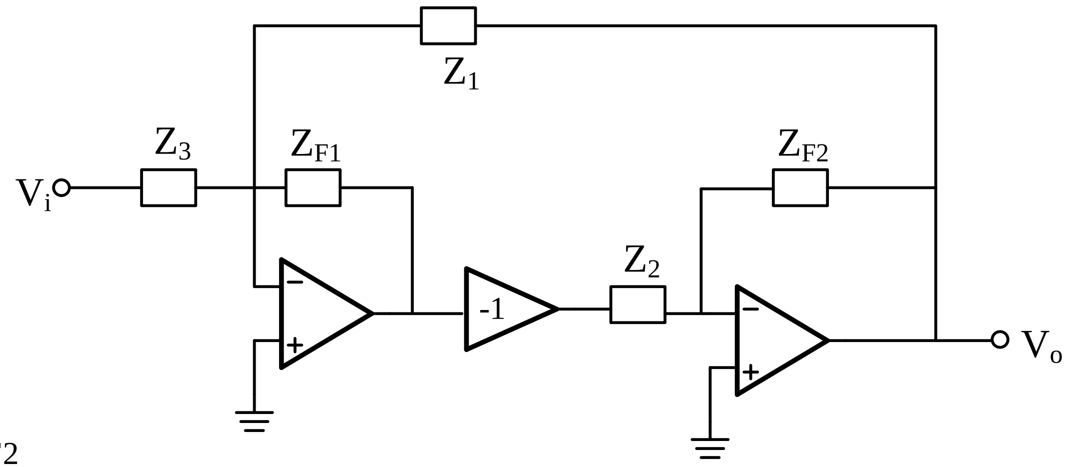
$$s_{p_{1,2}} = \frac{-(C_{F1}R_{F1}+C_{F2}R_{F2}) \pm \sqrt{(C_{F1}R_{F1}+C_{F2}R_{F2})^2 - 4C_{F1}R_{F1}C_{F2}R_{F2}}}{2C_{F1}R_{F1}C_{F2}R_{F2}}$$

To have complex poles it requires that

$$(C_{F1}R_{F1})^2 + (C_{F2}R_{F2})^2 - 2C_{F1}R_{F1}C_{F2}R_{F2} < 0\,?$$

Which it is impossible to satisfy. Therefore, cascading two first-order filter yield a second-order filter with only real poles.

The general form of the second order two-integrator loop has the following topology.



$$H = \frac{-\dfrac{Z_{F1}}{Z_3}\dfrac{Z_{F2}}{Z_2}}{1 + \dfrac{Z_{F1}}{Z_1}\dfrac{Z_{F2}}{Z_2}} \qquad (7a)$$

Note the similarity of Eq. (7a) with (2). Also observe that A "-1" needs to be inserted before or after the second inverter to yield a negative feedback loop.

Let us consider the following filter where

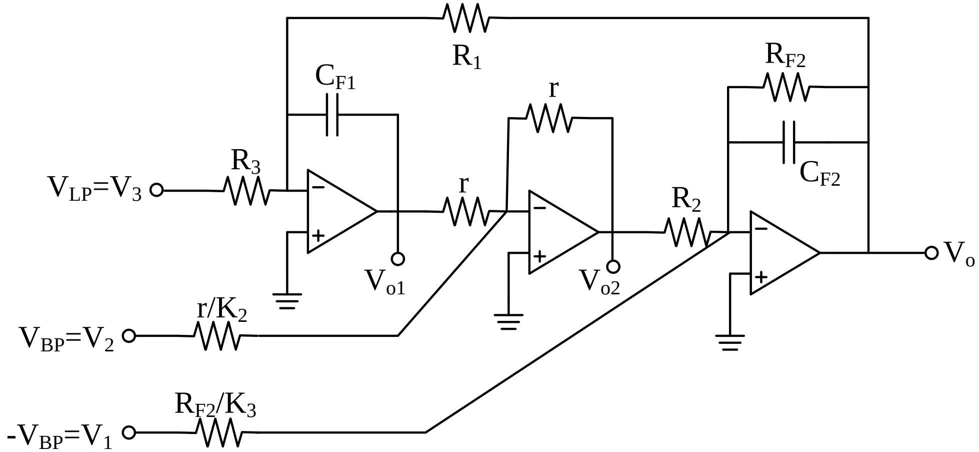
$$Z_3 = R_3,\ Z_1 = R_1,\ Z_2 = R_2,\ Z_{F1} = \frac{1}{sC_{F1}},\ Z_{F2} = \frac{R_{F2}}{1+sC_{F2}R_{F2}}$$

Thus Eq. (7a) yields:

$$H = \frac{-\dfrac{1}{sC_{F1}R_3}\dfrac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})}}{1+\dfrac{1}{sC_{F1}R_1}\dfrac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})}}$$

$$H = \frac{-\dfrac{1}{C_{F1}R_3C_{F2}R_2}}{s^2+\dfrac{s}{C_{F2}R_{F2}}+\dfrac{1}{C_{F1}R_1C_{F2}R_2}} = \frac{-\omega_{o1}^2}{s^2+\dfrac{\omega_o}{Q}s+\omega_o^2}$$

By injecting in different current summing nodes **a general biquad filter** can be obtained.

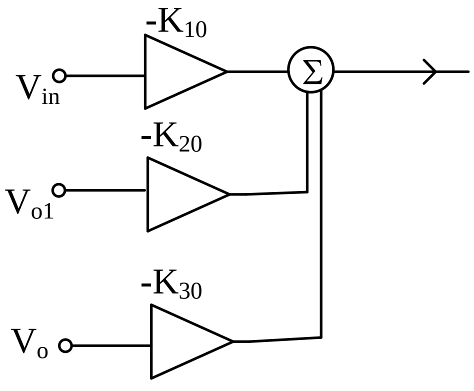


$$V_o = \frac{V_3 \frac{1}{sC_{F1}R_3} \frac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})} + \frac{-K_2 \frac{R_{F2}}{R_2}}{(1+sC_{F2}R_{F2})} - \frac{K_3 \frac{R_{F2}}{R_{F2}} V_1}{(1+sC_{F2}R_{F2})}}{1 + \frac{1}{sC_{F1}R_1} \frac{R_{F2}/R_2}{(1+sC_{F2}R_{F2})}}$$

$$V_o = \frac{V_3 \frac{1}{C_{F1}R_3C_{F2}R_2} + V_2 sC_{F1}R_1K_2 - V_3 sC_1R_1K_3}{s^2 + \frac{s}{C_{F2}R_{F2}} + \frac{1}{C_{F1}R_1C_{F2}R_2}}$$

Exercise 2. Obtain the expressions of V_{o1} and V_{o2}.

More general biquad expressions and topologies can be obtained by adding a summer.

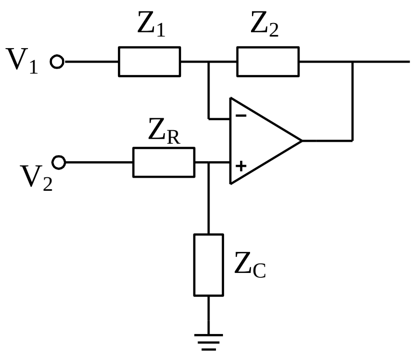


$$V_{oT} = -K_{10}V_{in} - K_{20}V_{o1} - K_{30}V_o$$

Exercise 3. Draw an active-RC topology of the block diagram show above.

Exercise 4 a) For only $V_1 \neq 0$ obtain V_o and V_{o1} when instead of the resistor R_{F2}/K_3 a capacitor $K_4\,C_{F2}$ is used. b) For only $V_3 \neq 0$ obtain V_{o1} when the resistor R_3 is replaced by a capacitor $K_{HP}C_{F1}$.

By using also the positive input of the op amp other useful filters can be obtained.

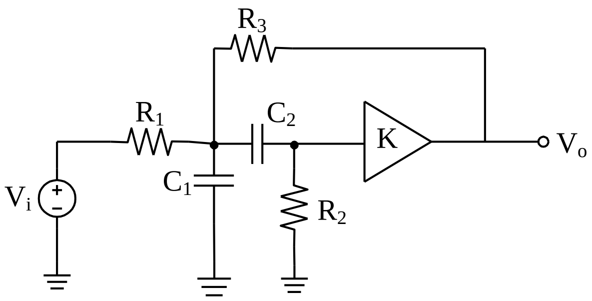


$$V_o = \frac{-\frac{Z_2}{Z_1}V_1 + \frac{Z_R}{Z_R + Z_C}V_2\left(1+\frac{Z_2}{Z_1}\right)}{1+\frac{1}{A}\left(1+\frac{Z_2}{Z_1}\right)}$$

Example. Phase shifter $Z_2=R_2=R_1$, $Z_1=R_1$, $Z_R=R$ $Z_C = \frac{1}{sC}$ and $A \to \infty$ with $V_1 = V_2$

$$\frac{V_o}{V_1} = -1 + \frac{sRC}{1+sRC}\cdot 2 = \frac{-1-sRC+2sRC}{1+sRC} = -\frac{1-sRC}{1+sRC}$$

Sallen and Key Bandpass Filter



K is a non-inverting amplifier

Using Nodal Analysis

$$V_1\left(s(C_1+C_2)+\frac{1}{R_1}+\frac{1}{R_3}\right)-sC_2V_2-\frac{V_o}{R_3}=\frac{V_i}{R_1} \qquad (1)$$

$$-V_1(sC_2) \quad + \quad V_2\left(sC_2+\frac{1}{R_2}\right)=0 \qquad (2)$$

$$V_o = KV_2 \qquad (3)$$

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{K\frac{s}{R_1C_1}}{s^2+\left[\frac{1}{R_2C_2}+\left(1-K+\frac{R_3}{R_1}+\frac{R_3}{R_2}\right)\frac{1}{R_3C_1}+\frac{R_1R_3}{R_1R_3R_2C_1C_2}\right]}$$

A particular case is for $R_1=R_2=R_3=R$, $C_1=C_2=C$

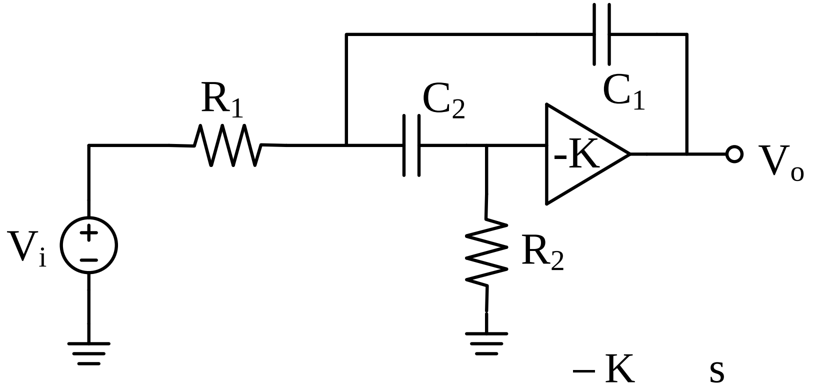
Then

$$\omega_o^2 = \frac{2}{(RC)^2} \quad ; \quad Q = \frac{\sqrt{2}}{4-K}$$

or for a given ω_o and Q

$$RC = \frac{\sqrt{2}}{\omega_o} \quad \text{and} \quad K = 4 - \frac{\sqrt{2}}{Q}$$

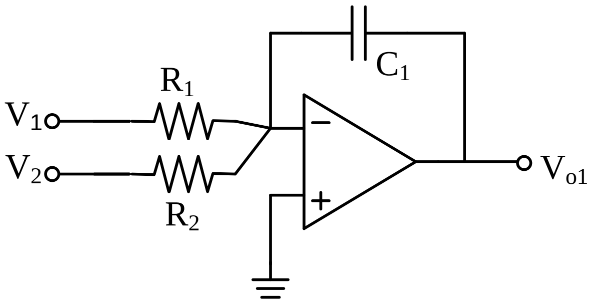
Exercise 5. Prove the transfer function is a BP filter of the following circuit



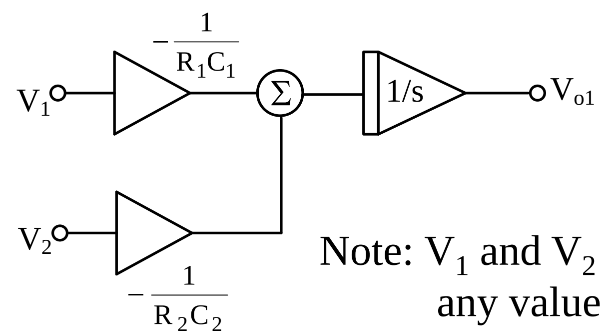
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{-K}{K+1}\,\frac{s}{R_1C_1}}{s^2 + \frac{1}{K+1}\left[\frac{R_1+R_2}{R_1R_2C_1} + \frac{1}{R_2C_2}\right]s + \frac{1}{(K+1)R_1R_2C_1C_2}}$$

Another technique for analysis and design based on state-variable uses building blocks.

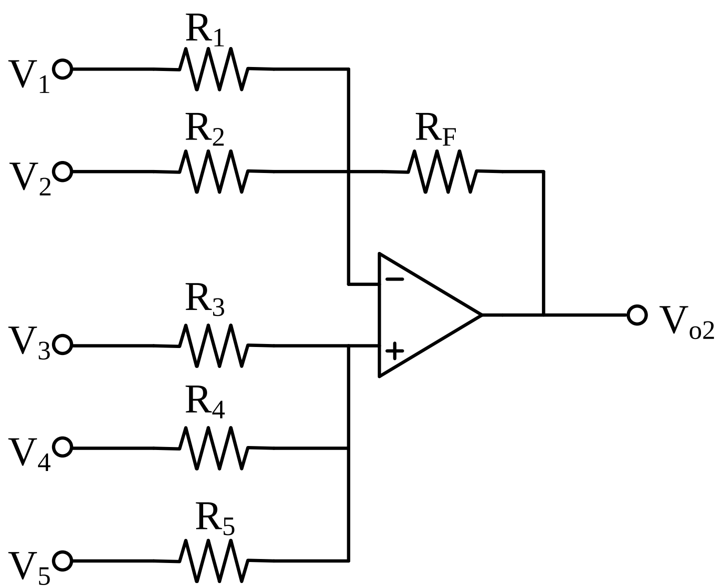
CIRCUIT

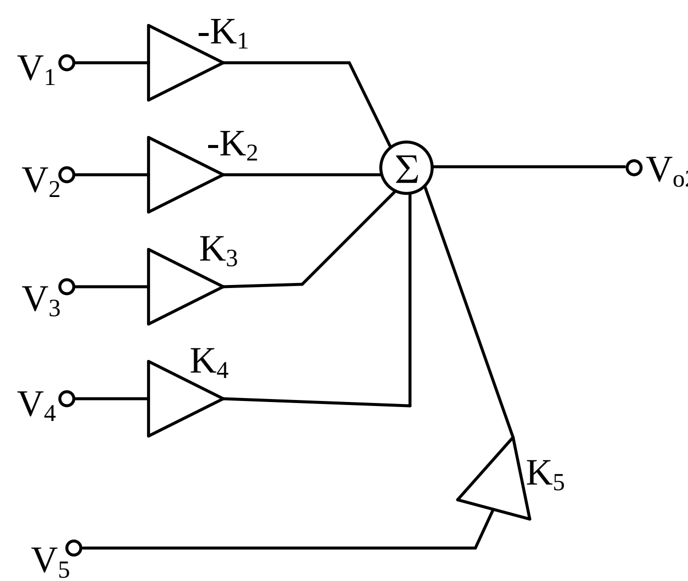


REPRESENTATION

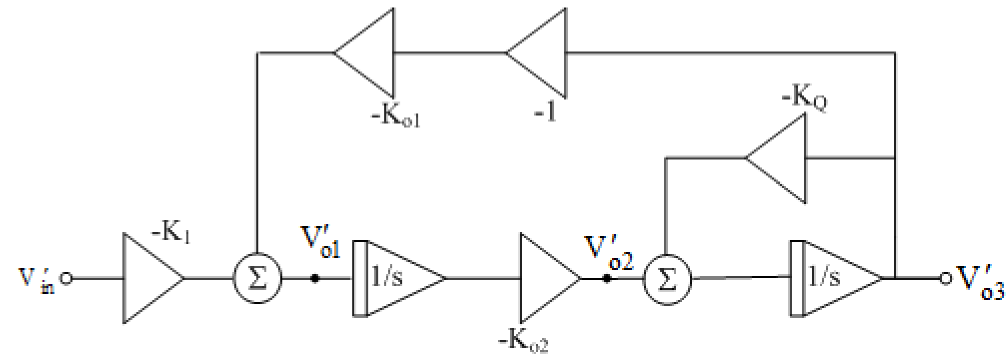


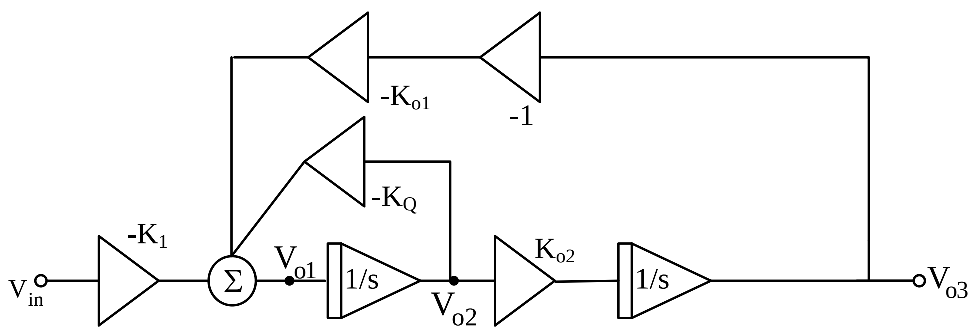
Note: V_1 and V_2 can take any value including V_o.





Let us apply to a two-integrator loop plus Mason’s Rule.



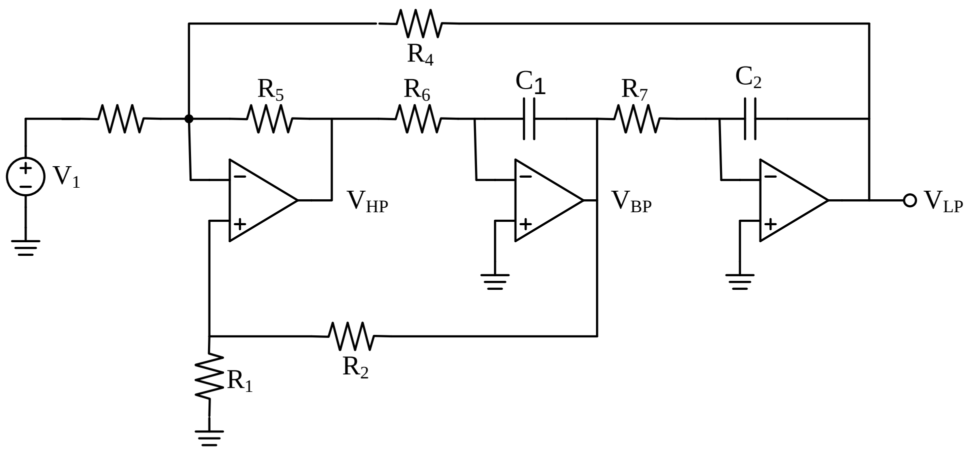


For Second-topology

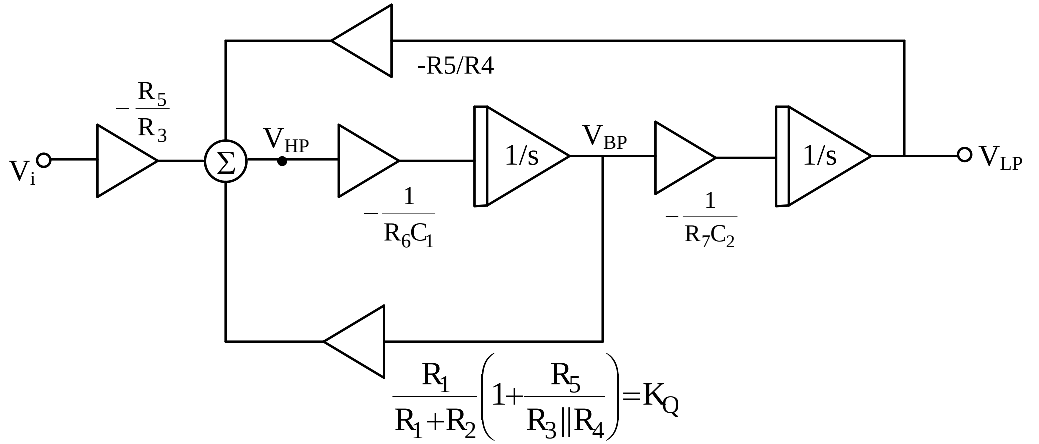
$$V_{o1} = \frac{-K_1 V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1}K_{o2}}{s^2}} = \frac{-K_1 s^2 V_{in}}{s^2 + K_Q s + K_{o1}K_{o2}} \qquad \text{HP}$$

$$V_{o2} = \frac{\frac{-K_1}{s} V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1}K_{o2}}{s^2}} = \frac{-K_1 s V_{in}}{s^2 + K_Q s + K_{o1}K_{o2}} \qquad \text{BP}$$

Next we show that we can go from an Active-RC representation into a block diagram or vice versa.



KHN Biquad Filter



$$\frac{V_{HP}}{V_i} = \frac{-\frac{R_5}{R_3}}{1 + \frac{K_Q}{R_6 C_1}\frac{1}{s} + \frac{R_5/R_4}{R_6 C_1 R_7 C_2 s^2}} = \frac{-\frac{R_5}{R_3} s^2}{s^2 + \frac{K_Q}{R_6 C_1} s + \frac{R_5/R_4}{R_6 C_1 R_7 C_2}}$$